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Production and energy mode control of a production-inventory system

Barış Tan [a], Oktay Karabağ [b,c,*], Siamak Khayyati [d]

[a] College of Administrative Sciences and Economics, College of Engineering, Koç University, Rumeli Feneri Yolu, İstanbul 34450, Turkey
[b] Erasmus School of Economics, Erasmus University Rotterdam, Burgemeester Oudlaan 50, Rotterdam 3000 DR, the Netherlands
[c] Department of Industrial Engineering, Yaşar University, Üniversite Caddesi Ağaçlı Yol No:37-39, Izmir 35100, Turkey
[d] Mannheim Business School, University of Mannheim, Mannheim 68161, Germany

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ABSTRACT

Energy efficiency in manufacturing can be improved by controlling energy modes and production dynamically. We examine a production-inventory system that can operate in Working, Idle, and Off energy modes with mode-dependent energy costs. There can be a warm-up delay to switch between one mode to another. With random inter-arrival, production and warm-up times, we formulate the problem of determining in which mode the production resource should operate at a given time depending on the state of the system as a stochastic control problem under the long-run average profit criterion considering the sales revenue together with energy, inventory holding and backlog costs. The optimal solution of the problem for the exponential inter-arrival, production and warm-up times is determined by solving the Markov Decision Process with a linear programming approach. The structure of the optimal policy for the exponential case uses two thresholds to switch between the Working and Idle or Working and Off modes. We use the two-threshold policy as an approximate policy to control a system with correlated inter-event times with general distributions. This system is modelled as a Quasi Birth and Death Process and analyzed by using a matrix-geometric method. Our numerical experiments show that the joint production and energy control policy performs better compared to the pure production and energy control policies depending on the system parameters. In summary, we propose a joint energy and production control policy that improves energy efficiency by controlling the energy modes depending on the state of the system.

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1. Introduction

The concerns related to rising energy prices, uncertain energy supply, and regulations related to controlling carbon emissions led manufacturers to focus on improving their energy efficiencies. The manufacturing industry accounts for 50% of primary energy use and 38% of the total greenhouse gas emission in the world (Wang et al., 2017). Many production facilities, especially those involving processes such as refining, casting, dyeing, and heating, consume energy intensively. For example, in an automotive assembly plant, more than 700 kilowatt hours of electrical energy is consumed for a single car assembly, while more than 60% of the total energy consumption of the factory is spent processing in paint booths and ovens (Yan & Zheng, 2020). Similarly, the steel industry consumes 6% of the energy used for production in the world. About 30% of this energy consumption is used for heating processes (Su et al., 2017). In this environment, developing an effective production control policy that controls production and energy modes dynamically in a way that reduces energy consumption while keeping the production targets has a potential to address the economic and environmental issues faced by manufacturers.

In order to balance the production targets and energy-saving objectives, there is a need for developing analytical models and using these models to design and implement data-driven control policies. Based on the need for developing effective production and energy control policies, our main research questions in this study are as follows: *How can production and energy mode of a production system be controlled dynamically in a way to balance energy cost benefits and production targets based on the state of the system at a given time?*; *What is the benefit of controlling production and energy mode of a production system depending on the system parameters?* In order to answer these questions, we study a manufacturing system that includes a single machine operating in a make-to-stock fashion. The machine can run in different energy/operation modes such as on, off, and idle modes that can be controlled in real time. Operating in each mode has a different energy consumption cost.

* Corresponding author.
E-mail address: karabag@ese.eur.nl (O. Karabağ).

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Additionally, there can be a warm-up delay to make a transition from one energy mode to another. We consider both the lost sales and backorder cases. For the backorder case, an arriving demand that is not met immediately from on-hand inventory is backordered, and it is satisfied when the production catches up with the demand. For the lost sales case, an arriving demand that is not met immediately from on-hand inventory is lost. The demand inter-arrival times, production times, and warm-up delays between different modes are modeled as Markov Arrival Processes (MAPs). The main objective is to maximize the expected profit rate that is the difference between the sales revenue and the sum of inventory/backlog- and energy-related costs per unit time by controlling production and energy mode of the machine based on the current state of the machine and the inventory position.

In this study, we first derive the optimal production and control policies for this system when the demand inter-arrival times, production times, and transition delays between different modes are assumed to be i.i.d. exponential random variables by using a linear programming (LP) approach to solve the associated Markovian Decision Process (MDP). The numerical analysis we conducted for the exponential case shows that a policy that uses two thresholds to switch between the Working and Off or Working and Idle modes is optimal. We use the structure of this policy as an approximate policy to control production and energy mode for a manufacturing system with correlated inter-arrival, processing, and phase-type warm-up times with general distributions. We model this general system as a Quasi Birth and Death Process and evaluate its performance by using a matrix-geometric method. Our numerical experiments show that optimizing energy and production control jointly improves the system performance significantly.

The main contribution of this study is three folds: First, we formulate and analyze the joint production and energy mode control problem as a stochastic control problem. In the literature, there are recent studies that developed various energy control policies for production systems and there are many studies for production and material flow control. However, to the best of our knowledge, controlling energy mode and production jointly has not been studied. Secondly, as opposed to suggesting using a particular policy, we analytically determine the optimal joint production and energy control policy for a system with exponentially distributed inter-event times. Lastly, as opposed to simulation-based approaches, we propose an analytical method to evaluate the performance of a system with correlated inter-arrival, service, and independent warm-up times with general distributions modelled as MAPs under the proposed joint production and energy control policy.

The organization of the remaining part of this paper is as follows: the pertinent literature is reviewed in Section 2. The model is introduced, and the joint energy and production control problem is defined in Section 3. The optimal control policy for a special case with exponential arrival, processing, and warm-up times is determined by solving the stochastic optimal control problem with an LP approach in Section 4. The approximate control policy for a system with correlated inter-event times that have general distributions is proposed in Section 5. An analytical model to evaluate the performance of the system under this policy and then determine its optimal parameters is also given in Section 5. Section 6 presents the numerical experiments. Finally, the conclusions are given in Section 7.

2. Literature review

Energy-aware control of production systems has attracted attention in recent years. The current state-of-art of this research stream is comprehensively reviewed in Biel & Glock (2016); Gahm et al. (2016); Suzanne et al. (2020), and Bänsch et al. (2021). Furthermore, there are a few reviews for performance evaluation of production systems under various production control policies, e.g., Dallery & Gershwin (1992); Li et al. (2009), and Papadopoulos et al. (2019). Here, we only focus on contributions of previous research with reference to the model proposed in this work. Specifically, we review only the pertinent studies that present energy-efficient production control strategies and energy-efficiency state control approaches that switch a single reliable machine or a set of reliable machines between different energy modes to save energy.

Mouzon et al. (2007) propose a switching policy for a machine that can be turned off when it becomes idle for electricity saving purpose and show that there exists a significant potential for reducing energy consumption especially in nonbottleneck machines. Mouzon & Yıldırım (2008) extend this framework to consider a multi-objective optimization problem aimed at minimizing total energy consumption and total tardiness on a single machine. Sun & Li (2012) develop an algorithm that predicts how long on/off times should be to control real-time energy in a production line. Mashaei & Lennartson (2012) derive a control policy that optimally determines when machine tools are turned on and off in a flow shop in which the number of pallets is limited. By using a simulation model, Dai et al. (2013) extend the problem setting of Mashaei & Lennartson (2012) to a multi-objective case where both the total energy cost and the makespan are minimized. The similar energy-efficient control strategies to the ones being introduced in these studies are also adapted to different real-life problems motivated by flow shops (Mashaei & Lennartson, 2014), paint shops (Cronrath et al., 2016), automotive plants (Katchasuwanmanee et al., 2017), CNC centers (Squeo et al., 2019), and circuit board manufacturer (Uit het Broek et al., 2020).

