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Wideband Magnetic Component Modeling in Power Electronics via Finite Element-Derived Equivalent Circuits

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Abstract

Magnetic components are essential in power electronic systems, where their behavior under high-frequency excitation plays a critical role in overall system performance. However, accurately modeling these components remains a challenge due to complex geometries, interactions between electromagnetic phenomena and frequency-dependent loss mechanisms. This thesis addresses the research problem of developing a general and robust methodology to derive accurate equivalent circuit models of magnetic components from physical principles, enabling fast and reliable simulations in frequency and time domains. The primary objective is to generalize the modeling of magnetic components through localized equivalent circuits across different levels of discretization, from the turn level to the winding level. The approach allows to identify circuit parameters representing both magnetodynamic and electrostatic behaviors by means of numerical resolutions. The method also makes an intensive use of ladder-type circuits in order to equip the model with frequency dependent behavior and still comply with the strong constraint of localized constant circuits. Data used in the study include detailed material characterizations (permeability, permittivity, losses) of ferrite cores and dielectric materials, as well as impedance measurements of various inductor and transformer prototypes studied under diverse configurations. The finite element method (FEM) is employed to extract local field quantities, which are then transformed into global circuit parameters: resistances, inductances, capacitances, and conductances. The method is first validated on inductors built with simple geometries (essentially exploiting rotational symmetry) and made of the materials characterized beforehand. It is then applied to multi-winding components with more complex geometries and design, such as industrial transformers. Key findings demonstrate that the proposed method and equivalent circuits can accurately predict the first few resonances. The tunable levels of discretization (by turn, by section, by winding) offer great flexibility to the designer, providing a trade-off between computational efficiency and model accuracy.

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Introduction

Power electronics is a well-established discipline that plays a critical role in a wide range of applications. Although the field is not new, its relevance continues to grow due to the increasing demand for high-performance systems in both domestic and industrial sectors [15, 64, 61]. The ongoing electrification of transportation, industrial microgrids, and decentralized energy production are just a few of the drivers pushing for more efficient, reliable, and compact power electronic systems [62]. Over recent decades, the widespread adoption of power electronics has significantly enhanced the potential for high-efficiency electrical energy conversion. At the heart of many power converters are magnetic components—transformers and inductors—which perform essential functions such as voltage conversion, filtering, galvanic isolation, and energy storage. While these components have been studied extensively [71, 97, 95, 78, 3, 45, 40], they have also had to evolve to meet the demands of modern power electronics.

One of the key trends in this evolution has been the steady increase in switching frequencies, aimed at reducing the size and improving the performance of power supplies. This trend has been accelerated by the advent of wide-band gap semiconductors such as SiC and GaN, which enable higher-frequency operation with lower switching losses than traditional silicon devices [4, 82, 76]. As a result, magnetic components are now subject to **more complex high-frequency phenomena**, including faster voltage transitions, stronger parasitic capacitive effects [6, 44], and increased frequency-dependent losses—such as hysteresis, eddy current [90], and dielectric losses [46, 38]. These effects lead to higher overall losses, as well as increased conducted and radiated electromagnetic interferences [49].

Although such challenges have long been recognized by magnetic component designers [33], the drastic rise in switching frequencies and power densities has amplified these issues [109]. **Accurately modeling and predicting these behaviors is essential to ensure the reliable and efficient operation of modern switch-mode power supplies.**

This research is driven not only by scientific curiosity but also by the needs of industrial applications, which demand practical, fast, and scalable modeling techniques. Electromagnetic finite element (FE) simulations, while accurate, are computationally expensive [80, 99] and often impractical for iterative design [87] or system-level integration. Moreover, coupling such models with the electrical environment (voltages, currents, control strategies) of the converter introduces significant complexity [91, 58, 26, 98]. On the other hand, purely circuit-based models—typically constructed from lumped parameters identified analytically or experimentally [92, 2]—lack the ability to account for complex geometries and frequency-dependent effects with sufficient fidelity. Analytical models are limited to idealized configurations, while experimental identification is time-consuming and often influenced by numerous external factors.

Analytical and **semi-analytical modeling** of ohmic and eddy current losses in winding structures has traditionally been addressed using closed-form expressions for 2D geometries and rectangular conductors to represent conduction and proximity effects. These formula-

tions are inherently compatible with equivalent circuit modeling strategies and provide a practical complement or alternative to full numerical extraction [95]. Such methods enable **rapid computations** and are frequently implemented in commercial simulation tools (e.g., Magnetics Designer). However, their accuracy remains limited, and not all geometrical configurations can be adequately represented using these simplified formulations.

Building upon these approaches, significant efforts have been made to bridge the gap between **experimental characterization** and circuit-oriented modeling of magnetic components. Techniques have been proposed to derive equivalent circuits from measured time- and frequency-domain responses, ensuring consistency with Maxwell-based interpretations and electromagnetic behavior [66, 78]. These methods allow the construction of **reduced-order models** that are physically meaningful and experimentally validated, particularly for wideband excitation and fast-switching waveforms encountered in power converters. Nevertheless, experimentally identified circuit models present several limitations: they may not be uniquely identifiable, their validity is typically restricted to the measurement frequency range and operating conditions, and nonlinear effects such as saturation or temperature dependence are often neglected. Moreover, the identification process can be sensitive to measurement noise, potentially leading to models that are accurate only within specific contexts rather than broadly predictive.

In parallel, **dielectric and capacitive effects**—particularly relevant under high-frequency or high dV/dt conditions—have been rigorously studied through electrostatic and full-wave simulations. Methodologies have been proposed to extract equivalent capacitance matrices from finite element models of multi-winding transformers, with experimental validation of the resulting networks [43, 44, 45]. This work is particularly important for **accurate modeling of insulation systems** and the construction of **detailed capacitive sub-networks**. However, these approaches often involve significant computational costs, and the resulting multi-turn models are typically synthesized into equivalent winding representations. Consequently, the spatial distribution of parasitic capacitances is partially lost, limiting the ability of such models to accurately predict higher-order resonant frequencies and localized field effects within the winding structure.

Physically-informed synthesis procedures have further advanced the modeling of magnetic components by mapping distributed electromagnetic phenomena onto lumped circuit equivalents. Semi-analytical methods have been used to derive inductive and capacitive behaviors in **medium-frequency transformers**, including turn-to-turn inductances, coupling factors, and capacitances based on geometrical and physical parameters, without resorting to full finite element simulation [40, 39]. These approaches enable **fast and parameterized model generation**, particularly for high-voltage and railway applications. Nonetheless, parasitic capacitive effects—such as inter-turn, inter-layer, and winding-to-core capacitances—are sometimes approximated or neglected, limiting the validity of these models over wide frequency ranges. Experimental and simulation-based methods have also been developed to characterize inter-winding and winding-to-core capacitances under fast voltage transients. By extracting impedance or admittance matrices from frequency-domain finite element simulations and fitting them with passive, rational multiport models [51], these techniques preserve the physical accuracy of the original field solution while enforcing passivity constraints. The resulting equivalent networks are suitable for direct integration into time- and frequency-domain simulations, allowing magnetic components to be simulated within their actual operating environment.

Finally, recent developments have focused on the **coupling of eddy currents** and **non-linear magnetic behavior**, through homogenized winding models and time-domain eddy

current formulations [96]. Providing methods to homogenize multi-turn windings, capturing the frequency-dependent reluctance and impedance effects. Similarly, another topic involves synthesizing **RL Cauer ladder** networks to represent the eddy current effects in windings in the time domain [84]. These methods combine accurate physical representation of hysteresis and dynamic losses with circuit-equivalent extraction. Ladder-type circuit networks, building on the foundational work of [67], are particularly suited to representing frequency-dependent parameters and capturing rapid transients induced by fast-switching semiconductor devices such as GaN and SiC [76]. By enabling the simulation of both frequency- and time-domain behavior within a compact circuit framework, these approaches provide a unified platform for accurate and efficient analysis of modern magnetic components. Nevertheless, despite their enhanced fidelity, these formulations remain computationally demanding and require detailed material characterization, particularly for anisotropic or composite media.

Building on existing research, we adopt a hybrid approach that extracts equivalent circuit models directly from finite element simulations. This strategy leverages the detailed geometrical and material information available in full-field simulations while providing a **compact circuit-level representation** suitable for system-level analysis. Unlike previous works that employ vector-fitting to identify rational multiport models and extract a limited set of node voltages [95, 51], our methodology focuses on generating full equivalent circuits. Frequency-dependent parameters are represented using **ladder-type networks**, building on the foundational concepts introduced by [67] to systematically capture distributed electromagnetic behavior in a compact form. In addition, following the insights of [76], ladder-type circuits are particularly effective for modeling rapid transients associated with fast-switching semiconductor devices such as GaN and SiC. This enables our models to accurately reproduce both frequency-dependent phenomena and time-domain switching behavior under realistic operating conditions. Our approach further integrates **numerical and experimentally validated techniques** inspired by prior research on impedance extraction, loss modeling, and physically informed synthesis. By incorporating **inductive, dielectric, and coupling effects** into a unified framework and applying the resulting parameters via ladder networks, we generate **broadband equivalent circuit models** capable of representing magnetic components across wide frequency ranges and under dynamic excitation. In this way, the proposed methodology bridges the limitations identified in previous analytical, semi-analytical, and experimentally derived models, offering a comprehensive solution for modern power electronics and high-frequency magnetic component simulation.

Objective

The objective of this thesis is to develop and validate equivalent circuit models for magnetic components that remain accurate over a wide frequency range and under realistic operating conditions of modern power electronic converters—with fast switching, non-sinusoidal signals, and high power density. We thus aim to capture not only the fundamental inductive behavior of transformers and inductors, but also their capacitive interactions, resonant phenomena, and both magnetic and dielectric losses. The models are derived from finite element simulations and translated into equivalent circuits, enabling fast and accurate simulation within standard circuit tools such as Spice-based simulations. The end goal is a practical and computationally efficient modeling toolbox for design engineers, allowing for reliable integration of high-frequency magnetic effects into full converter simulations in both time and frequency domains.

Outline

Chapter 1 introduces the main elements of magnetic components (cores, windings, and insulation) and reviews key design strategies to optimize their performance and compactness. The chapter explains material choice and concludes by highlighting the critical role of magnetic component design in power electronic applications through three converter topologies.

Chapter 2 presents the state of the art in physical modeling of electromagnetic phenomena. By separating the electromagnetic problem into magnetodynamic and electrostatic sub-problems, the chapter details the governing equations and associated loss mechanisms. It explains how global quantities such as voltages, currents, charges, and potentials can be used to derive circuit constants: resistances, inductances, conductances and capacitances.

Chapter 3 introduces a generalized discretization method for winding structures, defining sections at various levels, from individual turns to full windings. The chapter demonstrates how inductive and resistive behaviors can be easily deduced from impedance matrices, while electrostatic modeling requires more complex developments, which constitutes a first main contribution. A second major contribution is the construction of equivalent ladder-type circuits that model frequency-dependent effects, including skin and proximity losses and ferromagnetic losses.

Chapter 4 validates the theoretical models through material characterization and experimental testing. Various inductors, including single- and multi-layer designs, are studied. Measurement results confirm the models accuracy in predicting inductance, capacitance, and loss behaviors. The chapter also illustrates the trade-off between model detail and simulation time, emphasizing the role of the designer in choosing appropriate discretization.

Chapter 5 applies the methodology to two industrial transformer case studies. The first highlights capacitive coupling effects in a multi-winding step-up transformer, while the second, from an LLC resonant converter, confirms the compatibility of the model with both frequency and time domain simulations. These examples showcase the practical relevance of the approach and its adaptability to real-world problems.

Related Publications and Presentations

Journal Article

"Multiple Resonance Prediction through Lumped-Parameter Modeling of Transformers in High Frequency Applications" N. Davister; F. Henrotte; F. Frébel; C. Geuzaine. *IEEE Transactions on Magnetics*, 2022 (Volume: 59, Issue: 6) [20]

International Conferences

- International Conference Compumag 2019. "Comparison of Finite Element Formulations for the Modeling of High-Frequency Effects in Electromagnetic Coils" [18]
- International Conference Compumag 2021. "Accurate Resonances Prediction through Lumped-Parameter Modeling of Transformers for High Frequency Applications" [17]
- 12th International Symposium on Electric and Magnetic Fields (EMF 2021). "Electromagnetic Modeling of Magnetic Components for Increased Frequency Applications" [16]

- International Conference Compumag 2022. "Inductor modeling in circuit-type softwares (SPICE) including Energy-based hysteresis model" F. Sixdenier; R. Scorretti; N. Davister; C. Geuzaine; F. Henrotte. *Proceedings of the 23rd International Conference on the Computation of Electromagnetic Fields*, Jan 2022, Cancun, Mexico, 6 pages. [100]
- 13th International Symposium on Electric and Magnetic Fields (EMF2023). "Multi-Level Circuit Models of Transformers and Inductors for High Frequency Power Converters" [19]

Magnetic Components: Description and Operation

Magnetic components, such as inductors and transformers, are fundamental building blocks in a wide range of power electronic systems. Their construction, material properties, and geometric design significantly influence the performance, efficiency, and reliability of the final application. This chapter provides a detailed introduction to the nature and role of these components, laying the groundwork for the more advanced modeling and experimental validation discussed in the later chapters of this thesis.

The first section introduces the key terminology, material choices and technological constraints that guide the design of magnetic components. It also highlights the role of winding techniques—including the use of Litz wire and multilayer configurations—and their impact on parasitic effects such as proximity and skin effects.

The second section discusses representative application contexts where these components are essential, including Switch Mode Power Supplies (SMPS), resonant converters, and high-voltage generators. In each case, the magnetic components impose specific constraints related to switching frequency, waveform shape, and loss mechanisms.

Thus, this chapter not only introduces the physical and technological foundations of magnetic components but also establishes the motivation for the modeling strategies developed throughout the thesis, culminating in their application to real industrial systems where performance, precision, and reliability are of critical importance.

1.1 General Transformer Description and Terminology

In term of use and design, power inductors and transformers are often perceived as different items; and they are, since inductances present storing and filtering utilities¹ whereas transformers rather allow to raise or lower a voltage, or even to distribute its input power to several output circuits. Nevertheless, from an implementation and modeling point of view, such magnetic components are very similar: made of several electric conductors wound around a coil former and of a magnetic core in its center, possibly containing an air gap. In this manuscript, the general approach that is developed allows us to study inductors and transformers regardless of their specific use and regroup them under the term of **transformer**, assuming that an inductor can be defined as a single-winding transformer.

In this section, we deal with the different parts which constitute a transformer, their formal description, terminology and how they are referred to in the transformer model.

¹Amongst others: attenuating and filtering of high frequency noise, store and manage energy in power converters or even as tuned LC tank circuits.

1. MAGNETIC COMPONENTS: DESCRIPTION AND OPERATION

The reader will notice that different **levels of discretization**, based on geometrical considerations, are defined hereafter and used as alternative modeling approaches.

The structure of a transformer includes the windings – made of electrically conductive materials, the insulators – used to ensure galvanic isolation between coils and prevent electric discharges from conductors to others, and the magnetic core – which concentrates the induction generated by the currents flowing in the windings. The proper design of a transformer consists in selecting and arranging all these elements correctly in order to meet the specifications such as size, weight, efficiency, precise values of inductances, voltage ratio, etc.

1.1.1 Coils

Coils consist of arrangements of conductors used to produce a strong magnetic field in a given direction, by means of the electric current that flows through them (following Ampere law). They are well known parts, manufactured for all types of electric devices and whose geometries vary widely depending on the equipment they are found in. For transformers, the coils often take the form of concentric spirals, helices or flat layouts, as depicted in Figures 1.1 to 1.3. The choice of a specific layout generally depends on space requirements, voltage withstand, power, weight, heat transfer, etc. and power converter technologies are no exceptions. For instance, single sheet conductors are easily stacked which permit to reach higher filling factors (theoretically 100 %) compared to round conductors, hence delivering higher power densities and better heat transfer [94]. However, foils may also lead to bigger parasitic capacitances than round wires [119].

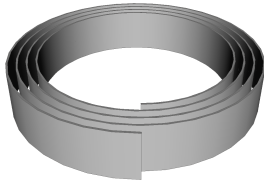


Figure 1.1: Single sheet 4-layer radial coil (4 turns).

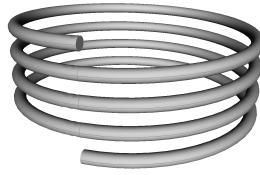


Figure 1.2: Single-layer axial coil (4 turns).

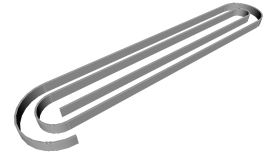


Figure 1.3: Longitudinal coil (2 turns).

Copper is the most common material used to create coils due to its high electric conductivity, availability and stability over time. **Aluminum** can also be used for this purpose, most often when weight constraints are involved (*e.g.* in aircrafts and overhead transmission lines) which will not be presumed in this work. Aluminum is a poorer conductor than copper, with a conductivity of about 65 % compared to copper's. However, it becomes an interesting candidate for mid-range frequencies, when skin and proximity effects come into play. Indeed, the lower conductivity of aluminum confers it a greater skin depth than copper which makes the skin and proximity effects losses less striking. The performance margin between aluminum and copper is therefore less pronounced than in direct current applications. Therefore, aluminum and copper are both useful in building coils for power electronics applications with increased switching frequencies. Nevertheless, because copper is processed and manufactured more easily than aluminum (regarding deformation, soldering, stability in time, ductility, etc.), there are many more applications that employ copper for coils, which is reflected in the test cases that we will study in what follows in this manuscript.

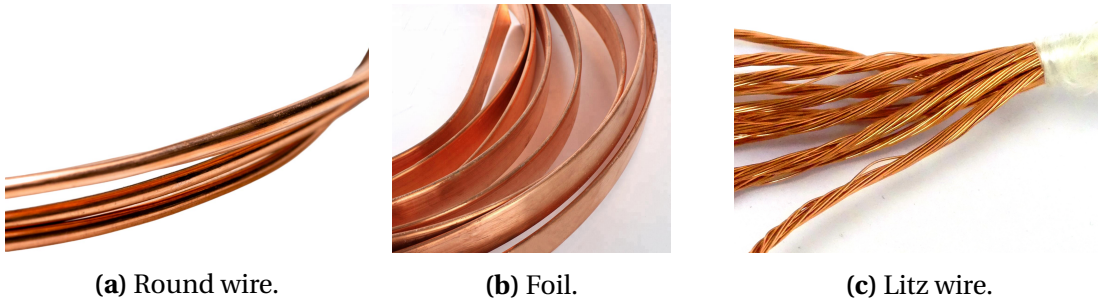
More than the material itself, it is the conductors shape that defines coils performance. The different formatting types include round wire, foil, Litz wire and flat enameled bar.

Table 1.1: Standardized electric conductivity of annealed copper [103] and aluminum [56].

Material	Electric conductivity σ at 20 °C [S/m]	Temperature coefficient [K^{-1}]
Copper	$5.801 \cdot 10^7$	$-4.04 \cdot 10^{-3}$
Aluminum	$3.767 \cdot 10^7$	$-3.90 \cdot 10^{-3}$

Round wire is a usual option for transformers, especially when the power rating and the filling factor can remain low; hence, it is often suitable for small applications. The two main advantages of the round wire are the ease it brings for manufacturing coils (counting the winding process, the electrical connections, soldering, etc.) and the **cost-effectiveness** due to its wide availability. Unfortunately, round wire also has limitations once the power density or the frequency increase. Eddy currents lead to more Joule losses for increased frequencies through skin and proximity effects. The impact of the former can be mitigated by reducing the conductors radius although very small radii cannot be obtained due to the increase of the DC resistance and mechanical constraints; and the effect of the latter cannot be resolved with simple winding methods. The fundamental reason is that if you reduce the radius to guard against skin and proximity effects, you increase the DC resistance which then limits the current carrying capacity and therefore the power.

When conductors must be closely stacked, it is preferable to use flat sections such as **foils**. They allow a theoretical filling factor of 100 % while the round section is limited to 90.7 %. Moreover, the small thickness of foils reduces the eddy current losses compared to round wires of equivalent cross-section. Copper foils are generally found in 0.1 to 1 mm thickness and compose high power ratings transformers [73]. If only skin effect is considered (*i.e.* for a single sheet), a 0.1 mm foil can carry electric current with almost no impact on Joule losses up to 1 MHz – as the sheet thickness remains larger than twice the skin depth (the exact ratio being $\pi/2$ for a rectangular section). And yet, the biggest problem comes from **proximity effects** that appear when multiple sheets, geometrically close to each other, carry electric current. Such an increase of losses in alternating regime goes hand in hand with, and is due to an increase in the number of layers. This had already been noticed very early by Dowell and accurately predicted for the transformer winding [27]. Other recent analytical models, based on a 2D approach, rather than a 1D one like Dowell, allow to refine the results for particular configurations such as air gaps consideration [68] or to compute the optimum thickness [65]. According to the optimal thickness calculation, it is necessary to divide the thickness by four when ten layers are gathered, in order to control the proximity losses. This amounts to 25 μm for 1 MHz currents which is a too small sheet thickness in order to implement the coils. In some cases, the foil technical limits are reached and another conductor design is used.

**Figure 1.4:** Typical conductor geometries used in high frequency transformer coils.

1. MAGNETIC COMPONENTS: DESCRIPTION AND OPERATION

If losses due to proximity effects play an important role (as in the high frequency multi-turns windings we consider), **Litz wire** is an appropriate choice to mitigate them. Litz wire comes in the form of multiple tiny strands isolated from each other and woven together so that, on average, they occupy all locations along the length of a wire. This arrangement's purpose is to distribute the effects of induced currents evenly over all strands, resulting in a uniform current density for the ideal case. Remark that even twisting imperfections in arrangements already lead to significantly reduced losses [47]. Litz wire is a particularly suitable solution for higher frequencies as the strands braiding drastically reduces losses from proximity effects.

The conductors within a Litz wire are generally organized in the following hierarchy: once grouped together, the strands form a bundle; several bundles can then be grouped in the same way to form a larger bundle; and these operations are repeated as many times as necessary in order to obtain the desired total number. Due to the large quantity of strands, bundles, sub-bundles and sub-sub-bundles, Litz wire is identified by the following notation $(m \times n \times \dots \times p)_t$ where m is the number of bundles containing n sub-bundles, etc., and t corresponds to the total number of strands in the wire. Such complex patterns are finally produced according to round and rectangular profiles ; the latter being often preferred for their higher filling factor. Figure 1.5 illustrates such complicate geometries for round profile Litz wires.

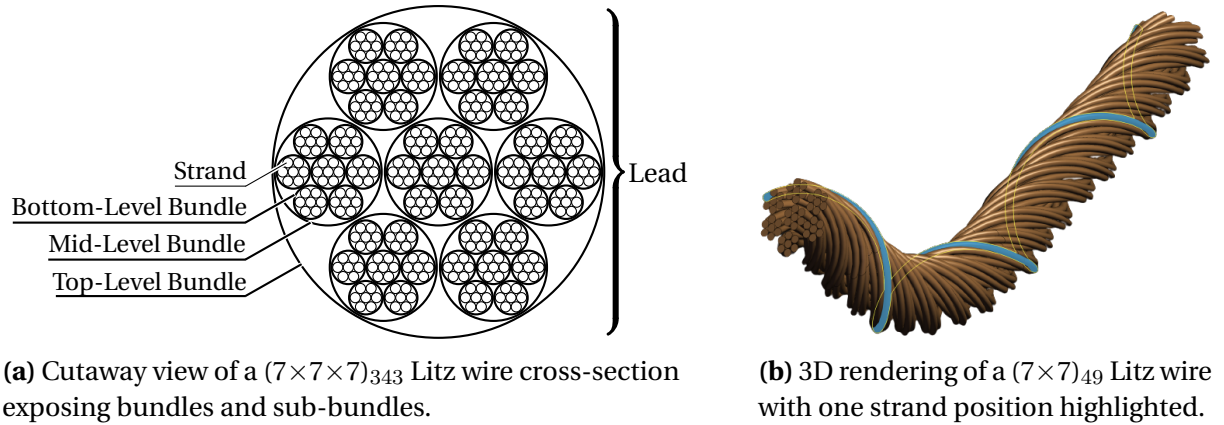


Figure 1.5: Round profiles Litz wire 2D and 3D [89] representations.

The strand diameter squared and the number of strands per wire roughly depend on the inverse of each other. The **number of strands** per wire is therefore an important **design parameter** since a diameter reduction allows, in first approximation, to reach higher limit frequencies than solid-round wire, for a given level of losses and at medium-frequencies. Thereafter and for high frequencies, the observation is reversed and multi-strands wires present higher level of losses [112]. As the diameter of the strands is reduced, the insulation takes up more and more of the bundle cross-section and deteriorates the efficiency of Litz wire. Moreover, multiplying the number of strands also leads to higher number of layers, which increases the proximity effect undergone by each strand. So, there exist an optimal number of strands that minimizes the losses, which can be determined analytically [106] or through numerical methods such as 2D Finite Elements, which is the approach followed in this work. Let us conclude by noting that the use of Litz wire is widespread for the realization of medium-frequency transformers, this is also illustrated in our three application test cases.

1.1.1.1 Conductor Nomenclature

We have just seen that the conductors used in the windings can have more or less complex geometries with different types of section and can present a particular organization in bundles and sub-bundles. Let us also note that in practical implementations, it happens that several conductors are wound simultaneously along the same path. Likewise, to make a winding, the assembly of several coils in series or in parallel is sometimes more efficient in terms of design. All these aspects lead to a very large variety of geometries to be distinguished and modeled. At this point, it is helpful to explain the nomenclature used to uniquely describe each conductor found in coils. The description follows a **bottom up** level of organization.

The smallest entity to be considered is the **strand**. It corresponds to a wire of extra small diameter which is insulated in enamel. In Litz wire, several strands are associated in **bundles** twisted in particular patterns ensuring that each strand occupies all the different positions on average over the wire length. At this stage, several levels of cable bundles can coexist. Nevertheless, they are still referred to as bundles and one can identify them according to their twisting-level as shown in Figure 1.5: strand-level, mid-level and top-level bundle. At a higher hierarchical level, we find the **leads** which correspond to the cables wrapped together. Such a procedure is often encountered in practice when we want to reduce the current density of a coil, then several wires are placed in parallel and wound together along the same path. When multi-leads wires are wound over several **turns** they make a **coil**. Such a coil possesses two ports through which several coils can be connected in series or in parallel. The ports of a coil are not necessarily accessible from the exterior of the transformer and are referred to as intermediate ports. The **winding** corresponds to the uppermost level, which consists of several coils connected in series, in parallel or a combination of both. To each winding correspond two ports: the positive and negative terminals, through which the winding is connected to the other elements of the circuit. Thereby **terminals** can be accessed to apply global electrical quantities such as voltages and currents. The succinct representation of this hierarchical structure is provided in Figure 1.6.

Remark that two different strands share only two physical connections at their ends. Indeed, it is at the two ports of the coil in which they are found that they will be soldered. This observation remains valid for all the levels below the coil, which are then put in parallel.

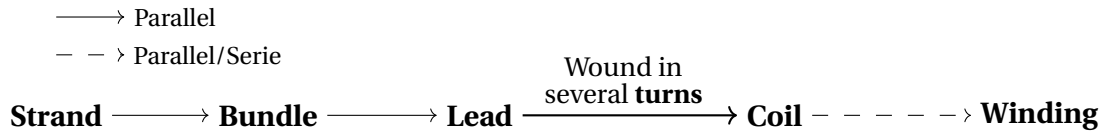


Figure 1.6: Hierarchical sequence of conductors making a winding.

It is important to note that winding and coil are two synonyms that are used interchangeably in the literature. Nevertheless, we have chosen to attribute two different hierarchical levels to them in our nomenclature, and this, in order to be able to describe the cases encountered in practice in the most general way possible.

1.1.1.2 Winding Patterns

The detailed nomenclature makes it possible to account for the structure of a winding but is still not sufficient to account for its **geometry**. When several layers of the same conductor follow each other, as in actual windings, the way to move from one **layer** to the other matters.

1. MAGNETIC COMPONENTS: DESCRIPTION AND OPERATION

Here we focus on the geometrical representation of multi-layer helical coils and for the sake of simplicity, we will represent the helix cross-section by a circle, but it can be rectangular or made of several strands (in the case of Litz wire). The geometrical description of foils is more straightforward than other shapes, due to the absence of a helix pitch; Figure 1.1 is rather explicit on this subject. Foils will therefore be considered as a simpler special case compared to solid-round and Litz cross-sections.

First, we can identify the case where the geometrical organization of conductors does present an irregular arrangement, it corresponds to the **wild winding** as illustrated in Figure 1.7. The only insulation between conductive parts is provided by the wire enamel. The goal here, is just to produce a winding with the right number of turns. This pattern's randomness leads to a greater dispersion of the component's parameters: the DC resistance which depends on the total wire length, the parasitic capacitance which depends on the conductors facing distance and the inductance but to a much lesser extent. In this design, the fill factor is around 65-75 % [113, p. 23]. This configuration is mostly found in transformers and machines of low power rating or in relays and contactors.

Then, we can identify the **layer winding** in which the conductors are placed vertically according to a regular arrangement. Such a winding pattern is represented in the center of Figure 1.7. The theoretical maximum filling factor is then $\frac{\pi}{4}$ or 78.5 %. In this configuration, it is then possible to intercalate an insulating layer between the conductor layers. Different arrangements exist in the way the conductors and insulators are placed between them. These choices are useful and important to the transformer designer but are not discussed in this work. We will simply cite Stearn [12] and Nuys [86] for their interleaved windings designs without going into further detail.

The last configuration considered corresponds to the **orthocyclic winding** (sometimes referred as hexagonal) for which the filling factor is the highest since it can theoretically reach 90.7 %. Unlike the layer winding, in this pattern, the conductors are actually parallel to each other along the axial direction. This is made possible by a crossover zone where the wires of successive layers cross each other. After this zone, each wire is inserted in the gap formed by 2 threads of the layer below. Note that the crossover occupies about 10 % of the coil [75] and that the coil needs to be wound along a **continuous** (back and forth) pattern in order to achieve the hexagonal layout. If the crossover zone is disregarded, this configuration can be represented very faithfully by a 2D sketch, as described in Figure 1.7.

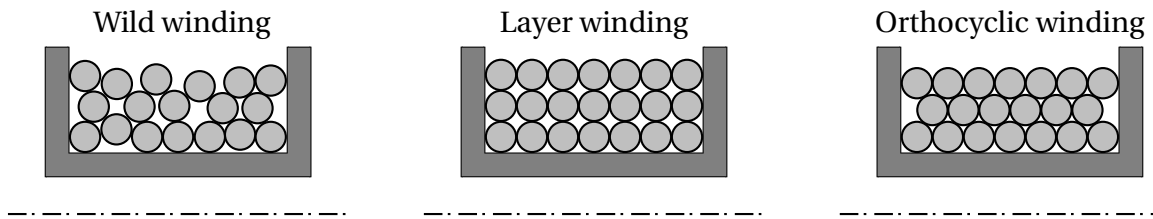


Figure 1.7: Winding organization types [113].

Considering that wild winding introduces too much variability to be correctly modeled without a large approximation and that the conductors within an orthocyclic winding are completely perpendicular to the axis of rotation of the coil, reducing it to a perfect 2D equivalent, only the layer winding case is left to be discussed. We will thus compare two patterns which depends on the layers transition choice.

When moving from one layer to another, two cases are possible: the continuous and the discontinuous patterns. In the **continuous** case (Figure 1.8), the conductor switches from

one layer to the other on the same side of the helices, whereas in the **discontinuous** case (Figure 1.9), the conductor switches from one layer to the other on opposite sides.

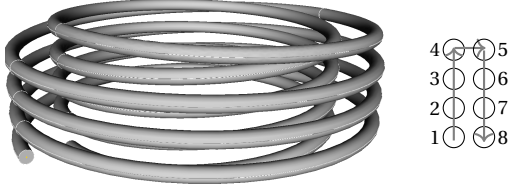


Figure 1.8: Multi-layer coil (8 turns) with **continuous** pattern.

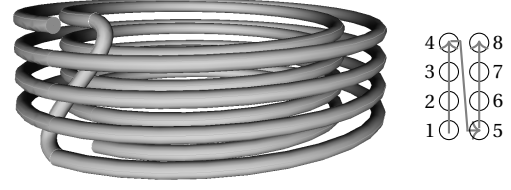


Figure 1.9: Multi-layer coil (8 turns) with **discontinuous** pattern

These two variants have a limited impact on the current and magnetic field distributions, but affect the electric field distribution. This is why it is important to differentiate between them when one wants to study the electric couplings within the coils.

In order to see what differences these two situations bring, it is possible to study a simplified case in which only the direct neighboring conductors (left, right, top, bottom) influence each other electrically [5]. This assumption is quite true when the space between conductors is small compared to the radius of the wires. For simplicity, the distance between two wires of the same layer and between two wires of two different layers is assumed to be identical.

The demonstration then assumes that the sum of squared voltage differences between neighboring conductors (top, bottom, left and right) is a valid picture of the equivalent capacitance as seen from the accesses. Comparing these sums for the continuous and discontinuous winding cases allows us to assess the capacitance dependency to the number of layers and turns per layer for each case. Considering that N_t is the total number of turns in the coil and that N_l corresponds to the number of complete layers, one can define N_{tl} as the number of turns per layer

$$N_{tl} = \frac{N_t}{N_l} . \quad (1.1)$$

Only complete layers of conductors are considered so as to avoid the evaluation of other special cases.

Assuming in both cases an applied voltage of 1 V across the winding terminals and a linear voltage drop along the coil [115], the electric potential in the middle of the i^{th} conductor can be evaluated by

$$V_i = \frac{2i - 1}{2N_t} . \quad (1.2)$$

The contribution of consecutive conductors within the same layer can be deduced as

$$N_l \sum_{i=1}^{N_{tl}-1} (V_{i+1} - V_i)^2 = N_l (N_{tl} - 1) \frac{1}{N_t^2} , \quad (1.3)$$

and leads to a simple expression since the voltage difference is always $\frac{1}{N_t}$ in that case. Expression (1.3) is observed for each winding pattern. Then, the contribution for adjacent conductors from consecutive layers (equivalently "left" and "right" neighbors) must also be taken

1. MAGNETIC COMPONENTS: DESCRIPTION AND OPERATION

into account and two cases can be distinguished: one for the **continuous** pattern

$$\begin{aligned}
 (N_l - 1) \sum_{i=1}^{N_{tl}} (V_{i+2N_{tl}-2i+1} - V_i)^2 &= (N_l - 1) \sum_{i=1}^{N_{tl}} \frac{(2N_{tl} - 2i + 1)^2}{N_t^2} \\
 &= (N_l - 1) \sum_{i=1}^{N_{tl}} \frac{(2i - 1)^2}{N_t^2} \\
 &= (N_l - 1) \frac{1}{3} N_{tl} \frac{4 N_{tl}^2 - 1}{N_t^2}, \tag{1.4}
 \end{aligned}$$

and for the **discontinuous** pattern, which yields

$$\begin{aligned}
 (N_l - 1) \sum_{i=1}^{N_{tl}} (V_{i+N_{tl}} - V_i)^2 &= (N_l - 1) \sum_{i=1}^{N_{tl}} \frac{N_{tl}^2}{N_t^2} \\
 &= (N_l - 1) N_{tl} \frac{N_{tl}^2}{N_t^2}. \tag{1.5}
 \end{aligned}$$

Expressions (1.4) and (1.5) commonly grow by a factor $\frac{(N_l-1) N_{tl}}{N_t^2}$ but differ for the rest. The squared voltage sum grows as

$$\frac{4 N_{tl}^2 - 1}{3} \tag{1.6}$$

for the **continuous pattern**, and as

$$N_{tl}^2 \tag{1.7}$$

for the **discontinuous pattern**.

After such a simplified mathematical development, we can conclude that the difference between continuous and discontinuous patterns is relatively small and should not exceed 33 % under the assumptions that we have made. The derived ratio is meaningful and consistent with previously found results which reported 2.83 pF and 2.2 pF respectively, for the continuous and discontinuous patterns in the case of a 20 turns winding arranged in 2 layers of 10 conductors [43]. Although this difference of about one third seems small at first glance, it can already lead to significant performance differences. For example, when the entire charge carried by the parasitic capacitances is dissipated during hard switching transitions, reduced parasitics (obtained with continuous pattern) will thus lead to an improved efficiency.

A defect of the 2D method is to assume that the conductors are parallel to each other in the plane or along the axial direction. This assumption, inherent to the 2D model, does not pose a problem in the case of discontinuous windings since the helices formed by the conductors have the same orientation and therefore yield to parallel wires – even if the exact distance between these wires can not be exactly determined. On the other hand, the 2D simplification is no longer true in the case of continuous windings, since the helices are oriented alternately from layer to layer. Nevertheless, the impact of the electric potential distribution for the continuous winding can be briefly studied by representing several 2D models for different angles. This effect is illustrated in the following figure.

First of all, we notice that at 180°, we obtain 37 pJ for the continuous case and 29.1 pJ for the discontinuous case. This corresponds well to the result expected by the previous mathematical development. Then, we notice that the electrostatic energy is maximized for 180°. Indeed, this corresponds to the case for which the conductors of different potential are

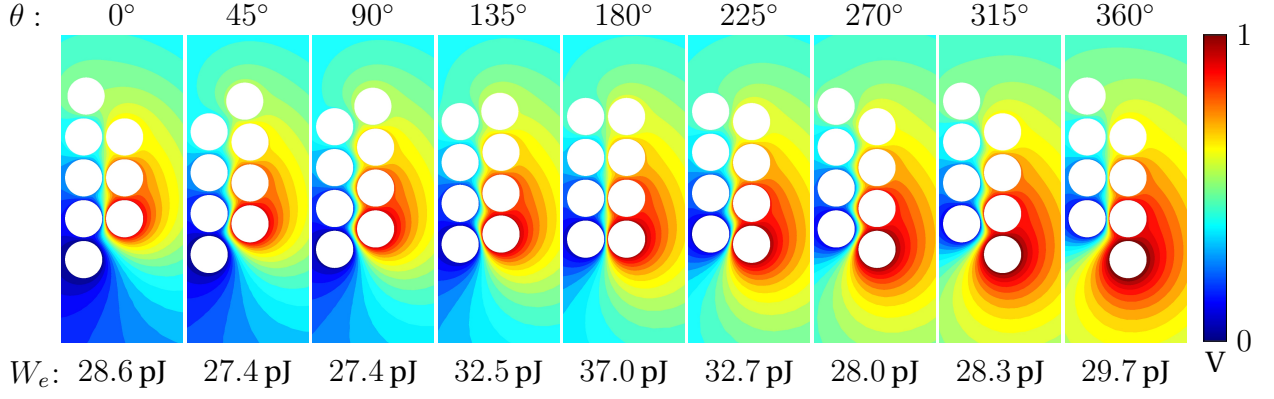


Figure 1.10: Voltage distributions along the continuous coil and total electrostatic energy W_e at different angular positions.

closest to each other. It is therefore the worst case for the continuous winding. Finally, the average value of 30.3 pJ is very close to the expected result for the discontinuous winding. Similar results were also obtained for a higher number of turns per layer². For 15 turns per layer, the values were 124.8 pJ for continuous case at 180°, 107.7 pJ on average and 93.9 pJ for the discontinuous case. The higher the number of turns per layer, the more these effects are compensated and the more the voltage difference is averaged over the height of the winding. The error tends to decrease for a large number of turns per layer since the side effects have less impact on the final value of the equivalent capacitance.

All these representations, as shown in Figure 1.7, are idealized 2D versions of the winding that is realized in practice. It is particularly difficult to realize the real complexity of the windings since on the one hand the successive wires form a helix (in which the azimuthal position of a conductor change over the helix pitch) and on the other hand the physical position is influenced by the winding method [8]. The exact coordinates of the conductors are therefore known to a limited extent. Nevertheless, some of the side effects can be obtained by the finite element method, as illustrated in Figure 1.10. This justifies its use in the design phase when specific arrangements, sometimes with few conductors, are evaluated.

1.1.2 Magnetic Cores

The association of several windings together can already lead to magnetically coupled circuit (as transformers) in the strict sense of the term. However, in practice, magnetic cores are used to confine and concentrate the flux shared by the windings in order to improve their coupling. The magnetic cores to which we refer have a **high magnetic permeability**, which means that the flux is collected in them rather than in the air; typical permeability values lie in the range of several thousands times higher than the one of vacuum.

There are quantities of materials with high magnetic permeability and each excels in different applications. The choice of one material over another depends on the saturation induction, frequency of use, losses, mechanical characteristics and core geometries for which the material is available. For instance, both electrical steels (Fe-Si and Fe-Ni), amorphous metals (Fe-based and Co-based) and nanocrystalline metals (Fe-Ni and Fe-Co) are the materials of choice for low frequency applications such as rotating machines, mains transformers,

²All those computations were made with conductors diameter of $\frac{4}{3}$, the inner winding radius set to 15 times the conductors radius and the space between consecutive turns set to 10 % of the conductors radius both vertically and horizontally.

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etc. Remark that amorphous and nanocrystalline metals perform very well up to 10 kHz [39] and 100 kHz respectively making them choice materials for medium frequency transformers. Although electrical steels are widely used for low- and medium-frequency magnetic components [39, 30], their use does not allow for high efficiency above 10-15 kHz [88, 7]. This material choice is therefore disregarded for the applications considered in this work: power electronics with switching frequencies of several tens of kHz.

Soft Ferrites

For power electronics applications, the currently most suitable magnetic cores are the **soft ferrites** which, despite lower saturation induction and higher coercive field than the softest metallic alloys, generate significantly less losses for frequencies above 100 kHz [72]. At room temperature, ferrites already perform better than amorphous irons in the same conditions³. Notice that nanocrystalline are competitive against ferrites up to 100 kHz and for high inductions. At the operating temperature of converters ($\sim 100^\circ\text{C}$), spinel ferrites and nanocrystalline materials show similar losses for an induction of 0.2 T and a frequency of 100 kHz. That is why, as a first approximation, we can consider that for lower inductions and higher frequencies, ferrites will perform better than nanocrystalline and conversely [72]. Keeping in mind that **nanocrystalline** materials are serious competitors for some applications ($f < 100 \text{ kHz}$ and $B > 0.2 \text{ T}$), one should not be quick to eliminate them as potential cores for future implementations. Nevertheless, the rest of this work still **focuses on ferrites** which remain the material of choice for magnetic cores in high frequency Switch Mode Power Supply.

Ferrites belong to the category of ferrimagnetic materials which behave like ferromagnetic materials at the macroscopic scale. Industrial ferrites are found in a wide variety of geometries as illustrated in Figure 1.11.



Figure 1.11: Typical ferrite geometries found in converters: E, EP, ETD, Pot, PQ, RM and toroidal cores from TDK Electronics [11].

The manganese-zinc (Mn-Zn) spinel ferrites are of interest for the energy conversion field. They are typically used for frequencies ranging from 10 kHz to 1 MHz and their electrical resistivity is superior to $4.1 \Omega \cdot \text{m}$ [9]. Ferrites have the particularity of being **magnetic** materials which, in addition to being slightly **conductive**, also behave as **dielectric** material and possess a significant electrical permittivity.

Ferrites are made from iron oxides mixed with other metals: Nickel, Manganese, Zinc, Copper. Their synthesis follows a classical ceramic process: grinding, bickering, compacting and sintering. The material thus obtained is presented in the form of a complex crystalline network in which metal radicals and oxides are alternately found. The iron ions retain their magnetic properties and are found trapped in a matrix of oxygen ions which isolate them from each other. This is what gives ferrites their low conductivity but also their dielectric properties. The other metal ions are located in the interstices and influence the magnetic

³Losses at 25°C , 20 kHz and 100 mT were measured as 4.4 W/kg (21.1 kW/m^3) for a 3C90 toroidal ferrite core (Annex B). Already low compared to 10 W/kg (83 kW/m^3) of a 2605-SA1 Metglas[®] amorphous core [32].

properties of ferrites according to their type and distribution. All these ions are strongly linked to each other which also gives the ferrites brittleness.

The dielectric behavior of ferrites is due to the mechanism of **electronic polarization** which corresponds to the movement of electrons in relation to their corresponding ion. When an external electric field is applied, it creates a series of dipoles which orient themselves in opposition to the applied field. At the same time, dielectric losses are due to the proximity of ions of the same element, with different valence, such as Fe^{2+} and Fe^{3+} [72]. When this pair of ions are in close crystallographic sites, this leads to a clear increase in conductivity and dielectric losses.

Note that the above macroscopic characteristics: permeability and permittivity, depend on the frequency. Actually, underlying microscopic mechanisms such as the movement of Bloch walls, electronic polarization, etc. generate these modifications but the only representation of macroscopic properties is sufficient. The **Debye relaxation** model is well suited to represent these frequency dependencies and a single relaxation-resonance is generally sufficient for power electronics applications. It is thus demonstrated that besides the standard magnetic and eddy current phenomena, the dielectric aspects must also be taken into account, in particular for the questions of electrical coupling and additional losses generated with increased operating frequency, as in actual power electronics.

The low manufacturing cost of ferrites adds to the beneficial physical properties described above. These elements put together explain the numerous industrial applications of these materials. Unfortunately, manufacturers' data are provided with high tolerances: permeabilities are indicated to $\pm 20\%$ and often differ from measurements. In addition, tolerances on losses figures are rarely specified.

To compensate for this lack of information, a measurement campaign was carried out on different formats and types of magnetic cores, in particular those used to wind the devices under test. The main results are summarized in Chapter 4, and details are available in Appendix B.

1.1.3 Insulating Materials

The insulating materials are inseparable from the conductors and magnetic cores described above. Naturally, in any real electrotechnical application, it is necessary to insulate the conductors from each other in order to prevent short-circuits that would impair efficiency and could destroy the equipment. In order to guarantee the galvanic isolation between the active parts of the transformer, including the magnetic core, insulating materials are placed upon and between them. We hereby discard the insulating oils which are found in oil-bath transformers and are rarely exploited in our applications since their use is justified by high voltage withstand and rated power.

In **dry-type** transformers, solid state insulators are applied, which include enamels, varnishes, films, coatings, resins and all mechanical insulating pieces which ensure the spacing between transformers active parts.

At the level of the conductors, we generally find one or more layers of enamel around the wire. These layers are made of polymers: usually polyurethane (PU), polyester-imide (PEI) or polyamide-imide (PAI). Then, in the windings, we find insulating films and insulating sheets between the different layers of conductors. Again, we find polymers such as nylon, polyester, polyvinyl, polyurethane, etc. It is also possible to embed the winding in the resin. Finally, between the different windings are placed solid parts in the form of dog bone, panel or corner angle. These parts have a high mechanical resistance because they support the stresses gen-

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erated by the concentric windings. They are therefore often made of fiberglass or fiberglass reinforced plastics and sometimes of mica.

The use of these insulating materials must allow both the holding in tension of the different conductive parts of the transformer and allow an adequate transfer of heat towards the environment in order to evacuate the losses. The maintained distance between 2 different windings can also allow the air to be channeled to transfer the heat from the windings to the outside of the transformer.

Considering the insulation is quite important in the modeling of high frequency effects because their high permittivity modifies the parasitic capacitances. Moreover, for very high frequencies, these dielectrics generate losses. The influence of insulating materials in power electronics transformers is therefore studied through the relative electrical permittivity. It should be noted that the precise knowledge of the dielectric parameters is a complex thing since they depend among others on hydrometry, frequency, temperature and that the aging of these materials also plays a role. Finally, we note that 2D models are not appropriate to account for effects related to point pieces such as corner angles and dog bones.

1.2 Power Electronics Applications - Converter Topology

Magnetic components are essential elements of the conversion chain in power electronics because they can store magnetic energy, adapt voltage levels and create galvanic isolation between different circuit parts. The transformers studied in this work are used in different converters that we will present below. Three topologies have been selected to illustrate the variety of uses of transformers in power electronics: the dual active bridge, the resonant LLC and the high step-up converter.

1.2.1 Dual Active Bridge Converter

The principle of the Dual Active Bridge (DAB) is inspired by the transfer of active power within electricity transmission networks in which the impedance of the lines is essentially inductive. As a result, in that configuration, the active power transfer is conducted by the phase shift between the two buses voltages rather than the amplitude difference. A simple transmission line between two network nodes is illustrated below, and the expression of the transmitted active power is given by (1.8).

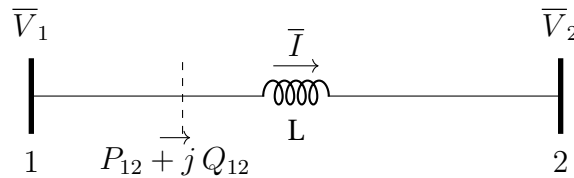


Figure 1.12: Power transfer across an inductance in the sinusoidal regime.

$$P_{12} = \frac{V_1 V_2 \sin(\phi_1 - \phi_2)}{\omega L} \quad (1.8)$$

A phase advance of Bus 1 voltage with respect to Bus 2 results in a positive active power from node 1 to node 2 and vice versa. The active power level evolves continuously with the phase

shift and the sign change occurs for a zero phase shift. A converter using this principle can therefore exchange power bi-directionally naturally.

In the Dual Active Bridge, two high frequency square wave signals replace the voltages of the two network buses. These signals are obtained thanks to two full-bridges (or half-bridges) placed on either side of the series inductor. In general, the inductance through which the power passes can be replaced by a transformer in order to adapt two different voltage levels between the input and the output of the DAB, as in Figure 1.13. In that case, the leakage inductance of the transformer makes the link from one bridge to the other and limits the active power transfer as expressed in (1.8).

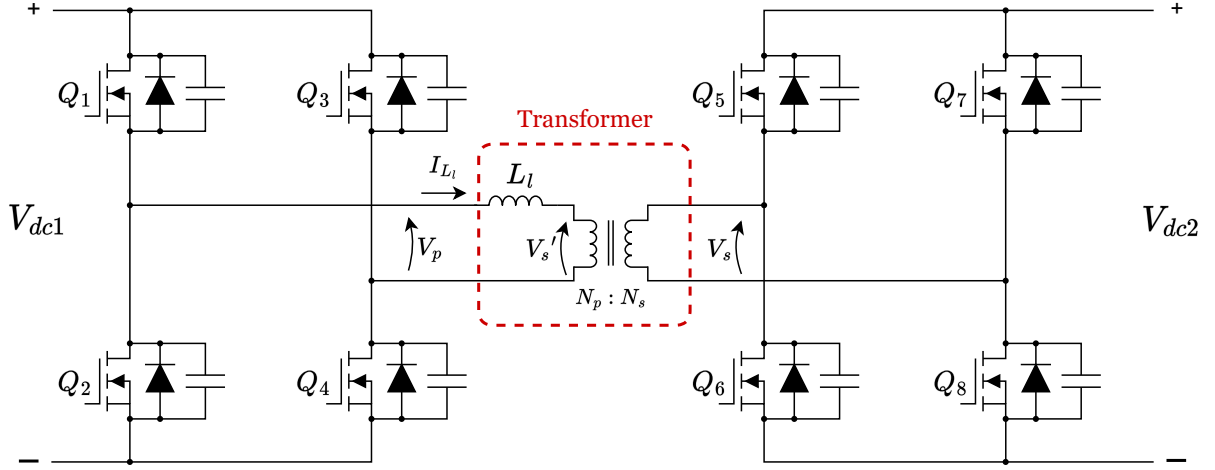


Figure 1.13: Dual Active Bridge topology, including MOSFET body diodes and parasitic capacitances.

The power transfer being always directed from the leading bridge to the lagging bridge [102], Bi-directional power transfer is easily obtained in a DAB by controlling only the phase shift between the 2 full-bridges [22].

Regarding the modulation, the two full-bridges generate full-wave PWM signals of 50 % duty cycle by controlling simultaneously the pairs of diagonal switches (Q_1 - Q_4 , Q_2 - Q_3 , Q_5 - Q_8 and Q_6 - Q_7). In Figure 1.14, we observe the effect of this modulation on the voltages on both sides of the inductance L_l and on the current flowing through it. At switching events, the energy stored in the inductor allows to discharge the parasitic capacitances of the MOSFETs, which has the effect of forcing the voltage to zero before the turn on. This phenomenon is called zero voltage switching (ZVS) and allows to reduce the switching losses in a significant way. In a DAB, all the switches of the lagging bridge operate in ZVS and some of the leading bridge as well [102].

The leakage inductance value must be minimized in order to obtain the highest possible power factor (*i.e.* the reactive power associated with the inductance must remain low compared to the active power). This means that very short rise times are associated with a low inductance and therefore a high power factor. However, the minimum value of the phase shift is limited by the control loop and therefore cannot be as small as possible. Another very limiting element of the dual active bridge is that the current excursion becomes more and more important as L_l is low when we have large DC voltage ranges (input and output). Consequently, the target value of the leakage inductance will be a compromise between the effect of the power factor, the time accuracy and the voltage levels. This important specification creates a strong constraint on the design of the transformer [39].

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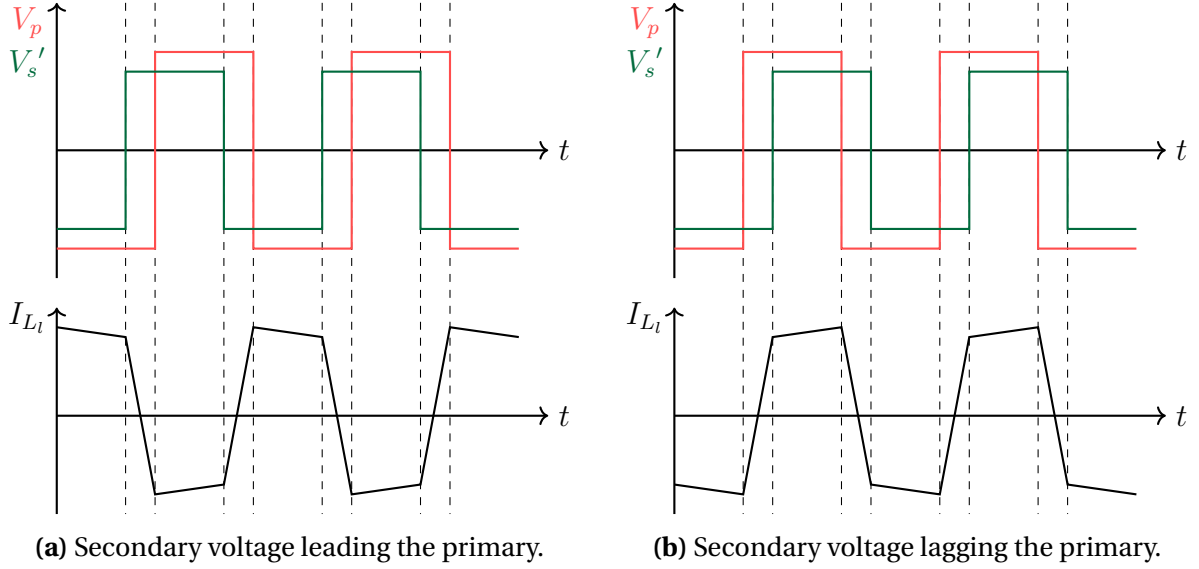


Figure 1.14: Waveforms of the transformer primary and secondary voltages and the leakage inductance current.

1.2.2 LLC Resonant Converter

The growing interest in resonant converters is due to their very advantageous performance. Specifically, these topologies allow to operate the converter at higher frequencies while maintaining lower switching losses. This is why resonant converters are good candidates for improving **efficiency** and **high power densities**. The LLC power converter takes its name from the three components that make up the resonant tank: a parallel inductor, a series inductor and a series capacitor [54]. The choice to integrate a transformer in this design is not trivial since the transformer's magnetizing and leakage inductances might play the role of the parallel and series inductors of the tank. And that, if the transformer is dimensioned properly.

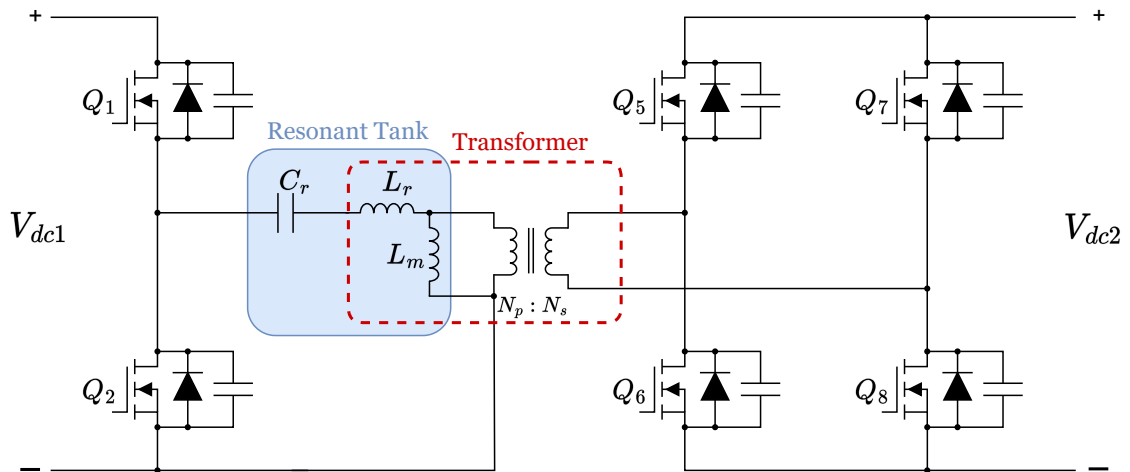


Figure 1.15: LLC resonant converter topology, including MOSFET body diodes and parasitic capacitances.

The LLC resonant converter has a topology close to the DAB except that in the application that we will consider, only a half-bridge was used on the primary side instead of of

full-bridge. The LLC tank has two main resonant frequencies: the first is linked to the resonance between the capacitance C_r and the series inductance L_r ; the second corresponds to the resonance between the capacitance and the magnetizing inductance L_m . This topology therefore makes it possible to use a switching frequency which is placed in the range of the two main resonance frequencies of the resonant tank. Similarly to the Dual Active Bridge, the LLC converter operates under ZVS conditions thanks to the resonance that is maintained.

In normal use, the switching frequency is above the first LLC tank self-resonant frequencies. It is therefore essential, when designing the transformer, to correctly estimate the magnetizing inductance and the leakage inductance which condition the resonances. It should also be noted that the estimation of parasitic capacitances of the transformer is important to understand their impact on the resonance frequencies.

Once again, strong constraints are observed for the transformer design specifications in order to operate the power electronic converter correctly which highlights the need to estimate the transformer parameters accurately.

1.2.3 High Step-Up DC-DC Converter

Other than the phase shift provided by the transformer leakage inductance or the resonances of the LLC tank, another common power electronics application is the high DC voltages step-up converter. Achieving a high voltage rise can be accomplished by using transformer based converters and integrating voltage multipliers as output stages [107, 25, 85]. Figure 1.16 shows a topology of this type which includes a **full bridge inverter** and a **voltage multiplier** connected to the secondary of the transformer. In such a configuration the transformer's turn ratio can advantageously serve to amplify the output voltage. The resonance associated with the secondary capacitance and leakage inductance is also exploited for amplification purposes.

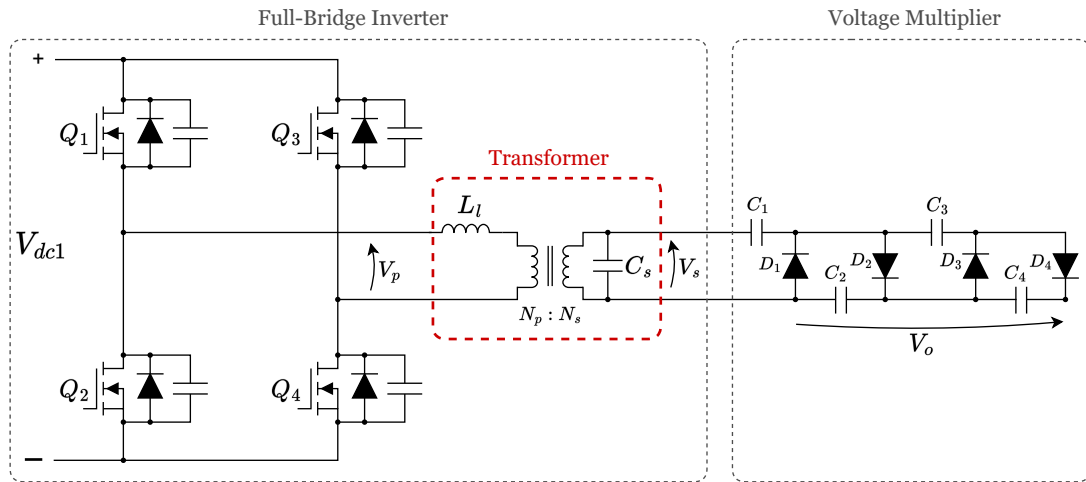


Figure 1.16: High step-up converter topology, including MOSFET body diodes and parasitic capacitances.

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Indeed, by simplifying the output stage into a parallel RC, we can identify the following resonant circuit, illustrated in Figure 1.17. Some parameters of this circuit are relevant for the analysis and working principle of the converter. Among those, we define the pseudo resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad (1.9)$$

and the quality factor of the RLC circuit

$$Q = \omega_0 RC = \frac{R}{\sqrt{\frac{L}{C}}} = R\sqrt{\frac{C}{L}}. \quad (1.10)$$

The quality factor is a figure of merit of the resonant current to the output current (resistive losses) ratio at the resonant frequency

$$Q = \frac{I_c}{I_r}. \quad (1.11)$$

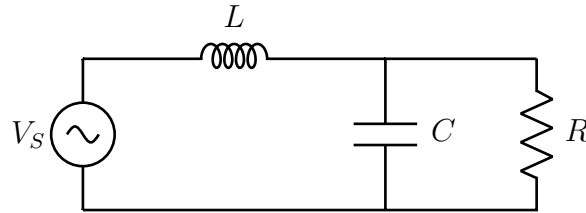


Figure 1.17: Simplified resonant circuit equivalent to the transformer and the output stage of the step-up converter.

In this case, the input signal is a full square wave which is taken up through the transformer to the voltage multiplier. The transformer therefore makes it possible to reduce the effort on the multiplier. The resonant circuit works with a high quality factor ($Q > 1$) which means that the resonant current is Q times bigger than the output current going to the charge. From this point of view only, the high quality factor seems sub-optimal. However, from the converter's point of view, the total efficiency is improved, minimizing both transformer and multiplier losses.

We note that the magnetizing inductance is supposed to be large enough so that it does not participate to the resonant circuit. The transformer design must therefore bring out a high magnetizing inductance and a precise value of both the leakage inductance and the output capacitance. Moreover, if the converter design supposes a quality factor $Q > 1$, then the transformer must be over sized and will work at Q times the output power which supplies the multiplier.

Electromagnetism applied to Magnetic Components

After having introduced and explained both the conception and use of transformers in the previous chapter, we will present the physical equations that govern the electrical and magnetic behavior of such devices. At the same time, we present the models applied to the elements that compose the transformers (conductors, magnetic cores and insulators) as well as the corresponding quantities of the equivalent localized constants circuits.

2.1 Introduction

Electromagnetic phenomena are governed by Maxwell's general equations [79] which are commonly expressed as a system of partial derivative equations. This set of laws allows to make the connection between electric phenomena and magnetic phenomena. Taken together, they form a mathematical representation of the same physical phenomenon: the electromagnetic fields [50, 105]. Maxwell's equations in differential form can be written as

$$\text{curl } \mathbf{h} = \mathbf{j} + \partial_t \mathbf{d}, \quad (2.1)$$

$$\text{curl } \mathbf{e} = -\partial_t \mathbf{b}, \quad (2.2)$$

$$\text{div } \mathbf{b} = 0, \quad (2.3)$$

$$\text{div } \mathbf{d} = \rho. \quad (2.4)$$

Equations (2.1) to (2.4) are respectively the Ampère-Maxwell law, the Faraday's law, the magnetic Gauss law and the electric Gauss law. These equations involve the four vector fields $\mathbf{h}$, $\mathbf{b}$, $\mathbf{e}$ and $\mathbf{d}$, respectively the magnetic field (A/m), the magnetic flux density (T), the electric field (V/m) and the electric displacement field (C/m²), as well as two other quantities: the conduction current density $\mathbf{j}$ (A/m²) and the electric charge density ρ (C/m³).

The electric field $\mathbf{e}$ and the magnetic field $\mathbf{h}$ involved in Maxwell's system of equations are nested together by means of the **curl** equations (2.1) and (2.2), associated to the circulation of these two vector fields. Such system of equations (2.1) to (2.4) remains true in any situation and is referred to as **fully-coupled** due to the entanglement of the $\mathbf{e}$ and $\mathbf{h}$ vector fields.

However, the resolution of the complete system is unnecessary for most of the electrical engineering applications, in which power electronic transformers and inductors are part of. The **displacement current** term ($\partial_t \mathbf{d}$) can be neglected if the biggest dimension of the equipment is much smaller than the wavelength of the signal: $\lambda = \frac{v}{f}$ [93, p. 123]. In free space, the wavelength of a 10 MHz frequency signal corresponds to 30 m which is much larger than the dimensions of the devices we deal with. Removing the displacement current

2. ELECTROMAGNETISM APPLIED TO MAGNETIC COMPONENTS

leads to the Ampere's law

$$\text{curl } \mathbf{h} = \mathbf{j}, \quad (2.5)$$

from which the magnetic field defined on the whole domain and the electric field defined on the conductors can both be computed.

From now on, the system becomes **uncoupled** as the electric field $\mathbf{e}$ and the magnetic field $\mathbf{h}$ in the non-conducting regions are obtained from two independent sets of equations: one for the **magnetic problem** and one for the **electric problem**. This allows to consider only a subset of Maxwell's equations and behavioral laws, which allows the definition of global magnetic and electric quantities, as we will see later. Note that these models allow to obtain the quantities representing the magnetic and electrical phenomena in a completely decoupled way. Nevertheless, the magnetic and electric interactions will be associated within localized constant circuits, which will be the topic of chapter 3. Finally, the separate formulations of the quasi-static states are developed in the next sections, as well as their global variables counterpart.

Whether the phenomena we study are periodic or not, it is always possible to represent them in the frequency domain. This is particularly suitable for the applications we are considering since the signals applied to the magnetic components are periodic. Maxwell's equations naturally remain valid but can be rewritten by expressing each field as a complex-valued phasor and will be specially noted with a bar above $\bar{\cdot}$ so as not to confuse them with fields expressed in the time domain. At the ω pulsation, the time derivative is replaced by $j\omega$ where j represents the pure imaginary unit. Maxwell's equations can thus be rewritten in the harmonic regime as follows:

$$\text{curl } \bar{\mathbf{h}} = \bar{\mathbf{j}}, \quad (2.6)$$

$$\text{curl } \bar{\mathbf{e}} = -j\omega \bar{\mathbf{b}}, \quad (2.7)$$

$$\text{div } \bar{\mathbf{b}} = 0, \quad (2.8)$$

$$\text{div } \bar{\mathbf{d}} = \rho. \quad (2.9)$$

The complex formalism is used to represent the time-harmonic behavior with the conjugate operator defined as $\cdot^*$. The convention used here consists of straight letters for real variables operated in the time domain (e.g. ρ) and italic letters for complex-valued variables operated in the frequency domain (e.g. ρ). In addition, fields whose value evolves over time can be represented by phasors in the frequency domain. To a sinusoidal function $x(t)$ defined through its amplitude X , its pulsation ω and its phase shift θ

$$x(t) = X \cos(\omega t + \theta) = \text{Re} \left(X e^{j(\omega t + \theta)} \right), \quad (2.10)$$

we can associate the phasor $\bar{x}$

$$\bar{x} = X e^{j\theta}, \quad (2.11)$$

defined by its amplitude and phase shift, all phasors oscillating at the harmonic pulse ω in the frequency domain.

For the sake of clarity and consistency, the **mathematical developments** of the magnetic problem will be expressed in the **time domain** since it is not excluded that the materials studied present certain non-linearities which invalidate the frequency domain transition. However, we will see that some simplifications assumed and applied a posteriori will allow us to adapt the developments easily to the frequency domain without loss of generality for the applications that interest us.

2.2 Magnetism

After decoupling Maxwell's equations into two independent problems, we study the magnetic aspect first and then the electric one. It makes sense to start with the magnetic model since the components we study are designed and operated as such, for filtering purposes, signal smoothing, voltage adaption, energy storage, etc. The electrical couplings are generally parasitic effects which intervene by the construction of the magnetic components, although it is sometimes possible to take advantage of these effects. This justifies developing the magnetic study first, as it corresponds to the main behavior of the inductive components (inductors and transformers) and the electric one subsequently since it comes into play afterwards.

In this first part, we study the dependency of the magnetic field $\mathbf{h}$ and the induction field $\mathbf{b}$ when an electric current flows through the inductive components, according to the hypotheses of uncoupling and harmonic regime. The electric field $\mathbf{e}$ also intervenes in this formulation via the Lenz-Faraday law in the established sinusoidal regime. This modifies the distribution of the currents in the conductors and consequently the distribution of the magnetic field. This problem of induced currents mainly affects the copper losses associated with the distribution of currents in the conductors and little the magnetic field level outside the conductors. Note that in this hypothesis of magnetoquasistatic, the electric field acts only through the conduction of the current and not outside the conductors, given that the variation of the electric displacement is neglected; thus discarding the wave propagation phenomena.

Solving Maxwell's equations requires the definition of the constitutive laws of materials. Such relations express the behavior of continuous media and complete the original set of equations (2.1) - (2.4) that would otherwise be indefinite. Concerning the representation of the magnetic problem, it is necessary to define the magnetic and electrical behavior laws generally expressed as

$$\mathbf{b} = \mu_0 (\mathbf{h} + \mathbf{m}) , \quad (2.12)$$

$$\mathbf{j} = \sigma \mathbf{e} + \mathbf{j}_s . \quad (2.13)$$

Those constitutive laws involve the magnetic permeability of vacuum μ_0 (H/m) and the electric conductivity σ (S/m) as well as other fields definition; being the magnetization $\mathbf{m}$ and the source current density $\mathbf{j}_s$. Generally speaking, those linking equations are tensor-based and can present non-simple dependencies as they may be non-linear (*e.g.* saturation), with anisotropy, influenced by the history (*e.g.* hysteresis) or even dependent on other physical quantities such as temperature or the position in space. Note that the constitutive laws represent the macroscopic behavior of materials from experimental observations and do not express the laws of physics. But they remain irreplaceable in the resolution of Maxwell's equations in order to fully determine the unknown electromagnetic fields.

The magnetic behavioral law, widely represented as in (2.12) exhibits the magnetization field $\mathbf{m}$ which characterizes the magnetic response of the material. In a homogeneous, linear, non-dispersive and isotropic magnetic medium, the induced magnetization (*i.e.* the magnetic moment per unit volume) is aligned with and proportional to the magnetic field [36]. On another note, it is common to define a permanent magnetization $\mathbf{m}_s$, relative to permanent magnets [69], and independent of the local magnetic field. However, such static magnetization $\mathbf{m}_s$ can be left aside for the representation of inductances and transformers in power electronics applications. Combining such assumptions leads to the magnetization $\mathbf{m}$

2. ELECTROMAGNETISM APPLIED TO MAGNETIC COMPONENTS

being expressed as

$$\mathbf{m} = \chi_m \mathbf{h} , \quad (2.14)$$

where χ_m is the magnetic susceptibility of the medium. Injecting (2.14) into (2.12) can be developed as

$$\begin{aligned} \mathbf{b} &= \mu_0 (\mathbf{h} + \chi_m \mathbf{h}) \\ &= \mu_0 (1 + \chi_m) \mathbf{h} \\ &= \mu_0 \mu_r \mathbf{h} \\ &= \mu \mathbf{h} . \end{aligned} \quad (2.15)$$

Defining at the same time μ , the magnetic permeability of the material and $\mu_r = (1 + \chi_m)$, the relative magnetic permeability of the material.

In the absence of static magnetization $\mathbf{m}_s$, which we consider, the material can still manifest three foremost deportments: diamagnetism, paramagnetism and ferromagnetism.

Prior to developing the equations that represent the magnetic deportments, it is necessary to highlight the behavior of magnetic materials at the heart of the model. Indeed, there are several families of magnetic materials which present different behaviors when they are subjected to a magnetic field. This is reflected in the value of their susceptibility χ_m as expressed in the equation 2.14.

First of all, there exists **diamagnetic** materials for which the magnetization they generate is opposite to the applied external field. Therefore, such behavior can be modeled by a slightly negative susceptibility of -10^{-5} - -10^{-4} , which also implies that the relative permeability μ_r is just below 1. Its origin is a quantum effect that can be associated with the modification of the orbital movement of electrons, which then behave like a microscopic solenoid that exhibits induced currents. According to Lenz's law, these induced currents generate an induction that opposes the field that gave them rise, which indeed corresponds to a negative susceptibility. Diamagnetism is a behavior that can be found in any type of matter but is often masked by the effects of paramagnetism or ferromagnetism that coexist with it.

Then, we can look at the **paramagnetic** materials where the reaction to the applied field produces a slightly higher induction in the same direction as the external field. Like diamagnetic materials, the magnetization is quite weak. It is characterized by a susceptibility of 10^{-5} in the paramagnetic case and a relative permeability slightly higher than 1. The idea that the magnetic moment of a substance can be assimilated to the sum of the moments of each of its atoms was introduced by Langevin [70]. According to this model, in the absence of a magnetic field, the vector sum of all the microscopic moments cancels and does not show any macroscopic magnetic moment. On the other hand, under the effect of an external field, these moments tend to align with it and reinforce the applied field. However, the resulting magnetization is very weak since the thermal agitation randomly reorients the microscopic moments, which is the predominant effect.

In summary, dia- and paramagnetic materials have relative permeabilities very close to unity and are of little use in electrical engineering because of their weak magnetization. It is generally accepted that their susceptibility can be neglected and that the vacuum permeability μ_0 is assimilated to these materials [57].

The cores used in power electronics are generally made of ferrites which are ferrimagnetic. From a macroscopic point of view, a **ferrimagnetic** material behaves exactly like a ferromagnetic one, the only difference being the microscopic phenomena that are responsible for the magnetic properties. Such particular properties of ferrites, compounds based on iron oxide and a metal M, are due to the elementary magnetic moments, or spins, carried

by the iron atoms and by those of the metal M. The metal ions occupy two different crystallographic sites and thus form two crystal sublattices. In each sub-lattice, the spins interact with each other in a ferromagnetic way, *i.e.* they tend to align themselves in the same direction, giving a non-zero total magnetization. The ferrite thus appears at low temperature as a permanent magnet. Above a certain temperature, called "Neel temperature", thermal agitation breaks the magnetic order and the body becomes paramagnetic. In the remaining of the manuscript, ferrites will be regarded as ferromagnetic materials for the sake of simplicity and to emphasize that only the macroscopic behavior is of interest for our models.

2.2.1 Ferromagnetic Materials

Of the three types of magnetic behavior: dia-, para- and ferromagnetism, we are interested in the representation of **ferromagnetic** materials that are used in practice to channel the magnetic induction $\mathbf{b}$ and to reduce the volume of the components. Ferromagnetic materials present strong magnetic response to external magnetic fields [37]. In addition, ferromagnetism is the ability of certain materials to maintain a substantial magnetization in the absence of an external source field and following previous exposure to a magnetic field. In that sense, one can say that ferromagnetic materials exhibit some memory related to the past history of the magnetization it has undergone. Such a behavior is certainly non-linear since the induction is no longer zero after the material has been magnetized.

This observation of non-linearity is made at the macroscopic scale. However, the causes of the complicated behavior of ferromagnetic materials lie at the microscopic scale. Any sample of macroscopic size is subdivided into smaller regions corresponding to magnetic domains. In these regions, called Weiss domains, the magnetic moment tends to align itself along the same direction under the effect of molecular forces whose extent is quite localized [57, p.16]. This is why different domains have different orientations of their magnetic moments. Two adjacent domains are separated by Bloch or Neel walls which allow the transition of the magnetic moment from one domain to the other. These walls have a certain thickness linked to this transition and can move under the effect of magnetic field variations, creating additional losses. The schematic representation of these walls is provided in figure 2.1 and we observe that the variation of magnetic moment is carried out by a rotational movement.

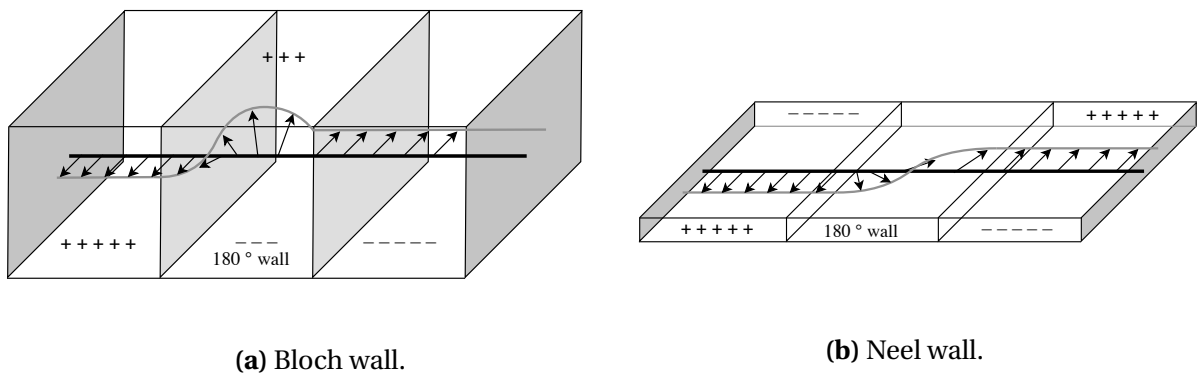


Figure 2.1: Magnetic domain transition via Bloch wall and Neel wall, excerpted from [63].

Under the effect of an external field, the magnetic domains deform and tend to align themselves to the applied field. Thus, the domains whose moment is oriented in the same direction as the field tend to expand and the others to shrink. For a low value of external

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field, the process is reversible and the walls deform slightly around their position without changing their pinning point.

However, this is no longer the case for high field values; the process is no longer reversible and is no longer continuous since the presence of defects in the material (cavities, grain boundaries, etc.) leads to abrupt jumps in the displacement of the walls, from pinning site to pinning site. These discrete variations correspond to the Barkhausen effect and the wall movements are then irreversible. This leads to the merging or splitting of the magnetic domains which then reorganize themselves. If the external field continues to increase, there will come a stage when all the magnetic moments are aligned and form a single domain, this is the maximum magnetization of the material. From then on, the strong response of the physical medium is no longer perceptible as no more magnetic moment can be reoriented.

It is the combination of these microscopic behaviors of the material that explains its non-linear characteristics and the saturation phenomenon observed at bigger scales. A typical ferromagnetic hysteresis curve, apparent on a macroscopic scale, is illustrated in Figure 2.2, where we can see that the induction associated to an increasing field does not cross the same points as the induction due to a decreasing field which corresponds well to the irreversibility of the walls motion. The area contained within the hysteresis cycle is associated with losses corresponding to microscopic phenomena (Bloch wall movements, wall rotation, defects, etc.).

Furthermore, the evolution of the magnetic domains of the material is represented for different magnetization levels. We notice that the magnetic domains aligned with the external field tend to maximize their distribution until they are totally aligned. Once all the domains are aligned with the external field, there is no more magnetization gain and the value of the permeability falls back to that of the vacuum μ_0 .

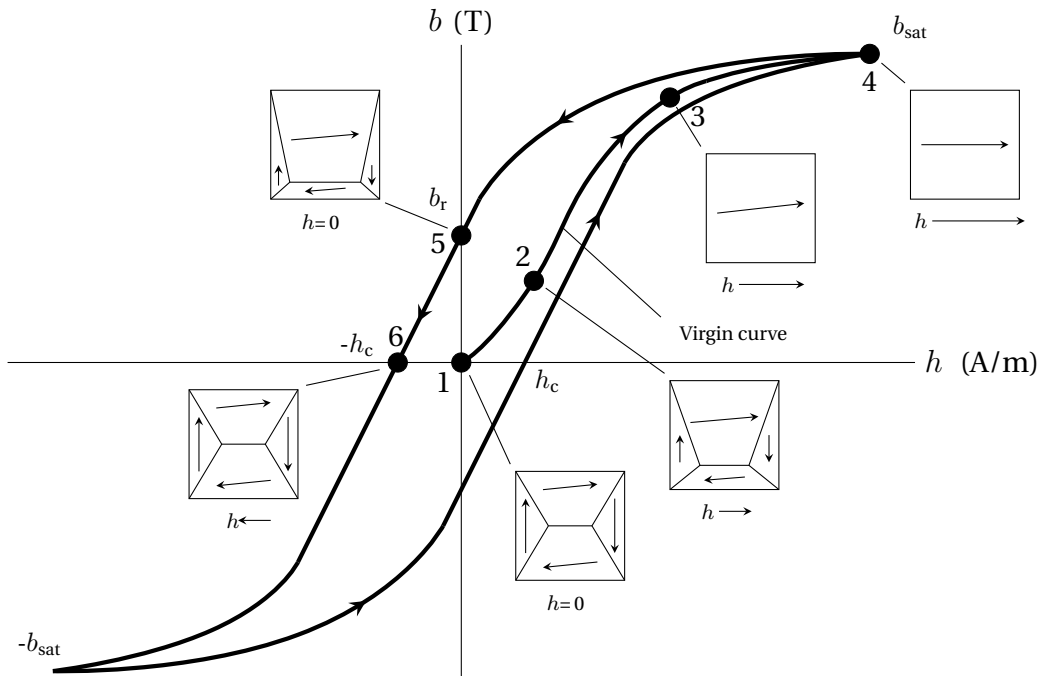


Figure 2.2: Magnetic domains evolution along the hysteresis cycle: at rest, first magnetization and alternating operation. Derived from [74].

