The Intriguing Relation Between Parenting Styles and Eldercare

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Abstract

This paper analyzes the effect of longevity on parenting choices from a life course perspective. We develop an overlapping generations model to address a core tradeoff that young parents face when investing in their children's human capital. They can choose a low-time-cost demanding strategy that risks straining intergenerational relations or a time-intensive pedagogical method that fosters familial bonds at the cost of reduced parental income. Aging parents value attention of their adult children, who uphold the eldercare norm while bringing the shared history of their relationship into the caregiving environment. The rising future need for eldercare heightens the value of pedagogical effort for building relational capital and reveals parental demandingness as counterproductive. Our analysis suggests that longer life expectancy reduces the prevalence of authoritarian parenting practices, while higher income promotes greater pedagogical effort. We characterize the steady states of parenting styles and human capital and then examine their dynamic responses to changes in longevity and eldercare time.

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1 Introduction

The home environment during childhood, particularly parenting, is vital to human development (Francesconi and Heckman 2016). To equip young children with the knowledge and skills for better future opportunities, parents are presented with a variety of educational methods to choose from. Developmental psychology research frames these methods along two dimensions—responsiveness (parental warmth, support, and attunement to a child's needs) and demandingness (behavioral control through discipline, expectations, and rules), suggesting that their interaction forms the basis of major parenting styles.[1] Weinberg (2001) highlights a socioeconomic divide in parenting: wealthy parents can leverage financial rewards to encourage desired behaviors, whereas low-income parents often resort to punitive harshness and authoritarian tactics. This paper aims to rationalize parenting choices from a complementary lens, focusing on the role of longevity and eldercare needs.

Family dynamics involve reciprocal processes, where interactions between parents and children have lasting impacts on each other's emotions and behaviors. Parental responsiveness often manifests as a thoughtful pedagogical effort—expressed through active listening, consistent structure, and teaching problem-solving—that nurtures intergenerational closeness. In contrast, authoritarian demandingness, which enforces behavioral control and limits autonomy through verbal aggression, corporal punishment, and silent treatment, tends to provoke hostility in children and undermine the quality of the parent-child relationship. As parents age, they desire the caregiving support provided by their grown children (Canta and Cremer 2019; Pestieau 2022). But the inclination of children to assume filial responsibilities depends on how they were treated by their parents during childhood.[2] The expectation that adult children will base future eldercare on past affective experiences plays a salient role in shaping parenting styles.

Given the potential merit of pedagogical effort in strengthening family bonds, the next question

[1]By crossing the two dimensions, each measured as high or low, Maccoby and Martin (1983) creates a 2×2 matrix that defines four parenting styles: authoritative parenting (high responsiveness, high demandingness), authoritarian parenting (low responsiveness, high demandingness), permissive parenting (high responsiveness, low demandingness), and neglectful parenting (low responsiveness, low demandingness).

[2]A growing body of empirical research addresses the importance of earlier family life in motivating adult children to support their elderly parents. Silverstein et al. (2002) show that children who spend more time in shared activities with their parents are more likely to provide care to them later in life in the US. Baumbach, Hughes, Derain, and Liu (2023) find that childhood experiences affect an individual's attitude toward caregiving support for aging parents in the US. Liu, Hughes, Baumbach, and Meng (2023) find that American family caregivers who had parents with intensive discipline in childhood report a high level of depressive symptoms in midlife. Vangen and Herlofson (2024) identify a high-quality parent-child relationship as a requisite for informal caregiving in Norway.

arises: Why are some parents motivated to employ demanding tactics? We propose two underlying reasons. First, parenting style is often conceptualized as the degree of parental patience in response to misbehavior (e.g., Burton, Phipps, and Curtis 2002); in particular, being demanding emerges as a time-minimal solution to disciplinary problems. Imagine that a mother attempts to motivate her eight-year-old son to apply greater effort at school. Demonstrating responsiveness—by communicating the value of effort, listening to the boy's perspective, cultivating a love of reading—involves a considerable and continuous time investment (Doepke and Zilibotti 2019; Kearney, Levine, and Pardue 2022). However, shouting at the boy in anger outsources behavioral and emotional regulations to him within seconds. Directives such as "Because I say so," "Homework first, then screens," and "Cry if you want, but the answer is no" cut straight to compliance and require no negotiation. Punitive actions, like forcing the boy to stand in the corner or spanking him, serve as an immediate signal that a lack of learning effort is unacceptable. Both responsive and demanding methods teach a lesson to the child, but they differ in their required time investment from the parent. For parents who prioritize income and career advancement, an authoritarian parenting style—characterized by limited responsiveness and a frequently demanding attitude—seems preferable, because it frees up time and energy for productive, income-boosting activities.

The other factor contributing to authoritarian parenting practices is low societal life expectancy. In societies with a short average lifespan, the role of children as future companions and sources of caregiving support holds less significance. As young parents anticipate a slim chance of surviving to old age, they have less incentive to invest in nurturing emotional bonds with their children and are more likely to overlook the future repercussions of a demanding and harsh attitude. This logic explains the prevalence of physical punishment within households in ancient times, a period when most individuals die young. Over the past century, life expectancy at birth has almost doubled in many economies (Hazan 2009; Haberkern 2023). While one in four Americans lost a parent by age 15 in 1900, it is now common for parents and children to share about 60 years together (Fingerman and Birditt 2011). By raising the likelihood that parents will require care from their children in old age, greater longevity encourages a shift toward less demanding and more pedagogical parenting to safeguard long-term relational harmony.

In this paper, we connect parenting to not only human capital formation but also aging and old-age support. An overlapping generations model is proposed, where individuals live for childhood, young adulthood, middle adulthood, and potentially old adulthood. Children accumulate human capital with the help of their parents, treating parental human capital and parenting styles as given. Young adults work for a wage, allocate their income between consumption and saving, and choose a parenting style defined by varying levels of responsiveness and demandingness. To be responsive, parents employ pedagogical practice, which deepens their connection with their children; however, this time-intensive commitment crowds out work hours, thereby reducing parental income. Being a demanding parent exhausts minimal resources but results in family estrangement. Both responsiveness and demandingness foster children's human capital development that concerns their parents. Upon entering middle adulthood, individuals live on savings and allocate their time between leisure and eldercare. While informal care can be motivated by altruism, reciprocal exchange, or societal norms (Klimaviciute and Pestieau 2023), we focus on the *normative* motive: children attend to their parents from a feeling of duty to fulfill societal expectations. Their attitudes and behaviors toward parents in the eldercare environment are contingent on the quality of their earlier relationships. The old adult derives utility from their child's attention, which is a function of both the time dedicated to eldercare and the affection the middle-aged child holds for the parent.

Our analysis identifies perceived longevity as a vital determinant of optimal parenting strategy. An increased projected probability of surviving to old age incentivizes forward-looking individuals to build stronger bonds with their children to secure companionship and support in later years. To cultivate an enduring intergenerational connection, parents eschew demandingness—which engenders resentment and damages the relational asset—and opt instead for more responsive approaches that prioritize an emotional tie and long-term attachment. This life-cycle perspective also provides a potential explanation for the paradox wherein increasing wage rates sometimes fail to boost labor supply. Concerned with their children's human capital formation and devotion to eldercare, parents may choose to work less and invest more time in their children's nurturance. This strategic change in parenting not only enhances childhood happiness but also increases care and attention that aging parents can expect to receive.

We then analyze the steady state—where human capital stock is constant across generations—using specific functional forms and numerical simulations. In the long run, each generation chooses the same parenting styles, which are determined by the likelihood of living to old age and the social norm governing time allocated to eldercare. We present a diagram showing the concurrent growth of pedagogical effort and human capital over time toward their own steady state. A central result is that an exogenous increase in longevity or eldercare requirements may prompt individuals to move away from authoritarian parenting under some configurations. This dynamic drives human capital accumulation, leading to a more educated and affluent population in the long run.

This paper contributes to the emerging economic theories of parenting styles. Weinberg (2001) constructs a model where a parent and her child interact for one period, offering an explanation for punitive harshness in low-income families. Akabayashi (2006) presents a dynamic model showing that a parent who initially holds high expectations regarding her child tends to understate the effort the child puts into accumulating his human capital, resulting in punitive interactions between them. Doepke and Zilibotti (2017) hold that the socioeconomic environment (e.g., income inequality and division of labor) is critical in shaping parenting, attributing the long-term decline of authoritarian parenting to rising returns to independence. Cobb-Clark, Salamanca, and Zhu (2019) suggest that heterogeneity in parenting comes from differences in the constraints parents face rather than their preferences. Rauh (2024) argues that parents navigate between encouraging children's autonomy and monitoring their choices to instill desired preferences. Building upon the literature, our OLG model focuses on intergenerational conflicts and reciprocity across the lifespan, weaving together research domains in parenting and eldercare.