Frigerio & Matta (2014) examine on/off policies for a single machine with a deterministic warm-up time and characterize the optimal control policy as a function of warm-up time and average part arrival time. Using a simulation model, Frigerio & Matta (2015) study the control problem of switching on/off a machine tool for energy saving during the idle times under the assumption of random warm-up times and fully observable buffer level, and they develop a hybrid policy that uses the time and the buffer level information together for the problem. Renna (2018) proposes a simulation model to investigate the performance of an adaptive control policy considering a pull system and compare it with some policies introduced in Frigerio & Matta (2015). Frigerio et al. (2021, 2020) extend the offline energy efficient control policy proposed by Frigerio & Matta (2015) by enabling its online application. Loffredo et al. (2021) configure an energy efficient control policy that utilizes buffer level information to switch on/off multiple identical machines operating in parallel workstations while satisfying a target level on the system availability. In a parallel machine line, Anghinolfi et al. (2021) and Heydar et al. (2021) also consider a minimization problem of total energy consumption and makespan. Brundage et al. (2013); Chang et al. (2012) and Li et al. (2016) and Loffredo et al. (2021) use similar policies for controlling machines in a serial line. Gahm et al., 2016 and Bänsch et al., 2021 present comprehensive literature classification tables comparing many of the studies given in this review.

Based on the review of the literature, we can restate the contribution of this study. First, as opposed to the existing studies that focus on either energy mode control (Cronrath et al., 2016; Frigerio et al., 2021; 2020; Frigerio & Matta, 2015; Mashaei & Lennartson, 2014; Sun & Li, 2012) or production control (e.g., Dizbin & Tan, 2019; Gavish & Graves, 1980; Khayyati & Tan, 2020; Sobel, 1982), we propose a joint optimal production and energy mode control policy. Secondly, to the best of our knowledge, this is the first attempt to determine the optimal joint production and energy control policy for a system with exponentially distributed inter-event times by using linear programming solution of the Markov Decision Process. Lastly, as opposed to simulation-based approaches (Dai et al., 2013; Frigerio & Matta, 2015; Renna, 2018), we propose an analytical method to evaluate the performance of a system with correlated inter-arrival, service, and independent warm-up times with general distributions modelled as MAPs under the proposed joint production and energy control policy. This analytical model allows us to determine the optimal parameters of the control policy in an efficient way.

3. Model

The main control problem addressed in this study is to decide whether the material flow in a machine will be allowed, whether the machine will be allowed to start production, and in which of the different energy modes the machine will operate at a given time dynamically based on the data collected from the system in order to maximize the expected profit that is the difference between the revenue generated through the sales and the total cost of the inventory carrying, not fulfilling the order and energy consumption. Fig. 1 depicts the production and energy mode control of a production-inventory system.

An operating machine can be switched from the on (also referred as the Working) mode to the standby (also referred as the Idle) mode or shut down (referred as the Off mode) depending on the demand, the inventory position of the finished products, and the energy mode of the machine. Similarly, a machine in the Idle or Off mode can be made operational according to the current state of the system. While the delay in the time required to turn on the machine from the Idle or the Off mode affects the flow of parts and thus the costs of carrying inventory and not fulfilling the order on time, its operation in these different energy modes will save energy. The main problem is to make these decisions together in the best way to maximize the expected profit. In this study, this problem is formulated and solved as a stochastic control problem. The assumptions of the model and the formal definition of the control problem are given as follows:

State. We consider a machine that operates in different energy modes. $\mathbf{M}$ denotes the set of different energy modes. The energy mode of the machine at time t is $P(t) \in \mathbf{M}$. The machine produces

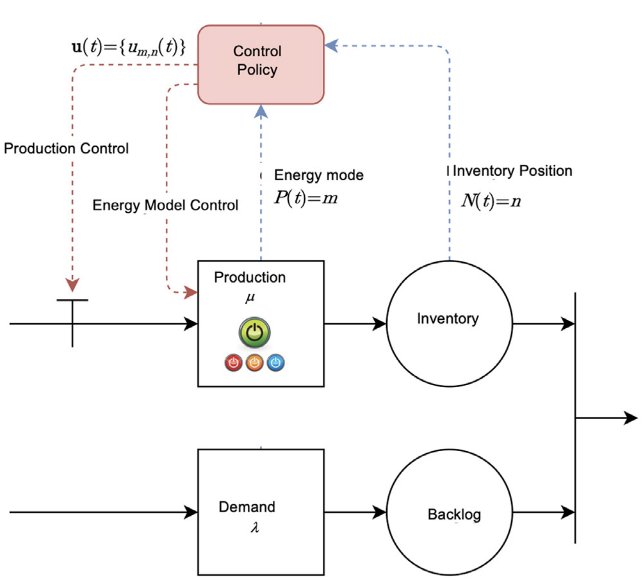

Fig. 1. Joint production and energy mode control of a production-inventory system.

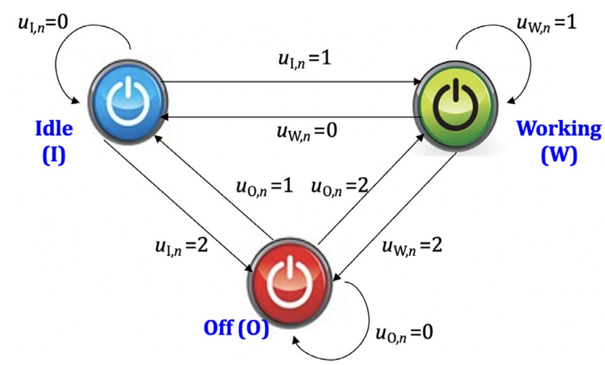

Fig. 2. Energy modes, state transitions and transition decisions.

to stock and the inventory/backlog level at time t is $N(t) \in \mathcal{N}$. We assume that the machine is never starved.

The state of the system at time t, $\mathbf{S}(t)$ includes the energy mode state of the machine, $P(t)$ and the inventory position at time t, $N(t)$, i.e., $\mathbf{S(t)} = (P(t), N(t))$. We consider both cases with the lost sales where $\mathcal{N} = \mathbb{N}$ and when the unsatisfied orders are backordered where $\mathcal{N} = \mathbb{Z}$. The state of the system in the steady-state is $\mathbf{S} = (P, N)$. The number of parts in the inventory is $N^+(t) = \max\{N(t), 0\}$ and the number of backorders is $N^-(t) = \max\{-N(t), 0\}$. The expected inventory and backlog levels are denoted with $\mathbb{E}[N^+]$ and $\mathbb{E}[N^-]$, respectively.

Energy modes and joint production and energy control. When the system is in mode m and the number of parts/backlogs is n at time t, $u_{m,n}(t) \in U_m$ represents the decision that can be taken among the permissible discrete actions given in the set of U_m at time t. The tuple $\mathbf{u}(t) = \{u_{m,n}(t)\}$ includes the decisions at all the possible energy modes and inventory/backlog levels for any given time t. Additionally, $u_{m,n}$ represents the decision taken in state (m, n) in the steady-state.

Fig. 2 shows the system analyzed in this study with three energy modes for Working (W), Idle (I), and Off (O) states, i.e., $i \in \mathbf{M} = \{\mathrm{W}, \mathrm{I}, \mathrm{O}\}$ and the transition controls between these modes. When the machine is in the Working mode, it processes a part. When it is in the Idle or Off modes, the production is stopped but it can be started after a delay.

The decision to switch from the Idle to the Working mode and from the Off to the Working mode ($u_{\mathrm{I,n}} = 1$ and $u_{\mathrm{O,n}} = 2$) for the system in Fig. 2 can be considered as a production control decision. If the machine is in the Idle mode, turning the machine into the Working mode ($u_{\mathrm{I,n}}(t) = 1$) takes the waiting part from the input buffer of the machine and starts processing on the machine. On the other hand, if the machine is not allowed to switch to the Working mode ($u_{\mathrm{I,n}}(t) = 0$), the material flow to the machine stops and the part is kept in the input buffer. Since the decisions described in $\mathbf{u}(t) = \{u_{m,n}(t)\}$ include both production and energy mode control decisions, $\mathbf{u}(t)$ is referred to as the joint production and control decision.

Transition times. When the decision $u_{i,n}(t)$ is taken at time t, the transition delay to switch energy mode i to mode j is denoted by τ_{ij}. The delay between some modes can be instantaneous. The non-zero delay τ_{ij} is modeled as a random variable.