On Figure 2.2, we can track the first magnetization curve (virgin curve) which follows the points 1 to 4. Point 1 corresponds to the demagnetized state for which the applied field is null

and the resulting magnetic moment is also null. At point 2, we already notice the displacement of the Bloch walls which is the source of a larger resultant moment in the direction of the applied field. At point 3, the merging of the domains produces a single domain of given magnetic moment. Finally, at point 4, the magnetic moment is perfectly aligned with the external field and the maximum magnetization is reached.

Owing to the irreversibilities, we notice that once the material is magnetized, the curve of first magnetization is not traversed anymore. For a given amplitude and frequency, the cycle will travel through the envelope by crossing the points 5 and 6. It then describes a hysteresis cycle (known as major) with a curve for the increasing phase and another curve for the decreasing phase. On this major cycle, point 5 corresponds to the point of **remanent induction**, *i.e.* the point at which the induction has a non-zero value when the external field falls to zero. This induction value is an image of the irreversible microscopic phenomena. It is this characteristic of remanence that explains how a ferromagnetic material can exhibit spontaneous magnetization in the absence of an external magnetic field. Different values of remanence are reached according to the different levels of maximum field applied and it can be observed very well in the ferrite measurements collected in the Appendix B. Point 6, on the other hand, corresponds to the **coercive field** level, *i.e.* the point at which the magnetic field value allows the material's induction to be canceled. This is information related to the strength of the field that must be applied in order to reverse the magnetization. In ferromagnetic materials, the coercive field and the size of the grains are linked by a close relationship [53].

Soft ferromagnetic materials are characterized by a low coercive field ($h_c < 10$ kA/m [39, p.61]) as opposed to hard ferromagnetic materials. The lower the coercive field and the remanent induction, the smaller the cycle area; which is what is sought for the cores of inductors and transformers in order to minimize the losses when these components are subjected to periodic regimes.

From their microscopic structure, ferromagnetic materials have a permeability that depends on both the magnetic field strength and the magnetization history. This is due to the orientation of magnetic domains at the grain scale, whose movement requires to overcome a certain energy barrier which is then dissipated as heat. This leads to a hysteresis phenomenon for which the magnetization and demagnetization of the material do not follow the same path in the b - h plane. The hysteresis effects that form a cycle are materialized by retardation effects between the magnetic field and the induction, due among other things to the movement of the walls. It is possible to formalize this relation by means of the magnetic constitutive law 2.15 (in the absence of static magnetization), by expressing the dependency between the induction b and the magnetic field h by means of the permeability μ which then depends on the magnetic field in the non-linear case. We recall that the relationship can be written as

$$\mathbf{b} = \mu(\mathbf{h}) \mathbf{h} . \quad (2.16)$$

We have seen that for ferromagnetic materials, the dependency between the magnetic field and the magnetic induction presents a strong non-linearity as well as retardation effects which are materialized by the hysteresis cycle. However, we can notice that in practical applications, inductors and transformers are considered as **linear** components with respect to the field amplitude [117, p. 182]. This means that their magnetization remains sufficiently small so as not to approach the saturation of the material and that ferromagnetic cores can be modeled by linear equations.

In the simplest case, we can for example consider that the fields $\mathbf{b}$ and $\mathbf{h}$ are collinear, which implies that the permeability μ is a scalar. We can also take a more intricate law of

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behavior in which the induction field $\mathbf{b}$ depends both on the instantaneous value of the magnetic field $\mathbf{h}$ and on its temporal derivative. Such law allows to take into account the retardation effects inherent to ferromagnetic materials and can be synthesized as

$$\mathbf{b} = \mu' \mathbf{h} - \alpha \partial_t \mathbf{h} \quad (2.17)$$

in the time domain. In the harmonic regime, such a behavioral law results in the use of a complex-valued permeability

$$\mu = \mu' - j\mu'' , \quad (2.18)$$

which therefore leads to a phase shift between the induction and the magnetic field when expressed as

$$\bar{b} = \mu \bar{h} , \quad (2.19)$$

phase lag that is the equivalent of a delay in the time domain. This phase shift leads to a quantity of ferromagnetic losses associated with the imaginary part of the permeability (μ''). As depicted in Figure 2.3, the use of the complex permeability as a dependency between the h field and the b induction describes an ellipse in the b - h plane that closely follows the data collected in practice for a hysteresis cycle at a given frequency.

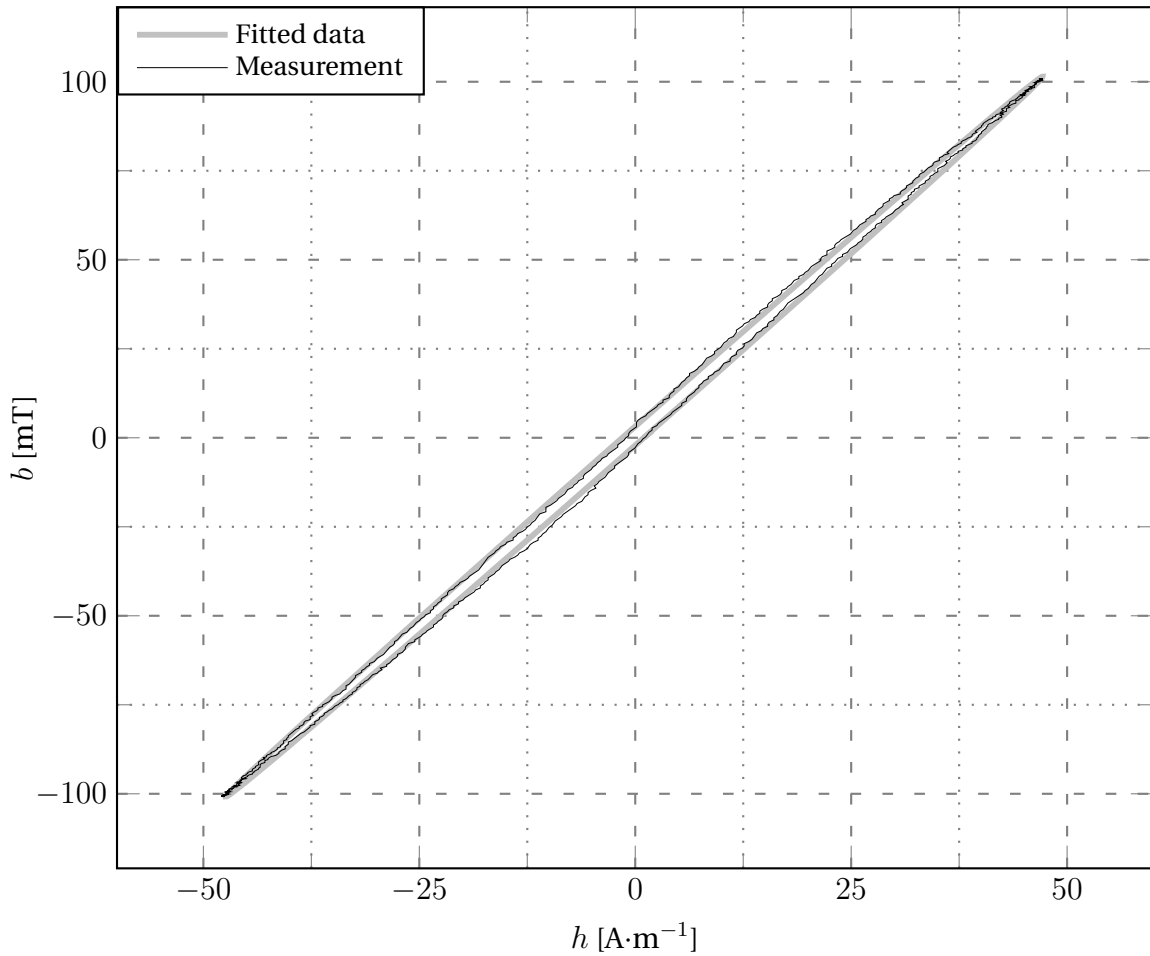


Figure 2.3: Induction to magnetic field dependency gathered for E43-10-28 core of 3C95 material type at 30°C, 5 kHz and 100 mT as well as the linear approximation of permeability providing $\mu'_r = 1700.7$ and $\mu''_r = 44.19$ represented by the light gray ellipse.

It is then necessary to determine the parameters μ' and μ'' of the complex permeability for a given curve measured. The choice is made here to consider the norm of the permeability as the value that maximizes the amplitude of the cycle in the b - h plane and the phase shift δ that provides the ellipse with the same surface as the collected data. The angle δ being the loss tangent of the permeability such that

$$\tan \delta = \frac{\mu''}{\mu'} . \quad (2.20)$$

Numerous hysteresis cycles have been measured on ferrite cores and are presented in Appendix B, in order to provide complete information about the characteristics and losses of the studied ferrite cores and similar geometries or materials.

2.2.2 Magnetodynamics

We have already seen that we consider the magnetic and electric fields as **partially coupled** and not completely coupled. This is a generally accepted assumption for the modeling of electromagnetic problems whose temporal evolution is not too sudden, *i.e.* the frequency of the phenomena remains low and consequently the wavelength remains high compared to the dimensions of the device.

Such specific study of electromagnetic fields is called magnetodynamics because it represents the evolution of magnetic phenomena while neglecting the impact of the displacement current. In fact, the magnetic field originating from the currents matters in order to determine the magnetic coupling of the studied component. The representation of fields for magnetic components also requires taking into account the impact of variable currents on electric and magnetic fields. The following developments will be carried out in time domain and then transposed to the frequency domain of use in the simulations.

In formal mathematical terms, the Ampere and the magnetic Gauss equations, to which one adds the magnetic behavior law form a reduced set of equations

$$\text{curl } \mathbf{h} = \mathbf{j} , \quad (2.21)$$

$$\text{curl } \mathbf{e} = -\partial_t \mathbf{b} , \quad (2.22)$$

$$\text{div } \mathbf{b} = 0 , \quad (2.23)$$

$$\mathbf{b} = \mu(\mathbf{h}) \mathbf{h} . \quad (2.24)$$

The evolution of the electric and magnetic fields over time implies to also include the Lenz-Faraday law 2.2 to our model which expresses that the time-varying magnetic flux induces an electric field. This same electric field can modify the distribution of currents within the conductive parts. Such behavior is reflected in the modeling of induced currents.

Although the knowledge of the $\mathbf{b}$ and $\mathbf{h}$ fields is a sufficient information to determine the magnetic state of a given system, it is often more convenient to introduce intermediate quantities to represent the problem. Here, we only discuss the use of magnetic vector potential $\mathbf{a}$ although other potentials can be defined. From the equation 2.3, it is possible to derive the magnetic induction $\mathbf{b}$, from a magnetic vector potential $\mathbf{a}$ [105] such that

$$\mathbf{b} = \text{curl } \mathbf{a} . \quad (2.25)$$

This vector potential is not unique, but is defined to the gradient of a scalar function. The vector potential reference should therefore be fixed by an appropriate gauge condition.

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The definition of this vector potential allows to combine the magnetic Gauss law and the Faraday law in the following way

$$\mathbf{curl}(\mu^{-1} \mathbf{curl} \mathbf{a}) = \mathbf{j} , \quad (2.26)$$

for which the current density $\mathbf{j}$ behaves as the magnetic field source of which $\mathbf{a}$ is the unknown resultant field. It is therefore possible to obtain the induction field $\mathbf{b}$ and the magnetic field $\mathbf{h}$ based on $\mathbf{a}$. We also notice that the permeability of the material takes part of this equation and that consequently, a material with a high permeability will concentrate a larger quantity of induction flux lines (and thus of vector potential) for a given current source.

We note, besides, that the flow of $\mathbf{b}$ crossing a surface S is equal to the circulation of $\mathbf{a}$ along the contour dS of this same surface such that

$$\phi_S = \iint_S \mathbf{b} \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{a} \cdot d\mathbf{l} . \quad (2.27)$$

Notice that this **flux** ϕ_S is defined on an arbitrary region and is therefore not linked to any conducting sub-region of the domain ; this is the definition of flux in the strict sense. Later, we will develop a definition of the flux associated to massive conductors. This alternative definition is more appropriate for the identification of our model parameters and should not be confused with the definition of induction flux for any arbitrary surface, expressed by Equation 2.27.

Whenever the fields change over time, either by the variation of excitation currents or the motion of magnets, induced currents appear in the conductors and change the magnetic field distribution as well as the current density distribution compared to the static case. Remark that magnets motion is not covered hereby.

Without further assumptions, we can assume that the resulting current density can be divided into two components

$$\mathbf{j} = \mathbf{j}_0 + \mathbf{j}_{ed} , \quad (2.28)$$

where the $\mathbf{j}_0$ current density, corresponds to the solution of an electrokinetic problem with the same total currents I_i in the different conductors. The specificity of the induced current $\mathbf{j}_{ed}$ is a **zero mean** component over the conductor section. We then have

$$\text{div } \mathbf{j} = 0 , \quad (2.29)$$

$$\text{div } \mathbf{j}_0 = 0 , \quad (2.30)$$

from which we deduce that the divergence of the zero mean component current is also zero,

$$\text{div } \mathbf{j}_{ed} = 0 . \quad (2.31)$$

The total current flowing through the i^{th} conductor is calculated as the integral of the current density over the conductor cross section:

$$\int_{\Omega_{c_i}} \mathbf{j}_{ed} d\mathbf{S} = 0 , \quad (2.32)$$

$$\int_{\Omega_{c_i}} \mathbf{j}_0 d\mathbf{S} = \int_{\Omega_{c_i}} \mathbf{j} d\mathbf{S} = I_i , \quad (2.33)$$

$$\|\mathbf{j}_0\| = \frac{I_i}{\Sigma_i} , \quad (2.34)$$

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where Ω_c represents the whole conducting domain, Ω_{c_i} represents the i^{th} conductor's domain and Σ_i is the cross-sectional area of said conductor. In an arbitrary plane of a straight conductor, $\mathbf{j}_0$ is simply the average current density over the conductor cross section.

From then on, we apply **Poynting's theorem** to the equations of magnetodynamics described above. This gives us

$$\underbrace{\int_{\Omega} -\mathbf{grad} \mathbf{v} \cdot \mathbf{j} d\Omega}_{\text{power supplied to the system by the accesses}} = \underbrace{\int_{\Omega} \partial_t \mathbf{a} \cdot \mathbf{j} d\Omega}_{\text{magnetic power}} + \underbrace{\int_{\Omega} \mathbf{e} \cdot \mathbf{j} d\Omega}_{\text{Joule losses}}, \quad (2.35)$$

expression which can itself be developed by substituting the current density $\mathbf{j}$ by its new definition 2.28. We then obtain

$$\int_{\Omega} -\mathbf{grad} \mathbf{v} \cdot (\mathbf{j}_0 + \mathbf{j}_{ed}) d\Omega = \int_{\Omega} \partial_t \mathbf{a} \cdot (\mathbf{j}_0 + \mathbf{j}_{ed}) d\Omega + \int_{\Omega} \frac{||(\mathbf{j}_0 + \mathbf{j}_{ed})||^2}{\sigma} d\Omega, \quad (2.36)$$

which simplifies into

$$\underbrace{\int_{\Omega} -\mathbf{grad} \mathbf{v} \cdot \mathbf{j}_0 d\Omega}_{\text{power supplied to the system by the accesses}} = \underbrace{\int_{\Omega} \partial_t \mathbf{a} \cdot (\mathbf{j}_0 + \mathbf{j}_{ed}) d\Omega}_{\text{magnetic power } P_{\text{mag}}} + \underbrace{\int_{\Omega} \frac{||\mathbf{j}_0||^2}{\sigma} d\Omega}_{\text{copper losses DC}} + \underbrace{\int_{\Omega} \frac{||\mathbf{j}_{ed}||^2}{\sigma} d\Omega}_{\text{supplemental copper losses}}; \quad (2.37)$$

since

$$\int_{\Omega} -\mathbf{grad} \mathbf{v} \cdot \mathbf{j}_{ed} d\Omega = 0 \quad (2.38)$$

and

$$\int_{\Omega} \mathbf{j}_0 \cdot \mathbf{j}_{ed} d\Omega = 0. \quad (2.39)$$

It is also possible to reach the same result based on the Faraday equation. Such a case is of interest to us because it is from the very same equation that the magnetodynamic finite element formulation is constructed and from which the global quantities defined at the accesses are derived. This approach will allow us to bring out the additional eddy currents losses term in our expression of the magnetic power. Starting from

$$-\mathbf{grad} \mathbf{v} = \partial_t \mathbf{a} + \mathbf{e}, \quad (2.40)$$

we can, on the one hand, multiply the expression 2.40 by the uniform current density $\mathbf{j}_0$ and integrate it over a conducting region Ω_i , and on the other hand multiply the same equation 2.40 by the non uniform current density $\mathbf{j}_{ed}$ and integrate it. These calculations indicate that

$$\int_{\Omega_{c_i}} -\mathbf{grad} \mathbf{v} \cdot \mathbf{j}_0 d\Omega = \int_{\Omega_{c_i}} \partial_t \mathbf{a} \cdot \mathbf{j}_0 d\Omega + \int_{\Omega_{c_i}} \frac{||\mathbf{j}_0||^2}{\sigma} d\Omega, \quad (2.41)$$

$$\underbrace{\int_{\Omega_{c_i}} -\mathbf{grad} \mathbf{v} \cdot \mathbf{j}_{ed} d\Omega}_{=0} = \int_{\Omega_{c_i}} \partial_t \mathbf{a} \cdot \mathbf{j}_{ed} d\Omega + \int_{\Omega_{c_i}} \frac{||\mathbf{j}_{ed}||^2}{\sigma} d\Omega. \quad (2.42)$$

It is then appropriate to define the concept of **magnetic power** P_{mag} as

$$P_{\text{mag}} := \int_{\Omega} \partial_t \mathbf{a} \cdot (\mathbf{j}_0 + \mathbf{j}_{ed}) d\Omega = -\partial_t E_{\text{mag}} + P_{\text{fe}} \quad (2.43)$$

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where E_{mag} represents the stored magnetic energy of the system and P_{fe} represents the power of the iron losses. By injecting Equation 2.42 into the magnetic power 2.43, we obtain the following expression

$$\int_{\Omega} \partial_t \mathbf{a} \cdot \mathbf{j}_0 d\Omega = -\partial_t E_{\text{mag}} + P_{\text{fe}} + \int_{\Omega} \frac{||\mathbf{j}_{\text{ed}}||^2}{\sigma} d\Omega. \quad (2.44)$$

The additional losses that occur in the harmonic regime are due to eddy currents that appear in the conductive parts. We choose to name them in reference to the **eddy currents** from which they originate:

$$P_{\text{ed}} = \int_{\Omega} \frac{||\mathbf{j}_{\text{ed}}||^2}{\sigma} d\Omega, \quad (2.45)$$

which finally provides

$$\int_{\Omega} \partial_t \mathbf{a} \cdot \mathbf{j}_0 d\Omega = -\partial_t E_{\text{mag}} + P_{\text{fe}} + P_{\text{ed}}. \quad (2.46)$$

This expression is verified for the whole system but can also be expressed for each conductor. In this case, the integral over the i^{th} conductor gives the magnetic energy variation, the iron power losses and the copper eddy current losses, associated with the current flowing in the conductor

$$\int_{\Omega_{c_i}} \partial_t \mathbf{a} \cdot \mathbf{j}_0 d\Omega = -\partial_t E_{\text{mag}_i} + P_{\text{fe}_i} + P_{\text{ed}_i}. \quad (2.47)$$

We can then extract the expression of the average current on the conductor section

$$\frac{I_i}{\Sigma_i} \int_{\Omega_{c_i}} \partial_t \mathbf{a} \cdot \mathbf{1} d\Omega = -\partial_t E_{\text{mag}_i} + P_{\text{fe}_i} + P_{\text{ed}_i}. \quad (2.48)$$

By noting the average vector potential over a conducting region Ω_{c_i} like

$$\Phi_i := \frac{1}{\Sigma_i} \int_{\Omega_{c_i}} \mathbf{a} \cdot \mathbf{1} d\Omega, \quad (2.49)$$

we can therefore write that the **magnetic power balance** 2.48 described for a conductor i , can be rewritten as

$$\partial_t \Phi_i I_i = -\partial_t E_{\text{mag}_i} + P_{\text{fe}_i} + P_{\text{ed}_i}, \quad (2.50)$$

which is expressed in the **harmonic regime** as

$$\frac{1}{2} j\omega \bar{\Phi}_i \bar{I}_i^* = -j\omega E_{\text{mag}_i} + \underbrace{P_{\text{fe}_i} + P_{\text{ed}_i}}_{P_{\omega_i}}, \quad (2.51)$$

$$\frac{1}{2} \bar{\Phi}_i \bar{I}_i^* = -E_{\text{mag}_i} - j \frac{P_{\omega_i}}{\omega}. \quad (2.52)$$

The power term P_{ω_i} corresponds to the part of the system losses that directly depends on the frequency. These are the losses in the magnetic core (hysteresis, eddy current and excess) and the eddy current losses in the conductors (skin and proximity effects).

The decomposition into real and imaginary parts of this last expression gives us the following equations

$$E_{\text{mag}_i} = -\frac{1}{2} \text{Re} \left(\bar{\Phi}_i \bar{I}_i^* \right), \quad (2.53)$$

$$P_{\omega_i} = -\frac{1}{2} \omega \text{Im} \left(\bar{\Phi}_i \bar{I}_i^* \right). \quad (2.54)$$

Under the linear assumption, the total magnetic energy of the system E_{mag} as well as the total power of the frequency losses P_{ω} are then calculated as the sum of each contribution from each conductor.

Finally, in harmonic regime, the iron losses alone can be deduced from the magnetic power by isolating its imaginary part such that

$$P_{\text{fe}} = \text{Re} (P_{\text{mag}}) , \quad (2.55)$$

$$P_{\text{fe}} = -\omega \text{Im} \left(\int_{\Omega} \bar{\mathbf{a}} \cdot \bar{\mathbf{j}} d\Omega \right) . \quad (2.56)$$

2.2.3 Global Quantities for Magnetic Parameters Identification

Although the distribution of local fields provides precise information about the state of an electromagnetic device, it is necessary to convert them into measurable quantities at the **macroscopic** level. We therefore consider the integrated forms of the local fields as **global quantities** which make sense from the perspective of engineers in charge of designing and optimizing the device. Moreover, such aggregated quantities correspond to the only physical accesses of the device that are measurable at our macroscopic scale, *i.e.* voltages and currents. For magnetic problems, we employ the total current $\bar{I}$ measured across the section of a given conductor

$$\bar{I}_k = \iint_{\Omega_{c_k}} \bar{\mathbf{j}} \cdot d\mathbf{S} , \quad (2.57)$$

the average magnetic vector potential crossing the conductor's surface

$$\bar{\Phi}_k = \frac{1}{\Sigma_k} \int_{\Omega_{c_k}} \bar{\mathbf{a}} \cdot \mathbf{1} d\Omega , \quad (2.58)$$

which will be considered as the total **magnetic flux** attached to the k^{th} conductor and the electrodes voltages

$$\bar{U}_k = \oint_{\mathcal{C}_k} \bar{\mathbf{e}} \cdot d\mathbf{l} . \quad (2.59)$$

The objective of the global quantities is to provide amounts that are comparable to the measurements that can be made in practice. They are used as the variables of a numerical test bench which allows to characterize a magnetic component by a series of independent identifications. One would proceed in the same way on a physical component by fixing a certain global quantity on one access and by observing a second global quantity on the same access or on another. The interest of the numerical method is on the one hand to be able to avoid the realization of a great number of prototypes to be evaluated, as well as to be able to discretize the component according to more accesses than those physically accessible.

To do this, we define a set of conductive regions in our domain. Let's suppose that each region corresponds to a massive conductor (a strand, a lead, a turn, etc.), we can then fix the global quantities of current, flux and voltage as defined above and reason according to them for the identification of the physical parameters of the component.

Assuming linearity between the fluxes and the currents that give rise to them, we can

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write that a flux is derived from each current through the complex parameter $\bar{\Lambda}_{kl}$ such that

$$\bar{\Phi}_k = \sum_l \Lambda_{kl} \bar{I}_l \quad (2.60)$$

$$\bar{\Phi}_k = \sum_l \left(L_{kl} - j \frac{R_{\omega kl}}{\omega} \right) \bar{I}_l . \quad (2.61)$$

Where Λ_{kl} represents a complex inductance whose real part corresponds to an inductance and whose imaginary part corresponds to a resistance assimilated to ferromagnetic losses. We therefore observe a phase difference between the flux and the current that gave it rise.

The inductances defined as in 2.61 contribute to the magnetic energy of the system. This can be formalized as follows

$$E_{\text{mag}} = -\frac{1}{2} \sum_k \text{Re} \left(\bar{\Phi}_k \bar{I}_k^* \right) , \quad (2.62)$$

$$E_{\text{mag}} = -\frac{1}{2} \sum_k \sum_l L_{kl} \text{Re}(\bar{I}_l \bar{I}_k^*) . \quad (2.63)$$

Similarly, the loss resistances of the induced effects (ferromagnetic losses and induced copper losses) are obtained as follows

$$P_{\omega} = -\frac{1}{2} \sum_k \text{Im} \left(\bar{\Phi}_k \bar{I}_k^* \right) \omega , \quad (2.64)$$

$$P_{\omega} = -\frac{1}{2} \sum_k \sum_l R_{\omega kl} \text{Im}(\bar{I}_l \bar{I}_k^*) . \quad (2.65)$$

It is always possible to write the Poynting theorem, which we do for a set of N conductors. This gives us

$$\underbrace{\sum_{k=1}^N \bar{U}_k \bar{I}_k^*}_{\text{power supplied to the system by the accesses}} = \underbrace{P_{\text{mag}}}_{\text{magnetic power}} + \underbrace{\sum_{k=1}^N R_{\text{DC},kk} I_k^2}_{\text{DC copper losses}} + \underbrace{\sum_{k=1}^N \sum_{l=1}^N R_{\text{ed},kl} \bar{I}_l \bar{I}_k^*}_{\text{frequency induced copper losses}} , \quad (2.66)$$

which corresponds to the equation 2.37 that was defined through the local fields, but which, this time, is applied through the associated global quantities and can be further developed as

$$\underbrace{\sum_{k=1}^N \bar{U}_k \bar{I}_k^*}_{\text{power supplied to the system by the accesses}} = \underbrace{-j\omega E_{\text{mag}} + P_{\text{fe}}}_{\text{magnetic power}} + \underbrace{\sum_{k=1}^N R_{\text{DC},kk} I_k^2}_{\text{DC copper losses}} + \underbrace{\sum_{k=1}^N \sum_{l=1}^N R_{\text{ed},kl} \bar{I}_l \bar{I}_k^*}_{\text{supplemental copper losses}} , \quad (2.67)$$

$$\underbrace{\sum_{k=1}^N \bar{U}_k \bar{I}_k^*}_{\text{power supplied to the system by the accesses}} = \underbrace{-j\omega \sum_{k=1}^N \sum_{l=1}^N L_{kl} \bar{I}_l \bar{I}_k^*}_{\text{magnetic power}} + \underbrace{P_{\text{fe}}}_{\text{DC copper losses}} + \underbrace{\sum_{k=1}^N R_{\text{DC},kk} I_k^2}_{\text{supplemental copper losses}} + \underbrace{\sum_{k=1}^N \sum_{l=1}^N R_{\text{ed},kl} \bar{I}_l \bar{I}_k^*}_{\text{supplemental copper losses}} . \quad (2.68)$$

Noting well that the ferromagnetic losses have already been defined in Equation 2.56 such that

$$P_{fe} = \omega \operatorname{Im} \left(\int_{\Omega} \bar{\mathbf{a}} \cdot \bar{\mathbf{j}} d\Omega \right) . \quad (2.69)$$

Unfortunately, this last expression cannot be isolated for each conductor. It is only valid for the whole domain considered. However, we choose to arbitrarily distribute the magnetic losses (hysteresis and eddy current) equally amongst all conductors based on the magnetomotive force they generate. This is a realistic assumption since two conductors generating the same magnetomotive force will generate the same magnetic induction and therefore the same amount of losses in the core.

If only one element is supplied with current, then all the iron losses can be inferred from this one. We can therefore attribute a resistance to it which represents the ferromagnetic losses, although these losses are actually located in the transformer core.

$$P_{fe} = R_{fe,kk} I_k^2 \Big|_{I_{k \neq l} = 0} . \quad (2.70)$$

The off-diagonal terms are then imposed such that

$$R_{fe,kl} = \sqrt{R_{fe,kk} R_{fe,ll}} . \quad (2.71)$$

Power transformers generally operate close to the open circuit equivalent, so this is a reasonable approach. Especially since it overestimates the iron losses rather than underestimating them.

When only one access is supplied, all the ferromagnetic losses are associated with the current flowing in that circuit. Moreover, only a current I_k flows through it and the voltage at its terminals can therefore be written in the following simplified form

$$\bar{U}_k = R_{DC,kk} \bar{I}_k + R_{ed,kk} \bar{I}_k + R_{fe,kk} \bar{I}_k + j\omega L_{kk} \bar{I}_k , \quad (2.72)$$

$$\bar{U}_k = R_{DC,kk} \bar{I}_k + R_{\omega,kk} \bar{I}_k + j\omega L_{kk} \bar{I}_k , \quad (2.73)$$

$$\bar{U}_k = R_{DC,kk} \bar{I}_k + j\omega \bar{\Phi}_k . \quad (2.74)$$

For all other conductors, the injected power is zero. Nevertheless, a voltage appears at their terminals following Lenz's law such that

$$\bar{U}_l = j\omega \bar{\Phi}_l , \quad (2.75)$$

$$\bar{U}_l = R_{ed,lk} \bar{I}_k + R_{fe,lk} \bar{I}_k + j\omega L_{lk} \bar{I}_k , \quad (2.76)$$

$$\bar{U}_l = R_{\omega,lk} \bar{I}_k + j\omega L_{lk} \bar{I}_k . \quad (2.77)$$

If we define $\underline{U}$ as the vector of voltages at the component ports and $\underline{I}$ as the vector of currents entering via these ports (considering the motor convention), it is then possible to obtain the following condensed form:

$$\underline{U} = \underline{Z}(\omega) \underline{I} , \quad (2.78)$$

$$\underline{U} = \left[\underline{R}_{DC} + \underline{R}_{ed}(\omega) + \underline{R}_{fe}(\omega) + j\omega \underline{L}(\omega) \right] \underline{I} \quad (2.79)$$

$$\underline{U} = \underline{R}_{DC} \underline{I} + j\omega \underline{\Phi}(\omega) \quad (2.80)$$

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Note that the resistance matrix $\underline{R}_{DC}$ is purely diagonal and corresponds in fact to the Joule effect losses in the conductor in continuous regime. There are no currents induced neither by skin effect nor by proximity effect and the conductors do not induce losses towards each other.

The identification of the elements of each matrix is done by injecting a current successively at each access and by observing the values of voltage, flux and magnetic losses at the terminals or on the whole domain such as:

$$R_{fe,kk} = \frac{P_{fe}}{I_k^2} \Big|_{I_{l \neq k} = 0}, \quad (2.81)$$

$$R_{fe,kl} = \sqrt{R_{fe,kk} R_{fe,ll}}, \quad (2.82)$$

$$\underbrace{R_{ed,kl} + R_{fe,kl}}_{R_{\omega,kl}} = \text{Im} \left(\frac{\bar{\Phi}_k}{\bar{I}_l} \Big|_{I_{k \neq l} = 0} \right) \omega, \quad (2.83)$$

$$\underbrace{R_{DC,kl} + R_{ed,kl} + R_{fe,kl}}_{R_{kl}} = \text{Re} \left(\frac{\bar{U}_k}{\bar{I}_l} \Big|_{I_{k \neq l} = 0} \right), \quad (2.84)$$

$$L_{kl} = \text{Im} \left(\frac{\bar{U}_k}{\bar{I}_l} \Big|_{I_{k \neq l} = 0} \right) \frac{1}{\omega}, \quad (2.85)$$

$$L_{kl} = \text{Re} \left(\frac{\bar{\Phi}_k}{\bar{I}_l} \Big|_{I_{k \neq l} = 0} \right). \quad (2.86)$$

The self-inductances of each conductor can be calculated in this way, as well as all mutual inductances. The resistances can be obtained in the same way. However it is a global resistance that is obtained. It can then be useful to decompose this resistance in several terms in order to find the origin of the losses (DC, eddy current or ferromagnetic core) and to improve the design.

These equations allow the identification of the resistive and inductive parameters for each frequency of interest. Note that it is also possible to obtain the parameters in the continuous mode, at zero frequency. In this case, the ratio of voltage to current provides the DC loss resistance and the ratio of flux to current provides the inductance.

2.3 Electricity

In the previous section, we presented the magnetoquasistatic approach of magnetic components using the associated global quantities to represent the interactions of magnetic kind. In addition to those, electrical interactions also appear between the different conductors that make up the magnetic components. In this part, we will present the underlying equations and the global quantities associated with the electrical phenomena, treated as a separate problem of the magnetic one.

Just as magnetic field variations generate losses in ferromagnetic materials, electric field variations are the source of localized losses in materials identified as **dielectric**. We will therefore start by explaining what these dielectric materials are and detail how the local fields behave within them.

2.3.1 Dielectric Materials

In the previous chapter, we explained the role of insulating materials in the manufacture of magnetic components in order to keep certain conductive parts separated from each other. The insulating characteristic of these materials prevents free charges from passing from one conductor to another in an undesirable way and this is indeed the intended goal. However, in industrial practice, most of the materials used for insulation purposes are also dielectric: ceramics, polymers (polypropylene, polycarbonate, polyethylene, polyvinyl chloride), alumina, quartz, mica, mineral oils, etc. Insulators are materials that are not able to conduct electric current efficiently on a macroscopic scale because of the low presence and mobility of free charges. In reality, insulators do have some conductivity, although very low. Dielectrics have the property of being **polarizable** when subjected to an external electric field. This occurs because dielectrics possess charges that are not capable of large displacements, associated with macroscopic currents, but these bound charges are capable of moving around their equilibrium position. Therefore, even if the dielectric materials do not contain free electric charges, they react to the electric field and present a certain polarization. When bound charges undergo the effect of an external electric field, they tend to align with it. This is the polarization phenomenon which induces the reorganization of the charges through motion and reorientation. Without going into details, the polarization can be of four types : space charge, dipolar, atomic and electronic. The total polarization P consists in the resultant of those four mechanisms and the origins of the different processes have already been described in [31, pp. 25-27]. Although insulating and dielectric effects are described differently depending on their use (the former to separate parts brought to different potentials, the latter to polarize under the effect of an electric field), they are generally synonyms [41].

One can differentiate two main categories of dielectric: the first gathers the media that inherently exhibit an asymmetrical distribution of charges with polar molecules. It is the case of the water for instance, which possesses a permanent dipole moment and absorbs very well the micro-waves energy. The reorientation mechanism of existing microscopic dipoles under the effect of an external electric field is illustrated in Figure 2.4.

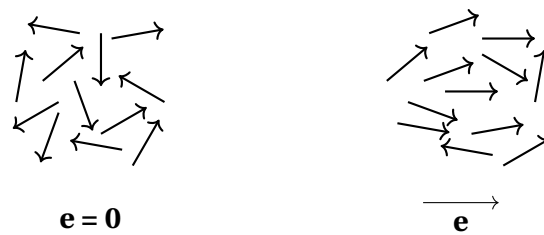


Figure 2.4: Illustration of the polarization effect by orientation of microscopic dipoles with zero net electric field on the left and positive electric field on the right part.

The second category gathers the media with the non polar characteristic and a high resistivity. Those materials, such as glass or ceramic, do not present a dipole moment in the absence of electric field due to the particles arrangement. Therefore, they do not absorb very well the energy from the micro-waves. The second kind (polymers, mica, Teflon, epoxy resin, etc.) are widespread in industrial domains and can be used as insulator. In the second category, which includes materials whose charge distribution remains symmetric in the absence of an external field, we distinguish two different polarization mechanisms: the first is ionic polarization and the second includes electronic and atomic polarization.

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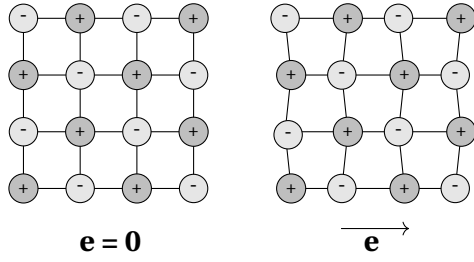


Figure 2.5: Atomic polarization.

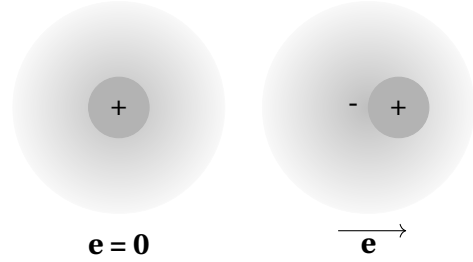


Figure 2.6: Electronic polarization.

As said before, one should remark that conduction currents can still appear within these media through different means such as surface currents due to impurities or partial discharges associated to holes and defects in the material. Such phenomena will not be studied further in this work.

For the representation of the electrical problem also, it is necessary to attach behavioral laws to Maxwell's equations in order to solve the system. Once again, a constitutive law is added to the original set of equations (2.1) - (2.4) that would otherwise be indefinite. In this case, the dielectric behavioral law is expressed as

$$\mathbf{d} = \varepsilon_0 \mathbf{e} + \mathbf{p} , \quad (2.87)$$

that relates the electric displacement $\mathbf{d}$ to the electric field $\mathbf{e}$.

Similar to the magnetization $\mathbf{m}$ included in the magnetic constitutive law, the dielectric behavioral law involves the polarization $\mathbf{p}$ of the material as described in the equation 2.87. This polarization corresponds to the reaction of non-moving charges in polarizable materials (otherwise called dielectrics), following the application of an external electric field. In the case of linear, homogeneous and isotropic materials, there exists a simple relation between the polarization and the electric field such that

$$\mathbf{p} = \varepsilon_0 \chi_e \mathbf{e} , \quad (2.88)$$

where the constant χ_e is called the electric susceptibility. Taking into account the behavioral law 2.87, we can deduce that

$$\mathbf{d} = \varepsilon_0 (1 + \chi_e) \mathbf{e} , \quad (2.89)$$

and pose

$$\epsilon_r = (1 + \chi_e) . \quad (2.90)$$

This allows us to write the behavioral law in the following form

$$\mathbf{d} = \varepsilon_0 \epsilon_r \mathbf{e} = \epsilon \mathbf{e} . \quad (2.91)$$

Note that more complicated relationships exist between electric field and polarization than the purely linear dependence. However, we will not assume these more complicated cases for this work. While the permittivity of a material is denoted ϵ in the time domain, it is denoted ε in the frequency domain by admitting the hypothesis of linearity allowing to directly transpose variables and parameters from one domain to the other.

2.3.2 Complex Permittivity and Dielectric Losses

From now on, we will assume that we are operating in the frequency domain with fields represented in the form of phasors and associated with complex type quantities. The polarization level of a dielectric medium is expressed by its permittivity. This parameter is considered here as a scalar in order to represent homogeneous, linear and isotropic materials. In the absence of material, as in vacuum, no permittivity is defined although the dielectric constant ε_0 exists. This parameter is linked to the Coulomb attraction model and defined as

$$\varepsilon_0 = 8.8541 \cdot 10^{-12} \text{ F} \cdot \text{m}^{-1} \quad (2.92)$$

and is often wrongly attributed to the permittivity of the vacuum and this, in a normalized way [41].

When a material is subjected to an electric field, its polarization is characterized by its permittivity which can be expressed through the relative permittivity ε_r of the material such that

$$\varepsilon = \varepsilon_r \varepsilon_0, \quad (2.93)$$

where ε_r is greater than 1 and ε corresponds to the absolute permittivity of the material.

When the electric field varies, the dipoles formed in the dielectric medium switch their orientation in order to follow the changes, this in turn creates losses and heat up the material. When we study the behavior of dielectric materials in the sinusoidal regime, we observe that the current and the voltage are not perfectly in quadrature, contrary to what a perfect capacitor model predicts. In fact, different phenomena are sources of losses which are characterized by a phase shift of less than 90° between the current and the voltage. In a capacitor modeled by a capacitance C_p and a parallel resistance R_p , we can define δ so that the losses can be characterized by a dissipation factor

$$\tan \delta = \frac{1}{\omega C_p R_p}, \quad (2.94)$$

which depends on the loss angle δ . In the harmonic condition, it is possible to represent this behavior through a complex relative permittivity in a similar way to what has been done for ferromagnetic materials in the previous section. Such a complex permittivity can be expressed as

$$\varepsilon_r = \varepsilon'_r - j \varepsilon''_r \quad (2.95)$$

and the permittivity of the electric behavioral law can then be expressed based on the complex relative permittivity

$$\varepsilon = \varepsilon_0 \varepsilon_r, \quad (2.96)$$

that relates the displacement field $\vec{d}$ to the electric field $\vec{e}$ in the electric model that we describe hereafter. With this approach in complex arithmetic, the tangent loss appears naturally as the ratio of the imaginary part to the real part such that

$$\tan \delta = \frac{\varepsilon''_r}{\varepsilon'_r}. \quad (2.97)$$

Furthermore, we have already noted that the polarization of materials can be due to different physical mechanisms that give rise to different types of polarizations: ionic, dipolar, atomic and electronic; and in turn leads to losses of different intensity. The activation of

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one polarization phenomenon or another depends on the frequency at which the material is excited and successive relaxation mechanisms are observed as the frequency increases. This first model, inherited from Debye [21], expresses the effects of each phenomena with a characteristic relaxation frequency: “*Each dielectric mechanism effect has a characteristic resonant frequency or relaxation frequency. As the frequency increases, the slower mechanisms drop out, leaving the faster ones to contribute to the storage.*” [14, p.6]. The illustration of the relaxation principle is provided in Figure 2.7 for the real and the imaginary parts of the complex permittivity.

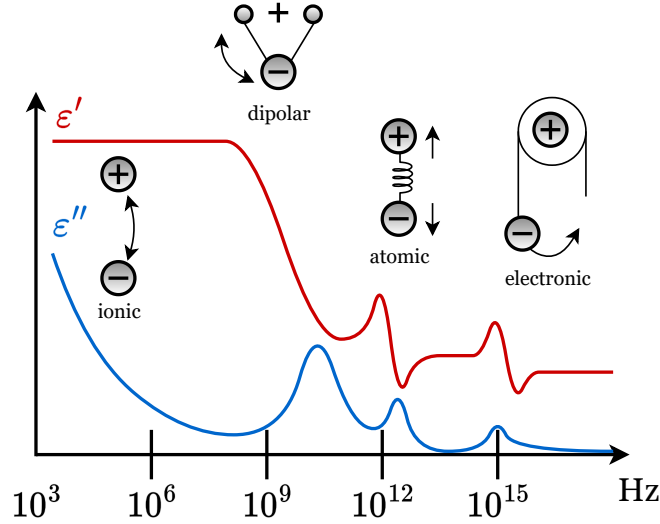


Figure 2.7: Illustration of the complex dielectric response for several Debye relaxations.

According to this model, each relaxation is represented by a complex permittivity approximated by a general equation. An extension of the Debye relaxation principle, the Cole-Cole model [13], expresses the frequency dependency of the permittivity as

$$\varepsilon^*(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (j\omega\tau)^{1-\alpha}}, \quad (2.98)$$

where ε^* is the complex dielectric constant, ε_s and ε_∞ are the "static" and "infinite frequency" dielectric constants and τ is the relaxation time constant. Remark that the Cole-Cole model, excerpted from the original paper [13, p. 346], narrowed to the Debye model as $\alpha = 1$.

It is worth closing this section on dielectrics, their polarization, their losses and their models by noting first that in harmonic regime, the permittivity reduces to a complex scalar under the linear, homogeneous and isotropic assumptions of the material. This simplifies the model we introduce in the next section into a **complex-valued** electrostatic model. Secondly, it is usually only a matter of considering the first relaxation mechanism that occurs at lower frequencies, since power electronics applications operate at frequencies that typically do not reach higher frequencies where relaxation mechanisms appear in the GHz range. Finally, it is quite rare to have access to the dependency between complex permittivity and frequency (or other parameters) without resorting to measurements. Manufacturers sometimes provide a relative permittivity range for the static case and rarely a loss tangent. These elements justify the use of a simple model to represent dielectric materials when frequency and temperature characterizations are not considered.

2.3.3 Electrostatic Type Model

In the section 2.1, we made the quasi-static hypothesis, which assumes that the fields vary slowly enough with time that the influence of the displacement current $\partial_t \mathbf{d}$ can be neglected; and this strong assumption is maintained throughout the manuscript. We have also seen that dielectric materials exhibit losses when stimulated in the harmonic regime through various polarization phenomena. These behaviors are correctly approximated at a **given frequency** by the use of a complex permittivity. In view of the two assumptions above, the electrical model we develop here can be compared to an electrostatic model because of the suppression of the displacement current. Nevertheless, the underlying harmonic hypothesis appears through the use of a complex permittivity and consequently the calculation of the $\bar{\mathbf{e}}$ and $\bar{\mathbf{d}}$ fields changes as they are then complex-valued.

As this second model focuses on the electric part, the magnetic phenomena are not considered and the assumptions imply no moving charges and no time-varying phenomena (*i.e.* $\mathbf{j} = 0$ and $\partial_t \cdot = 0$). The time dependency does not appear explicitly in the model, even if it is held implicitly through the dielectric losses. The electric field $\bar{\mathbf{e}}$ and the electric displacement field $\bar{\mathbf{d}}$ obey Maxwell's equations (2.2) and (2.4), respectively the Faraday and the electric Gauss equations. As the time dependency vanishes, the Faraday equation simplifies into

$$\text{curl } \bar{\mathbf{e}} = \mathbf{0} , \quad (2.99)$$

from which we can deduce that the electric field derives from a scalar potential $\bar{v}$ such that

$$\bar{\mathbf{e}} = -\text{grad } \bar{v} . \quad (2.100)$$

However, note that the potential is known to within a scalar value that will have to be fixed as the reference electric potential. A commonly accepted choice is to fix a zero potential on the domain boundary. This boundary can be located at a certain physical distance or at infinity, depending on whether an appropriate geometrical transformation is used. Moreover, the electric Gauss' Law yields to

$$\text{div } \bar{\mathbf{d}} = \rho . \quad (2.101)$$

The combination of these equations with the constitutive law of dielectric materials gives us Poisson's equation which models the electrostatic system in terms of scalar electric potential

$$-\varepsilon \text{ div grad } \bar{v} = \rho , \quad (2.102)$$

$$\Delta \bar{v} = -\frac{\rho}{\varepsilon} . \quad (2.103)$$

With Poisson's equation, it is possible to calculate the electric potential distribution $\bar{v}$ and by extension the electric field distribution $\bar{\mathbf{e}}$ based on a charge density ρ fixed as the source of the electric field. In the same way as ferromagnetic materials whose high permeability concentrates the magnetic induction, dielectric materials with permittivity greater than 1 concentrate a larger electric displacement $\bar{\mathbf{d}}$. The modeling follows the same logic as for magnetic phenomena and exploits the definition of potentials and charges integrated on given conductor volumes. Poisson's equation is not sufficient on its own to fully determine the problem, it must be assorted with limit conditions on the boundary of the domain, for instance Dirichlet or Neuman conditions. All together, they make up a complete set of equations sufficient to describe the electrostatic behaviors. An **electrostatic** model

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expressed with the time notation (despite the absence of time derivative) would provide a Poisson equation such as

$$\Delta v = -\frac{\rho}{\epsilon} , \quad (2.104)$$

implying real values. This justifies the name of the complex-valued model as **electrostatic type** model in view of their shared similarities.

In the same way as was done for the magnetic case, the electrostatic potential energy can be expressed in terms of the local fields $\mathbf{e}$ and $\mathbf{d}$, respectively the electric field and the electric displacement field, with Ω being the domain of definition of the system. The definition of electric power P_{elec} follows naturally from this in the temporal case, just as it did for magnetic power

$$P_{\text{elec}} := \int_{\Omega} \partial_t \mathbf{d} \cdot \mathbf{e} d\Omega = -\partial_t E_{\text{elec}} + P_{\text{diel}} . \quad (2.105)$$

Here, E_{elec} represents the electric energy of the system and P_{diel} represents the power dissipated by the dielectric losses. In the harmonic regime, this definition can be directly adapted as

$$\int_{\Omega} \bar{\mathbf{d}} \cdot \bar{\mathbf{e}} d\Omega = -E_{\text{elec}} - j \frac{P_{\text{diel}}}{\omega} . \quad (2.106)$$

One can therefore notice a large number of similarities between the magnetodynamic and electrostatic models in complex arithmetic. The definitions of powers as a function of local fields lead to the same interpretations in both cases, which facilitates the exploitation of the results. In the same way, the definition of intermediate concepts such as global quantities is interpreted in an almost identical way in both models. However, the resolution of the two systems is quite different and leads to physical results that are complementary to each other; the magnetic couplings in one case and the electrical couplings in the other. Only the underlying concepts share the same representation.

2.3.4 Voltages and Electric Charges as Global Quantities

The same approach as the one followed in the magnetic modeling is taken in order to convert the local fields into **global quantities**, measurable at the macroscopic level. In this case, we choose the electrical voltage maintained between two conductors. Considering that the reference of voltages can be located arbitrarily, we choose to put the zero reference at the infinite, which correspond to the external boundary of the domain under the infinite transform [52]. The potential of a given point in space can therefore be defined in relation to the reference potential as

$$\bar{V}_a = - \int_{\infty}^a \bar{\mathbf{e}} \cdot d\mathbf{l} . \quad (2.107)$$

If a is a point associated with an ideal conductor, then $\bar{V}_a$ is also the potential of this same conductor at equilibrium. Then, it is possible to calculate any potential difference between two conductors based on the same definition. We will note $\bar{V}_{ab}$ the voltage difference between conductors a and b :

$$\bar{V}_{ab} = \bar{V}_a - \bar{V}_b = - \int_a^b \bar{\mathbf{e}} \cdot d\mathbf{l} . \quad (2.108)$$

To this global quantity of potential $\bar{V}_a$, we can associate a dual quantity (as in the case of the electric current and the magnetic flux), in this case the total charge carried by a conduc-

tor a , and defined as

$$\bar{Q}_a = \int_{V_a} \rho \, dV, \quad (2.109)$$

for space charges. And that can also be expressed as the electric displacement flux around the conducting region

$$\bar{Q}_a = \int_{\partial V_a} \bar{\mathbf{d}} \cdot d\mathbf{S}. \quad (2.110)$$

The global electrical quantities have the same interest as those defined in magnetism. The studied effects can be easily modeled at different scales (one turn, one lead, etc.) and their physical interpretation is immediate. For each chosen region, we can associate a **global voltage** quantity and a **global charge** quantity which are used to identify the electrical parameters of the component. Assuming linearity between the charges and the voltages that give rise to them, we can write that a total charge is derived from each voltage via a complex parameter. Once those parameters have been identified, such linear equations allow to deduce the charge accumulated on each conductor, only based on the knowledge of their voltages. Two different approaches, yet equivalent, are presented in the next sections and the relationships between voltages and charges of a system of conductors can be established by the following conjunction as developed in [28, pp.79-80].

2.3.4.1 Influence Coefficients

In the same way that the complex inductance is a parameter which links the electric current and the magnetic flux according to a linear dependency hypothesis, we can define a quantity which links the total electric potential of an electrode to the charge it carries. For that matter, we introduce the influence coefficients $\bar{K}_{kl}$ that are complex-valued and that allow to write

$$\bar{Q}_k = \sum_l K_{kl} \bar{V}_l, \quad (2.111)$$

$$\bar{Q}_k = \sum_l \left(\kappa_{kl} - j \frac{\gamma_{kl}}{\omega} \right) \bar{V}_l. \quad (2.112)$$

Equation 2.111 can be referred to as electrostatic equations of second kind [29, p.155]. The $\bar{K}_{kl}$ coefficient represents the charge carried by the k^{th} conductor when the l^{th} conductor is raised at 1 V, all other conductors being at the 0 reference potential. “The $\bar{K}_{kl}|_{k \neq l}$ is the influence coefficient between two conductors and the $\bar{K}_{kk}$ coefficient is the own capacity of the conductor in the presence of all the others” [43, p.33]. Remark that the $\bar{K}_{kk}$ coefficient, being the own capacity of the conductor, is defined positive, while the $\bar{K}_{kl}$ coefficients are defined negative.

Based on the definition of the influence coefficients, the expression of the electric energy of the system can be set up as

$$\begin{aligned} E_{\text{elec}} &= \frac{1}{2} \sum_k \text{Re} \left(\bar{Q}_k \bar{V}_k^* \right) \\ &= \frac{1}{2} \sum_k \sum_l \kappa_{kl} \text{Re} \left(\bar{V}_l \bar{V}_k^* \right), \end{aligned} \quad (2.113)$$

and a similar expression can be obtained for the dielectric losses, by involving the imaginary

part of the influence coefficients in the expression such that

$$\begin{aligned} P_{\text{diel}} &= \frac{1}{2} \sum_k \text{Im} \left(\bar{Q}_k \bar{V}_k^* \right) \omega \\ &= \frac{1}{2} \sum_k \sum_l \gamma_{kl} \text{Im} \left(\bar{V}_l \bar{V}_k^* \right) \omega . \end{aligned} \quad (2.114)$$

It is thus observed that the charges, potentials and complex influence coefficients are the electrostatic equivalents of the fluxes, currents and complex inductances previously defined in magnetoquasistatic. We will now evaluate another method that relates charges and potentials together via parameters other than influence coefficients.

2.3.4.2 Partial Capacitances

It is possible to express the relations between charges and voltages through the voltage differences between the conductors, rather than through the absolute voltages. This approach leads to the definition of partial capacitances which are exploitable in the sense of the classical circuit theory [1]. It is therefore a question of expressing the charge as a function of the differences of potential as follows

$$\bar{Q}_k = \sum_l \bar{T}_{kl} (\bar{V}_k - \bar{V}_l) + \bar{T}_k \bar{V}_k . \quad (2.115)$$

These equations are referred to as the electrostatic equations of the third kind [29] and involve the complex parameters $\bar{T}_{kl}$ and $\bar{T}_k$. The first one corresponds to the partial capacitance linking the two conductors k and l to each other; and the second one corresponds to the partial capacitance linking the conductor k to the reference region (for example the external boundary of the domain), of zero potential by definition. These parameters $\bar{T}$ are therefore similar to **complex capacitances**, the real part of which corresponds to a capacitance associated to a stored electrical energy and the imaginary part of which corresponds to a conductance associated to dielectric losses. We find again the parallel with the complex inductance $\bar{L}$ linking the fluxes and the currents. We can also develop the expression 2.115 in order to make the real and imaginary parts of the complex capacitances $\bar{T}$ appear:

$$\bar{T} = \left(C - j \frac{G}{\omega} \right) , \quad (2.116)$$

where the ideal capacitances are represented by the parameters C (respectively C_k and C_{kl}) and the conductances by the parameters G (respectively G_k and G_{kl}). These electrostatic equations of the third kind, expressed as via 2.115 are deduced from the equations of the second kind 2.111. To do this, we can simultaneously subtract and add the term $\sum_l \bar{K}_{kl} \bar{V}_k$, which leads to

$$\bar{Q}_k = \sum_l \bar{K}_{kl} \bar{V}_l - \sum_l \bar{K}_{kl} \bar{V}_k + \sum_l \bar{K}_{kl} \bar{V}_k , \quad (2.117)$$

$$= \sum_l -\bar{K}_{kl} (\bar{V}_k - \bar{V}_l) + \sum_l \bar{K}_{kl} \bar{V}_k , \quad (2.118)$$

and from this relation, it is then possible to observe that the partial capacitance of the conductor k with respect to the reference region of the potential such that

$$\bar{T}_k = \sum_l \bar{K}_{kl} , \quad (2.119)$$

as well as the partial capacitances between different conductors such as

$$\bar{I}_{kl} = -\bar{K}_{kl} . \quad (2.120)$$

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From these last two equations, we can obtain the same identification for the real and imaginary parts such that

$$C_{kl} = -\kappa_{kl} \quad (2.121)$$

$$C_k = \sum_l \kappa_{kl} , \quad (2.122)$$

$$G_{kl} = -\gamma_{kl} \quad (2.123)$$

$$G_k = \sum_l \gamma_{kl} . \quad (2.124)$$

It is sometimes useful to exploit a condensed form in order to convert the influence coefficients into partial capacitances. We then have to define a tensor χ_{ijkl} such that

$$\bar{\Gamma}_{ij} = \chi_{ijkl} K_{kl} , \quad (2.125)$$

with

$$\chi_{ijkl} = \begin{cases} 1 & \text{if } i = j = k \\ -1 & \text{if } i = k \text{ and } j = l \neq k \\ 0 & \text{otherwise} \end{cases} . \quad (2.126)$$

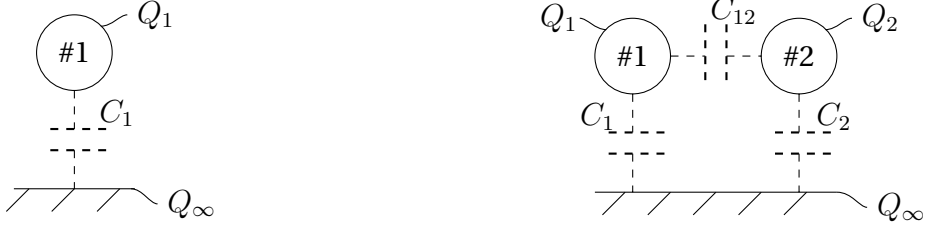
It is then possible to set up the expression of the electric energy in terms of partial capacitances such that

$$\begin{aligned} E_{\text{elec}} &= \frac{1}{2} \sum_k \text{Re} \left(\bar{Q}_k \bar{V}_k^* \right) \\ &= \frac{1}{2} \sum_k \sum_l C_{kl} \text{Re} \left((\bar{V}_k - \bar{V}_l) \bar{V}_k^* \right) + \frac{1}{2} \sum_k C_k V_k^2 \\ &= \frac{1}{2} \sum_k \sum_l C_{kl} V_k^2 + \frac{1}{2} \sum_k C_k V_k^2 - \frac{1}{2} \sum_k \sum_l C_{kl} \text{Re} \left(\bar{V}_l \bar{V}_k^* \right) , \end{aligned} \quad (2.127)$$

as well as the dielectric losses if the conductances are involved instead of the ideal capacitances which leads to

$$\begin{aligned} P_{\text{diel}} &= \frac{1}{2} \omega \sum_k \text{Im} \left(\bar{Q}_k \bar{V}_k^* \right) \\ &= \frac{1}{2} \omega \left(\sum_k \sum_l G_{kl} \text{Im} \left((\bar{V}_k - \bar{V}_l) \bar{V}_k^* \right) + \sum_k G_k V_k^2 \right) \\ &= \frac{1}{2} \omega \left(\sum_k \sum_l G_{kl} V_k^2 + \sum_k G_k V_k^2 - \sum_k \sum_l G_{kl} \text{Im} \left(\bar{V}_l \bar{V}_k^* \right) \right) . \end{aligned} \quad (2.128)$$

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Influence Coefficient	Partial Capacitance	Influence Coefficient	Partial Capacitance
$Q_\infty = -\kappa_{11} V_1$	$Q_\infty = -C_1 V_1$	$Q_\infty = -(\kappa_{11} + \kappa_{21}) V_1 - (\kappa_{12} + \kappa_{22}) V_2$	$Q_\infty = -C_1 V_1 - C_2 V_2$
$Q_1 = \kappa_{11} V_1$	$Q_1 = C_1 V_1$	$Q_1 = \kappa_{11} V_1 + \kappa_{12} V_2$	$Q_1 = C_{12} (V_1 - V_2) + C_1 V_1$
		$Q_2 = \kappa_{21} V_1 + \kappa_{22} V_2$	$Q_2 = C_{21} (V_2 - V_1) + C_2 V_2$

Figure 2.9: Charges upon conductors at equilibrium expressed with influence coefficients and partial capacitances.

The approach of partial capacitances offers the interesting property that the Γ_{kl} parameters are positive. Indeed, if the k^{th} conductor is in the presence of another conductor, it will exhibit a positive charge if $V_k > V_l$ (and conversely). Therefore, the partial capacitance between two conductors must be positive: $\Gamma_{kl} > 0$. The same reasoning still applies for the partial capacitances between conductors and the reference. The electric charges that are established on conductors in equilibrium with their environment can be represented with the influence coefficients as well as with the partial capacitances as long as these expressions verify that the total charge is zero. We can illustrate these conductors at equilibrium in the lossless case for an isolated conductor and for a set of two conductors, as in Figure 2.9.

Transformer Modeling Using Lumped-Parameter Circuits

The physical models already developed in the previous chapter enable us to establish the relationships between the **global quantities** such as from the electric current to the magnetic flux on one hand and from the voltage to the electric charges on the other. It is possible to do this by solving the corresponding systems: the **magnetic** one for the current-flux dependency and **electric** one for the voltage-charge dependency. With the approach adopted here, it is the finite element method that provides the values of the exploitable global quantities that can be identified to perform frequency or time domain simulations afterwards. By doing so, it is possible to break the components topology into smaller parts and proceed to the identification of the magnetic and electric parameters through different subsets of groups of turns: the **sections**.

In this chapter, we first present the method developed to obtain the different levels of discretization available and how these different representations of the magnetic component provide varying amounts of information. Such discretization levels are expressed for both the magnetic and electric models of the studied devices and coupled through circuit equations subsequently. Then, the second part focuses on explaining how these physical quantities can be exploited through a circuit simulator and the choices made in order to exploit the same circuit model for the frequency and time domains. The search for a consistent model and the desire to express it by mean of linear components that could be easily interpreted by designers guided the implementation of this second part.

3.1 Notation Conventions of Magnetic Components

It is important to adopt an effective and unambiguous nomenclature to designate the various conductive elements making up the winding topology: coils, leads, strands. A clear notation must not only highlight the discretization of the magnetic component: windings, turns, sections, but also indicate whether reference is being made to the **finite element model** of the component or to its **circuit equivalent**. For the sake of consistency, the notation is identical for inductors and transformers, the only difference being that an inductor has only one winding.

Windings are designated by the letter **W** and numbered in ascending order. Similarly, the turns that make up a winding are designated by the letter **T** and numbered in ascending order, resetting the count with each winding. The same logic applies to the leads that make up a winding, designated by the letter **L**. For greater simplicity, we deliberately omit to indicate the lead if there is only one, as this is the case for many components. An example is given in

the following figure to illustrate the notation adopted for a transformer with two windings, the first of which consists of 5 turns, wound up on 2 leads, and the second of 4 turns with a single lead.

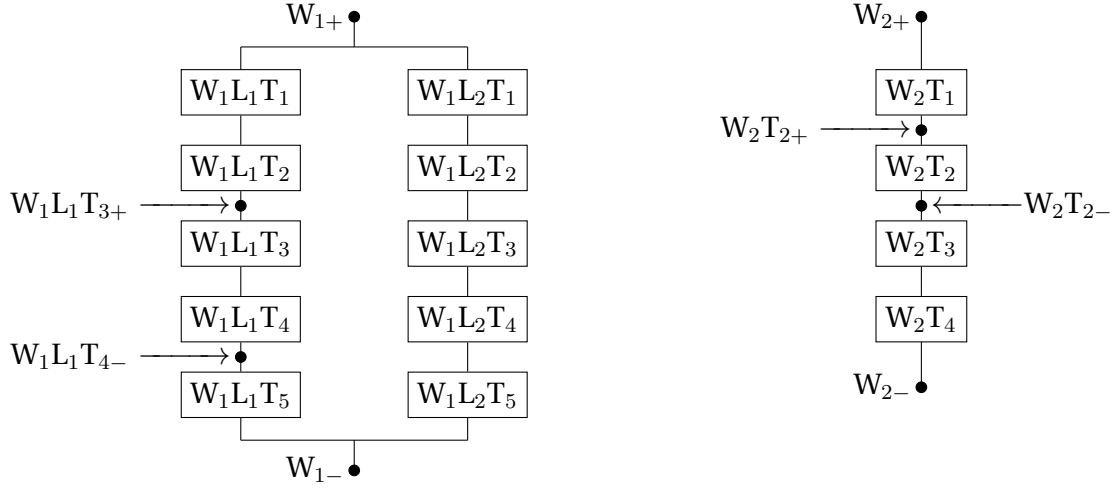


Figure 3.1: Transformer notation convention for physical conducting parts.

We can already point out that magnetic components windings are arbitrarily cut at a turn level, for two main reasons. Firstly, slicing between different turns is easily represented in a 2D model, since the cross-sectional view provides a surface area per turn. Secondly, slicing is mandatory in order to be able to use localized constant circuit components, which require two terminals (a + and a -). The turn level corresponds to the smallest 2-dimensional cutout level. Note that it is also possible to consider other, coarser divisions, such as a complete winding or any group of turns. We will use the term "**section**" to describe a group of turns considered to be unbreakable from the model point of view. This will be important later, when we compare these different levels of discretization of magnetic components.

Each winding element is represented by a passive one-port delimited by its two terminals, a positive and a negative one. For example, we can take a look at the second turn of the second winding, whose element corresponds to W_2T_2 , which we know how to represent within a physical model (*e.g.* FEM) and its two accesses W_2T_{2+} and W_2T_{2-} , which can be used for the equivalent electric circuit. Like all one-ports, it is characterized by the voltage difference across its ports and the current flowing through it. In a similar way, any winding or lead is likened to a one-port, to which we associate a current and a voltage difference.

Since successive turns of a lead are part of the same piece of copper wound on itself, it is only logical that two consecutive turns should be directly connected and share the same node. Thus, the negative pole of turn i will be the same node as the positive pole of turn $i + 1$, within the same lead. Every first turn of a lead shares its positive terminal with the positive terminal of the winding in which it is located; it is a single circuit node. The same applies to the negative terminal of each last turn of any lead. As a result, all the leads of a single winding are always connected in parallel at their access points, the aim being to share the current of a winding equally between its different leads.

In practice, for a Litz wire, it is quite possible that these different strands, even belonging to the same turn, are not at the same potential. However, for the sake of simplicity, we will assume that all strands carry the same potential and share the current optimally. In the absence of this assumption, we would have to consider each strand in the same way as each lead is considered, *i.e.* independently of each other, with the ends as the only com-

3. TRANSFORMER MODELING USING LUMPED-PARAMETER CIRCUITS

mon connections. This would raise other concerns, such as the very large number of nodes to be taken into account on the one hand, and the difficulty of determining the exact geometric position of each strand within each turn of the winding on the other. It therefore makes sense to simplify this part of the model and idealize the Litz wire and its performance. Conductive parts external to the windings (*e.g.* shielding, mounting device or chassis) are considered equipotential and represented by a single circuit node interconnected via partial capacitances to the other conductive elements.

The main information is that, whatever the chosen level of discretization, each element (a turn, a section, a winding) possesses a positive pole (+) and a negative pole (-). The notation is straightforward: the + or - symbol directly follows the element to which it refers. It is therefore possible to associate an electrical potential with a given node of the magnetic component. If we denote the circuit element under consideration by i , its positive node by $i+$ and its negative node by $i-$, we can associate the potential V_{i+} with the positive pole and the potential V_{i-} with the negative pole. We also need to define the **voltage** associated with a given **turn**. Since we are considering a 2D representation of the magnetic component, we need to fix the position at which the cut takes place. To do this, we use the variable ζ to designate the azimuthal coordinate of the chosen reference frame. The ζ variable varies between 0 and 1 for a complete revolution. Thus, and under the assumption of a linear voltage decrease along the turn, one can write that the voltage at the position ζ of a turn is

$$V_i = (1 - \zeta) V_{i+} + \zeta V_{i-} , \quad (3.1)$$

where i necessarily designates a turn's name, for instance W_2T_2 . V_i is therefore an average value over a turn that can be exploited in a 2D representation of the device (for FEM identification), whereas V_{i+} and V_{i-} are electrical quantities that can be measured at the accesses and obtained or imposed via a circuit simulation. Such a design directly corresponds to a modeling choice. It is important to note that the equation 3.1 allows us to switch from circuit-type quantities (V_{i+} , V_{i-}) to physic-type quantities (V_i) that can be used for discretization of the magnetic component and vice versa. Hence, V_i is not a potential defined at the accesses of the i element, but a value associated with a particular abscissa, value that will be considered as constant along a turn's revolution for our 2D representation.

In contrast, we define the potential difference between the accesses of an element as such that

$$U_i = V_{i+} - V_{i-} , \quad (3.2)$$

$$U_i = - \int_{\infty}^{i+} \vec{e} \cdot d\vec{l} + \int_{\infty}^{i-} \vec{e} \cdot d\vec{l} , \quad (3.3)$$

$$U_i = - \int_{i-}^{i+} \vec{e} \cdot d\vec{l} , \quad (3.4)$$

which, as it happens, is an **experimentally measurable quantity**.

This equation is useful for representing the voltage at the accesses of any element (turn, section, winding) and is extensively used for putting the magnetoquasistatic system into equations. Furthermore, this is where the link between the magnetic and electrical systems comes into play. Indeed, the voltage across a magnetic one-port is made up of two potentials, themselves also defined for the electrostatic problem, and the partial capacitances therefore anchor themselves between the nodes of the magnetic elements and short-circuit them at higher frequencies. The change in regime is reflected in terms of physical fields. At higher frequencies, energy is no longer transferred from one turn to another via the magnetic field

they share, but through the inter-turn electric field. Notice that Equations 3.1 and 3.4 are linear and can readily be converted to the steady-state sinusoidal regime using phasors.

3.2 Discretization Levels

In the previous part, we introduced a generic notation for magnetic components, based on the physical division of windings into different turns. The "per-turn" level of discretization corresponds to the most precise level for 2D representation, and the "per-winding" level corresponds to the coarsest discretization. However, it is also possible to set intermediate discretization levels by grouping several consecutive turns together [20]. This offers the advantage of reducing the number of system variables compared with the **turn-level**, while retaining more magnetic/electrical couplings than the **winding-level**. It is therefore an interesting trade-off for components with a large number of turns, in order to reduce calculation times (for both finite element identifications and circuit simulations) while retaining the first few resonances.

Each element of a winding can thereby be transposed into a localized constant circuit component. In a winding of N_L leads with N_T turns each, there are up to

$$N_o = N_L N_T \quad (3.5)$$

independent one-ports and

$$N_n = (N_T + 1) N_L - 2(N_L - 1) \quad (3.6)$$

independent nodes, to which must be added the nodes corresponding to external conductive parts (shielding, chassis, etc.). Thus, in our basic example from figure 3.1, there are 10 elements and 10 nodes in the first winding, and 4 elements and 5 nodes in the second; for a total of 14 distinct elements and 15 nodes for the complete component.

We then obtain a matrix of dimensions $(N_o \times N_o)$ to represent the magnetic system and a matrix of dimensions $(N_n \times N_n)$ to represent the electrical system. Assuming that these matrices are symmetrical¹, this provides n_m parameters for the magnetic system and n_e for the electrical system such that

$$n_m = \frac{N_o (N_o + 1)}{2} , \quad (3.7)$$

$$n_e = \frac{N_n (N_n + 1)}{2} . \quad (3.8)$$

Put another way, 105 different inductive terms and 120 partial capacitances are needed to model the magnetic component of Figure 3.1 turn by turn with an equivalent localized constant circuit.

If the model is simplified as much as possible, only two separate windings remain, with two one-ports and four nodes. This provides 3 different inductive terms (two self-inductance and one mutual) and 10 partial capacitances.

The number of capacitances may seem high for only two windings, but it should be pointed out that the reference of the potentials is considered external to the windings by default. As soon as this same reference is assigned to one of the nodes, for example the negative pole of the first winding, only 6 different capacitances remain, the others being short-circuited or paralleled. Similarly, if the negative poles of the two windings are connected

¹which is not necessarily the case for the real part of the impedance, but remains generally true.

together, only 3 partial capacitances remain, which corresponds to the model generally encountered in the literature for a two-winding transformer: one capacitance for each winding and one inter-winding capacitance.

3.3 Discretization of the Magnetic Part

Considering first the magnetic aspect, there are as many independent voltages as there are elements in the circuit, and the equations governing them can be expressed in the following generic matrix form

$$\underline{U} = (\underline{R} + j\omega \underline{L}) \underline{I}, \quad (3.9)$$

where each component of the vector $\underline{U}$ represents the voltage across a circuit element and $\underline{I}$ represents the current in each circuit elements..

For greater clarity between the different levels of discretization, an additional letter is used in the subscript to indicate whether it involves quantities per turn, per section, per lead or per winding. Thus, we express equation 3.9 in the following forms

$$\underline{U}_T = (\underline{R}_T + j\omega \underline{L}_T) \underline{I}_T, \quad (3.10)$$

$$\underline{U}_S = (\underline{R}_S + j\omega \underline{L}_S) \underline{I}_S, \quad (3.11)$$

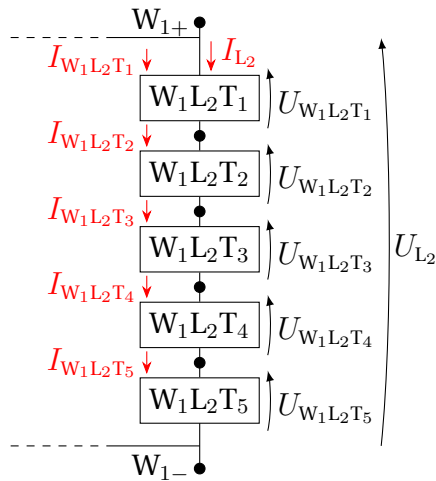
$$\underline{U}_L = (\underline{R}_L + j\omega \underline{L}_L) \underline{I}_L, \quad (3.12)$$

$$\underline{U}_W = (\underline{R}_W + j\omega \underline{L}_W) \underline{I}_W, \quad (3.13)$$

respectively for turns, sections, leads and windings. Note that the turn-level, lead-level and winding-level cutouts are unique, whereas the section-levels are arbitrarily fixed. There can therefore be several different ones.

Moving from a **finer** to a **coarser** level of discretization is always possible with a few algebraic operations. We can illustrate this by reducing the turn model (the finest) to the winding model (the coarsest). Knowing the inductance matrix per turn $\underline{L}_T$, we can deduce the inductance matrix per winding $\underline{L}_W$. First, note that all turns within a lead share the same current, and that the total lead voltage is the sum of the voltages of each turn.

An example can be provided for the second lead of our basic example from Figure 3.1 for which the voltage difference across the lead corresponds to the sum of the voltage differences defined turn by turn as expressed in Equations 3.14 to 3.16,



$$U_{L_2} = V_{L_2+} - V_{L_2-} \quad (3.14)$$

$$= V_{W_1L_2T_{1+}} - V_{W_1L_2T_{5-}} \quad (3.15)$$

$$= \sum_{i=1}^5 U_{W_1L_2T_i}, \quad (3.16)$$

$$I_{L_2} = I_{W_1L_2T_i}, \quad \forall i \in \{1, \dots, 5\}, \quad (3.17)$$

Figure 3.2: Lead voltage and current spread by turn.

and whose lead and turn currents are all identical (Eq. 3.17).

From these purely algebraic relations, we can deduce that the lead's equivalent inductance is the sum of all the inductances (self and mutual) of the individual turns that make up the lead². As the voltages are sum up turn by turn, the lines can be added together and as the currents are shared by turns of the same lead, the columns can also be added within the same block. One can also show these different blocks of matrices for the component of Figure 3.1, which gives

$$\underline{\underline{L}}_T = \begin{bmatrix} L_{1,1} & L_{1,2} & L_{1,3} & L_{1,4} & L_{1,5} & L_{1,6} & L_{1,7} & L_{1,8} & L_{1,9} & L_{1,10} & L_{1,11} & L_{1,12} & L_{1,13} & L_{1,14} \\ L_{2,1} & L_{2,2} & L_{2,3} & L_{2,4} & L_{2,5} & L_{2,6} & L_{2,7} & L_{2,8} & L_{2,9} & L_{2,10} & L_{2,11} & L_{2,12} & L_{2,13} & L_{2,14} \\ L_{3,1} & L_{3,2} & L_{3,3} & L_{3,4} & L_{3,5} & L_{3,6} & L_{3,7} & L_{3,8} & L_{3,9} & L_{3,10} & L_{3,11} & L_{3,12} & L_{3,13} & L_{3,14} \\ L_{4,1} & L_{4,2} & L_{4,3} & L_{4,4} & L_{4,5} & L_{4,6} & L_{4,7} & L_{4,8} & L_{4,9} & L_{4,10} & L_{4,11} & L_{4,12} & L_{4,13} & L_{4,14} \\ L_{5,1} & L_{5,2} & L_{5,3} & L_{5,4} & L_{5,5} & L_{5,6} & L_{5,7} & L_{5,8} & L_{5,9} & L_{5,10} & L_{5,11} & L_{5,12} & L_{5,13} & L_{5,14} \\ L_{6,1} & L_{6,2} & L_{6,3} & L_{6,4} & L_{6,5} & L_{6,6} & L_{6,7} & L_{6,8} & L_{6,9} & L_{6,10} & L_{6,11} & L_{6,12} & L_{6,13} & L_{6,14} \\ L_{7,1} & L_{7,2} & L_{7,3} & L_{7,4} & L_{7,5} & L_{7,6} & L_{7,7} & L_{7,8} & L_{7,9} & L_{7,10} & L_{7,11} & L_{7,12} & L_{7,13} & L_{7,14} \\ L_{8,1} & L_{8,2} & L_{8,3} & L_{8,4} & L_{8,5} & L_{8,6} & L_{8,7} & L_{8,8} & L_{8,9} & L_{8,10} & L_{8,11} & L_{8,12} & L_{8,13} & L_{8,14} \\ L_{9,1} & L_{9,2} & L_{9,3} & L_{9,4} & L_{9,5} & L_{9,6} & L_{9,7} & L_{9,8} & L_{9,9} & L_{9,10} & L_{9,11} & L_{9,12} & L_{9,13} & L_{9,14} \\ L_{10,1} & L_{10,2} & L_{10,3} & L_{10,4} & L_{10,5} & L_{10,6} & L_{10,7} & L_{10,8} & L_{10,9} & L_{10,10} & L_{10,11} & L_{10,12} & L_{10,13} & L_{10,14} \\ L_{11,1} & L_{11,2} & L_{11,3} & L_{11,4} & L_{11,5} & L_{11,6} & L_{11,7} & L_{11,8} & L_{11,9} & L_{11,10} & L_{11,11} & L_{11,12} & L_{11,13} & L_{11,14} \\ L_{12,1} & L_{12,2} & L_{12,3} & L_{12,4} & L_{12,5} & L_{12,6} & L_{12,7} & L_{12,8} & L_{12,9} & L_{12,10} & L_{12,11} & L_{12,12} & L_{12,13} & L_{12,14} \\ L_{13,1} & L_{13,2} & L_{13,3} & L_{13,4} & L_{13,5} & L_{13,6} & L_{13,7} & L_{13,8} & L_{13,9} & L_{13,10} & L_{13,11} & L_{13,12} & L_{13,13} & L_{13,14} \\ L_{14,1} & L_{14,2} & L_{14,3} & L_{14,4} & L_{14,5} & L_{14,6} & L_{14,7} & L_{14,8} & L_{14,9} & L_{14,10} & L_{14,11} & L_{14,12} & L_{14,13} & L_{14,14} \end{bmatrix}, \quad (3.18)$$

in which we can observe the different blocks relating to the three different leads of the component. Reducing the turns level to the leads level provides a 3 by 3 matrix as follows

$$\underline{\underline{L}}_L = \begin{bmatrix} L_{L1,1} & L_{L1,2} & L_{L1,3} \\ \cdot & L_{L2,2} & L_{L2,3} \\ \cdot & \cdot & L_{L3,3} \end{bmatrix}, \quad (3.19)$$

where $L_{L1,1}$ corresponds to the self-inductance of lead 1 and $L_{L1,2}$ corresponds to the mutual inductance between leads 1 and 2. Inductances can be obtained from the turn model, by summing the components of all the turns that make up a lead. It follows that

$$L_{L1,1} = \sum_{i=1}^5 \sum_{j=1}^5 L_{i,j}, \quad (3.20) \quad L_{L1,2} = \sum_{i=1}^5 \sum_{j=6}^{10} L_{i,j}, \quad (3.21) \quad L_{L1,3} = \sum_{i=1}^5 \sum_{j=11}^{14} L_{i,j}, \quad (3.22)$$

and so on for the turn indices relative to each lead.