2 The Analytical Framework

Consider an overlapping generations economy populated by a continuum of individuals, each living for either three or four periods and endowed with one unit of time per period. In childhood, each individual accumulates human capital without making any decision. During young adulthood, they raise a single child and choose their parenting styles, which in turn influence their work time. Moreover, they earn a wage from working and optimize consumption and saving choices subject to intertemporal budget constraints. Upon reaching middle adulthood, they retire from employment, financing consumption through savings while allocating their time between leisure and eldercare. The probability of living to old adulthood, denoted by $\pi \in (0, 1)$, is exogenous.[3] The elderly derive

[3]Our assumption of an exogenous survival probability is empirically grounded, as genes are a primary determinant of aging (Browner et al. 2004). Factors like lifestyle and healthcare access also play a role. In Appendix B, we follow the literature on endogenous lifetime (e.g., Chakraborty 2004; de la Croix and Ponthière 2010) to consider the positive effect of personal health investments on longevity.

utility from private consumption and their children's attention.

Denote individuals who were born and spend childhood in period t as "generation t." In period $t{+}1$, a representative young member of generation t possesses human capital stock H_t. She devotes a fraction $p_t \in (0,1)$ of her time to pedagogical practices while exercising parental demandingness with intensity $d_t \geqslant 0$. We assume that d_t is an attitude that involves no time input. With the wage rate per efficiency unit of labor normalized to unity, the young adult's total earned income is equal to $(1-p_t)H_t$. Her budget constraint amounts to

$$c_t^y + s_t = (1-p_t)H_t, \tag{1}$$

where c_t^y and s_t denote her consumption and saving, respectively.

In period $t+2$, the member of generation t spends a proportion $\tau_t \in [0,1]$ of time on eldercare, receiving leisure utility $\mathbf{u}(1-\tau_t)$, with $\mathbf{u}' > 0$. Given parental survival probability of π, the middle-aged adult expects their utility from leisure to be $\mathbb{V}(\tau_t, \pi) = \pi\mathbf{u}(1-\tau_t) + (1-\pi)\mathbf{u}(1)$. Moreover, in exchange for $\varphi > 0$ units of potential elderly consumption, the middle-aged adult purchases an actuarially fair longevity insurance by paying a premium of $\pi\varphi$. For simplicity, we assume that the intertemporal interest rate is zero. Consumption during middle adulthood, denoted by c_t^m, equals savings net of expected old-age consumption, as specified below:

$$c_t^m = s_t - \pi\varphi. \tag{2}$$

If a member of generation t dies at the beginning of period $t+3$, her utility is normalized to 0. If she survives to old adulthood, she consumes at a fixed level of $\varphi > 0$, yielding utility $\Phi(\varphi) > 0$, where $\Phi' > 0$.[4] In addition, the elderly adult receives attention A_t from her child. With the survival probability π, the expected old-age utility can be written as a weighted sum of $\pi[A_t + \Phi(\varphi)]$.

Taken together, the lifetime utility of a representative member of generation t is given by

$$U_t = -z(d_{t-1}) + v(c_t^y) + [u(c_t^m) + \mathbb{V}(\tau_t, \pi)] + \pi[A_t + \Phi(\varphi)] + \delta H_{t+1}. \tag{3}$$

[4]We simplify our discussion about old-age consumption to emphasize that the elderly derive utility primarily from the spiritual fulfillment of familial care. For the elderly, material consumption becomes less important in many aspects: the sensory pleasure derived from food attenuates (Spence and Youssef 2021), the social competition driving demand for luxury items (e.g., cars, attire) recedes (Schade, Hegner, Horstmann, and Brinkmann 2016), and health and mobility constraints often make extensive travel impractical (Luiu, Tight, and Burrow 2017).

The first term on the right hand side (RHS) represents the childhood disutility inflicted by parental demandingness, where $z' > 0$ and $z'' \geqslant 0$. The second term is young-age utility from consumption, which exhibits diminishing marginal utility ($v' > 0$ and $v'' < 0$). The third term, which captures middle-aged utility, consists of two additively separable elements: utility from consumption $u(.)$—where $u' > 0$ and $u'' < 0$—and expected utility from leisure $\mathbb{V}(\tau_t, \pi)$. The fourth term measures expected old-age utility from consumption and eldercare. The final term reflects parental concern for the child's human capital, with the parameter $\delta > 0$ denoting the strength of such concern.

The attention received by an elderly parent is determined by two complementary components: (i) the time investment of the child (τ_{t+1}), representing physical companionship, and (ii) the child's affective devotion (denoted by a_{t+1}), capturing emotional bond. Formally, we write A_t as

$$A_t = \tau_{t+1} a_{t+1}. \tag{4}$$

Throughout this paper, we assume that $\tau_{t+1} = \tau$ is exogenous and determined by a social norm.[5] The child's affection for their parent depends on parental behavior during upbringing. Specifically, a pedagogical approach cultivates filial love, while an demanding attitude breeds estrangement and alienation.[6] The function $a_{t+1} = a(p_t, d_t)$ satisfies the following properties:

$$\frac{\partial a}{\partial p_t} > 0 \qquad \frac{\partial a}{\partial d_t} < 0 \qquad \frac{\partial^2 a}{\partial p_t^2} \leqslant 0 \qquad \frac{\partial^2 a}{\partial d_t^2} \leqslant 0.$$

The utility derived from eldercare in equation (4) can thus be rewritten as

$$A_t = \tau a(p_t, d_t). \tag{5}$$

Building on earlier empirical research (Black, Devereux, and Salvanes 2005; Fiorini and Keane 2014; Doepke, Sorrenti, and Zilibotti 2019), we model a child's human capital as

[5]For instance, in many southern and central European countries, adult children are normatively or legally obligated to aid their aging parents in need (Haberkern and Szydlik 2010). Daughters often conform to family norms of long-term care and feel guilty if they provide less eldercare than the average level (Barigozzia, Cremer, and Roeder 2020). In Appendix C, we consider that eldercare provision is endogenously determined as a dynamic Nash equilibrium in an intergenerational supergame.

[6]This formulation is empirically supported. For instance, Trommsdorff (2013) finds that German and Japanese women intend to provide more support to their mothers who exhibited more sensitivity during their childhood. Özmete and Pak (2022) reveal a negative link between childhood stress and filial piety among Turkish adults.

$$H_{t+1} = h(H_t, p_t, d_t), \tag{6}$$

which increases with parental human capital (H_t), pedagogical engagement (p_t), and demandingness intensity (d_t). Each of these inputs exhibits diminishing returns, namely

$$\frac{\partial h}{\partial H_t} > 0 \qquad \frac{\partial h}{\partial p_t} > 0 \qquad \frac{\partial h}{\partial d_t} > 0 \qquad \frac{\partial^2 h}{\partial H_t^2} < 0 \qquad \frac{\partial^2 h}{\partial p_t^2} < 0 \qquad \frac{\partial^2 h}{\partial d_t^2} < 0.$$

For analytical tractability, we assume that cross derivatives are zero, as follows:

$$\frac{\partial^2 a}{\partial d_t \partial p_t} = 0 \quad \text{and} \quad \frac{\partial^2 h}{\partial d_t \partial p_t} = 0, \tag{7}$$

implying that the marginal contribution of one parental input to children's affective experience and human capital is independent of the other input.

3 The Determination of Parenting Styles

Given the analytical framework in Section 2, we proceed to derive the endogenous parenting variables and conduct comparative static analyses. By substituting (1), (2), (5), and (6) into (3), we rewrite the lifetime individual utility as

$$U_t = -z(d_{t-1}) + v[(1-p_t)H_t - s_t] + u(s_t - \pi\varphi) + \mathbb{V}(\tau, \pi) + \pi[\tau a(p_t, d_t) + \Phi] + \delta h(H_t, p_t, d_t). \tag{8}$$

In period $t+1$, each member of generation t optimally chooses their parenting tactics (d_t, p_t) and savings (s_t), taking H_t and other parameters as given.