At a given time, the machine is either operating in an energy mode $i \in \mathbf{M}$ or in transition from mode i to j, $i \neq j$ going through a transition delay τ_{ij} following a production or energy mode change decision. In the model depicted in Fig. 2 and analyzed in Section 4, the times required to turn the machine Off when it is Idle or Working, turn the machine Idle when it is Working, and start the production when it is Idle are assumed to be instantaneous, i.e., $\tau_{\mathrm{IO}} = \tau_{\mathrm{WO}} = \tau_{\mathrm{WI}} = \tau_{\mathrm{IW}} = 0$. However, when the machine is off, the time it takes for the machine to be ready for production in the Idle mode, τ_{OI} is a random variable with expectation $\mathbb{E}[\tau_{\mathrm{OI}}] = 1/\delta_{\tau_{\mathrm{OI}}}$. Similarly, when the machine is off, the time it takes for the machine to start production in the Working mode, τ_{OW} is a random variable with expectation $\mathbb{E}[\tau_{\mathrm{OW}}] = 1/\delta_{\tau_{\mathrm{OW}}}$. Additionally, the time to finish processing a part, denoted by τ_{WW} is also a random variable and its expectation is $\mathbb{E}[\tau_{\mathrm{WW}}] = 1/\mu$. Lastly, the time between the two successive parts that arrive at the system, namely the inter-arrival time, is also a random variable with expectation $\mathbb{E}[\tau_{\mathrm{a}}] = 1/\lambda$.

We assume that the warm-up periods, the processing times, and the arrival times are independent of each other. However, the sequence of processing times, and the arrival times can be correlated with the given autocorrelation structures. We assume that the warm-up times are i.i.d. We model the processing times, the arrival times and the warm-up times as Markov Arrival Processes.

Costs and revenue. The energy consumption in each mode is different. When the machine is off, the energy consumption will be minimal and limited to the essential activities such as network communications. In the Idle mode, some energy will be consumed to adapt the machine mode to the operation to be performed. In this case, the amount of energy consumed will be slightly more than in the Off mode, but less than in the Working mode. While the machine is in the Working mode, all its functions will be active, and this will make the corresponding state the highest energy-consuming state compared to the other two states. We also consider that the energy consumption in the Working mode is proportional to the processing time.

The cost of the energy consumed by the machine per unit time in the energy mode i is given as c_i. The cost of the energy consumed by the machine per unit time in transition from the energy mode i to the energy mode j is given as c_{ij}. Let $I_i(t)$ be an indicator function that is equal to 1 if the energy mode of the machine at time t, $P(t) = 1$ and 0 otherwise. Similarly, let $D_{ij}(t)$ be an indicator variable if the machine is in transition from energy mode i to j. $D_{ij}(t)$ is set to 0 if $\tau_{ij} = 0$ or if $i = j$; otherwise, it is set to 1. Then, the instantaneous energy cost of the machine at time t can be written as $c_i I_i(t) + c_{ij} D_{ij}(t)$.

The inventory carrying cost is c^+. We consider both the backorder and the lost sales cases. For the backlog case, the backlog cost is c^-. The revenue per part, which is the difference between the sales price and the production cost, is denoted with r and the revenue at time t is denoted with $R(\mathbf{S}(t))$. If backorder is allowed, then each arriving order will either be satisfied immediately from the inventory if there is an available part or later if the inventory is empty. For the lost sales case, since backorder is not allowed, the order is lost when the inventory level is zero upon the arrival of an order. The inventory carrying and backorder cost is $C(\mathbf{S}(t)) = c^+ N^+(t) + c^- N^-(t)$.

Objective function and the control problem. The stochastic control problem we consider is determining the joint production and energy mode control $\mathbf{u}(t)$ that determines the transitions among the different modes at a given time depending on the state of the system $\mathbf{S}(t)$ to maximize the expected profit rate in the long run:

$$\chi^{\mathbf{u}(t)} = \mathbb{E}^{\mathbf{u}(t)}\left[\lim_{T\to\infty}\frac{1}{T}\int_{t=0}^{T} R(\mathbf{S}(t)) - C(\mathbf{S}(t)) - \sum_{m\in\mathbf{M}}\left(c_m I_m(t) + \sum_{n\in\mathbf{M}} c_{mn} D_{mn}(t)\right) dt \,|\mathbf{S}(0)\right] \tag{1}$$

where $\chi^{\mathbf{u}(t)}$ is the expected profit rate obtained by using the state-dependent control policy $\mathbf{u}(t)$.

Then, the objective of the problem is to identify the optimal policy $\mathbf{u}^*(t)$ defined as:

$$\mathbf{u}^*(t) = \inf_{\mathbf{u}(t)} -\chi^{\mathbf{u}(t)} \tag{2}$$

Table 1 gives the description of the notation used throughout the paper.

4. Optimal joint production and energy control of a system with exponential arrival, processing and warm-up times

In this section, we consider a special case of the optimal joint production and energy control problem given in Section 3. The model we consider includes 3 energy modes as given in Fig. 2. The inter-arrival, processing, and warm-up times between the Off mode and the Working mode and between the Off mode and the Idle mode are assumed to be i.i.d. exponential random variables. In this model, the delays between different modes excluding τ_{OI} and τ_{OW} are instantaneous. Therefore, the expected profit function only includes the per unit time transition costs c_{OI} and c_{OW} together with the energy costs in W, I, and O.

Showing the structure of the optimal policy analytically even for the exponential case is an open research problem due to the profit function that includes terms that depend on the residence times in different states in addition to other costs. We will use the linear programming formulation of the Markov Decision Process to obtain the optimal policy for the problem. The optimal joint production and energy control policy obtained for this special case will be used to propose a production and energy control policy for a system with the correlated inter-arrival, processing, and phase-type warm-up times that have general distributions modelled as Markov Arrival Processes.

4.1. Linear programming formulation

Under the assumption of exponential inter-arrival, processing and warm-up times, the problem given in Section 3 is a Markovian Decision Process. More specifically, we consider the problem of controlling a continuous time discrete state space MDP to maximize long-run average profit. Our problem can be classified as a finite MDP because it has a finite action space and includes strictly positive, bounded rewards and costs. Considering these problem characteristics, existence of an optimal deterministic stationary policy for the given problem is ensured (Puterman, 2014).

We utilize a linear programming formulation to solve the MDP and obtain the optimal production and energy control policies. In order to utilize this solution approach, we first transform this continuous-time process into an equivalent discrete-time one by using uniformization. We then establish the dual linear programming formulation of this equivalent discrete-time problem. The LP formulation of the problem treats the long-run fraction of the time that the system spends in different states when a particular action is taken when exiting that state. As a result of this effort, duality theory allows us to characterize the optimal decisions of the MDP model we consider in the manuscript via the optimal solution of the associated dual problem. Namely, the solution of the problem yields the systems steady-state distribution under the optimal policy. The optimal action policy is obtained by mapping the decision variables that have a non-zero value in the solution to their corresponding actions. When LP yields an optimal solution, we are sure that it is the optimal solution of the MDP (Bertsekas, 2015).

This solution approach has been recently used in the literature to find the optimal control policies for MDP problems in production systems (see, e.g., Hosseini & Tan, 2019; Karabağ & Tan, 2019). Alternatively, a value iteration or policy iteration approach can also

Table 1
Description of notation.