The system can be further reduced, as leads 1 and 2 are part of the same winding and connected in parallel. Conversion to equivalent impedance seen from the primary (W1) is therefore no longer as immediate and intuitive as it was for series-connected turns. The first step is to invert the impedance matrix, defined as follows

$$\underline{\underline{Z}}_L = \underline{\underline{R}}_L + j \omega \underline{\underline{L}}_L, \quad (3.23)$$

whose inverse, the admittance matrix, is expressed as

$$\underline{\underline{Y}}_L = (\underline{\underline{Z}}_L)^{-1}. \quad (3.24)$$

The admittance matrix per winding is obtained by grouping together the admittances of the leads belonging to the same winding, the total admittance corresponding to the sum of the

²The identical deduction can be made for the elements of the resistance matrix.

3. TRANSFORMER MODELING USING LUMPED-PARAMETER CIRCUITS

leads' admittances. Based on the circuit shown in Figure 3.1, we can define the admittance matrix per winding more precisely as follows

$$\underline{Y}_{\underline{W}} = \left[\begin{array}{c|c} \sum_{i=1}^2 \sum_{j=1}^2 Y_{L_{i,j}} & \sum_{j=1}^2 Y_{L_{3,j}} \\ \hline \sum_{i=1}^2 Y_{L_{i,3}} & Y_{L_{3,3}} \end{array} \right], \quad (3.25)$$

and it is perfectly possible to obtain the impedance matrix per winding $\underline{Z}_{\underline{W}}$ by inverting the admittance matrix $\underline{Y}_{\underline{W}}$.

$$\underline{Z}_{\underline{W}} = (\underline{Y}_{\underline{W}})^{-1}. \quad (3.26)$$

Interpreting winding-level parameters on the basis of lead-level parameters is complicated when several leads make up a winding, as the equations 3.24 to 3.26 are inevitable. It is nevertheless possible to analyze the effect of two leads used for a single winding, setting losses aside. In this case, the equivalent inductance of the complete winding is expressed as

$$L_{W_1} = \frac{L_{L_{1,1}} L_{L_{2,2}} - L_{L_{1,2}} L_{L_{2,1}}}{L_{L_{1,1}} + L_{L_{2,2}} - L_{L_{1,2}} - L_{L_{2,1}}} \quad (3.27)$$

$$L_{W_1} = \frac{L_{L_{1,1}} L_{L_{2,2}} - (L_{L_{1,2}})^2}{L_{L_{1,1}} + L_{L_{2,2}} - 2 L_{L_{1,2}}} \quad (3.28)$$

We can introduce $k_{L_{1,2}}$ the coupling coefficient between leads 1 and 2 such that

$$k_{L_{1,2}} = \frac{L_{L_{1,2}}}{\sqrt{L_{L_{1,1}} L_{L_{2,2}}}}. \quad (3.29)$$

Which reduces the expression of the winding inductance L_{W_1} to

$$L_{W_1} = \frac{L_{L_{1,1}} L_{L_{2,2}} (1 - k_{L_{1,2}}^2)}{L_{L_{1,1}} + L_{L_{2,2}} - 2 k_{L_{1,2}} \sqrt{L_{L_{1,1}} L_{L_{2,2}}}}. \quad (3.30)$$

Then, we can assume that both leads have the same self-inductance. This is a perfectly realistic assumption, given that the leads have the same number of turns and are wound side by side along the same curvilinear abscissa. The self-inductance of a single lead is therefore L_L . Finally, we obtain a reduced expression of the winding inductance as a function of the self-inductance of a lead and the coupling coefficient between the two leads, as follows

$$L_{W_1} = L_L \frac{1 + k_{L_{1,2}}}{2} \quad (3.31)$$

In the ideal case, the coupling coefficient is unitary and the winding inductance is equal to the inductance of a single lead. This is no surprise, since the purpose of associating several leads in parallel is not to modify the inductive parameters of the winding, but rather to allow more current to flow through it while limiting losses. Using n leads theoretically multiplies the effective conductor cross-section by n . Note that in practice, the coupling coefficient is not unitary, but nevertheless very close to unity, as the leads are wound together and become tightly interlaced.

We have seen that it is entirely possible to switch from an inductive model defined at the turn level to an inductive model defined at the winding level, via an intermediate level per

lead. We have seen that the impedance of a given lead corresponds to the sum of the components of each of the turns that make up that lead, which follows the logic that the inductance of several turns in series will grow quadratically with the number of turns (except for the coupling coefficients between turns). We have also seen that the association of several leads in parallel can be converted into winding-level quantities, although the immediate interpretation of their performance is less obvious, except under a few strong simplifying assumptions.

3.3.1 Section Level Discretization

In this work, we introduce a new approach to partitioning leads in a totally arbitrary way, by grouping together a certain number of consecutive turns: the concept of **section**. A section can therefore contain any number of turns from 1 to N_T . It is possible to illustrate a section level discretization for the component in Figure 3.1 by defining 3 sections for each of the primary leads (two sections of two turns and one section of one turn), as well as two sections of two turns for the secondary winding lead.

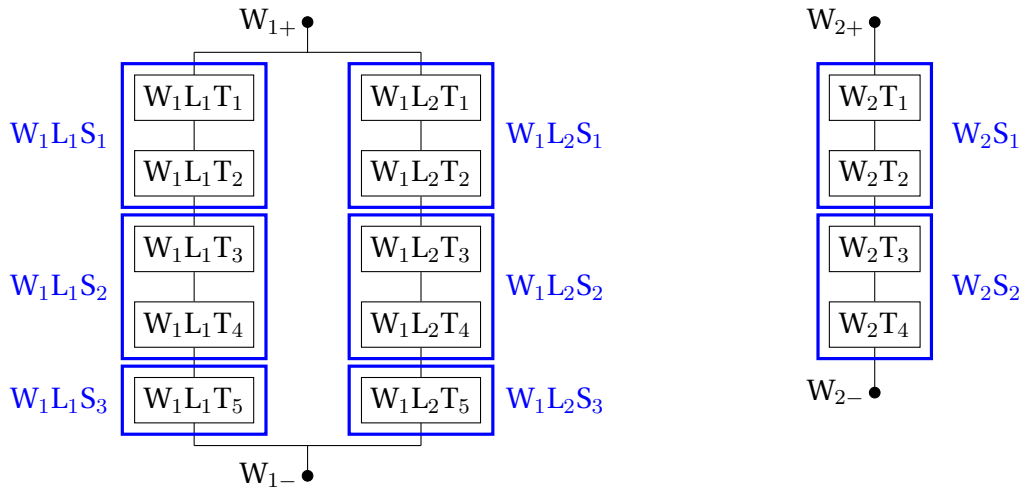


Figure 3.3: Discretization of magnetic component windings into arbitrary sections.

Such level of discretization as illustrated in Figure 3.3 can be schematized as 64 blocks within the inductance matrix per turn $\underline{L}_T$:

$$\underline{L}_{\text{T}} = \begin{matrix} L_{S_{1,1}} \\ \left[\begin{array}{cc|cc|cc|cc|cc|cc|cc} L_{1,1} & L_{1,2} & L_{1,3} & L_{1,4} & L_{1,5} & L_{1,6} & L_{1,7} & L_{1,8} & L_{1,9} & L_{1,10} & L_{1,11} & L_{1,12} & L_{1,13} & L_{1,14} \\ \cdot & L_{2,2} & L_{2,3} & L_{2,4} & L_{2,5} & L_{2,6} & L_{2,7} & L_{2,8} & L_{2,9} & L_{2,10} & L_{2,11} & L_{2,12} & L_{2,13} & L_{2,14} \\ \cdot & \cdot & L_{3,3} & L_{3,4} & L_{3,5} & L_{3,6} & L_{3,7} & L_{3,8} & L_{3,9} & L_{3,10} & L_{3,11} & L_{3,12} & L_{3,13} & L_{3,14} \\ \cdot & \cdot & \cdot & L_{4,4} & L_{4,5} & L_{4,6} & L_{4,7} & L_{4,8} & L_{4,9} & L_{4,10} & L_{4,11} & L_{4,12} & L_{4,13} & L_{4,14} \\ \cdot & \cdot & \cdot & \cdot & L_{5,5} & L_{5,6} & L_{5,7} & L_{5,8} & L_{5,9} & L_{5,10} & L_{5,11} & L_{5,12} & L_{5,13} & L_{5,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & L_{6,6} & L_{6,7} & L_{6,8} & L_{6,9} & L_{6,10} & L_{6,11} & L_{6,12} & L_{6,13} & L_{6,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{7,7} & L_{7,8} & L_{7,9} & L_{7,10} & L_{7,11} & L_{7,12} & L_{7,13} & L_{7,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{8,8} & L_{8,9} & L_{8,10} & L_{8,11} & L_{8,12} & L_{8,13} & L_{8,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{9,9} & L_{9,10} & L_{9,11} & L_{9,12} & L_{9,13} & L_{9,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{10,10} & L_{10,11} & L_{10,12} & L_{10,13} & L_{10,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{11,11} & L_{11,12} & L_{11,13} & L_{11,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{12,12} & L_{12,13} & L_{12,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{13,13} & L_{13,14} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{14,14} \end{array} \right] \end{matrix}, \quad (3.32)$$

Given that the turns of the same section are always consecutive, as was already the case for the turns of the same lead, the model is immediately reduced by summing all the terms of the same block, as previously done for the impedance of the leads. This provides an inductance matrix per section, of dimensions $(N_s \times N_s)$ with N_s defined as the total number of

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sections defined over the entire device. As this division is arbitrary, several models of different dimensions may exist. In fact, turn-based and lead-based models can be seen and defined as special cases of section-based division. Splitting windings on an arbitrary basis (by section) thus joins geometry-based cutting (by turn, by lead).

Since the inductance matrices are symmetrical, switching from the turn-level model to the section-level model reduces the number of model parameters from 105 to 36 for the example from Figure 3.3. Reducing the inductance matrix per turn (3.32) to a smaller matrix per section

$$\underline{\underline{L}}_S = \begin{bmatrix} L_{S1,1} & L_{S1,2} & L_{S1,3} & L_{S1,4} & L_{S1,5} & L_{S1,6} & L_{S1,7} & L_{S1,8} \\ \cdot & L_{S2,2} & L_{S2,3} & L_{S2,4} & L_{S2,5} & L_{S2,6} & L_{S2,7} & L_{S2,8} \\ \cdot & \cdot & L_{S3,3} & L_{S3,4} & L_{S3,5} & L_{S3,6} & L_{S3,7} & L_{S3,8} \\ \cdot & \cdot & \cdot & L_{S4,4} & L_{S4,5} & L_{S4,6} & L_{S4,7} & L_{S4,8} \\ \cdot & \cdot & \cdot & \cdot & L_{S5,5} & L_{S5,6} & L_{S5,7} & L_{S5,8} \\ \cdot & \cdot & \cdot & \cdot & \cdot & L_{S6,6} & L_{S6,7} & L_{S6,8} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{S7,7} & L_{S7,8} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & L_{S8,8} \end{bmatrix} \quad (3.33)$$

is already progress in itself towards simplifying the inductive model. In this way, the winding information extracted from the turns can be reduced and synthesized into different groups of turns for which a physical meaning can be sought (different layers of wire, conductors close to air gaps, etc.).

Whilst it is already interesting to be able to algebraically compress the inductance matrix per turn into an inductance matrix per section in order to reduce the complexity of resolving the system (or equivalent circuit), it is even more useful to directly identify the elements of the inductance matrix per section via the finite element method. To do this, it is necessary to couple the turns unknowns by means of **global quantities** that ensure that the total current shared by the turns of the same section is identical from one turn to the next, and that the voltage at the section's accesses corresponds to the sum of the voltages of each of the turns. Such global quantities have already been introduced in the previous chapter through the equations 2.57 to 2.59 for the magnetic problem. All that remains is to identify the components section by section. To do so, each individual section must be stimulated by a given voltage (1 V, for example), while imposing a zero current on all other sections. We obtain the impedance associated with a section by dividing its voltage by the total current flowing through the stimulated section, just as we would proceed turn by turn. We can then express that

$$Z_{S_{i,j}} = R_{S_{i,j}} + j \omega L_{S_{i,j}} = \frac{\overline{U}_{S_i}}{\overline{I}_{S_j}}, \quad (3.34)$$

including both the magnetic couplings and the losses at the same time. By doing so, the number of finite element problems that need to be solved to identify the system parameters is directly reduced. In this case, for n sections, we need to solve n problems (one per stimulated section) and calculate $\frac{n(n+1)}{2}$ post-operations. In view of the small number of continuity equations required for the global quantities, and the number of degrees of freedom of the problems we have to deal with ($10^5 - 10^7$ DoFs), the additional complexity of the new system to be solved is more than negligible.

Dividing the component into several arbitrary sections can already be useful from a magnetic point of view, for designers wishing to better understand certain phenomena and isolate different geometric effects.

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To illustrate this, consider an air inductor with 30 turns divided into 3 sections of 10 turns. Two designs are to be compared: the first is a single-layer inductor in which the 3 sections are arranged one after the other in the same plane, and the second design places the 3 sections on 3 separate layers, one on top of the other, as shown in Figure 3.4. Both layouts present perfect rotation symmetry around a longitudinal y -axis.

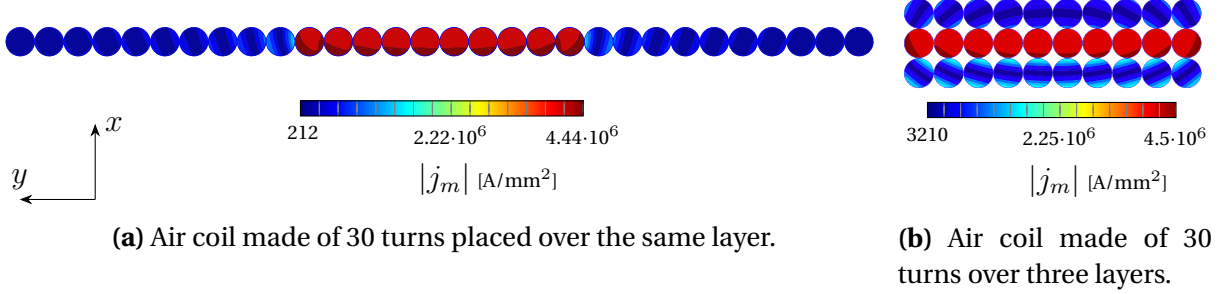


Figure 3.4: Current density from two layouts of a 30-turn air coil. The middle 10-conductor section is stimulated by a voltage of 1 V at 4200 Hz, the other two sections being open.

The average radius of the central layer turns is set at 25 mm, and the wire diameter corresponds to 1 mm. The penetration depth is equal to the 1 mm diameter at 4200 Hz, frequency used for numerical computations. Henceforth, identifications provide inductance matrices³ per section for the two cases for which it is possible to compare the values

$$\underline{\underline{L}}_{S, 1\text{lay}} = \begin{bmatrix} L_{S1,1} & k_{S1,2} & . \\ 7.3226 \mu\text{H} & 0.486414 & 0.223258 \\ . & 7.3145 \mu\text{H} & 0.486267 \\ . & . & 7.3217 \mu\text{H} \end{bmatrix}, \quad \underline{\underline{L}}_{S, 3\text{lay}} = \begin{bmatrix} 6.8302 \mu\text{H} & 0.917535 & 0.822694 \\ . & 7.3044 \mu\text{H} & 0.919589 \\ . & . & 7.7727 \mu\text{H} \end{bmatrix}. \quad (3.35) \quad (3.36)$$

The total inductance of the first case is $39.4647 \mu\text{H}$ and that of the second case is $60.7157 \mu\text{H}$ (respectively $40.58 \mu\text{H}$ and $64.9981 \mu\text{H}$ according to Wheeler's empirical formula [111]). It can be seen straight away that the low inductance of the single-layer case stems from the low coupling coefficients between sections. In fact, two windings placed end-to-end share much less flux than concentrically wound windings, as for the three sections of the multilayer case.

After the inductive behavior, we can analyze the resistive behavior, noting that the resistance matrices are symmetrical, since the conductivity is the same for all conductors:

$$\underline{\underline{R}}_{S, 1\text{lay}} = \begin{bmatrix} 34.323 \text{ m}\Omega & -0.071 \text{ m}\Omega & -0.100 \text{ m}\Omega \\ . & 34.538 \text{ m}\Omega & -0.072 \text{ m}\Omega \\ . & . & 34.323 \text{ m}\Omega \end{bmatrix}, \quad \underline{\underline{R}}_{S, 3\text{lay}} = \begin{bmatrix} 33.528 \text{ m}\Omega & 0.996 \text{ m}\Omega & -0.044 \text{ m}\Omega \\ . & 36.081 \text{ m}\Omega & 1.807 \text{ m}\Omega \\ . & . & 37.998 \text{ m}\Omega \end{bmatrix}. \quad (3.37) \quad (3.38)$$

The total resistance of the first case is $102.52 \text{ m}\Omega$ and that of the second case is $113.125 \text{ m}\Omega$. The observations are less obvious for the resistive behavior. Firstly, it should be noted that the single-layer case generates fewer losses than the multi-layer case for the same current. This is due to the proximity effect, which is more pronounced in the latter case. Then, note that off-diagonal terms can take on a positive or negative sign, and that this sign is difficult to predict in advance. Finally, for the three-layer case, one can attribute an excess loss that is represented by about $1 \text{ m}\Omega$ due to the interaction of the two lower layers, as well as $1.8 \text{ m}\Omega$ due to the interaction of the two upper layers.

³The off-diagonal terms have been replaced by their coupling coefficient within the matrix for better readability and understanding.

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In direct current, the resistance of the middle section corresponds to $33.65 \text{ m}\Omega$. At 4200 Hz , a slight increase is already noticeable due to proximity effects. This rise is more marked in the second case with three layers, since the magnetic couplings between the different sections are stronger.

The above simple example of an open-air inductor shows how useful it can be to divide a winding into different sections in order to better understand how they interact; not only for simple questions of winding arrangement, but also for other geometrical considerations, such as the vicinity of conductors with a ferromagnetic core air gap, or more exotic geometrical distributions. It is easy to put yourself in the shoes of the magnetic component designer, who, using these methods, will be able to determine which layer of the winding dissipates the most power, whether conductors close to an air gap are more prone to Joule losses, or even to what extent losses due to coupling between sections (diagonal terms of the resistance matrix) reinforce the total losses.

3.4 Discretization of the Electric Part

We would like to apply the same levels of discretization, this time considering the electric part of our model. In this case, there is nothing to indicate that it is possible to reduce our model per turn, especially as the capacitances between the different turns are not in series, as were the inductances of consecutive turns belonging to the same section.

To begin with, it should be noted that the number of unknowns in the finite element model does not correspond directly to the number of nodes in the equivalent circuit. In fact, for each circuit element, two nodes are assigned (the positive and negative poles), each of which is assigned an electrical potential. In the 2D physical representation (FEM), there is only one surface on which the potential is based. The electrical potential is thus unique for a given turn, and it is then necessary to express the potential of the turn as a linear combination of the potentials of its nodes.

The expression 3.1 is therefore useful for this purpose, since it indicates the potential of each turn corresponds to a linear combination of the potentials of the positive and negative poles, based on the azimuthal coordinate $\zeta \in [0, 1]$:

$$V_{T_i} = (1 - \zeta) V_{T_{i+}} + \zeta V_{T_{i-}} . \quad (3.39)$$

First, consider the case of a single, completely isolated turn. It can be given the following electrical model and representation in the 2D plane.

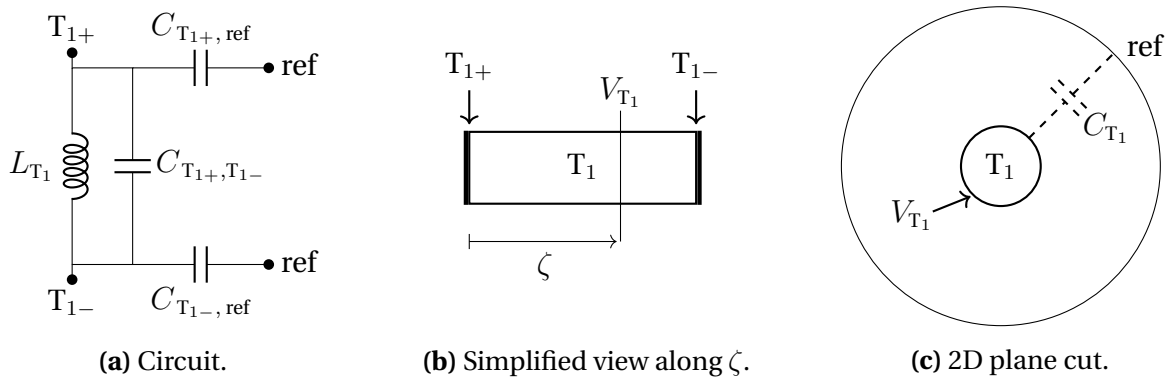


Figure 3.5: Single-turn winding representations.

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Note that the circuit representation does not involve the same number of capacitances (minimum 3 for one turn) as the 2D physical model representation (one capacitance between the turn and the voltage reference). Nevertheless, we can match their electrostatic energies, assuming they are equal. In so doing, this energy can be expressed on the basis of circuit quantities

$$\begin{aligned} W^e &= \frac{1}{2} \left(C_{T_{1+}, \text{ref}} V_{T_{1+}}^2 + C_{T_{1+}, T_{1-}} (V_{T_{1+}} - V_{T_{1-}})^2 + C_{T_{1-}, \text{ref}} V_{T_{1-}}^2 \right), \\ &= \frac{1}{2} \left(\underbrace{(C_{T_{1+}, \text{ref}} + C_{T_{1+}, T_{1-}})}_{\square} V_{T_{1+}}^2 - \underbrace{2 C_{T_{1+}, T_{1-}}}_{*} V_{T_{1+}} V_{T_{1-}} + \underbrace{(C_{T_{1-}, \text{ref}} + C_{T_{1+}, T_{1-}})}_{\triangle} V_{T_{1-}}^2 \right), \end{aligned} \quad (3.40)$$

as well as on the basis of the quantities linked to the 2D cut

$$\begin{aligned} W^e &= \frac{1}{2} C_{T_1} V_{T_1}^2, \\ &= \frac{1}{2} C_{T_1} ((1 - \zeta) V_{T_{1+}} + \zeta V_{T_{1-}})^2, \\ &= \frac{1}{2} \left(\underbrace{C_{T_1} (1 - \zeta)^2}_{\square} V_{T_{1+}}^2 + \underbrace{2 C_{T_1} (1 - \zeta) \zeta}_{*} V_{T_{1+}} V_{T_{1-}} + \underbrace{C_{T_1} \zeta^2}_{\triangle} V_{T_{1-}}^2 \right). \end{aligned} \quad (3.41)$$

On the basis of these two developments, we can derive from the equations 3.40 and 3.41 that

$$C_{T_{1+}, \text{ref}} = C_{T_1} (1 - \zeta) \quad (3.42)$$

$$C_{T_{1-}, \text{ref}} = C_{T_1} \zeta \quad (3.43)$$

$$C_{T_{1+}, T_{1-}} = C_{T_1} (\zeta - 1) \zeta \quad (3.44)$$

As a result, it can be seen that it is possible to express the 3 capacitances of the circuit on the basis of the capacitance provided by the 2D model alone, as well as the azimuthal coordinate ζ of the 2D section. Note that for the special case where $\zeta = 0$, only the node T_{1+} is useful in the circuit, and all the electric charge carried by the conductor relates to this same node. The same analysis applies to the case where $\zeta = 1$, but this time with reference to the node T_{1-} .

Passing through electrostatic energies ensures that both representations of the magnetic component (a single turn for a single winding) are consistent and store the same amount of energy when subjected to the same external conditions *i.e.* applied voltages. However, this approach requires developments whose length explodes as the number of turns increases. Indeed, in order to reduce the number of expressions, it would be useful to introduce a more efficient notation to represent the transition from circuit-type quantities to 2D quantities (FEM).

To do this, we introduce the vector ω which converts circuit quantities into 2D finite element quantities and vice versa, such that

$$\omega = \begin{pmatrix} (1 - \zeta) \\ \zeta \end{pmatrix}. \quad (3.45)$$

We can therefore express the equation 3.39, via the transpose of this vector ω

$$(V_{T_1}) = \begin{pmatrix} (1 - \zeta) & \zeta \end{pmatrix} \begin{pmatrix} V_{T_{1+}} \\ V_{T_{1-}} \end{pmatrix}, \quad (3.46)$$

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and use ω as a counterpart to distribute the 2D electric charge between the two nodes of the equivalent circuit:

$$\begin{pmatrix} Q_{T_{1+}} \\ Q_{T_{1-}} \end{pmatrix} = \begin{pmatrix} (1 - \zeta) \\ \zeta \end{pmatrix} (Q_{T_1}) . \quad (3.47)$$

Note that in this case, the W_1 winding is also the S_1 section and the T_1 turn. As we have already seen (Figure 3.3), a section can represent all the turns of a lead, a subset of the turns of a lead, or a single turn.

We can thus refer to $\bar{V}_s$ as the vector of potentials associated to the section nodes and $\bar{Q}_s$ as the vector of charges associated to the section nodes. In the particular case of a single turn, there will be

$$\bar{V}_s = \begin{pmatrix} V_{T_{1+}} \\ V_{T_{1-}} \end{pmatrix} , \quad (3.48)$$

and

$$\bar{Q}_s = \begin{pmatrix} Q_{T_{1+}} \\ Q_{T_{1-}} \end{pmatrix} . \quad (3.49)$$

We will then refer to the vector of potentials associated to the physical regions of the finite element model as $\bar{V}$, and to the vector of loads associated to the physical regions of the finite element model as $\bar{Q}$:

$$\bar{V} = (V_{T_1}) , \quad (3.50)$$

$$\bar{Q} = (Q_{T_1}) . \quad (3.51)$$

Expressing the influence coefficient matrix per section as K_s and the influence coefficient matrix of the 2D model as $\bar{K}$, we can write the following relationships:

$$\bar{Q} = \bar{K} \bar{V} \quad (3.52)$$

$$\bar{Q}_s = \bar{K}_s \bar{V}_s \quad (3.53)$$

$$\bar{V} = \omega^T \bar{V}_s \quad (3.54)$$

$$\bar{Q}_s = \omega \bar{Q} . \quad (3.55)$$

Finally, by rearranging these equations, it is possible to express the matrix of influence coefficients per section as a function of the matrix of influence coefficients per finite element, such that

$$\bar{K}_s = \omega \bar{K} \omega^T . \quad (3.56)$$

Since there is only one turn to consider⁴, the finite element matrix $\bar{K}$ has only one real value: $\kappa_{T_1} = C_{T_1}$. Expanding the equation 3.56, one can write that

$$\begin{pmatrix} \kappa_{S_{1+,1+}} & \kappa_{S_{1+,1-}} \\ \cdot & \kappa_{S_{1-,1-}} \end{pmatrix} = \begin{pmatrix} (1 - \zeta) \\ \zeta \end{pmatrix} (\kappa_{T_1}) \begin{pmatrix} (1 - \zeta) & \zeta \end{pmatrix} \quad (3.57)$$

$$= \begin{pmatrix} (1 - \zeta)^2 \kappa_{T_1} & (1 - \zeta)\zeta \kappa_{T_1} \\ (1 - \zeta)\zeta \kappa_{T_1} & \zeta^2 \kappa_{T_1} \end{pmatrix} . \quad (3.58)$$

⁴and considering that dielectric losses are not included in the influence coefficient matrix for the sake of clarity

From this point, it remains to convert the newly-obtained influence coefficients into partial capacitances via the relationships 2.121 and 2.122, previously established in such a way that

$$C_{T_{1+}, \text{ref}} = \kappa_{S_{1+,1+}} + \kappa_{S_{1+,1-}} \quad (3.59)$$

$$C_{T_{1-}, \text{ref}} = \kappa_{S_{1+,1-}} + \kappa_{S_{1-,1-}} \quad (3.60)$$

$$C_{T_{1+}, T_{1-}} = -\kappa_{S_{1+,1-}} . \quad (3.61)$$

Recall that by the same relations, the partial capacitance C_{T_1} is directly equal to the corresponding influence coefficient κ_{T_1} ,

$$C_{T_1} = \kappa_{T_1} . \quad (3.62)$$

After these substitutions, we finally obtain that

$$C_{T_{1+}, \text{ref}} = C_{T_1} (1 - \zeta) \quad (3.63)$$

$$C_{T_{1-}, \text{ref}} = C_{T_1} \zeta \quad (3.64)$$

$$C_{T_{1+}, T_{1-}} = C_{T_1} (\zeta - 1) \zeta . \quad (3.65)$$

which is exactly the same result as the one obtained by the energy approach of the two representations (circuit and 2D physics) as shown in equations 3.42, 3.43 and 3.44.

Rather than using an energetic approach whose notations and identification complicate the procedure for obtaining circuit parameters, a matrix approach is used to convert physical input values into data per section that can be processed as a circuit. Systematic writing and consideration of consecutive turns within the same section is formalized and explained below via the newly introduced shape functions.

3.4.1 Improved Sections Representation with Shape Functions

In the previous part, we saw that it is possible to obtain equivalence between the circuit approach and the 2D physical approach by means of two techniques: on the one hand, by developing the electrostatic energy expression for each approach and inspecting the term-by-term values in each equation (Equations 3.40 and 3.41), and on the other hand by introducing the omega transformation (Equation 3.45), which provides the same result as the energy method, given the linearity of the system. The problem with using electrostatic energy is that the expression increases quadratically with the number of independent conductors, making inspection and development immeasurably more complex. What is more, the grouping of several turns within a single section is not explicitly stated. This justifies our choice of the second method, which involves the application of an **affine transformation**. In this part of the manuscript, we will explain how this transformation can be derived from the shape functions well known in the field of finite elements, and through which intermediaries consecutive turns of the same section are grouped together.

Shape functions are functions that **interpolate** a numerical solution relating discrete values associated with elements of a mesh. Independent of the finite element problem, Galerkin's method allows us to discretize the solution u of a continuous problem by choosing a discrete space where the approximate discrete solution u_h exists. By imposing that these functions be polynomials with non-zero values for the reference element and zero values for the other elements, and by associating an appropriate mapping to link the base element to each element of the geometric triangulation, we obtain the discrete space on which u_h is based, with which we can approximate the continuous solution. This is what we call shape

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functions, for which the **nodal basis** is generally chosen, *i.e.* the function takes the value 1 on the vertex and 0 everywhere else. And so, a polynomial of degree 1 on a given interval is completely determined by the values of two distinct points; in this case, we can speak of the element's degrees of freedom, which are then associated with its nodes.

It is in this sense that the idea of shape functions is useful, and appeared to us as a suitable way of representing the **linear dependency** between the **potential** of a turn as a function of the potentials of its two accesses (in this case, the vertices). We then use this shape function idea to apply it to the sections of our model. The accesses of a section correspond to the nodes of a base element (if we refer to a finite element mesh) and the section itself corresponds to the mesh edge.

There are several differences between the two approaches. Firstly, the shape functions employed are not based on geometric mesh elements, but rather on a **topological partitioning** (into sections) of the windings. Secondly, the major difference between the two comes from the fact that there is a **subset** associated with a section. Indeed, where the shape function grows or shrinks linearly, we find the turns (and their accesses) that make up the relevant section, as illustrated in Figure 3.6.

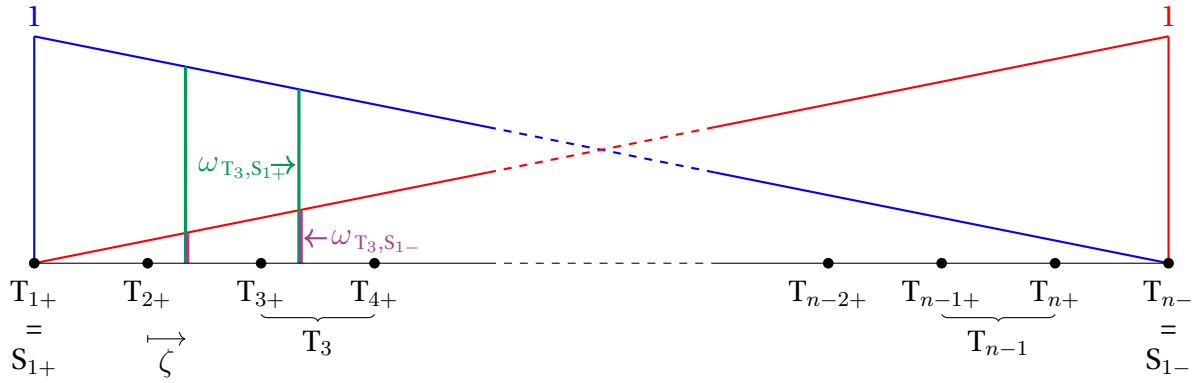


Figure 3.6: First order Lagrange reference element for the n -turn section S_1 .

And this is where shape functions come into their own. Here, using shape functions allows us to assign potential values to turns as a function of potential values at section nodes. For all turns within the same section, it is possible to calculate the potentials of the nodes of each turn on the basis of the potentials of the section nodes, such that

$$V_{T_{i+}} = \frac{n+1-i}{n} V_{S_{j+}} + \frac{i-1}{n} V_{S_{j-}}, \quad (3.66)$$

$$V_{T_{i-}} = \frac{n-i}{n} V_{S_{j+}} + \frac{i}{n} V_{S_{j-}}, \quad (3.67)$$

thus considering the i^{th} turn of the j^{th} section. Note that these expressions are independent of ζ , since the nodes relating to the turns are set uniquely, respectively at the beginning and end of each turn.

Based on the node voltages of the turns, we can express the physical voltage of a turn using once again the expression 3.39, which depends on the variable ζ such that

$$V_{T_i} = \omega_{T_i, S_{j+}} V_{S_{j+}} + \omega_{T_i, S_{j-}} V_{S_{j-}}, \quad (3.68)$$

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by introducing the terms $\omega_{T_i, S_{j+}}$ and $\omega_{T_i, S_{j-}}$ as the coefficients that multiply the voltages at the section nodes

$$\omega_{T_i, S_{j+}} = \frac{(n+1) - i - \zeta}{n}, \quad (3.69) \quad \omega_{T_i, S_{j-}} = \frac{(i-1) + \zeta}{n}. \quad (3.70)$$

From here, we have the matrix of ω coefficients of a section, allowing us to transform voltages at circuit nodes into the voltage of a physical model (in this case 2D). This matrix ω_{S_j} intervenes such that

$$\bar{V}_{S_j} = \omega_{S_j} \bar{V}_{T_{S_j}}, \quad (3.71)$$

where the vector $\bar{V}_{T_{S_j}}$ here represents the vector of voltages of the turns belonging to the section S_j (physical FEM point of view) and the matrix ω_{S_j} then represents a $(2 \times n)$ matrix, given that a section S_j has 2 terminals (S_{j+} and S_{j-}) and is made up of n physical turns, which is represented as follows

$$\omega_{S_j} = \begin{pmatrix} \omega_{S_{j+}} \\ \omega_{S_{j-}} \end{pmatrix} = \begin{pmatrix} \omega_{T_1, S_{j+}} & \omega_{T_2, S_{j+}} & \omega_{T_3, S_{j+}} & \cdots & \omega_{T_n, S_{j+}} \\ \omega_{T_1, S_{j-}} & \omega_{T_2, S_{j-}} & \omega_{T_3, S_{j-}} & \cdots & \omega_{T_n, S_{j-}} \end{pmatrix}. \quad (3.72)$$

In this matrix, the first row corresponds to the $\omega_{S_{j+}}$ function, whose values allow the relationship between the voltage (respectively the charge) at node S_{j+} and the voltages (respectively the charge) of the n turns of section S_j . The second line corresponds to the $\omega_{S_{j-}}$ function, which itself relates to node S_{j-} .

We have seen how the matrix of shape functions ω_{S_j} , defined for a given section of n turns, allows us to convert the **circuit quantities** defined at the nodes of the section into **physical quantities** to be applied to each turn of our 2D model. It remains to define these same interactions between different sections, noting that they can be either independent (if they are part of two different windings) or linked by circuit equations, for example if they are connected in series within the same lead.

If we consider m totally **independent sections**, *i.e.* whose nodes are not connected to each other at all (no series or parallel connection of sections), we can write that the matrix of shape functions ω which links all the physical turns of the magnetic component to all the nodes of the equivalent circuit (per section), can be written as the assembly of the m matrix ω_{S_j} for j ranging from 1 to m . Note that, due to the independence of the different sections, the total matrix is block-diagonal such that

$$\omega = \begin{pmatrix} \overset{(2 \times n_{S_1})}{\boxed{\omega_{S_1}}} & 0 & \cdots & 0 \\ 0 & \overset{(2 \times n_{S_2})}{\boxed{\omega_{S_2}}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \overset{(2 \times n_{S_m})}{\boxed{\omega_{S_m}}} \end{pmatrix}, \quad (3.73)$$

whose each block corresponds to an elementary matrix ω_{S_j} of a given section.

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When several sections are linked together, they share common nodes and the ω matrix is then reduced. Take, for example, the case where several sections are in series within the same lead, as illustrated in Figure 3.7.

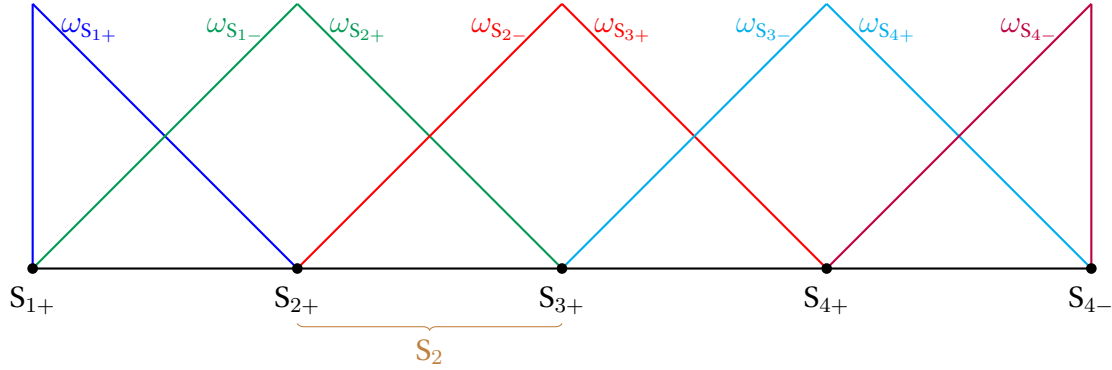


Figure 3.7: Shape functions associated to sections in series.

It can be seen that the shape functions attached to a node may relate to several sections, except when the node is isolated, such as at the end of a single lead. Like adjacent elements in a finite element mesh, the degree of freedom of a node is associated with two shape functions: one from the element on the left, the other from the element on the right. In the ω matrix

$$\omega = \begin{pmatrix} \omega_{S1+} & 0 & 0 & 0 \\ \omega_{S1-} & \omega_{S2+} & 0 & 0 \\ 0 & \omega_{S2-} & \omega_{S3+} & 0 \\ 0 & 0 & \omega_{S3-} & \omega_{S4+} \\ 0 & 0 & 0 & \omega_{S4-} \end{pmatrix}, \quad (3.74)$$

one can see, for instance, that the positive node in the second section (S_{2+}) is linked to the ω_{S1-} shape function, derived from the preceding element, and to the ω_{S2+} shape function, derived from the element following this node.

A numerical example illustrates the assembly of different matrices into a global matrix for sections of arbitrary and different sizes. We assume a single-lead winding made up of 5 turns divided into two sections: a 3-turn section and a 2-turn section. There are therefore 5 physical potentials attached to the turns, 3 section node potentials and the 3 associated shape functions. In Figure 3.8, we first represent the turns by their azimuthal coordinate ζ where they appear in series, one after the other. Subsequently, they are represented according to the 2D cut plane used for FEM modeling. Finally, the 3 shape functions are drawn, along with the values taken by each shape function and assigned to each physical turn. The numerical values in the example are obtained by choosing $\zeta = 0.5$, *i.e.* in the middle of each turn.

With Equation 3.75, it can be seen that the voltages of the regions of the turns are determined on the basis of the voltages assigned to the section nodes and vice versa. We choose to write it in this way, since for a given section node stimulation, we can directly determine the voltages to be applied to each of the physical turns in the 2D model.

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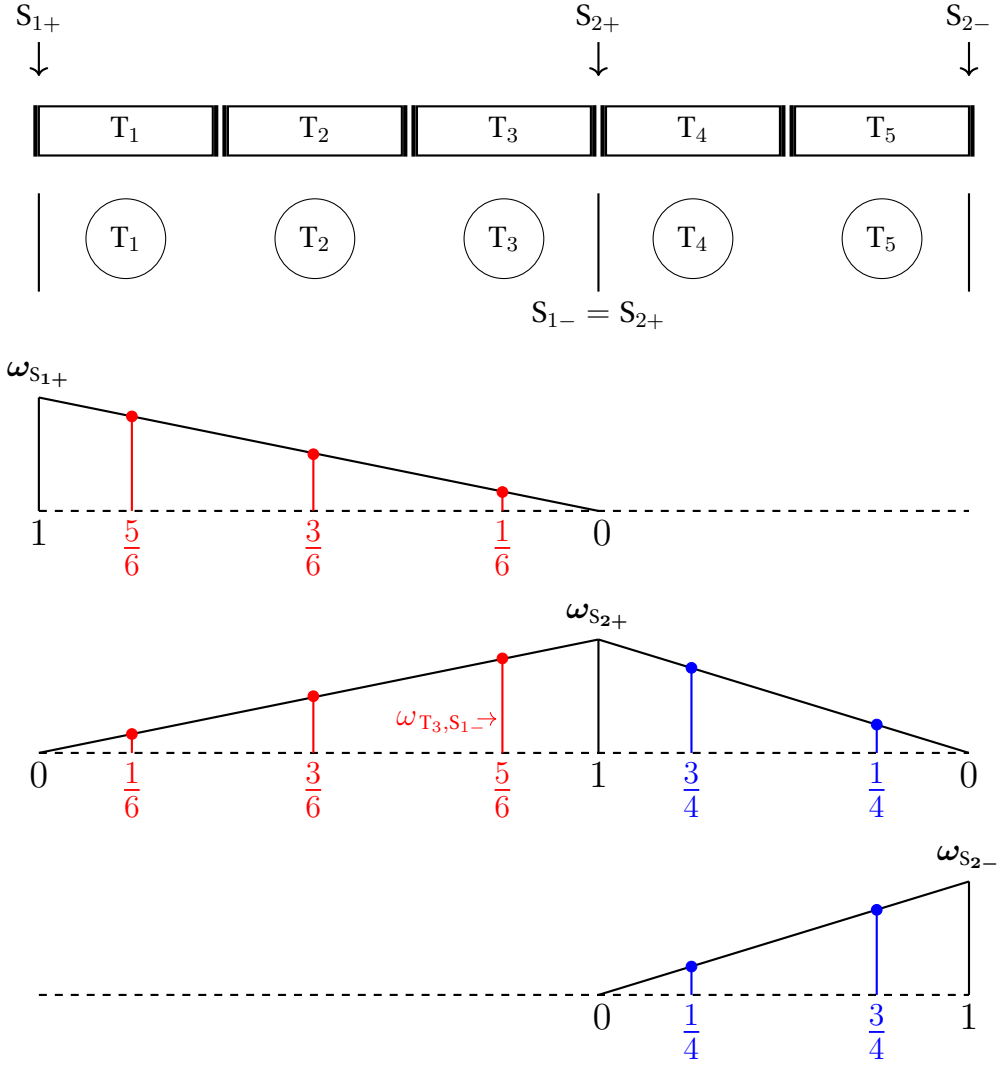


Figure 3.8: Shape functions developments supposing $\zeta = 0.5$.

The dependence of the charges associated with the section nodes with respect to the charges carried by the physical turns is taken up in Equation 3.76. Similarly, it is chosen to be written in this direction, since it enables the charges supported by the section nodes to be calculated on the basis of the charges on the turns, calculated for a given stimulation via finite elements such that

$$\begin{pmatrix} V_{T_1} \\ V_{T_2} \\ V_{T_3} \\ V_{T_4} \\ V_{T_5} \end{pmatrix} = \begin{pmatrix} \omega_{S_1}^T & & \\ \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{3}{6} & \frac{3}{6} & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{pmatrix} \begin{pmatrix} V_{S_{1+}} \\ V_{S_{2+}} \\ V_{S_{2-}} \end{pmatrix}, \quad (3.75)$$

$$\begin{pmatrix} Q_{S_{1+}} \\ Q_{S_{2+}} \\ Q_{S_{2-}} \end{pmatrix} = \underbrace{\begin{pmatrix} \omega_{S_1} & & \\ \frac{5}{6} & \frac{3}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & \frac{3}{6} & \frac{5}{6} & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{pmatrix}}_{\omega} \begin{pmatrix} Q_{T_1} \\ Q_{T_2} \\ Q_{T_3} \\ Q_{T_4} \\ Q_{T_5} \end{pmatrix}. \quad (3.76)$$

Each row of the ω matrix corresponds to the values of the shape function associated with the corresponding node. For example, the second row corresponds to the $\omega_{S_{2+}}$ function described in Figure 3.8. In this same ω matrix, we find the block matrices for each section, in this case ω_{S_1} and ω_{S_2} .

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Alternatively, the matrix structure may be illustrated by the situation where two sections share a node, but are not in series within the same lead which is the case for the first and last sections of different leads from the same winding. Let's take the simple case of two sections of three turns, as illustrated in the following figure,

$$\begin{pmatrix} Q_{S_{1+}} \\ Q_{S_{3+}} \\ Q_{S_{4+}} \end{pmatrix} = \underbrace{\begin{pmatrix} \omega_{S_1} & & \\ & \omega_{S_2} & \\ & & \omega \end{pmatrix}}_{\omega} \begin{pmatrix} Q_{T_1} \\ Q_{T_2} \\ Q_{T_3} \\ Q_{T_4} \\ Q_{T_5} \\ Q_{T_6} \end{pmatrix} \quad (3.77)$$

we can directly remark that there is no difference with the case of several sections within the same lead. In fact, these relationships are managed in an identical way in both cases, with the difference that the sequence of physical variables and circuit variables changes, forcing some blocks of the global matrix ω to split, as it can be seen for the ω_{S_2} block. The matrix ω is therefore no longer block-diagonal as soon as there are connections between different sections, whether they are in series or in parallel.

3.4.2 Section Level Discretization

In this section, the discretization of the magnetic component into sections for electrical phenomena will be expressed. To begin with, let us take the illustrative circuit in Figure 3.1, which involves two independent windings, and assign it the unique node names as shown in Figure 3.9. From now on, we want to express the transformation matrix ω_T , defined at the

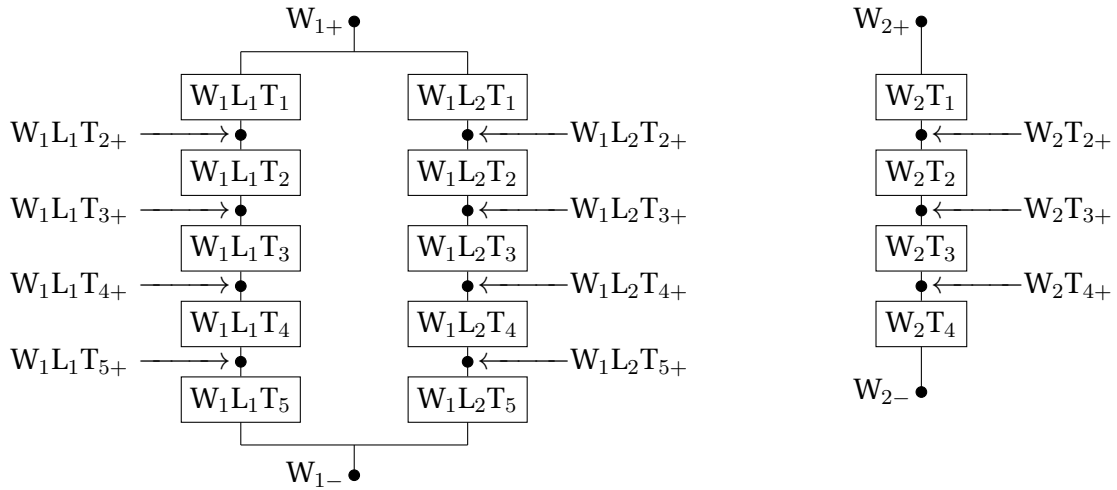


Figure 3.9: Transformer notation convention for physical conducting parts.

turn level, such that

$$\omega_T = \begin{pmatrix} \omega_{T,W_1} & 0 \\ 0 & \omega_{T,W_2} \end{pmatrix}, \quad (3.78)$$

which is then written as the assembly of a matrix ω_{T,W_1} associated with the first winding and a matrix ω_{T,W_2} associated with the second, given the independence between W_1 and W_2 .

These transformation matrices are taken from Equations 3.79 and 3.80.

$$\begin{pmatrix} Q_{W_{1+}} \\ Q_{W_{1L_1T_{2+}}} \\ Q_{W_{1L_1T_{3+}}} \\ Q_{W_{1L_1T_{4+}}} \\ Q_{W_{1L_1T_{5+}}} \\ Q_{W_{1-}} \\ Q_{W_{1L_2T_{2+}}} \\ Q_{W_{1L_2T_{3+}}} \\ Q_{W_{1L_2T_{4+}}} \\ Q_{W_{1L_2T_{5+}}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-\zeta & 0 & 0 & 0 & 0 & 1-\zeta & 0 & 0 & 0 & 0 \\ \zeta & 1-\zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \zeta & 1-\zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \zeta & 1-\zeta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta & 1-\zeta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \zeta & 1-\zeta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \zeta & 1-\zeta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta & 1-\zeta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta & 1-\zeta \end{pmatrix}}_{\omega_{T,W_1}} \begin{pmatrix} Q_{W_{1L_1T_1}} \\ Q_{W_{1L_1T_2}} \\ Q_{W_{1L_1T_3}} \\ Q_{W_{1L_1T_4}} \\ Q_{W_{1L_1T_5}} \\ Q_{W_{1L_2T_1}} \\ Q_{W_{1L_2T_2}} \\ Q_{W_{1L_2T_3}} \\ Q_{W_{1L_2T_4}} \\ Q_{W_{1L_2T_5}} \end{pmatrix} \quad (3.79)$$

The W_1 winding has two leads, and their terminals are connected to the winding access nodes: W_{1+} and W_{1-} . In the above equation, we can see that the charge $Q_{W_{1+}}$ associated with the first access depends both on the charge $Q_{W_{1L_1T_1}}$ that comes from the first turn of the first lead, and on the charge $Q_{W_{1L_2T_1}}$ that comes from the first turn of the second lead. Similarly, the load $Q_{W_{1-}}$ associated with the other access depends both on the load $Q_{W_{1L_1T_5}}$ carried by the last turn of the first lead, and on the load $Q_{W_{1L_2T_5}}$ carried by the last turn of the second lead.

$$\begin{pmatrix} Q_{W_{2+}} \\ Q_{W_{2T_{2+}}} \\ Q_{W_{2T_{3+}}} \\ Q_{W_{2T_{4+}}} \\ Q_{W_{2-}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-\zeta & 0 & 0 & 0 \\ \zeta & 1-\zeta & 0 & 0 \\ 0 & \zeta & 1-\zeta & 0 \\ 0 & 0 & \zeta & 1-\zeta \\ 0 & 0 & 0 & \zeta \end{pmatrix}}_{\omega_{T,W_2}} \begin{pmatrix} Q_{W_{2T_1}} \\ Q_{W_{2T_2}} \\ Q_{W_{2T_3}} \\ Q_{W_{2T_4}} \end{pmatrix} \quad (3.80)$$

The ω_{T,W_2} matrix is simpler, as the second winding has only one lead. Consequently, the matrix corresponds to the direct assembly of 5 blocks associated with the nodes of the 4 turns making up the winding. We also note that the blocks lie on the diagonal of the matrix, as the turns are connected in series within the same lead, as already described by equation 3.74.

Having analyzed the per-turn division, the finest of our discretization models, we can proceed as for the magnetic part by defining arbitrary sections grouping together 1 or more consecutive turns of the same lead. To enable comparison between the inductive and capacitive circuit sections, the same sections are used as for the magnetic part (Figure 3.3). All that remains is to add unique section nodes names, as shown in Figure 3.10, for the model to be complete. Once these intermediate nodes have been defined, all that remains is to apply the method as it stands.

With the chosen division, the result is again two sections of two turns and one section of one turn for each lead from the first winding, and two sections of two turns for the second winding. The number of section nodes is 9 for 14 physical regions associated with the different turns, providing us with the dimensions (9×14) for our ω_s transformation matrix from

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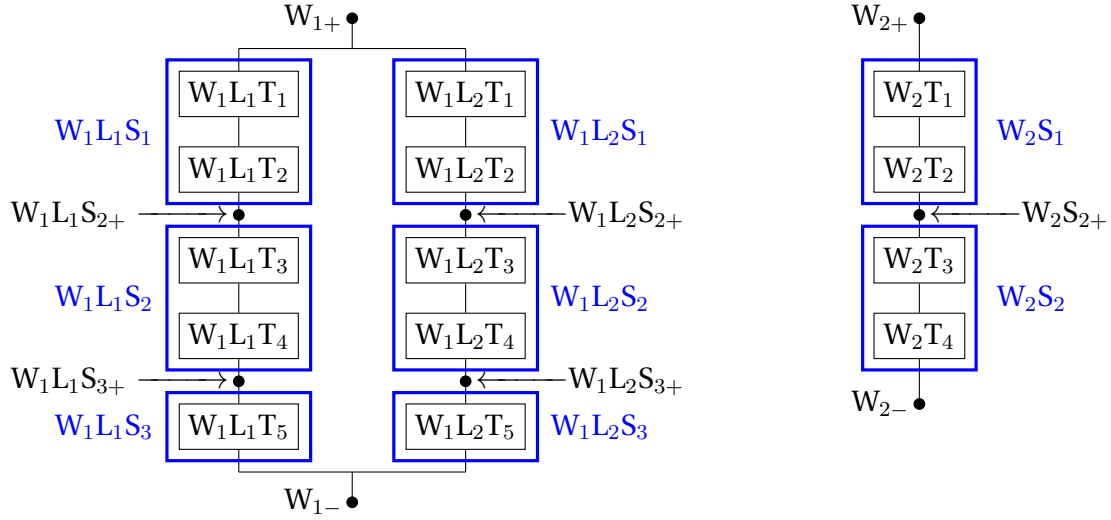


Figure 3.10: Transformer discretization in arbitrary sections.

physical regions to the equivalent circuit split in sections. Once again, the transformation matrix ω_S

$$\omega_S = \begin{pmatrix} \omega_{S,W_1} & \mathbf{0} \\ \mathbf{0} & \omega_{S,W_2} \end{pmatrix}, \quad (3.81)$$

is composed of two principal blocks, each relative to one winding and associated together along the matrix diagonal.

$$\begin{pmatrix} Q_{W_{1+}} \\ Q_{W_{1L_1S_{2+}}} \\ Q_{W_{1L_1S_{3+}}} \\ Q_{W_{1-}} \\ Q_{W_{1L_2S_{2+}}} \\ Q_{W_{1L_2S_{3+}}} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{2-\zeta}{2} \frac{1-\zeta}{2} & 0 & 0 & 0 & \frac{2-\zeta}{2} \frac{1-\zeta}{2} & 0 & 0 & 0 \\ \frac{\zeta}{2} & \frac{1+\zeta}{2} & \frac{2-\zeta}{2} & \frac{1-\zeta}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\zeta}{2} & \frac{1+\zeta}{2} & \frac{1-\zeta}{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \zeta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\zeta}{2} & \frac{1+\zeta}{2} & \frac{2-\zeta}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\zeta}{2} & \frac{1+\zeta}{2} \end{pmatrix}}_{\omega_{S,W_1}} \begin{pmatrix} Q_{W_{1L_1T_1}} \\ Q_{W_{1L_1T_2}} \\ Q_{W_{1L_1T_3}} \\ Q_{W_{1L_1T_4}} \\ Q_{W_{1L_1T_5}} \\ Q_{W_{1L_2T_1}} \\ Q_{W_{1L_2T_2}} \\ Q_{W_{1L_2T_3}} \\ Q_{W_{1L_2T_4}} \\ Q_{W_{1L_2T_5}} \end{pmatrix} \quad (3.82)$$

Note that the charge carried by the first access ($Q_{W_{1+}}$) depends both on the charges of the first two turns of the first lead, since they are grouped together in the same section, and on the first two turns of the second lead, for the same reason. The coefficients related to the charge ($Q_{W_{1-}}$) remain unchanged compared to the per-turn discretization. Nothing changes here, since only one turn (for each lead) is connected to the negative terminal of the first winding (W_{1-}).

$$\begin{pmatrix} Q_{W_{2+}} \\ Q_{W_{2S_{2+}}} \\ Q_{W_{2-}} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{2-\zeta}{2} \frac{1-\zeta}{2} & 0 & 0 \\ \frac{\zeta}{2} & \frac{1+\zeta}{2} & \frac{2-\zeta}{2} \\ 0 & 0 & \frac{\zeta}{2} \end{pmatrix}}_{\omega_{S,W_2}} \begin{pmatrix} Q_{W_{2T_1}} \\ Q_{W_{2T_2}} \\ Q_{W_{2T_3}} \end{pmatrix} \quad (3.83)$$

The matrix of influence coefficients per section $\bar{K}_s$ can therefore be obtained from the matrix of influence coefficients per turn $\bar{K}$, based on the affine transformation newly introduced, such as

$$\bar{K}_s = \omega \bar{K} \omega^T. \quad (3.84)$$

As an example, taking $\zeta = \frac{1}{2}$ we obtain the value of the first term of the matrix of influence coefficients per section as follows

$$\begin{aligned} \bar{K}_{s_{1,1}} = \frac{1}{16} & (9\bar{K}_{1,1} + 3\bar{K}_{1,2} + 9\bar{K}_{1,6} + 3\bar{K}_{1,7} + 3\bar{K}_{2,1} + \bar{K}_{2,2} + 3\bar{K}_{2,6} + \bar{K}_{2,7} \\ & + 9\bar{K}_{6,1} + 3\bar{K}_{6,2} + 9\bar{K}_{6,6} + 3\bar{K}_{6,7} + 3\bar{K}_{7,1} + \bar{K}_{7,2} + 3\bar{K}_{7,6} + \bar{K}_{7,7}) . \end{aligned} \quad (3.85)$$

Where $\bar{K}_{s_{1,1}}$ represents the influence coefficient of the section node W_{1+} with itself. We can immediately see that the influence coefficients used in this expression are those associated with the turns adjacent to the section node, in this case turns 1, 2, 6 and 7, *i.e.* $W_1L_1T_1$, $W_1L_1T_2$, $W_1L_2T_1$ and $W_1L_2T_2$. The same observation can be made upon a non-diagonal term of the section-level influence coefficients matrix:

$$\begin{aligned} \bar{K}_{s_{3,8}} = \frac{1}{16} & (\bar{K}_{3,11} + 3\bar{K}_{3,12} + 3\bar{K}_{3,13} + \bar{K}_{3,14} + 3\bar{K}_{4,11} + 9\bar{K}_{4,12} \\ & + 9\bar{K}_{4,13} + 3\bar{K}_{4,14} + 2\bar{K}_{5,11} + 6\bar{K}_{5,12} + 6\bar{K}_{5,13} + 2\bar{K}_{5,14}) . \end{aligned} \quad (3.86)$$

Where $\bar{K}_{s_{3,8}}$ represents the influence coefficient between section node $W_1L_2S_{3+}$ and section node W_2S_{2+} . Similarly, we note that the influence coefficients used in this expression are those associated with the turns adjacent to the section node, in this case, turns 3, 4, 5, 11, 12, 13 and 14, *i.e.* $W_1L_1T_3$, $W_1L_1T_4$, $W_1L_1T_5$, W_2T_1 , W_2T_2 , W_2T_3 and W_2T_4 .

The way in which the number of parameters of the modeled system is reduced is therefore consistent with the physical understanding that can be given to the various coupling terms (electrical, but also magnetic as seen in the previous part).

3.4.3 Model Reduction by Matrix Transformations

It may be useful to convert the extracted circuit model back to the physical quantities obtained by finite element calculation. Within the same lead, it is always possible to switch back from the **circuit** model per turn to the **physical** model per turn. Indeed, there are always $n + 1$ nodes for a lead of n turns, which means that the amount of information in the circuit model exceeds that in the physical model, and can therefore be reduced by introducing its left inverse matrix. Knowing that the influence coefficient matrix per turn $\bar{K}_T$ used for the circuit representation is deduced from the influence coefficient matrix $\bar{K}$ of the 2D Finite element turns as

$$\bar{K}_T = \omega_T \bar{K} \omega_T^T, \quad (3.87)$$

where the matrix ω_T is of dimension $(m \times n)$ with $m = n + 1$, so we can calculate its left inverse matrix $\omega_{T,\text{left}}$ such that

$$\omega_{T,\text{left}}^{-1} = (\omega_T^T \omega_T)^{-1} \omega_T^T. \quad (3.88)$$

With this inverse matrix on the left, we can fall back on the physical model, as it would be obtained using the 2D finite element method ⁵,

$$\bar{K} = \omega_{T,\text{left}}^{-1} \bar{K}_T (\omega_{T,\text{left}}^{-1})^T. \quad (3.89)$$

⁵Note that this no longer applies when several leads are assembled in parallel

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The left inverse matrix of a given A matrix of rank n can be defined from the product of A^T and A , which gives a square matrix of rank n that can be inverted. This can be written as follows

$$A = \begin{pmatrix} \\ \\ \end{pmatrix}_{\substack{(m \times n) \\ m > n}} \quad (A^T A)^{-1} \underbrace{(A^T A)}_{(n \times n)} = \mathbf{I} \quad (3.90)$$

which leads to

$$A_{\text{left}}^{-1} = (A^T A)^{-1} A^T, \quad (3.91)$$

which then corresponds to the left inverse of matrix A . This holds true when we go back from the circuit model to the physical model, when there has been no loss of information during the first transformation.

Just as it is possible to define a left-hand inverse matrix of dimensions $(m \times n)$ for which $m > n$, it is also possible to define a right-hand inverse

$$A_{\text{right}}^{-1} = A^T (A A^T)^{-1}, \quad (3.92)$$

for a matrix A of dimensions $(m \times n)$ with $m < n$. This corresponds, for example, to the reduced matrix obtained in per section models.

The definition of the inverse on the right is then necessary when we wish to reduce a model by section into a more condensed model, for example by lead or winding. Indeed, for instance, if we wish to reduce a model by section at winding level, the relationship between the influence coefficient matrices is as follows

$$\bar{\mathbf{K}}_W = \overbrace{\begin{matrix} \omega_W & \omega_{S,\text{right}}^{-1} \end{matrix}}^{\omega_{S,W}} \bar{\mathbf{K}}_S (\omega_{S,\text{right}}^{-1} \omega_W)^T, \quad (3.93)$$

$(N_W \times N_W) \quad (N_W \times n_T) \quad (n_T \times N_S) \quad (N_S \times N_S)$

and the following matrix appears

$$\omega_{S,W} = \omega_W \omega_{S,\text{right}}^{-1}, \quad (3.94)$$

which corresponds to the change in discretization level from section-level to winding-level.

If we define by n the number of physical elements of the system to be modeled (n being for example n_T turns, n_S sections, n_W windings) and by N the number of associated circuit nodes (similarly N being possibly N_T turns nodes, N_S sections nodes or N_W windings nodes), the number of simulation and postprocessing operations minimally required corresponds to the value of Table 3.1.

Table 3.1: Number of system resolutions and post operations required, depending on the discretization level choice.

	Number of system resolutions	Total number of post operations
Inductive	n	$C_n^2 = \frac{1}{2}(n+1) n$
Capacitive	N	$C_N^2 = \frac{1}{2}(N+1) N$

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To illustrate this, we can refer back to our case study of the 30-turn air inductor, for which two layouts exist: the first imposes all turns consecutively on the same layer, and the second features 3 layers of 10 conductors placed one on top of the other.

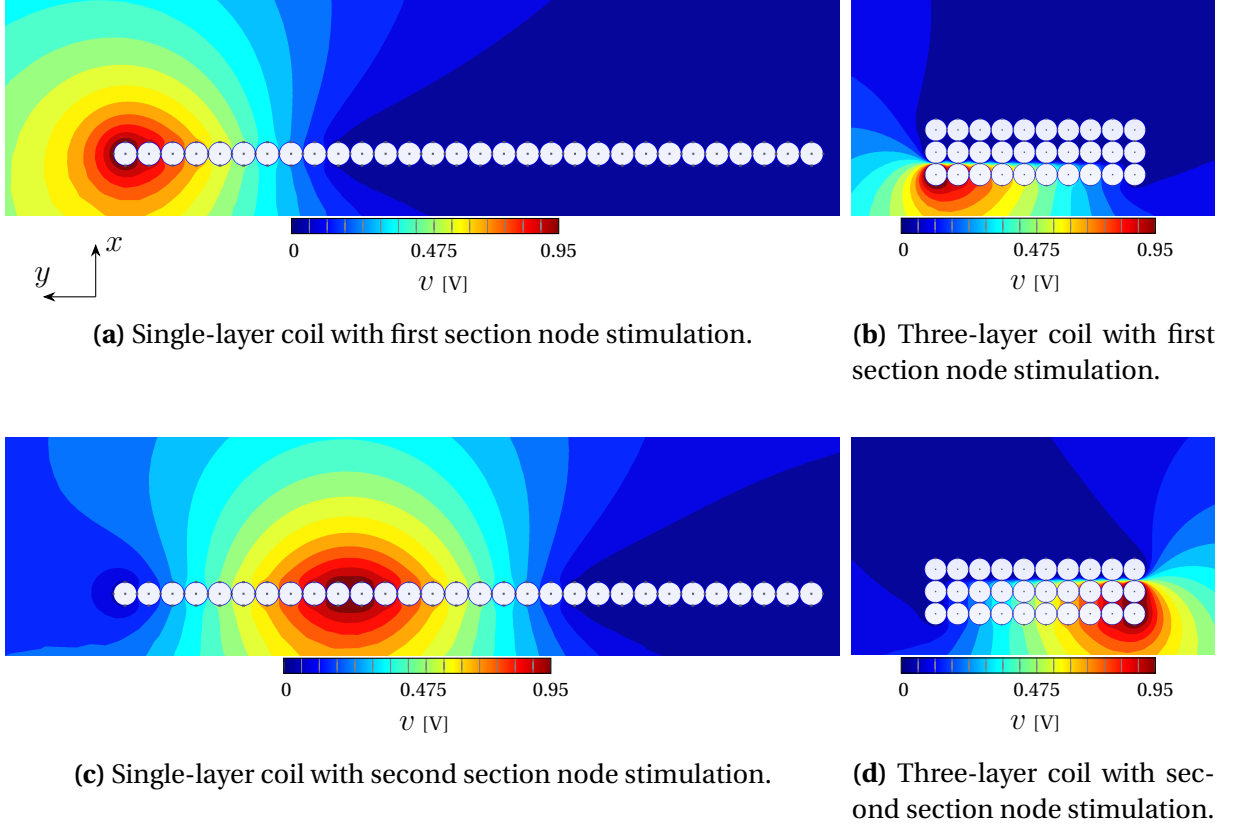


Figure 3.11: Voltage distributions comparison from two different layouts of a 30-turn air coil.

Figure 3.11 shows the stimulation of the first and second section nodes for both geometries. This illustrates the linear evolution of the potential within the same section and the electrostatic identifications provide the following capacitance matrices for each geometry:

$$\begin{aligned}
 \underline{\underline{C}}_{S, 1\text{lay}} &= \begin{bmatrix} \overset{C_{S_{1+}, \text{ref}}}{0.873 \text{ pF}} & \overset{C_{S_{1+}, 2+}}{2.052 \text{ pF}} & 0.410 \text{ pF} & 0.128 \text{ pF} \\ \cdot & \cdot & 2.052 \text{ pF} & 0.410 \text{ pF} \\ \cdot & \cdot & 0.773 \text{ pF} & 2.053 \text{ pF} \\ \cdot & \cdot & \cdot & 0.873 \text{ pF} \end{bmatrix}, & \underline{\underline{C}}_{S, 3\text{lay}} &= \begin{bmatrix} 0.389 \text{ pF} & 1.597 \text{ pF} & \overset{C_{S_{1+}, 3+}}{52.107 \text{ pF}} & 0.164 \text{ pF} \\ \cdot & 0.505 \text{ pF} & 1.738 \text{ pF} & 54.279 \text{ pF} \\ \cdot & \cdot & 0.879 \text{ pF} & 1.653 \text{ pF} \\ \cdot & \cdot & \cdot & 0.762 \text{ pF} \end{bmatrix}.
 \end{aligned}
 \tag{3.95} \tag{3.96}$$

We notice that the capacitive coupling is clearly more pronounced for the three-layer case than for the single-layer case. This is particularly noticeable for the terms $C_{S_{1+}, 3+}$ and $C_{S_{2+}, 3-}$ where the electrostatic energy is concentrated.

We can also seek to reduce this model by section into a more compact model by winding. Rather than reiterating new identifications with the potentials imposed by winding, we implement the affine transformations defined by right inverse matrices, as described above.

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In this case, the matrix ω_s

$$\omega_s = \begin{pmatrix} \frac{19}{20} & \frac{17}{20} & \frac{15}{20} & \frac{13}{20} & \frac{11}{20} & \frac{9}{20} & \frac{7}{20} & \frac{5}{20} & \frac{3}{20} & \frac{1}{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{20} & \frac{3}{20} & \frac{5}{20} & \frac{7}{20} & \frac{9}{20} & \frac{11}{20} & \frac{13}{20} & \frac{15}{20} & \frac{17}{20} & \frac{19}{20} & \frac{19}{20} & \frac{17}{20} & \frac{15}{20} & \frac{13}{20} & \frac{11}{20} & \frac{9}{20} & \frac{7}{20} & \frac{5}{20} & \frac{3}{20} & \frac{1}{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{20} & \frac{3}{20} & \frac{5}{20} & \frac{7}{20} & \frac{9}{20} & \frac{11}{20} & \frac{13}{20} & \frac{15}{20} & \frac{17}{20} & \frac{19}{20} & \frac{19}{20} & \frac{17}{20} & \frac{15}{20} & \frac{13}{20} & \frac{11}{20} & \frac{9}{20} & \frac{7}{20} & \frac{5}{20} & \frac{3}{20} & \frac{1}{20} \\ 0 & \frac{1}{20} & \frac{3}{20} & \frac{5}{20} & \frac{7}{20} & \frac{9}{20} & \frac{11}{20} & \frac{13}{20} & \frac{15}{20} & \frac{17}{20} & \frac{19}{20} \end{pmatrix}, \quad (3.97)$$

which allows physical quantities to be converted into circuit quantities by section, couples itself with the ω_{W} matrix

$$\omega_{\text{W}} = \begin{pmatrix} \frac{59}{60} & \frac{57}{60} & \frac{55}{60} & \frac{53}{60} & \frac{51}{60} & \frac{49}{60} & \frac{47}{60} & \frac{45}{60} & \frac{43}{60} & \frac{41}{60} & \frac{39}{60} & \frac{37}{60} & \frac{35}{60} & \frac{33}{60} & \frac{31}{60} & \frac{29}{60} & \frac{27}{60} & \frac{25}{60} & \frac{23}{60} & \frac{21}{60} & \frac{19}{60} & \frac{17}{60} & \frac{15}{60} & \frac{13}{60} & \frac{11}{60} & \frac{9}{60} & \frac{7}{60} & \frac{5}{60} & \frac{3}{60} & \frac{1}{60} \\ \frac{1}{60} & \frac{3}{60} & \frac{5}{60} & \frac{7}{60} & \frac{9}{60} & \frac{11}{60} & \frac{13}{60} & \frac{15}{60} & \frac{17}{60} & \frac{19}{60} & \frac{21}{60} & \frac{23}{60} & \frac{25}{60} & \frac{27}{60} & \frac{29}{60} & \frac{31}{60} & \frac{33}{60} & \frac{35}{60} & \frac{37}{60} & \frac{39}{60} & \frac{41}{60} & \frac{43}{60} & \frac{45}{60} & \frac{47}{60} & \frac{49}{60} & \frac{51}{60} & \frac{53}{60} & \frac{55}{60} & \frac{57}{60} & \frac{59}{60} \end{pmatrix}, \quad (3.98)$$

which is its counterpart for the windings.

$$\omega_{S,W} = \omega_W \omega_{S,\text{right}}^{-1} \quad (3.99)$$

And finally, we obtain the change from the quantities defined by sections into quantities defined by windings by applying the following transformation

$$\omega_{S,W} = \begin{pmatrix} 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}. \quad (3.100)$$

The operation then provides the capacitance matrices for the complete windings such that

$$\underline{C}_{\text{W, 1lay}} = \begin{bmatrix} 1.646 \text{ pF} & 0.843 \text{ pF} \\ \cdot & 1.646 \text{ pF} \end{bmatrix}, \quad (3.101) \quad \underline{C}_{\text{W, 3lay}} = \begin{bmatrix} 1.0187 \text{ pF} & 47.6933 \text{ pF} \\ \cdot & 1.5164 \text{ pF} \end{bmatrix}. \quad (3.102)$$

If we reduce the system as much as possible to have only one equivalent capacitance⁶, we then obtain

$$C_{\text{eq}} = \frac{C_{W_{1+},\text{ref}} C_{W_{1-},\text{ref}}}{C_{W_{1+},\text{ref}} + C_{W_{1-},\text{ref}}} + C_{W_{1+},1-} \quad , \quad (3.103)$$

capacitance which is worth 1.666 pF for the first case and 48.291 pF for the second. Once coupled with the inductive model already identified previously (respectively 39.4647 μ H and 60.7157 μ H), we deduce that the first resonance frequency is of 19.63 MHz for the single-layer geometry versus 2.93 MHz for the multi-layer geometry. So that makes a significant difference.

Obviously, with these very reduced models, we can only deduce the first resonance frequency. This is the whole point of the section-level discretization: providing a reduced model of the magnetic component which makes it possible to calculate several resonance phenomena, while maintaining an acceptable level of complexity.

⁶We will assume here that the potentials of each winding are floating with respect to the reference.

3.5 Circuit Representation and Frequency Dependency

In the electrical devices we study, the behavior is affected by the frequency at which the phenomena occur. This particularly applies to magnetic phenomena and the associated joule losses (both in conductors and ferromagnetic cores). Indeed, many phenomena affect the distribution of currents, and consequently the equivalent resistance and inductance of the device. Already in a single conductor, the **skin effect** modifies the distribution of current density within the conductor, and consequently its resistance and equivalent inductance. With the skin effect, the variable magnetic field induced by the conductor's own currents causes the current density to react and spread out around the periphery of the conductor, as shown in Figure 3.12.

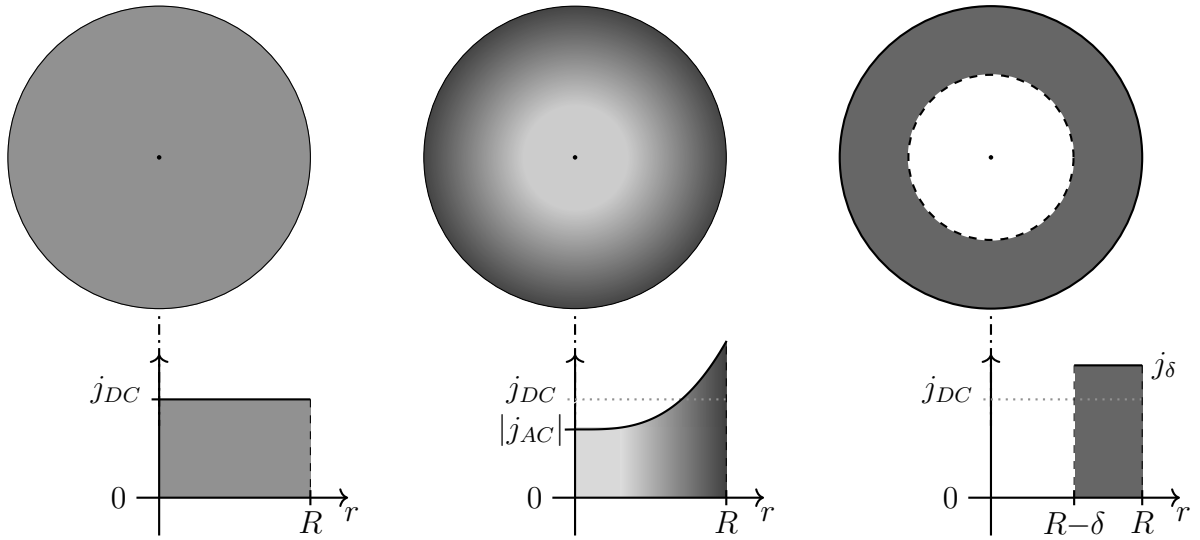


Figure 3.12: Illustration of the skin effect upon a single conductor.

In this graph, we observe a different current density distribution according to the regime applied to the conductor: direct current (DC) or alternating (AC). On the very left, for direct current I_{DC} , the current density is uniform over the entire surface of the conductor. In the center of the figure, an alternating current is applied, and the amplitude of the current density $|j_{AC}|$ changes from a low value at the center of the conductor to a high value at the periphery. This non-uniform current density results in higher Joule losses than in DC, since losses depend quadratically on current density. The equivalent resistance of the conductor is therefore higher in the event of skin effect (AC) than in DC. The part of the graph on the very right corresponds to the simplified representation of the skin effect, for which the current density is uniform over a peel of thickness δ and whose losses are equivalent to those of the non-uniform j_{AC} distribution. It is commonly used in electrical engineering.

In direct current, if a total current I is injected into a straight conductor with a circular cross-section, the current density is constant over the entire surface of the conductor and is equal to

$$j_{DC} = \frac{I}{\pi R^2} , \quad (3.104)$$

where R designates the radius of the conductor.

In AC conditions, the current density inside the conductor is no longer constant, due to the effect that the varying magnetic field has on the distribution of the current. This is known as the skin effect, which increases current density at the periphery of the conductor and

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decreases it at its center, as illustrated in the middle of Figure 3.12. In this case, the current density varies along the radius of the conductor according to the following relationship

$$j_{AC} = \frac{I}{2\pi R} k \frac{J_0(kr)}{J_1(kR)}, \quad (3.105)$$

$$j_{AC} = \underbrace{\frac{I}{\pi R^2}}_{j_{DC}} \underbrace{\frac{1}{2} \frac{1-j}{\alpha} \frac{J_0\left(\frac{1-j}{\alpha} \frac{r}{R}\right)}{J_1\left(\frac{1-j}{\alpha}\right)}}_{d_{AC}}, \quad (3.106)$$

in which we find the direct current density j_{DC} , multiplied by a complex distribution factor, d_{AC} , whose value depends on the position along the radius r . This expression involves the dimensionless factor $\alpha = \frac{\delta}{R}$, which refers to the ratio of the penetration depth δ to the radius R of the conductor, and the first kind Bessel functions of order 0 and 1, respectively J_0 and J_1 .