The interior solution to d_t satisfies the first order condition of U_t in (8) with regard to d_t. Parents face a critical trade-off: being more demanding boosts their children's human capital development at the cost of the intergenerational emotional bond. The strained relationship may result in poorer care provided to parents when they age. Clearly, the probability of living to old adulthood, π, leads an important role in governing this tradeoff, as we discuss in the next proposition:

Proposition 1 *d_t decreases with π.*

Proof. See Appendix A. ■

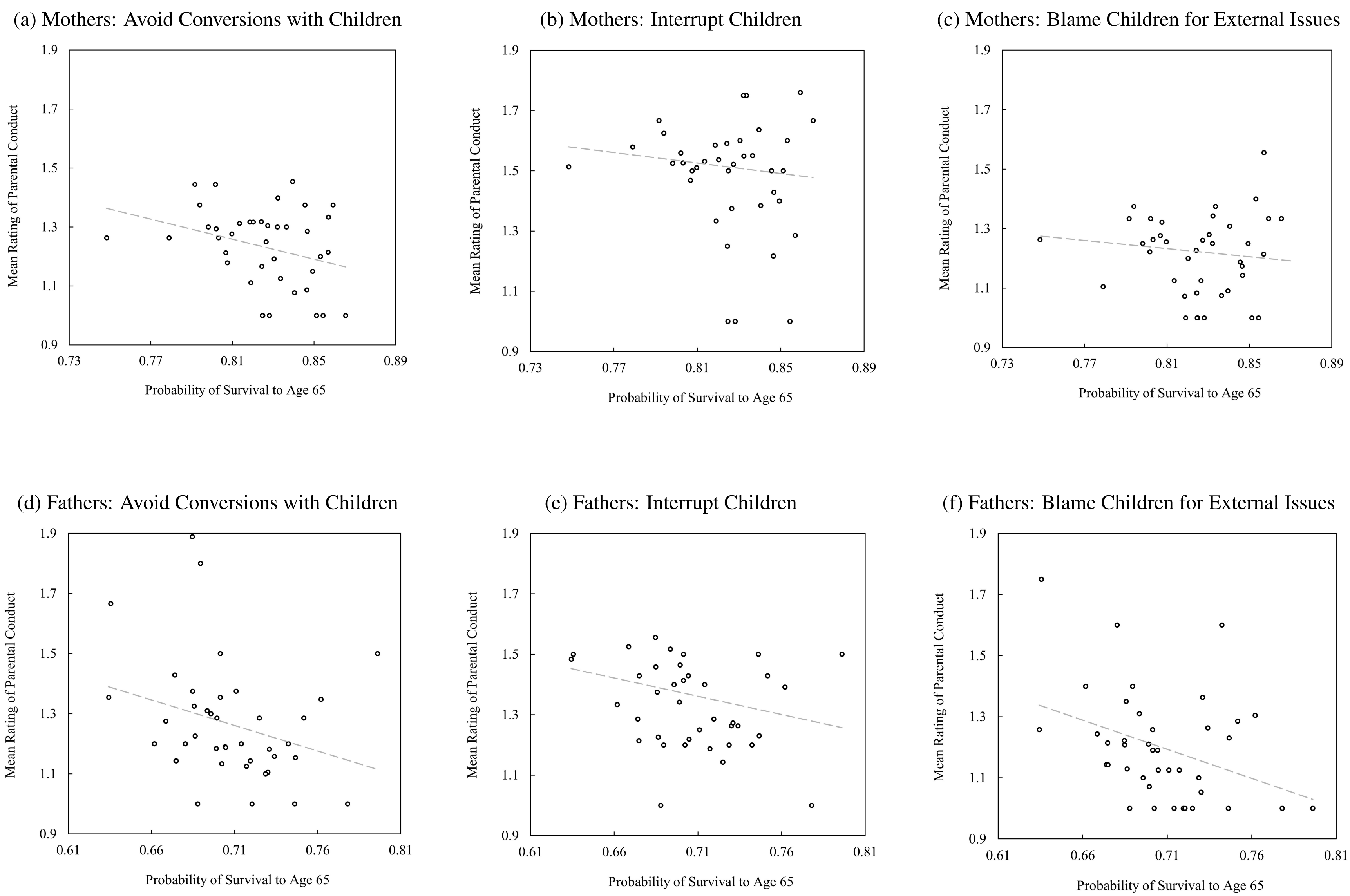
Proposition 1 predicts a negative relationship between parental lifespan and demandingness. High expectations of living into old adulthood make parents concerned about their future eldercare. Since parents anticipate their aging and potential need for assistance, they become more aware of the role that their adult children will play in the caregiving dynamics. Recognizing that maintaining a harmonious relationship secures their children's attention in later life, parents reduce aggression and conflicts in their child-rearing approaches. This prediction potentially explains the prevalence of corporal punishment inflicted by parents in ancient times when global life expectancy was short. For instance, certain Bible verses could be interpreted as endorsing punitive harshness.[7]

The U.S. observations are broadly consistent with Proposition 1. We measure parental demandingness using data from the 2002 survey of the Panel Study of Income Dynamics–Child Development Supplement (PSID-CDS) database, where children aged 12 to 18 are asked to evaluate their parents' attitudes and behaviors on a three-point Likert scale (1 = *not like them*, 2 = *somewhat like them*, 3 = *a lot like them*). We define old adulthood as age 65 and above and calculate each parent's survival probability to age 65 by combining their gender, age, and state of residence (from PSID) with state-level life tables from the U.S. Mortality Database. By averaging individual data within each state, we plot Figure 1 to illustrate a visual summary of state-level variations in the probability of surviving to age 65 for mothers and fathers and children's perception of maternal and paternal demandingness along three dimensions.

The first dimension addresses the question whether the parent *changes the subject whenever the child has something to say* (plots (a) and (d)). A higher average rating means a larger probability that the parent shuts down conversations and avoids interactions, creating implicit rules about what the child is allowed to express or feel. The second dimension investigates whether the parent *often interrupts the child* (plots (b) and (e)). A higher rating reflects stricter parental control, including regulated communication, requirements for immediate compliance, and suppressed autonomy. The third dimension assesses the extent to which the parent *blames the child for other family members' problems* (plots (c) and (f)). A higher rating indicates that the parent expects their child to manage things outside of their control, placing an unrealistic emotional or behavioral burden on the child. In all scatterplots of Figure 1, the best-fit line has a negative slope, implicating that higher parental

[7] "Folly is bound up in the heart of a child, but the rod of discipline will drive it far from him." (Proverbs 22:15); "Do not withhold discipline from a child; if you punish them with the rod, they will not die. Punish them with the rod and save them from death." (Proverbs 23: 13-14).

Figure 1: Life Expectancy and Parental Demandingness across U.S. States in 2002



survival probabilities correlate with lower perceived demandingness.

The intensity of demandingness chosen by parents generates profound effects on children's development and future behaviors. First, when parents employ a less demanding approach, children experience greater happiness during childhood (smaller $z(.)$); conversely, excessive parental control engenders psychological stress and erodes children's sense of autonomy. Second, high parental demandingness yields a short-term benefit in children's human capital accumulation (larger $h(.)$). Finally, high levels of parental demandingness can lead to children's resentment and emotional distancing within the household (smaller $a(.)$), which, in turn, undermines their inclination to provide caregiving support later in life.

Next, we examine the optimal allocation of parental time to children's education. Equation (8) shows that increased pedagogical engagement with children comes at the cost of reduced labor supply, thereby lowering parental earnings. However, this investment creates two gains. First, a higher degree of parental involvement cultivates the parent-child emotional tie, enhancing children's affection for their parents and motivating them to provide eldercare in the future. Second, parental involvement contributes to children's human capital formation, improving their future productivity and socioeconomic outcomes. The optimal time commitment to children's education satisfies the first order condition of U_t in (8) with respect to p_t. We develop the following proposition to discuss the changes in p_t in response to the changes in π and H_t.

Proposition 2 *(i) p_t increases with π if and only if*

$$\tau \frac{\partial a}{\partial p_t} > \varphi H_t \left(\frac{1}{\frac{1}{-v''} + \frac{1}{-u''}} \right); \tag{9}$$

(ii) p_t increases with H_t if

$$v' \leqslant \delta \frac{\partial^2 h}{\partial p_t \partial H_t}. \tag{10}$$

Proof. See Appendix A. ■

Part (i) states that longer life expectancy (larger π) may incentivize parents to invest more effort in their children's education (larger p_t) under certain circumstances. Equation (9) outlines the sufficient and necessary condition for maintaining this positive correlation. The left-hand side (LHS) of condition (9)—measuring the marginal effect of pedagogical practice on the child's affective experience, conditional on the parent living to old age—reflects the emotional returns from parental responsiveness. The RHS of (9) sets a threshold concerning the material well-being. When elderly consumption is very low (i.e., $\varphi \to 0$), children's attention becomes the primary source of old-age utility. In this case, the LHS dominates the RHS, making it more likely that an increase in lifespan induces greater parental time investment. However, income (H_t) generates an ambiguous impact on the RHS. Higher income tends to reduce the curvature of preferences over consumption during young and middle adulthood (e.g., in a logarithmic utility function), which decreases the term in parentheses. In sum, parents with longer life horizons will invest more time in children's education only when the emotional payoff is worth the cost to their own consumption.