Notation	Description
$P(t) = m$	Status of the machine at time t, $m \in \mathbf{M}$
$N(t) = n$	Number of parts\backlogs at time t
$N^{+}(t)$	Number of parts in the inventory at time t
$N^{-}(t)$	Number of backlogs at time t
$\mathbf{S}(t) = (P(t), N(t))$	State of the system at time t
$\mathbf{M}$	Set of machine statuses $\mathbf{M}$={W(Working), I(Idle), O(Off)}
$u_{m,n}(t)$	Action taken at time t when the state is (m, n), $u_{m,n} \in U_m$
$\mathbf{u}(t) = \{u_{m,n}(t)\}, m \in \mathbf{M}, n \in \mathcal{N}$	Tuple of actions at all states
U_{I}	Permissible action set for the Idle mode, U_{I}={0(I), 1(W), 2(O)}
U_{W}	Permissible action set for the Working mode U_{W}={0(I), 1(W), 2(O)}
U_{O}	Permissible action set for the Off mode, U_{O}={0(O), 1(I), 2(W)}
τ_a	Demand inter-arrival time
$\lambda = 1/\mathbb{E}[\tau_a]$	Demand rate
τ_{ij}	The transition delay to switch from energy mode i to j
$\mu = 1/\mathbb{E}[\tau_{\mathrm{WW}}]$	Production rate
$\delta_{ij} = 1/\mathbb{E}[\tau_{ij}]$	Average transition delay (warm-up) rate from mode i to j
r	Unit sales revenue
c^{+}	Unit holding cost
c^{-}	Unit backlog cost
$C(\mathbf{S}(t))$	The inventory carrying and backorder cost at time t
$R(\mathbf{S}(t))$	The revenue rate at time t
c_i	Energy cost per unit time in the energy mode i
c_{ij}	Energy cost per unit time transiting from state i to j

be used to determine the optimal policy. We verified the results obtained by using the LP approach by solving the same problem by using a value iteration algorithm.

Let $X_{m,n,u_{m,n}}$, $m \in \mathbf{M}, n \in \mathcal{N}, u_{m,n} \in U_m$ be the long-run fraction of the time that the system spends in the state (m, n) where the energy mode of the machine is m, the inventory position is n, and the action $u_{m,n}$ is taken when exiting the state (m, n). For the sake of brevity, in the LP formulation, $X_{m,n,u_{m,n}}$ is replaced with X_{m,n,v_m} without any loss of generalization.

In order to formulate the problem as a linear program with a finite number of constraints and variables, the inventory level is truncated at level K, i.e. $n \leq K$ and K is chosen in a way that the steady-state probabilities at the upper truncation levels are below a given tolerance level.

The formulations for the lost sales case is given in Appendix A and the formulation for the backlog case is given in Appendix B.

In the given problem setting, there exist $\mathbf{M}$ machine modes, $|U_m|$ state-dependent actions and $K+1$ inventory levels for the lost-sales case and $K-L+1$ inventory levels for the backlog case. Accordingly, the linear programming formulation for the lost-sales case has $\mathbf{M} \times (K+1) \times |U_m|$ different decision variables and $5 + |U_m| + 3K$ different constraints in addition to the non-negativity constraint. Similarly, the linear programming formulation for the backlog case has $\mathbf{M} \times (K-L+1) \times |U_m|$ decision variables and $5 + |U_m| + 3 \times (K-L)$ constraints.

Considering the problem instances given in Section 6, each LP formulation has $3 \times 20 \times 3 = 180$ decision variables and $5 + 3 + (3 \times 20) = 68$ constraints in addition to the non-negativity constraints.

4.2. Structure of the optimal policy for the system with exponential demand inter-arrival, production and warm-up times

The solution of the LP formulation of the MDP shows that the transitions between the different energy modes are controlled with a threshold-policy based on the inventory position. The optimal policy determines the energy and production control decisions based on different thresholds to decide whether to switch from one mode to another. Since there are three modes, in principle, the solution of the stochastic control problem can have up to 6 thresholds. The inventory threshold that determines when the energy mode is switched from mode i to mode j is denoted with S_{ij}, $i, j \in \{\mathrm{W}, \mathrm{I}, \mathrm{O}\}$. However, considering the dynamics of the system, a number of these thresholds will not be used simultaneously in the steady state for a given system.

When the machine is working, it switches to the Idle mode when the inventory position reaches S_{WI} or it switches to the Off mode when the inventory position reaches S_{WO}. Since the inventory position increases only when the machine is working, as soon as the mode switches to either Idle or Off, the inventory level starts decreasing. When the machine is off, it switches either to the Working mode when the inventory level decreases to S_{OW} or to the Idle mode when the inventory level decreases to S_{OI} depending on the parameters. When the machine is idle, it starts producing when the inventory level decreases to S_{IW}. Note that when the machine is idle, it is not expected to make a transition to the Off mode, since staying in the Off mode during the previous period would give a lower cost. Therefore, the system operates between the Working and Idle modes with two thresholds $(S_{\mathrm{WI}}, S_{\mathrm{IW}})$, between the Working and Off modes with two thresholds $(S_{\mathrm{WO}}, S_{\mathrm{OW}})$, or with transitions among the Working, Off, and Idle modes with three thresholds $(S_{\mathrm{WO}}, S_{\mathrm{OI}}, S_{\mathrm{IW}})$.

Fig. 3 depicts the structure of the optimal policy for these three alternatives. The direction of the change in the inventory position when the machine is operating in a given mode is also indicated with an arrow. Although, the optimal policy can either use two or three thresholds as shown in Fig. 3, depending on the system parameters, the solutions of the LP formulation for the lost sales case given in Section 4.1 for the range of system parameters given in Table 3 show that the system operates either in the Working and Idle modes or in the Working and Off modes under the optimal policy. For a few cases where the warm-up time is long and the inventory carrying cost is high, an alternative optimal solution is found where the policy uses three thresholds with the Working-Idle-On models but yields the same average profit rate obtained with an alternative solution that uses only two modes simultaneously. The decision to use a joint production and energy control policy that uses the Working-Off modes or the Working-Idle modes depends on the system parameters.

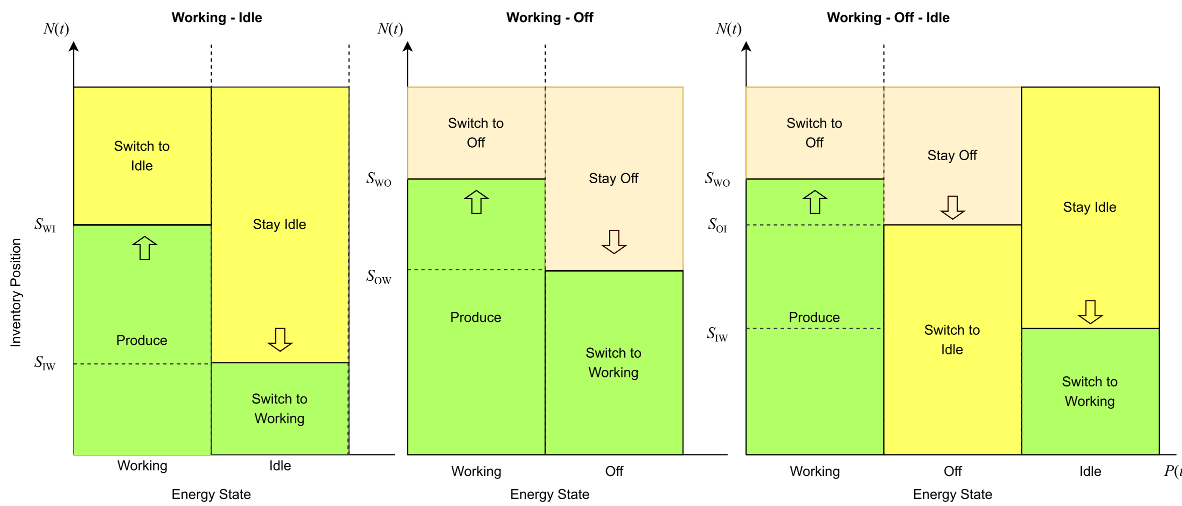

Fig. 3. The structure of the optimal policy for the system with exponential demand inter-arrival, production, and warm-up Times depending on the energy mode and the inventory position.

5. An approximate joint production and energy control policy for a system with general production and inter-arrival and warm-up time processes

The optimal control problem analyzed in Section 4 assumes exponentially distributed inter-arrival, production, and warm-up times. When these times have general distributions and when they are correlated, the structure of the optimal policy changes. When the inter-arrival, production, and warm-up times are modelled as MAPs, the control problem given in Eq. (2) will also be an MDP. The solution of this MDP will be a state-dependent threshold policy where a different inventory threshold will be set for combination of different phases of inter-arrival, processing, and warm-up times. Since the MAP representation is used to model general distributions with possible autocorrelation, the phases of the inter-event times cannot be observed and used for a real-time production and energy mode control. As a result, there is a need to develop an easily implementable production and energy mode control policy for the general case. In the next section, we propose such an approximate policy to control production and energy for a system with general and correlated inter-arrival, production, and independent warm-up times and present an analytical model to determine its optimal parameters.