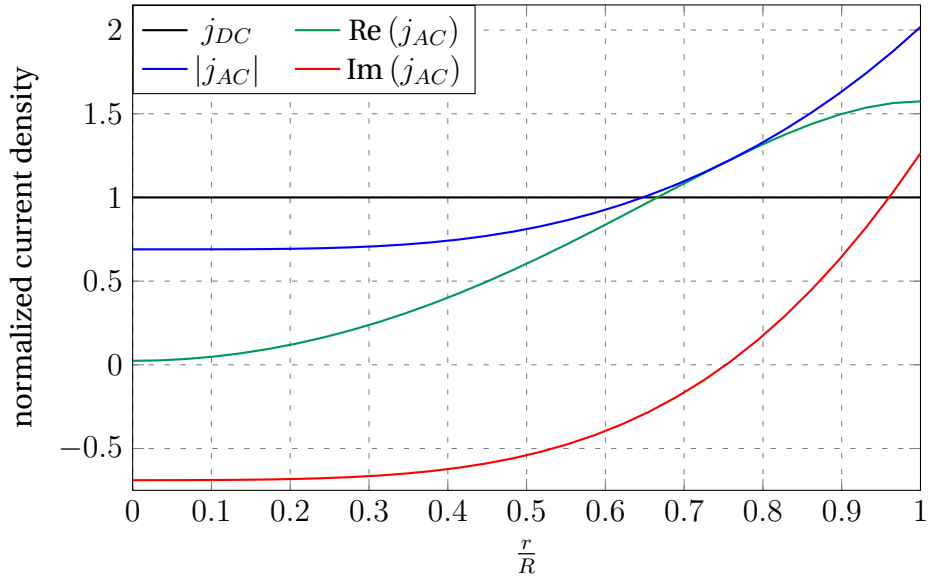


Figure 3.13: Current density distribution along the radius of a straight circular wire.

Note that for these expressions, the skin depth δ is defined

$$\delta = \sqrt{\frac{1}{\pi \sigma \mu f}}, \quad (3.107)$$

as well as the wave number k

$$k = \frac{1-j}{\delta}. \quad (3.108)$$

As the signal frequency increases, the majority of the current flows to the periphery of the conductor, over a smaller area than the total conductor surface. This uneven current distribution, which occurs in AC operation, has several consequences: on the one hand, it increases the equivalent resistance as the effective cross-section decreases, and on the other, it reduces the conductor's internal inductance, which then tends towards 0. The frequency

dependence of these two behaviors (resistive and inductive) can be seen in Figure 3.14 for a normalized conductor size.

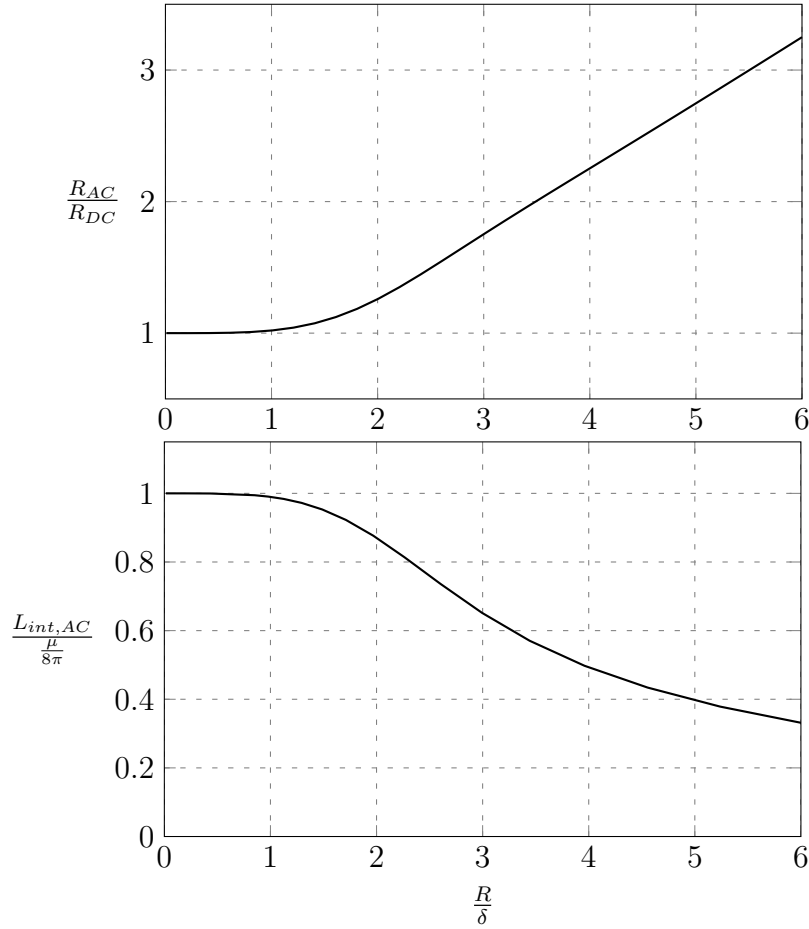


Figure 3.14: Equivalent normalized resistance and inductance evolution with respect to the square root of the frequency.

When the skin depth is half the conductor radius, the resistance value already increases by 25.8 % and the internal inductance is 87 % of its initial value for very low frequencies. Thereafter, the resistance continues to increase according to a square-root dependence of f for higher frequencies, and the inductance continues to decrease, until it tends towards 0 when the current is pushed to the extreme circumference of the conductor.

We have seen that the skin effect reduces the internal inductance and increases the equivalent conductor resistance at higher frequencies. Although this increase may seem minor, it is only the impact of induced currents on a single conductor. When several conductors are arranged side by side, they influence each other, and the effects of the induced currents upon the losses are amplified. In the case of several conductors influencing each other, we refer to this as the **proximity effect**. Figure 3.15 shows the current density distribution for two closely spaced rectilinear conductors in the case where the currents are imposed in the same direction and the case where they are of opposite sign. In the first case, with currents of the same sign, we notice that the current density is pushed towards the opposite ends of the two conductors. As if the two conductors formed a single surface, with the current distributed around its periphery. The opposite effect is observed when the currents are of opposite sign and current densities tend to converge. In both configurations, the losses are greater than the sum of the skin effect losses in each individual conductor.

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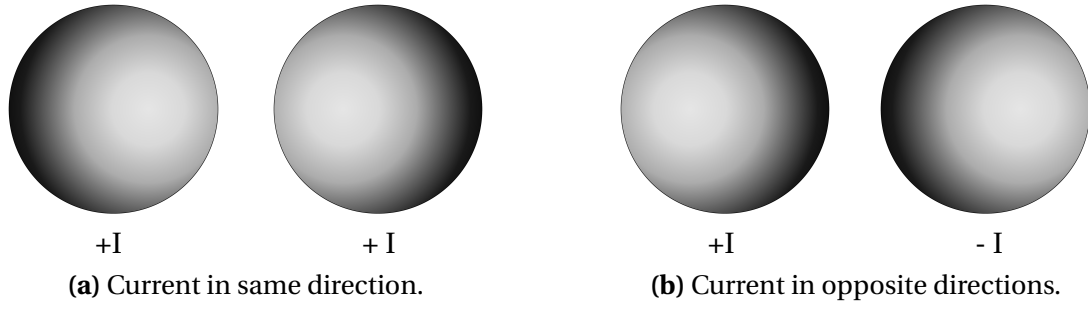


Figure 3.15: Current density distribution under proximity effect between two single conductors.

The **proximity effect** is exacerbated when several layers of conductors interact, and the contributions of each layer add up in terms of current density. Losses, meanwhile, can increase as the square of the frequency in the worst cases. The increase in **losses** with frequency is therefore much **greater** with the proximity effect than with the skin effect, where losses only increase with the square root of the frequency.

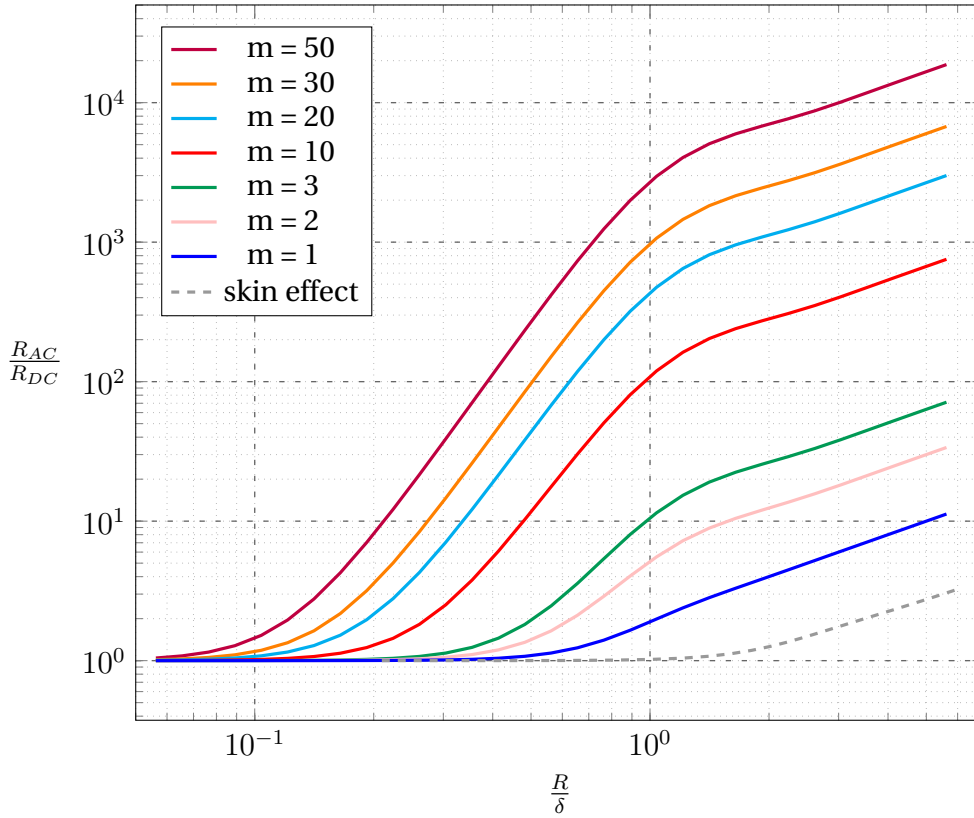


Figure 3.16: Proximity effect equivalent resistance levels for different number of layers, corresponding to the analytical results, obtained with R being the radius of the conductor and δ being the skin depth.

A distinction can therefore be made between the **skin effect**, whose source corresponds to the magnetic field induced by currents in the conductor itself, and the **proximity effect**, whose magnetic field source is external to the conductor, typically another nearby conductor. These two effects belong to the category of induced currents that appear in conductors at high frequencies.

Litz wire is generally adopted to mitigate the additional losses due to these induced currents. However, the skin effect and the proximity effect can also appear in a litz wire for sufficiently high frequencies or a large number of layers. These particular current distributions are illustrated in Figure 3.17.

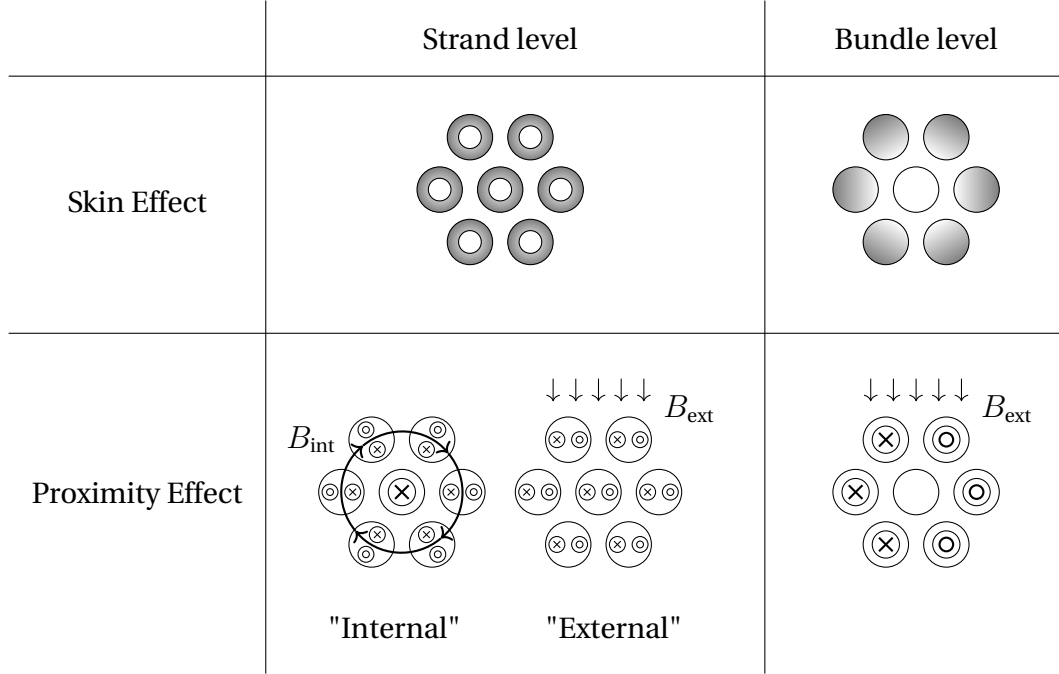


Figure 3.17: Effects of induced currents within a litz wire, obtained from [106].

Here you can see the **skin effect** that can affect current distribution at the level of individual strands, as illustrated in the top left-hand corner, or the skin effect at the level of a complete bundle (top right-hand corner), for which the distribution evolves from the strands at the center of the bundle to the outer strands. The **proximity effect** can also occur. Firstly, at strand level, with different effects for an internal magnetic field, due to the effects of strands in the bundle on others, and for a magnetic field external to the bundle. Secondly, the proximity effect at the level of the bundle itself, for which the sign of the strand current varies between different zones of the bundle.

It is possible to explain AC current distribution using simple **analytical models**, such as those described above for the skin effect, or Dowell's method for the proximity effect. Although these methods provide a valuable insight into conductor behavior and losses as a function of frequency, the results are rather limited, since they assume a certain conductor geometry and layered winding organization. It is therefore difficult to represent every possible transformer configuration without resorting to **approximations** on conductor organization and geometry.

This is where the **2D finite element model** comes into its own, as it allows us to account for more complex geometries, such as specific geometrical arrangements, the absence of a magnetic core or the presence of large air gaps, which locally modify the magnetic field distribution, therefore the electric current density and associated copper losses. By taking advantage of finite elements and the magnetodynamic model from Chapter 2, we can extract circuit parameters as localized constants of the device under study, for each frequency of interest. In the case of magnetic phenomena, this means obtaining self-inductances and mutual inductances, as well as individual conductor resistances and coupled resistances.

3.5.1 Two Wires Case Study

To familiarize oneself with the concepts of mutual inductances and resistances, we can study the case of two neighboring conductors, placed in air and interacting with each other through the magnetic flux. The dependency of the circuit's magnetic parameters on the applied frequency can therefore be illustrated. By imposing a given electric current in a wire and observing the voltage on this wire and on its neighbor, we can determine the mutual inductances and resistances such as

$$Z_{ij} = \frac{\bar{U}_i}{\bar{I}_j}, \quad (3.109)$$

with

$$R_{ij} = \text{Re}(Z_{ij}), \quad (3.110)$$

$$X_{ij} = \text{Im}(Z_{ij}). \quad (3.111)$$

In the case of two wires, for which we define two independent accesses, we therefore have the (2×2) matrix

$$\underline{\underline{Z}} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} + j\omega \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}. \quad (3.112)$$

For this geometry, a low-frequency case (1 Hz) and a high-frequency case (100 kHz) are compared. In Figure 3.18 on the left, we can see the lack of induced currents in the bottom conductor when the top conductor is supplied with 1 ampere at 1 Hz, given the low flux variation associated with this low frequency. In contrast, on the right-hand side, currents are induced in the bottom conductor by the top conductor, at the higher frequency of 100 kHz.

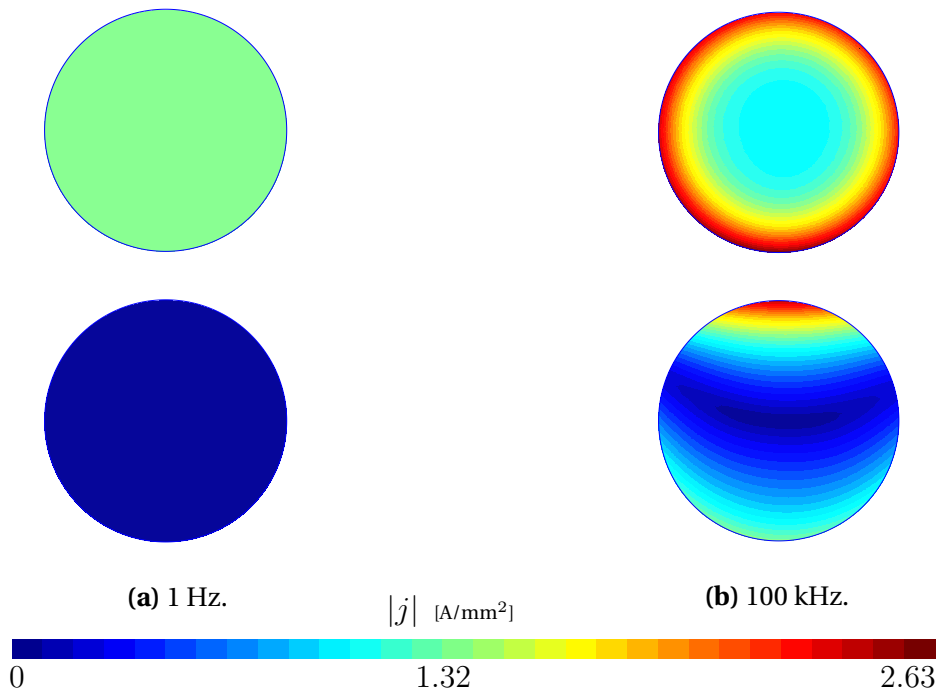


Figure 3.18: Current density amplitude $|j|$ over two round conductors (1 mm Ø) influencing each other, with net current of 1 ampere imposed at the top conductor and 0 at the other.

Since the system here is defined as symmetrical with respect to the two wires, it is sufficient to study variations in the resistances and inductances of just one of the two wires, to

simplify and reduce the analysis.

In direct current, the resistance of a single conductor is $R_{DC} = 21.37 \text{ m}\Omega$ and this remains true for each of the wires: $R_{11} = R_{22} = 21.37 \text{ m}\Omega$ at 1 Hz.

At 100 kHz, the self-resistance of the first conductor increases to $R_{11} = 39.49 \text{ m}\Omega$, which is higher than the self-resistance of the same conductor alone (in the absence of the second), whose value at 100 kHz is $R_1 = 31.36 \text{ m}\Omega$. This substantial increase between the case of the single conductor and the case of the conductor in the presence of the second exposes the **additional losses** associated with **induced currents**. Indeed, maintaining induced currents in the bottom conductor (whose total net current is zero) requires additional power to be injected into the top conductor, which is what the higher resistance value R_{11} represents.

The mutual resistance value is negative in this case: $R_{21} = -1.85 \text{ m}\Omega$ at 100 kHz. R_{21} is a non-diagonal term in the resistance matrix whose sign can be either negative or positive, depending on the device geometry. If we express the active power dissipated in this circuit as

$$P = \text{Re} \left(\sum_i \bar{U}_i \bar{I}_i^* \right) \quad (3.113)$$

$$P = R_{11} I_1^2 + (R_{12} + R_{21}) \text{Re}(\bar{I}_1 \bar{I}_2^*) + R_{22} I_2^2 \quad (3.114)$$

$$P = R_{11} I_1^2 + (R_{12} + R_{21}) I_1 I_2 \cos(\phi_2 - \phi_1) + R_{22} I_2^2, \quad (3.115)$$

where ϕ_1 and ϕ_2 correspond to the phase angles of the currents I_1 and I_2 . We notice that the contribution of the off-diagonal terms (R_{12} and R_{21}) depends on the phase shift between the currents I_1 and I_2 . If we consider that both turns share the same current (as if they were part of the same winding), then the contribution to the losses is of the same sign as R_{12} and R_{21} , *i.e.* negative in this case. On the other hand, if we assume that the two currents are in phase opposition (as if they were forming a transformer with a short-circuited secondary), then the contribution changes sign and becomes positive. In other words, for this device, there will be more copper losses in the conductors if the currents are in phase opposition than if they are in phase, for the same current amplitudes.

Figure 3.19 shows the evolution of resistances as a function of frequency, for a single wire (R_1) and for the first two terms of the resistance matrix (R_{11} and R_{12}).

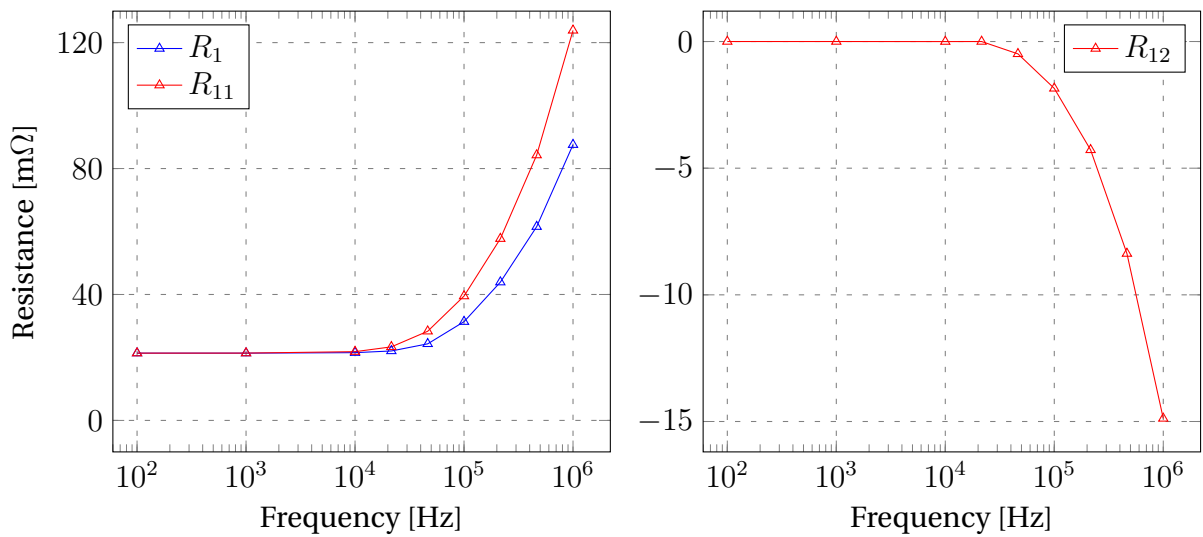


Figure 3.19: Frequency dependency of the resistances from the two-wire system.

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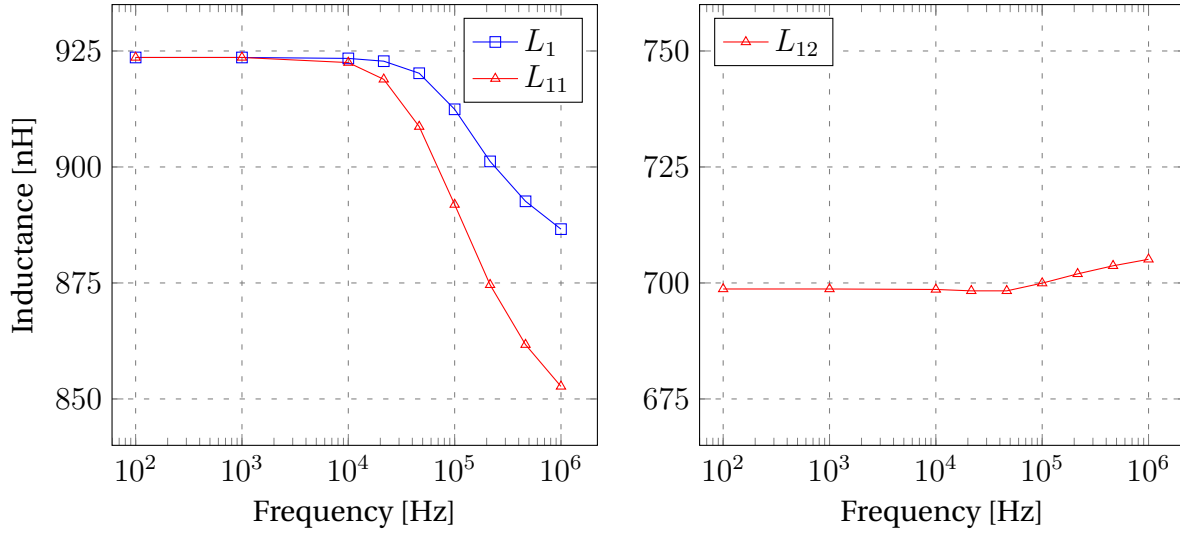


Figure 3.20: Frequency dependency of the inductances from the two-wire system.

After observing the behavior of the real part of the impedance, we can study its inductance, whose evolution as a function of frequency is shown in Figure 3.20. The first thing to note is that the inductance of the single conductor (L_1) starts from the same value as the self-inductance of the top conductor (L_{11}) in the presence of the second. The starting value being $L_1 = L_{11} = 923.6$ nH at a frequency of 1 Hz. The value of the mutual inductance is lower: $L_{12} = L_{21} = 698.7$ nH at 1 Hz, corresponding to a coupling coefficient of 75.65 %. At 100 kHz, the value of the self-inductance $L_{11} = 891.9$ nH is much lower than the value of the inductance of an insulated conductor $L_1 = 912.4$ nH. On the other hand, the value of the mutual inductance remains almost unchanged: $L_{12} = 699.96$ nH.

As the frequency increases and the current density is increasingly pushed to the periphery of the conductors, the inductance declines at the same time as the resistance begins to rise. In the case of the conductor alone, without the effects of induced currents linked to the presence of the bottom conductor, we notice that the value of the inductance L_1 decreases less rapidly, just as the resistance R_1 increased less rapidly.

3.5.2 Frequency Dependency of Magnetic Parameters Represented through Ladder Circuits

The inductive parameters of magnetic components vary according to the frequency applied to them, both in terms of the resistances associated with losses and the inductances associated with field distribution in space. Localized constant circuits (inductance and resistance) cannot represent these variations, at least not via the use of a single resistance and a single inductance. Parameters **variations in frequency** can be approximated more or less accurately by the combination of several linear components (R, L, C). It is then conceivable to represent the real component, whose parameters depend on frequency, by a **ladder circuit**. A whole series of ladder circuits exist, such as Foster networks, Cauer networks or other combinations of R-L-C in pi, T, etc. Generic circuits based on the Cauer network have already been implemented in order to model the windings of electrical devices (e.g. for inverter fed AC machines) and proved to be an appropriate fit for this purpose [77, 84].

R-L ladder circuits usage is a classic technique from radio frequency applications [110], where constant steps were used to generate the echelons, leading to large ladder sizes. A sim-

ilar approach involving variable stepping between resistances on the one hand, and inductances on the other, was subsequently introduced [67] and it is precisely such an approach that has been chosen for this work. The 4-level Cauer ladder represents the physical phenomena of the skin effect through different resistances and inductances, which can then be interpreted. Such a circuit is shown in Figure 3.21 and features a fairly small number of circuit elements.

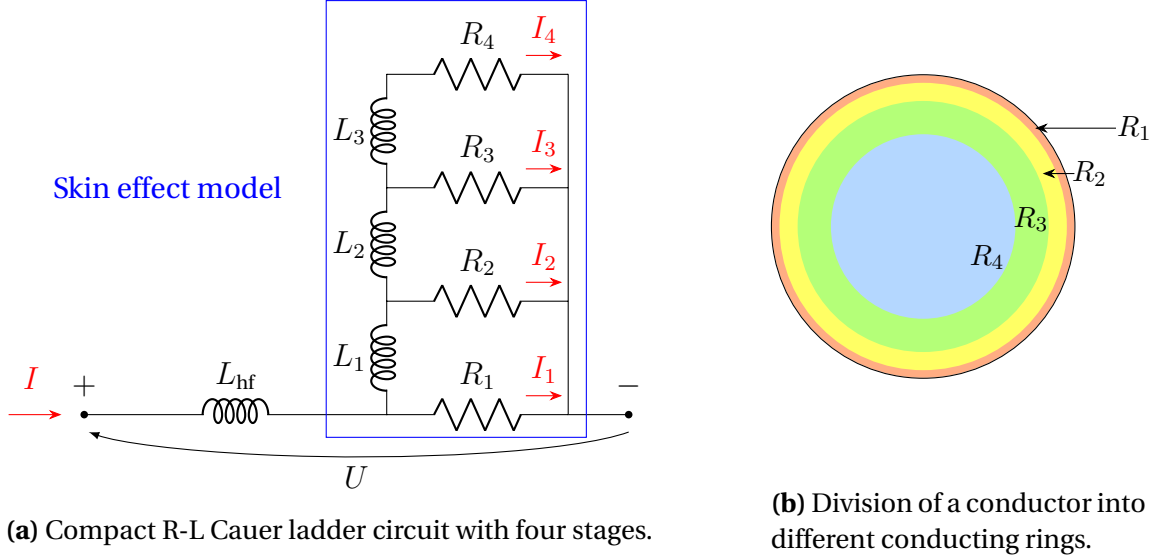


Figure 3.21: Compact Cauer ladder circuit and equivalent conducting ring zones.

With this approach, it becomes clear that the different rings carry different fractions of the total current in the conductor. At low frequencies, all the current is distributed over the 4 shell rings, and the conductor resistance corresponds to the 4 resistances paralleled. At high frequencies, the inductances limit the current flow in the upper stages, forcing the current through R_1 only, which is a reliable representation of current distribution in the presence of skin effect. The same holds true for the inductance as at low frequencies, the entire cross-section of the conductor is useful and the inductance is at its maximum value; at high frequencies, the current density is pushed to the periphery of the conductor, leaving only the external inductance, associated with the magnetic field located around the wire.

The ladder circuit that models frequency variations is characterized by 4 parameters: the base resistance R_1 , the base inductance L_1 , the resistance ratio r_R and the inductance ratio r_L . The parameters of an echelon are then multiples of the parameters of the previous echelon, such as

$$R_{i+1} = \frac{R_i}{r_R} \quad (3.116)$$

$$L_{i+1} = \frac{L_i}{r_L}. \quad (3.117)$$

Such a circuit then generates a resistance whose value increases with frequency, and an inductance whose value decreases with frequency. This corresponds well to the effects observed on complete windings and on the diagonal terms of the inductance matrix. However, our simple two-turn example shows us that off-diagonal terms can follow the opposite behavior, *i.e.* a resistance that decreases and even becomes negative, as well as an inductive part that increases. The ladder circuit described above is not sufficient to directly represent such behavior. Fortunately, the use of **controlled voltage sources** allows us to get around

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this problem. This is achieved by simply defining a constant R-L pair whose value is transmitted by a controlled source, and defining the corresponding ladder circuit whose value is transmitted by a source of **opposite polarity**. In this way, resistances and inductances are subtracted respectively, resulting in zero resistance at low frequencies and negative resistance at higher frequencies, and an inductance whose value increases with frequency.

Based on the compact ladder circuit [67], we can develop a generic circuit useful for both frequency behaviors: positive resistance that increases with decreasing inductance, and negative resistance with increasing inductance. To do this, we define the following ladder resistance, which can be used for both positive and negative polarity.

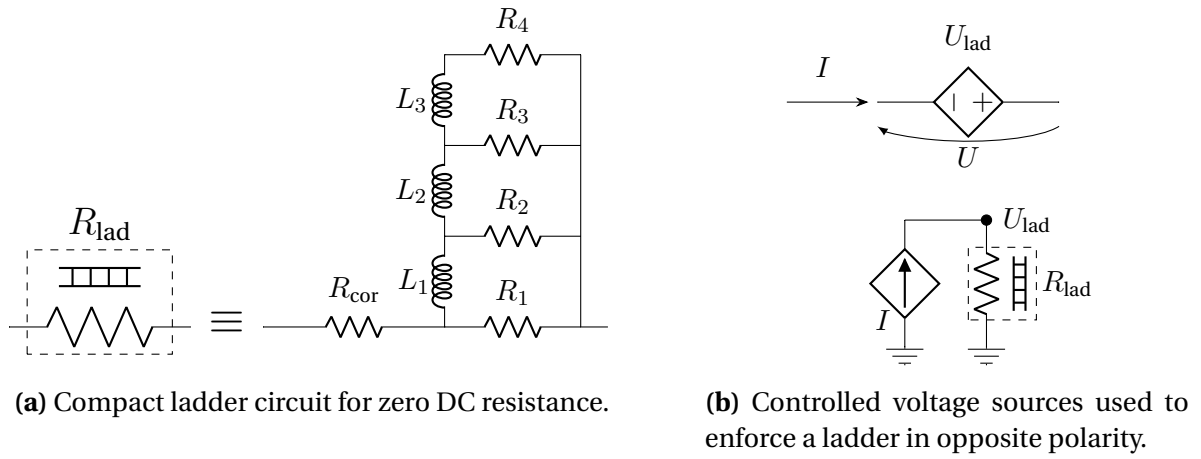


Figure 3.22: Generic ladder circuit for positive and negative polarities, enforcing zero DC resistance.

The behavior of the ladder circuit impedance as a function of frequency is illustrated below for the diagonal term R_{11} - L_{11} and for the off-diagonal term R_{12} - L_{12} . The parameters values are taken from Table 3.2 for the R_{11} - L_{11} case and from Table 3.3 for the R_{12} - L_{12} case. Then, the impedances of the equivalent circuits are compared with the impedances obtained by direct identification in FEM.

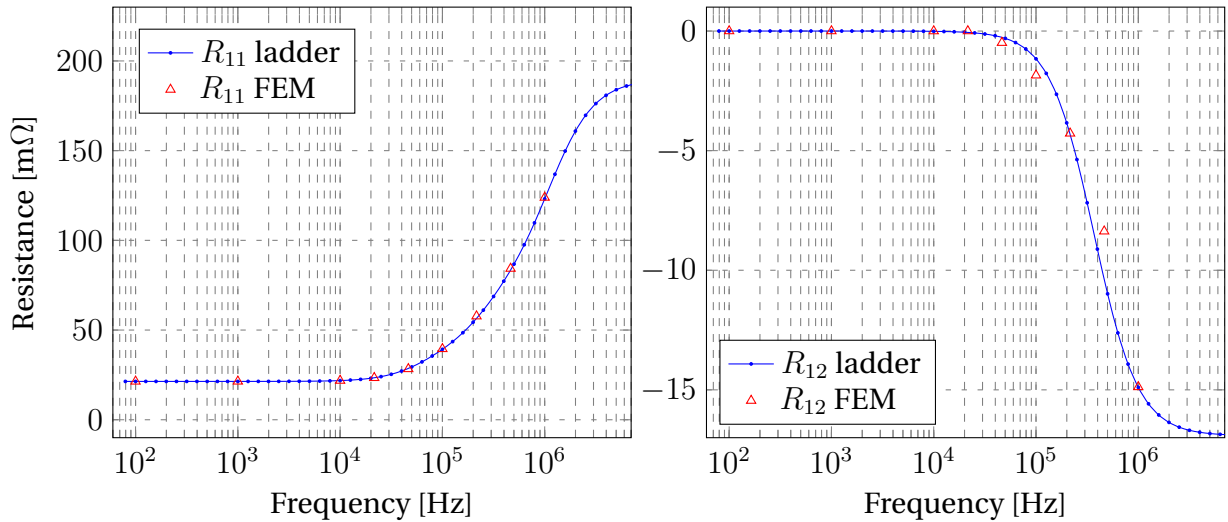


Figure 3.23: Resistance dependency with frequency.

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One can remark a pretty strong agreement for the resistive part with the appropriate evolution along the frequency. The ladder circuit approach remains valid for both the diagonal term and the off-diagonal one.

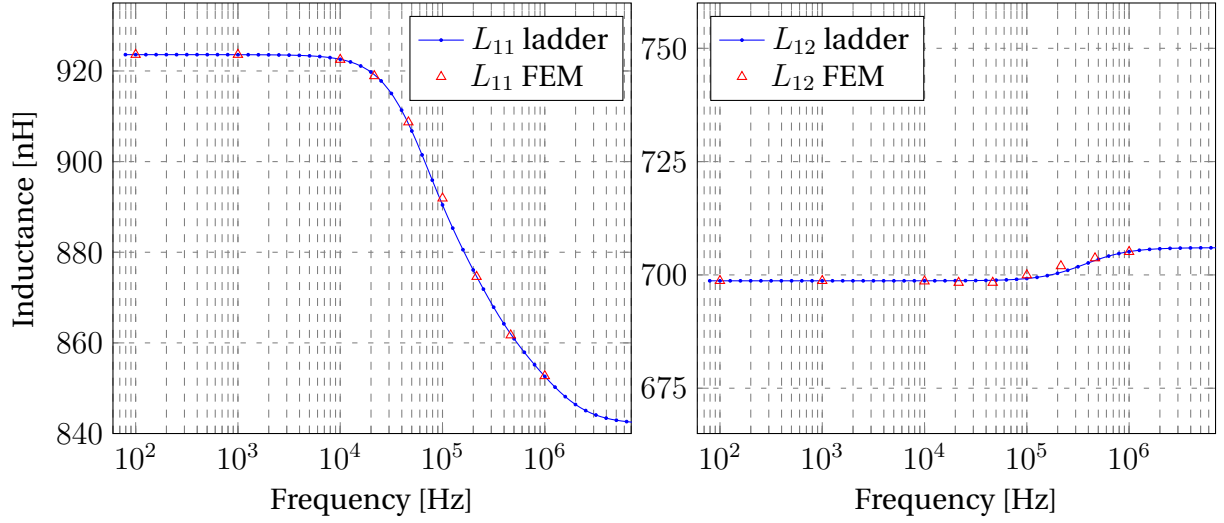


Figure 3.24: Inductance dependency with frequency.

The same observation can be made for the inductance of the ladder circuit, in relation to the inductances identified. It can be seen that the evolution and values follow the previously identified data very well.

Table 3.2: Optimized ladder parameters of $R_{11} - L_{11}$ impedance.

$L_{1,11}$	$r_{L,11}$	$R_{1,11}$	$r_{R,11}$	$R_{cor,11}$	$R_{DC,11}$	$L_{hf,11}$
314.681 nH	0.096275	528.008 mΩ	0.326648	-359.630 mΩ	21.37 mΩ	842.085 nH

Table 3.3: Optimized ladder parameters of $R_{12} - L_{12}$ impedance.

$L_{1,12}$	$r_{L,12}$	$R_{1,12}$	$r_{R,12}$	$R_{cor,12}$	$R_{DC,12}$	$L_{hf,12}$
820.478 nH	3.621246	179.436 mΩ	0.0943564	-162.518 mΩ	0.0 mΩ	706.011 nH

Having observed that the ladder circuit gives the expected behavior for each term of the impedance matrix, we have every right to expect that putting several elements in series will also give the right impedance without deviating too much from the identified values. In order to evaluate these performances, we can assume the following case and consider that the two turns are in series and are part of the same winding whose total impedance corresponds to the sum of all the terms of the impedance matrix per turn. We then compare the total impedance obtained by serializing the ladder circuits for Z_{11} and Z_{22} , with the impedance of the ladder circuit identified for the complete winding, denoted by Z_{tot} . The parameters of the complete winding-level circuit are as follows,

Table 3.4: Optimized ladder parameters of $R_{tot} - L_{tot}$ impedance.

$L_{1,tot}$	$r_{L,tot}$	$R_{1,tot}$	$r_{R,tot}$	$R_{cor,tot}$	$R_{DC,tot}$	$L_{hf,tot}$
740.309 nH	1.346422	214.320 mΩ	3.868447	-2.7574 mΩ	42.74 mΩ	3101.496 nH

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and the comparison is shown in Figures 3.25 and 3.26. Both approaches provide satisfactory resistance and inductance values over the frequency range for which they are optimized, meaning 100 Hz to 1 MHz. Outside this range, high-frequency values differ, particularly for the resistance in this case.

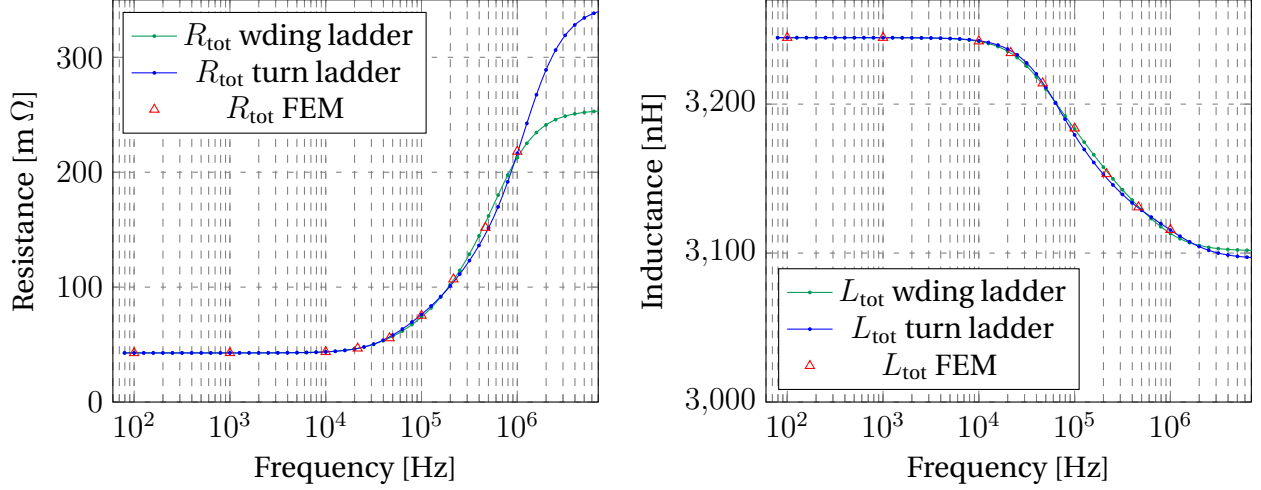


Figure 3.25: Resistance and inductance comparison between the turn-level and the winding-level models.

Assuming FEM identification as the reference, and successively comparing the winding model and the turn model, we obtain the following relative errors for which the maximum resistance deviation is $\sim 3.9\%$ for the turn model at 464 kHz and $\sim 2.8\%$ for the winding model at the same frequency. The inductance deviation is even smaller, never exceeding $\sim 0.16\%$.

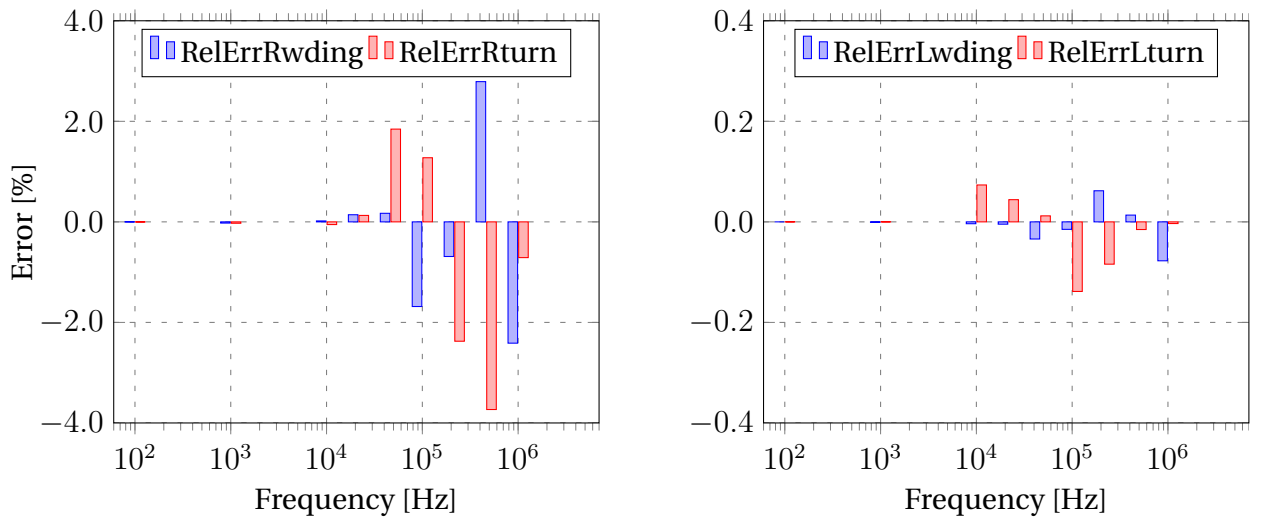


Figure 3.26: Relative errors comparison between the turn-level and the winding-level for the resistances and the inductances. The relative errors are computed based on the FEM identification values.

3.5.3 Inductor Ringing Illustration

The ringing of inductors is a widespread phenomenon in switch-mode power supplies, which occurs when a L - C circuit becomes isolated and resonates. The circuit is typically formed by an inductance (magnetizing or leakage) and a parasitic capacitance, which can come from either a magnetic component or a semiconductor. The resonant frequency associated with inductor ringing is often much higher than the switching frequency, due to low parasitic capacitances. It is useful to be able to correctly estimate this phenomenon, on the one hand for the possible electromagnetic compatibility problems linked to the interference it creates [34, 108, 114] and on the other hand for the additional losses associated with ringing.

The frequencies that are problematic for electromagnetic compatibility can be estimated analytically, provided that the parasitic elements are well estimated, and this is generally taken into account when designing the converter. In contrast, signal amplitudes (currents, voltages) and losses are generally obtained by measurement [118] or by estimation at a few point frequencies [55] or even by time-domain finite element simulations, which are very time-consuming. Estimating the attenuation and losses associated with ringing presents a major difficulty: the equivalent resistance of the inductor can vary by several orders of magnitude between the switching frequency of the converter and the natural frequency of the L - C circuit, resulting in a loss differential between low-frequency and high-frequency phenomena.

To illustrate this, we study a L - C circuit stimulated by a square-wave voltage with two variants: the first exploits a simple model of the inductance with its DC resistance, while the second takes advantage of the ladder model developed in the previous part to better represent the losses of the magnetic component at high frequencies.

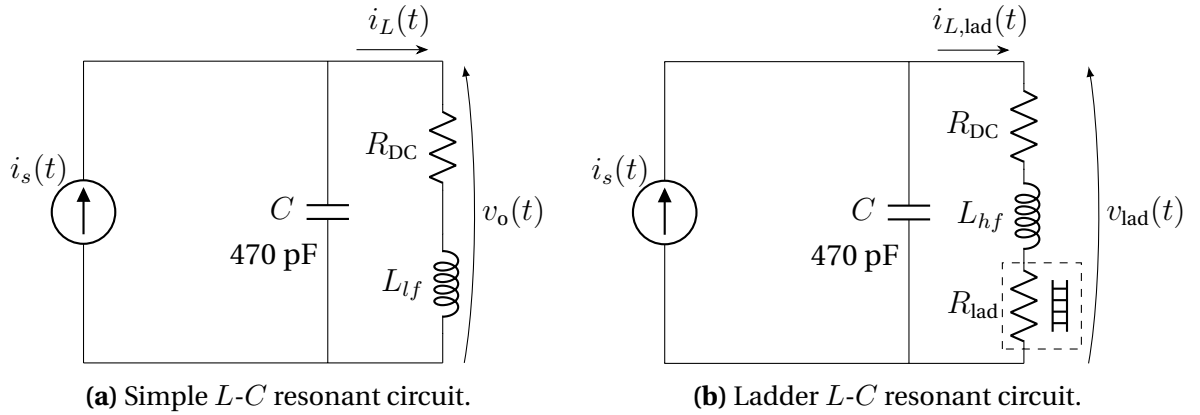


Figure 3.27: Inductor ringing circuits for classical localized constant impedance and an equivalent ladder.

The inductance being modeled is paralleled by a 470 pF capacitor representing any parasitic capacitance. A square-wave PWM signal of 1 A and 1 kHz is applied across this resonant circuit to simulate the behavior of a switching power supply.

Table 3.5: Ladder parameters of the ringing inductance.

$L_{1,\text{ring}}$	$r_{L,\text{ring}}$	$R_{1,\text{ring}}$	$r_{R,\text{ring}}$	$R_{\text{cor},\text{ring}}$	R_{DC}	L_{hf}	L_{lf}
25.551 μH	1.0000108	51.349 Ω	1.00228	-12.7935 Ω	50 m Ω	977.57 μH	1 mH

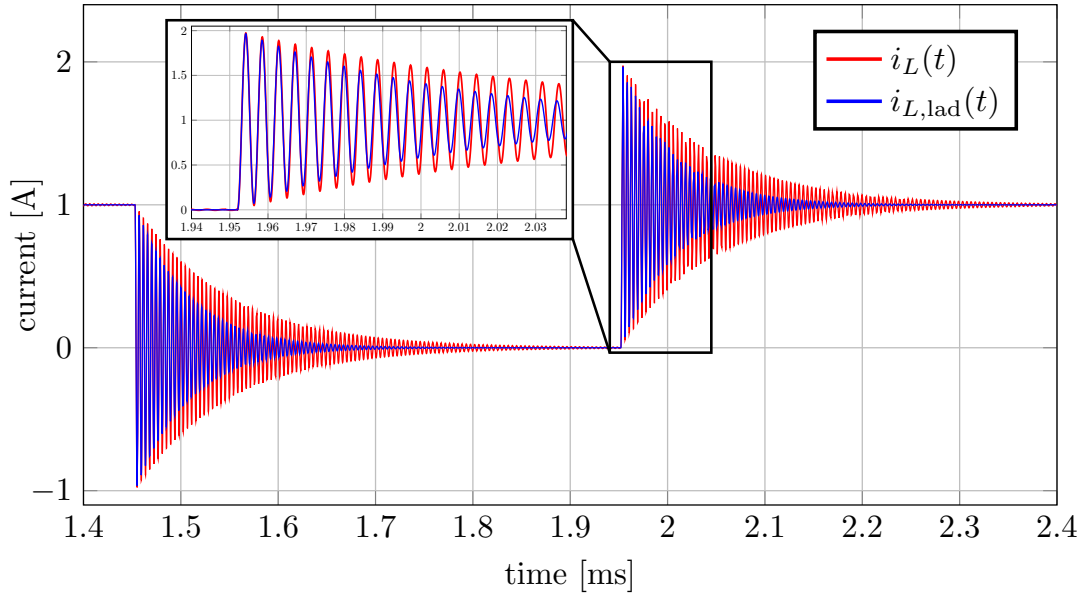


Figure 3.28: Ringing currents evolution over a complete time period (1 ms), for the classical R - L circuit and the ladder network.

In both configurations (classical and ladder), the ringing current possesses two main components: a first component at 1 kHz linked to the switching frequency, and a second, damped component, at 232 kHz resulting from the ringing of the circuit. In figure 3.28, we first notice that the resonance attenuation is clearly more pronounced for the ladder circuit model. This result is quite normal and expected, given that the resistance of the ladder circuit changes and increase with frequency. This resistance is 50 m Ω at DC, 50 m Ω at 1 kHz and 15 Ω at 232 kHz, whereas the simple inductive model has a single resistance value of 50 m Ω for all frequencies.

The choice of the component values is purely arbitrary and for the sake of demonstrating the behavior of the ladder circuit. 50 m Ω corresponds to a small wire cable and 15 Ω corresponds to the value that can be seen at high frequency through the wire. The higher attenuation of the ladder circuit results in a slightly lower RMS current value than for the simple circuit: 726.79 mA and 739.93 mA respectively. However, the losses still remain higher for the ladder network's case as the 232 kHz signal component stimulates a resistance higher than the one from the simple R - L circuit. This leads to 28.46 mW of losses for the constant circuit and 446.98 mW for the ladder network. This difference between the losses is not trivial and can be taken into account easily with a suitable ladder circuit for future applications.

The use of ladder circuits is not limited to a particular type of losses. It can be used for both copper losses and iron losses and consider to create a ladder equivalent per type of loss in order to be able to study them separately. In the exemplary case of core-less two-turn components, we have already presented the interpretation of off-diagonal resistances in terms of copper losses within the circuit. In the following part, the inclusion of iron losses is developed and will give the reader the correct interpretation of the behavior of magnetic losses via series resistance in the model.

3.5.4 Magnetic Losses Circuit Inclusion

In the previous chapter, we presented a method to obtain the iron losses by FEM and to distribute them between the different terms of the resistance matrix. Their inclusion in the

proposed circuit is quite similar to the copper losses, *i.e.* by mean of a series resistive term. However, the behavior is different from the case of copper losses given that the off-diagonal terms of the iron losses are only positive. We will therefore use the extreme cases of the open circuit of the short circuit in order to illustrate the differences between copper losses and iron losses depending on the operating point of a simple transformer with two windings.

For the case of the transformer with two windings, we can express the relationships between the voltages and the currents according to the following equations:

$$\underline{\underline{U}} = \underline{\underline{Z}} \underline{\underline{I}} \quad (3.118)$$

with

$$\underline{\underline{U}} = \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \quad (3.119)$$

$$\underline{\underline{I}} = \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}, \quad (3.120)$$

and

$$\underline{\underline{Z}} = \begin{bmatrix} R_{cu,11} & R_{cu,12} \\ R_{cu,21} & R_{cu,22} \end{bmatrix} + \underbrace{\begin{bmatrix} R_{fe,11} & R_{fe,12} \\ R_{fe,21} & R_{fe,22} \end{bmatrix}}_{=\underline{\underline{R}}_{fe}} + j\omega \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}. \quad (3.121)$$

In this case, we can express the iron losses of the device as follows:

$$P_{fe} = R_{fe,11} I_1^2 + (R_{fe,12} + R_{fe,21}) \operatorname{Re}(\bar{I}_1 \bar{I}_2^*) + R_{fe,22} I_2^2 \quad (3.122)$$

$$P_{fe} = R_{fe,11} I_1^2 + (R_{fe,12} + R_{fe,21}) I_1 I_2 \cos(\phi_2 - \phi_1) + R_{fe,22} I_2^2, \quad (3.123)$$

where ϕ_1 and ϕ_2 correspond to the phases of the currents I_1 and I_2 . This expression always remains valid and when the phase shift between the currents exceeds $\frac{\pi}{2}$, the second term becomes negative and therefore moderates the total losses. In other words, when the phase shift between the currents I_1 and I_2 exceeds $\frac{\pi}{2}$, the amplitude of the total magneto-motive force decreases and consequently the amplitude of the magnetic induction and the associated losses.

For the sake of simplicity, we consider the resistances of the iron losses as proportional to the magneto-motive forces ($N_1 I_1$ and $N_2 I_2$) from which they emerge. It corresponds to the 1D assumption of the ferromagnetic core behavior. This provides us

$$\underline{\underline{R}}_{fe} = \begin{bmatrix} N_1^2 R_{mag} & N_1 N_2 R_{mag} \\ N_1 N_2 R_{mag} & N_2^2 R_{mag} \end{bmatrix} \quad (3.124)$$

where N_1 and N_2 correspond to the number of turns of each winding of the transformer and R_{mag} a constant associated with the iron losses of the magnetic core common to both windings. In this case, we can adapt the equation 3.123 of iron losses as follows

$$P_{fe} = N_1^2 R_{mag} I_1^2 + 2N_1 N_2 R_{mag} I_1 I_2 \cos(\phi_2 - \phi_1) + N_2^2 R_{mag} I_2^2. \quad (3.125)$$

Regardless of the coupling factor between primary and secondary, in the case of the open circuit seen from the primary, the ferromagnetic losses will always have the value

$$P_{fe} = N_1^2 R_{mag} I_1^2, \quad (3.126)$$

given that the secondary current is zero (*i.e.* $I_2 = 0$).

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Let us now consider that the circuit we are studying is not perfectly coupled, but that the mutual inductance between the primary and the secondary can be expressed as an average of the inductances of the primary and the secondary limited by a coupling coefficient k such that $k \in [0; 1]$, this is then written

$$\underline{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} \frac{L_{11}}{k \sqrt{L_{11} L_{22}}} & \frac{k \sqrt{L_{11} L_{22}}}{L_{22}} \end{bmatrix}. \quad (3.127)$$

Adding that for these coupled circuits, we have

$$\frac{L_{11}}{L_{22}} \simeq \frac{N_1^2}{N_2^2}. \quad (3.128)$$

By imposing a short circuit on the secondary, we can deduce the ratio between the secondary and primary current by taking the simplifying assumption that the reactances are much greater than the resistances. This is a completely realistic hypothesis for transformers whose losses are low compared to the magnetizing inductance and it is written as follows:

$$I_2 = -k \sqrt{\frac{L_{11}}{L_{22}}} I_1 \simeq -k \frac{N_1}{N_2} I_1, \quad (3.129)$$

to the condition that the frequency is high enough to neglect the resistive terms and that the primary and secondary are still coupled. Finally, we can rewrite the ferromagnetic losses for a short circuit configuration such that

$$P_{fe} = R_{mag} \left(N_1^2 - 2k \frac{N_1}{N_2} N_1 N_2 + N_2^2 k^2 \frac{N_1^2}{N_2^2} \right) I_1^2 \quad (3.130)$$

$$P_{fe} = N_1^2 R_{mag} (1 - k)^2 I_1^2. \quad (3.131)$$

In this expression 3.131, we notice that the ferromagnetic losses for the short circuit configuration are restricted by the term $(1 - k)^2$, which follows well the physical understanding that we have of magnetic couplings. Indeed, if the coupling is perfect (*i.e.* $k = 1$), the losses drop to zero, which is logical since then the secondary current perfectly opposes the primary current and the induction becomes then null in the ferromagnetic core. Furthermore, if the coupling coefficient is inexistent (*i.e.* $k = 0$), imposing a current on the primary has no impact on the secondary and no current appears there. Induction then finds its source only via the primary current and the magnetic losses are identical to the case of the open circuit seen from the primary. For all other values of the coupling coefficient, we note that the ferromagnetic losses in short circuit are always lower than in open circuit, given that part of the secondary current counterbalances the magneto-motive force of the primary.

3.5.5 Circuit Representation with Controlled Voltage Sources

Now that we have explained how we can divide our model into different arbitrary sections and replace the circuits in simple localized constants with their ladder equivalent, it remains for us to provide a representation of what such a circuit can be for the magnetic components that we study.

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To do this, we start from the synthetic equations obtained at the end of Chapter 2, in the magnetodynamics section which indicates that

$$\bar{U}_k = R_k \bar{I}_k + j \omega \bar{\Phi}_k \quad (3.132)$$

$$\bar{U}_k = R_k \bar{I}_k + j \omega \sum_l \Lambda_{kl} \bar{I}_l \quad (3.133)$$

$$\bar{U}_k = R_k \bar{I}_k + j \omega \sum_l \left(L_{kl} - j \frac{R_{\omega kl}}{\omega} \right) \bar{I}_l \quad (3.134)$$

$$\bar{U}_k = \sum_l (R_{kl} + j \omega L_{kl}) \bar{I}_l \quad (3.135)$$

where R_{kl} represents both direct current copper losses in the conductor itself as well as copper losses due to induced currents or even iron losses. We can break down this expression into three different terms in order to facilitate the interpretation of the various mechanisms and we then write that

$$\bar{U}_k = R_{DC,k} \bar{I}_k + \sum_l R_{ed,kl} \bar{I}_l + \sum_l R_{fe,kl} \bar{I}_l + \sum_l j \omega L_{kl} \bar{I}_l . \quad (3.136)$$

This expression can be translated into a localized constant circuit with controlled sources of currents and voltages. The diagonal terms are directly represented in the corresponding branch of the circuit and the off-diagonal terms are introduced via the controlled sources. An illustration of this circuit is provided in the following figure for the case of a magnetically coupled circuit formed by two windings.

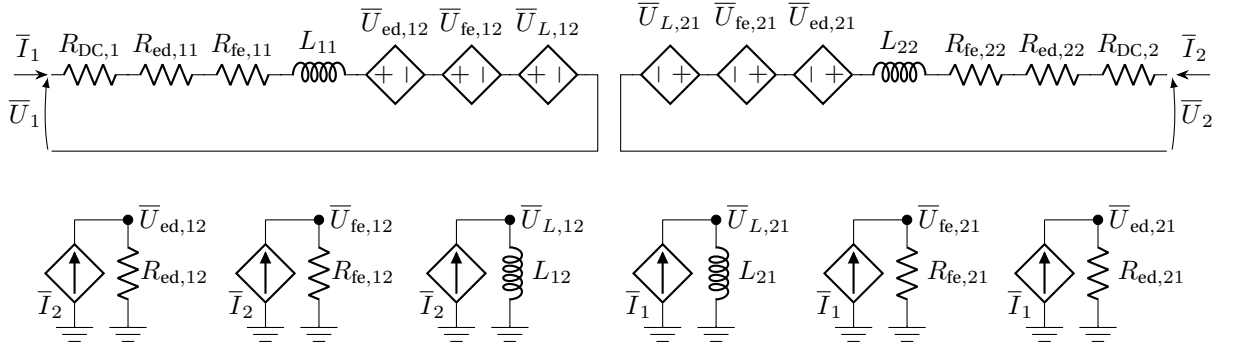


Figure 3.29: Controlled current and voltage sources model of two magnetically coupled inductors.

The dependence of the parameters as a function of frequency can easily be applied to this circuit. It is then sufficient to replace each resistance R_{ed} and R_{fe} by its equivalent ladder network while maintaining an identical circuit topology. This therefore makes it possible to establish the losses separately on each component of the circuit while integrating their dependence on the frequency.

Validation Cases

In this chapter, the different steps performed to experimentally validate the equivalent models are described. To do so, the first part focuses on the **characterization** of the **materials** involved in inductive and capacitive coupling, as well as additional losses (ferromagnetic and dielectric) related to them. The explanations specify which samples are measured, how material parameters (for soft ferrites and insulators) are estimated, and how they are subsequently used in finite element analysis.

In the rest of this chapter, **canonic cases** are used to illustrate the various phenomena that occur within magnetic parts: copper losses, iron losses, dielectric losses; also, to compare the different levels of discretization of parts into sections and explain how to take advantage of them, and finally, to illustrate the favorable properties of ladder circuits useful for representing electrical phenomena whose parameters depend on frequency. At the end of this chapter, the reader will see that these experimental measurements on well-controlled test cases validate the numerical models used to describe magnetic components, which we presented in Chapters 2 and 3.

4.1 Materials Characterization

The various physical devices (inductors and transformers) used to validate the numerical models involve different materials that need to be characterized in advance. On the one hand, ferromagnetic materials (ferrites) are used to concentrate magnetic induction within the device, and for which we want to determine the relative permeability $\bar{\mu}_r$. On the other hand, dielectric materials are used to separate conductive parts from each other, and for which we want to determine the relative permittivity $\bar{\epsilon}_r$. First, will be presented a study of different ferrites from the magnetic point of view, followed by a study of a dielectric used in our validation cases: the PVC.

The parameters to be identified are **complex quantities** associated with a presupposed steady-state sinusoidal regime. For both magnetic and electric behavior, the real part of the complex parameter ($\bar{\mu}_r$ or $\bar{\epsilon}_r$) is associated with the energy per cycle passing through the system, and the imaginary part corresponds to the losses within the material, which appear concomitantly with a phase shift between the magnetic and electric fields and their respective inductions.

This first section will successively present the ferromagnetic materials characteristics, the different samples and the method used to measure them, as well as the dielectric materials characteristics, the corresponding samples and the measurements associated to them.

4.1.1 Ferromagnetic Materials

In order to be able to compare the results of numerical simulations with measurements on the physical component, it was necessary to characterize the ferromagnetic cores beforehand in order to inject the measured parameters into the model. Indeed, a wide variety of different ferrite types and geometries are available. For this work, we gathered together the usual cores for a number of applications: toroids, U's, ETD's, E's, etc., as well as a particular core geometry, the P70, which has the advantage of presenting a symmetry of revolution, and which were opted for in order to validate the axisymmetric model. The chosen core materials are also often encountered in power electronics practical applications: N87, N97, 3C90, 3C95, 3C97, N22, etc. and different temperature ranges were covered as well. A summary of all the measurements is given in Appendix A.

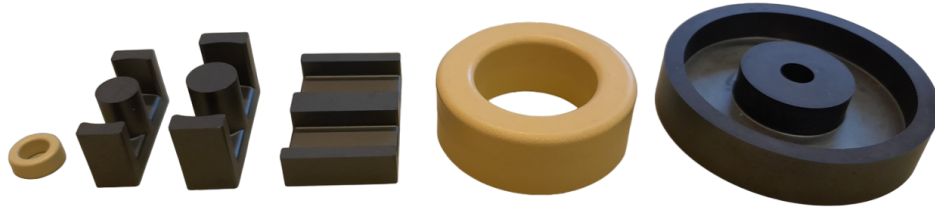


Figure 4.1: Subset of ferrite cores characterized with AMH-200k permeameter: TX13, ETD34, ETD39, E43, TX51, P70 ; among the samples whose data are included in the Appendix A.

Permeability measurements were carried out using an AMH-200k permeameter (Laboratorio Eletroffisico brand) in the Ampère laboratory (*Université Claude Bernard Lyon - UCBL*). The measurement is performed using two windings around the sample to be characterized: one winding is used to measure the current, the other to measure the voltage. Current and voltage values are converted into magnetic field and magnetic induction by applying a 1D assumption and specifying sample measurements (magnetic path length, cross-section and number of primary and secondary turns).

The sinusoidal regime is considered for these measurements, *i.e.* a **sinusoidal voltage** is applied by the permeameter. When distortions occur with saturation of the magnetic core, the device iteratively corrects the voltage waveforms so that they remain sinusoidal [23]. As a result, the shape of the current no longer appears sinusoidal, but the voltage is kept as close as possible.

The device adopts the principle of the Epstein frame commonly used to characterize laminated magnetic sheets. Around the magnetic circuit (*i.e.* the sample to be tested), there is a primary winding through which the device is powered and in which the injected current is measured by means of a shunt resistor, as well as a secondary winding used to measure the induced voltage. The experimental circuit setup is provided in Figure 4.2 in which one retrieves the arbitrary waveform generator, a power amplifier, voltage/current probes, the acquisition system and finally the ferrite sample.

In addition, the ferromagnetic core is placed in an autoclave to set its operating temperature. The temperature of the sample is controlled by the person carrying out the experimental manipulation, and the waveform of the voltage is controlled by the permeameter itself in a closed loop. Moreover, Litz wire is used to create the windings in order to limit copper losses and, consequently, the heating of the ferrite sample by contact with the windings, thereby preventing unwanted hot spots. The number of turns required in the primary and secondary depends on both the core and the test parameters (frequency, induction). The

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number of turns calculation is specified below, after developing the magnetic field and induction equations and providing the physical constraints of the output amplifier (maximum current and voltage).

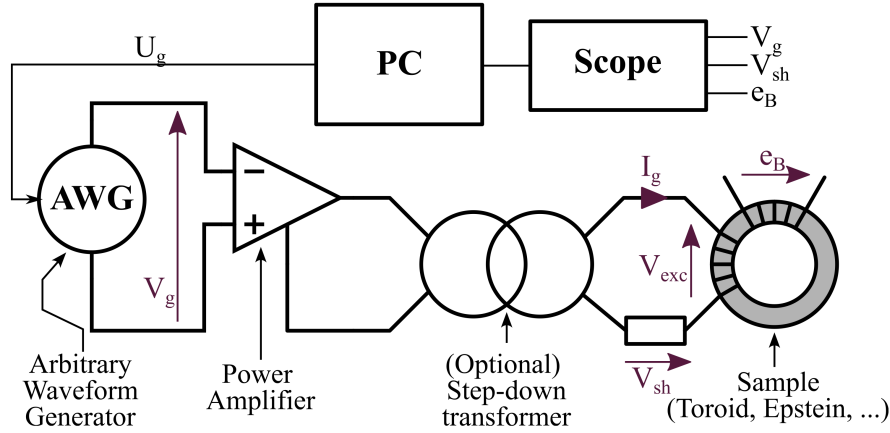


Figure 4.2: Experimental circuit schematic used for the ferromagnetic material characterization, excerpted from [23].

From the measured voltage and current, the induction and magnetic field could be deduced, assuming a 1D field within the magnetic circuit. The magnetic field is expressed as

$$h(t) = \frac{N_1 i(t)}{l_m}, \quad (4.1)$$

where $h(t)$, the amplitude of the magnetic field, depends on the number of turns N_1 of the primary, the mean magnetic path length l_m and the current $i(t)$ flowing through the winding. Similarly, one can obtain

$$b(t) = -\frac{1}{N_2 A_c} \int_0^t v(u) du + \underbrace{b_0}_{=0}, \quad (4.2)$$

where the amplitude of the magnetic induction $b(t)$ depends on the number of turns N_2 on the secondary side, the effective cross-section of the magnetic circuit A_c and the integral of the voltage $v(t)$ across the secondary winding.

Measurement of the secondary voltage raises relatively few issues, since it is possible to connect a differential probe that accepts high voltage levels (500 V, 1 kV) to the secondary terminals. The user simply needs to add the conversion ratio of the probe to the equation 4.2, either by explicitly specifying this ratio or by integrating it into the equivalent cross-section value. However, both primary current and voltage must be limited. In fact, the amplifier can accept a maximum current of

$$I_{\max} = 16 \text{ A}, \quad (4.3)$$

and a maximum voltage of

$$V_{\max} = 140 \text{ V}. \quad (4.4)$$

It is therefore necessary to adapt the number of turns of the primary circuit to the measurement parameters: frequency, maximum induction, length and cross-section of the magnetic circuit. The maximum primary voltage $\hat{V}_{\text{meas}}$ can be deduced from the test parameters as follows:

$$\hat{V}_{\text{meas}} = 2\pi f A_c N_1 \hat{B}_{\text{meas}}. \quad (4.5)$$

Concerning the current, the maximum primary current can be expressed as

$$\hat{I}_{\text{meas}} = \frac{l_m}{N_1} \hat{H}_{\text{meas}} . \quad (4.6)$$

The aim is to ensure that the measurement voltage and current do not exceed the maximum values of the power amplifier. Finally, the constraint on the number of turns to be wound at the primary is provided by the two inequalities

$$\frac{l_m}{I_{\text{max}}} \hat{H}_{\text{meas}} \leq N_1 \leq \frac{V_{\text{max}}}{2\pi f A_c \hat{B}_{\text{meas}}} . \quad (4.7)$$

Using the expression 4.7, one can see that for a given number of turns, the product of frequency and maximum induction cannot exceed a certain value, at the risk of exceeding the voltage allowed by the measuring device. This explains why some measurements carried out at higher frequencies have involved reducing the amplitude of the maximum induction.

Generally, for this type of characterization, the aim is to describe the behavior of the material for a given magnetic induction. This makes it possible to be accurate about the value of the voltage that the measuring device will have to handle, but not about the value of the current. Without knowing the permeability and saturation level of the material, it is indeed difficult to quantify the magnetic field associated with a given induction, even if estimates can be obtained from manufacturers' data sheets which remain uncertain as typical values are provided but no tolerances. To be on the safe side, is first recommended to characterize the material for low induction and a high number of primary turns. It allows to obtain typical permeability values without exceeding the maximum amperage of the power amplifier, and then obtain saturation values by increasing the induction. Knowing these physical quantities, it is then possible to adapt the number of turns to be wound at the primary.

Once the primary and secondary turns have been set correctly, the permeameter deduces the magnetic field and induction from the primary current and secondary voltage respectively. These b and h data are stored, and it is then possible to draw the material's hysteresis cycle for known measurement conditions (temperature, frequency, maximum induction). On the basis of each cycle obtained, it is possible to extract certain parameters useful for describing the material: maximum induction, maximum field and energy dissipated per cycle and per unit volume E_v . The remanant induction b_r and the coercive field h_c are quantities generally exploited for the nonlinear representation of ferromagnetic materials [59]. It was opted for a linear model and therefore chosen not to exploit them.

To represent the behavior of ferrites, a loss model was chosen that only requires knowledge of the relative permeability and is subsequently expressed as a complex-valued quantity:

$$\bar{\mu}_r = \mu'_r - j \mu''_r . \quad (4.8)$$

This model is justified by the use of harmonic simulations to determine the system parameters; system linearity is therefore assumed as a preliminary assumption. The real part of the permeability is considered to dominate the behavior, and can be obtained via the maximum magnetic field and induction values

$$\mu'_r = \frac{b_{\text{max}}}{h_{\text{max}}} \frac{1}{\mu_0} . \quad (4.9)$$

The imaginary part concentrates the losses of the hysteresis cycle. It is therefore linked to the energy (E_v) lost during one cycle, and is estimated as follows

$$\mu''_r = \frac{E_v}{\pi h_{\text{max}}^2} \frac{1}{\mu_0} . \quad (4.10)$$

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Other methods exist for identifying the complex permeability associated with a hysteresis cycle [48]. Nevertheless, our direct approach, although less generic, is sufficient for the case of weakly saturated cores as used in power electronics applications.

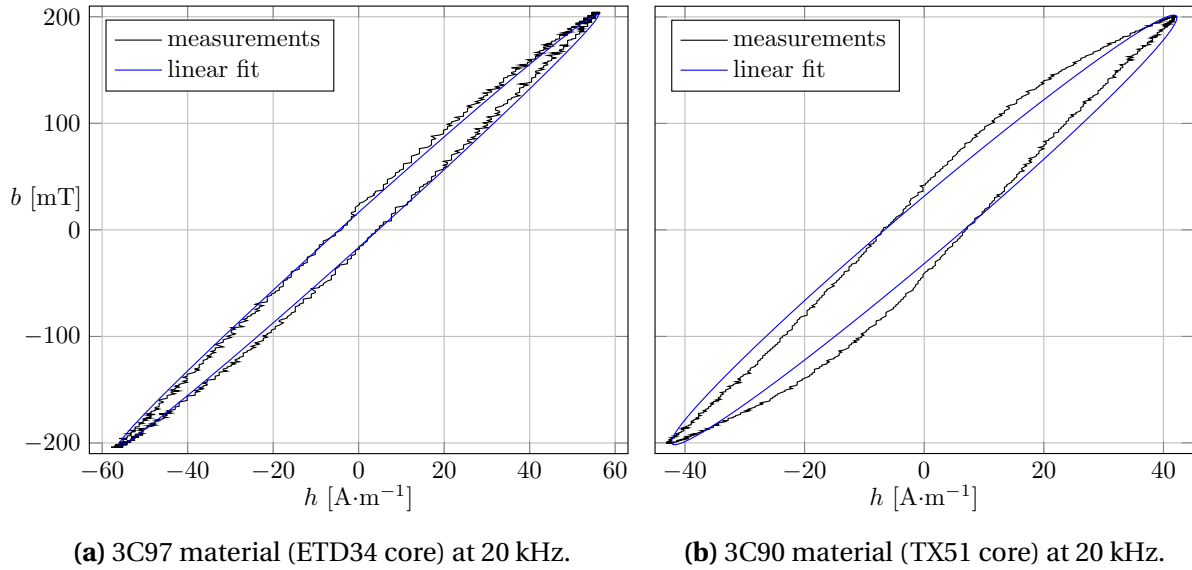


Figure 4.3: Hysteresis cycles of different ferrite materials at room temperature and 200 mT. Subfigure (a) presents the linear behavior of the 3C97 and subfigure (b) shows the 3C90 material exhibiting non-linear behavior [100].

By superimposing the measured cycle and the cycle obtained with the linear parameters a strong agreement between the two is immediately noticed. However, there are some cases where the linear assumption is challenged. For example, in the case of 3C90 material, the saturation bend begins at around 200 mT. As a result, the ellipse deduced from the complex permeability is not superimposed on the measured cycle (Figure 4.3b). As the surface area is the same, for a given operating point, losses will therefore be identical. In this case, we felt that the approximation gave satisfactory results and preferred to keep the model simple and linear. Indeed, linear parameters are necessary for the harmonic regime approach chosen for finite element models. A non-linear approach would not be compatible and would require modeling the device in the time domain, which is much more costly in terms of resolution.

Concerning the two materials described in Figure 4.3, the measured parameters $b_{\max}$, $h_{\max}$ and E_v and the deduced complex permeability are given in Table 4.1.

Table 4.1: Characterization parameters of the 3C97 and 3C90 materials for 25°C, 200 mT and 20 kHz conditions.

Material	$b_{\max}$ [mT]	$h_{\max}$ [A·m ⁻¹]	E_v [J·m ⁻³]	μ'_r	μ''_r	$\tan \delta$
3C97	201.61	56.43	2.5373	2836.1	201.6	0.07108
3C90	200.51	42.40	4.4364	3722.4	624.6	0.16779

If one wishes to understand how ferrite equivalent parameters evolve as a function of test conditions (frequency, magnetic induction, etc.), one can, for example, display the behavior of the real and imaginary parts of the permeability as a function of these same conditions. Figure 4.4 shows the real part of the relative permeability μ'_r for 3C90 and N22. It can be seen that the value depends mainly on the induction amplitude, and therefore on the influence of the saturation of the material; and relatively little on the frequency.

With little loss of information, we can simplify the behavior and consider a constant value regardless of frequency, as long as the induction does not approach saturation too much. This assumption has little influence on core designs with an air gap, since the main obstacle to the passage of magnetic flux is the air gap, not the core. Which is the case for the devices studied in this work. This assumption should be treated with caution in the case of cores with no air gap, e.g. toroidal cores.

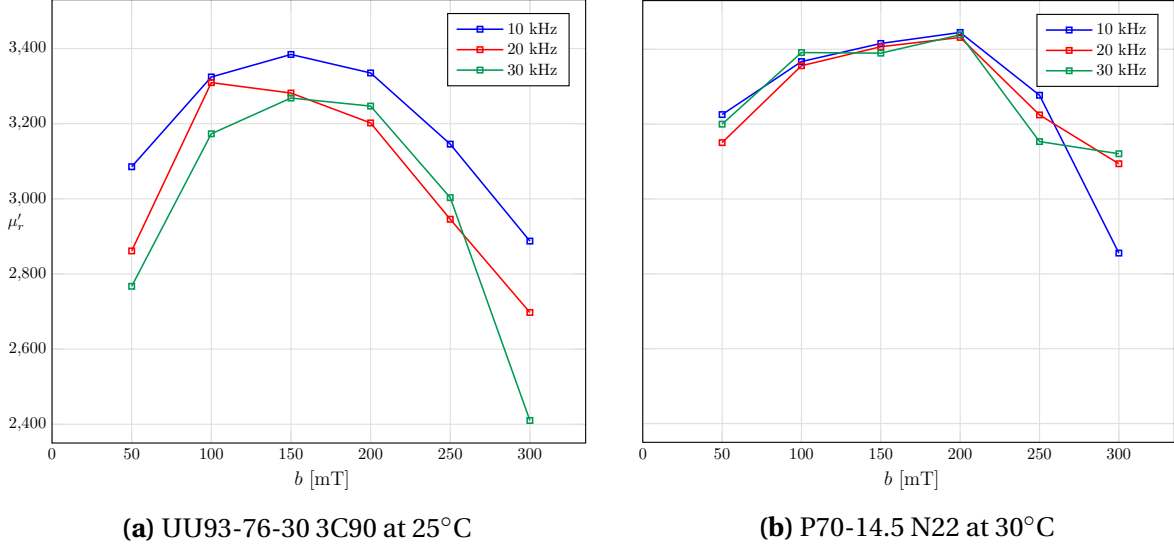


Figure 4.4: Real part of the relative permeability measured for the 3C90 and N22 materials between 10 and 30 kHz and between 50 and 300 mT of magnetic induction.

For ferromagnetic losses, the evolution of the imaginary part μ''_r as a function of induction and frequency is first presented. Here, it is preferred to study the evolution of the loss tangent ($\tan \delta$), as this allows us to normalize losses with respect to the real part. Let us denote the magnetic loss angle δ such that

$$\tan \delta = \frac{\mu''_r}{\mu'_r} . \quad (4.11)$$

The loss ratios for 3C90 and N22 are shown in the following figure.

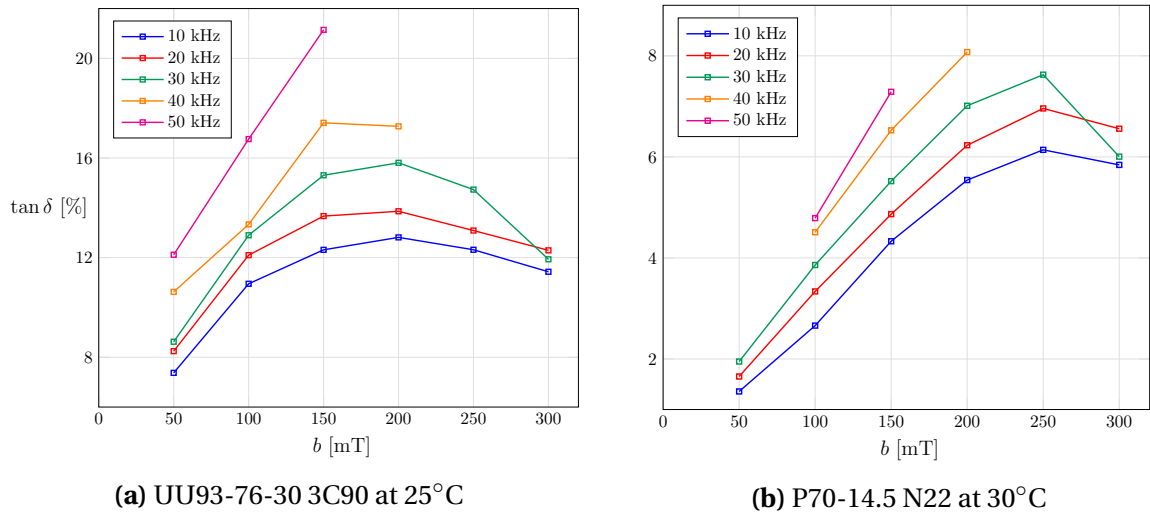


Figure 4.5: Loss tangent for two ferrites at different frequencies and magnetic inductions.