The positive effect of π on p_t in Proposition 2(i) finds empirical support. Using Japanese survey data, Yasuda et al. (2024) show that the projected shortening of lifespan due to COVID-19 reduces parental effort in educating children specifically for those who anticipate relying on their children for future care. Given that a higher survival probability reduces parental demandingness and boosts pedagogical engagement (see Propositions 1 and 2(i)), we can infer from equation (6) that the next generation's human capital stock (H_{t+1}) may also increase under certain conditions.

Note that H_t—which represents parental human capital or income level—is a state variable for generation t. Part (ii) suggests that a larger H_t may stimulate involvement in children's education. On the one hand, young adults tend to increase labor supply to take advantage of higher wages. On the other hand, increased parental human capital reinforces the efficiency of pedagogical methods, thereby inducing parental time inputs. Condition (10) presents a sufficient condition for the latter effect to outweigh the former effect, resulting in a net increase in the time allocated to children's development. The positive correlation between p_t and H_t has received extensive empirical backing. Guryan, Hurst, and Kearney (2008) show that mothers with a college degree or higher spend 4.5 hours more per week in child care and learning activities than mothers with a high school diploma or below. Ramey and Ramey (2010) report that from the 1980s to the mid-1990s, college-educated American mothers increased their weekly time with children by nine hours (a rise of 70%), despite a drastic pay raise. Doepke and Zilibotti (2017) find that parents with higher educational attainment exhibit a greater propensity for responsive parenting. Agostinelli, Doepke, Sorrenti, and Zilibotti (2022) estimate that a \$10,000 increase in family income is associated with additional 3.5 minutes spent on child care per day in 2020 in the US.

In addition to parental longevity and human capital, parental aspiration and social expectation may also influence parenting styles. We present the following proposition to analyze the comparative statics of the equilibrium (d_t, p_t) with respect to the two parameters (δ, τ).

Proposition 3 *(i) An increase in δ leads to an increase in both d_t and p_t;*
(ii) An increase in τ leads to an decrease in d_t and an increase in p_t.

Proof. See Appendix A. ■

By part (i), we can compare parenting styles across cultural contexts that differ in the emphasis placed on children's human capital accumulation. In societies that assign a significant weight on offsprings' academic achievement and economic advancement (large δ), parents are more likely to adopt demanding practices—such as imposing high standards and enforcing strict discipline—to exert pressure on their children to excel. Meanwhile, a stronger concern about children stimulates parents to participate more intensively in children's education (de la Croix and Michel 2002; Galor 2011). The tendency of reallocating time from work toward nurturing ultimately benefits children's development. Both positive relationships indicated in Proposition 3(i) reflect the altruism effect: a genuine motivation to ensure children flourish, even in the absence of direct personal gain. In sum, societies that highly value children's human capital tend to prioritize authoritative parenting styles (characterized by high demandingness and high responsiveness).

Part (ii) addresses how anticipated role of children in supporting the elderly motives parents to adjust their child-rearing strategies. In societies where adult children are expected to offer intensive caregiving support to their aging parents (large τ), the potential long-term relational cost associated with parental demandingness (i.e., reduced filial love) becomes significant, thereby discouraging young parents from treating their children in a demanding manner. Instead, parents are incentivized to demonstrate increased responsiveness and warmth toward their children during childhood. This fosters an implicit intergenerational exchange, where time investments made today are intended to be repaid through eldercare offered tomorrow. Conversely, a weak filial norm (small τ) gives rise to the prevalence of authoritarian parenting styles (characterized by high demandingness and low responsiveness).

We now move to the specific functional forms to translate the above predictions into concrete results and conduct a numerical analysis. To explicitly present the equilibrium solutions, our analysis employs the following functions for $v(.)$, $u(.)$, $h(.)$, and $a(.)$:

$$v(c_t^y) = \ln(c_t^y), \qquad u(c_t^m) = \ln(c_t^m), \qquad H_{t+1} = H_t^{\alpha} p_t^{\beta} + d_t^{\gamma}, \qquad a_{t+1} = \mu + \lambda p_t - \frac{d_t^2}{2}. \tag{11}$$

The first two expressions define logarithmic utility functions for consumption in young and middle adulthood, which exhibit the property of diminishing marginal utility. The latter two expressions satisfy condition (7). The third expression shows the evolution of human capital, where parameters $\alpha, \beta, \gamma \in (0,1)$ represents diminishing returns to each input; in particular, the better educated the parents are, the more the children benefit from instructional time. The last expression reflects how parenting influences a child's affection for their parent. The parameter $\mu > 0$, which depicts innate filial love in the absence of parent-child interactions, is assumed to be sufficiently large to ensure $a_{t+1} > 0$. The parameter $\lambda > 0$ measures the marginal effect of parental pedagogical practice on intergenerational solidarity.

Given these explicit functions, we derive that the optimal solutions to d_t and p_t satisfy

$$d_t = \left(\frac{\delta\gamma}{\pi\tau}\right)^{\frac{1}{2-\gamma}}, \tag{12}$$

$$p_t + \frac{2}{\pi\tau\lambda + \beta\delta H_t^{\alpha} p_t^{\beta-1}} = 1 - \frac{\pi\varphi}{H_t}. \tag{13}$$

By using equations (11) and (12), we rewrite A_t and H_{t+1} as

$$A_t = \tau\left[\mu + \lambda p_t - \frac{1}{2}\left(\frac{\delta\gamma}{\pi\tau}\right)^{\frac{2}{2-\gamma}}\right], \tag{14}$$

$$H_{t+1} = H_t^{\alpha} p_t^{\beta} + \left(\frac{\delta\gamma}{\pi\tau}\right)^{\frac{\gamma}{2-\gamma}}, \tag{15}$$

where p_t is implicitly solved in equation (13).

Using the baseline parameters in Table 1, Figure 2 illustrates the numerical comparative statics of three key endogenous variables—pedagogical effort (p_t), received eldercare (A_t), and the child's human capital (H_{t+1})—with respect to changes in parental life expectancy (π) and parental human capital (H_t). For a constant level of parental human capital ($H_t = 2$), plots (a), (b), and (c) depict how variations in π influence p_t, A_t, and H_{t+1}, respectively. As π increases from 0.35 to 0.95, all

Table 1: Benchmark Values for Key Parameters

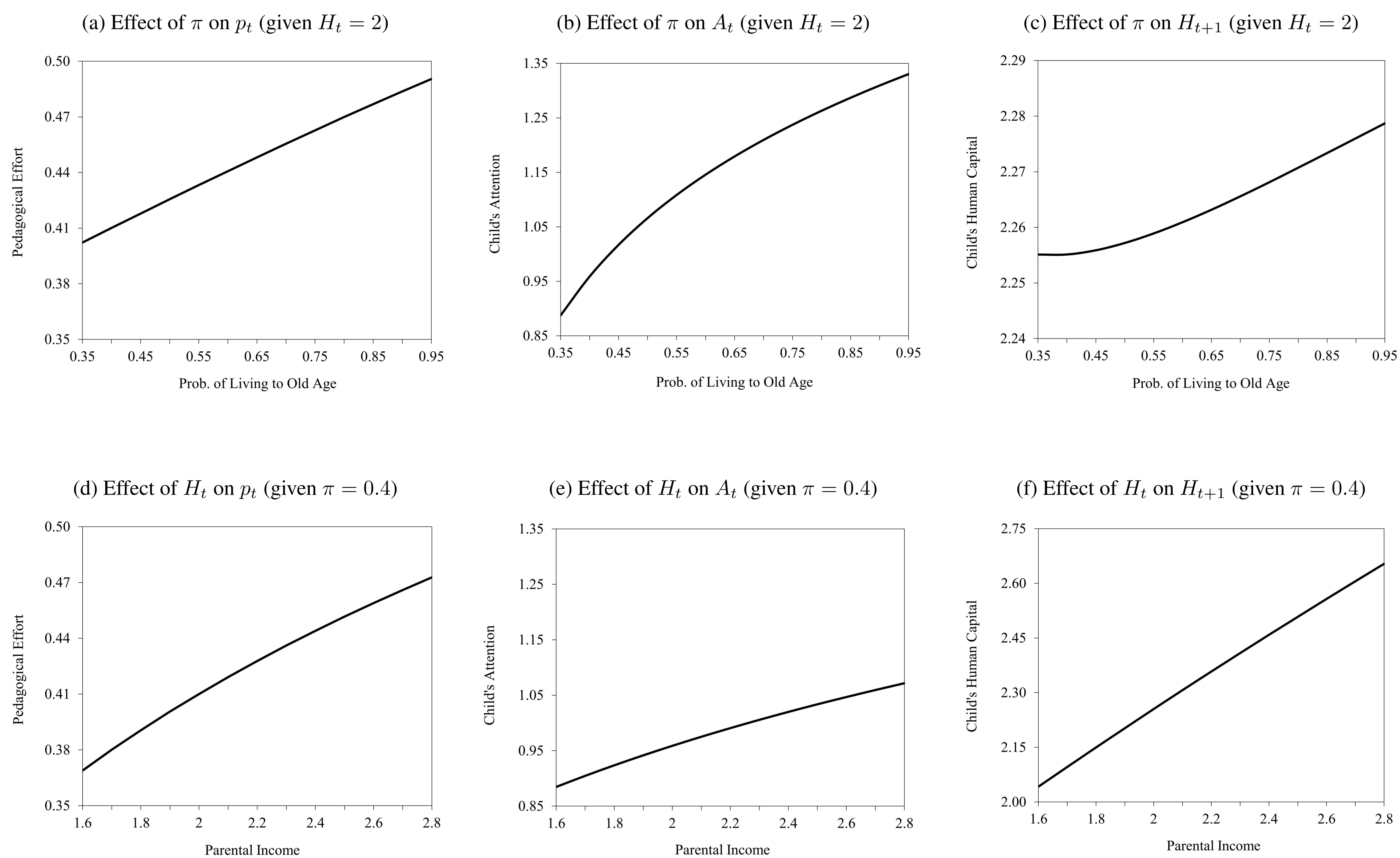
Notation	Description	Value
α	Efficiency of intergenerational human capital transmission	0.7
β	Efficiency of pedagogical effort	0.3
γ	Efficiency of demandingness	0.1
δ	Preference over the quality of the child	3
φ	Elderly consumption	0.05
τ	Time invested in eldercare	0.6
μ	Innate filial love	1
λ	Effect of pedagogical effort on the child's affection	3