5.1. Two-threshold production and energy control policy

Based on the results given in Section 4.2, we propose a threshold-type policy that uses two thresholds to control production and energy mode jointly for a production-inventory system with correlated inter-arrival, service, and phase-type warm-up times. These thresholds determine when to switch between the Working and the Off modes or between the Working and the Idle modes depending on the inventory position and the energy mode of the machine. The policy that uses the Working and the Off modes is referred to as the Working-Off policy. Since this policy prioritizes energy saving costs by determining when to turn off and on the machine, it can be considered as *the pure energy control*. Alternatively, the policy that uses the Working and the Idle modes keeps the machine always ready for production and it does not allow the machine to turn off and then go through a warm-up period. Therefore, it prioritizes meeting the demand with low inventory holding and backlog costs. The policy that uses the Working and the Idle modes is referred to as the Working-Idle policy. The Working-Idle policy can be considered as *the pure production control policy*. The pure production control is also referred to as *the always on* policy in the literature when the energy control problem is analyzed in production lines with finite inter-station buffers. In this setting, the finite buffers limit the production when the machines are controlled with *the always on* policy. The expected profits for the two alternatives of operating with the Working and Off or with the Working and the Idle modes will be determined and the one that gives a higher profit will be used.

When the policy uses the Working and the Off modes, according to this policy when the upper threshold S_{WO} is reached, the machine is turned off and when the lower threshold S_{OW} is reached, the machine is turned on. This policy does not use the Idle mode but operates with the Off and Working modes. Formally, it can be defined as:

$$u_{W,n} = \begin{cases} 2, & \text{if} \quad n \geq S_{WO} \\ 1, & \text{if} \quad n < S_{WO} \end{cases}, \qquad u_{O,n} = \begin{cases} 2, & \text{if} \quad n \leq S_{OW} \\ 0, & \text{if} \quad n > S_{OW} \end{cases}. \tag{3}$$

Similarly, when the policy uses the Working and the Idle modes, when the upper threshold S_{WI} is reached, the machine is switched to the Idle mode and when the lower threshold S_{IW} is reached, the machine is switched to the Working mode. This policy does not use the Off mode but operates with the Idle and Working modes. Formally, it can be described as:

$$u_{W,n} = \begin{cases} 0, & \text{if} \quad n \geq S_{WI} \\ 1, & \text{if} \quad n < S_{WI} \end{cases}, \qquad u_{I,n} = \begin{cases} 1, & \text{if} \quad n \leq S_{IW} \\ 0, & \text{if} \quad n > S_{IW} \end{cases}. \tag{4}$$

Let χ^*_{WO} be the maximum profit that can be obtained under the Working-Off policy by using the optimal threshold levels S_{WO} and S_{OW}. Analogously, let χ^*_{WI} be the maximum profit that can be obtained under the Working-Idle policy by using the optimal threshold levels S_{WI} and S_{IW}. If $\chi^*_{WO} \geq \chi^*_{WI}$ then the optimal two-threshold policy uses the Working and Off modes and makes the production and energy mode control decisions according to Eq. (3). On the other hand, if $\chi^*_{WO} < \chi^*_{WI}$ then the optimal two-threshold policy uses the Working and Idle modes and makes the production and energy mode control decisions according to Eq. (4). As a result, the two-threshold policy that sets the state-dependent con-

Table 2
Description of notation of the MAP model.

Notation	Description
S^u	Upper threshold level
S^d	Lower threshold level
$\tilde{d}$, $(\mathbf{D0}, \mathbf{D1})$, $\mathbf{d}$	The MAP of the arrival process, its size, transition matrices and its initial probability vector
cv_a	The coefficient of variation of the inter-arrival times
L_1^a	The first-lag autocorrelation of the inter-arrival times
$\tilde{p}$, $(\mathbf{P0}, \mathbf{P1})$, $\mathbf{p}$	The MAP of the production process, its size, transition matrices and its initial probability vector
cv_s	The coefficient of variation of the production times
L_1^s	The first-lag autocorrelation of the production times
$\tilde{w}$, $(\mathbf{W0}, \mathbf{W1})$, $\mathbf{w}$	The MAP representation of the phase-type distribution of the warm-up times, its size, transition matrices and its initial probability vector
cv_w	The coefficient of variation of the warm-up times
$\mathbf{e}_{i,x}$	Row vector of length x with a one in dimension i and zeros elsewhere
$\mathbf{J}_{i,j,x}$	Square matrix of size x with a one at row i and column j and zeros elsewhere
$\mathbf{0}_{x,y}$	Matrix of zeros with x rows and y columns
$\mathbf{I}_x$	Identity matrix of size x
$\delta_{\{x\}}$	Indicator function that is equal to one if the statement x is true and zero otherwise
$\mathbf{x}'$	The transpose of vector $\mathbf{x}$

trol vector $\mathbf{u} = (u_{\mathrm{W},n}, u_{\mathrm{I},n}, u_{\mathrm{O},n})$ can be described as follows:

$$u_{\mathrm{W},n} = \begin{cases} 2, & \text{if} \quad n \geq S_{\mathrm{WO}}, \ \chi^*_{\mathrm{WO}} \geq \chi^*_{\mathrm{WI}} \\ 1, & \text{if} \quad n < S_{\mathrm{WO}}, \ \chi^*_{\mathrm{WO}} \geq \chi^*_{\mathrm{WI}} \\ & \text{or} \quad n < S_{\mathrm{WI}}, \ \chi^*_{\mathrm{WO}} < \chi^*_{\mathrm{WI}}, \\ 0, & \text{if} \quad n \geq S_{\mathrm{WI}}, \ \chi^*_{\mathrm{WO}} < \chi^*_{\mathrm{WI}} \end{cases} \tag{5}$$

$$u_{\mathrm{I},n} = \begin{cases} 1, & \text{if} \quad n \leq S_{\mathrm{IW}}, \ \chi^*_{\mathrm{WO}} < \chi^*_{\mathrm{WI}} \\ 0, & \text{if} \quad n > S_{\mathrm{IW}}, \ \chi^*_{\mathrm{WO}} < \chi^*_{\mathrm{WI}} \end{cases}, \tag{6}$$

$$u_{\mathrm{O},n} = \begin{cases} 2, & \text{if} \quad n \leq S_{\mathrm{OW}}, \ \chi^*_{\mathrm{WO}} \geq \chi^*_{\mathrm{WI}} \\ 0, & \text{if} \quad n > S_{\mathrm{OW}}, \ \chi^*_{\mathrm{WO}} \geq \chi^*_{\mathrm{WI}} \end{cases}. \tag{7}$$

5.2. An analytical model for performance evaluation of a production-inventory system controlled with the two-threshold policy

We now present an analytical model to analyze the performance of a production/inventory system under the production and energy mode control based on the two-threshold policy introduced in Section 5.1.

We propose a model that can be employed to evaluate the performance of a system that uses either the Working-Off policy or the Working-Idle policy. In order to propose a common model for both cases, the system states are separated into three different groups: a system with a working state denoted with W, a non-working state (either Off or Idle depending on the model to be analyzed) denoted with N, and a warm-up state denoted with R. The warm-up state is added to the state space description in order to determine the energy cost during the warm-up period for the transition from the non-working state to the working state.

We consider correlated inter-arrival, processing, and independent warm-up times with general distributions that are modelled as Markov Arrival Processes (MAPs). Markov arrival processes (MAP) are arrival processes in which the inter-arrival times are interrelated and the process development is modeled using a continuous time Markov chain (Neuts, 1979). MAPs also contain information about which transitions will result in an arrival event with its associated probabilities.