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Another alternative is to express the power of losses dissipated in the sinusoidal regime by Steinmetz's equation [104], which states that the power losses per unit volume P_v corresponds to

$$P_v = k \cdot f^\alpha \cdot b_{\max}^\beta, \quad (4.12)$$

where k , α and β are constants identified for each material with $\alpha > 1$ and $\beta > 2$. Steinmetz coefficients (k , α and β) can be calculated on the basis of measured hysteresis cycles, using a regression for instance. In figure 4.6, the measured power corresponds fairly well to the power losses estimated with the Steinmetz model.

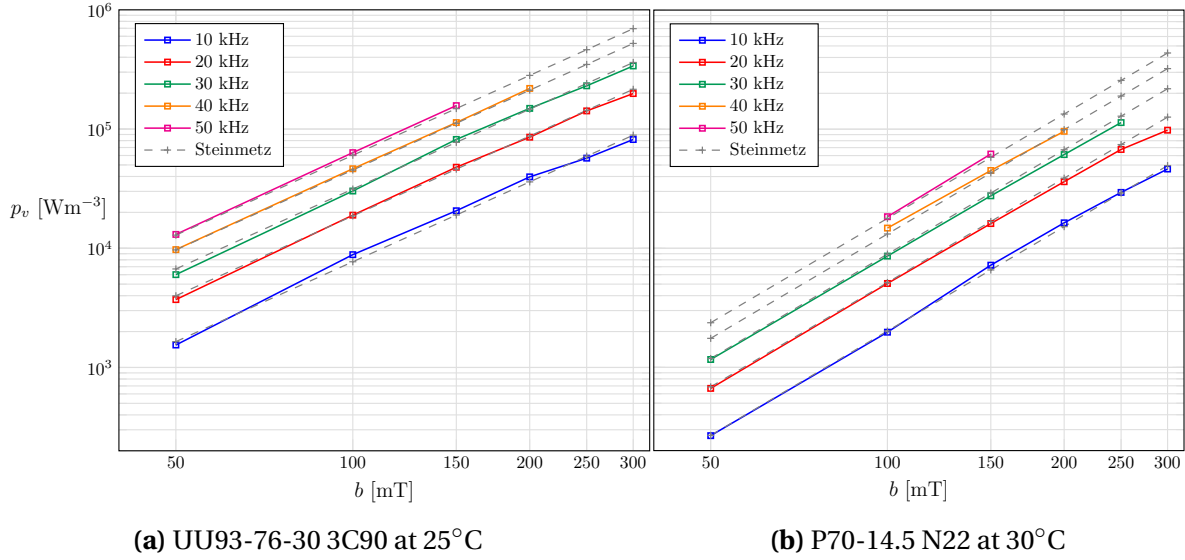


Figure 4.6: Loss characteristics of 3C90 and N22 materials at ambient temperature, compared to Steinmetz approximation.

The parameters of the Steinmetz equation for 3C90 and N22 are shown in the following table, after a regression applied to the measurement points.

Table 4.2: Steinmetz parameters of the 3C90 and N22 materials.

	k	α	β
3C90	10.02121	1.2779	2.22573
N22	6.37395	1.3529	2.91077

In the case where the Steinmetz equation is used to evaluate ferromagnetic losses, it is still possible to obtain an equivalent complex permeability, under the same assumptions as for fitting the ellipse to the cycle. Knowing the real part, it is then possible to express the imaginary part as

$$\mu_r'' = \frac{P_v}{\pi f b_{\max}^2} \mu_0 \mu_r'^2, \quad (4.13)$$

$$\mu_r'' = \frac{k}{\pi} f^{\alpha-1} b_{\max}^{\beta-2} \mu_0 \mu_r'^2. \quad (4.14)$$

Knowing the losses and behavior of the various materials previously characterized, the choice was made to convert the hysteresis cycle data into a complex permeability that can be used immediately in a harmonic model. The complex relative permeability of each measured cycle is given in the appendix. The real and imaginary parts are calculated on the basis of the equations 4.9 and 4.10. Steinmetz coefficients for their part are calculated on a case-by-case basis, since they depend directly on the operating points used to calculate the regression.

4.1.2 Dielectric Materials

For dielectric materials, we focused on characterizing the complex permittivity of PVC, which is used as a coil former for several of our validation cases. This choice is justified by the availability of PVC samples of sufficiently large dimensions to perform dielectric tests on them. For this purpose, two samples taken from two different parts were used: the first is a plate. The first consists of a plate used to construct a planar capacitor, while the second consists of a tube used to construct a cylindrical capacitor.

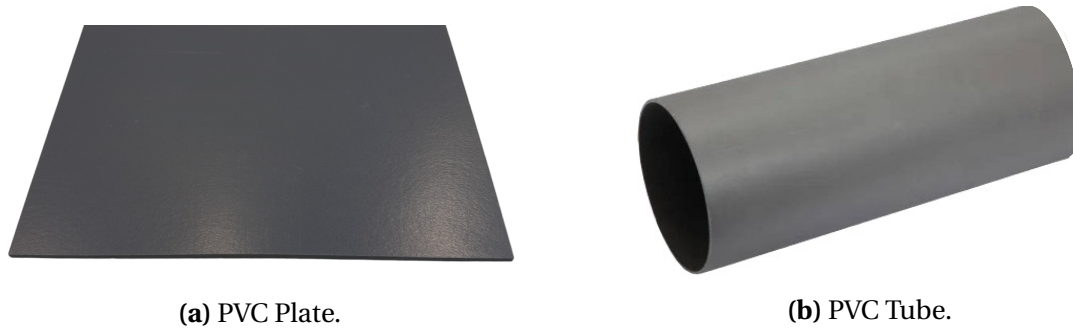


Figure 4.7: PVC sample pieces for capacitor realization.

To create a planar capacitor whose dielectric is the PVC sample, we place self-adhesive conductive tape on both surfaces of the PVC plate. These strips correspond to the capacitor electrodes accessed during measurement to determine device currents and voltages. Care should be taken to add a guard electrode around the perimeter of one of the two faces. The guarded electrode and the guard electrode are brought to the same potential during measurement, and only the current from the guarded electrode is used to calculate the admittance. The purpose of the guard electrode is to limit fringing flux at the edge of the sample; it therefore serves to maintain a rectilinear field at the edges of the capacitor under test.

A cross-sectional view of the planar capacitor is provided in Figure 4.8. The measuring electrodes are two wide aluminum strips centered on the PVC sample, and the guard electrode is made of copper adhesive tape running all around the plate.

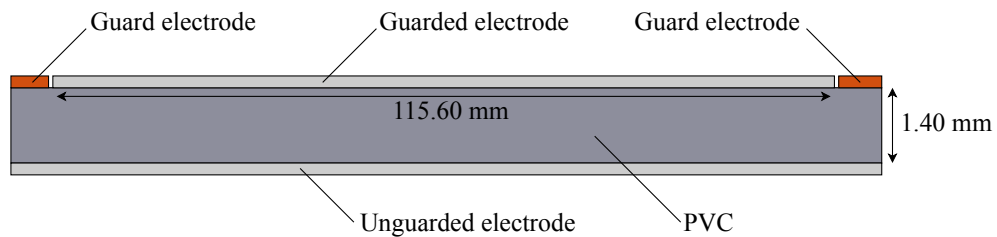


Figure 4.8: Cross-section view of the plate capacitor with the electrodes and PVC sample.

Views of the guarded electrode and the unguarded electrode are illustrated in the following figure, which shows the measuring and guard electrodes, as well as the copper clip-on-edge mounting part, allowing access to the guard electrode. A strip of insulating material is used to prevent contact between this part and the unguarded electrode. A nylon bolt-nut assembly is used to tighten two brass washers which provide the contact points between the external circuit and the measuring electrodes.

This sample is particularly useful because its dimensions and assembly are well mastered. Indeed, the geometric dimensions are large enough to be measured accurately, and

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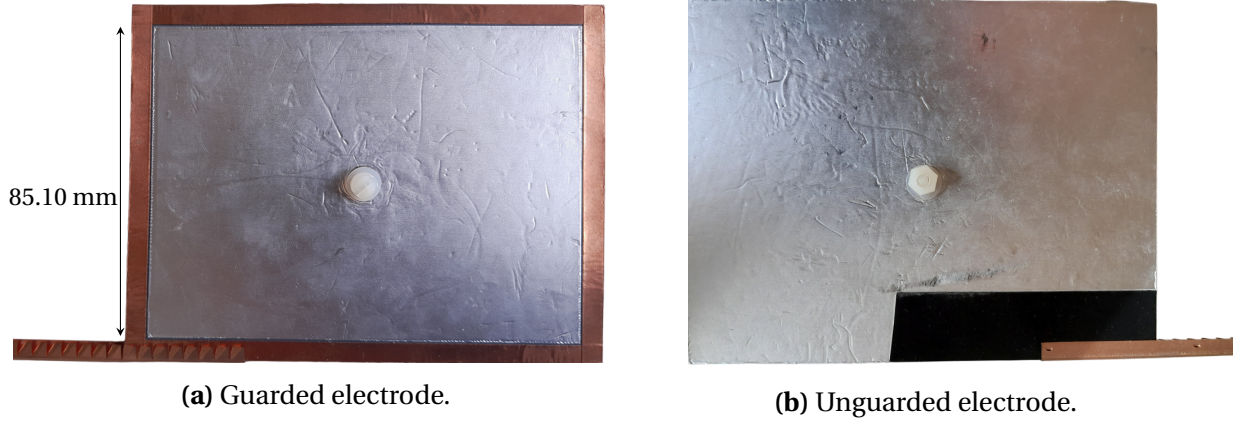


Figure 4.9: Parallel-plate capacitor electrodes pictures.

the electrodes are fixed as close as possible to the sample, with no gaps between them. It should be noted, however, that our model neglects the presence of glue between each electrode and the dielectric under test, as well as the central hole (4.16 mm in diameter) required to fix the current-carrying wires. The dimensions of the plane capacitor are provided in the table below for its height h , width w , surface $S (= h \cdot w)$ and thickness t .

Table 4.3: Physical dimensions of the parallel-plate capacitor.

h [mm]	w [mm]	S [mm ²]	t [mm]
115.60	85.10	9837.56	1.40

The analytical formula linking the capacitance of a planar capacitor to its physical parameters (properties, dimensions) indicates that

$$C_{\text{plate}} = \frac{\varepsilon_0 \varepsilon_r S}{t} , \quad (4.15)$$

where $\varepsilon_0 = 8.8541 \cdot 10^{-12}$ corresponds to the permittivity of vacuum, S is the surface area of each capacitor plate, and t is the distance between them. Using this formula, can be deduced the relative permittivity of the material ε_r by measuring the capacitance of the constructed capacitor. Considering that the measurement also provides the dielectric losses, then it should be borne in mind that permittivity is a complex quantity as

$$\bar{\varepsilon}_r = \varepsilon'_r - j \varepsilon''_r . \quad (4.16)$$

The formula is then updated by noting that relative permittivity and capacitance are complex quantities.

$$\bar{\varepsilon}_r = \frac{t \bar{C}_{\text{plate}}}{\varepsilon_0 S} . \quad (4.17)$$

The measured admittance can therefore be written as a complex capacitance such as

$$Y_{\text{plate}} = G_{\text{plate}} + j \omega C_{\text{plate}} = j \omega \bar{C}_{\text{plate}} , \quad (4.18)$$

from which are deduced the real and imaginary parts of the relative permittivity

$$\varepsilon'_r = \frac{t}{\varepsilon_0 S} C_{\text{plate}} \quad (4.19)$$

$$\varepsilon''_r = \frac{t}{\varepsilon_0 S} \frac{G_{\text{plate}}}{\omega} . \quad (4.20)$$

If the real and imaginary parts of the relative permittivity are constant, a constant capacitance is observed, along with a conductance whose value increases linearly with frequency.

In a similar fashion, a cylindrical capacitor was assembled around a tube-shaped PVC sample. To build it, an aluminum electrode was placed in the center of the tube, as well as a measuring electrode on the periphery, surrounded by the two guard electrodes. The external electrodes are made of adhesive conductive strips. This sample is interesting in that it comes from the PVC rod used to make the coil former for one of our validation inductors (the multi-layer axisymmetric). The capacitance provided by a cylindrical capacitor can be written using the following analytical formula

$$C_{\text{tub}} = \frac{h 2 \pi \varepsilon_0 \varepsilon_r}{\ln \left(\frac{D_{\text{ext}}}{D_{\text{int}}} \right)} , \quad (4.21)$$

where here h represents the height of an electrode, and D_{ext} , D_{int} the outside and inside diameters of the tube.

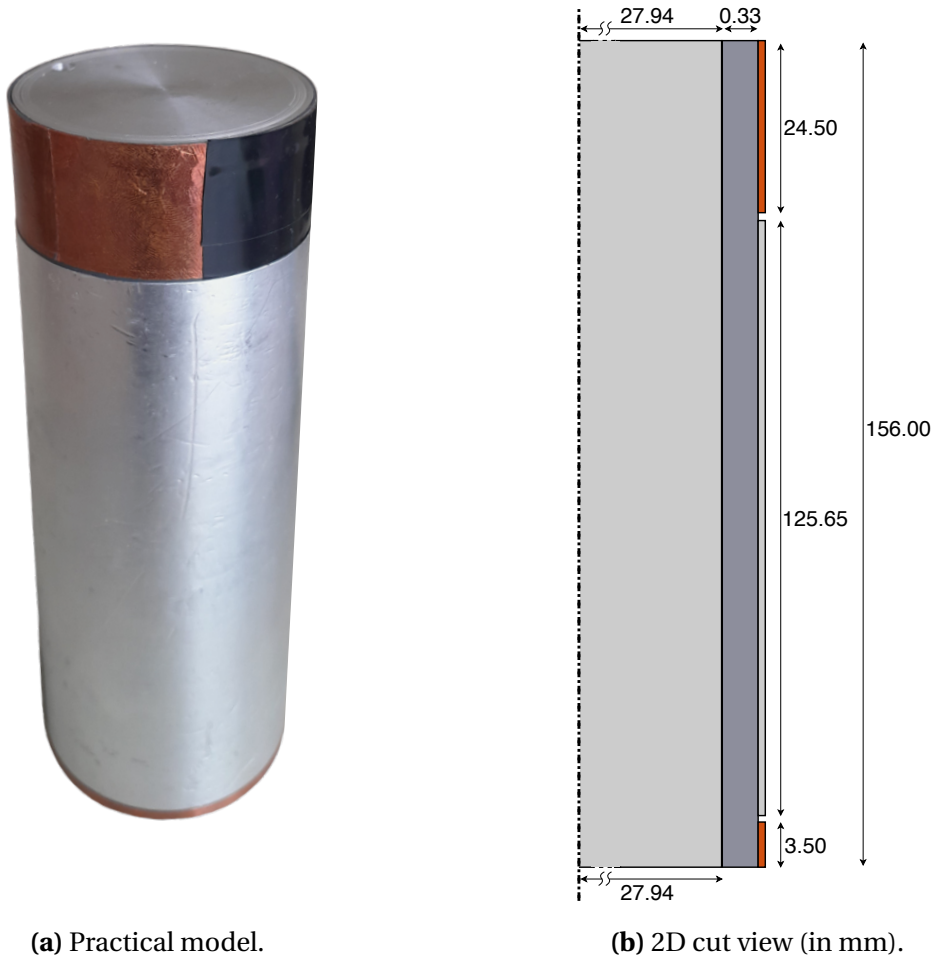


Figure 4.10: Cylindrical capacitor.

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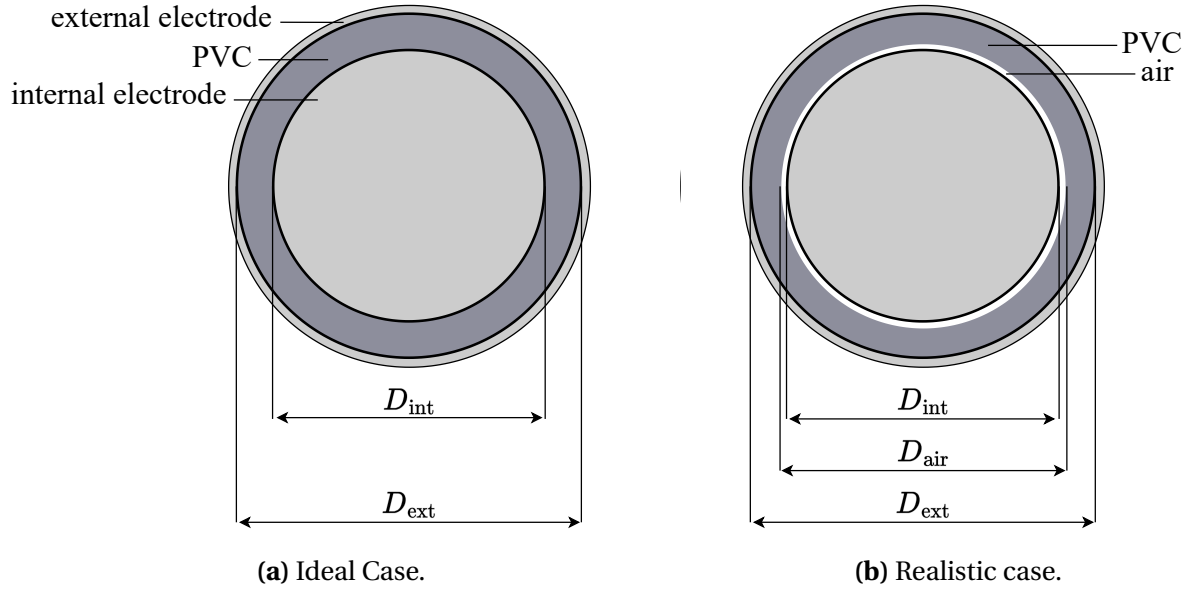


Figure 4.11: Cross-section view of the cylindrical capacitor.

In this configuration, due to mechanical tolerances, there is a small space between the PVC and the internal electrode, allowing the latter to be easily inserted. There is therefore a thin layer of air between the two parts, which distorts the measurement. Unfortunately, this is due to the fact that the cylinder's surface is not perfectly smooth due to its machining. The capacitor can therefore be considered as comprising two materials in two successive layers: air and PVC. The thickness of the air is between D_{int} and D_{air} and the PVC between D_{air} and D_{ext} . If air is present between the electrode and the material to be tested, the total capacitance of this capacitor is expressed as the series connection of two capacitors: the first whose dielectric is air and the second PVC. This gives

$$C_{\text{cyl}} = \left(\frac{1}{C_{\text{air}}} + \frac{1}{C_{\text{PVC}}} \right)^{-1}, \quad (4.22)$$

where

$$C_{\text{air}} = \frac{h 2 \pi \varepsilon_0}{\ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right)}, \quad (4.23)$$

and

$$C_{\text{PVC}} = \frac{h 2 \pi \varepsilon_0 \varepsilon_r}{\ln \left(\frac{D_{\text{ext}}}{D_{\text{air}}} \right)}. \quad (4.24)$$

The new analytical formula, taking into account a certain air thickness, is expressed as follows

$$C_{\text{cyl}} = \frac{h 2 \pi \varepsilon_0 \varepsilon_r}{\varepsilon_r \ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right) + \ln \left(\frac{D_{\text{ext}}}{D_{\text{air}}} \right)}. \quad (4.25)$$

Considering that permittivity is a complex number and that the measured admittance can be converted into a complex capacitance $\overline{C}_{\text{cyl}}$, we can then express permittivity as a function of this capacitance,

$$\overline{\varepsilon}_r = \frac{\overline{C}_{\text{cyl}} \ln \left(\frac{D_{\text{ext}}}{D_{\text{air}}} \right)}{h 2 \pi \varepsilon_0 - \overline{C}_{\text{cyl}} \ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right)}. \quad (4.26)$$

Which, when split into real and imaginary parts, gives us

$$\varepsilon'_r = \frac{h 2 \pi \varepsilon_0 \ln \left(\frac{D_{\text{ext}}}{D_{\text{air}}} \right) C_{\text{cyl}} - \ln \left(\frac{D_{\text{ext}}}{D_{\text{air}}} \right) \ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right) \left(C_{\text{cyl}}^2 + \left(\frac{G_{\text{cyl}}}{\omega} \right)^2 \right)}{\left(h 2 \pi \varepsilon_0 - \ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right) C_{\text{cyl}} \right)^2 + \left(\ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right) \frac{G_{\text{cyl}}}{\omega} \right)^2}, \quad (4.27)$$

and

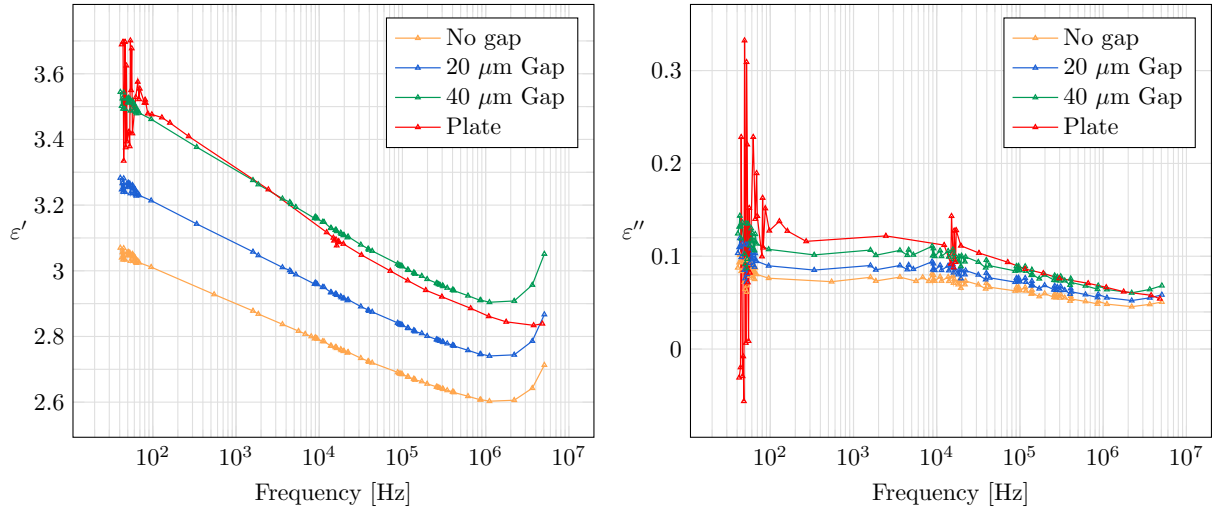
$$\varepsilon''_r = \frac{h 2 \pi \varepsilon_0 \ln \left(\frac{D_{\text{ext}}}{D_{\text{air}}} \right) \frac{G_{\text{cyl}}}{\omega}}{\left(h 2 \pi \varepsilon_0 - \ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right) C_{\text{cyl}} \right)^2 + \left(\ln \left(\frac{D_{\text{air}}}{D_{\text{int}}} \right) \frac{G_{\text{cyl}}}{\omega} \right)^2}. \quad (4.28)$$

Table 4.4: Dimensions of the cylinder capacitor.

D_{int} [mm]	$D_{\text{air},20}$ [mm]	$D_{\text{air},40}$ [mm]	D_{ext} [mm]	h [mm]
55.88	55.92	55.96	56.21	125.65

We decided to test this correction taking into account the air depth for two thicknesses: 20 and 40 μm in order to compare them with the result of the raw formula without correction.

The admittances of the two capacitors were measured using an Agilent 4249A impedance analyzer, and the permittivities calculated on the basis of the measurements are shown in the following figure.



(a) Real part of PVC relative permittivity.

(b) Imaginary part of PVC relative permittivity.

Figure 4.12: Relative permittivity of PVC material obtained for the cylinder and the plate samples.

The rule matching the real part of the relative permittivity of the PVC to the frequency corresponds to

$$\varepsilon'_{\text{PVC}} = 3.8 f^{-0.0208}, \quad (4.29)$$

with f representing the frequency in Hz. And the rule matching the imaginary part of the relative permittivity of the PVC to the frequency is piecewise linear with a the constant value of 0.1 from DC to 10 kHz and

$$\varepsilon''_{\text{PVC}} = 0.243 f^{-0.096}, \quad (4.30)$$

for frequencies higher than 10 kHz and with the frequency f expressed in Hz.

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The dielectric properties of PVC¹ vary with frequency, but only slightly. Indeed, between 100 Hz and 1 MHz, the real part of the permittivity only decreases by around 20 %, and the same applies to the imaginary part. This means that the capacitance remains stable as a function of frequency. However, while the imaginary part remains constant, this is not the case for the equivalent conductance, which increases almost linearly.

A ladder circuit is again well suited to model the impedance because the capacitance is nearly constant over frequency and that the conductance varies greatly. To achieve this, we set the limit that this circuit should only be used to connect two points spaced one or two decades apart in frequency, with as few elements as possible. The aim is not to achieve the same complexity as the circuit developed for inductances, but to be able to estimate dielectric losses for the first resonant frequency and one or two decades higher. In principle, there is no point in trying to represent these losses (and consequently this conductance) for frequencies below the first resonance frequency, since other phenomena then dominate the behavior of the device.

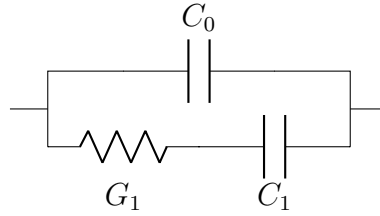


Figure 4.13: Simple capacitive ladder circuit.

The goal is to determine the elements of the following circuit, whose values guarantee the proper admittance at low and high frequencies. Assuming that at pulsation ω_{LF} , it is possible to match the admittance

$$\bar{Y}_{LF} = G_{LF} + j \omega_{LF} C_{LF} , \quad (4.31)$$

and similarly, at the ω_{HF} pulsation, is expected the value

$$\bar{Y}_{HF} = G_{HF} + j \omega_{HF} C_{HF} . \quad (4.32)$$

With this, can be determined

$$C_1 = \sqrt{\frac{G_{HF}^2 \frac{G_{LF}^2 G_{HF} (\omega_{LF}^2 - \omega_{HF}^2)^2}{G_{LF} \omega_{HF}^2 - G_{HF} \omega_{LF}^2}}{\omega_{LF}^2 \omega_{HF}^2 (G_{HF} - G_{LF})}} , \quad (4.33)$$

then

$$C_0 = C_{HF} + \frac{C_{LF} - C_{HF}}{2} - \frac{C_1}{2} . \quad (4.34)$$

Finally, the last value to be estimated, G_1 , is obtained through a minimization scheme to ensure that the real part of the ladder admittance is as close as possible to G_{LF} and G_{HF} for the corresponding frequencies. The expression of the equivalent admittance of the ladder capacitive circuit can be written as

$$\bar{Y}_{eq} = \frac{G_1 (\omega C_1)^2}{G_1^2 + (\omega C_1)^2} + j \frac{G_1^2 \omega C_1}{G_1^2 + (\omega C_1)^2} + j \omega C_0 , \quad (4.35)$$

¹ Ferrites are also dielectric. Unfortunately, experimental testing of their relative permittivity was not possible, so existing measurement data were used. Typical value of permittivity and permeability have been obtained [60].

where the expression for the real part, which depends on the parameters G_1 , C_1 and the pulsation ω , is clearly shown.

4.2 Description and Contributions of Validation Cases

The first part of this chapter provided the properties needed to model the materials that made up the validation devices. The second part focused on the description, modeling, and measurement of these validation cases, in order to enable a comparison between experimental results and those of numerical simulations.

To achieve this, only **axisymmetric geometries** were produced, in order to be as close as possible to 2D models with symmetry of revolution, and consequently to minimize approximation errors of a purely geometric nature. In other words, physical devices were produced whose representation by an axisymmetric 2D model was as faithful as possible to the real geometry on which the measurements were made.

It should be noted, however, that any 2D model was inevitably a simplification and idealization of the real component. Nevertheless, the undesirable effects produced by non-idealities were minimized as far as practical. Among other measures, electrical accesses were kept as short as possible, helix pitches were designed small, and all out-of-plane elements such as layer-to-layer connections were kept as short as possible relative to the dimensions of each test case.

Four devices were studied in increasing order of complexity and physical phenomena involved. First, a **multi-layer inductor** with a relatively low number of turns was studied. The choice of a multi-layer winding was justified by the emphasis on layer-by-layer capacitive coupling (where the electrical potential gradient was most intense) and the intention to achieve a relatively high equivalent capacitance for a given low number of turns. This case had three variants: the first without a core, the second with a half-core, and the third with the complete core made up of two half-cores facing each other. In the first case, the air inductor thus created enabled a more specific study of the evolution of copper losses at frequencies below the first resonance. It also enabled validation of the level of dielectric losses, which were generally overshadowed by ferromagnetic losses, absent in this case. The second case, with a half-core, was useful for validating the ferromagnetic model of the ferrite, without assumptions about a possible air gap (avoiding tolerance problems). For the third case, with the magnetic path closed by a second half-core, the influence of iron losses compared to total losses could be discussed, as well as the evolution of capacitive/inductive coupling phenomena in the presence of a full core.

The latest test case was a **single-layer air inductor**. The aim was to verify whether the localized constant circuit representation was sufficient to simulate devices whose capacitive coupling was mainly turn-by-turn, rather than between successive layers of conductors.

The higher number of turns of this winding resulted in a larger series of consecutive resonances over the measurement range under examination. It was therefore also an appropriate validation case for the comparison of different discretization levels, as presented in the previous chapter.

This case was also used to carry out a small sensitivity study on the equivalent winding capacitance as a function of the relative permittivity of the two dielectrics involved: enamel and PVC. This enabled a verification of the electrostatic modeling hypothesis².

²prevailing dielectric properties of the coil former (PVC) over those of the wire enamel.

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To conclude this aside, impedance measurements for these systems were taken with different instruments, to provide greater confidence in the measurements. The multi-layer air inductor was used to compare measurements made with both the Bode 100 analyzer (Omicron Lab) and the Agilent 4294A. In addition, DC resistance measurements were made using a Metrahit 271 milliohmmeter. Comparisons of these various measurements with analytical estimates and numerical simulations formed the basis of the validations.

4.3 Multi-layer Air Inductor with Rotational Symmetry

The first device studied corresponds to a 21-turn multi-layer air inductor (7 turns distributed over three layers) consisting of 3 concentric coil formers, each with a helical groove designed to precisely fix the conductor position on the PVC coil former. Each stage clamps the conductor to the previous stage, without the need to glue the copper wire to the coil former or introduce any other dielectric material. A final PVC outer cylinder clamps the three lower stages. It should be noted that the PVC parts machined to form the coils former are extracted from the same bar as the sample on which the dielectric characterization was performed. The physical properties of these parts can therefore be expected to be shared with the sample. The geometry and a 2D cross-section with dimensions are shown in the following figure.

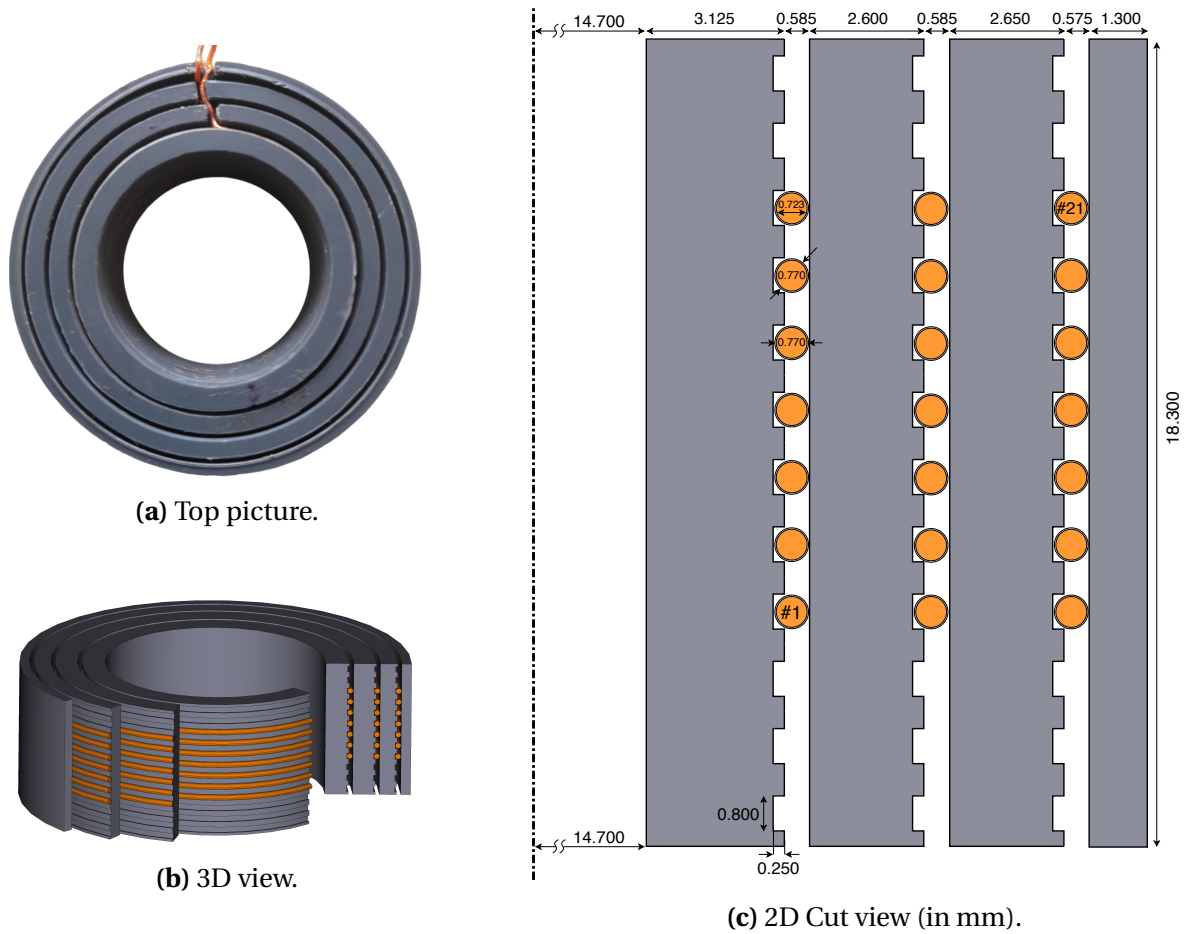


Figure 4.14: Multi-layer air inductor with rotational symmetry with positive terminal defined at spire #1.

The winding follows a **discontinuous pattern** as presented in Figure 1.9 which implies that the turns of successive layers are superimposed along the radial axis and enforce a proper rotational symmetry. An additional groove has been made vertically on each cylinder to allow the wire to pass back to the foot of the coil former after each winding layer.

The copper conductor is AWG 21 (American Wire Gauge) with a 0.410 mm^2 cross-section and a total diameter of $\sim 770 \text{ }\mu\text{m}$. The wire length of the axisymmetric model is 2795 mm, giving a resistance of $117.3 \text{ m}\Omega$ according to Pouillet's law. The access wires have a total length of 120 mm, and the 3 vertical sections needed to pass from one layer to the next account for a total length of $3 \times 12.2 \text{ mm} = 36.6 \text{ mm}$. The additional resistance of these cable segments is therefore $6.6 \text{ m}\Omega$. The finite element model provides the equivalent resistance for the symmetrical part of the device with a value of $117.8 \text{ m}\Omega$, fairly close to the analytical calculation. If we take into account the resistance of the wires between the 3 layers and of the access wires, we can draw up a comparison between the analytical value of the resistance versus the finite element calculation, as well as the various measurements: Bode 100 and Agilent 4294A analyzers and Metrahit milliOhmmeter. The values are shown in the subsequent table.

Table 4.5: Comparison of DC resistance estimations and measurements for the axisymmetric multi-layer air inductor.

MetraHit (DC)	Agilent (40 Hz)	Bode 100 (20 Hz)	Analytical (DC)	FEM (10 Hz)
130.6 m Ω	132.4 m Ω	115.8 m Ω	124.3 m Ω	124.8 m Ω
reference	1.4 %	-11.3 %	-4.8 %	-4.4 %

There are several possible reasons for discrepancies between analytical and measured values. First of all, it must be pointed out that copper conductivity is sensitive to temperature variations, about 0.39 % per degree. As the temperature of the sample is that of the part being tested, it may be higher or lower than the reference 20°C . Then, the analyzers (Bode 100 and Agilent) measure the voltage/current ratio in the device under test, and for low frequencies, the voltage amplitude is relatively low for a current imposed at a few milliamperes. Finally, there are imperfections such as contact resistances, which we try to minimize, for example by using test fixtures with the Agilent 4249A and suitable clamps for the Bode 100, but which are nonetheless present. Deviations of a few percent are not surprising for measurements of such low values.

Measuring DC resistance is not an end in itself, but it does help to ensure that impedance analyzers or finite element methods are correctly estimated, and to guarantee both conductor continuity and the absence of short-circuits within the winding.

4.3.1 Impedance Analyzers: Measurements and Comparisons

Prior to directly comparing measurement results with simulations, it is important to have a thorough understanding of what has been measured and the influence that certain parameters have on the results. First, it makes sense to compare the measurements made with the Bode 100 and the Agilent 4294A. The amplitude of the impedance is shown on the following Bode diagram, on which the three curves are plotted: Bode 100 with positive terminal single ended (referred as *positive polarity*), Bode 100 with negative terminal single ended (referred as *negative polarity*) and Agilent 4294A (*positive polarity*). Indeed, when one terminal is single ended (also called the hot wire), it means that the other one is grounded with respect to the measurement device. For the same connection, in this case the positive polarity,

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there is a close agreement between the measurements of the two devices, in terms of amplitudes. Note that the Agilent sweeps frequencies from 40 Hz to 100 MHz, while the Bode 100 only measures from 20 Hz to 50 MHz. Measurements at higher frequencies revealed four additional resonances in the 50-100 MHz band. In terms of impedance values, we can easily identify three zones: the first corresponds to low frequencies, below 300 Hz, where the behavior is mainly resistive; the second corresponds to an inductive behavior up to 10 MHz; and the third corresponds to capacitive behavior above ~ 15 MHz, followed other resonances of the system.

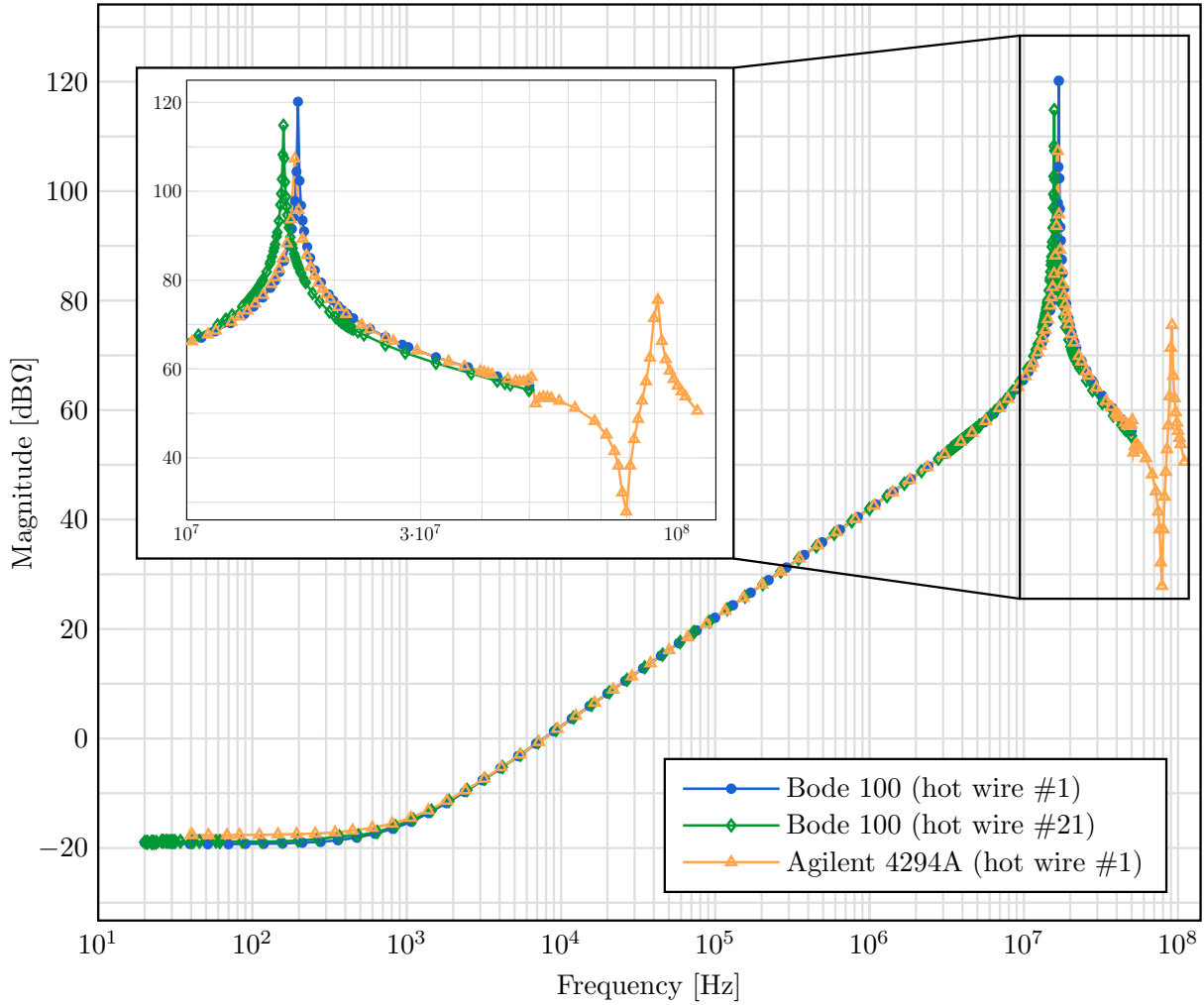


Figure 4.15: Impedance amplitude measurements of the multi-layer air inductor with the Bode 100 and the Agilent 4294A impedance analyzers.

For phase measurements, there are greater differences and more surprising behaviors, particularly for Bode 100. Over the 500 kHz to 10 MHz range, we observe that, for measurements with positive polarity, the phases match well for both the Bode 100 and the Agilent. However, an unexpected behavior occurs on the Bode 100 measurement with the hot wire exchanged. We notice that the phase continues to increase until it exceeds the value of 90° , which is an inconsistent value for a passive component.

On the other hand, for resonances, the change in phase is clearly observed for all three measurements, with nothing surprising for the Agilent and Bode 100 in reverse polarity. In other words, the phase changes continuously from 90° to -90° , passing through 0° , whereas this is not the case for the third measurement. We can clearly see the phase rising above 90° and

then, after resonance, dropping abruptly below -90° . This is hardly surprising. Indeed, phase measurements close to 90° are more difficult to obtain. We simply note that measurements made with the Agilent 4294A seem more reliable and less erroneous than those made with the Bode 100.

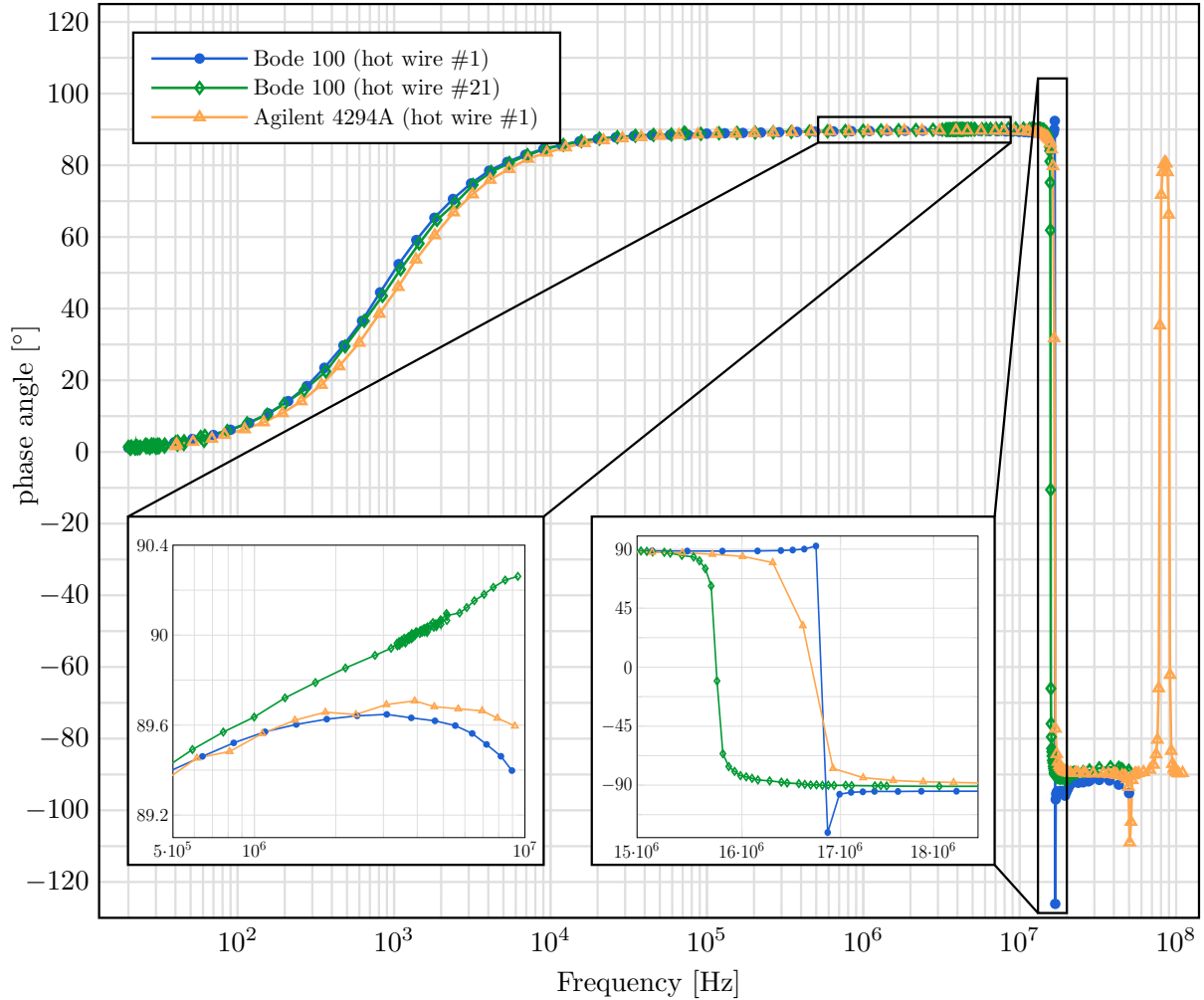


Figure 4.16: Phase angle measurements for the impedance of the multi-layer air inductor with the Bode 100 and the Agilent 4294A impedance analyzers.

Since the measurements made with the Agilent exhibit less pronounced phase artifacts, at least at the resonant frequency, and given that the amplitudes measured by both the Bode 100 (positive polarity configuration) and the Agilent are very close, we will consider that the Agilent fulfills the conditions for valid measurements, which we will compare with the extracted models later on. What is more, the Agilent 4294A allows measurements up to 100 MHz, showing additional behaviors that were not possible with the Bode 100 because it is limited to 50 MHz. However, we will keep the Bode 100 measurements in order to check whether the effect of the polarity change observed with the measurements is indeed found in the equivalent model of the component.

4.3.2 Influence of Components Polarity

For Bode 100 measurements, we were also able to compare two configurations: one for a given polarity and the other for its reverse polarity. The main observation is that the first

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resonance depends on the polarity with which the inductor is connected to the measuring device. The first resonant frequencies of the system are shown in the table below. One corresponds to the positive polarity connection $f_{\text{res}+}$ and the other to the negative polarity connection $f_{\text{res}-}$.

Table 4.6: First resonant frequencies of the multi-layer air inductor for the negative (hot wire #21) and positive (hot wire #1) polarities.

negative terminal grounded $f_{\text{res}-}$	positive terminal grounded $f_{\text{res}+}$
15.756 MHz	16.866 MHz

The model we have chosen allows us to explain this relatively easily. Indeed, the simplest model we implement is winding discretization, which provides a single inductance in the case of a single winding, but three distinct capacitances. This is explained by the fact that there are always at least two distinct nodes per winding in addition to the reference node (potential 0), as illustrated in the following figure.

This circuit is symmetrical with respect to its terminals, provided that the potentials of the terminals are floating with respect to the reference potential. In practice, one of the winding ports is at the same potential as the reference, typically via a connection to the measurement device chassis, which is also connected to earth. In doing so, we short-circuit the capacitance $C_{W_{1+}, \text{ref}}$ or the capacitance $C_{W_{1-}, \text{ref}}$ depending on the polarity of the wiring. In fact, nothing requires that the capacities $C_{W_{1-}, \text{ref}}$ and $C_{W_{1+}, \text{ref}}$ have the same value. This is actually not the case for this 21-turn inductor. This can be understood by observing the arrangement of the magnetic component's turns and accesses. Figure 4.17 clearly shows that the arrangement of the conductors is not symmetrical with respect to the reference potential, in this case placed at the outer level of the infinite layer.

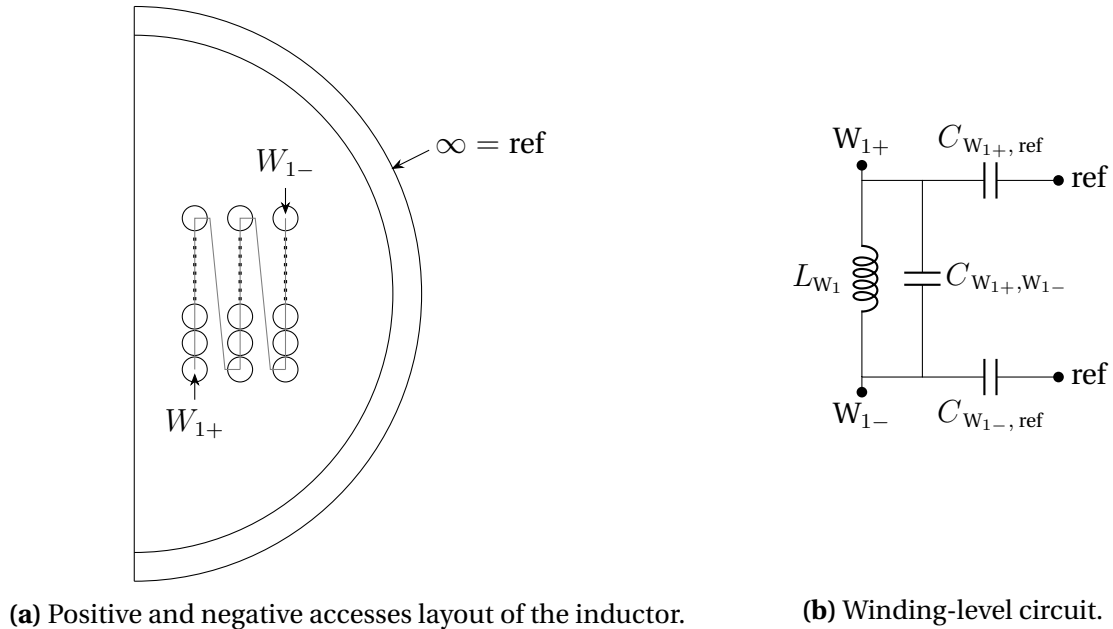


Figure 4.17: Layout and winding-level circuit representation of the 21-turn air inductor.

Thus, the positive pole, designated by the W_{1+} node, is located on the inner side of the device and is therefore exposed to less capacitive coupling with respect to the infinite layer.

On the other hand, the negative pole, designated by the W_{1-} node, is on the outer surface of the component and therefore sees a high capacitive coupling with its external environment.

The axis of symmetry carries no charge (a Neuman condition is assigned to it) and therefore plays no part in electrostatic coupling. The infinite boundary of the model (at the right edge of the Figure 4.17(a)), on the other hand, does carry a charge and shares capacitive couplings with the conductive parts of the device under test.

To be able to compare impedance measurements with finite element calculations, the parasitic capacitance produced by the winding accesses must be taken into account. The length of the current-carrying wires has been kept to a minimum. However, an additional parasitic capacitance of 0.4 pF has been estimated for these wires. This will have to be added to the FEM values to make the comparison with the physical device meaningful.

The electrostatic finite element model gives us the following capacitance values at 10 MHz frequency: $C_{W_{1+},\text{ref}} = 1.09$ pF, $C_{W_{1-},\text{ref}} = 1.92$ pF and $C_{W_{1+},W_{1-}} = 3.12$ pF. Clearly, the capacitances of the two ports are very different, and the highest capacitance is found between the two winding ports. If we assume that the negative terminal is connected to ground, we obtain an equivalent capacitance of

$$C_+ = C_{\text{term}} + C_{W_{1+},W_{1-}} + C_{W_{1+},\text{ref}} \quad (4.36)$$

$$= 4.61 \text{ pF} , \quad (4.37)$$

and if we consider reverse polarity, the total capacity is

$$C_- = C_{\text{term}} + C_{W_{1+},W_{1-}} + C_{W_{1-},\text{ref}} \quad (4.38)$$

$$= 5.44 \text{ pF} . \quad (4.39)$$

If we evaluate the ratio

$$\sqrt{\frac{C_-}{C_+}} = 1.086 , \quad (4.40)$$

we obtain a difference of the order of 8 %, close to the difference measured between the two resonance frequencies of each polarity

$$\frac{f_{\text{res}+}}{f_{\text{res}-}} = 1.070 , \quad (4.41)$$

of around 7 %. There are, of course, measurement errors that taint this result, but we consider that the main effect of the change in polarity materializes at the level of the equivalent capacitance and, consequently, at the first resonant frequency.

Therefore, we introduced the method used to differentiate between the first two resonance frequencies related to the positive and negative wiring. The simple model of the component with two independent nodes carrying different charges, depending on the wiring conditions is enough to reproduce the shifting effect and is quite convincing.

4.3.3 Low Frequency Phenomena: Inductance and Copper Losses

First and foremost, our model needs to identify low-frequency behavior. Transformers are generally used in these frequency ranges, and we will need to provide precise values for the corresponding inductance and resistance.

If we first consider the inductance, we can already estimate its value at low frequency using Wheeler's analytical formula. This analytical approach yields a value of 20.7 μH [111]

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which is very close to the value identified by finite elements: $20.23 \mu\text{H}$. However, the analytical approach is limited to a single value and does not take into account the evolution of inductance with frequency. In fact, when induced currents begin to appear, in this case around 10 kHz, the current density distribution is such that the value of the inductance decreases. It is the portion of the inductance associated with the flux embraced by the massive conductor that decreases due to a current density increasingly concentrated at the periphery of the conductors because of the effects of induced currents (skin and proximity effects). This is clearly seen in the following figure, where the inductance drops to $19.71 \mu\text{H}$ at 1 MHz for the finite element evaluation.

If we now wish to compare the values identified by finite elements with the measured values, we can already observe that they all follow the same evolution. However, all three measurements strongly overestimate the inductance value for frequencies below 1 kHz. This is probably due to the lack of precision associated with the low impedance of the device under test at these frequencies. In addition, integration times are sometimes too long at low frequencies, making measurements sensitive to external interference. The value then stabilizes at around $20.5 \mu\text{H}$ for both Bode 100 and Agilent 4294A measurements, as shown in Figure 4.18. In addition, the decrease in inductance beyond 10 kHz is observed at the same rate for both measurements and finite element simulation results. This supports the use of the 2D FEM model to evaluate the inductance of a device with axisymmetric geometry. Finally, beyond 800 kHz, measured inductance values increase again with frequency, due to capacitive coupling and the approach of the resonant frequency. In practice, the inductance does not increase, but rather the impact of capacitive coupling is observed.

In the case of resistance, the evolution is very similar despite some amplitude differences between the curves (measured and finite element). As with inductance, a change is observed from 10 kHz upwards, due to induced currents which impose a greater current density at the periphery of the conductor, and consequently a greater equivalent resistance. The increase in resistance is due to the skin and the proximity effects between adjacent turns of the winding. The skin effect alone cannot explain a value of $900 \text{ m}\Omega$ at 1 MHz, but only $350 \text{ m}\Omega$ for a round conductor of $723 \mu\text{m}$ diameter. As expected, the 2D finite element approach for axisymmetric geometry is particularly well suited to extracting magnetic and resistive parameters.

However, measurements are shifted up at higher frequencies by the system's first resonance. Indeed, if we want to be able to evaluate the resistance value for frequencies above 1 MHz, we have to subtract the impact of the capacitance on the measured values. In other words, knowing the parameters of the resonant circuit: inductance and equivalent capacitance, we can substitute the effect of resonance for the real part of the impedance measured with an analyzer. This correction can therefore be applied for frequencies close to the resonance. To remove the effect of the capacitance on impedance measurements, we can study the following RLC equivalent circuit.

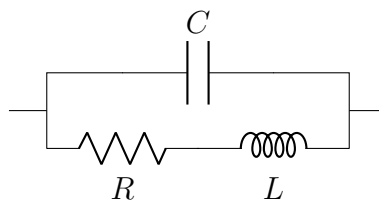


Figure 4.19: Simplest RL circuit with a parasitic capacitance across the accesses.

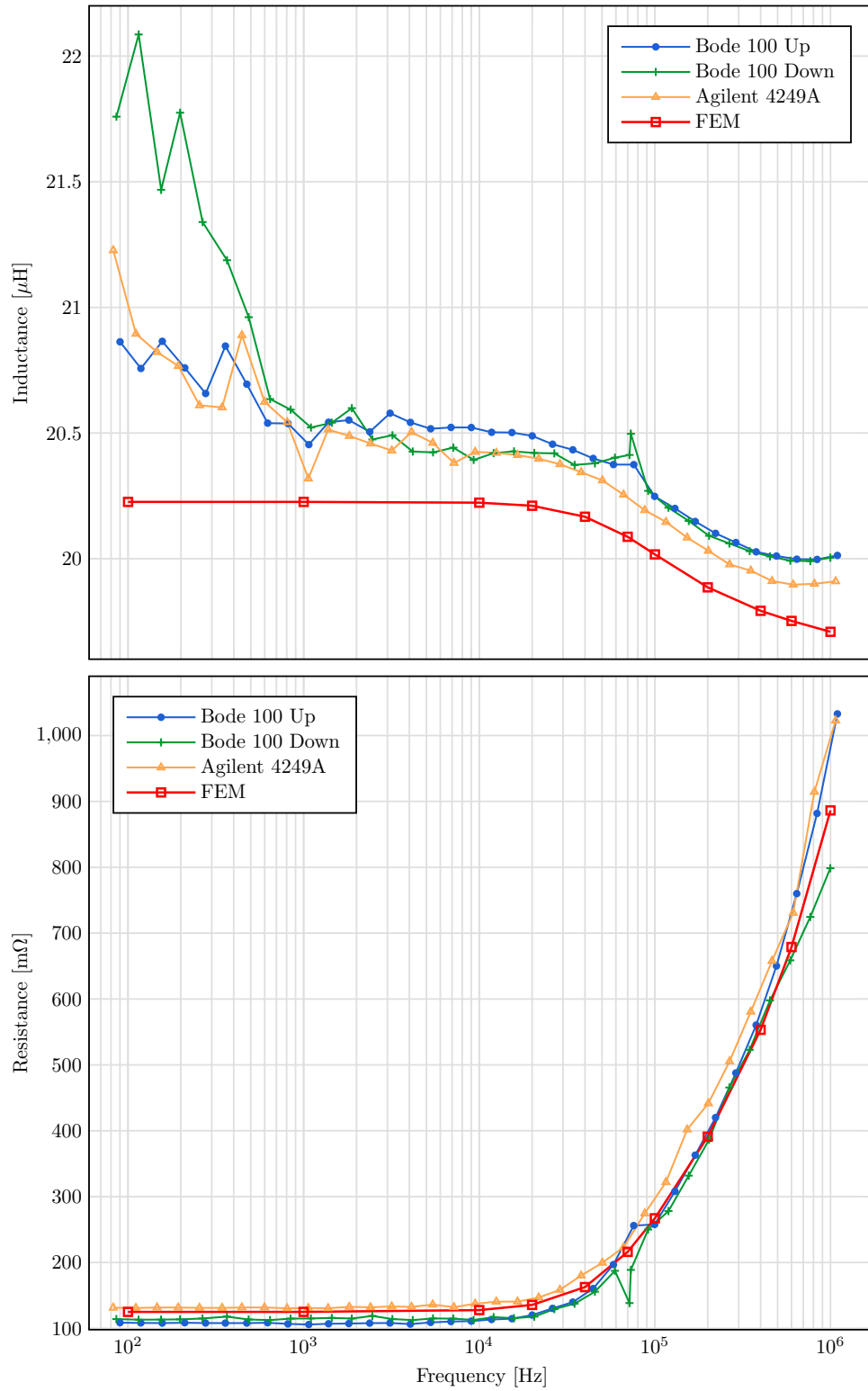


Figure 4.18: Inductance and resistance evolution with frequency for the 21-turn air inductor.

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In this case, the impedance is written as

$$Z = \frac{(R + j \omega L) \frac{1}{j \omega C}}{(R + j \omega L) + \frac{1}{j \omega C}}, \quad (4.42)$$

which can be developed a little further to clearly show the real and imaginary parts, such as

$$Z = \frac{R}{(1 - \omega^2 L C)^2 + (\omega R C)^2} + j \frac{(\omega L (1 - \omega^2 L C)^2 - \omega R^2 C)}{(1 - \omega^2 L C)^2 + (\omega R C)^2} \quad (4.43)$$

by isolating the real part of the impedance, one will obtain a second-degree equation in R

$$\text{Re}(Z) = \frac{R}{(1 - \omega^2 L C)^2 + (\omega R C)^2}, \quad (4.44)$$

whose solutions are

$$R = \frac{1 \pm \sqrt{1 - 4 (\omega C \text{Re}(Z) (1 - \omega^2 L C))^2}}{2 \omega^2 C^2 \text{Re}(Z)}. \quad (4.45)$$

Therefore this measured resistance can be displayed with the correction applied and compare it with the finite element model over a wider range of frequencies. Indeed, the instrument measures the real part of the impedance which is not the resistance of the model. However, with Equation (4.45), the value of the resistance from the equivalent model can be deduced. The comparison was carried further for this test case, after the inclusion of the dielectric losses in the model.

4.3.4 High Frequency Dielectric Losses

In practice, dielectric losses are no longer negligible and need to be taken into account in the calculation. As an example, consider what happens at 10 MHz. At this frequency, the permittivity of PVC is $\bar{\epsilon} = 2.8 - j 0.040$, according to the characterizations previously carried out. To appreciate the impact of complex-valued permittivity, we can display the voltage distribution curves, which are now also in the complex plane. Since the real part of the permittivity dominates the material's behavior, there is very little difference in the real part of the voltage. For the imaginary part, on the other hand, the field lines are particularly concentrated in the dielectrics, and even more so in the PVC, which concentrates a large proportion of the losses, due to its dielectric characteristics and its large volume within the device. Finally, we can add that in the lossless case, the relative permittivity would be real-valued and so would the voltage, by extension.

In Figure 4.20, these effects on the voltage distribution within the winding are clearly visible. Dirichlet's constraints are well respected, with a voltage exhibiting a decreasing real value along the winding and an imaginary value of zero throughout the winding. Within the dielectric of the coil former (the PVC), numerous field lines are clearly visible, indicating that a strong electric field is present in this material, thus generating losses. What is most striking is that many of the field lines are concentrated in the outer corners of the device, on the most peripheral layers.

These isovalue maps therefore also have the advantage of highlighting not only dielectric losses, but also the regions of the magnetic component where these losses reach high and,

conversely, low values. It can be expected that this information can be used to improve the performance of an air-coiled component for which the aim is to minimize dielectric losses, and to determine which parts of a coil to form are the most loss-generating, and how to modify a geometry to mitigate them.

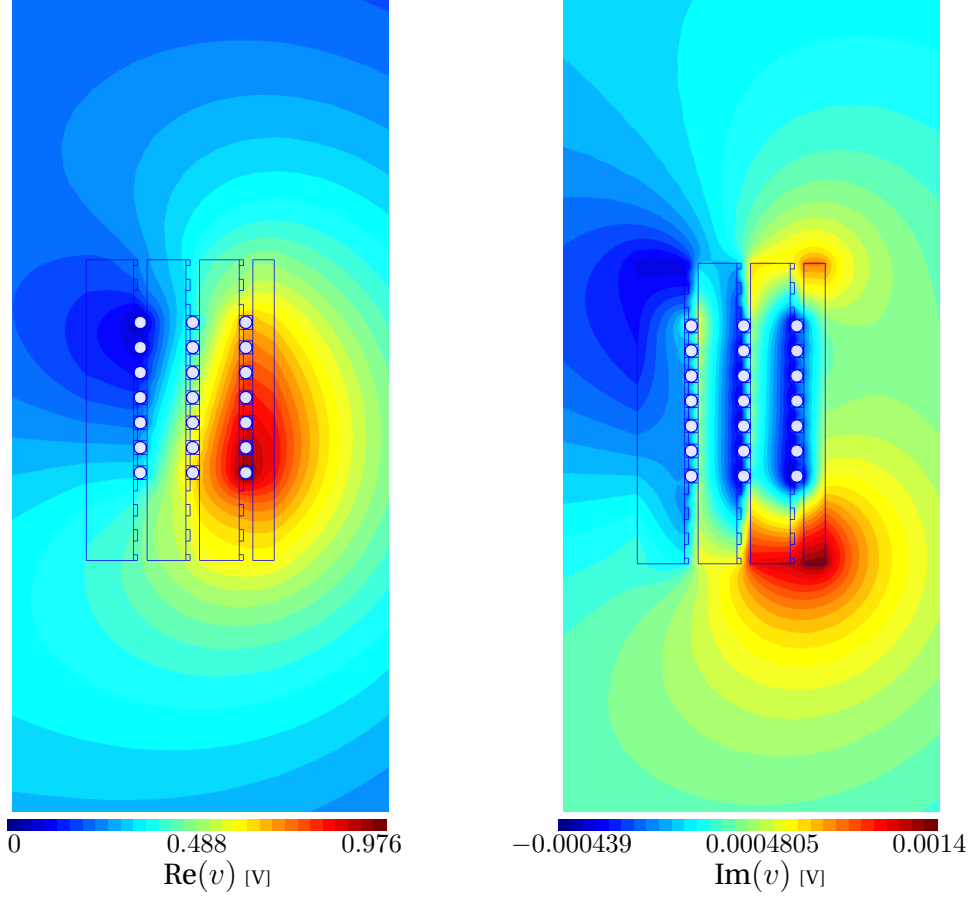


Figure 4.20: Simulation of voltage field distribution for permittivity parameters identified at 10 MHz.

The equivalent capacitances and conductances defined at the winding terminals and obtained with these models have the following values for a given frequency of 10 MHz.

$$\underline{C}_{W, 10 \text{ MHz}} = \begin{bmatrix} 1.09396 \text{ pF} & 3.12126 \text{ pF} \\ \cdot & 1.91944 \text{ pF} \end{bmatrix}, \quad (4.46)$$

$$\underline{G}_{W, 10 \text{ MHz}} = \begin{bmatrix} 73.766 \text{ nS} & 2374.707 \text{ nS} \\ \cdot & 147.263 \text{ nS} \end{bmatrix}. \quad (4.47)$$

The evolution of these parameters for different frequencies and the value of the relative permittivity, which depends on this frequency, are shown in the following table.

Table 4.7: Relative dielectric permittivity and circuit parameters evolution regarding frequency increase.

f [MHz]	$\bar{\epsilon}$	$C_{1+, \text{ref}}$ [pF]	$C_{1-, \text{ref}}$ [pF]	$C_{1+, 1-}$ [pF]	$G_{1+, \text{ref}}$ [nS]	$G_{1-, \text{ref}}$ [nS]	$G_{1+, 1-}$ [nS]
1	$2.9 - j0.060$	1.09682	1.92519	3.21525	10.381	21.086	351.58
10	$2.8 - j0.040$	1.09396	1.91944	3.12126	73.766	147.263	2 374.71
100	$2.7 - j0.020$	1.09092	1.91342	3.02608	393.631	772.661	12 031.30

The effect of frequency on the material's dielectric characteristics, and ultimately on localized constants circuit parameters, is clear. Among other things, we can see that by multi-

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plying the frequency by 10, the equivalent conductance values are less than 10 times higher, since the imaginary part of the dielectric permittivity has not kept the same value, but has decreased over the frequency interval under consideration.

By correcting for the effect of capacitance on the losses, we can study the non-capacitive current of the I_{RLG} device, which corresponds to the conduction current in the copper and the leakage current associated with the dielectric losses (*i.e.* the imaginary part of the displacement current).

A circuit equivalent to the case under consideration, without the capacitor, is shown in the figure below. Remark that an LTspice circuit has no Siemens conductance element already defined, but conversion to equivalent resistance is immediate, since one is the inverse of the other (and vice-versa).

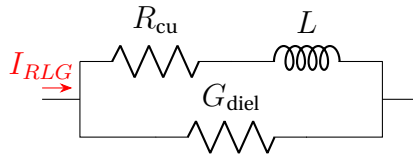


Figure 4.21: R-L-G one-port equivalent circuit.

The analytical expression for the impedance of this dipole is obtained as follows

$$Z = \frac{(R_{cu} + j \omega L) R_{diel}}{(R_{cu} + R_{diel}) + j \omega L} \quad (4.48)$$

for which the real and imaginary parts can be shown more clearly, such as

$$Z = \frac{(R_{cu} + R_{diel}) R_{cu} R_{diel} + R_{diel} (\omega L)^2}{(R_{cu} + R_{diel})^2 + (\omega L)^2} + j \frac{\omega L R_{diel}^2}{(R_{cu} + R_{diel})^2 + (\omega L)^2} . \quad (4.49)$$

To determine the portion of the total non-capacitive current (I_{RLG}) that flows through the dielectric, we can use the current divider formula, assuming that the impedance of R_{cu} is very small compared to the impedance of ωL , and calculate this fraction as follows

$$\frac{I_{diel}}{I_{RLG}} = \frac{G_{diel}}{\sqrt{G_{diel}^2 + \frac{1}{(\omega L)^2}}} , \quad (4.50)$$

which indicates 0.004 % of the total current at 1 MHz, 0.298 % at 10 MHz and 14.97 % at 100 MHz. Note that in this case, this current is practically in quadrature with the current in the conductors.

Now that we have determined the correction to be applied to the impedance measurements with Equation (4.45), and established an alternative method for estimating total circuit losses by subtracting the influence of the first resonance, we can compare the corrected measurements with the circuit model from which the equivalent capacitances have been removed. Note that the dielectric loss conductances are still present, only the capacitances have been removed from the model. This is shown in Figure 4.22 for frequencies up to the first resonance frequency and even slightly beyond. In this figure, we can see that copper losses predominate the behavior up to 1 MHz. Then, we can clearly see that the added dielectric losses greatly increase the value of the equivalent series resistance. Finally, we can

see that the losses measured with the correction and the model losses without the equivalent capacitances provide curves very close to each other. Our estimates of the equivalent resistance of total losses (copper and dielectrics) are even slightly higher than the values obtained from experimental measurements. This difference is not surprising, given the amplitude differences and the high frequencies considered, *i.e.* above 10 MHz.

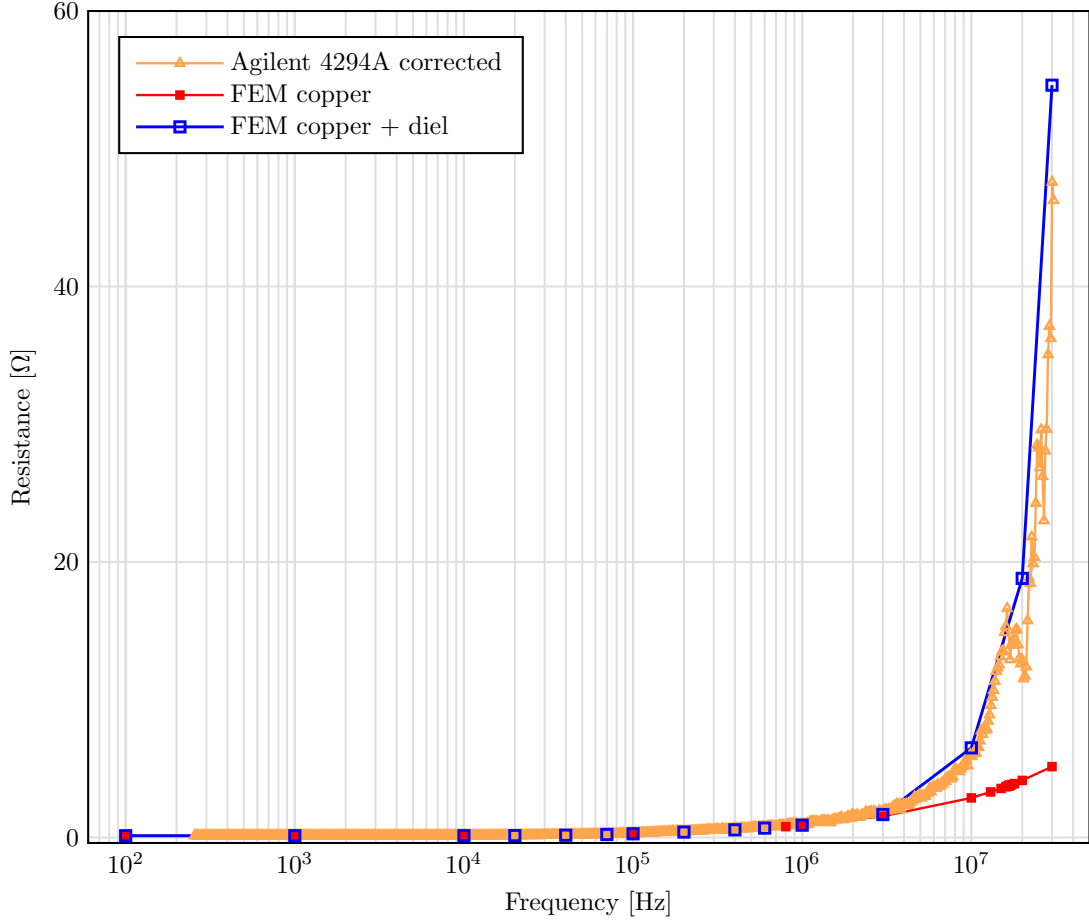


Figure 4.22: Equivalent series resistance depending on the frequency for the measured case (corrected with Equation (4.45), the model with only copper losses and the model with both copper and dielectric losses.

Remember that values above 10 MHz should be treated with caution, as dielectric loss resistances depend directly on the imaginary part of the complex permittivity, which has not been well characterized for these frequencies. In this example, we have kept $\varepsilon'' = 0.020$ for frequencies above 10 MHz.

At this level, we can already boast of having been able to both measure and represent low-frequency behavior (inductance evolution and copper losses), high-frequency losses of the dielectric type in this case, as well as coupling capacitances between the various nodes of the winding discretization. All that remains now is to compare this complete model with impedance measurements for different levels of discretization, and to analyze the impact that these levels of discretization have on the results.

4.3.5 Discretization Levels Comparison

The complete winding discretization model has been validated for the various aspects mentioned above (copper losses, dielectric losses, inductance evolution, equivalent capacitance), and we now need to test this same model for other discretization levels and compare these results with each other. This should highlight the benefits of each of them, and the advantages of choosing one level of discretization over another.

To carry out the comparisons, we have chosen three different discretization levels for the magnetic component. The circuit comprises 21 turns spread over 3 layers, as shown in Figure 4.14c. It therefore seems useful to compare a model per winding (1 element of 21 turns), a model per layer (3 elements of 7 turns) and a model per turn (21 elements of 1 turn). Other representations are still possible, but these 3 cases already illustrate the advantages of the method we introduced in the previous chapter, and enable us to keep to divisions that follow the geometric organization of the component. A comparison of the numerical models with measurements made on the real component is shown in the following figure. The model used for comparison also includes simple capacitive ladders as described in Figure 4.13. On the component impedance bode plot, the first thing to notice is that all 3 nu-

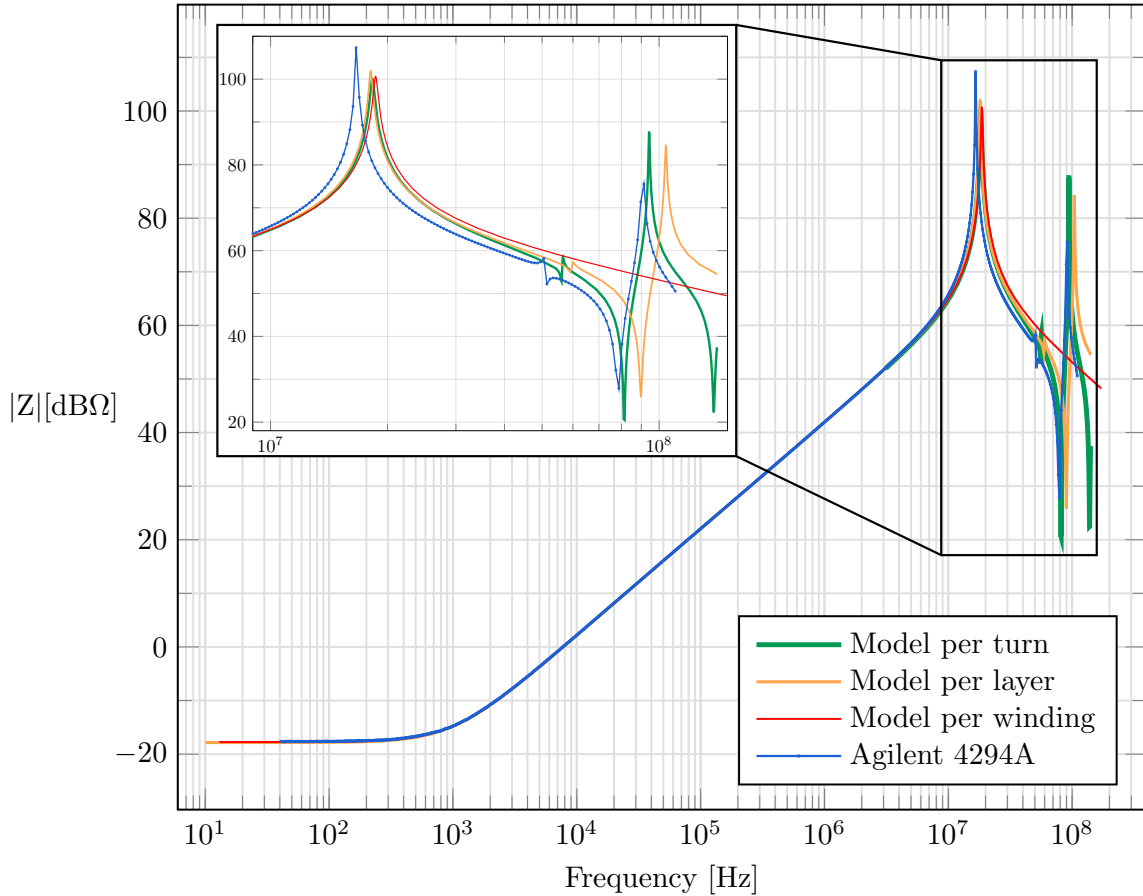


Figure 4.23: Comparison of the measured impedance against simulations with three discretization model cases: per turn, per layer and per winding.

merical models provide the first resonance with the same precision and similar amplitude. This is a significant aspect for the optimization procedure of the different ladders, which, when put together, provide the same losses for different representations of the equivalent circuit (winding, layer, turn). Then, the winding model only allows us to observe the first res-

onance, without surprise, since there is only one L-C coupling in this circuit. The model per layer provides richer and more accurate information on subsequent resonances, although slightly less precise than the model per turn. We note that the high-frequency amplitude follows the measurement fairly closely, thanks to the capacitive ladders that model dielectric losses at very high frequencies. Finally, the most precise level of discretization, the per-turn model, provides us with the most complete information on successive resonances at higher frequencies. In this case, the amplitude deduced from this model follows very closely the value measured by the analyzer. Reassuringly, the layer level already correctly indicates the first five resonances/anti-resonances at a lower computational cost than the turn model. It is therefore up to the designer who uses these representations to choose the most appropriate discretization model for his pre-design, and therefore to potentially choose a less discretized level if the need is not felt, and vice-versa.