three curves exhibit a positive slope. The positive link between π and p_t aligns with the prediction of Proposition 2(i), that is, condition (9) is met. Intuitively, greater life expectancy raises a parent's expected payoff from time investments in teaching their child. The increase in pedagogical effort strengthens parent-child bond, thereby encouraging the child to reciprocate in future informal care. A longer life horizon further enhances this emotional tie by discouraging parental demandingness (see equation (12)). By combining the two effects on parenting, plot (b) shows that A_t increases with π, which echoes Perrig-Chiello and Höpflinger's (2005) finding that an extension of common lifespan of parents and children leads to increasing parent-support ratio in Switzerland.

Furthermore, by the human capital production function in (11), deeper pedagogical engagement accelerates child development, although lower demandingness stifles it. The net positive impact of π on H_{t+1} fits historical observations. Between the 1850s and 1970s, American men saw parallel gains in longevity and education: their life expectancy at age five climbed from 52.5 to 70.73 years, while their average years of schooling rose from 8.71 to 15.87 (Hazan 2009). Besides, an extensive body of literature confirms that short life expectancies create a prohibitive barrier to human capital formation (Cervellati and Sunde 2005; Hazan and Zoabi 2006; de la Croix and Licandro 2013).

Holding the probability of living to old adulthood fixed at $\pi = 0.4$, plots (d), (e), and (f) show the response of p_t, A_t, and H_{t+1} to the changes in parental human capital (H_t), respectively. First, a higher level of parental human capital increases pedagogical effort by improving the productivity of the learning environment, as reflected in the third expression of equation (11). Parents allocate more time in educating their children when their effort becomes more effective at improving their

Figure 2: Survival Probability, Parenting, Human Capital, and Eldercare



children's human capital. All else equal, increased pedagogical effort, which enhances parent-child closeness, leads to better caregiving support from children in later years (larger A_t). Finally, higher parental human capital and intensified pedagogical effort are mutually reinforcing, amplifying their combined impact on the child's development (larger H_{t+1}).

4 The Steady State

Our analysis has thus far focused on solving the model within a generation, where individuals make intertemporal optimal decisions based on the state variable H_t. We now turn to characterizing the long-run equilibrium. The economy reaches a steady state when human capital stock stabilizes at a constant level (i.e., $H_t = H$ for any t). In the steady state, each generation chooses the same parenting style and saving level. In other words, the optimal individual choices are time-invariant over time (i.e., $p_t = p$, $d_t = d$, $s_t = s$ for all t).

To explicitly derive the steady-state outcomes, we resort to specific functions in equation (11). It follows immediately from equation (12) that d_t is constant and equal to its steady-state value in each period. We establish the following proposition to characterize the human capital stock in the steady state:

Proposition 4 *Given (11), the steady-state human capital* (H) *is determined by*

$$p = \left[H^{1-\alpha} - \frac{1}{H^{\alpha}} \left(\frac{\delta\gamma}{\pi\tau} \right)^{\frac{\gamma}{2-\gamma}} \right]^{\frac{1}{\beta}}, \tag{16}$$

$$p + \frac{2}{\pi\tau\lambda + \beta\delta H^{\alpha} p^{\beta-1}} = 1 - \frac{\pi\varphi}{H}. \tag{17}$$

Proof. See Appendix A. ■

Proposition 4 states that the steady-state human capital, H, is obtained by solving a system of two equations, (16) and (17). Equation (16) expresses p explicitly as a function of H. Substituting (16) into (17) yields an equation that has only one variable (H). The solution to H is implicit here, which depends on the values of eight parameters, namely $(\alpha, \beta, \gamma, \delta, \pi, \tau, \lambda, \varphi)$.

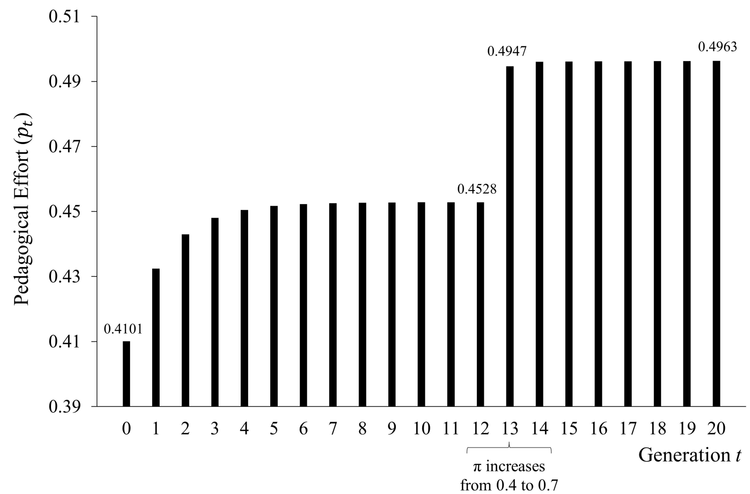
To further explore the features of transitional dynamics and steady state, we numerically simulate the dynamic path of equilibrium outcomes using the parameter values in Table 1 and the initial values $\{H_0 = 2, \pi = 0.4\}$. In Figure 3, we illustrate the evolution of pedagogical effort (p_t) and human capital (H_t) across generations and their responses to a demographic and social shift. Plot (a) shows that Generation 0 allocates 41.01% of their time to educate their children ($p_0 = 0.4101$). This commitment displays a growing trend over time, as every successive generation, who benefit from a higher inherited human capital stock, optimally devotes more time to children's education. By Generation 12, the time investment converges to its steady-state value of $p_{12} = p = 45.28\%$. Mirroring this trend, the human capital stock (or income level) of the economy progresses from its initial value of $H_0 = 2$ toward its steady-state value of $H = 2.5158$.

We then introduce an increase in life expectancy of Generation 13 and their descendants, i.e., π increases from 0.4 to 0.7. The heightened likelihood of survival to old adulthood enhances the marginal return on pedagogical investment, as it raises the expected value of future old-age support. In response, Generation 13 extend their educational time to an optimal fraction of $p_{13} = 49.47\%$, creating a discrete upward jump observed in plot (a). From equation (12), we also infer a decrease in parental demandingness from 1.1246 to 0.8377. While Generation 13's human capital stock is kept at the steady-state value ($H_{13} = H = 2.5158$), their adoption of a less authoritative parenting style helps increase their children's human capital to $H_{14} = 2.5363$. Our simulation results, which show divergent trajectories of H_t and d_t, are in line with Weinberg's (2001) finding that income is a key determinant of the child-rearing strategy, with high earners relying less on punitive harshness compared to low earners. The human capital stock of the economy then gradually climbs to a new steady state of 2.5403 over time.