The MAP model of the demand arrival process has a size of $\tilde{d}$ and described by the matrices $(\mathbf{D0}, \mathbf{D1})$ and the initial state probability vector $\mathbf{d}$. The MAP model of the production time process has size $\tilde{p}$ and described with the matrices $(\mathbf{P0}, \mathbf{P1})$ and the initial state probability vector $\mathbf{p}$.

The two-threshold policy described in Section 5 either uses the Idle mode or the Off mode together with the Working mode. The two thresholds used by the policy are referred to as S^u and S^d. For the Working-Idle policy, $S^u = S_{\mathrm{WI}}$ and $S^d = S_{\mathrm{IW}}$. Similarly, for the Working-Off policy, $S^u = S_{\mathrm{WO}}$ and $S^d = S_{\mathrm{OW}}$. When the Idle mode is used as the non-working state, the warm-up delay corresponds to τ_{IW} and when the Off mode is used as the non-working state, the warm-up delay corresponds to τ_{OW}. These warm-up delay times are i.i.d. random variables that have a phase-type (PH) distribution with $\tilde{w}$ phases described by the matrices $(\mathbf{W0}, \mathbf{W1})$ and the initial state probability vector $\mathbf{w}$. Table 2 gives the notation used in this section.

In the model described in Section 3, the time to switch from the Working mode to the Off mode, from the Working mode to the Idle mode, and from the Idle mode to the Working mode are instantaneous. The instantaneous transitions are approximated by using a high transition rate denoted with η. Accordingly, the transition rates from the Off or the Idle mode to the warm-up state, from the Working mode to the Idle or to the Off mode are set to η to approximate the instantaneous transition. In this case, $\tilde{w}$ is set to 1, $\mathbf{W0} = -\eta$ and $\mathbf{W1} = \eta$. Alternatively, a different continuous time Markov Chain (CTMC) model with instantaneous transitions can be used. Note that the method described in this section allows analysis of systems with instantaneous or non-zero transition times modelled as MAPs.

5.3. Continuous-time Markov chain model of the system with MAP inter-arrival, MAP production, and PH warm-up times

The system with MAP inter-arrival, MAP production, and PH warm-up times can be modeled as a continuous-time Markov chain (CTMC). By determining the steady-state distribution of the continuous-time Markov chain, the expected profit and other performance measures can be calculated for the two-threshold policies for the Working-Off and Working-Idle cases with the given different characteristics of the warm-up delay. The optimal threshold values can be determined to maximize the expected profit for each case accordingly. Comparing the optimal expected profits for both cases yields the preferred two-threshold policy that uses either the Off mode or the Idle mode as the non-working state.

5.3.1. Formation of the state transition matrix from the MAP representations of the inter-arrival, production and warm-up times

The transition rate of the continuous time Markov chain of the system is denoted by $\mathbf{Q}$. $\mathbf{Q}$ has a block tridiagonal structure including the transitions that lead to a decrease in the inventory position,

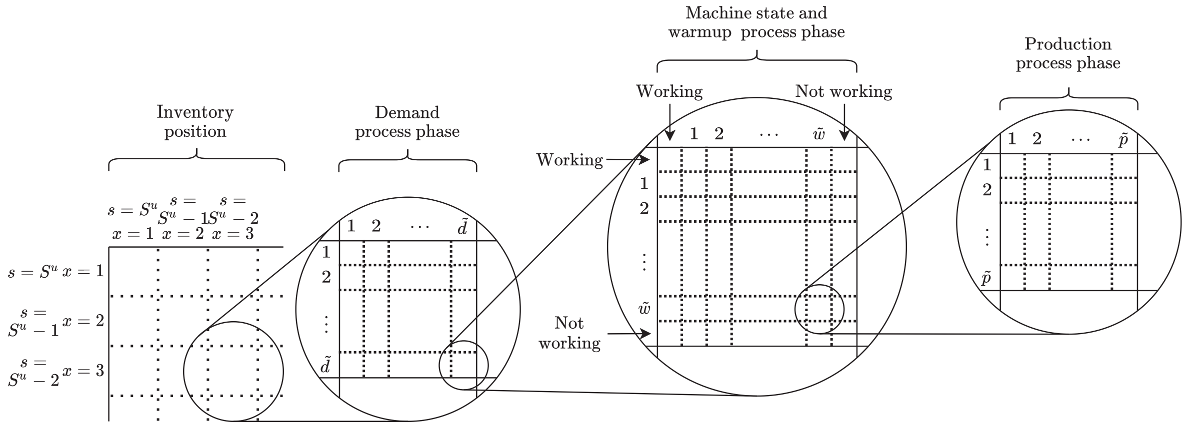

Fig. 4. The structure of the rate matrix of the CTMC model of a production-inventory system with MAP inter-arrival, MAP production and PH warm-up times controlled with the two-threshold policy.

no change in the inventory position, and an increase in the inventory position. For a block that corresponds to a given inventory level, the states are ordered as the phase of the demand process, the machine state (working, warm-up and its phase, not working), and the phase of the production process. Fig. 4 depicts the structure of the state space of the CTMC. We use a method to build $\mathbf{Q}$ based on the matrices that describe the MAP models of the inter-arrival, processing and warm-up times. The details of this method are given in Appendix C.

5.3.2. Determining the steady-state probabilities by using a matrix-geometric approach

Once the transition rate matrix $\mathbf{Q}$ is formed, the steady-state probability distribution for the system denoted with π is determined by solving the following set of equations:

$$\pi \mathbf{Q} = \mathbf{0} \tag{8}$$

$$\pi \mathbf{1} = 1 \tag{9}$$

In general, due to the size of $\mathbf{Q}$, solving the set of equations given in Eqs. (8) and (9) is computationally demanding. In fact, for the backlog case, the size of $\mathbf{Q}$ is infinite. Therefore, its steady-state probability distribution cannot be obtained without truncating the infinite state space that yields an approximate solution.

Instead of solving the set of equations directly and truncating the state space for the backlog case, we use the matrix geometric method that yields the exact solution in a computationally efficient way by exploiting the special structure of $\mathbf{Q}$. The matrix geometric method divides $\mathbf{Q}$ to a number of boundary and repeating blocks. The division of states into repeating and boundary blocks is done in a way that the transitions within and between consecutive repeating blocks are not dependent on the location of the blocks (Ost, 2013). The inventory level is used to place the states into blocks. Namely, all states with the same inventory level belong to the same block. Then, an iterative method is used for solving the balance equations that involve the repeating blocks and a set of linear equations is solved for solving the balance equations that involve the boundary blocks. The mathematical details of the matrix geometric method we employ to derive the relevant steady-state probabilities are given in Appendix D.

5.3.3. Determining the performance of the system

Once the steady-state probability distribution is determined with the given matrix geometric method, the average profit rate and other performance measures of the system can be evaluated.

The expected inventory and backlog levels, denoted as $\mathbb{E}[N^+]$ and $\mathbb{E}[N^-]$ are determined as

$$\mathbb{E}[N^+] = \sum_{s=0}^{S^u} s\,\mathbf{prob}[N = s], \tag{10}$$

$$\mathbb{E}[N^-] = \sum_{s=-\infty}^{0} s\,\mathbf{prob}[N = s]. \tag{11}$$

where $\mathbf{prob}[N = s]$ denotes the probability that the inventory position is s. The mathematical details of how we calculate the relevant probability are given in Appendix E. Note that $\mathbb{E}[N^-] = 0$ for the lost sales case.

The average profit rate is dependent on the steady-state probabilities in the working, non-working, and standby states in addition to the average inventory and backlog levels. Let π_{W}, π_{N}, and π_{R} denote the steady state probabilities for the system states Working (W), the Not-working state (N), and warm-up (R). The determination of π_{W}, π_{N}, and π_{R} from the steady-state probabilities are given in Appendix E.