In a second step, we will discuss the computation times for the different parts: identifications, capacitive and inductive ladder optimization and circuit simulation (LTspice). The number of unknowns and computation times for each model step are given in Table 4.8.

Table 4.8: Computation times comparison of three different levels of discretization (turn, layer and winding) of the 21-turn air inductor. LTspice resolutions were carried in the frequency domain from 10 Hz to 100 MHz with 3300 points per decade.

		Winding	Layer	Turn
Number of magnetic identifications	[-]	1	3	21
Total magnetic identification time	[s]	2.977	7.627	56.620
Number of inductance matrix elements	[-]	1	6	231
Total RL ladders optimization time	[s]	1.081	3.635	30.269
Number of electric identifications	[-]	2	4	22
Total electric identification time	[s]	2.237	4.483	35.540
Number of capacitance matrix elements	[-]	3	10	253
Total GC ladders optimization time	[s]	1.353	4.634	118.924
LTspice resolution time	[s]	0.261	1.457	2038.934

As provided in the above table, the values increase significantly with the level of discretization of the component. The number of finite element identifications increase with the number of elements per matrix. The same applies to the optimization of inductive and capacitance ladder circuits, with a particularly long time for the level per turn. The most drastic increase in calculation time concerns LTspice resolution, which takes negligible time for winding-level and layer-level. On the other hand, the turn-level model requires a prohibitively long calculation time. Especially since we are only considering a component with a low number of turns, what would be the case for a component with several hundred turns, as can be found in practice. A priori, the LTspice resolution time would become very long and unusable in practice for designers. Hence the obvious advantage of intermediate levels of discretization, such as the layer level in this case. We end up with a good trade-off between computation time and the useful information that the model allows us to extract.

4.4 Multi-Layer Inductor with Half Ferrite Core

For this validation case, we opted for an intermediate geometry, in which half a ferrite core is added to the 21-turn winding previously validated. This core, the P70, is almost completely

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rotationally symmetrical, with the exception of the small bore that provides access to the wires. Naturally, we chose this unusual core to validate our axisymmetric numerical models. This configuration is not representative of a real case. Its purpose is simply to verify the ferromagnetic parameters of the N22 half-core used here, without having to take into account (or being able to do without) a possible air gap in the case of cores assembled one against the other. Obviously, it would be risky to assume that the parameters of the second half-core will be identical to those of the first, on which the measurement was carried out, since there is a wide dispersion of ferrite permeability values. In this case, the manufacturer indicates a tolerance of $\pm 20\%$. Nevertheless, the assumption that different ferrite cores share the same properties is made to facilitate our approach. The real part of the measured permeability of N22 varies between 2800 and 3400. It is therefore advisable to generate finite element models for different values of this permeability and check which is the closest to the low-frequency inductance measurement.



(a) Multi-layer core fixed on top of half ferrite core.

(b) Half a ferrite core.

Figure 4.24: Multi-layer inductor with rotational symmetry and half ferrite P70 core.

At first, we are mainly interested in the value of the real part of the relative permeability of the ferrite. For this reason, we focus on the values and evolution of inductance as a function of frequency, as shown in Figure 4.25. For this comparison with measurements, we tested several relative permeabilities within the finite element model with μ'_r equal to 2000, 2800 and 3200.

Inductive calculations based on impedance amplitude and phase measurements are prone to error, particularly at low phase values. Indeed, a given phase-related error will have a much greater impact on the imaginary part of the impedance for phase values close to 0, rather than for values close to $\frac{\pi}{2}$. The inductance measurement is rather variable below 2 kHz, despite the fact that the finite element model is quite consistent. So we should not rely too much on the measurement below 2 kHz.

For the mid-frequency bands between 3 and 10 kHz, the inductance value is stable, and we note that the relative permeabilities of 2800 and 3200 (purely real) are centered on the measurement data. This confirms that the ferromagnetic data collected on the N22 material are not far from the values of the magnetic models, which are close to the measurements. Indeed, for low fields around 5 kHz, the magnetic characterizations indicate $\mu'_r \simeq 3240$.

At higher frequencies, between 30 and 400 kHz, a drop in inductance is observed in both measurements and numerical models. We immediately notice that models with relative permeabilities above 2000 slightly overestimate the inductance value. There are other explanations than the permeability value. For example, a higher conductivity or larger conductor cross-section may explain a shift towards lower frequencies in the transition of the inductance value, as observed by measurement. According to this hypothesis, the geometry of the

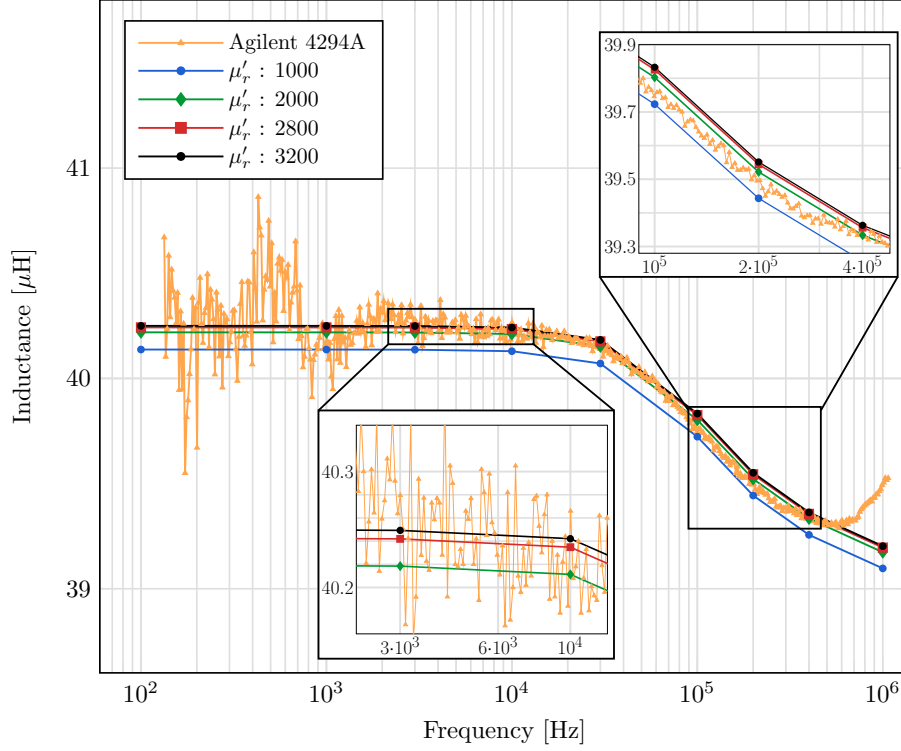


Figure 4.25: Inductance dependency of the frequency for the half core multi-layer inductor.

model would somewhat underestimate the conductor cross-section, which is quite plausible.

Having studied the inductance, we can now focus on the resistance of the component. If we display the resistance value as a function of frequency, we always observe an increase at the same time as the inductance decreases. In this case, this occurs around 30 kHz. In addition, the resistance increases more than in the case without a core. This effect may be due to additional ferrite losses. Further on, we will see that these additional losses are not iron losses, but copper losses, which manifest themselves in a more uneven distribution of current density in the conductor. It is to avoid this kind of misunderstanding about the origin of losses that our finite element models are very useful, as they enable us to know which phenomenon is at the origin of which loss, unlike impedance measurements, which only provide a global magnitude of losses. In the following figure, the increase in resistance with frequency is clearly visible, and there is a strong agreement between the measured values and the circuit simulations of the layer-level discretization model.

At low frequencies, there are only copper losses and a resistance whose value matches that of the DC resistance. At higher frequencies, numerical simulations slightly overestimate the equivalent resistance of copper losses.

Lastly, we once again displayed the amplitude bode diagram of the component impedance. On this graph, we notice a shift in the first resonance frequency between the measurement and our model per layer. An explanation can be found in the fact that ferrites also exhibit dielectric behavior in addition to their ferromagnetic behavior. Indeed, the relative permittivity of ferrites reaches values of the order of 10 to 100 depending on the frequency [10, 35, 60].

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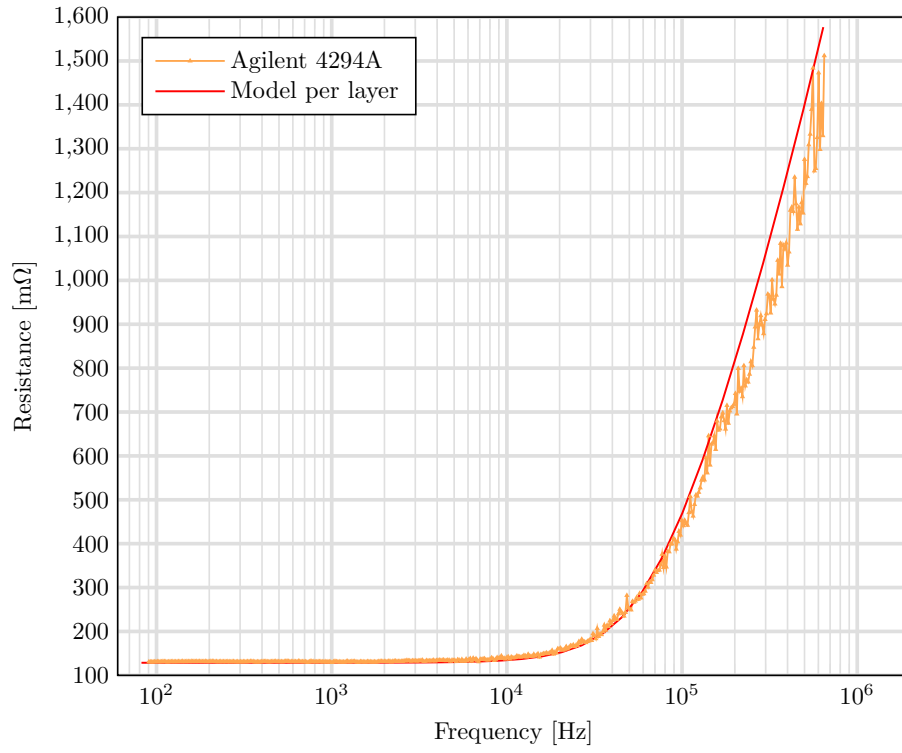


Figure 4.26: Resistance dependency on the frequency for the half core multi-layer inductor.

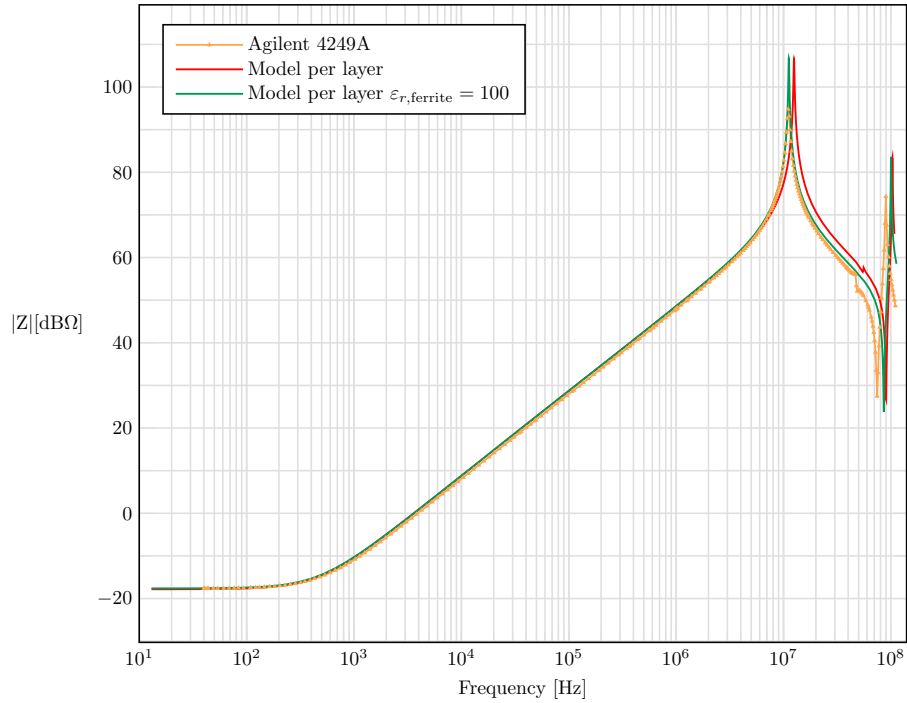


Figure 4.27: Comparison of the measured impedance from the half core setup with the discretization level per layer. The second model includes a high relative permittivity for the ferrite core.

4.5 Multi-Layer Inductor with Full Ferrite Core

Having investigated the multi-layer inductor in air and with the half-core, it is only natural to focus on another configuration in which the magnetic circuit is closed with a second core. This is a more realistic setup of an inductance consisting of a winding and a closed magnetic circuit. The component can be seen from the outside in the following figure just below, the copper winding being completely surrounded by the ferrite cores.

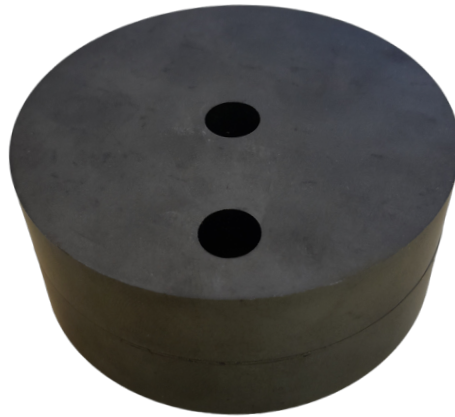


Figure 4.28: Physical device of the multi-layer inductor associated with full core configuration.

This component therefore concentrates all the aspects that need to be evaluated when modeling more sophisticated components such as transformers: copper losses, ferromagnetic losses, capacitive couplings and dielectric losses. This component features a conductor arranged on several layers the coil former shown in Figure 4.14c and two P70 half-cores. With this configuration, all elements have near-ideal symmetry of revolution, enabling 2D finite element modeling without further geometric approximation.

The two previous validation cases involve all the components in this case (coil former, conductor, core), and each enabled us to validate low-frequency behavior (inductance variation, copper losses), high-frequency behavior (capacitive coupling, dielectric losses) and to take into account a ferromagnetic core with high permeability.

The only aspect that remains to be taken into account, and not the least, concerns ferromagnetic losses. In the case of half-core validation, copper losses on their own were sufficient to explain the increase in high-frequency resistance. This is justified by the low level of magnetization of the core, given a magnetic circuit with very little flux channeling. In fact, without the second core, the induction flux disperses into the air around the winding, and the low current levels are not sufficient to strongly magnetize the half-core present. As a result, there are ferromagnetic losses that we were able to neglect without concern in the previous case. In the case of validation with the complete core, the losses within the ferrites are no longer negligible, so we will take the opportunity to evaluate the performance of our approach.

The difficulty in modeling iron losses lies firstly in their non-linear behavior, influenced both by the amplitude of the induction and the value of the applied frequency, but also in their strong dependence on external conditions such as temperature. This last point has been deliberately omitted in order to focus on the losses estimation at a given temperature.

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With the magnetic characterizations we have carried out on various materials (cf. section 4.1.1 and Appendix B), we can establish the relationship between magnetic induction and the real part of the permeability. We did not consider a frequency dependence for the real part of relative permeability for the sake of simplicity on the one hand, but mainly because the frequency dependence was weak over the measured frequency range (10, 20, 30 and 40 kHz), as can be seen in Figure 4.4b. We therefore preferred to extract the average value for these frequencies and then look for the relationship $\mu'_r(\hat{B})$ which links the real part of the relative permeability to the maximum induction obtained for a given harmonic regime.

In Figure 4.29, we have plotted the values of permeability calculated on the basis of the hysteresis cycles measured for the N22 core, together with the second-order fit deduced from these measurement points. We can see that permeability varies from 3000 to 3450 when induction varies from 0 to 200 mT in the harmonic regime.

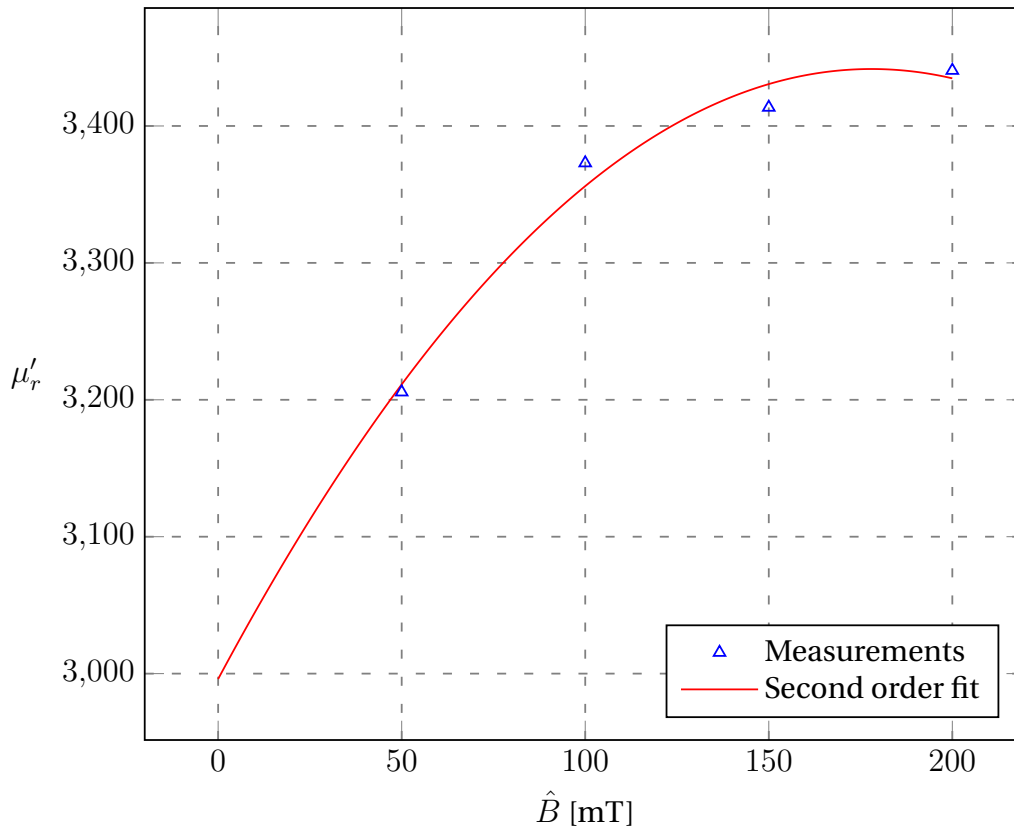


Figure 4.29: Dependence of the real part of the relative permeability of the N22 material with respect to the magnetic induction magnitude.

With the choice of a polynomial fit of order 2 over the studied induction range, we can therefore establish that

$$\mu'_r = F + G \hat{B} + H \hat{B}^2, \quad (4.51)$$

with $F = 2996$, $G = 5.00665$ and $H = -0.014065$. As a consequence, we now have a relationship allowing to relate the permeability to the maximum induction $\hat{B}$ with the prior assumptions of a harmonic regime, a known frequency interval considered and the linear dependence between the magnetic induction b and the magnetic field h .

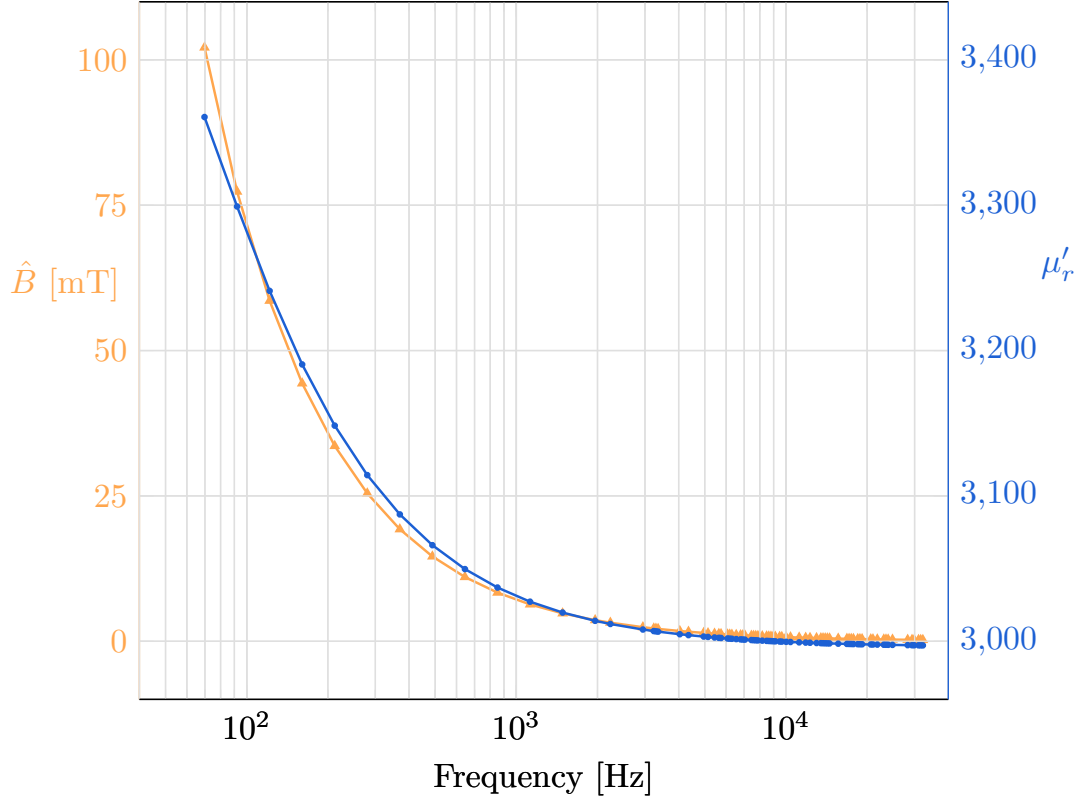


Figure 4.30: Magnetic induction and relative permeability evolution with frequency, considering the measured impedance of the Full-Core validation case. The voltage stimulation is constant over frequency, resulting in a magnetic field that decreases as a function of frequency, and a decreasing permeability measured.

Knowing this relationship, as well as the current imposed by the Agilent 4294A impedance analyzer, we can now estimate the value of induction in the core for the various operating points covered by the analyzer. This information is shown in the following figure, where permeability decreases at the same rate as induction as frequency increases. Beyond 5 kHz, the induction tends rapidly towards 0 and the permeability stagnates at around 3000 for this and all higher frequencies.

4.5.1 Inductance Analysis

If we display the inductance calculated on the basis of the relative permeability estimates, we obtain the following figure, in which we notice a few differences. First of all, we can clearly

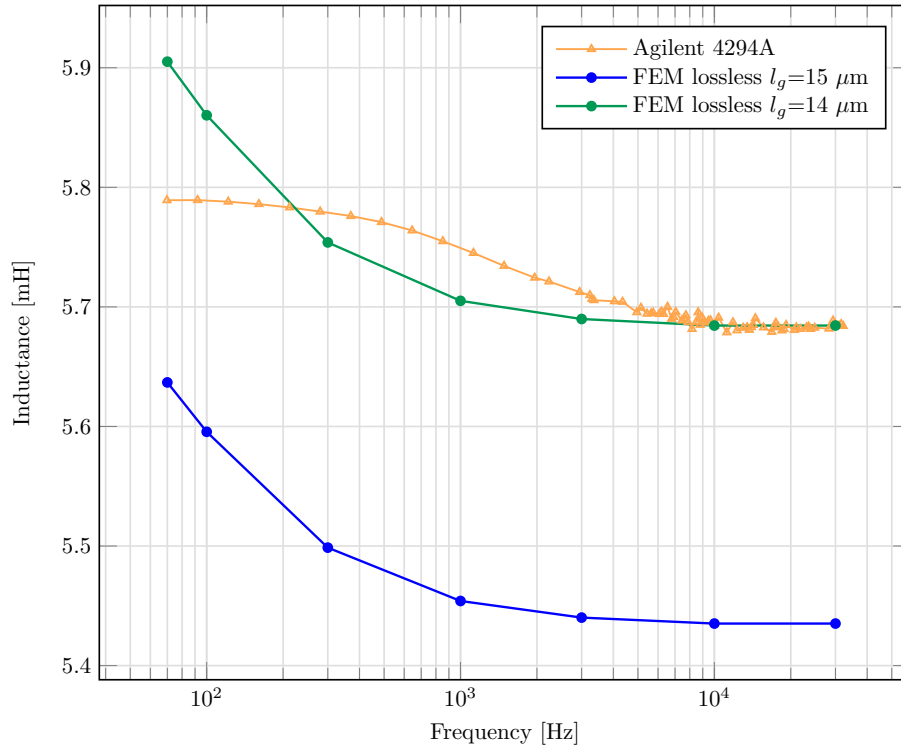


Figure 4.31: Inductance evolution of the full core validation case (14 μm and 15 μm air gaps).

see that the decrease in inductance at low frequency is linked to the drop in relative permeability, although the calculated drop does not exactly follow the measured drop. The drop in the inductance is not related to the skin effect (see resistance from Figure 4.32, the drop is well dependent on the applied field)³. Secondly, the inductance value stabilizes from 10 kHz onwards, in both measurements and numerical simulations. This corresponds to the frequencies at which induction becomes insignificant and, consequently, relative permeability stagnates around the value of 3000. Finally, we modified the value of the air gap used for the geometry of our finite element model. We observe that a small variation of 1 μm has a strong influence on the result. This is no surprise, as it is well known that magnetic circuits with a small air gap exhibit a strong dependence relative to this air gap length. It should just be noted that the estimates are closer to the measurements for an air gap of 14 μm than 15 μm . This high sensitivity to air gap indicates that a larger, fixed air gap length would have been preferable to validate this model. Nevertheless, comparisons between measurements and numerical simulations are still usable, given that the air gap is 14 μm . In practice, this air gap should have been zero, but this remains complicated to implement. The air gap is chosen to be matched, as we were unable to measure it within acceptable range.

³This means that there is a limit to the measured induction value greater than that estimated, perhaps due to the low-frequency series resistance; this could not be verified.

4.5.2 Losses Analysis

The computed and measured inductance were observed to be in reasonable agreement to a certain extent. The same question also arises for the losses. We first decide to investigate the influence of ferromagnetic material on total losses. To do this, we compared the equivalent resistance of the model for different imaginary parts of the relative permeability from the core. The results are shown in Figure 4.32 and cover the frequency range well below the first resonance (from DC to 100 kHz) as well as the range of imaginary parts of the relative permeability varying from 0 to 45. We immediately notice that the measured equiv-

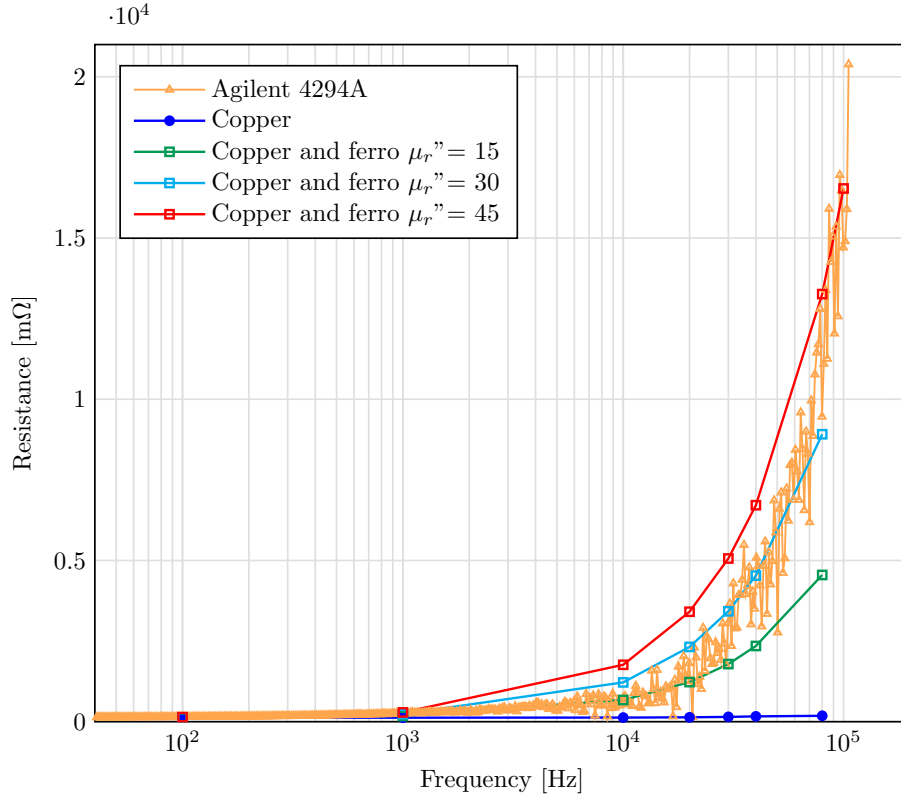


Figure 4.32: Frequency dependency of the equivalent resistance for the Full-Core test case, considering different imaginary parts for the relative permeability and comparing the simulation to the resistance measured with the Agilent 4294A impedance analyzer.

alent resistance does not correspond to a single numerically simulated permeability value, but to several values that follow one another with frequency. This observation stems directly from the non-linear frequency dependence of ferromagnetic losses (Steinmetz equation). However, the frequency dependence of the imaginary part of permeability (μ_r'') is difficult to establish. Indeed, the more the frequency increases, the more the magnetic induction b decreases and the more the imaginary part of permeability should decrease, due to the impedance analyzer behavior. As an example, frequencies above 10 kHz lead to the induction being very close to zero and the ferromagnetic losses should also decrease accordingly. Nevertheless, the increase in frequency also leads to higher losses and the exact trend cannot be determined with sufficient certainty. Unlike the real part of the permeability, it is therefore necessary to update the value of the imaginary part as a function of frequency and the measurements that we realized proved it to be difficult to preliminarily assess the imaginary part of relative permeability.

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In Figure 4.32, we can clearly see that the imaginary part varies from approximately 0 to 45 over the interval from 0 to 100 kHz, as a rough approximation. Knowing that the first resonance frequency occurs at around 700 kHz and that we want to obtain the correct amplitude for this frequency, we have therefore chosen to set the imaginary part of the permeability to 100 in the simulations (regardless of the frequency) in order to identify the equivalent circuit elements and verify whether the model can approximate the losses at the first resonance frequency.

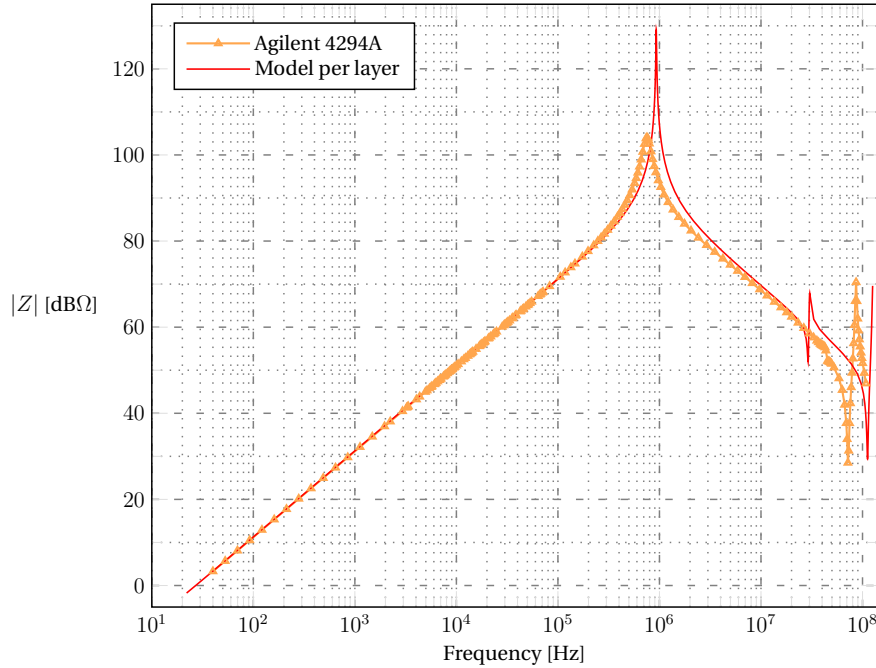


Figure 4.33: Comparison of the measured impedance from the full core setup with the discretization level per layer.

This was the basis for the comparison shown in Figure 4.33. However, we immediately observe that the ferromagnetic losses at resonance frequency have been underestimated by our model, although the dielectric losses linked to higher frequencies have been well estimated in view of the next resonances around 100 MHz. The results of this test show that the inclusion of ferromagnetic losses in the model requires prior verification of the material characteristics according to external conditions: frequency, temperature, magnetic induction field. Another approach consists of modifying the equivalent circuit model to include the effect of material non-linearities via other localized constants and the use of a transformation via a gyrator [101], [100]. The guarantee of being able to use the same model in both the time and frequency domains disappears at the same time.

4.5.3 Current Distribution Comparison of the 3 Cases

For the half-core configuration, we noted that the equivalent resistance was higher than for the coreless configuration, even though the model did not incorporate any ferromagnetic losses. It is time to explain, now that the three validation cases have been covered: coreless, half core and full core. We compare these three situations on the copper loss plane only. This is why the current density of each respective case is shown in Figure 4.34, for a frequency of 100 kHz and the winding-level discretization. The latter shows different current densities between the three cases. In the first case, without a core, the current is distributed around

the periphery of the area containing the conductors. In the second case, with a half core, we notice a clear increase in current density in certain areas at the bottom of the winding. In the latter case, with the full core, the current density only increases in the area close to the air gap, towards the middle of the winding height and towards the center of the device.

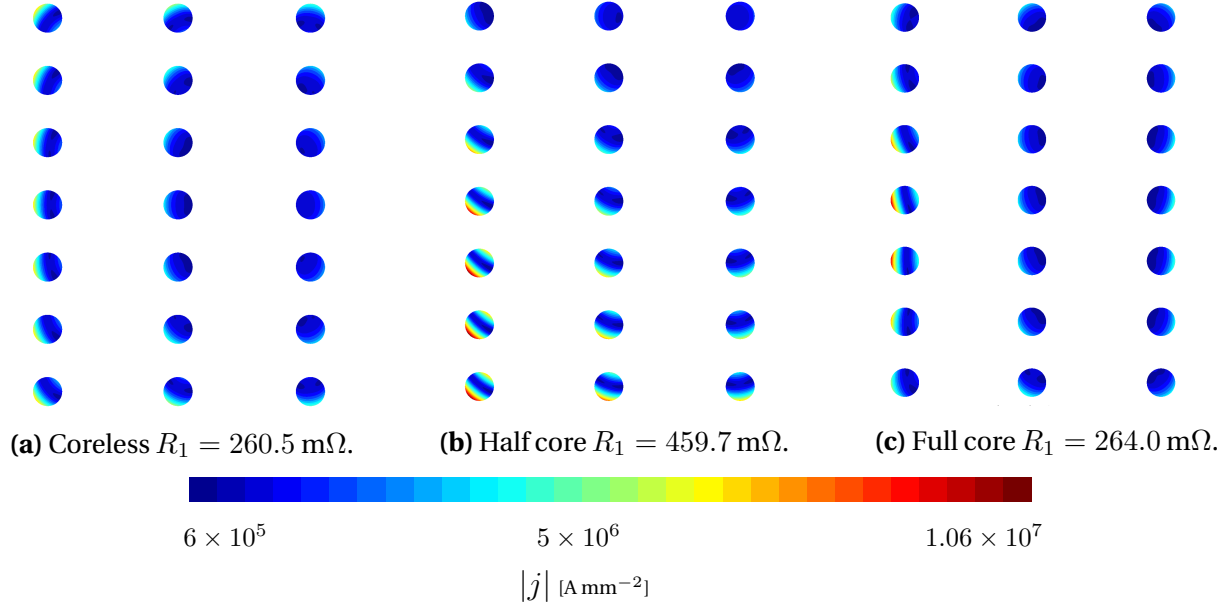


Figure 4.34: Current densities at 100 kHz for the three different test cases considering no ferromagnetic losses ($\mu'' = 0$).

In the case of the half-core, the equivalent resistance of copper losses is much higher than in the cases with and without the core, by almost twice the value. This is due to the very high current density in the conductors in the lower half of the winding. This distribution comes from the magnetic flux lines that cross these conductors, as shown in Figure 4.35. In

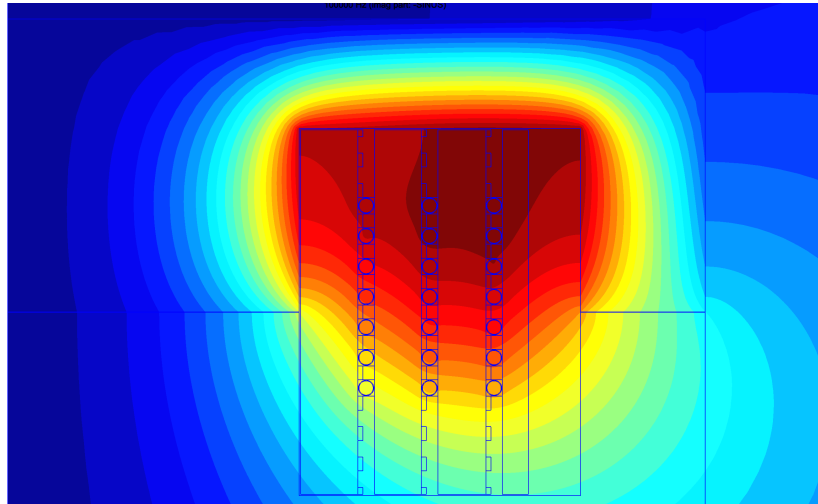


Figure 4.35: Magnetic vector potential distribution in the half-core configuration. Each zone surrounded by two lines corresponds to a constant magnetic flux tube.

this figure, we can clearly see that the magnetic induction flux crosses the cross-section of the bottom conductors, inducing currents. It is this mechanism of induced currents that is

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responsible for the additional losses of the half core case compared to the case without core. The same explanation applies to the case with the complete core, where flux lines extending from the air gap cross conductors and impose induced currents on them, creating additional losses, but to a lesser extent.

4.6 Single-layer Air Inductor with Rotational Symmetry

After having reviewed the three variants of the multi-layer inductor to compare the different parts of the model (inductance value, copper losses, iron losses, capacitive couplings and dielectric losses) with the measurements carried out on the samples, we wanted to study a final validation case with a single-layer inductor. Indeed, capacitive couplings are stronger between successive layers, since the potential difference between adjacent conductors of different layers is greater than that between two conductors within the same layer. For this reason, we wanted to test our method on a single-layer winding, in order to verify the validity of our approach for weaker capacitive couplings.

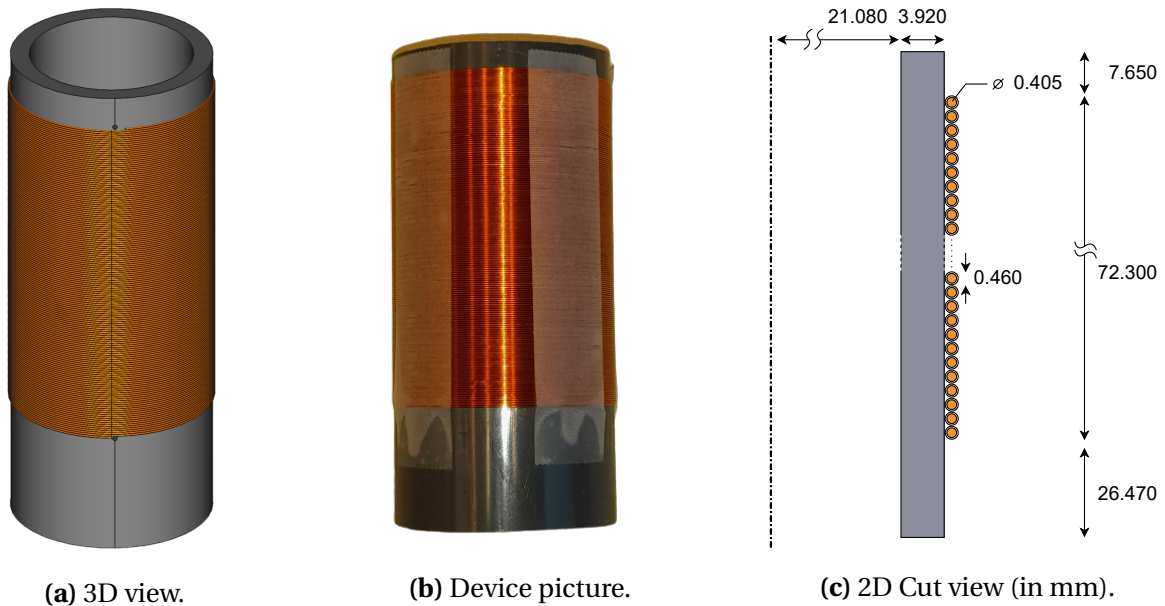


Figure 4.36: Single-layer air inductor with rotational symmetry.

Information on the device geometry is shown in the figure above. The winding consists of 156 turns of enameled AWG26 copper wire. The access length is 8 cm, which corresponds to the length of half a turn of the winding. This is very small compared with the total length of the winding, and the resistance of the accesses is therefore 10.7 m Ω . The total winding wire length is 24.73 m. The size of the accesses is therefore minimal compared to the total length.

Table 4.9: Comparison of DC resistance estimations and measurements for the axisymmetric single-layer air inductor.

MetraHit (DC)	Agilent (40 Hz)	Bode 100 (20 Hz)	Analytical (DC)	FEM (1 kHz)
3386 m Ω	3403 m Ω	3334 m Ω	3227 m Ω	3252 m Ω
reference	0.5 %	-1.54 %	-4.7 %	-3.9 %

Table 4.9 above shows the equivalent resistance values at very low frequencies for various measuring instruments (ohmmeter, impedance analyzer) and calculation methods (analytical, FEM). We note that the dispersion of values is very low for this device, mainly because the geometry is very simple and few approximations are involved.

In this case, we start by comparing the equivalent resistance of the device, measured with an impedance analyzer, with the value calculated by finite elements. As in the three previous cases, we observe very reliable agreement between measured and estimated resistance, up to a frequency of 1 MHz. Above this frequency, measured losses increase significantly faster than copper losses alone. If we add the dielectric losses to the copper losses, we can see that we get closer to the measured resistance for frequencies above 1 MHz. Nevertheless, the value obtained by simulation always underestimates the measured value. This is a slight difference from previous cases. Knowing that couplings between consecutive turns are more preponderant in this case, we may wonder whether additional losses in the wire enamel may not be the source of this underestimation of total losses. In any case, dielectric

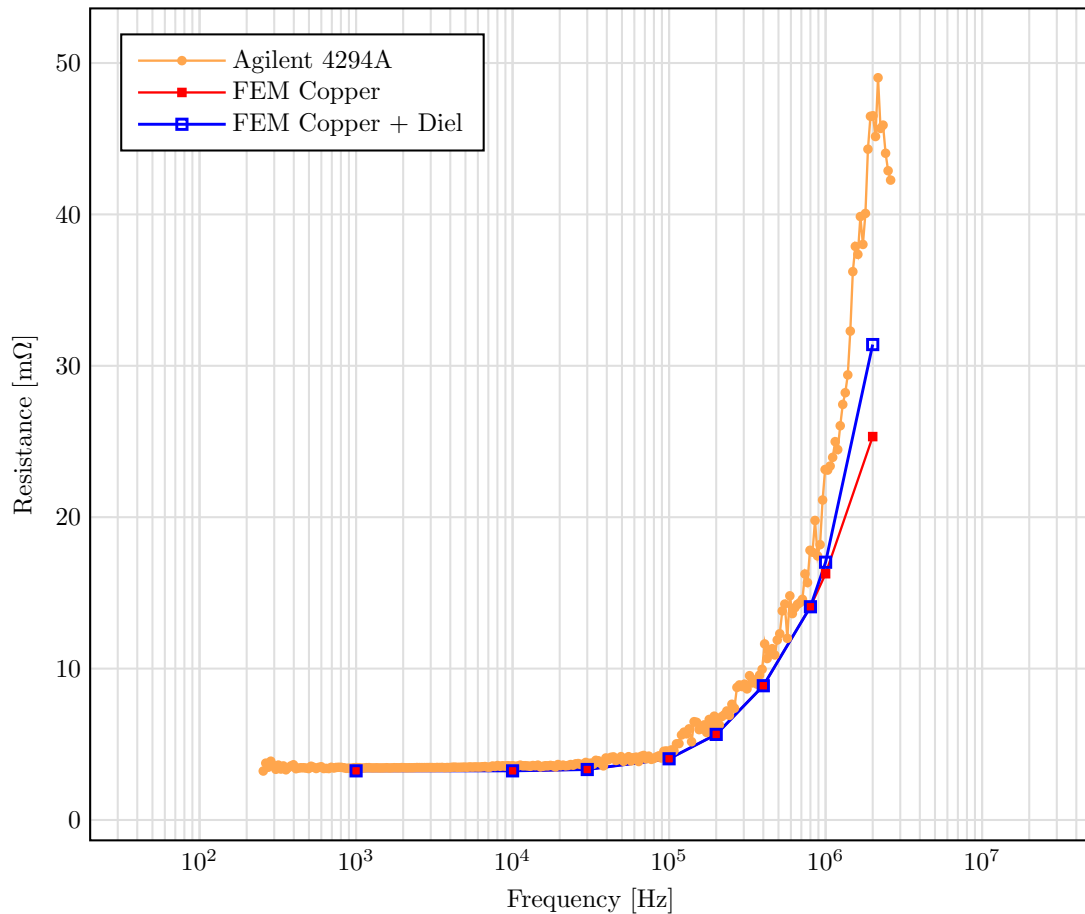


Figure 4.37: Single-layer air inductor resistance evolution over frequency.

losses are lower in this validation case than in the previous ones, since no interlayer coupling is involved. Finally, although the deviation appears small on the graph, the addition of dielectric losses reduces the approximation error from 84 % to 46 % at the 2-MHz frequency, which is not negligible in itself.

Having confirmed what we had already obtained for the previous validations, we can now take advantage of this case to study other aspects of modeling. For example, our simulation approach makes it possible to perform a sensitivity analysis on certain model parameters to

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determine their importance on the overall behavior of the studied device. Here, we decided to study the dependence of the capacitances seen on the winding accesses on the relative permittivity values of the two dielectric materials present: the wire enamel and the PVC coil former. These capacitances can be decomposed into three equivalent circuit values: $C_{1+,ref}$, $C_{1-,ref}$, $C_{1+,1-}$, and the value of the equivalent capacitance seen from the accesses when their electric potentials are left floating, C_1 such that

$$C_1 = C_{1+,1-} + \frac{C_{1+,ref} \cdot C_{1-,ref}}{C_{1+,ref} + C_{1-,ref}} \quad (4.52)$$

To do this, we sweep the ε_r values ranging from 1 to 5 for each material separately. The curves from this sensitivity analysis are shown in the following figure.

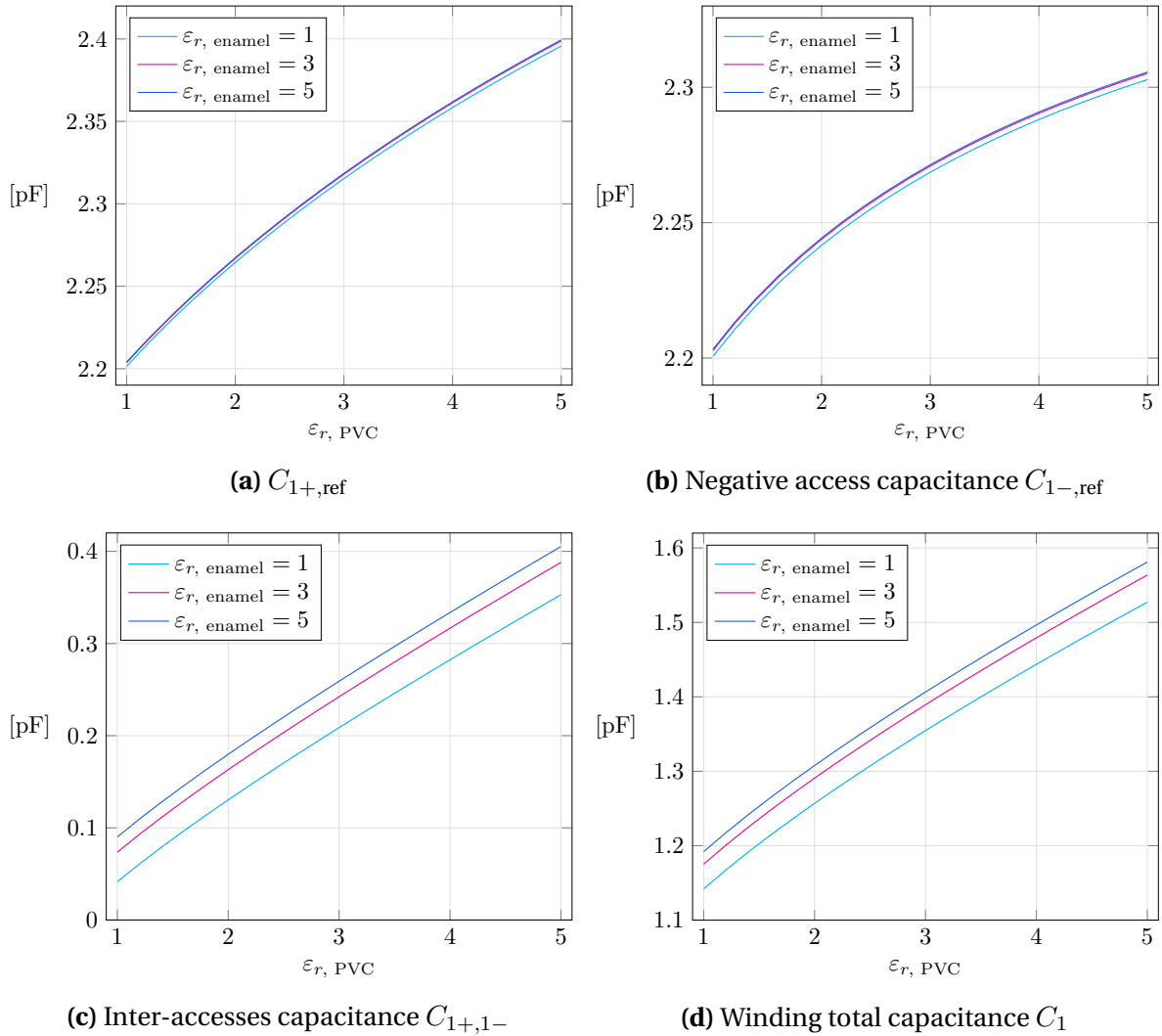


Figure 4.38: Capacitances dependency to the relative permittivity of the PVC and the enamel.

In this figure, we can see that for very different permittivity values of enamel, the behavior remains almost unchanged in terms of equivalent capacitance. The most striking point corresponds to the capacitance $C_{1+,1-}$, where the difference in value is around 50 fF between the relative permittivity of enamel ranging from 1 to 5. This is still low, but the greater dependence can be explained by the fact that for the $C_{1+,1-}$ capacitance, the electric field

that comes into play is that which is distributed between successive turns of the winding. Whereas for the two capacitances of the two accesses ($C_{1+,ref}$ and $C_{1-,ref}$), the electric field rather passes through the coil former and the characteristics of the wire enamel come into play only very minimally. Finally, the total capacitance value depends much more on the permittivity of PVC than on that of wire enamel. Indeed, for the same variation in relative permittivity (from 1 to 5), the variation in capacity is 4.3 % for enamel permittivity change and 31.9 % for PVC change. We can see, therefore, that it was entirely justified to study the characteristics of PVC on the basis of the samples, rather than those of enamel. All the more so as reliable measurement conditions for enamel are more difficult to gather.

The single-layer air inductor lends itself particularly well to the study of the different discretization levels introduced in Chapter 3. This is due to the number of turns (156), which can be easily divided into integer multiples, facilitating comparison between discretization levels. Figure 4.39 shows the amplitude and phase of the measured impedance and simulated ones obtained for different discretizations: 4, 12 and 39 sections. Comparison with measurements shows that the greater the number of sections, the better the high-frequency resonances are obtained. Indeed, on the one hand, a model with few sections predicts only a limited number of resonances. This is the case with the 4-section model, whose usefulness diminishes above 20 MHz. On the other hand, by increasing the number of sections to 12, we find more resonances, but with approximate spacing in the frequency domain. Finally, a high number of sections, in this case 39 sections of 4 conductors, made it possible to estimate resonant frequencies up to 50 MHz.

In all cases, the first resonance shows a large error with respect to the measurement, while the high-frequency behavior is well estimated and the geometry is simplistic. We deduced that a parasitic capacitive common-mode coupling had certainly disturbed our measurement and shifted the measured frequency lower. Such a parasitic coupling would not impact the higher frequencies behavior, mainly dictated by the turn-to-turn interactions.

Finally, we could have increased the number of sections even further, but this would not have provided any more information on the frequency range studied and would have led to a prohibitive resolution time for the circuit solver, as was shown for the 21-turn air inductor.

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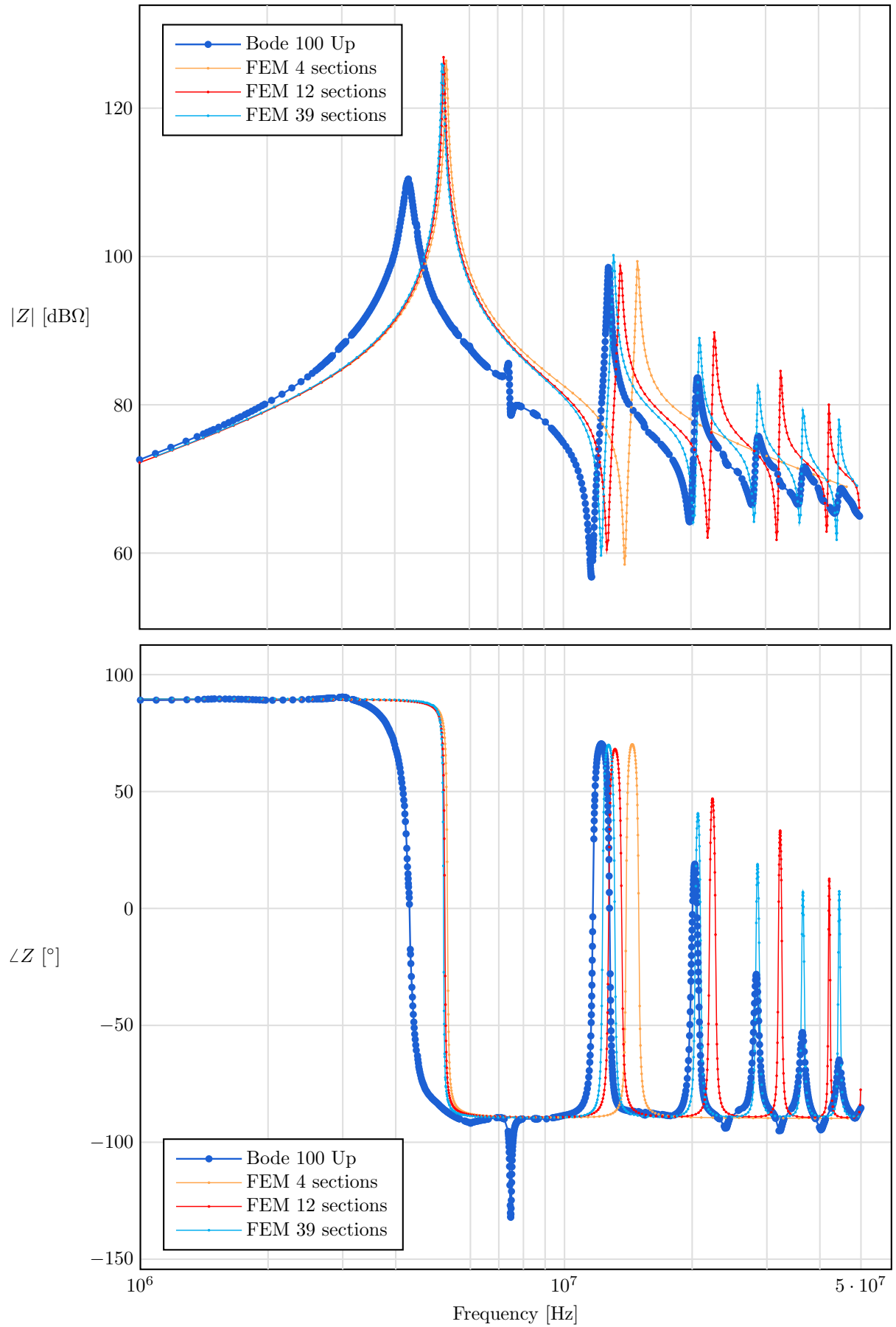


Figure 4.39: High frequencies resonances comparison between measurements and simulations of the 156-turn single-layer air inductor for different discretization levels.

4.7 Conclusion

To conclude this chapter, it is worth pointing out that to validate our discretization and equivalent circuit models, a major effort was first made to characterize lossy materials: ferromagnetic and dielectric materials. In the second part, four simple validation cases with rotational symmetry were used to compare measurements with simulation results. These comparisons highlighted the accurate simulation results for capacitive and inductive coupling, as well as for copper and dielectric losses. The most difficult point to validate remains ferromagnetic losses, for which the dependence on field amplitude still poses too many problems for comparison with impedance analyzers. To overcome this problem, simulated and measured values must be compared at a known field level. This is the subject of one of the case studies in the next chapter, for which the estimation of iron losses is critical.

We were thus able to implement our discretization method, test its flexibility and highlight its limitations, when, for example, the computation time of the circuit solver becomes too long for high discretization levels. All this leads us to want to test the simulations on less obvious cases taken from industrial power electronics applications, where the correct design of the magnetic component was of prime importance. This topic will be dealt with in the next chapter.

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Industrial Cases

In this final chapter, we take inspiration from real-life industrial applications to show that the equivalent circuit models, validated on simple inductances in the previous chapter, can be applied to practical cases of increasing complexity. For this purpose, two different high-frequency transformers have been selected for use in different converter topologies and ranges. Before anything else, at the beginning of this chapter, we recall the basics of a generic 2-winding transformer model to explain the frequency-dependent behavior of this component and to understand how the first resonant frequencies, due to capacitive/inductive coupling with magnetizing and leakage inductances, are estimated; the concrete case studies follow afterwards.

The first component studied is a two-winding transformer whose purpose is to step up the voltage level of the first stage of a high-voltage generator. To accomplish this, a large number of turns are wound in the secondary, in order to maximize the **transformer ratio**. In doing so, the resulting electrostatic couplings in the secondary are thus made stronger.

The last case is a two-winding transformer operated in a resonant converter topology. The interest of this case comes from the measurements that could be carried out on it. Indeed, for this case, it was possible to acquire the actual time signals from the converter down to the transformer. As a result, we were able to verify our model on time-domain signals, rather than just frequency-domain ones.

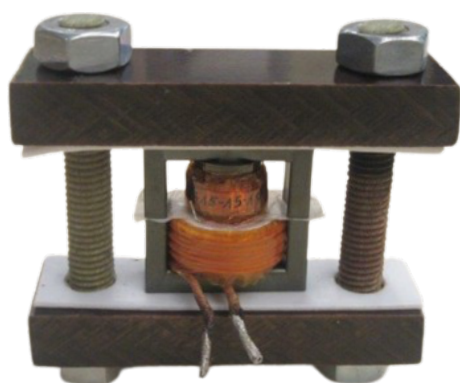
All these examples illustrate that the circuit models we have introduced are by nature extensible to complex topologies, are not limited to a certain number of windings and, above all, allow us to transpose the study from the **frequency domain** to the **time domain**. This last point is particularly important for Switch Mode Power Supply converters, where the signals are not sinusoidal and the interaction between switching components and filtering components (inductors and transformers) is often neglected as a first approximation.

In the previous chapter, we focused on understanding the phenomena that localized constant circuit models can capture¹. Here, we extend this study to other aspects that impact our models, such as geometries that differ by the presence of Litz wire or no-symmetric ferrite cores, or transformers with extended number of windings.

¹inductances, capacitances, copper losses, iron losses, dielectric losses, and their frequency dependence were modeled using ladder circuits, and showed close agreement between measurements and models.

5.1 High-Voltage Step-Up Transformer

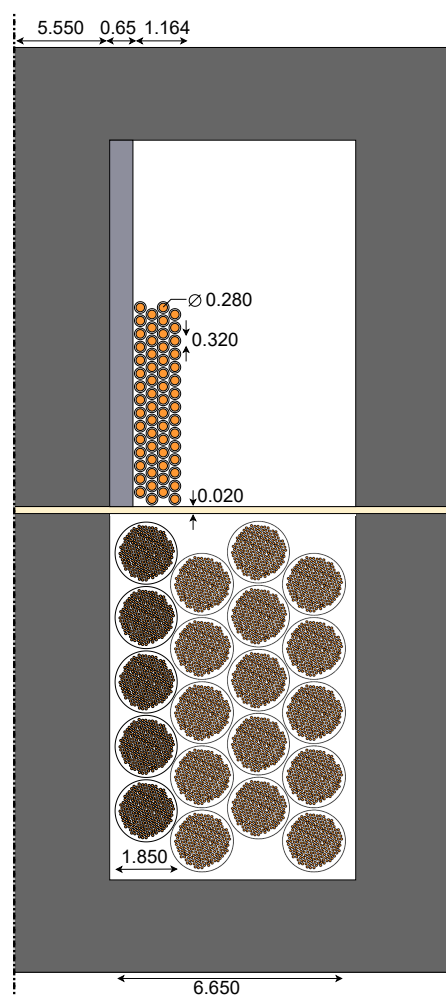
The first industrial case we will study involves a two-winding resonant transformer. This transformer can be found in a high-ratio step-up DC-DC converter, and is used to increase the converter's input voltage in order to supply the input of a voltage multiplier. The converter's operating principle requires a resonant circuit, and therefore the knowledge of the transformer's resonant frequency. The main design objective is to avoid having to add a resonant element to the transformer (external inductor or external capacitor) to obtain the desired frequency. The component shown in Figure 5.1a corresponds to a scaled down, simplified version of such a transformer encountered in practice for this application, modified as not to disclosed any confidential information, in particular, with respect to the actual winding, air gap and coil former. The figure here below also provides information on the geometry of this transformer, whose primary consists of 20 turns of Litz wire and secondary of 60 turns of round wire.



(a) Device picture.

Transformer data	
Core Material	3C97
Core Type	ETD 34/17/11
Primary type	Litz wire ($200 \times \varnothing 100 \mu\text{m}$)
Secondary type	Round wire ($280 \varnothing \mu\text{m}$)
N_1	20
N_2	60

(b) Transformer Data.



(c) 2D Cut view (in mm).

Figure 5.1: Step-up transformer with 20 turns at the primary and 60 turns at the secondary.

For the first time in the practical cases we encounter, Litz wire is used to make one of the windings. In this case, the primary winding, with its larger copper cross-section, generates more copper losses due to induced current, which must be mitigated by the use of Litz wire.

Such a usage is widespread in practice, and serves to validate our approach upon use cases. The choice of round wire or Litz wire has no impact on the identification method described in the previous chapters. In fact, the assumptions made do not impose any restriction on the wire or core geometry, provided that it can be represented in 2D.

Although homogenization techniques exist for copper losses of Litz wire [81, 24, 42, 83, 116], we have decided to model this geometry faithfully by using the number of Litz wire strands (200 for this wire) in the finite elements that we exploit. This choice results in a large mesh size, which greatly increases the resolution time of the finite element system. In the scope of the finite element models, we also assume that the wire is ideal and that the current is shared equally between all the strands of the same Litz wire. To do this, we impose that the current is the same, by means of additional circuit equations.

Furthermore, the geometry of the transformer does not exhibit rotational symmetry in all its component parts. Only the central leg of the ferromagnetic core and the windings exhibit rotational symmetry. The outer legs of the core have plane symmetry. To integrate them into an axi-symmetrical 2D model, we have adapted their geometry within the model so that the equivalent sections through which the magnetic flux travels have the same surface area as on the real core.

5.1.1 From the Simplified 3-Capacitance Model to Generic Model

We take advantage of this first transformer case to illustrate how a simple equivalent model (3 inductive and 3 capacitive parameters) is sufficient to represent certain magnetic component behaviors; and subsequently, how the proposed model allows to represent the same behaviors with a systematic identification of circuit elements, but also to go further regarding the amount of information (additional behaviors) that such detailed model and simulation can provide.

One of the benefits of the generic models explained in previous chapters is that they preserve the same consistency and generality regardless neither of the number of windings in the magnetic circuit nor the level of discretization chosen (turns, sections, windings).

A popular model used to represent a 2-winding transformer consists of a circuit comprising 3 inductive and 3 capacitive parameters [66]. Such a circuit can be made symmetrical with respect to the 2 accesses (primary and secondary) and is represented as the model shown in Figure 5.2.

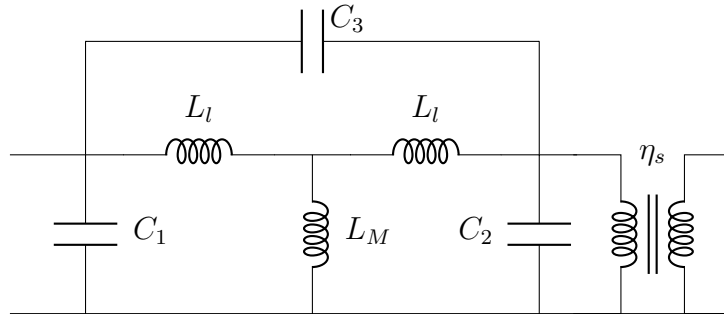


Figure 5.2: Two-winding transformer equivalent circuit, discretized at the winding-level, including parasitic capacitances referenced to the transformer primary and symmetrized with regard to its two accesses.

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The developments that lead to such a model can be found in Appendix A. The simplified 3-capacitance model is very useful for understanding how the different resonances of a 2-winding transformer appear. These resonance frequencies are obtained as follows:

$$f_1 = \frac{1}{2\pi\sqrt{L_o(C_1 + C_2)}} \quad (5.1)$$

$$f_2 = \frac{1}{2\pi\sqrt{L_{cc}(C_1 + C_3)}} \quad (5.2)$$

$$f_3 = \frac{1}{2\pi\sqrt{L_{cc}(C_2 + C_3)}} \quad (5.3)$$

$$f_4 = \frac{1}{2\pi\sqrt{L_{cc}\left(C_3 + \frac{C_1 C_2}{C_1 + C_2}\right)}}, \quad (5.4)$$

where L_o represents the inductance seen from the primary with the secondary in open circuit and L_{cc} represents the inductance seen from the primary with the secondary in short circuit.

Following the approach of K  radec [66], the capacitances C_1 , C_2 and C_3 can be identified on the basis of experimental measurements, provided that the connections between the different accesses are fixed, maintained (*i.e.* that a common voltage reference is chosen for both windings and that they are physically connected without galvanic isolation). This is a limitation in itself, since not all transformer connections can be represented by this model, including those with galvanic isolation between the primary and secondary windings.

The 3-capacitance model gives a meaningful insight of the frequency behavior of the 2-winding transformer without requiring too much complexity, and provides simple analytical expressions. However, given that this model is restricted by certain conditions (limited to 2 windings, dependent on primary/secondary wiring), that the calculation of primary, secondary and primary-secondary capacitances requires further analytical development on one hand, and on the other hand, that solving more complex circuits with numerous parasitic elements is not a problem as long as you are willing to solve them with a circuit simulator (*e.g.* LTspice) as proposed, then it makes much more sense to exploit the generic circuit obtained by applying the systematic identifications described in Chapter 3

In the case of a two-winding transformer with winding-level discretization, we obtain the circuit shown in Figure 5.3. First of all, we note that the magnetic part remains unchanged for this 2-winding transformer: the symmetrical circuit as described above [66] and the proposed controlled sources circuit are completely equivalent for the case of a 2-winding component, as far as the magnetic part is concerned.

In Figure 5.3, we can see that each circuit node has a partial capacitance towards the reference of potentials, still considered here as external to the transformer windings, and therefore without any prior assumption about the access (or accesses) physically connected to the ground (*i.e.* this voltage reference). Of course, to obtain the capacitances to the ground ($C_{W_{1+},0}$, $C_{W_{1-},0}$, $C_{W_{2+},0}$ and $C_{W_{2-},0}$) with the finite element method, assumptions had to be made about the region over which the reference of potentials is placed. For this work, we have considered the infinite boundary of the 2D domain, but other applications may require the inclusion of a conductive housing close to the transformer within the finite elements model, thus exacerbating the capacitive couplings of the device.

In addition to the capacitances to ground, we need to include the capacitances that link the nodes of the transformer in pairs. It includes the capacitances specific to each winding

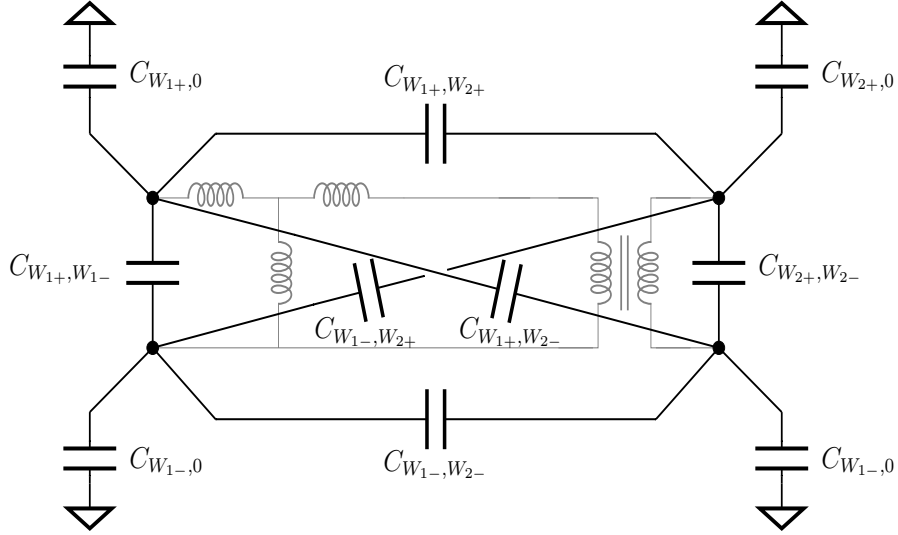


Figure 5.3: Winding-level representation of magnetic and electric couplings of the two-winding transformer model. The representation implements the generic approach for the capacitances and the symmetric magnetic model conserved from the 3-capacitance model.

($C_{W_{1+},W_{1-}}$, $C_{W_{2+},W_{2-}}$) and the capacitances that connect the windings together ($C_{W_{1+},W_{2+}}$, $C_{W_{1+},W_{2-}}$, $C_{W_{1-},W_{2+}}$ and $C_{W_{1-},W_{2-}}$).

On the basis of this two-winding transformer model, a comparison can be made with the simplified 3-capacitance model presented earlier. At least 4 resonant frequencies are expected to be comparable for both models. Knowing the limitations and results provided by the two different approaches, we can now review the measurements to verify the applicability of these models.

5.1.2 Resonant Frequencies Measurements

The transformer was characterized using the Agilent 4294A impedance analyzer, already used for the validations in the previous chapter. To do so, four standard measurements were carried out: a measurement at the primary with the secondary open (Primary 2O), a measurement at the secondary with the primary open (Secondary 1O), a measurement at the primary with the secondary in short-circuit (Primary 2S) and a measurement at the secondary with the primary in short-circuit (Secondary 1S).

In Figure 5.4, one can observe that the frequencies predicted by the 3-capacitance model appear clearly over the frequency range of the measurements and that f_1 , f_2 and f_4 are related to phenomenon commonly shared by the side from which the measurements were made (from primary or from secondary). The only exception comes from the third frequency, f_3 that differs depending on the side where the measurements were captured.

We will then denote f_i for frequencies measured from the primary and f'_i for frequencies measured from the secondary, for the sake of clarity of notation and comparison of measurements seen from the primary or secondary side.

The difference is slight, but stems from the fact that the 3-capacitance model over-simplifies the capacitive couplings between the two windings. A more detailed model is thus required to explain this difference. In Table 5.1, the various resonant frequency values are listed, along with their relative deviations. We notice that the frequencies f_1 , f_2 and f_4 deviate by less than 1 to 3 % from their secondary view counterparts, while the frequency f_3 deviates by more

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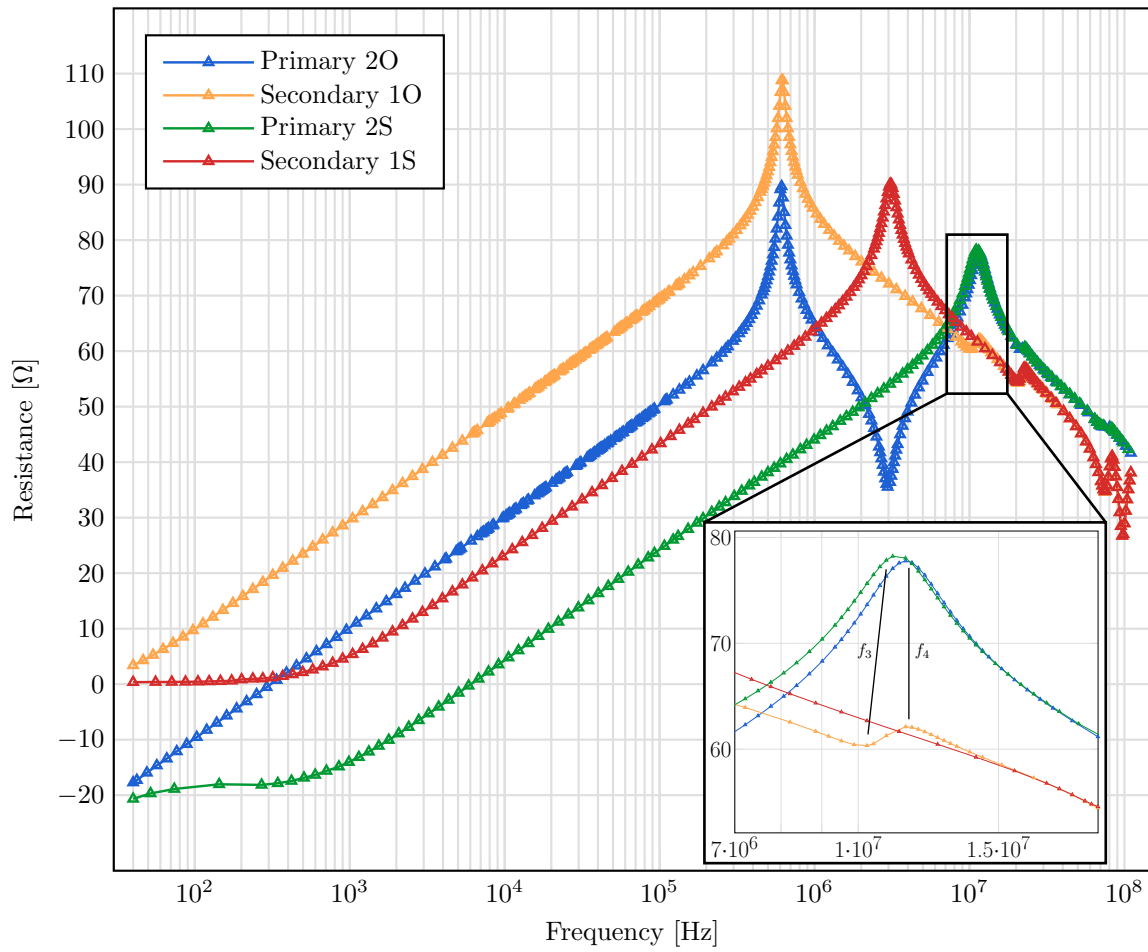


Figure 5.4: Reduced transformer impedance measurements at the primary and secondary under open circuit and short circuit conditions.

Table 5.1: Measured frequencies of the two-winding step-up transformer and relative difference computation of secondary relative to the primary.

Measured frequency [MHz]	f_1 0.6102	f_2 2.9902	f_3 11.1415	f_4 11.4181
	f'_1 0.6196	f'_2 3.0910	f'_3 10.2018	f'_4 11.1529
Relative difference [%]	1.54	3.37	-8.43	-2.32

than 8 %. One can argue that a difference of 8 % is not much, especially regarding values obtained by measurements. But compared to the other measurements errors that all lie within 1-3 %, it is not negligible.

On the other hand, if we wanted to identify the 3 capacitances of the reduced equivalent model [66], we could do so by means of equations (A.5), (A.6) and (A.7), by substituting frequencies with measured values. Looking from the primary point of view (with frequencies f_1, f_2, f_3), we would then obtain $C_1 = 126.07$ pF, $C_2 = 5.96$ pF and $C_3 = 3.37$ pF. On the other hand, from the secondary point of view (with frequencies f'_1, f'_2, f'_3), we would then obtain $C_1 = 119.15$ pF, $C_2 = 9.07$ pF and $C_3 = 2.05$ pF. The differences are not striking, but it does show that it is impossible to match the various resonances on the basis of the simplified 3-capacitance model.

The main reason for this is the simplified circuit diagram's assumption that both windings have a common potential reference. Yet, in this case, the winding being measured sees its reference imposed by the measuring device, while the other winding remains floating. This configuration is not compatible with the 3-capacitance model.

The 3-capacitance transformer model shown in Figure 5.2 has the great merit of simplicity. Nevertheless, it has major shortcomings when it comes to studying high-frequency behavior, compared with the controlled-source circuit (Figure 3.29). For example, on the magnetic part, the absence of coupled losses resistances (associated with induced current losses) and on the electrostatic part, the constraint of a common potential reference for both windings. The 3-capacitance transformer model does not allow us to represent all the phenomena studied as we would like, but it does provide a clearer representation to illustrate inductive-capacitive couplings, while neglecting the loss aspect. An excellent starting point for a two-winding transformer whose resonant frequencies we want to identify and understand. Furthermore, circuit simulation tools such as LTspice allow to expand the capacitive part of this model and solve it without adding any complexity to the method; only the number of identifiable parameters increases. We are therefore free to add as much capacitances to the model as we need, in other words, as much capacitance as the finite element identifications provide.

By doing so, the static parameters identified for this transformer make it possible to identify the different resonant frequencies of the component and to differentiate between the behavior of one access or the other. The inductive and capacitive parameters are respectively given in Tables 5.5 and 5.6 obtained in the next section. The frequencies associated with this circuit are shown in the following table. There are clear differences depending on the access

Table 5.2: Multiple resonant frequencies observed on the two-winding step-up transformer (' symbol referred to the quantities measured from the secondary).

	Measure [MHz]	Lossless model [MHz]	Relative Error [%]
f_1	0.6102	0.602	-1.34
f'_1	0.6196	0.600	-3.16
f_2	2.9902	2.914	-2.55
f'_2	3.0910	2.893	-6.41
f_3	11.1415	10.782	-3.23
f'_3	10.2018	10.589	3.80
f_4	11.4181	11.040	-3.31
f'_4	11.1529	10.891	-2.35

through which the test is performed. The magnitude of the relative errors between the loss-

less circuit model and the measurements remains high. However, this model already makes it possible to obtain resonance frequencies with relative precision (less than 5 %, with the exception of f_2') and, above all, to differentiate the result seen from one access or the other with a single model.

5.1.3 Model Complexity Gradation

In this part, we study the high-frequency transformer using 4 different models, each with a greater degree of detail, to illustrate the impact of modeling choices on frequency impedance curves. The 4 models chosen are as follows:

- 1) no ladder with DC copper losses only
- 2) RL circuit ladder for frequency-dependent copper losses
- 3) RL circuit ladder for copper and ferromagnetic losses as a function of frequency
- 4) RL circuit ladder (copper and ferro. losses) and GC circuit ladder (dielectric losses)

Based on the simplified 2D geometry, described in Figure 5.1c, we identify different inductive and capacitive parameters of the transformer. The previous part of the study of the two-winding transformer focused on comparing experimental results to numerical results (FEM) for the determination of resonant frequencies. Such part of the work highlighted the agreement between measurements and simulations, and made the link between the 3-capacitance circuit and the generic capacitive circuit. In addition, for this section, we detail the numerical model used for the use case and provide more details about the different results obtained according to the circuit elements used for each model (RLC, inductive ladder, capacitive ladder, copper losses alone or with ferromagnetic losses). To perform the numerical analysis, the geometry of the transformer was discretized using 1,543,000 elements. A portion of the geometric discretization can be seen in Figure 5.5, which shows the air-gap area. Also visible, the details of the secondary's round conductors and the primary's Litz wire conductors.