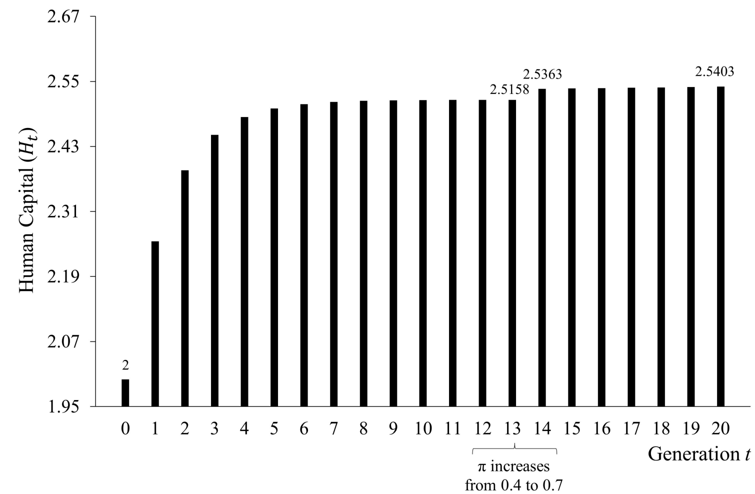
Plots (c) and (d) differ from plots (a) and (b) only in the exogenous shift that occurs for Generation 13 and all subsequent generations: a permanent increase in the time allocated to eldercare from 0.6 to 0.85. The increased time spent together with adult children tends to gravitate a change in parenting styles. It follows from equation (12) that d falls from 1.1246 to 0.9362: the prospect of old-age dependency serves as a catalyst for parents to treat their children in a less demanding way. As shown in plot (c), an increase in eldercare time induces parents to adopt pedagogical methods: Generation 13 allocates 47.86% of their time to children's education during young adulthood, a greater investment than their parents made ($p_{12} = 45.28\%$). Consequently, a net increase in human capital of their children ($H_{14} = 2.5337$) is observed in plot (d), as the positive impact of increased pedagogical effort outweighs the negative consequence of reduced demandingness. This gain fur-

Figure 3: Transitional Dynamics and Steady States in Parenting and Human Capital: Shifts in Longevity and Eldercare Norms

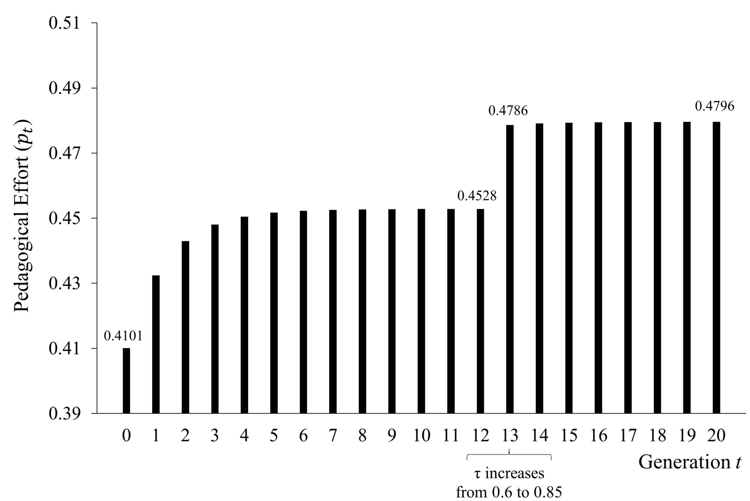
(a) Pedagogical Effort Adjusts to Increased Longevity



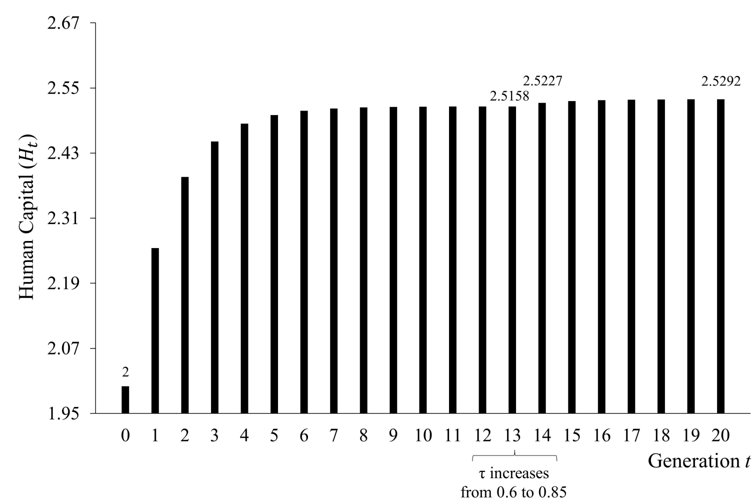
(b) Human Capital Adjusts to Increased Longevity



(c) Pedagogical Effort Adjusts to Longer Eldercare Time



(d) Human Capital Adjusts to Longer Eldercare Time



ther facilitates the human capital acquisition in all subsequent generations, which asymptotically approaches a new steady state of $H = 2.5292$. Higher parental human capital raises the marginal productivity of pedagogical practice, leading parents to intensify such efforts over time toward the new steady state of $p = 47.96\%$.

5 Conclusion

Population aging has emerged as a prominent global demographic trend marked by the increase in life expectancy, which creates a growing need for eldercare in many countries (Pestieau 2022). Extended lifespans are prolonging the overlap of parents' and children's lifetimes into adulthood. This paper formalizes the transition from being a child to assuming the obligation of a caregiver for old parents. Children exposed to pedagogical practices that prioritize responsiveness often sustain positive interactions with their parents after they transition into adulthood. Filial affection serves as the foundation for the willing provision of attention and assistance to parents in times of need. Conversely, children who vividly remember demanding and harsh treatment during childhood are more prone to depression or feelings of disrespect, thereby harboring enduring resentment toward their parents. To the extent that repairing a damaged relationship is challenging, these children will pay less attention to their parents.

We establish an overlapping generations model, wherein parenting styles matter to children's human capital development, familial bond, and parental income. Educating the child in a pedagogical way is time intensive, which reduces the parent's income by crowding out their work hours; however, it can effectively enhance the child's affection toward the parent. In contrast, being a demanding parent is the easy way out but creates a more authoritarian atmosphere within the family. Aging parents desire attention and caregiving support from their adult children, who in turn carry the shared history of their relationship into the care environment. Our model formalizes such intergenerational reciprocity and conflicts associated with the choices of parenting styles. With longer life expectancy, forward-looking parents choose to engage more in pedagogical teaching and less in strict demands. Simply put, increased longevity discourages the use of authoritarian parenting. A promising direction for future work would be a rigorous empirical investigation into the causal link from parental life expectancy to the incidence of authoritarian parenting.

From a lifespan perspective, our analysis also examines how parental altruism and human capital as well as filial norms shape parenting styles. We suggest that societies with a stronger parental emphasis on child development exhibit a higher prevalence of authoritative parenting, and parental human capital is positively related to the adoption of pedagogical practices. Our simulations further show that more intensive filial care requirements for aging parents can increase the steady-state levels of pedagogical engagement and human capital stock.

This paper aims to propose a positive theory that connects parenting styles to eldercare and human capital development. The interconnections between these factors underscores the significance of societal progress in safeguarding the well-being of future generations. In this study, we explore the implications of these connections absent government interventions. Yet it is noteworthy to acknowledge that determining the optimal policies regarding parenting and eldercare is a normative question of equal importance. Understanding these policy implications will be the focus of our future research.

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Appendix A: Mathematical Proofs of Propositions

Proof of Proposition 1

Given d_{t-1} and H_t, the first-order condition of U_t in (8) with respect to d_t obtains

$$\frac{\partial U_t}{\partial d_t} = \pi\tau\frac{\partial a}{\partial d_t} + \delta\frac{\partial h}{\partial d_t} = 0 \qquad \Leftrightarrow \qquad \delta\frac{\partial h}{\partial d_t} = -\pi\tau\frac{\partial a}{\partial d_t}. \tag{A1}$$

Totally differentiating (A1) with respect to d_t, p_t, and π obtains

$$\delta\left(\frac{\partial^2 h}{\partial d_t \partial p_t}dp_t + \frac{\partial^2 h}{\partial d_t^2}dd_t\right) = -e\frac{\partial a}{\partial d_t}d\pi - \pi\tau\left(\frac{\partial^2 a}{\partial d_t \partial p_t}dp_t + \frac{\partial^2 a}{\partial d_t^2}dd_t\right). \tag{A2}$$

Under condition (7), we can rearrange (A2) as

$$\frac{dd_t}{d\pi} = -\underbrace{\frac{\partial a}{\partial d_t}}_{+}\left(\underbrace{\delta\frac{\partial^2 h}{\partial d_t^2} + \pi\tau\frac{\partial^2 a}{\partial d_t^2}}_{-}\right)^{-1} < 0. \tag{A3}$$