For the policy that uses the Off mode as the non-working state, the steady-state probability that the system spends in the Off mode is $\pi_{\mathrm{O}} = \mathbf{prob}[P = \mathrm{O}] = \pi_{\mathrm{N}}$. Since the system operates only with the Working and the Off modes, the steady-state probability in the Idle mode is 0, i.e., $\pi_{\mathrm{I}} = \mathbf{prob}[P = \mathrm{I}] = 0$. Then, the expected total profit rate of the system under the Working-Off policy (χ_{WO}) is:

$$\chi_{\mathrm{WO}}(S^u, S^d) = r\mu\pi_{\mathrm{W}} - \big(c^+\mathbb{E}[N^+] + c^-\mathbb{E}[N^-] + c_{\mathrm{W}}\pi_{\mathrm{W}} + c_{\mathrm{O}}\pi_{\mathrm{O}} + c_{\mathrm{OW}}\pi_{\mathrm{R}}\big). \tag{12}$$

Similarly, for the policy that uses the Idle mode as the non-working state, the steady-state probability that the system spends in the Idle mode is $\pi_{\mathrm{I}} = \mathbf{prob}[P = \mathrm{I}] = \pi_{\mathrm{N}}$ and $\pi_{\mathrm{O}} = \mathbf{prob}[P = \mathrm{O}] = 0$. Furthermore, when the transition from the Idle mode to the Working mode is instantaneous, the steady-state probability of being in the warm-up state will be 0, i.e $\pi_{\mathrm{R}} = 0$. Accordingly, the expected total profit rate for the Working-Idle policy (χ_{WI}) is given as:

$$\chi_{\mathrm{WI}}(S^u, S^d) = r\mu\pi_{\mathrm{W}} - \big(c^+\mathbb{E}[N^+] + c^-\mathbb{E}[N^-] + c_{\mathrm{W}}\pi_{\mathrm{W}} + c_{\mathrm{I}}\pi_{\mathrm{I}}\big). \tag{13}$$

When backlog is allowed, since all demand arrivals are satisfied either from the inventory or after a delay, in Eq. (13), $\mu\pi_{\mathrm{W}}$ equals the demand arrival rate for the backlog case. However, for the lost sales case, a part of the demand that arrives when the inventory level is 0 is lost. In this case, $\mu\pi_{\mathrm{W}} < \lambda$.

The expected profit functions given in Eqs. (12) and (13) are used to obtain the optimal thresholds that maximize the expected profits. A search algorithm is used to determine the optimal thresholds that maximize the expected profit. Once the optimal profits for Working-Off and Working-Idle policies, χ^*_{WO} and χ^*_{WI} are determined, the approximate two-threshold policy operates following Eqs. (5)–(7).

6. Numerical experiments

We now investigate the performance of the production-inventory system under the proposed production and energy mode control policy through numerical experiments. We report the results only for the lost sales case for brevity. We present the experiments for the model with exponential inter-event times first and then for the model with general and correlated inter-event time processes. The LP solution is obtained by using GAMS and MATLAB using CPLEX. The LP solution is verified by solving the MDP by using a value iteration algorithm. The MAP analysis is implemented in MATLAB.

6.1. Performance of joint production and energy control for a system with exponential inter-arrival, production and warm-up times

We first compare the performance of the proposed joint production and energy control policy that uses two thresholds with the optimal policy obtained by using the LP solution of the MDP for a system with exponential inter-arrival, production, and warm-up times with the parameter set given in Table 3. The parameter set is constructed by considering the range of relative energy savings in working, off, and idle modes reported in different studies and rescaling the energy unit in a way to make the energy consumption in the off mode to be 0 and the energy consumption in the working mode to be 1 and normalizing the time unit in order to make the average production time to be 1.

Table 3
Parameter set for the numerical experiments.

Parameter	Set	Parameter	Set
μ	$\{1\}$	c^+	$\{0.1, 0.3, 0.5\ldots, 1.9\}$
λ	$\{0.1, 0.3, 0.5, 0.7, 0.9\}$	c_I	$\{0, 0.2, 0.4, 0.6, 0.8, 1\}$
δ_{OW}	$\{0.1, 0.3, 0.5, 0.7, 0.9\}$	c_W	$\{1\}$
δ_{IW}	$\{\infty\}$	c_O	$\{0\}$
r	$\{0, 1, 2, 3, 4, 5\}$	c_{OW}	$\{0, 1\}$

For all of the instances given in Table 3, the expected profit rate obtained by using the two-threshold policy proposed in Section 5.1 and the optimal profit rate obtained by using the LP solution given in Section 4.1 were the same. That is, for the systems with exponential inter-event times, the two-threshold policy yields the optimal solution for all the cases given in Table 3. Note that, the optimal policy can use more thresholds depending on the system parameters as described in Section 4.2. However, our numerical experiments show that the two-threshold policy yields the highest average profit rate for a wide range of system parameters.

The model considered in this study allows the production to be halted in the Idle mode for a period of time as well as switching off the production. We compare the optimal policy and the two-threshold policy with two alternative policies that are the Working-Off policy that can be considered as the pure energy control and the Working-Idle policy that can be considered as the pure production control. Since the two-threshold policy is constructed by comparing the expected profits for these two policies, its performance is always better than these pure policies. We next analyze the effect of the system parameters on the performance differences among these policies.

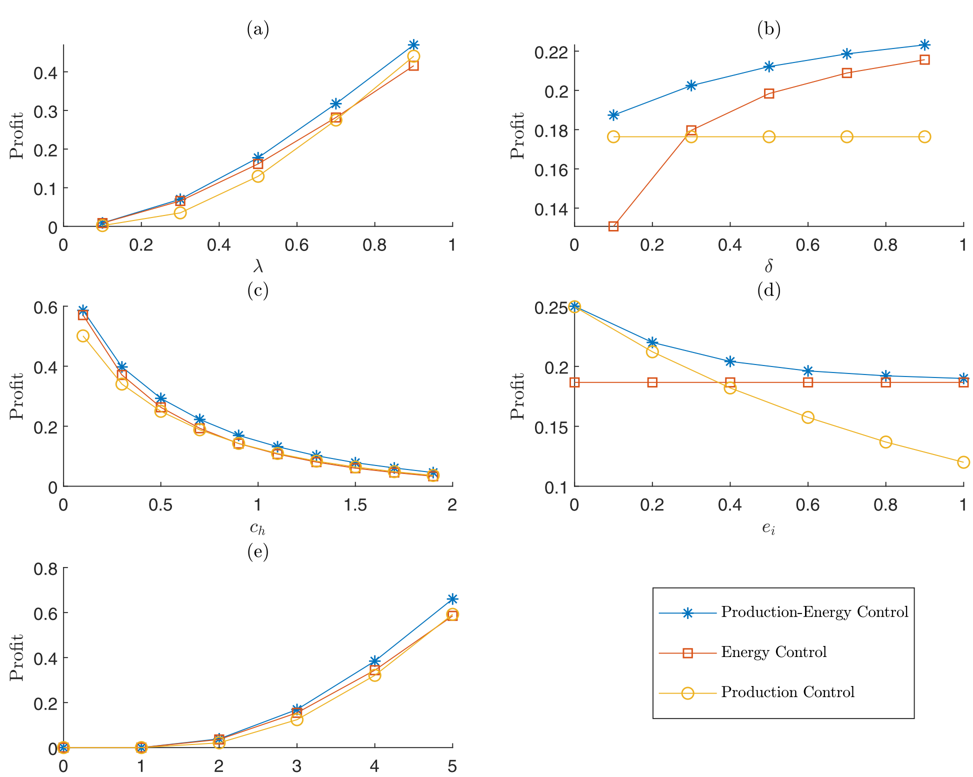

Fig. 5. The effect of system utilization, warm-up times, inventory cost and revenue on the expected profits obtained by using the joint production-energy control, pure production control, and pure energy control for the case with exponential inter-event times ($c_{OW} = 0$, all the other parameters are given in Table 3).

Fig. 5 shows the effect of the system parameters on various performance measures. The average of the performance measures obtained for 9000 different cases are plotted. Fig. 5(a) shows the effect of utilization on the profits obtained by using the joint Production-Energy Control, Pure Production Control, and Pure Energy Control over the parameter set specified in Table 3. Fig. 5(a) depicts that when the demand arrival rate and therefore the utilization is low, the joint production and energy control and pure energy control have a significantly better performance compared to the pure production control. This is due to the fact that when arrivals are rare, it is not desirable to keep the machine idle for a long time waiting for a demand arrival. Therefore, the pure energy control that operates with an off mode gives a higher profit.