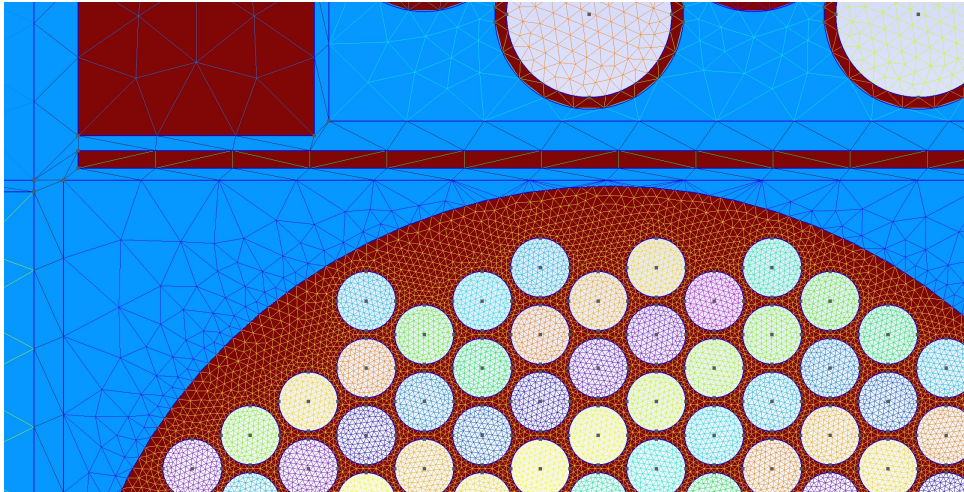


Figure 5.5: Detail of the mesh geometry around the air gap of the transformer. The total mesh includes 1,543,000 elements.

As aforementioned, the geometry of the Litz wire is faithfully represented with each strand defined separately in 2D. Note that, for the sake of simplicity, the strands follow a regular honeycomb arrangement. In terms of the equivalent circuit, the winding discretization has been adopted because of its acceptable level of simplicity for a 2-winding component.

Therefore, the system requires 3 inductive and 10 capacitive parameters for its representation on a winding-level discretization. Typical resolution times observed for such a mesh and with the winding discretization are summarized in Table 5.3. The total resolution time

Table 5.3: Computation times observed for the high-voltage step-up transformer, considering magnetic and electric problems.

Number of magnetic stimuli	[-]	2
Total magnetic identification time	[s]	43.5
Number of inductance matrix elements	[-]	3
Number of electric stimuli	[-]	4
Total electric identification time	[s]	88.2
Number of capacitance matrix elements	[-]	10

for the 4 electrostatic stimuli (one per node of the circuit), was 88 seconds and the total resolution time for the 2 magneto-dynamic stimuli (one per winding) was 43.5 seconds. These observations are independent of the assumptions on losses made for our 4 different models, and only the values of the materials characteristics and the values of the parameters derived from identifications are modified by the assumptions made on losses. It should be noted, however, that models incorporating ladders require such identifications to be carried out for multiple frequencies, thus multiplying the number of stimuli and the total time required for identifications. But the resolution time of each magnetic and electric problem still remains the same.

The subsequent sections describe the various models and compare them in terms of the impedance curves. The models are compared with each other, as well as with transformer measurements, highlighting the impact of each loss assumption on the curves obtained.

5.1.3.1 Frequency-Independent Parameters Including DC Copper Losses only

In a first step, we focus on the simplified model with no ladder, neglecting the parameters dependency with respect to the frequency and therefore no iron losses, no induced current losses and no dielectric losses are included.

The first objective is to observe the order of occurrence of resonance frequencies, to validate that the lossless model accounts for the same elements as the measurements and the simple model of [66]. As a result, we can verify that the expected phenomena (resonance between open-circuit inductance and capacitance, resonance between short-circuit inductance and capacitance, etc.) do indeed occur at the expected frequencies. We also expect resonances to be practically unattenuated, as only DC copper losses are included.

The physical characteristics of the ferrite and dielectric used in the numerical models are provided below in Table 5.4. Indeed, the frequency at which the identifications were applied is taken into account for determining the characteristics: 1 kHz for magnetic characteristics and 1 MHz for dielectric characteristics.

Table 5.4: Magnetic and electric material properties of ferrite 3C97, PVC and enamel.

$\mu_{r,3C97}$	$\epsilon_{r,PVC}$	$\epsilon_{r,enamel}$
2600	2.6	2.0

Under the assumption of frequency-independent parameters, we note that the resistances associated to induced current losses (R_{12} and R_{21}) are not part of the model and can

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therefore be neglected at such low frequency (they are of the order of $\mu\Omega$). Table 5.5 contains the values of inductive and resistive parameters identified at 1 kHz for the frequency-independent model.

Table 5.5: Resistances and inductances of the high-voltage step-up transformer identified with FEM at the winding-level for the low frequency behavior (1 kHz).

Resistances [m Ω]		Inductances [μ H]		
R_1	R_2	L_{11}	L_{12}	L_{22}
12.3	685.4	515.6	1508.1	4621.5

Capacitances are also identified to complete the model, but at a higher frequency: 1 MHz. Table 5.6 provide the 1 MHz capacitances identified at the winding-level.

Table 5.6: Capacitances of the high-voltage step-up transformer identified with FEM at the winding-level for the high frequency behavior (1 MHz).

Winding to ground capacitances [pF]			
$C_{W_{1+},0}$	$C_{W_{1-},0}$	$C_{W_{2+},0}$	$C_{W_{2-},0}$
0.29	1.07	0.16	0.16
Inter-winding capacitances [pF]			
$C_{W_{1+},W_{2+}}$	$C_{W_{1-},W_{2-}}$	$C_{W_{1+},W_{2-}}$	$C_{W_{1-},W_{2+}}$
1.33	3.72	2.15	1.16
Winding capacitances [pF]			
$C_{W_{1+},W_{1-}}$	$C_{W_{2+},W_{2-}}$		
8.52	131.78		

All the resistive, inductive and capacitive elements obtained with FEM identifications (included in Tables 5.5 and 5.6) can be substituted within the circuit presented in Figure 5.3. The curves obtained by simulating this circuit (*e.g.* with LTspice), are shown in Figures 5.7 and 5.8 for two different cases: impedance seen from the primary with secondary open and impedance seen from the secondary with primary open. These figures also display the results obtained with the more elaborate models described in next sections.

5.1.3.2 Ladder-Type RL Circuit Including Copper Losses Frequency Dependency

The second level of modeling we have chosen incorporates the frequency dependency of copper losses, but keeps capacitances constant and does not include iron losses in the model. As in the previous case, we have identified the transformer's various inductive and capacitive parameters, as well as the associated losses. Such component values of the localized constant circuit were identified at different frequencies, and the associated ladder circuits were subsequently derived. The resulting circuit therefore shows a dependency of its copper losses as a function of frequency by means of the inductive ladder circuit. The inductive ladders are optimized on the basis of identifications made at 1 kHz, 100 kHz and 1 MHz. Iron and dielectric losses are still not integrated in the model for now.

Same as for the preceding model, the curves obtained by simulating this circuit (*e.g.* with LTspice), are shown in Figures 5.7 and 5.8 for two different cases: impedance seen from the primary with secondary open and impedance seen from the secondary with primary open.

Table 5.7: Optimized ladder parameters of the high-voltage step-up transformer.

$L_{1,11}$	$r_{L,11}$	$R_{1,11}$	$r_{R,11}$	$R_{\text{cor},11}$	$R_{DC,11}$	$L_{hf,11}$
37.2688 μH	1.264	117.9029 Ω	1.1841	-22.4736 Ω	13.12 m Ω	479.4193 μH

$L_{1,12}$	$r_{L,12}$	$R_{1,12}$	$r_{R,12}$	$R_{\text{cor},12}$	$R_{DC,12}$	$L_{hf,12}$
22.3666 mH	49.2365	943.1784 Ω	0.2152	-741.7641 Ω	0.0 m Ω	1.4078 mH

$L_{1,22}$	$r_{L,22}$	$R_{1,22}$	$r_{R,22}$	$R_{\text{cor},22}$	$R_{DC,22}$	$L_{hf,22}$
2.5848 mH	43.6900	1910.4497 Ω	0.3669	-1231.8540 Ω	67.43 m Ω	4.2999 mH

Comparisons with simpler and more elaborated models are handled in the corresponding section.

5.1.3.3 Ladder-Type RL Circuit Including Copper and Ferromagnetic Losses Frequency Dependency

Once again, we use the previous model as a basis and complexity it a little more; this time, by integrating ferromagnetic losses into the new model. For this purpose, identifications are carried out in the same way as for the previous model, adding an imaginary part to the relative permeability of the ferrite. The various finite element simulations are therefore carried out with the following complex permeability for the ferrite core:

$$\mu_r = 2500 - j * 200 . \quad (5.5)$$

The topology of the equivalent circuit obtained is then identical to that of the previous model; only the parameter values are different, since they include ferromagnetic losses in addition to copper losses. The values of the inductive ladders parameters are listed in Table 5.8.

Table 5.8: Optimized ladder parameters of the high-voltage step-up transformer, including ferromagnetic losses.

$L_{1,11}$	$r_{L,11}$	$R_{1,11}$	$r_{R,11}$	$R_{\text{cor},11}$	$R_{DC,11}$	$L_{hf,11}$
3.917 μH	104.518	73.023 Ω	3.084	-1.701 Ω	76.661 m Ω	478.529 μH

$L_{1,12}$	$r_{L,12}$	$R_{1,12}$	$r_{R,12}$	$R_{\text{cor},12}$	$R_{DC,12}$	$L_{hf,12}$
3.731 mH	42.419	227.845 Ω	6.056	-1025.847 Ω	0.0 m Ω	1.409 mH

$L_{1,22}$	$r_{L,22}$	$R_{1,22}$	$r_{R,22}$	$R_{\text{cor},22}$	$R_{DC,22}$	$L_{hf,22}$
4.970 mH	2.727	2660.209 Ω	3.9868	-1981.358 Ω	1249 m Ω	4.296 mH

The curves obtained by integrating the losses associated with ferrite are also shown in Figures 5.7 and 5.8 for comparison with the other models, simpler and more complex ones.

5.1.3.4 Ladder-Type RL Circuit and Ladder-Type GC Circuit

Similarly, dielectric losses and their frequency dependency are obtained via a capacitive ladder circuit, as described in previous chapter.

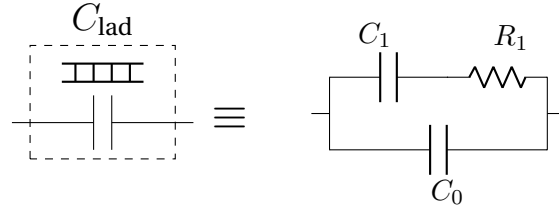


Figure 5.6: Resistive-capacitive ladder used for modeling the frequency dependence of dielectric losses.

Therefore, each capacitance is replaced by a ladder of resistive-capacitive type (denoted GC ladder) as presented in Figure 5.6 and detailed in Chapter 4. For this transformer, the GC ladders were optimized between the frequencies of 1 and 10 MHz. Measuring the relative permittivity of dielectric for frequencies higher than the 1 MHz has proven to be a challenge with the use of impedance analyzers and presence of parasitic resonances of the sample under test.

Table 5.9: Capacitive ladders of the high-voltage step-up transformer obtained for the 1-10 MHz range.

Winding capacitive ladders				
	W1+_W1-	W2+_W2-		
C_0 [pF]	8.407	13.024		
R_1 [M Ω]	0.215	0.163		
C_1 [pF]	0.234	0.308		

Inter-winding capacitive ladders				
	W1+_W2+	W1+_W2-	W1-_W2+	W1-_W2-
C_0 [pF]	1.317	2.130	0.115	0.371
R_1 [M Ω]	2.150	1.630	51.062	25.762
C_1 [pF]	0.025	0.033	$1.09 \cdot 10^{-3}$	$2.17 \cdot 10^{-3}$

Winding to ground capacitive ladders				
	W1+_0	W1-_0	W2+_0	W2-_0
C_0 [pF]	0.286	1.071	0.155	0.165
R_1 [M Ω]	879.5	560.1	43.9	-154.6
C_1 [pF]	$6.27 \cdot 10^{-5}$	$3.44 \cdot 10^{-5}$	$1.31 \cdot 10^{-3}$	$3.68 \cdot 10^{-4}$

As with previous models, the curves obtained by circuit resolution are shown in Figures 5.7 and 5.8. The various results obtained are described below, together with comparisons between the curves.

5.1.3.5 Comparison of Gradual Complexity Models

The four models presented above express how the complexity of the equivalent circuits we proposed is built up incrementally. The initially simple circuits, which can be connected to existing models, become more complex with the addition of elements such as inductive and capacitive ladders. After that, we illustrated that the inductive ladders can incorporate different loss components (copper, ferromagnetic) depending on the materials properties

integration. Finally, the capacitive ladder, presented last, allows to add additional loss elements such as dielectric losses whose impact is significant for the highest frequencies. The different models are compared in Figures 5.7 and 5.8, on the basis of the impedance measurement.

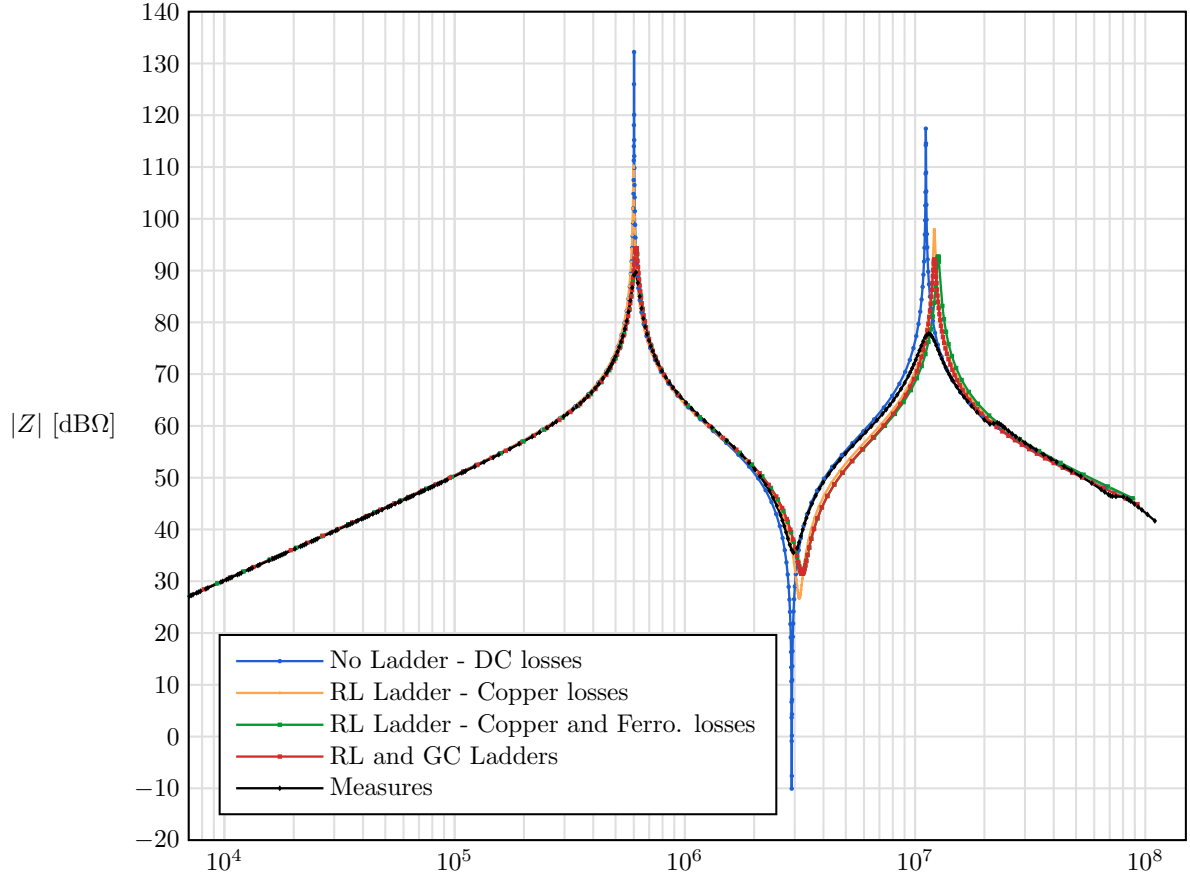


Figure 5.7: Impedance observed at the primary with the secondary left open.

The Figure 5.7 displays the observation made from the primary with the secondary open. First, we notice that the non-laddered circuit resonances are very high in term of amplitude, notably 132 dBΩ for the first resonance and -10 dBΩ for the third, compared to 90 dBΩ and 35 dBΩ obtained from the measurements. It is obviously due to the fact that the only damping elements included in this circuit model are the DC resistances (Table 5.5) .

In the second model, ladder circuits are added but only for the induced current losses. Neither ferromagnetic nor dielectric losses are not taken into account in this model. Comparing the circuit with the non-laddered one, we notice two main points. First, the obvious one, is that resonance amplitudes are more attenuated. In fact, the loss resistances increase with frequency (a phenomenon represented by the ladder) and dampen the resonances all the more, thereby reducing the amplitude of each resonance. For example, the first resonance only peaks at 110 dBΩ. The second observation, which is not particularly striking, comes from the resonance **frequencies** 2 and 3, which are **shifted** in comparison with the model without ladder. These resonances depend on the **leakage inductance** value (primary or secondary) and the leakage inductance is tainted by an error for models with ladder. Indeed, the leakage inductance is obtained from the difference between two inductances, represented here via ladders, which are only approximations of the behavior identified at each given frequency, thus exhibiting an erroneous value of leakage inductance and, conse-

quently, of the associated resonance.

In general, higher-frequency resonances are less attenuated in the simulations. This is because the inductive ladder circuit is optimized from DC up to the 1 MHz frequency and no longer plays its role above that. At higher frequencies, dielectric losses gain in importance, hence the interest in including them in a fourth model by means of a capacitive ladder. In so doing, we can see that the model behavior change very little, but that resonances above 1 MHz are more attenuated due to the integration of dielectric losses. As this effect is not very pronounced, it is very likely that the loss characteristics of dielectric materials have been underestimated in this work. To the material characterization efforts that have been made, we should add loss studies at higher frequencies, as we have not been able to characterize dielectrics beyond 1 MHz with a simple impedance analyzer.

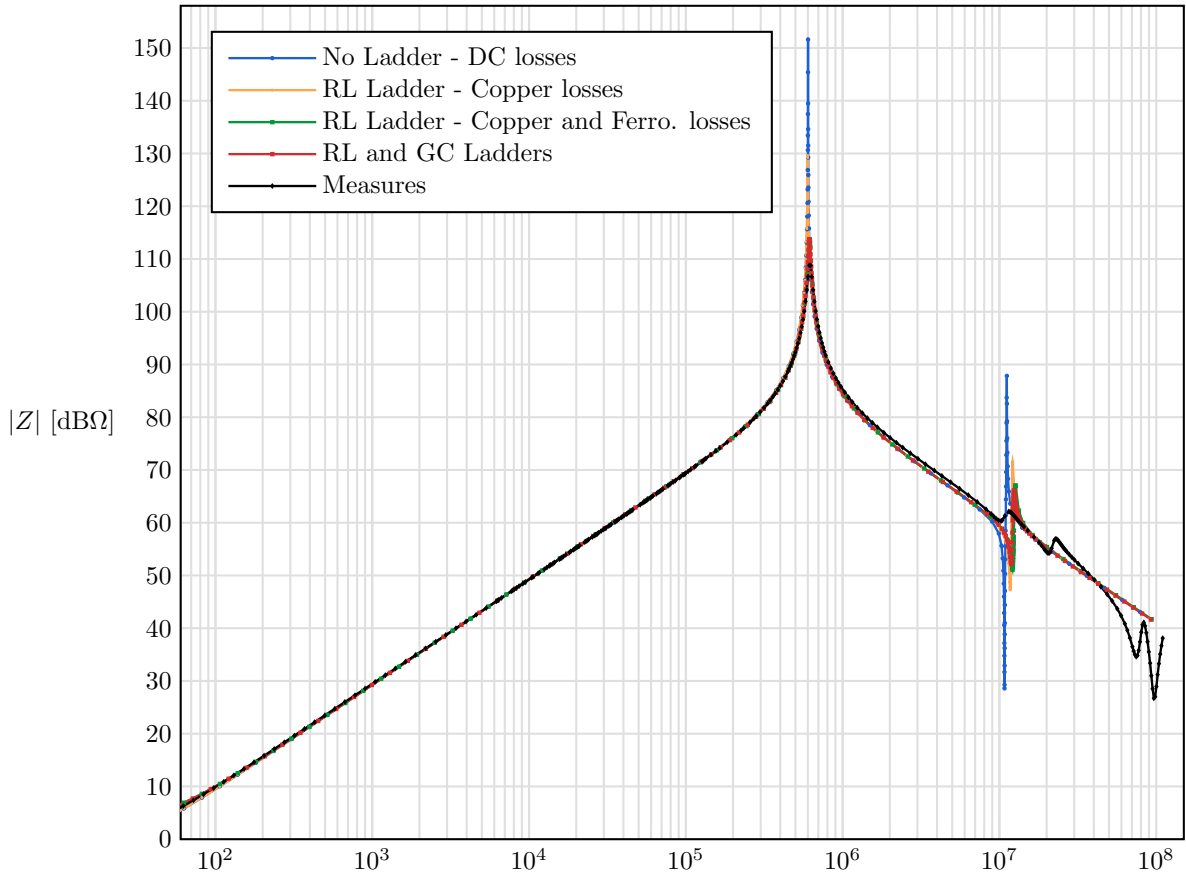


Figure 5.8: Impedance observed at the secondary with the primary left open.

On Figure 5.8, the observations are largely identical to what was observed on the primary. In other words, the more extensive the models, the more damped the resonances and the more closely they match the measurements. This is especially noticeable for the first resonance, which rises progressively from 150 dBΩ (for the DC copper loss model), to 130 dBΩ (for the copper loss ladder model only), to 114 dBΩ (for the ferromagnetic loss ladder model and the dielectric loss model), while the measured amplitude is 109 dBΩ. Note that dielectric losses are not very significant at this frequency of 610 kHz, which explains why the models with and without dielectric losses provide a similar amplitude. Similar to the measurements made on the primary side, we also note the frequency shift of resonances 2 and 3 for all models incorporating ladder circuits. On Figure 5.8, for the measurement curve, we can see more clearly the successive resonances above 10 MHz, which, by choice, are not included in our

models, since the assumption of winding-level discretization was made at the outset. Such resonances are highly damped by loss phenomena, and before exploiting a more finely discretized model, it would be wise to correctly represent losses at these high frequencies, then integrate them into a finer model.

5.2 Transformer of Resonant LLC Converter

The last case of application we study in this work is a resonant converter transformer, for which the analysis consists in extracting the time signals associated with switching. This more advanced analysis (application to the time domain) enables us to validate the equivalent circuit model for the application cases closest to industrial reality, *i.e.* the extraction of signal waveforms for design validation (particularly for electromagnetic compatibility).

The converter studied is of resonant type, an increasingly common topology for switch mode power supplies. This is justified by the favorable properties of the resonant circuit, the use of which drastically reduces switching losses in semiconductors. For the converter to work properly, it is necessary to know the transformer parameters precisely. The magnetizing inductance and leakage inductance contribute to the resonance of the complete circuit, as do the parasitic capacitances of the transformer. These capacitances also induce other resonant phenomena such as ringing, which it is useful to take into account when estimating losses with precision, or when identifying problematic frequencies in terms of electromagnetic compatibility.

For all these reasons, the latter case lends itself particularly well to our study, especially as the transformer was measured in its environment, within the converter itself, at different power levels. In this way, we were able to access the temporal signals and verify our localized constant model for these complex temporal signals.

5.2.1 LLC Converter Principle and Time Signals

First, we focus on the operating principle of the LLC converter in normal operations. For this purpose, we take the diagram from the first chapter in Figure 5.9.

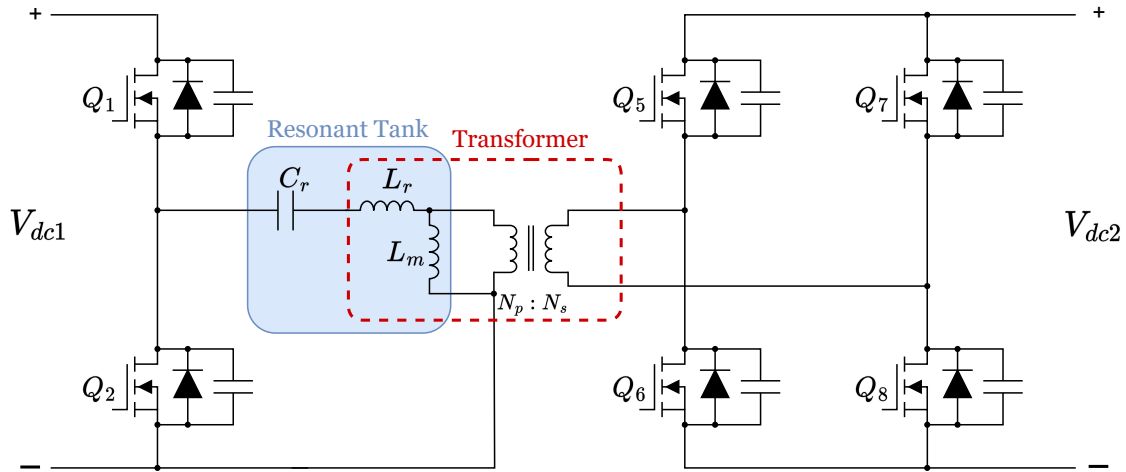


Figure 5.9: LLC resonant converter topology, including MOSFET body diodes and parasitic capacitances.

These include three resonant elements: a series capacitance, a series inductance and a parallel inductance. In practice, the transformer's magnetizing inductance is used as the parallel inductance, and another inductance is added in series with the leakage inductance to obtain the right value for the resonant frequency being considered. On the DC1 input side, a half-bridge MOSFET is used to hash the input voltage to the correct converter resonance frequency. On the DC2 output side, a rectifier bridge is present, which can be a diode bridge

or, in this case, a bridge of four MOSFETs, whose use is exactly the same. The input signal to the resonant circuit is therefore a square-wave signal with a duty cycle of 50 % and successive values of 0 and V_{dc1} . This same signal feeds the resonant circuit. At the transformer primary, the square-wave voltage signal resonates the LLC tank, producing a sinusoidal signal. At switching frequencies above the resonant frequency, the bridge output voltage can be regulated. Under power, the amplitude of the signals varies, and the switching frequency of the circuit is adapted accordingly to ensure that the MOSFETs switch in a ZVS manner.

5.2.2 Transformer Under Test

The conversion equipment includes a transformer whose inductances contribute to the resonance of the LLC tank located between the input half-bridge and the output full-bridge. This step-down transformer is made up of 19 turns on the primary side and 5 turns on the secondary side. Each winding is made of Litz wire to minimize copper losses, given the high switching frequency of the converter (between 100 and 150 kHz). A view of this transformer is shown in the following figure.



Figure 5.10: Resonant transformer used in the LLC converter.

The magnetic circuit is made up of two PQ-32/30 geometry magnetic half-cores in “T” magnetic material (Magnetics brand), with the central leg reduced to create the air gap that will provide the desired value for the magnetizing inductance.

As in the previous cases, the information about this transformer allowed the creation of a finite element model, from which the values of the localized constants circuit elements could be identified. And this equivalent circuit is inserted into the complete transformer environment (*i.e.* other circuit elements such as MOSFETs, sources and loads) to perform the time simulation.

5.2.3 Voltage and Current Waveforms Analysis

One of the aims of the industrial test case is to demonstrate the ease of use of the equivalent circuit models for time-domain simulations. Indeed, while it is already an advantage to be able to characterize the magnetic component beforehand in the frequency domain, it

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is an even better opportunity to exploit the same model in the time domain in order to observe the impact that non-idealities have on the converter's behavior. For example, we can observe the high-frequency resonance of magnetic components during switching, or the effect of induced current losses on signal attenuation at these same high frequencies. Voltage and current signals from the primary and secondary windings were recorded for different converter output power levels: 100, 300, 500 and 750 W.

5.2.3.1 Low Power Transfer

To begin with, we decided to check the model for the intermediate 300 W power level. On the one hand, we have the signals measured directly at the transformer ports when the converter is operating at a load of $9.3\ \Omega$ and, on the other hand, these same signals obtained by LTspice simulation, which integrates the transformer model (circuit in localized constants).

Figure 5.11 shows the voltage curve measured at the primary of the transformer for a power of 300W. We can see the main signal, which has a square waveform, to which a high-frequency signal of the order of ten MHz is added. This high-frequency signal decreases exponentially after each transition. This phenomenon is observed in both measurements and simulations, although the rate of decay is not identical. The decay rate is significantly higher in the measurements. Similarly, the measured high frequency differs slightly from the calculated value. The measured curves show a frequency of approx. 8.5 MHz, while the simulations show a value of approx. 7.9 MHz. On figure 5.11, we can clearly see that the measured phenomenon evolves at a slightly higher frequency than the LTspice estimate.

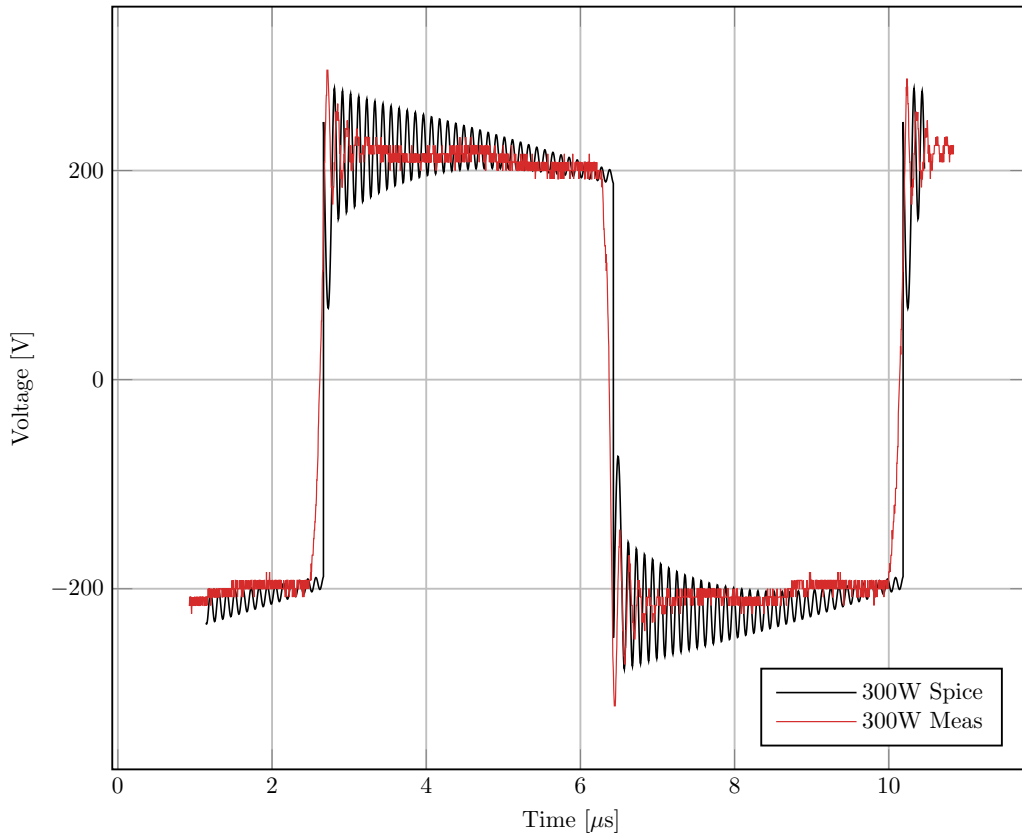


Figure 5.11: Transformer primary voltage waveform at 300 W.

This frequency of 7.9 MHz is not the resonant frequency of the transformer. In fact, the resonant frequency of the transformer alone is 2.8 MHz, as measured and identified by finite

element analysis. In reality, the environment in which the transformer is installed interacts with it. Indeed, after analysis and measurement, we identified that the line coming from the MOSFET bridge adds an additional RL//C impedance in series with the LLC tank. The resonant frequency of this line alone is 9.1 MHz, and we were able to characterize it by means of measurements. Once characterized, this line is modeled by an inductive ladder circuit and a parasitic capacitor in parallel. The integration of this line impedance means that the measured and simulated signals match. This was not the case beforehand, and prompted us to investigate the reasons for the discrepancy. This further illustrates the usefulness of being able to simulate the transformer in its environment and in the time domain, rather than on a table, in the frequency domain.

Figure 5.12 shows the voltage waveform measured at the transformer secondary for a power of 300W. With the exception of measurement noise, a square waveform is found at the secondary in both measurements and simulations.

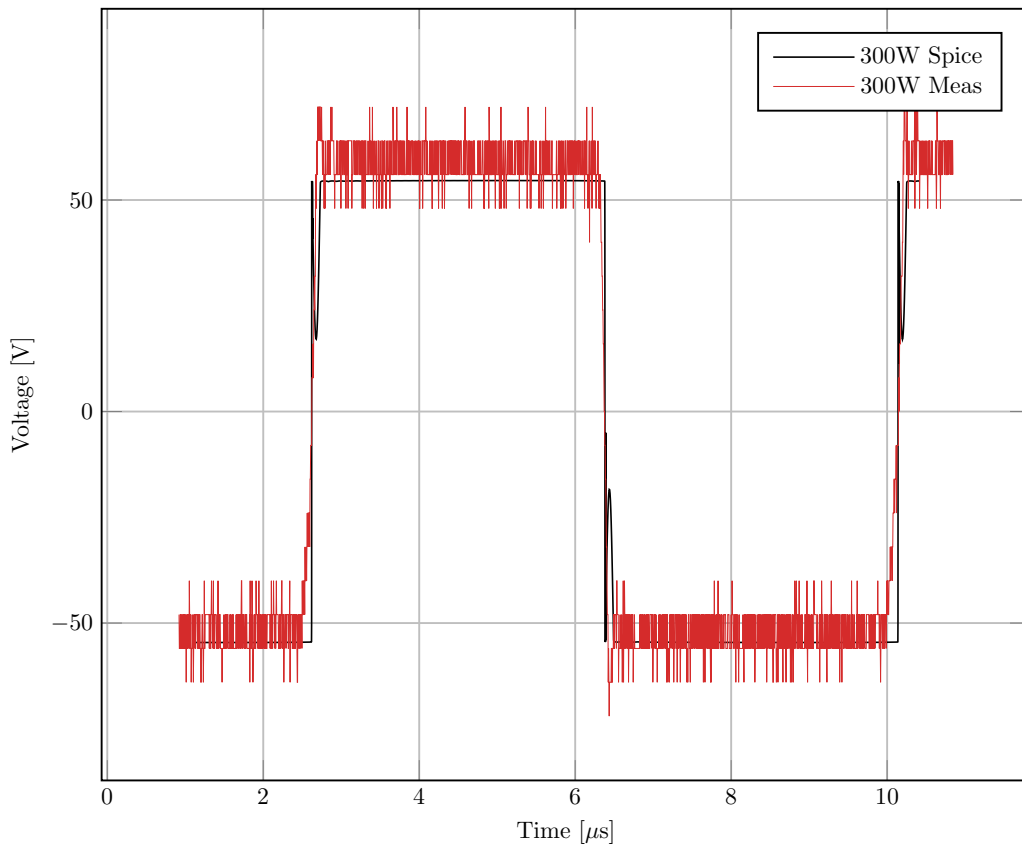


Figure 5.12: Transformer secondary voltage waveform at 300 W.

For secondary measurements, parasitic elements are reduced by the absence of LLC tanks, and no significant high-frequency resonance is observed on voltage signals.

Current waveforms have also been studied. In Figure 5.13, the current waveform of the primary side of the transformer is shown when 300W of power is transferred. The amplitude and waveform are in strong agreement. The simulations also show the 7.9 MHz interference frequency previously identified on the primary voltage. The current measurement quantization noise does not allow us to confirm or deny whether the frequency observed on the measurements is close to the resonance frequency of 7.9 MHz, at least on the primary current measurement. On the measured signal, the amplitude is lower than on the simulated signal. This can be explained by proportionally higher inline losses at this low power level,

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such as the drop in diode voltage.

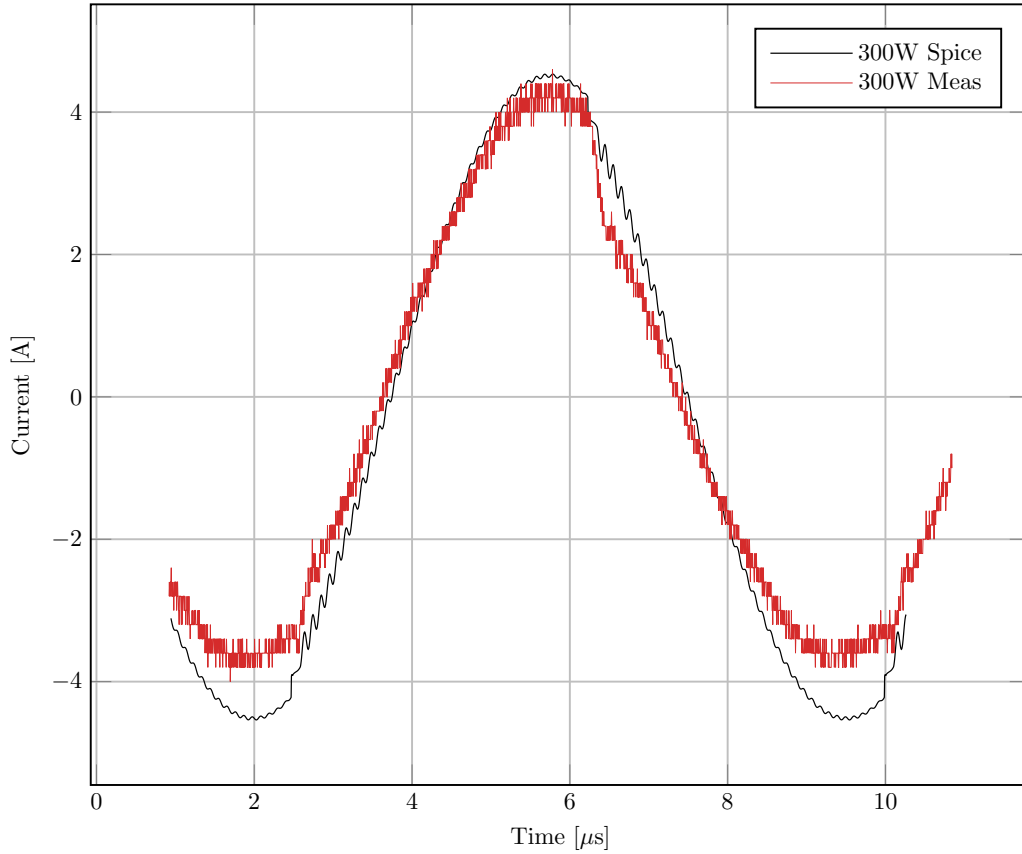


Figure 5.13: Transformer primary current waveform at 300 W.

The amplitude of the secondary current is greater than the amplitude of the primary current, and the signal-to-noise ratio is better. On Figure 5.14, the 8.5 MHz high-frequency signal is clearly visible after each transition.

In addition, the secondary current measurement signal shows an abrupt current jump after each transition. This jump is not reflected in the simulated curves. Once again, this discrepancy is marked at lower levels of exchanged power.

5.2.3.2 High Power Transfer

Having evaluated the model for a low level of transferred power, we modified the charge level of the setup and transferred 750 W, a power level close to the nominal value of the system. The aim is to demonstrate that the same circuit model can reproduce the behavior of the physical system for different stimuli, in this case a higher amplitude. To do this, we compare the signals measured directly at the transformer ports when the converter is flowing on a $3.8\ \Omega$ load with the same signals obtained by LTspice simulation. Just as was done previously for the lower power transfer.

First of all, it should be noted that the voltage signals are independent of the power level, since they are imposed by the input bridge of the converter. The same square-wave voltage is therefore applied to the transformer. Consequently, the secondary voltage also remains identical to the previous case. The curves are identical at 300 W and 750 W, so there is little point in discussing voltages again. On the other hand, for higher exchanged power, the primary and secondary currents will be higher, and their waveforms will also be different.

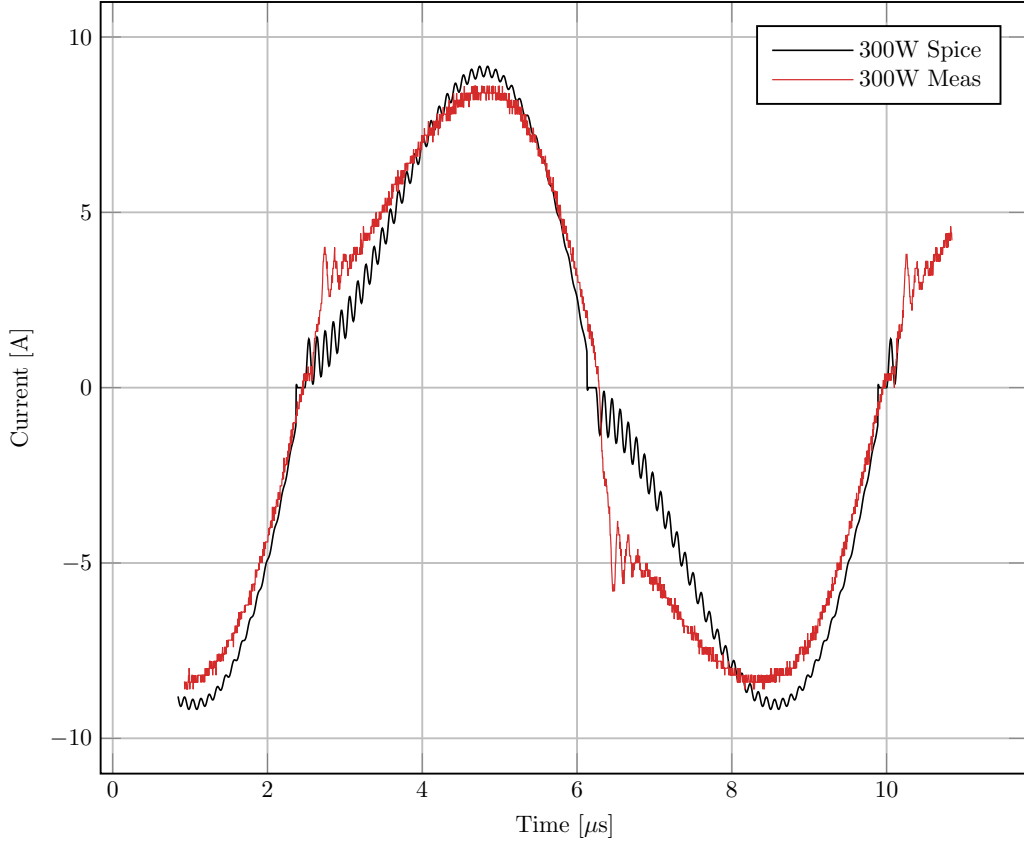


Figure 5.14: Transformer secondary current waveform at 300 W.

In Figure 5.15, we see the transformer primary current curve for simulations and measurements. The two waveforms are extremely well matched. In fact, both the amplitude and the shape of the curves match very well and overlap almost perfectly. On both curves, you can see a current jump during voltage transitions. The two curves differ somewhat around this phenomenon. As in the previous case, the current amplitude is too low to see the high-frequency phenomenon clearly, and the signal-to-noise ratio is not high enough to highlight the part of the signal at 8.5 MHz. The simulations show a very slight oscillation at 7.9 MHz.

Again, as in the previous case, the amplitude of the secondary current is greater than the amplitude of the primary current, and the signal-to-noise ratio is better. In Figure 5.16, the 8.5 MHz high-frequency signal is clearly visible after each transition. This signal is slightly more difficult to discern here, as the current amplitude is higher than at 300 W (~ 20 A peak instead of 8 A), but the phenomenon is clearly visible. As with primary current, there is a discrepancy between measured and simulated signals during transitions. It can be seen that the current jump does not occur at exactly the same instant, although the difference is very small: ~ 200 ns.

In summary, by analyzing this transformer in its environment (*i.e.* its DC-DC converter), we were able to extract the voltage and current signals established during operation, for different values of exchanged power. On the one hand, it demonstrates that the circuit models proposed in this work can be used in both harmonic and time-domain regimes, and that it is indeed the same model in both cases. On the other hand, comparisons of measurement curves and simulations show high consistency. Firstly, in terms of the general appearance of the signals, but above all for the detection of the first resonant frequency of the complete setup (transformer and its resonant LLC circuit).

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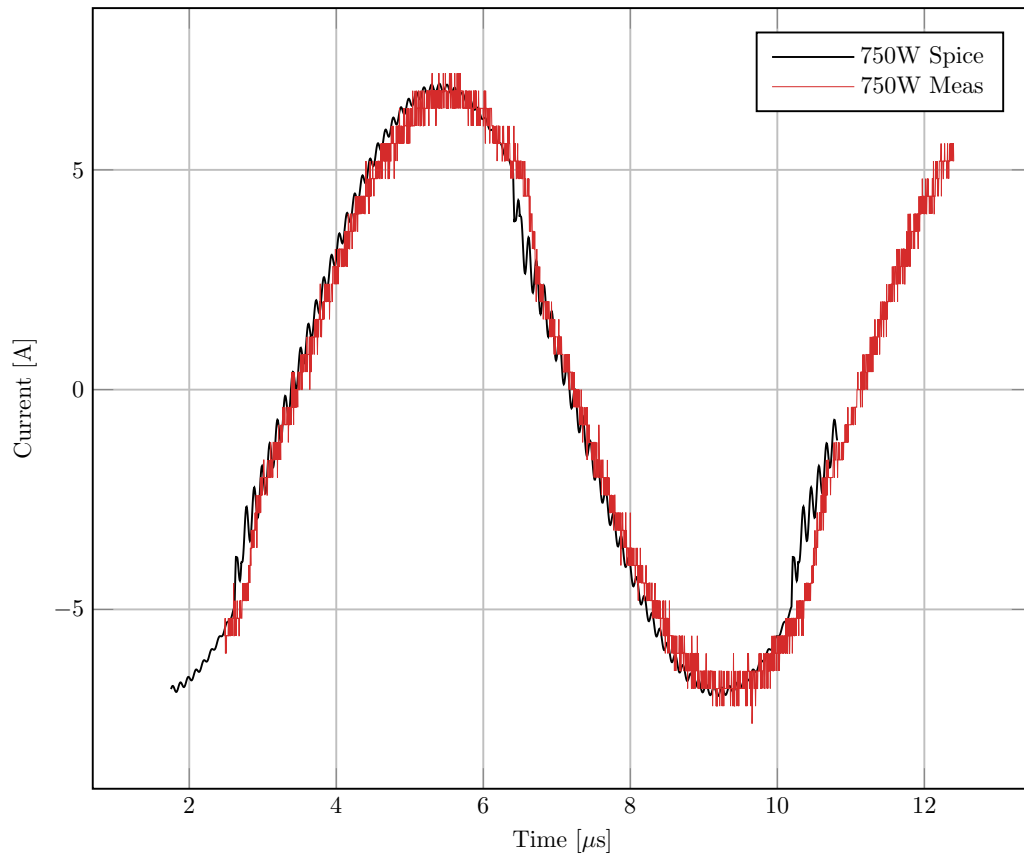


Figure 5.15: Transformer primary current waveform at 750 W.

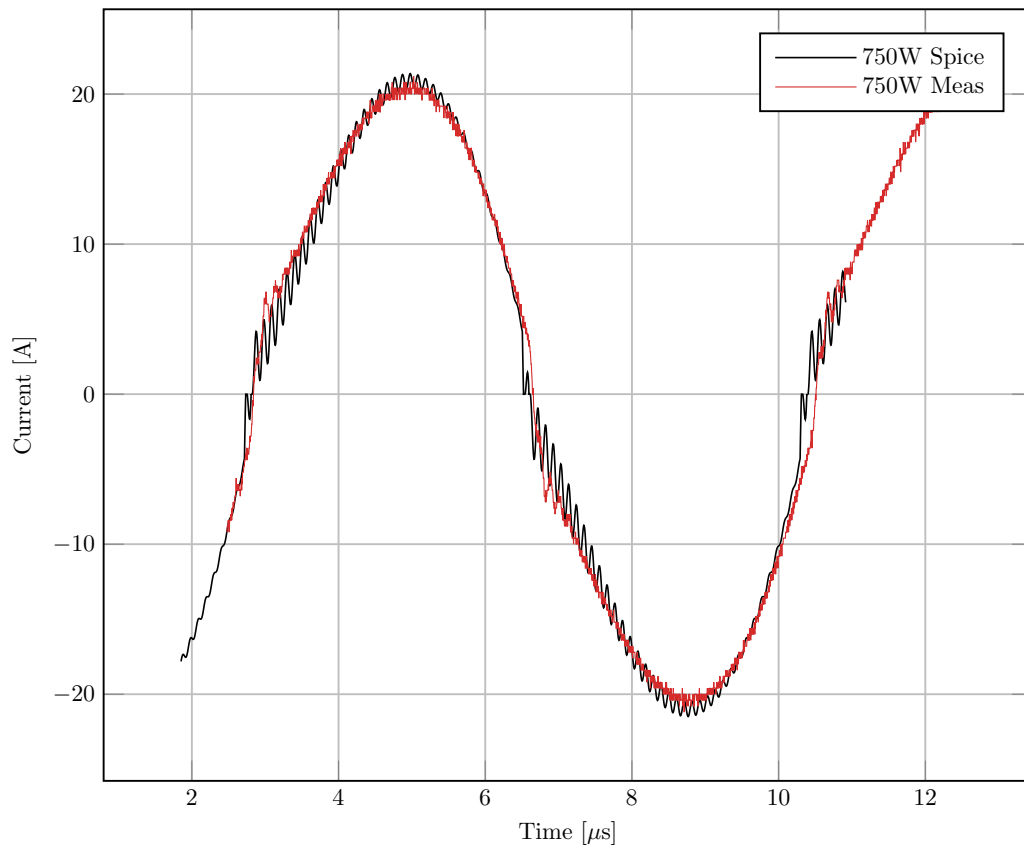


Figure 5.16: Transformer secondary current waveform at 750 W.

The last application case illustrates that our proposed solution meets industrial requirements. In other words, the need to represent a design with complex geometry by means of an equivalent circuit. An equivalent circuit which itself integrates inductive coupling elements, as well as parasitic capacitive couplings, and which can be operated via a circuit simulator in both frequency and time regimes.

5.3 Conclusion

In this chapter, we focus on two application cases that come directly from industry. These two cases present more complex geometries than the validation setups presented in the previous chapter. Nevertheless, we have been able to demonstrate that the solution we propose, and which was validated in the previous chapter, can be applied to actual devices, as encountered in industry.

In the first case, a study was carried out in the frequency regime, and we demonstrated that our models enable us to recover the various resonant phenomena of a transformer. In addition, the integration of losses was investigated in detail, and we highlighted the impact of different losses on impedance amplitude. From copper losses to dielectric losses and ferromagnetic losses. The inclusion of frequency-dependent losses is an improvement on the simplified circuit model of the two-winding transformer, especially since it exploits ladder circuits that are compatible in both frequency and time regimes.

The second case focused on the time-dependent study of a transformer in its real environment, in this case an LLC-type converter. We were able to demonstrate excellent agreement between simulated and measured signals. In addition, the presence of the first harmonic of the complete circuit could be highlighted both in simulation and measurements. This indicates that with the proposed model, the same circuit identified by means of harmonic stimuli, assuming a frequency regime, can also be used as it is in simulations of circuits in a time regime. The advantages of a single model are clear. On the one hand, in frequency terms, it makes it possible to evaluate the behavior of the component on the bench top: for example, to assess resonance frequencies; and then to exploit the same model in the time regime: typically to assess the performance of a complete design, which includes the magnetic component and its conversion environment. As a result, technicians wishing to evaluate the performance of their design can do so in both frequency and time regimes, using a single circuit model identified only once beforehand.

In addition to linking up with existing simplified models, our models allow to go further. Firstly, to simulate a component using a single equivalent circuit in different grounding configurations. Secondly, to be able to represent magnetic components with more than two windings without modifying either the method or the assumptions, *i.e.* the approach is not limited by a maximum number of windings. Thirdly, the component can be discretized more finely than at the winding level, enabling higher resonant frequencies to be determined in advance.

General Conclusion

The main objective of this thesis was to provide a practical toolbox for power electronics design engineers, enabling the modeling of magnetic components through equivalent circuits that remain valid over wide frequency ranges and capture key physical phenomena: magnetic coupling, copper losses, skin and proximity effects, ferromagnetic losses, parasitic capacitances, and dielectric losses. This work is grounded in a detailed understanding of high-frequency behavior in magnetic components used in power electronics. The thesis focused on generalizing localized constant-parameter circuit models, based on various levels of discretization. The core methodology relied on identifying circuit parameters using a finite element (FE) approach, applied to both magneto-dynamic and electrostatic physical models.

A key innovation of this work is the division of windings into discrete sets of turns—referred to as sections—which can range from a single turn (turn-level) to the entire winding (winding-level). This flexible discretization allows the designer to tailor the model to the level of detail needed. In the magnetic case, we demonstrated that the different discretization levels correspond to specific subdivisions of the turn-level impedance matrix, with each block's equivalent impedance derived directly. In contrast, the electrostatic case presents more complexity, as charges cannot be aggregated as directly. Here lies the first original contribution of the thesis: an algebraic method using shape functions and transformations to map finite element quantities into localized circuit constants, enabling the formal link between 2D FE models and circuit-level representations. This formulation naturally led to an innovative method for adapting and selecting the discretization level according to the needs of magnetic circuit designers.

We then extended the method to account for frequency-dependent losses. First, skin and proximity effects were captured using ladder circuit representations, allowing each impedance matrix entry to be replaced by its frequency-dependent equivalent. The same approach was applied to model ferromagnetic losses. The result is a generic equivalent circuit architecture capable of handling multiple windings and embedding ladder-type subcircuits—constituting the second major contribution of this work.

Based on these developments, we conclude that the thesis achieved its primary objective: to produce equivalent circuit models that comprehensively incorporate the relevant magnetic and electric phenomena.

Two major validation cases were investigated: a multi-layer inductor (with three variants: air-core, half-core, and full-core) and a single-layer air-core inductor. Components were characterized using impedance analyzers and compared with the results of the proposed modeling method. The models accurately captured inductance, capacitance, copper and core losses, dielectric losses, and associated resonances. By evaluating multiple levels of discretization, we demonstrated the trade-off between model detail and circuit simulation time. More refined models preserved high-frequency resonances, while coarser models led to loss of detail—highlighting the importance of carefully selecting discretization levels

based on simulation goals. Regarding the validation, we concluded that the method performs exceptionally well for axisymmetric geometries, while results for ferromagnetic losses must be interpreted with care due to the assumption of material linearity, which holds only for limited frequency ranges and low induction amplitudes.

Finally, the models were applied to two industrial transformers with more complex geometries. One was a step-up transformer, studied in the frequency domain to investigate capacitive couplings and to benchmark against a state-of-the-art model. The second transformer, part of an LLC resonant DC-DC converter, was analyzed in the time domain using the same modeling approach. The agreement between simulation and measurement confirmed the model's validity across both domains.

In conclusion, this thesis demonstrates that the proposed method offers a versatile and effective modeling strategy. Its ability to handle multi-winding configurations, operate in both frequency and time domains, and adapt to various levels of discretization provides design engineers with a powerful tool. The equivalent circuits derived via finite element analysis enable accurate and efficient simulation over a wide frequency range, meeting the practical demands of modern power electronics design.

5.4 Perspectives

This thesis has laid the groundwork for accurate equivalent circuit modeling of magnetic components across a broad frequency range. Several promising directions emerge from this work, both in terms of fundamental research and practical industrial applications.

At **low frequencies**, magnetic core losses remain the dominant loss mechanism. A natural next step involves the development of hybrid models that combine physical principles with data-driven approaches. For instance, finite element simulations incorporating hysteresis models could be used to derive simplified circuit-level representations. These models could be further enhanced using machine learning techniques, such as a generalized Steinmetz model trained on experimental datasets, enabling efficient yet accurate modeling of non-linear, history-dependent loss behavior.

At **medium frequencies**, dielectric losses become increasingly significant, as evidenced by the validation cases involving air-core inductors. We believe that further experimental efforts—particularly extended measurements on components without magnetic cores—will deepen our understanding of dielectric behavior in this frequency range. Accurate characterization of insulation materials will require precise measurements of their loss tangent in the 100 kHz to 10 MHz range. This information is especially relevant for the design of multi-layer windings, film insulation systems, and dielectric interfaces commonly found in industrial transformers and high-voltage transformers in which the electric field is particularly high. Similar perspectives hold for the characterization of ferromagnetic losses for which dedicated experimental setups must be used [23].

At **high and very high frequencies**, the traditional assumptions of lumped-parameter modeling begin to break down. The separate treatment of inductive and capacitive effects becomes inadequate, necessitating more advanced modeling strategies. One approach is the **Darwin** model, which introduces coupled electromagnetic behavior while retaining a circuit-based formulation. A more general solution is the **full-wave** model, which involves direct numerical resolution of Maxwell's equations without decoupling electric and magnetic phenomena. This raises a fundamental question: To what extent can we still rely on lumped circuit models at high frequencies or small spatial scales? At some point, magnetic

components behave more like transmission lines, and the modeling paradigm must shift to reflect this transition.

From a **practical and industrial standpoint**, three major directions for further improvement have been identified:

1. **Amplitude-Dependent Losses:** This work introduced frequency-dependent loss mechanisms via ladder circuits. However, in the presence of nonlinear magnetic materials, losses also depend on the signal amplitude. In such cases, using the same model for both time- and frequency-domain simulations becomes invalid. Future work should focus on developing representations that can handle amplitude-dependent behavior—either through nonlinear time-domain models or new formulations that bridge both domains consistently.
2. **Computation Time Reduction:** In an industrial context, usability is closely tied to simulation speed. While computation times on the order of 90 seconds are acceptable in academic studies, industrial engineers typically require results within a few seconds. Achieving this without sacrificing accuracy may involve strategies such as model order reduction, parametric precomputation, or integration with optimized solvers.
3. **Thermal Losses Estimation:** A preliminary study, conducted in collaboration with UGent and Prof. Alex Vanden Bossche, investigated the thermal behavior of an industrial transformer using a precision calorimeter. While satisfactory agreement with the equivalent circuit model was observed at low power, discrepancies appeared at full load. These deviations likely stem from unmodeled effects such as temperature-dependent material properties, non-uniform heat distribution, or interactions within the experimental setup. These findings point to the need for coupled electro-thermal modeling, particularly for high-power applications.

In summary, this work opens several promising research avenues that extend beyond the frequency-domain equivalent circuit modeling of magnetic components. Addressing nonlinear, thermal, and ultra-high-frequency behaviors will be key to further improving both the accuracy and the industrial applicability of these models.

Appendices

Two-Winding Transformer Simplified Model

We hereby present the brief review of a simple circuit model of two-winding transformers. The aim is to provide an understanding of the physical origin of the various resonances observed on a transformer, between its windings. This model assumes a per-winding cut: a primary and a secondary, both magnetically and electrically coupled. To understand the various resonances that appear in a two-winding transformer, it is appropriate to introduce a simplified circuit diagram that explains the origins of the resonances with a reduced number of parameters. To this aim, we have chosen to use the symmetrical circuit model shown in Figure A.1, for which the following characteristics are known

Table A.1: Two-winding transformer definitions of measured/identified parameters.

Parameter	Definition
L_o	Open circuit inductance, seen from the primary
L'_o	Open circuit inductance, seen from the secondary
L_{cc}	Short circuit inductance, seen from the primary

From these three known values, we can express the circuit below¹

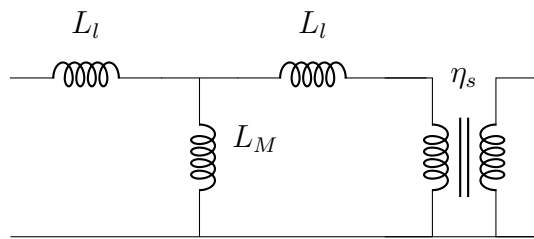


Figure A.1: Two-winding transformer symmetric equivalent circuit, discretized at the winding-level. The primary corresponds to the left part of the circuit.

where appear the deduced characteristics as indicated in the following table. The transformer ratio η_s (of the ideal transformer) is not equal to the ratio of the turns numbers, as often fixed by definition, but to the square root of the ratio of the open-circuit inductances measured at the two ports,

$$\eta_s = \sqrt{\frac{L'_o}{L_o}}. \quad (\text{A.1})$$

¹exhibits the identical behavior of the matrix form described in Equation (3.127)

Table A.2: Two-winding transformer definitions of circuit parameters.

Parameter	Definition
η_s	Ratio of the equivalent ideal transformer
k	Coupling coefficient (between primary and secondary)
L_M	Magnetizing inductance, seen from the primary
L_l	Leakage inductance, seen from the primary

The coupling coefficient k between the two windings is defined on the basis of the short-circuit inductance L_{cc} and open-circuit inductance L_o , measured at the primary (or equivalently at the secondary), such that

$$k = \sqrt{1 - \frac{L_{cc}}{L_o}} . \quad (\text{A.2})$$

The magnetizing inductance L_M can then be deduced as follows

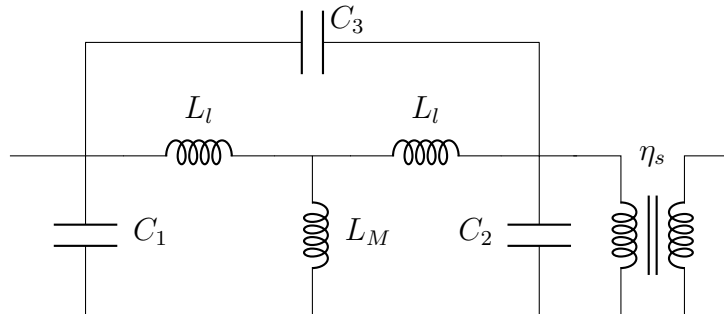
$$L_M = (1 - k) L_o , \quad (\text{A.3})$$

and the leakage inductance L_l of the equivalent circuit can be deduced as

$$L_l = k L_o . \quad (\text{A.4})$$

The T-shaped circuit shown in Figure A.1 is symmetrical by construction, and ensures that the short-circuit inductances seen in one winding or the other are identical, give or take the transformation ratio (*i.e.* η_s , the open-circuit inductances ratio square root). It is a simple and convenient model of the two-winding transformer, allowing us to demonstrate leakage and magnetizing inductances, concepts appreciated by electrical engineers.

To this strictly magnetic model, we can add parasitic capacitances between the various nodes of the circuit. In order to keep the number of parameters to a minimum and facilitate the analysis of such a circuit, we apply the following reasoning [66]: we choose to drastically reduce the number of parasitic capacitances to 3 (for a simplified analysis of the transformer resonances) and to keep them all on one side of the transformer, here on the primary side, as for the inductors.


Figure A.2: Two-winding transformer equivalent circuit, discretized at the winding-level, including parasitic capacitances referenced to the transformer primary.

This yields the circuit shown in Figure A.2, for which capacitances C_1 and C_2 are relative to the coils (one for the primary and one for the secondary) and capacitance C_3 corresponds

to the electrical coupling between the transformer's two coils (*i.e.* the inter-winding capacitance). Such a circuit allows us to explain the origin of the first resonance frequencies that already occur at low frequencies within a transformer. This is achieved with relatively few parameters: 3 magnetic and 3 electric.

At this point, it's worth noting that the capacities C_1 to C_3 are global capacitances, equivalent to the capacitances identified by finite elements. The values assumed by the capacitances C_1 to C_3 depend not only on the physical characteristics (just like the capacitances identified by numerical method), but also on the transformers' cabling (*i.e.* whether the windings share a potential reference and which winding terminal is grounded).

To this model are associated four distinct resonances [66] which are calculated as followed

$$f_1 = \frac{1}{2\pi\sqrt{L_o(C_1 + C_2)}} \quad (\text{A.5})$$

$$f_2 = \frac{1}{2\pi\sqrt{L_{cc}(C_1 + C_3)}} \quad (\text{A.6})$$

$$f_3 = \frac{1}{2\pi\sqrt{L_{cc}(C_2 + C_3)}} \quad (\text{A.7})$$

$$f_4 = \frac{1}{2\pi\sqrt{L_{cc}\left(C_3 + \frac{C_1 C_2}{C_1 + C_2}\right)}} \quad (\text{A.8})$$

The first resonance, with frequency f_1 , occurs due to coupling between the open-circuit inductance and the intra-winding parasitic capacitances, C_1 and C_2 . The second resonance occurs when the secondary is short-circuited, or equivalently, when the impedance of C_2 is lower than L_o and allows current to pass. The resulting inductance L_{cc} (inductance observed at the primary when the secondary is short-circuited) resonates with the capacitances C_1 and C_3 , which are then connected in parallel. The same behavior occurs for the third frequency resonance f_3 , except that we are now on the secondary side and the primary side is short-circuited. We then end up with the same equation, except that C_1 and C_2 are reversed.

Note that frequencies f_2 and f_3 are placed in arbitrary order here. The order in which they appear depends on the intra-winding capacitances of the primary and secondary windings. There is no way to determine in advance whether the primary or secondary winding will be higher, although generally, the winding with the most turns is also the one with the highest capacitance. The capacitance of the primary winding is determined by the number of turns in the winding. The capacitance of the secondary winding is determined by the number of turns in the winding and the capacitance of the primary winding. Finally, the f_4 resonant frequency occurs when the components of the equivalent circuit form a loop with the short-circuit equivalent inductance, C_1 , C_2 in series and with C_3 in parallel with the inductance. The resonant frequency f_4 therefore corresponds to the combination of the short-circuit inductance and an equivalent capacitance made of C_3 in parallel with C_1/C_2 in series.

In Figure A.3, the impedance norm of the transformer as a function of frequency is represented schematically, both from the primary and secondary perspectives. This graph illustrates where the different resonances occur and what connections can be made between the phenomena seen from the primary and secondary perspectives. In fact, the first and last resonance frequencies are common and do not depend on which side of the transformer you are looking at. All components drawn on the primary side can be transferred to the secondary side without affecting these two frequencies. The inductances will then be multiplied by the square of the transformation ratio and the capacitances by the inverse ratio,

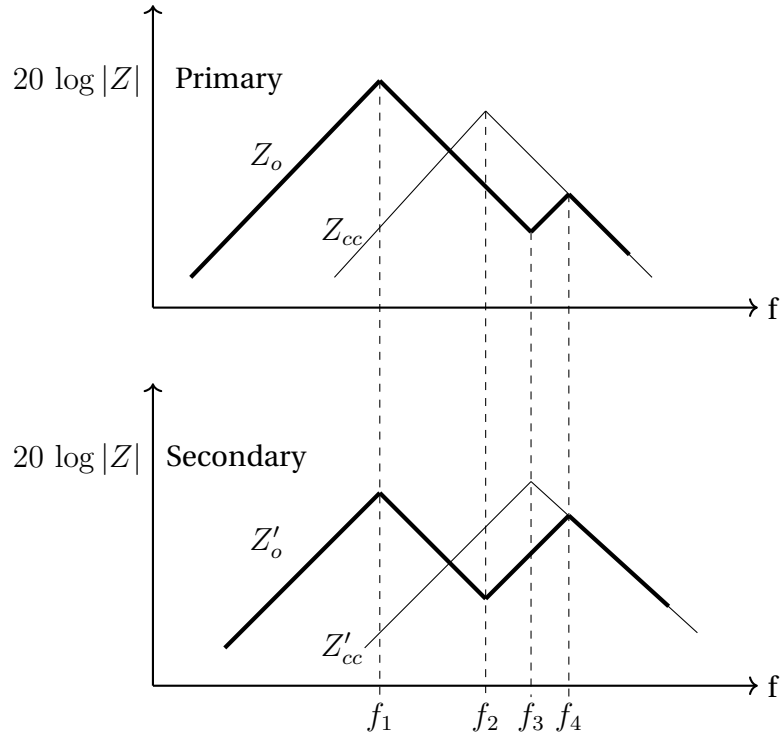


Figure A.3: Two-winding transformer resonant frequencies schematized, adapted from [66].

which therefore cancel each other out for resonance.

For the other two frequencies f_2 and f_3 , the behavior is different. To explain why, from one transformer's access, it is seen as a resonance and from the other access, it is an anti-resonance (transition from capacitive to inductive behavior), let us consider the case f_2 . In this case, the short-circuit inductance of the primary resonates with $C_1 + C_3$, and this occurs when the secondary is short-circuited, which means that C_2 is equivalent to a short circuit at this frequency when viewed from the primary side. In a completely equivalent manner, at this same frequency, but this time seen from the secondary side, all elements (except C_2) will resonate and together produce an infinite impedance. At the same time, C_2 will be equivalent to a short circuit and therefore to zero impedance, which amounts to anti-resonance from the secondary point of view.

A.1 Inter-winding Capacitances for Ideal Transformer Model

The model of an ideal transformer can be extended with parasitic capacitances between the windings. Those capacitance can be placed as in the following figure, between the top parts of the two windings and between the bottom parts.

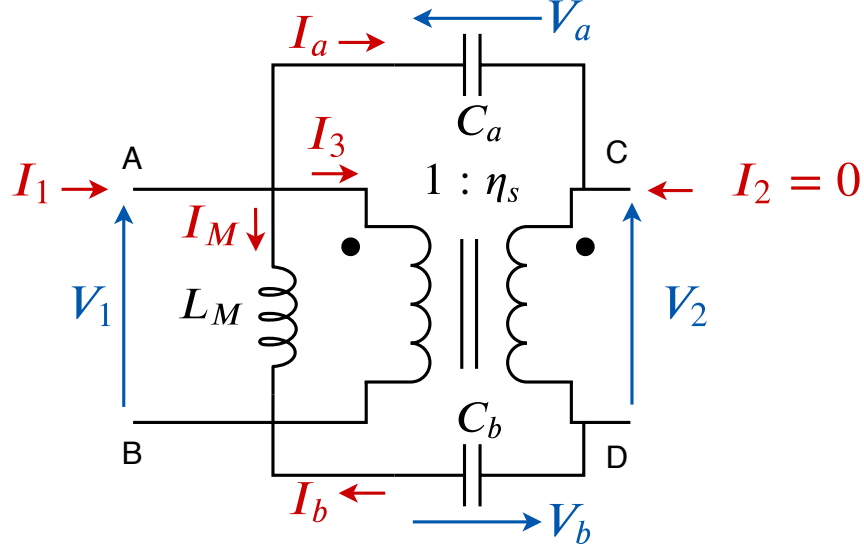


Figure A.4: Ideal transformer with parasitic inter-winding capacitances.

The equations describing such circuit can be developed as following. The ideal transformer part states that:

$$\overline{V_2} = \eta_s \overline{V_1} \quad (\text{A.9})$$

$$\overline{I_3} = \eta_s \overline{I_a} \quad (\text{A.10})$$

$$\overline{I_a} = \overline{I_b} = -\frac{\overline{I_1}}{\eta_s} \quad (\text{A.11})$$

Applying the Kirchhoff's Voltage Law leads to:

$$\overline{V_1} = \overline{V_a} + \overline{V_2} + \overline{V_b} \quad (\text{A.12})$$

$$\overline{V_a} + \overline{V_b} = (1 - \eta_s) \overline{V_1} \quad (\text{A.13})$$

And applying the Kirchhoff's Circuit Law leads to:

$$\overline{I_1} = \overline{I_M} + \overline{I_a} - \eta_s \overline{I_a} \quad (\text{A.14})$$

$$\overline{I_1} = \overline{I_M} + (1 - \eta_s) \overline{I_a} \quad (\text{A.15})$$

Then, the equations defining the capacitor's behaviour provides:

$$\overline{I}_a = j \omega C_a \overline{V}_a \quad (\text{A.16})$$

$$\overline{I}_b = j \omega C_b \overline{V}_b \quad (\text{A.17})$$

$$\overline{V}_b = \frac{C_a}{C_b} \overline{V}_a \quad . \quad (\text{A.18})$$

Then, the equation linked to the magnetizing inductance provides:

$$\overline{V}_1 = j \omega L_M \overline{I}_M \quad . \quad (\text{A.19})$$

Injecting Equation A.18 into A.13

$$\overline{V}_a \left(1 + \frac{C_a}{C_b} \right) = (1 - \eta_s) \overline{V}_1 \quad (\text{A.20})$$

$$\overline{V}_a = \frac{1 - \eta_s}{1 + \frac{C_a}{C_b}} \overline{V}_1 \quad (\text{A.21})$$

$$\overline{V}_1 = \overline{V}_a \frac{1 + \frac{C_a}{C_b}}{1 - \eta_s} = j \omega L_M \overline{I}_M \quad (\text{A.22})$$

Injecting Equation A.15 into A.22

$$j \omega L_M \overline{I}_1 - j \omega L_M \overline{I}_a (1 - \eta_s) = \overline{V}_a \frac{1 + \frac{C_a}{C_b}}{1 - \eta_s} \quad (\text{A.23})$$

Injecting Equation A.16 into A.23

$$j \omega L_M \overline{I}_1 - \omega^2 L_M C_a (1 - \eta_s) \overline{V}_a = \overline{V}_a \frac{1 + \frac{C_a}{C_b}}{1 - \eta_s} \quad (\text{A.24})$$

$$j \omega L_M (1 - \eta_s) \overline{I}_1 = \overline{V}_a \left(\left(1 + \frac{C_a}{C_b} \right) - \omega^2 (1 - \eta_s)^2 L_M C_a \right) \quad (\text{A.25})$$

Injecting Equation A.21 into A.25

$$j \omega L_M (1 - \eta_s) \overline{I}_1 = \left(1 + \frac{C_a}{C_b} - \omega^2 (1 - \eta_s)^2 L_M C_a \right) \frac{(1 - \eta_s)}{1 + \frac{C_a}{C_b}} \overline{V}_1 \quad (\text{A.26})$$

$$j \omega L_M \overline{I}_1 = \left(1 + \frac{C_a}{C_b} - \omega^2 (1 - \eta_s)^2 L_M C_a \right) \frac{1}{1 + \frac{C_a}{C_b}} \overline{V}_1 \quad (\text{A.27})$$

$$j \omega L_M \overline{I}_1 = \left(1 - \frac{\omega^2 (1 - \eta_s)^2 L_M C_a C_b}{1 + \frac{C_a}{C_b}} \right) \overline{V}_1 \quad (\text{A.28})$$

$$j \omega L_M \overline{I}_1 = \left(1 - \frac{\omega^2 (1 - \eta_s)^2 L_M C_a C_b}{C_a + C_b} \right) \overline{V}_1 \quad (\text{A.29})$$

$$j \omega L_M (C_a + C_b) \overline{I}_1 = (C_a + C_b - \omega^2 (1 - \eta_s)^2 L_M C_a C_b) \overline{V}_1 \quad (\text{A.30})$$

Finally, the equivalent impedance seen from the primary can be expressed as:

$$Z_1 = \frac{\overline{V}_1}{\overline{I}_1} = \frac{j \omega L_M (C_a + C_b)}{(C_a + C_b) - \omega^2 (1 - \eta_s)^2 L_M C_a C_b} . \quad (\text{A.31})$$

This expression simplifies into $j \omega L_M$ when $\eta_s = 1$, $C_a = 0$ or $C_b = 0$.

Remark that the denominator vanishes for:

$$(C_a + C_b) - \omega^2 (1 - \eta_s)^2 L_M C_a C_b = 0 \quad (\text{A.32})$$

$$\omega^2 (1 - \eta_s)^2 L_M C_a C_b = (C_a + C_b) . \quad (\text{A.33})$$

Leading to a resonant pulsation ω_0 of:

$$\omega_0 = \sqrt{\frac{1}{L_M C_{eq}}} \quad (\text{A.34})$$

with

$$C_{eq} = \frac{C_a C_b}{C_a + C_b} (1 - \eta_s)^2 . \quad (\text{A.35})$$

Ferromagnetic Materials Characterization

The ferromagnetic cores used for the various case studies were first characterized using an AMH-200K-S permeameter, as shown in the following figure. These data can be used to estimate the magnetic parameters of a given type of material.



Figure B.1: Permeameter AMH-200K-S from Laboratorio Eletroffisico.

B.1 Characterization of E core 43/10/28 in 3C95 Material

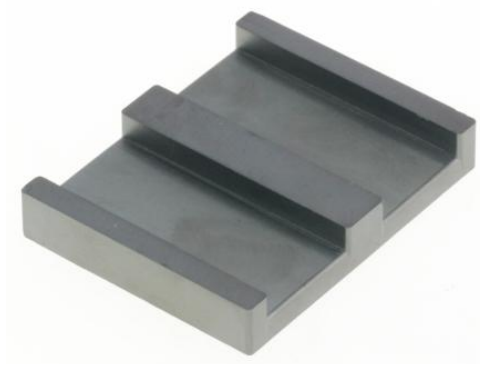
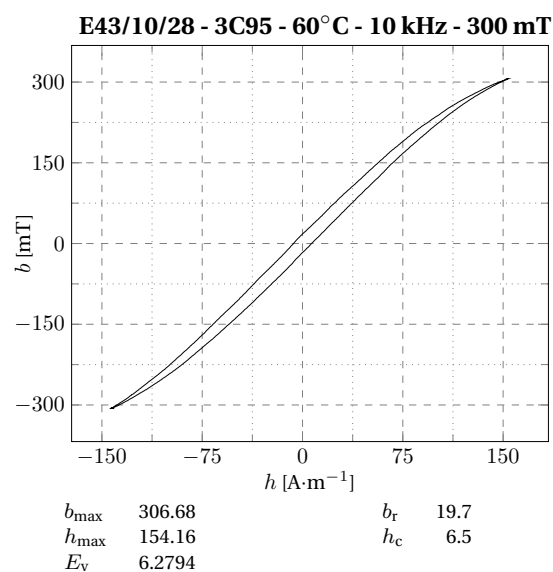
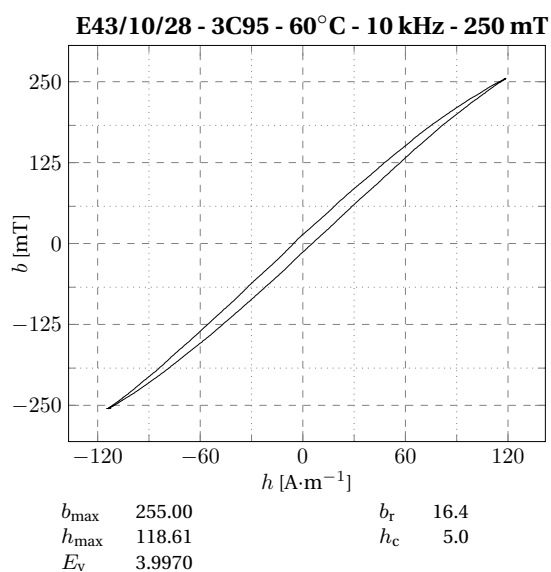
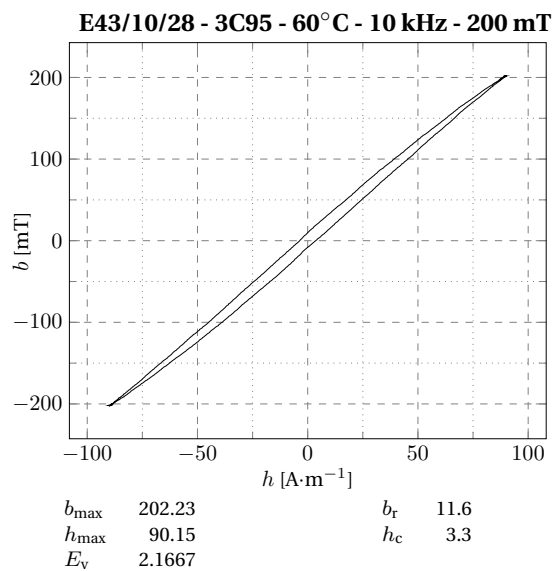
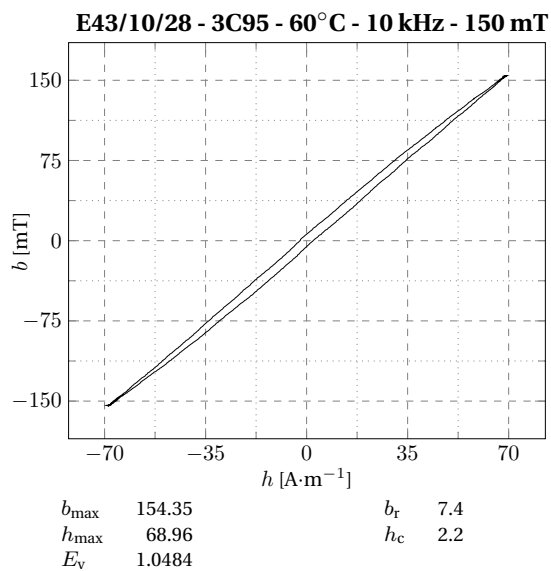
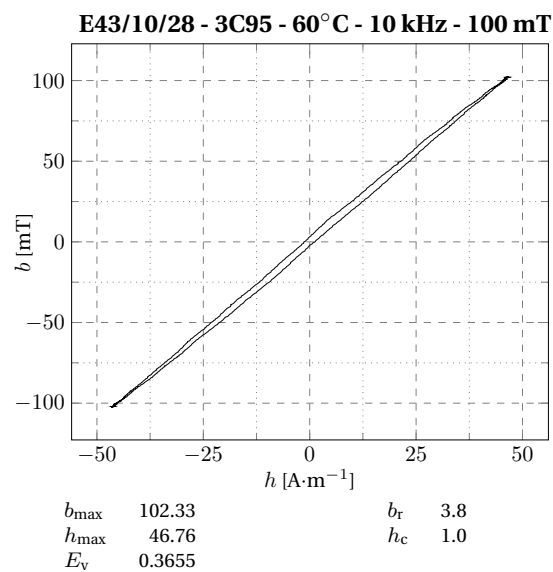
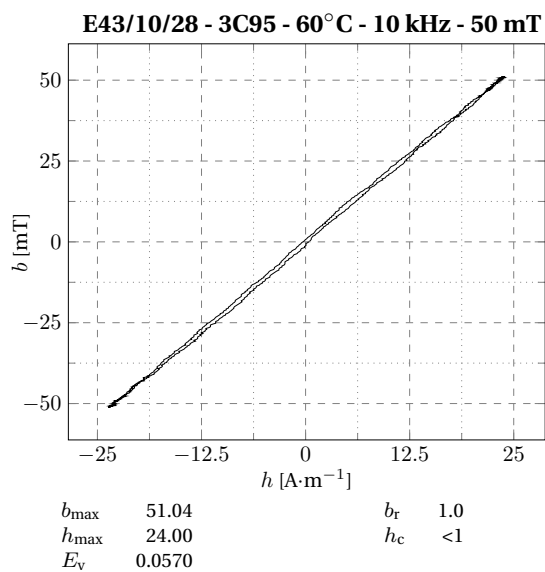


Figure B.2: Illustration of half a 43/10/28 E core.