Proof of Proposition 2

The first-order conditions of U_t in (8) with respect to p_t and s_t yield

$$\frac{\partial U_t}{\partial p_t} = v'(-H_t) + \pi\tau\frac{\partial a}{\partial p_t} + \delta\frac{\partial h}{\partial p_t} = 0 \qquad \Leftrightarrow \qquad \delta\frac{\partial h}{\partial p_t} = H_t v' - \pi\tau\frac{\partial a}{\partial p_t}, \tag{A4}$$

$$\frac{\partial U_t}{\partial s_t} = -v' + u' = 0 \qquad \Leftrightarrow \qquad v' = u'. \tag{A5}$$

Totally differentiating (A4) and (A5) with respect to d_t, p_t, s_t, and π and using condition (7) yields

$$\delta\left(\frac{\partial^2 h}{\partial p_t^2}dp_t + \frac{\partial^2 h}{\partial p_t \partial d_t}dd_t\right) = H_t v''(-H_t dp_t - ds_t) - \tau\frac{\partial a}{\partial p_t}d\pi - \pi\tau\left(\frac{\partial^2 a}{\partial p_t \partial d_t}dd_t + \frac{\partial^2 a}{\partial p_t^2}dp_t\right)$$

$$\Leftrightarrow \quad \left(\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + H_t^2 v''\right)\frac{dp_t}{d\pi} + H_t v''\frac{ds_t}{d\pi} = -\tau\frac{\partial a}{\partial p_t}, \tag{A6}$$

$$v''(-H_t dp_t - ds_t) = u''(ds_t - \varphi d\pi) \qquad \Leftrightarrow \qquad H_t v''\frac{dp_t}{d\pi} + (v'' + u'')\frac{ds_t}{d\pi} = \varphi u''. \tag{A7}$$

By combining the above two equations, we derive

$$-\Big[\underbrace{\frac{v''+u''}{v''u''}\left(\delta\frac{\partial^2 h}{\partial p_t^2}+\pi\tau\frac{\partial^2 a}{\partial p_t^2}\right)+H_t^2}_{+}\Big]\frac{dp_t}{d\pi}=\tau\frac{\partial a}{\partial p_t}\frac{v''+u''}{v''u''}+\varphi H_t, \tag{A8}$$

which shows that $\frac{dp_t}{d\pi}>0$ if and only if $\tau\frac{\partial a}{\partial p_t}\frac{v''+u''}{v''u''}+\varphi H_t<0$, which can be rearranged as (9).

Totally differentiating (A4) and (A5) with respect to d_t, p_t, s_t, and H_t and then using (7) yields

$$\begin{aligned}
&\delta\left(\frac{\partial^2 h}{\partial p_t^2}dp_t+\frac{\partial^2 h}{\partial p_t\partial d_t}dd_t+\frac{\partial^2 h}{\partial p_t\partial H_t}dH_t\right)=H_tv''[(1-p_t)dH_t-H_tdp_t-ds_t]\\
&+v'dH_t-\pi\tau\left(\frac{\partial^2 a}{\partial p_t\partial d_t}dd_t+\frac{\partial^2 a}{\partial p_t^2}dp_t\right)\\
&\Leftrightarrow\quad\left(\delta\frac{\partial^2 h}{\partial p_t^2}+\pi\tau\frac{\partial^2 a}{\partial p_t^2}+H_t^2v''\right)\frac{dp_t}{dH_t}+H_tv''\frac{ds_t}{dH_t}=v'+(1-p_t)H_tv''-\delta\frac{\partial^2 h}{\partial p_t\partial H_t}, && \text{(A9)}\\
&v''[(1-p_t)dH_t-H_tdp_t-ds_t]=u''ds_t\quad\Leftrightarrow\quad H_tv''\frac{dp_t}{dH_t}+(v''+u'')\frac{ds_t}{dH_t}=(1-p_t)v''. && \text{(A10)}
\end{aligned}$$

By combining the above two equations, we derive

$$\begin{aligned}
&\left[\left(\delta\frac{\partial^2 h}{\partial p_t^2}+\pi\tau\frac{\partial^2 a}{\partial p_t^2}+H_t^2v''\right)(v''+u'')-(H_tv'')^2\right]\frac{dp_t}{dH_t}\\
&=\left[v'+(1-p_t)H_tv''-\delta\frac{\partial^2 h}{\partial p_t\partial H_t}\right](v''+u'')-(H_tv'')(1-p_t)v''\\
\Leftrightarrow\quad&\Big[\underbrace{\frac{v''+u''}{v''u''}\left(\delta\frac{\partial^2 h}{\partial p_t^2}+\pi\tau\frac{\partial^2 a}{\partial p_t^2}\right)+H_t^2}_{+}\Big]\frac{dp_t}{dH_t}=\left(\delta\frac{\partial^2 h}{\partial p_t\partial H_t}-v'\right)\underbrace{\frac{v''+u''}{-v''u''}}_{+}+\underbrace{(1-p_t)H_t}_{+}, && \text{(A11)}
\end{aligned}$$

which shows that $\frac{dp_t}{dH_t}>0$ if $\delta\frac{\partial^2 h}{\partial p_t\partial H_t}-v'\geqslant 0$, which can be rearranged as (10).

Proof of Proposition 3

Under condition (7), totally differentiating (A1) with respect to d_t, p_t, and δ and rearranging obtains

$$\frac{dd_t}{d\delta}=-\underbrace{\frac{\partial h}{\partial d_t}}_{-}\Big(\underbrace{\delta\frac{\partial^2 h}{\partial d_t^2}+\pi\tau\frac{\partial^2 a}{\partial d_t^2}}_{-}\Big)^{-1}>0. \tag{A12}$$

Totally differentiating (A1) with respect to d_t, p_t, and τ and then rearranging derives

$$\frac{dd_t}{d\tau} = \underbrace{-\pi\frac{\partial \pi}{\partial d_t}}_{-}\Big(\underbrace{\delta\frac{\partial^2 h}{\partial d_t^2} + \pi\tau\frac{\partial^2 a}{\partial d_t^2}}_{-}\Big)^{-1} > 0. \tag{A13}$$

Totally differentiating (A4) and (A5) with respect to d_t, p_t, s_t, and δ and then using (7) yields

$$\frac{\partial h}{\partial p_t}d\delta + \delta\left(\frac{\partial^2 h}{\partial p_t^2}dp_t + \frac{\partial^2 h}{\partial p_t \partial d_t}dd_t\right) = H_t v''(-H_t dp_t - ds_t) - \pi\tau\left(\frac{\partial^2 a}{\partial p_t \partial d_t}dd_t + \frac{\partial^2 a}{\partial p_t^2}dp_t\right)$$

$$\Leftrightarrow \quad \left(\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + H_t^2 v''\right)\frac{dp_t}{d\delta} + H_t v''\frac{ds_t}{d\delta} = -\frac{\partial h}{\partial p_t}, \tag{A14}$$

$$v''(-H_t dp_t - ds_t) = u'' ds_t \quad \Leftrightarrow \quad H_t v''\frac{dp_t}{d\delta} + (v'' + u'')\frac{ds_t}{d\delta} = 0. \tag{A15}$$

By combining the above two equations, we derive

$$\left[\left(\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + H_t^2 v''\right)(v'' + u'') - (H_t v'')^2\right]\frac{dp_t}{d\delta} = -\frac{\partial h}{\partial p_t}(v'' + u'')$$

$$\Leftrightarrow \quad \frac{dp_t}{d\delta} = \underbrace{-\frac{\partial h}{\partial p_t}}_{-}\Bigg(\underbrace{\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + \frac{v'' u'' H_t^2}{v'' + u''}}_{-}\Bigg)^{-1} > 0. \tag{A16}$$

Totally differentiating (A4) and (A5) with respect to d_t, p_t, s_t, and τ and then using (7) yields

$$\delta\left(\frac{\partial^2 h}{\partial p_t^2}dp_t + \frac{\partial^2 h}{\partial p_t \partial d_t}dd_t\right) = H_t v''(-H_t dp_t - ds_t) - \pi\tau\left(\frac{\partial^2 a}{\partial p_t \partial d_t}dd_t + \frac{\partial^2 a}{\partial p_t^2}dp_t\right) - \pi\frac{\partial a}{\partial p_t}d\tau$$

$$\Leftrightarrow \quad \left(\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + H_t^2 v''\right)\frac{dp_t}{d\tau} + H_t v''\frac{ds_t}{d\tau} = -\pi\frac{\partial a}{\partial p_t}, \tag{A17}$$

$$v''(-H_t dp_t - ds_t) = u'' ds_t \quad \Leftrightarrow \quad H_t v''\frac{dp_t}{d\tau} + (v'' + u'')\frac{ds_t}{d\tau} = 0. \tag{A18}$$

By combining the above two equations, we derive

$$\left[\left(\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + H_t^2 v''\right)(v'' + u'') - (H_t v'')^2\right]\frac{dp_t}{d\tau} = -\pi\frac{\partial a}{\partial p_t}(v'' + u'')$$

$$\Leftrightarrow \quad \frac{dp_t}{d\tau} = \underbrace{-\pi\frac{\partial a}{\partial p_t}}_{-}\Bigg(\underbrace{\delta\frac{\partial^2 h}{\partial p_t^2} + \pi\tau\frac{\partial^2 a}{\partial p_t^2} + \frac{v'' u'' H_t^2}{v'' + u''}}_{-}\Bigg)^{-1} > 0. \tag{A19}$$

Proof of Proposition 4

In the steady state, equation (13) can be rewritten as equation (17). The third equality of equation (11) can be rewritten as

$$H = H^{\alpha} p^{\beta} + d^{\gamma} \qquad \Leftrightarrow \qquad p = \left[(H - d^{\gamma}) H^{-\alpha} \right]^{\frac{1}{\beta}} . \tag{A20}$$

Substituting (12) into (A20) and then rearranging yields equation (16).