Fig. 5 (b) depicts the effect of the average warm-up period on the performance of these policies. The pure energy control is sensitive to the duration of warm-up times as it uses many warm-up times whereas the pure production control only uses the Idle mode and therefore it is not affected by the changes in the warm-up period. Fig. 5(c) shows that all these methods have a lower optimal profit when the inventory cost rate increases due to increasing inventory holding cost as expected. Likewise, Fig. 5(d) shows that the energy cost of being in the Idle mode does not affect the pure energy control as it does not use this state. Furthermore, when the energy cost in the Idle mode is very small, the profit obtained by using the pure production control is close to the profit obtained by using the joint production and energy control. In Fig. 5(e), the sales price is varied for a constant production cost to analyze the effect of the revenue per unit produced on the performance of the policies. If the revenue is low per unit produced, it will not be desirable to always have the machine on and incur the comparatively larger energy costs.

6.2. Performance of joint production and energy control for a system with MAP inter-arrival, MAP production, PH warm-up times

After analyzing the system with exponential inter-event times, we now investigate the effects of system parameters on the performance of production and energy control policies for a system with MAP inter-arrival, MAP Production, and PH warm-up Times. For this case, the effects of the system utilization, the inventory holding cost rate, the average warm-up period, the idle energy cost, and the revenue rate on the profit obtained by using the joint production and energy mode control policy are similar to those given in Section 6.1 and not reported again.

In this set of experiments, we focus on the effects of variability and correlation in demand inter-arrival, production, and warm-up times on the performance of the two-threshold policy. For assessing the effect of the variability of the inter-event times on the performance of the policy, we vary the coefficient of variation of the demand inter-arrival, processing, and warm-up times of the cases generated based on Table 3 with an additional energy cost of $c_{OW} = 1$ for the warm-up period. The figures that describe these experiments are given in Appendix G. The results we obtained from these experiments show that as the coefficient of variation of the inter-arrival and processing times decreases, i.e the inter-event times become less variable, the profit obtained by using the energy and production control policies linearly increases. However, the variability of the warm-up times does not affect the performance of the control policies significantly (see Appendix G). This is partly because there is an option for using the Idle and Working modes (production control) and avoiding the effect of the variability of the warm-up times and in cases where this variability is very costly, production control will be chosen.

We also examine the effects of first lag autocorrelation of the inter-arrival times (L_1^a) and the processing times (L_1^s) on the performance of the method. For this examination, we use 4 state MAPs that are built using the matrices that are given in Appendix F. The results we get from the examination reveal that as the first-lag autocorrelation of the inter-arrival time increases, the profit obtained by using the energy and production control policies slightly decreases. Additionally, it is observed that the first-lag autocorrelation of the processing time does not affect the profit obtained by the energy and production control policies significantly.

7. Conclusions

In this study, we investigate joint production and energy control of a make-to-stock production system that can operate in different energy modes such as Working, Idle, and Off modes. In each energy mode, the system consumes a different amount of energy, so their relevant energy costs are considered to be different. There can be a warm-up delay to switch from one mode to another. While operating a production resource in a low-energy consuming mode saves energy, the delays to switch back to the Working mode may affect the production and service level targets. Therefore, production and energy mode switching decisions should be given considering the overall performance of the system including energy costs and production and service level targets. We formulate the problem of maximizing the long-term average profit rate by deciding on switching different modes dynamically based on the state of the system as a stochastic control problem.

We determine the optimal control policy of this problem when the demand inter-arrival, processing, and warm-up times are i.i.d. exponential random variables by solving an LP formulation of the associated Markov Decision Process. Our numerical results for the exponential case show that, for the range of parameters observed in practice, the optimal energy and production control policy uses either only the Working-Off or the Working-Idle modes with two thresholds or the Working-Off-Idle modes with three thresholds. However, our numerical results show that the two-threshold policy yields the optimal profit that can also be obtained with a three-threshold policy for the range of system parameters considered.

Furthermore, for a system with general and correlated inter-event times, we propose using the two-threshold policy as an approximate joint energy and production control policy. Since this policy only requires the real-time energy mode and inventory position information, it can be implemented easily.

As opposed to other studies that use simulation to analyze the performance of a suggested policy, we analyze a system with inter-arrival, production and warm-up times modelled as Markov Arrival Processes and MAP warm-up times analytically. For this analysis, we construct the state transition matrix of the underlying continuous time Markov Chain directly from the matrices that determine the MAP representations of these inter-event times. Then, the steady-state probabilities are determined efficiently by using a Matrix Geometric approach. The average profit rate of the system under the suggested policy is then determined from the steady-state probabilities. The two thresholds that maximize the average profit rate are determined in a computationally efficient way by using a search algorithm.

We also empirically examine how system characteristics affect this integrated energy and production control policy and its benefit compared to the pure energy and production control policies. When the utilization is low, energy control that does not use the idle mode is preferable since it does not force the machine to operate in the idle mode for a long time waiting for a demand arrival. Similarly, as the difference between the off and idle costs decreases, production control becomes more attractive since keeping the machine in the idle mode becomes less costly compared to keeping it in the off mode. The difference of the upper and lower thresholds affects the number of items that are produced between two machine shutdowns. A larger average warm-up time makes producing in large quantities between shutdowns more advantageous. However, this must be balanced with the effect it has on the inventory and backlog costs. When the inventory cost is high, chasing demand yields better profits as opposed to going through working and non-working periods where the machine is turned on and off to save energy. Note that all these observations hold for cases both with exponential and MAP inter-event times.

Considering only the case with MAP inter-event times, we also study how variability in demand inter-arrival, production, and warm-up times affects the benefits of our approximate policy compared to the pure energy and production control policies. The experiments reveal that with a decrease in the coefficient of variation of the inter-arrival and processing times, i.e., when the inter-event times become less variable, the profit obtained by using the energy and production control policies linearly increases. Furthermore, it is observed that the variability of the warm-up times has a very limited effect on the performance of the control policies. In addition, the first lag autocorrelation of the arrival and service processes on the performance of our approximate policy are numerically analyzed; eventually, the analysis shows that the relevant system parameters have no significant effect.

This study can be extended in several ways. Showing the structure of the optimal policy analytically is challenging due to the form of the objective function that includes the time spent in different states. Proving the optimality of the threshold-type policy for the joint energy and production control problem for a production and inventory system with exponential inter-event times is an open research problem.

Another research direction is developing a data-driven method to implement the two-threshold policy in the absence of detailed information about the random processes that govern the inter-event times. The proposed control policy uses two thresholds that are set based on the system parameters. In general, the thresholds used in the two-threshold policy can be set differently depending on the clusters of signals referred to as the markings (Khayyati & Tan, 2020). Using more parameters in the control policy based on the markings and controlling the system dynamically according to the data collected from the system allows this policy to yield better results. However, the benefit of using more complex policies must be balanced with the difficulty of optimizing their parameters.

In addition, in this study we considered the joint production and energy control of a production system in isolation. Controlling more than one station simultaneously requires extending this model to consider arrivals from an upstream station. In this case, the optimal policy that depends on both the inventory position and also the number of parts in its input buffer together with the machine state needs to be derived. Once the optimal policy for the system with upstream arrivals is available, the optimal policy for each station in a multi-station system can be analyzed with the flow coming from its upstream station and the overall desired production rate of the system shared as the demand arrival rate. These extensions are left for future research.

As a summary, we propose an effective joint energy mode and production control policy. The proposed policy balances the energy cost benefits that can be obtained from turning the machine off or idle during certain times with the costs associated with the differences between the cumulative production and demand. Using this joint energy mode and control policy increases the profit that is the difference between the revenue obtained from sales and the costs associated with energy consumption, inventory holding and backlogs. Furthermore, optimizing the energy consumption provides environmental benefits in addition to the cost benefits that are balanced with the need to meet customer demand on a timely basis.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.12.021.

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