Appendix B: Endogenous Lifespan and Educational Expenditure

In this appendix, we introduce two extensions to the model. First, individuals can extend their lifespan through intentional health investments devoted during young adulthood. In practice, such investments may include engaging in regular aerobic exercise, undergoing routine medical check-ups, and consuming dietary supplements. Second, we allow parents to contribute to their children's human capital not only through parenting but also through financial investments in education.

The model structure is similar to that in Section 2, except that a member of generation t allocates m_t to improve their health and e_t to their child's educational fund in period $t+1$. In other words, each young adult optimally chooses parenting strategies (p_t, d_t), health investment (m_t), educational spending (e_t), and saving (s_t), subject to the following budget constraint:

$$c_t^y = (1 - p_t)H_t - m_t - e_t - s_t. \tag{B1}$$

Health investments enhance the probability of survival to old adulthood. This probability is given by $k(m_t)$, where $k' > 0$ and $k'' < 0$, indicating positive but diminishing returns. The consequent increase in life expectancy raises the insurance premium to $\varphi k(m_t)$.

Educational expenditure facilitates children's human capital development, namely

$$H_{t+1} = h(H_t, p_t, d_t, e_t), \tag{B2}$$

where h is increasing and concave in e_t (i.e., $\frac{\partial h}{\partial e_t} > 0$ and $\frac{\partial^2 h}{\partial e_t^2} < 0$). Besides, parental investment in education can shape a child's gratitude and reinforce his perception of being valued and supported. For example, financial investment in education is a tangible expression of parental commitment and love, i.e., $a_{t+1} = a(p_t, d_t, e_t)$ where $\frac{\partial a}{\partial e_t} > 0$.

A representative member of generation t expects her lifetime utility to be

$$\begin{aligned} U_t = &-z(d_{t-1}) + v[(1 - p_t)H_t - m_t - e_t - s_t] + u[s_t - \varphi k(m_t)] + \mathbb{V}[\tau, k(m_{t-1})] \\ &+ k(m_t)[\tau a(p_t, d_t, e_t) + \Phi(\varphi)] + \delta h(H_t, p_t, d_t, e_t). \end{aligned} \tag{B3}$$

The rational forward-looking individual maximizes U_t by making a life plan. Taking the first order conditions of (B3) with respect to $(d_t, p_t, s_t, e_t, m_t)$ obtains

$$\frac{\partial U_t}{\partial d_t} = k(m_t)\tau\frac{\partial a}{\partial d_t} + \delta\frac{\partial h}{\partial d_t} = 0, \tag{B4}$$

$$\frac{\partial U_t}{\partial p_t} = v'(-H_t) + k(m_t)\tau\frac{\partial a}{\partial p_t} + \delta\frac{\partial h}{\partial p_t} = 0, \tag{B5}$$

$$\frac{\partial U_t}{\partial e_t} = -v' + k(m_t)\tau\frac{\partial a}{\partial e_t} + \delta\frac{\partial h}{\partial e_t} = 0, \tag{B6}$$

$$\frac{\partial U_t}{\partial m_t} = -v' + u'(-\varphi)k' + k'[\tau a(p_t, d_t, e_t) + \Phi(\varphi)], \tag{B7}$$

$$\frac{\partial U_t}{\partial s_t} = -v' + u' = 0. \tag{B8}$$

The optimal interior solutions to $(d_t, p_t, s_t, e_t, m_t)$ are given by the following two expressions:

$$k(m_t) = -\frac{\delta}{\tau}\frac{\partial h}{\partial d_t}\left(\frac{\partial a}{\partial d_t}\right)^{-1}, \tag{B9}$$

$$v' = u' = \delta\frac{\partial h}{\partial e_t} + \tau k(m_t)\frac{\partial a}{\partial e_t} = [\tau a(p_t, d_t, e_t) + \Phi(\varphi) - \varphi u']k' = \frac{1}{H_t}\left[\delta\frac{\partial h}{\partial p_t} + \tau k(m_t)\frac{\partial a}{\partial p_t}\right]. \tag{B10}$$

Appendix C: The Credible Social Norms of Eldercare

Eldercare is often motivated by social norms, which encompass explicit and implicit guidelines within a culture, imposing invisible constraints on the behaviors of family members (Barigozzia, Cremer, and Roeder 2020; Klimaviciute and Pestieau 2023). In the main text, middle-aged adults are assumed to allocate an exogenous share of time to eldercare ($\tau_t = \tau$ for all t). In this appendix, we aim to determine the conditions under which the social norm of eldercare can be sustained in a Nash equilibrium. Following the literature on intergenerational trade (e.g., Ehrlich and Lui 1991), we assume that generation $t+1$ conforms to the social norm of choosing $\tau_{t+1} = \tau$ if they observe generation t devoting a share $\tau_t = \tau$ of time caring for generation $t-1$; otherwise, they leave their parents uncared for. This assumption aligns with empirical evidence that adult children give more socio-emotional support to mothers who support their own parents (Silverstein et al. 2025).

Rewrite the lifetime utility of a member of generation t in equation (8) as

$$U_t = -z(d_{t-1}) + v[(1-p_t)H_t - s_t] + u(s_t - \pi\varphi) + \mathbb{V}(\tau_t, \pi) + \pi[\tau_{t+1} a(p_t, d_t) + \Phi(\varphi)] + \delta h(H_t, p_t, d_t). \tag{C1}$$

The member of generation t who conforms to the norm by choosing $\tau_t = \tau$ in period $t+2$ expects her child to choose $\tau_{t+1} = \tau$ in period $t+3$. In this case, (C1) can be rewritten as

$$\begin{aligned} U_t = &-z(d_{t-1}) + v[(1-p_t)H_t - s_t] + u(s_t - \pi\varphi) + \mathbb{V}(\tau, \pi) \\ &+ \pi[\tau a(p_t, d_t) + \Phi] + \delta h(H_t, p_t, d_t). \end{aligned} \tag{C2}$$

Because U_t is a continuous function of p_t, d_t, and s_t, all of which are defined over compact sets, an optimal solution must exist. We denote the optimal solutions as p_t^*, d_t^*, and s_t^*, respectively, each of which depends on τ. Substituting $p_t^*(\tau)$, $d_t^*(\tau)$, and $s_t^*(\tau)$ into (C1) obtains the maximum utility of conforming to the norm, denoted by $U_t^C(\tau)$, where superscript C denotes conformity.

Conversely, if the individual deviates from the filial norm in period $t+2$, she will optimally choose $\tau_t = 0$, which leads her child to set $\tau_{t+1} = 0$, too. In this case, (C1) can be rewritten as

$$U_t = -z(d_{t-1}) + v[(1-p_t)H_t - s_t] + u(s_t - \pi\varphi) + \mathbb{V}(0, \pi) + \pi\Phi + \delta h(H_t, d_t, p_t). \tag{C3}$$

Because U_t is a continuous function of p_t, d_t, and s_t, all of which are defined over compact sets, an optimal solution must exist. We denote the optimal solutions as p_t', d_t', and s_t', respectively, each of which depends on τ. Substituting p_t', d_t', and s_t' into (C1) obtains the maximum utility of deviating from the norm, denoted by $U_t^D(\tau)$, where superscript D denotes deviation.

Clearly, the individual finds that an deviation makes herself worse off if and only if $U_t^C \geqslant U_t^D$. In other words, the social norm is credible if and only if

$$U_t^C(\tau) \geqslant U_t^D. \tag{C4}$$

Any τ that satisfies (C4) can be sustained as an intergenerational Nash equilibrium. Consequently, multiple Nash equilibria may exist, each corresponding to a different social norm of eldercare. Our analysis in Section 2 represents one such equilibrium, characterized by a particular value of τ.