B.1.1 60°C and 10 kHz



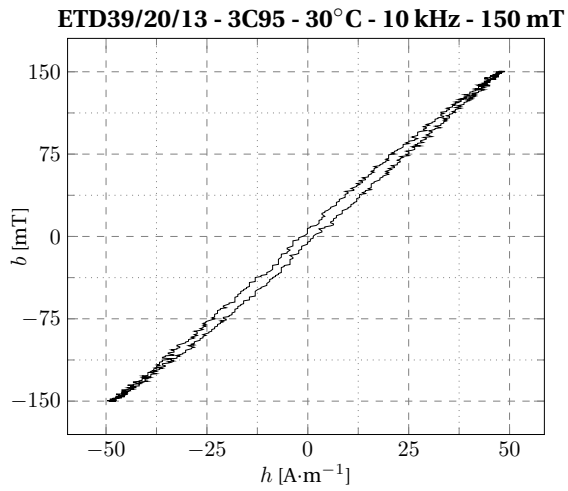
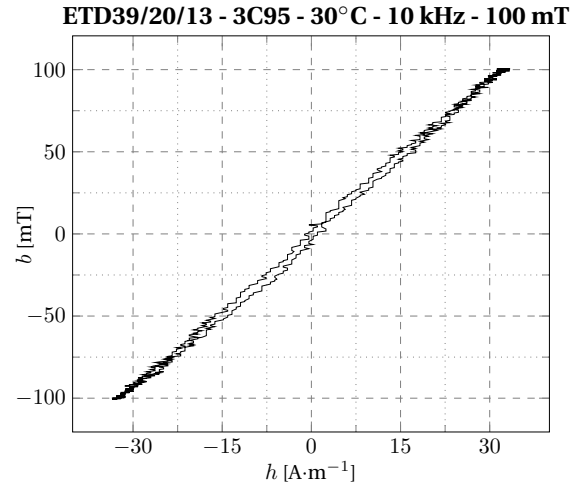
B.2 Characterization of ETD core 39/20/13 in 3C95 Material



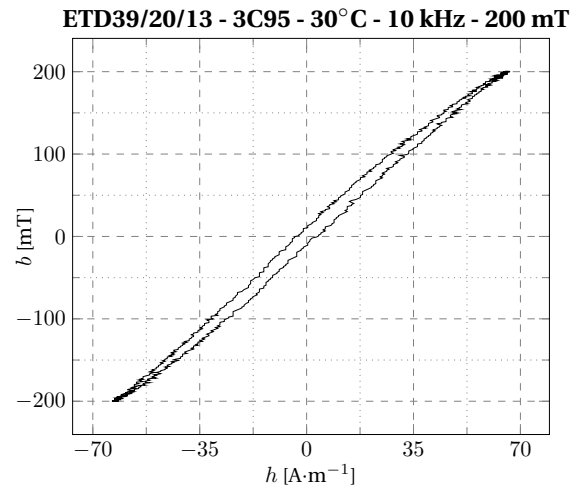
Figure B.3: Illustration of half a 39/20/13 ETD core.

B.2.1 30°C and 10 kHz

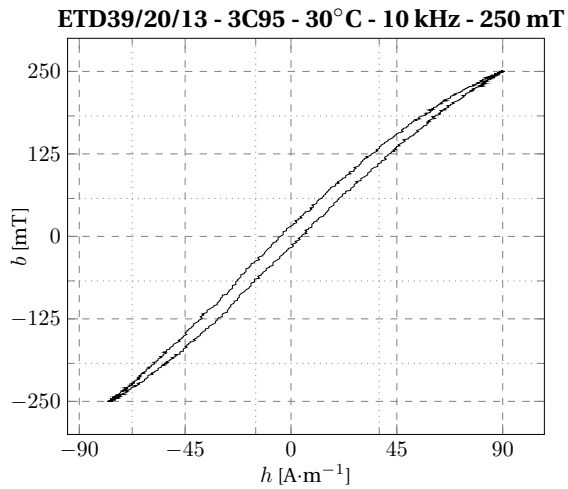
$b_{\max}$	100.47	b_r	7.0
$h_{\max}$	33.36	h_c	<1
E_v	0.2828		



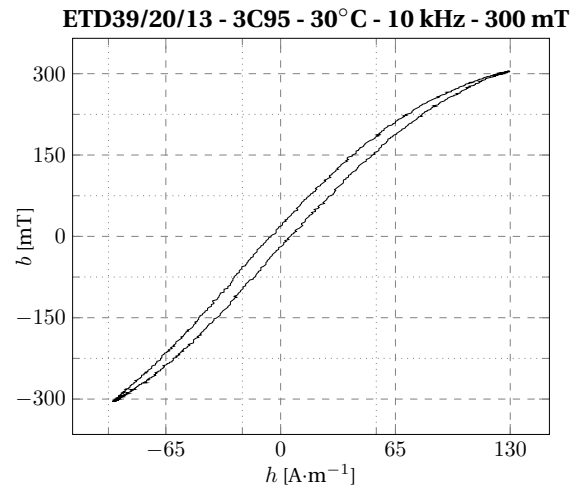
$b_{\max}$	150.33	b_r	8.6
$h_{\max}$	48.78	h_c	<1
E_v	0.8394		



$b_{\max}$	200.33	b_r	10.8
$h_{\max}$	66.49	h_c	2.5
E_v	1.8732		

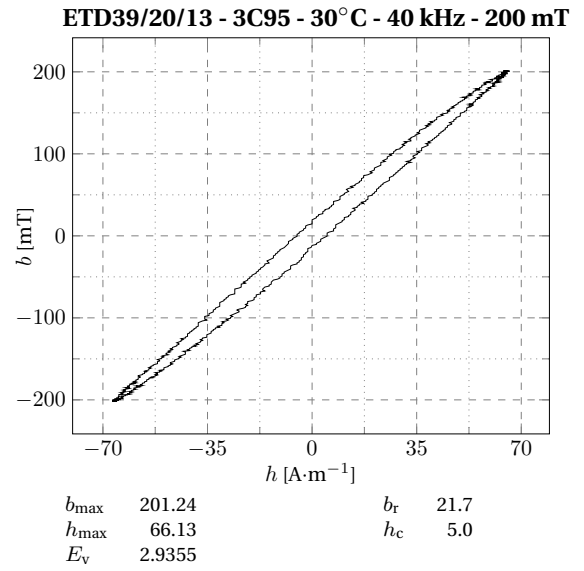
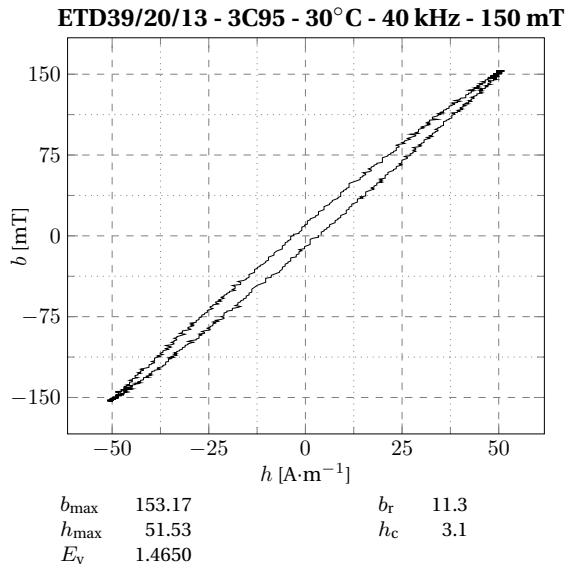
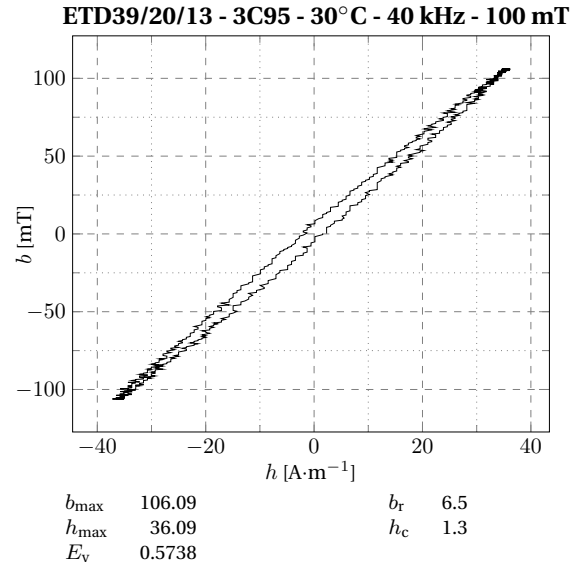
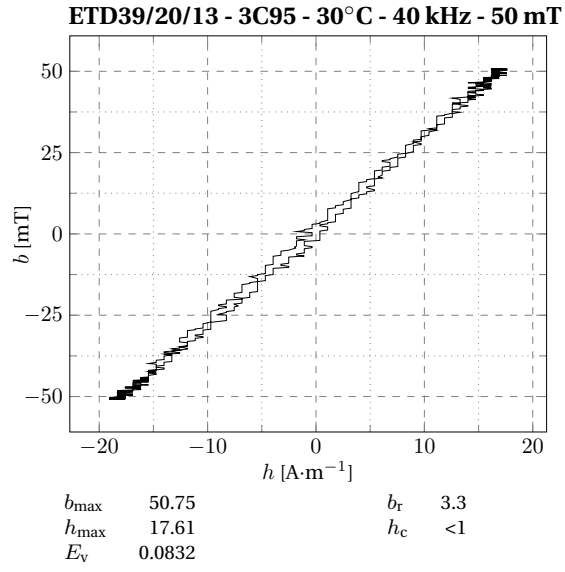


$b_{\max}$	250.11	b_r	16.2
$h_{\max}$	90.82	h_c	4.1
E_v	3.2859		



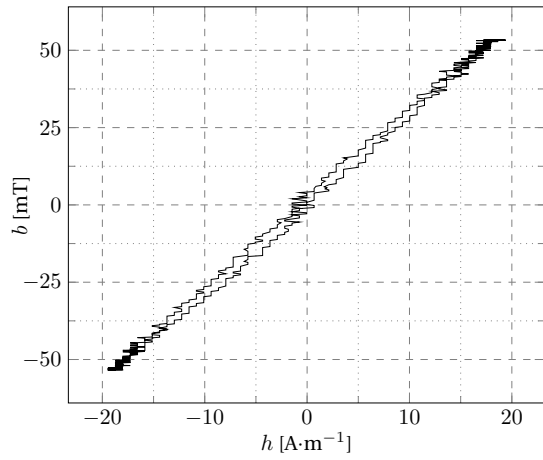
$b_{\max}$	304.34	b_r	19.3
$h_{\max}$	129.69	h_c	5.4
E_v	5.5085		

B.2.2 30°C and 40 kHz



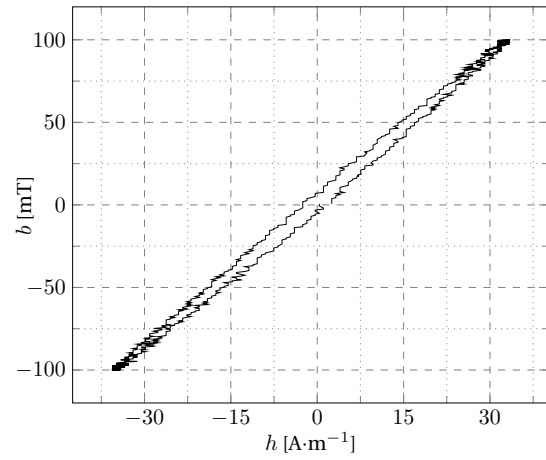
B.2.3 30°C and 50 kHz

ETD39/20/13 - 3C95 - 30°C - 50 kHz - 50 mT



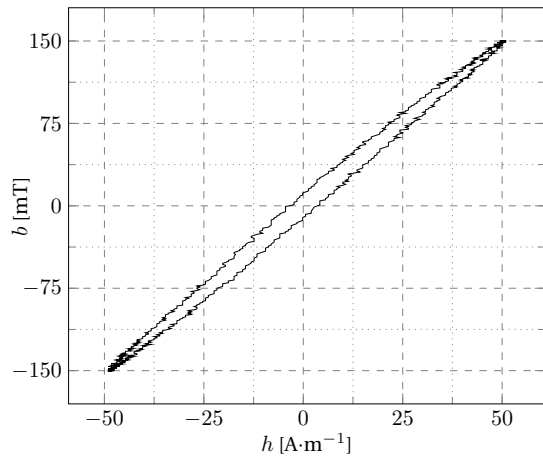
$b_{\max}$	53.38	b_r	<1
$h_{\max}$	19.37	h_c	1.5
E_v	0.1113		

ETD39/20/13 - 3C95 - 30°C - 50 kHz - 100 mT



$b_{\max}$	99.97	b_r	8.1
$h_{\max}$	33.42	h_c	2.5
E_v	0.5377		

ETD39/20/13 - 3C95 - 30°C - 50 kHz - 150 mT



$b_{\max}$	150.20	b_r	13.0
$h_{\max}$	50.88	h_c	3.0
E_v	1.4843		

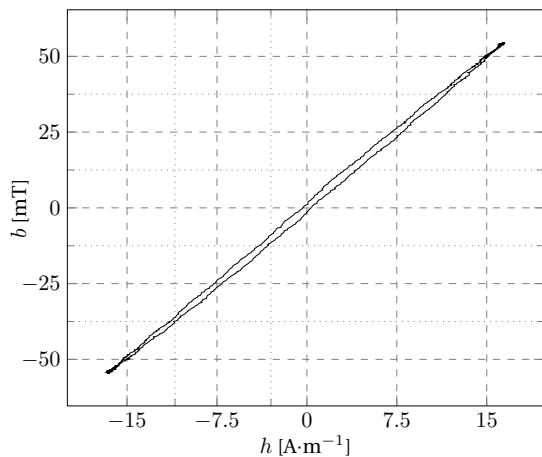
B.3 Characterization of ETD core 34/17/11 in 3C97 Material



Figure B.4: Illustration of half a 34/17/11 ETD core.

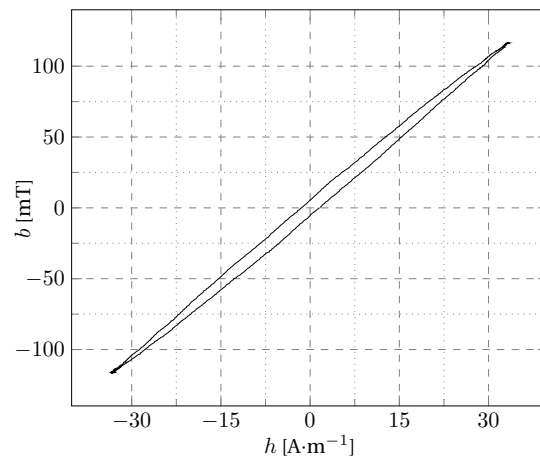
B.3.1 25°C and 10 kHz

ETD34/17/11 - 3C97 - 25°C - 10 kHz - 50 mT



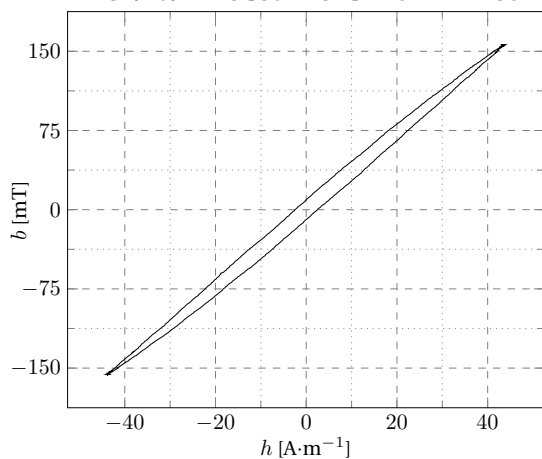
$b_{\max}$	54.47	b_r	1.1
$h_{\max}$	16.48	h_c	<1
E_v	0.0734		

ETD34/17/11 - 3C97 - 25°C - 10 kHz - 100 mT



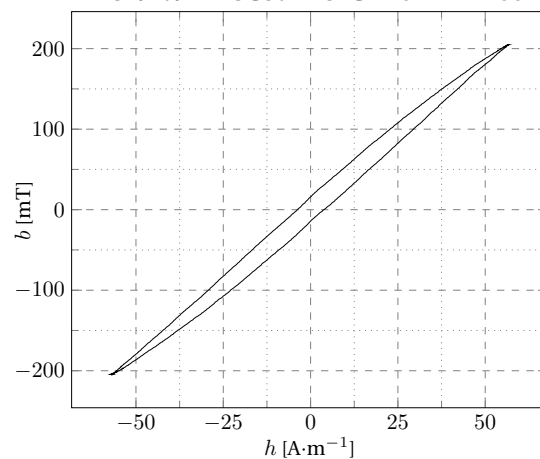
$b_{\max}$	116.50	b_r	6.2
$h_{\max}$	33.39	h_c	1.4
E_v	0.5223		

ETD34/17/11 - 3C97 - 25°C - 10 kHz - 150 mT



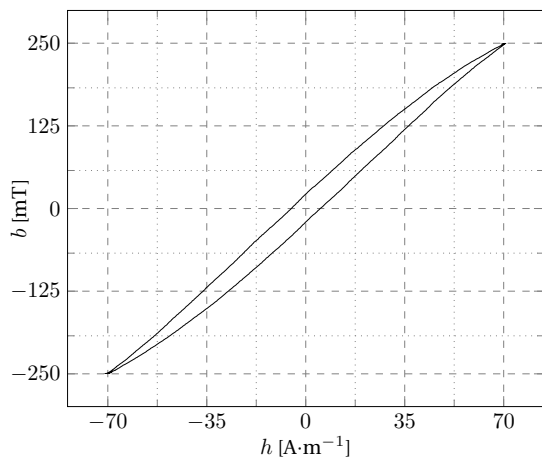
$b_{\max}$	156.29	b_r	9.9
$h_{\max}$	43.74	h_c	2.5
E_v	1.1301		

ETD34/17/11 - 3C97 - 25°C - 10 kHz - 200 mT



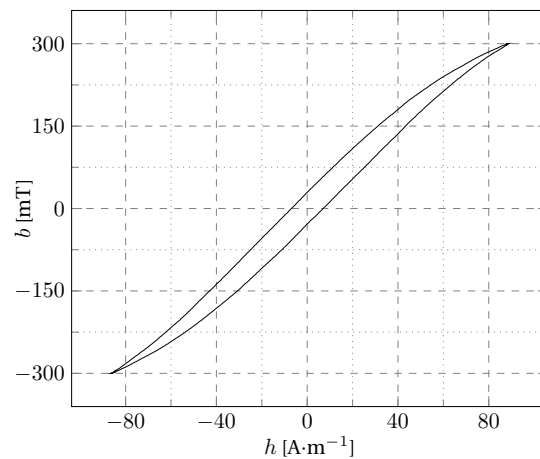
$b_{\max}$	204.96	b_r	15.9
$h_{\max}$	56.72	h_c	3.6
E_v	2.3414		

ETD34/17/11 - 3C97 - 25°C - 10 kHz - 250 mT



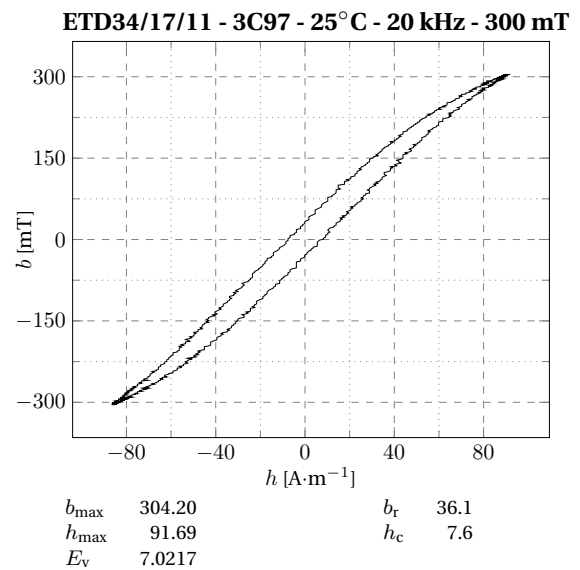
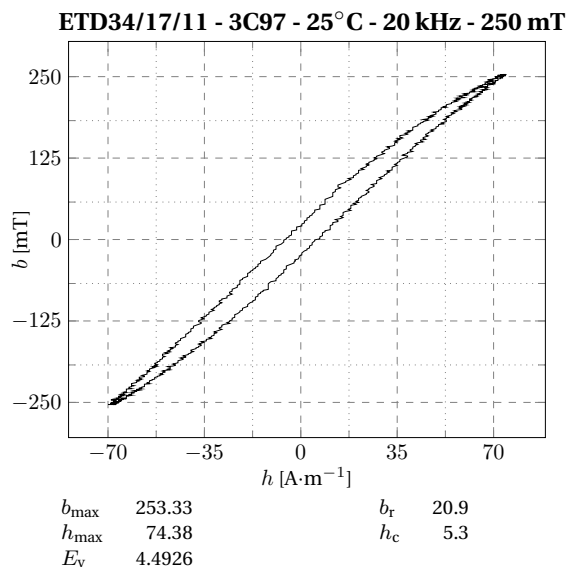
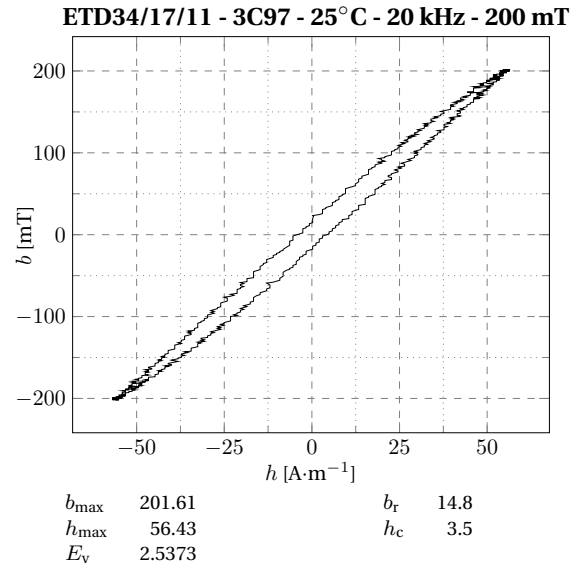
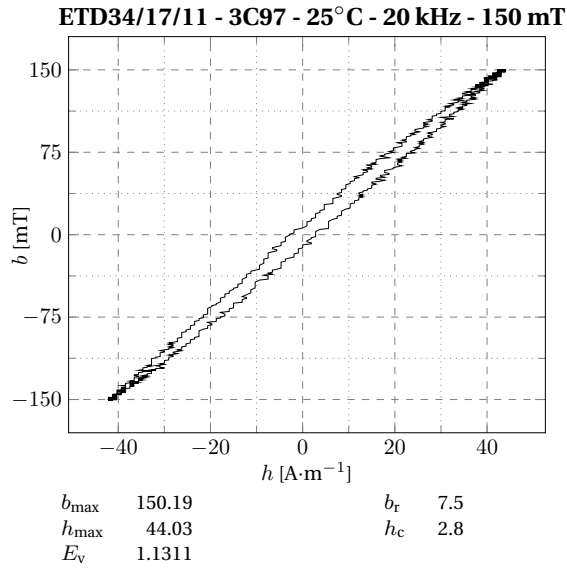
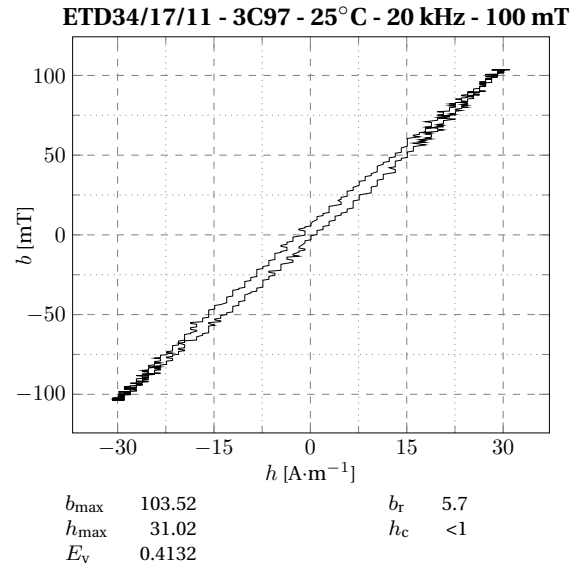
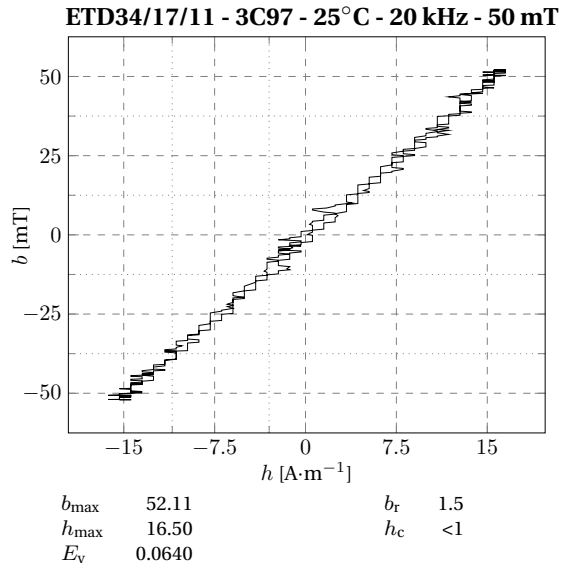
$b_{\max}$	249.60	b_r	23.7
$h_{\max}$	70.34	h_c	5.1
E_v	3.8760		

ETD34/17/11 - 3C97 - 25°C - 10 kHz - 300 mT



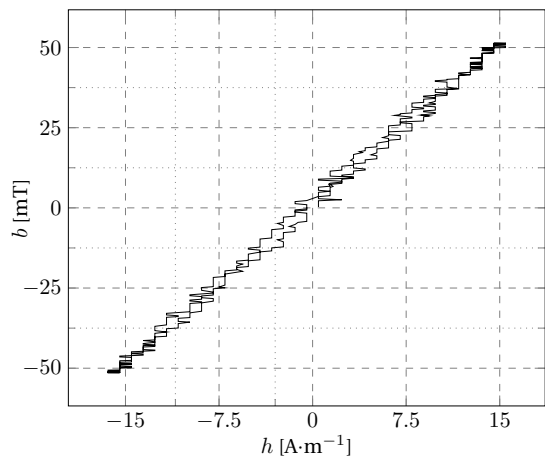
$b_{\max}$	300.41	b_r	29.7
$h_{\max}$	88.78	h_c	6.5
E_v	6.4181		

B.3.2 25°C and 20kHz



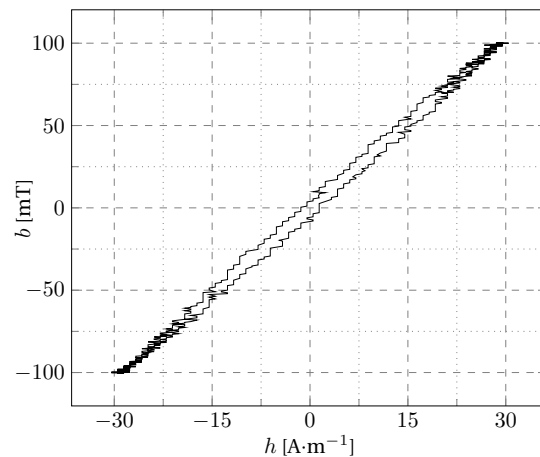
B.3.3 25°C and 30kHz

ETD34/17/11 - 3C97 - 25°C - 30 kHz - 50 mT



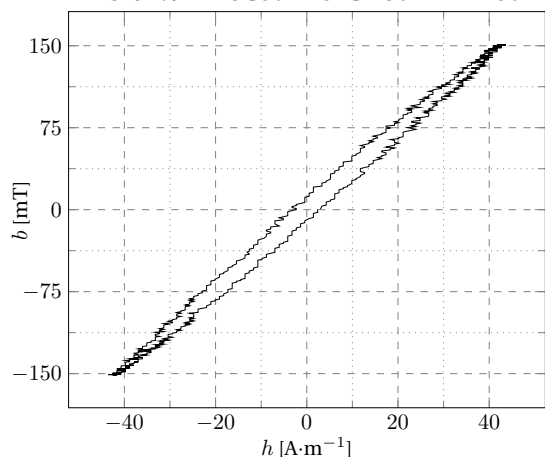
$b_{\max}$	51.47	b_r	3.3
$h_{\max}$	15.46	h_c	<1
E_v	0.0880		

ETD34/17/11 - 3C97 - 25°C - 30 kHz - 100 mT



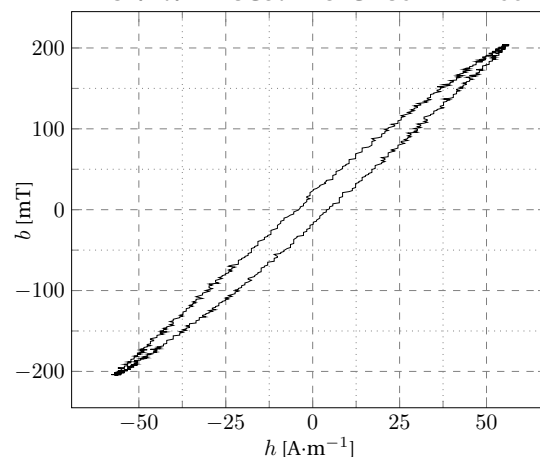
$b_{\max}$	100.23	b_r	4.4
$h_{\max}$	30.45	h_c	1.4
E_v	0.4258		

ETD34/17/11 - 3C97 - 25°C - 30 kHz - 150 mT



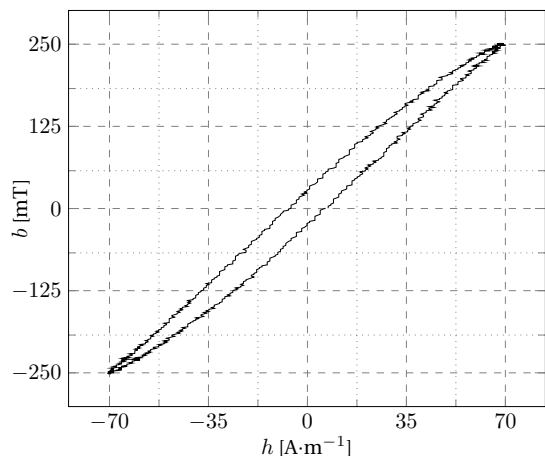
$b_{\max}$	151.01	b_r	12.5
$h_{\max}$	43.60	h_c	2.3
E_v	1.3181		

ETD34/17/11 - 3C97 - 25°C - 30 kHz - 200 mT



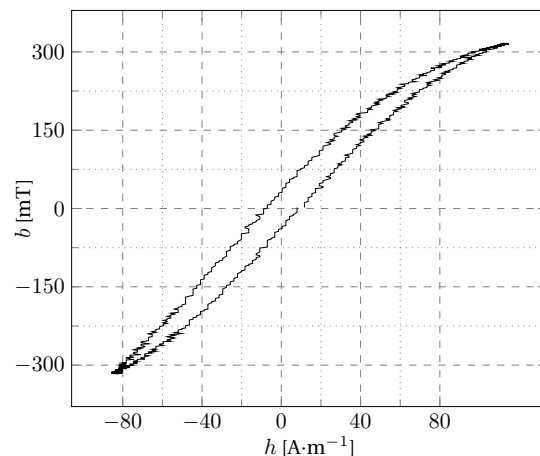
$b_{\max}$	204.12	b_r	23.8
$h_{\max}$	56.38	h_c	3.8
E_v	2.9038		

ETD34/17/11 - 3C97 - 25°C - 30 kHz - 250 mT



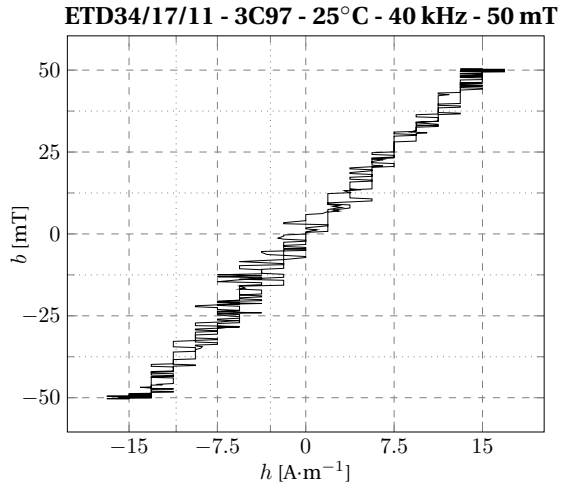
$b_{\max}$	251.10	b_r	30.6
$h_{\max}$	70.10	h_c	5.8
E_v	4.9972		

ETD34/17/11 - 3C97 - 25°C - 30 kHz - 300 mT

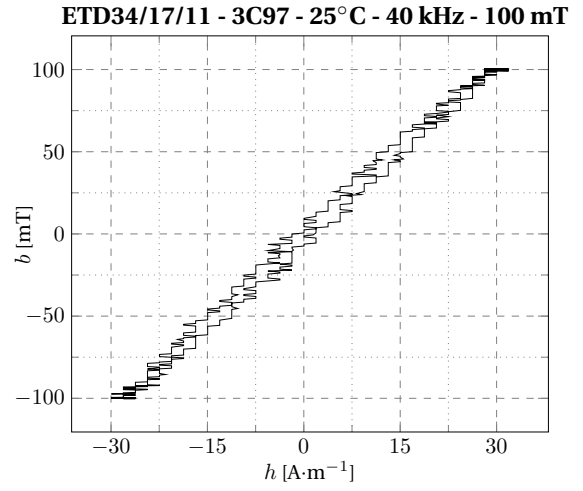


$b_{\max}$	316.16	b_r	31.4
$h_{\max}$	114.73	h_c	7.1
E_v	8.5144		

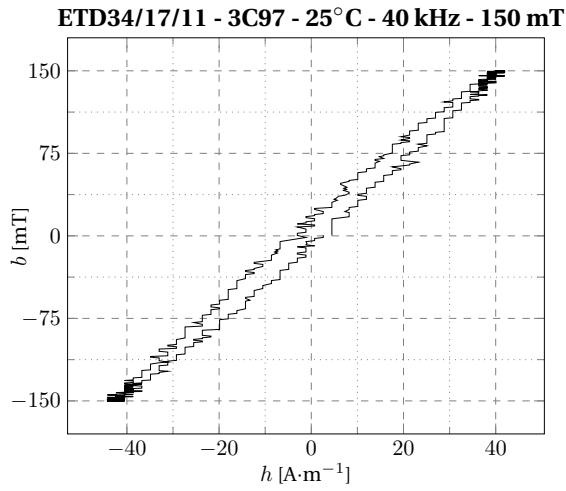
B.3.4 25°C and 40kHz



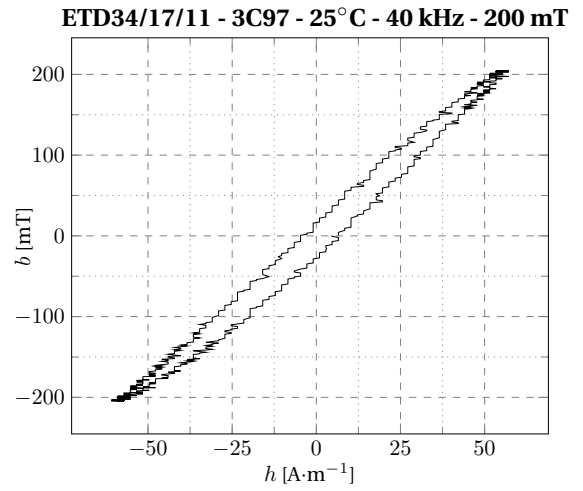
$b_{\max}$	50.39	b_r	<1
$h_{\max}$	16.86	h_c	<1
E_v	0.1025		



$b_{\max}$	100.41	b_r	<1
$h_{\max}$	31.88	h_c	<1
E_v	0.4986		

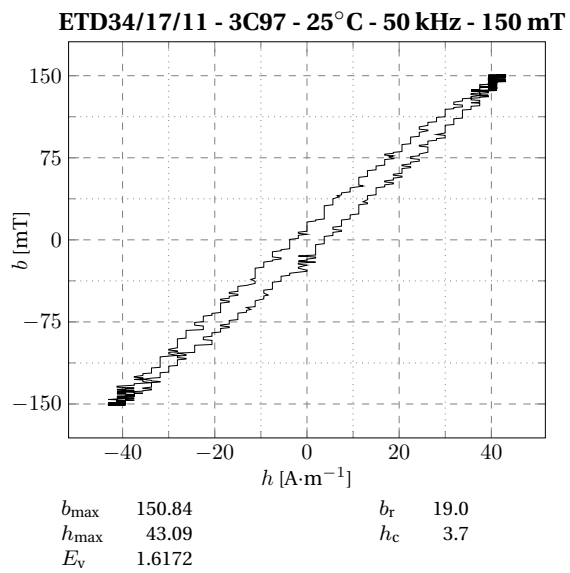
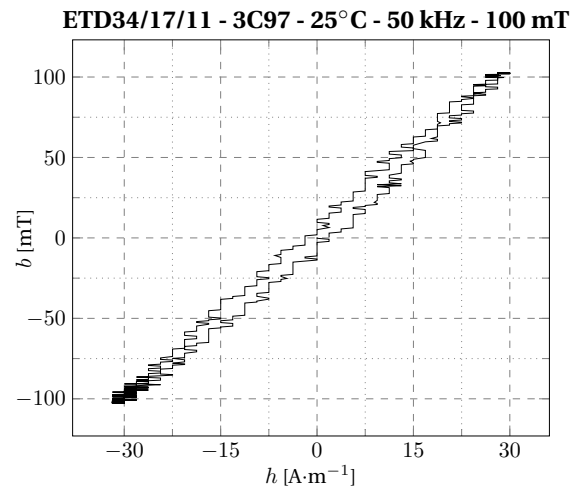
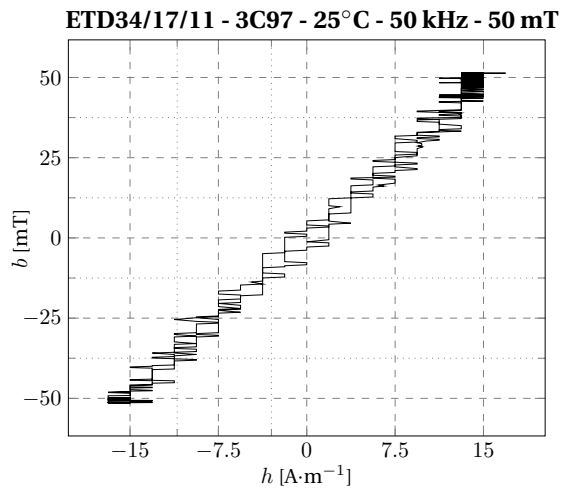


$b_{\max}$	150.01	b_r	18.7
$h_{\max}$	41.94	h_c	3.0
E_v	1.4135		



$b_{\max}$	204.41	b_r	17.6
$h_{\max}$	57.15	h_c	4.7
E_v	3.2498		

B.3.5 25°C and 50kHz

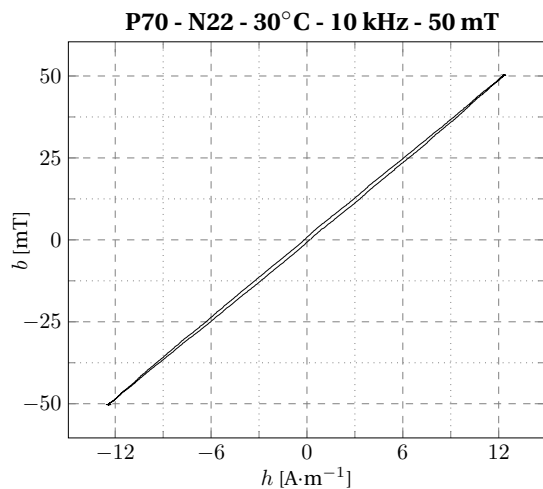


B.4 Characterization of Pot core P70×14.5 in N22 Material

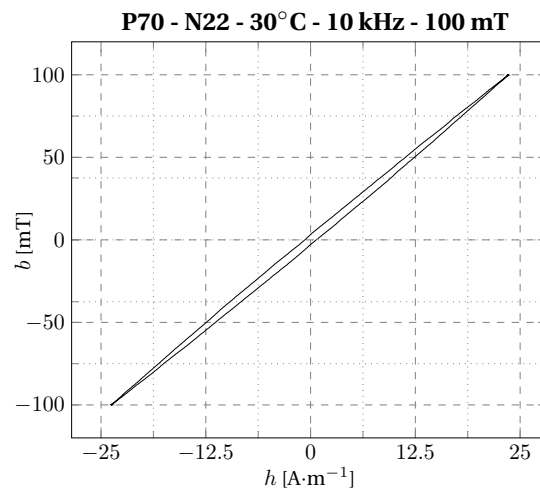


Figure B.5: Illustration of half a P70 Pot core.

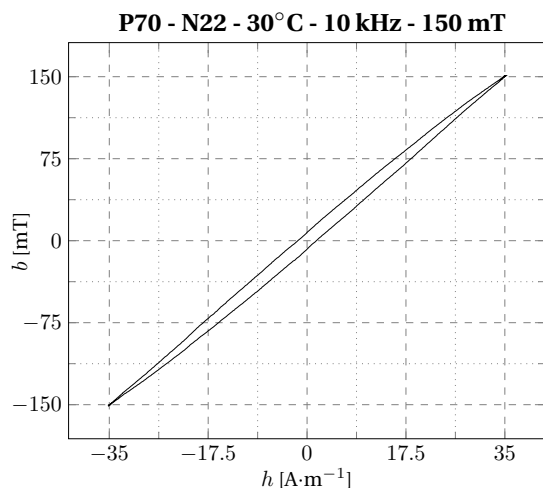
B.4.1 30°C and 10 kHz



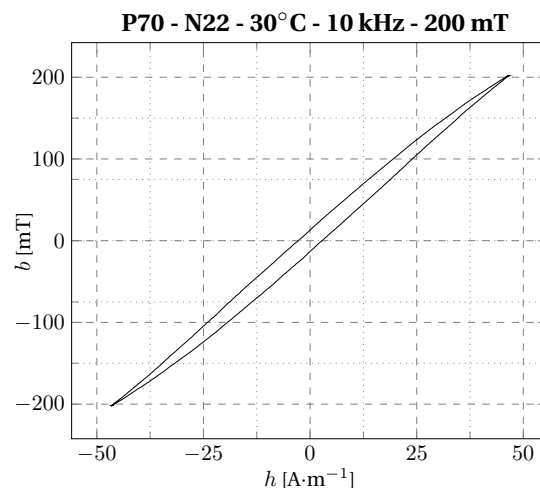
$b_{\max}$	50.36	b_r	<1
$h_{\max}$	12.42	h_c	<1
E_v	0.0267		



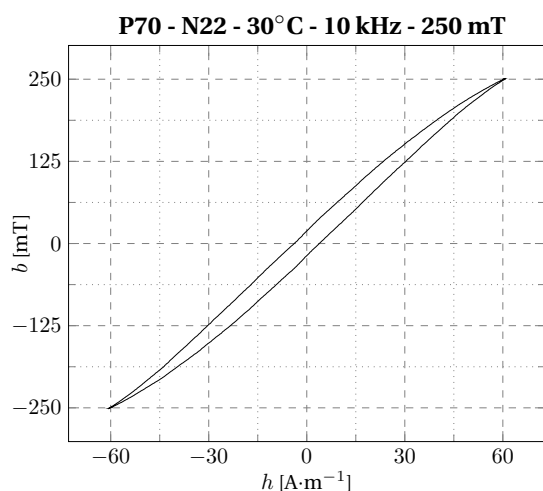
$b_{\max}$	99.98	b_r	3.7
$h_{\max}$	23.63	h_c	<1
E_v	0.1974		



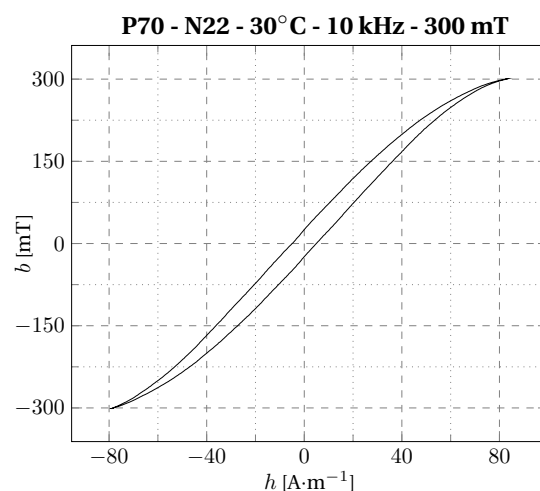
$b_{\max}$	150.97	b_r	8.8
$h_{\max}$	35.14	h_c	1.6
E_v	0.7213		



$b_{\max}$	201.95	b_r	14.3
$h_{\max}$	46.56	h_c	2.7
E_v	1.6353		

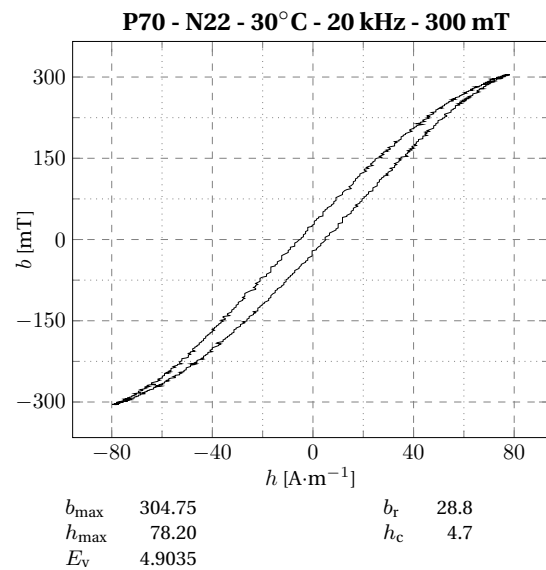
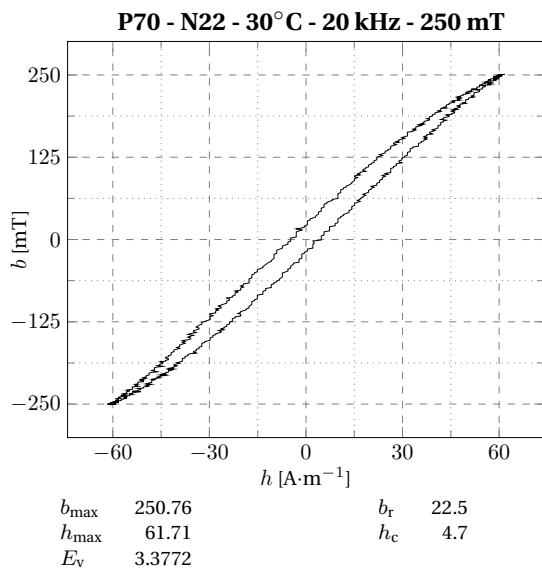
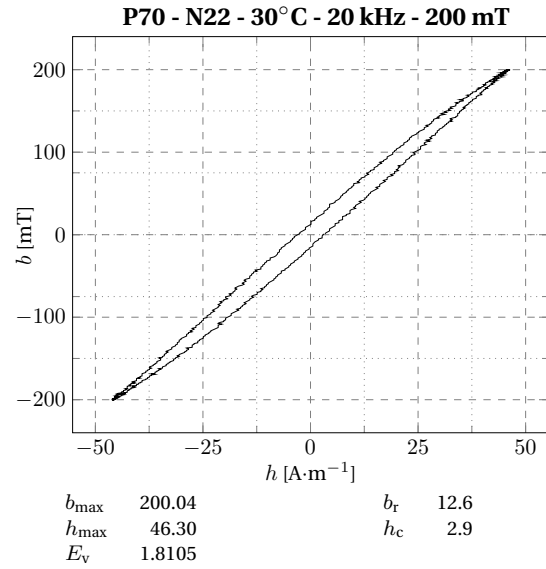
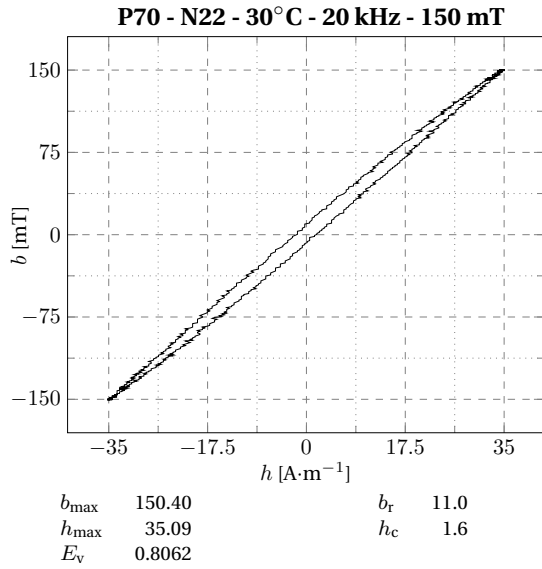
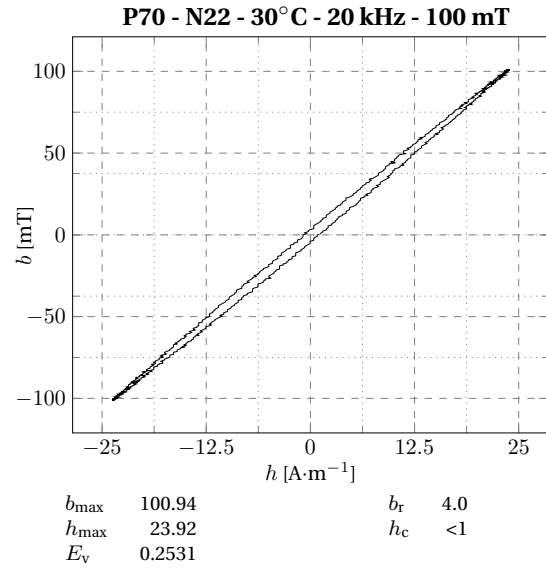
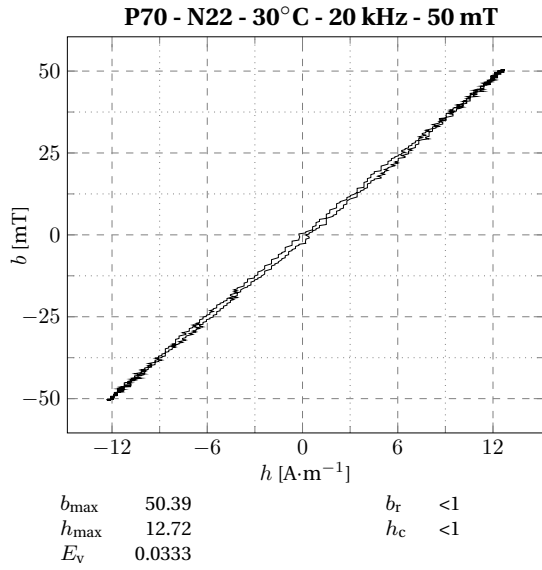


$b_{\max}$	251.12	b_r	19.4
$h_{\max}$	60.86	h_c	3.7
E_v	2.9447		

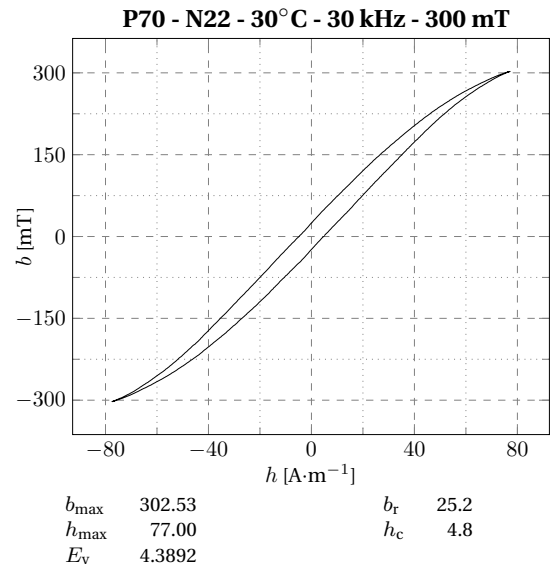
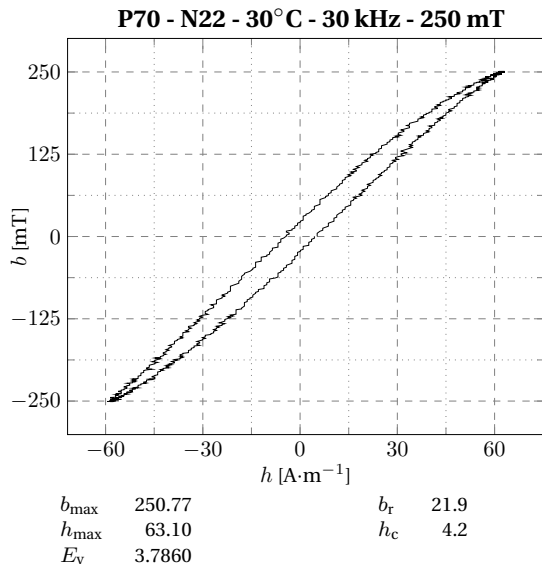
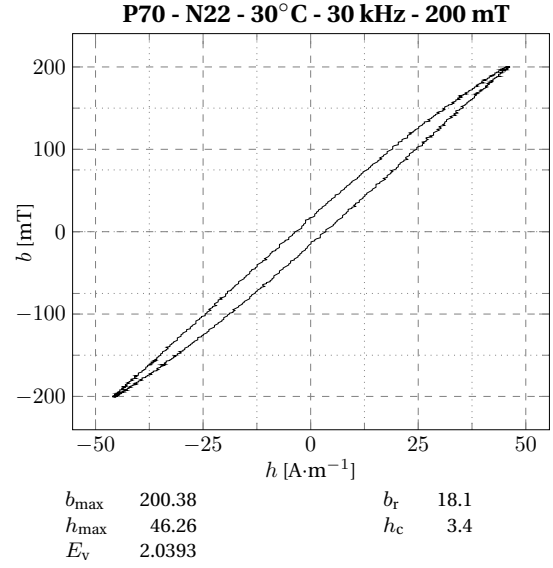
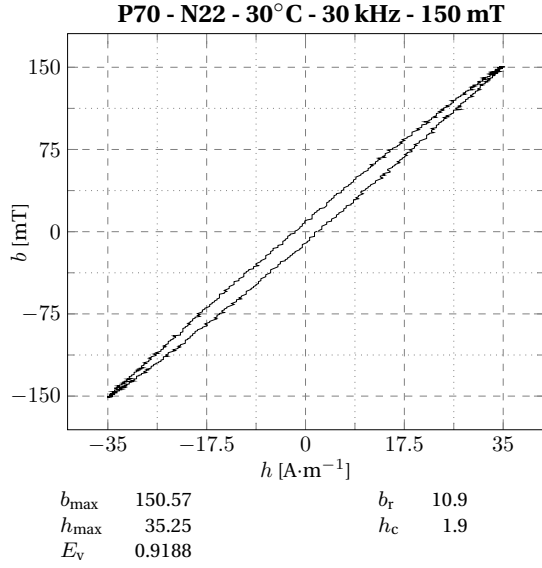
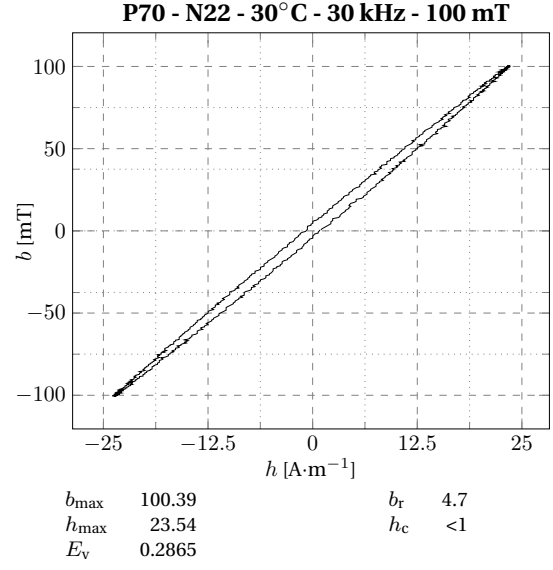
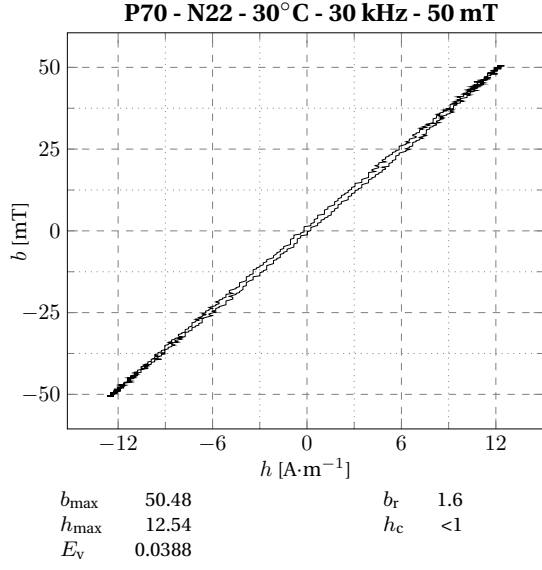


$b_{\max}$	301.17	b_r	25.5
$h_{\max}$	83.78	h_c	4.6
E_v	4.6273		

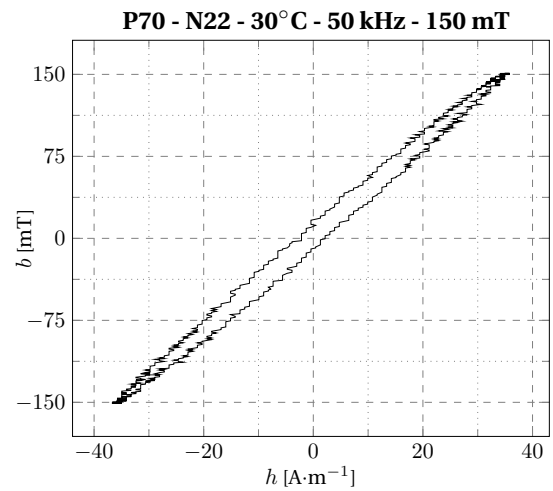
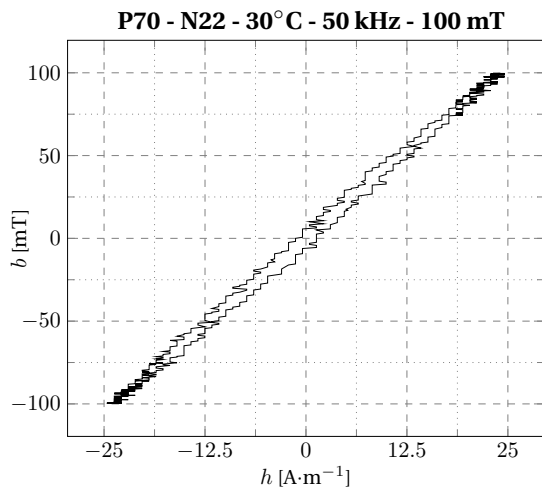
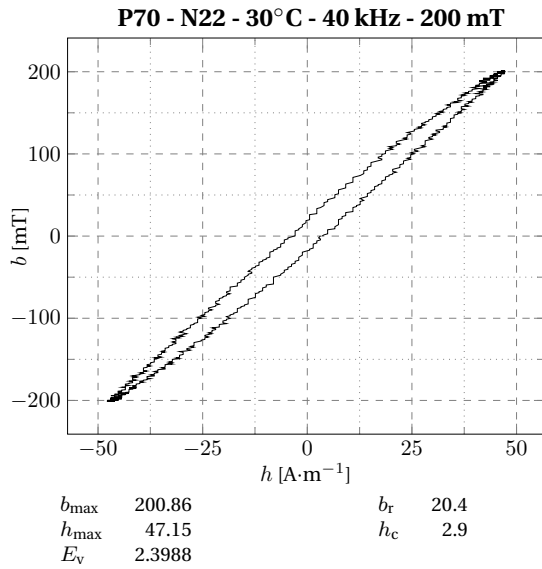
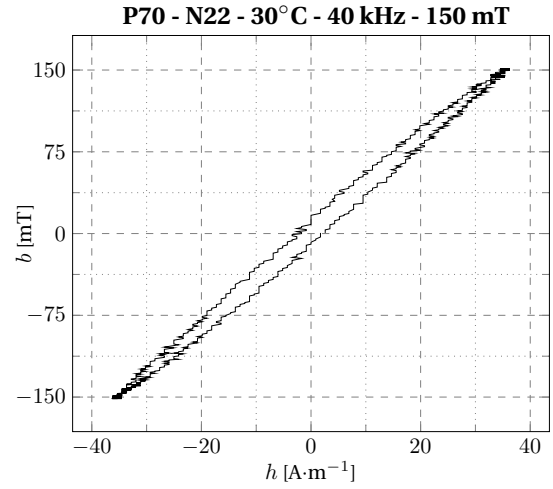
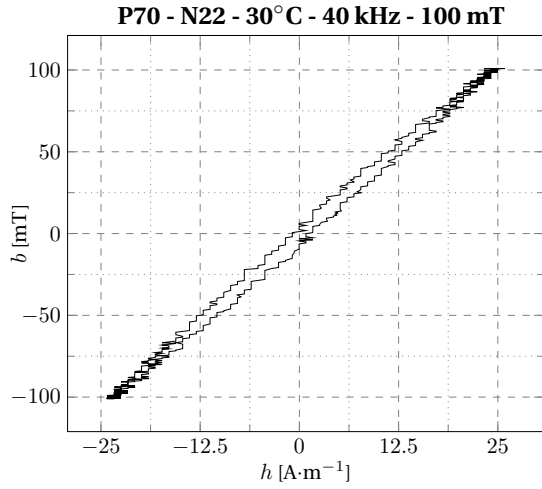
B.4.2 30°C and 20 kHz



B.4.3 30°C and 30 kHz



B.4.4 30°C and 40 kHz and 50 kHz

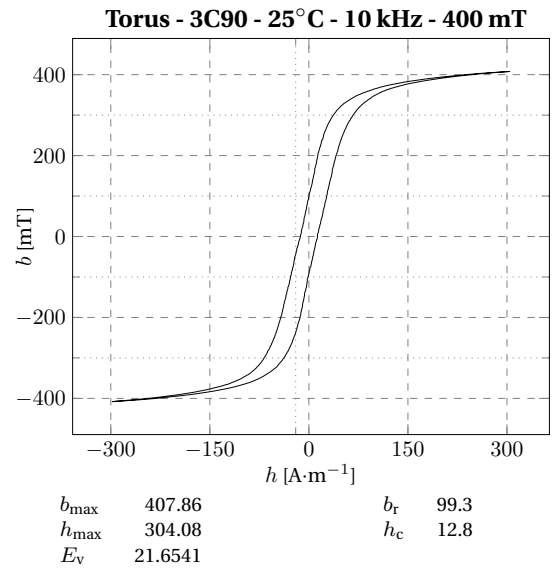
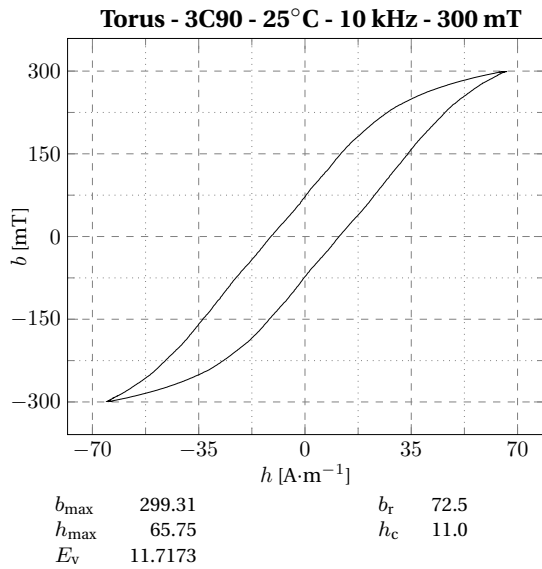
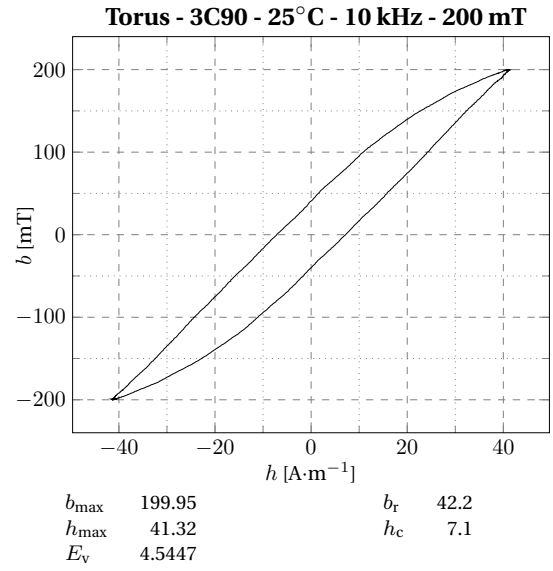
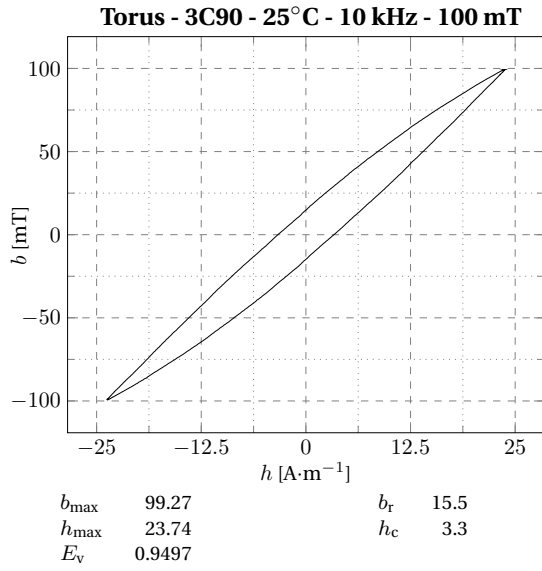
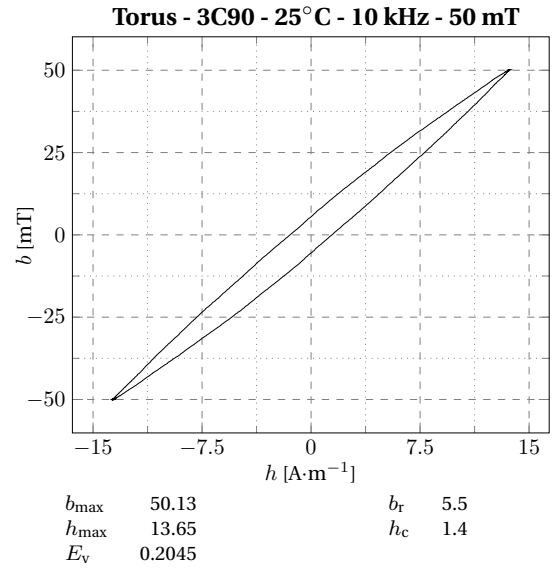
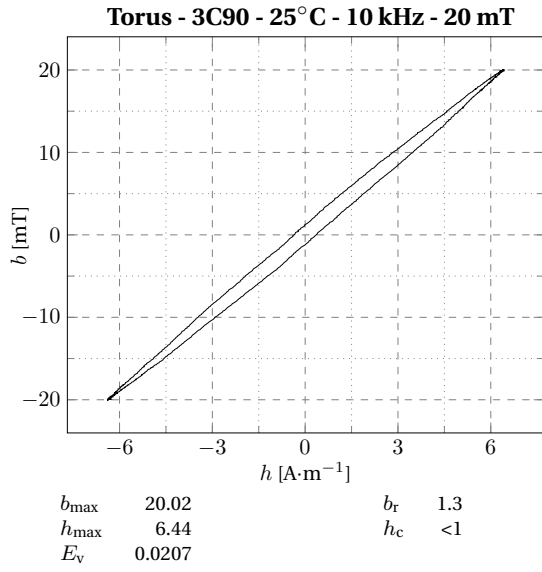


B.5 Characterization of Torus core in 3C90 Material

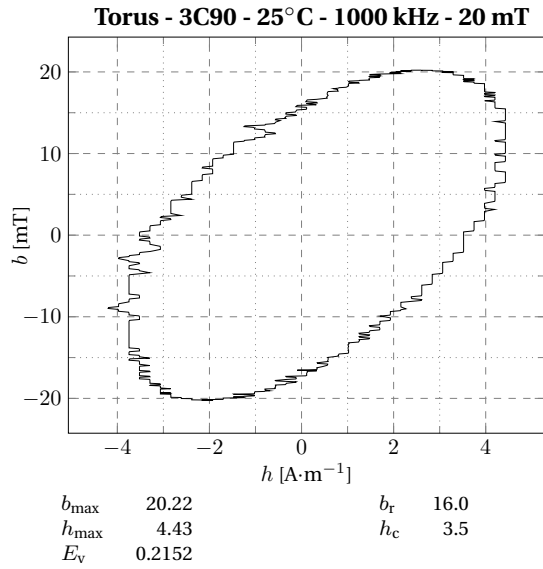
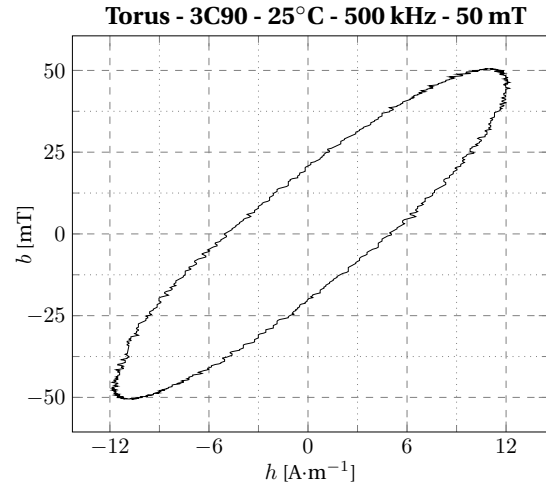
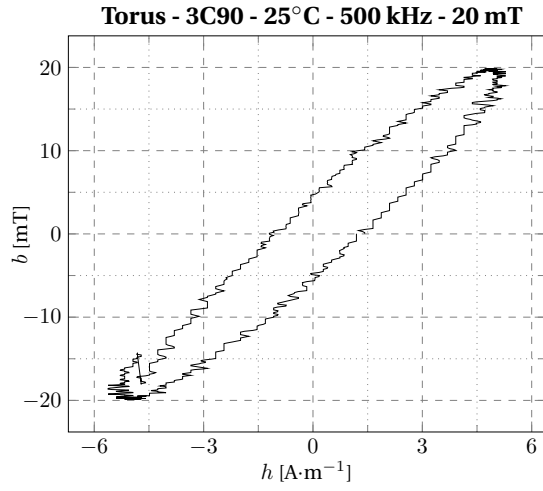


Figure B.6: Illustration of torus core of type T157/52.

B.5.1 25°C and 10 kHz



B.5.2 25°C and 500 kHz and 1 MHz



B.6 Complex Permeability Estimations

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
UU93-76-30	N87	25	5	50	2474.4	150.0	60	5	50	2908.8	98.4	100	5	50	3215.0	69.7
UU93-76-30	N87	25	5	100	2722.3	237.5	60	5	100	3032.2	156.0	100	5	100	3299.2	127.7
UU93-76-30	N87	25	5	150	2861.5	310.3	60	5	150	3077.7	207.4	100	5	150	3261.1	185.7
UU93-76-30	N87	25	5	200	2951.3	353.0	60	5	200	3087.8	239.9	100	5	200	3189.5	219.5
UU93-76-30	N87	25	5	250	2921.2	360.6	60	5	250	2996.9	253.9	100	5	250	2963.7	219.3
UU93-76-30	N87	25	5	300	2774.6	338.4	60	5	300	2763.9	235.7	100	5	300	2547.0	177.3
UU93-76-30	N87	25	10	50	2495.1	163.3	60	10	50	2909.0	110.6	100	10	50	3212.8	84.0
UU93-76-30	N87	25	10	100	2726.6	258.4	60	10	100	3030.2	178.8	100	10	100	3285.2	154.1
UU93-76-30	N87	25	10	150	2892.1	328.1	60	10	150	3105.8	227.3	100	10	150	3289.7	209.3
UU93-76-30	N87	25	10	200	2966.9	370.7	60	10	200	3102.9	260.9	100	10	200	3201.5	243.3
UU93-76-30	N87	25	10	250	2893.1	373.5	60	10	250	2976.3	269.8	100	10	250	2963.0	238.2
UU93-76-30	N87	25	10	300	2712.7	340.8	60	10	300	2619.0	229.3	100	10	300	2414.2	171.4
UU93-76-30	N87	25	15	50	2304.3	141.9	60	15	50	2763.3	115.1	100	15	50	2949.2	78.2
UU93-76-30	N87	25	15	100	2673.7	256.3	60	15	100	3004.6	187.4	100	15	100	3253.7	161.4
UU93-76-30	N87	25	15	150	2838.7	327.2	60	15	150	3080.5	234.6	100	15	150	3214.5	213.7
UU93-76-30	N87	25	15	200	2957.2	386.9	60	15	200	3086.4	280.0	100	15	200	3105.1	250.2
UU93-76-30	N87	25	15	250	2903.8	392.0	60	15	250	3008.5	298.6	100	15	250	2844.6	240.2
UU93-76-30	N87	25	15	300	2605.7	330.2	60	15	300	2539.6	230.1	100	15	300	2349.8	179.6
UU93-76-30	N87	25	20	50	2386.5	165.4	60	20	50	2882.5	140.3	100	20	50	3102.6	103.6
UU93-76-30	N87	25	20	100	2669.4	267.9	60	20	100	2960.1	196.6	100	20	100	3260.4	186.9
UU93-76-30	N87	25	20	150	2828.3	343.0	60	20	150	3072.5	258.3	100	20	150	3297.6	259.5
UU93-76-30	N87	25	20	200	2944.1	397.4	60	20	200	3053.7	294.3	100	20	200	3019.9	259.8
UU93-76-30	N87	25	20	250	2864.2	397.1	60	20	250	2888.3	292.1	100	20	250	2960.0	285.4
UU93-76-30	N87	25	20	300	2605.4	339.7	60	20	300	2603.0	257.6	100	20	300	2148.7	159.2
UU93-76-30	N87	25	25	50	2441.1	182.7	60	25	50	2785.5	139.8	100	25	50	2711.5	90.1
UU93-76-30	N87	25	25	100	2689.1	282.8	60	25	100	2984.7	219.6	100	25	100	3165.5	192.9
UU93-76-30	N87	25	25	150	2905.8	377.4	60	25	150	3093.2	285.6	100	25	150	2986.5	235.3
UU93-76-30	N87	25	25	200	2985.0	433.1	60	25	200	2946.9	295.3	100	25	200	3162.8	312.4
UU93-76-30	N87	25	25	250	2790.6	393.6	60	25	250	2985.5	333.6	100	25	250	2912.2	297.6
UU93-76-30	N87	25	25	300	2600.9	352.6	60	25	300	2573.9	266.1	100	25	300	2381.5	213.5
UU93-76-30	N87	25	30	50	2427.7	185.5	60	30	50	2856.9	161.1	100	30	50	3258.8	146.9
UU93-76-30	N87	25	30	100	2672.9	292.3	60	30	100	3039.9	244.2	100	30	100	3142.1	212.3
UU93-76-30	N87	25	30	150	2885.3	380.7	60	30	150	3090.0	304.0	100	30	150	3198.5	293.7
UU93-76-30	N87	25	30	200	2965.9	445.8	60	30	200	2995.9	325.0	100	30	200	3168.6	340.8
UU93-76-30	N87	25	30	250	2844.3	422.5	60	30	250	2712.7	288.4	100	30	250	2770.4	286.6
UU93-76-30	N87	25	30	300	2577.4	360.7	60	30	300	2685.0	308.8	100	30	300	2454.1	242.6
UU93-76-30	N87	25	40	50	2522.8	224.1	60	40	50	2851.1	177.1	100	40	50	3214.5	164.4
UU93-76-30	N87	25	40	100	2738.2	336.0	60	40	100	3028.4	269.6	100	40	100	3248.7	255.8
UU93-76-30	N87	25	40	150	2685.3	371.0	60	40	150	3107.1	344.6	100	40	150	3372.3	365.6
UU93-76-30	N87	25	40	200	2885.6	458.0	60	40	200	3089.2	386.2	100	40	200	3176.4	381.8
UU93-76-30	N87	25	50	50	3067.3	231.5	60	50	50	2870.6	215.5	100	50	50	3171.5	194.8
UU93-76-30	N87	25	50	100	3051.2	304.5	60	50	100	3079.6	320.0	100	50	100	3284.9	300.5
UU93-76-30	N87	25	50	150	3007.3	367.0	60	50	150	3110.2	391.2	100	50	150	3264.3	389.4

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
UU93-76-30	N97	25	5	50	2922.9	222.0	100	5	50	3266.2	61.1	100	5	50	3266.2	61.1
UU93-76-30	N97	25	5	100	3222.0	356.3		5	100	3381.8	118.6	100	5	100	3381.8	118.6
UU93-76-30	N97	25	5	150	3362.8	453.8		5	150	3355.8	174.4	100	5	150	3355.8	174.4
UU93-76-30	N97	25	5	200	3423.0	489.9		5	200	3298.7	211.7	100	5	200	3298.7	211.7
UU93-76-30	N97	25	5	250	3364.5	488.0		5	250	3056.9	210.4	100	5	250	3056.9	210.4
UU93-76-30	N97	25	5	300	3121.7	426.2		5	300	2562.7	160.8	100	5	300	2562.7	160.8
UU93-76-30	N97	25	10	50	2895.6	229.5		10	50	3273.6	70.3	100	10	50	3273.6	70.3
UU93-76-30	N97	25	10	100	3232.5	383.0		10	100	3380.1	140.1	100	10	100	3380.1	140.1
UU93-76-30	N97	25	10	150	3392.5	471.0		10	150	3400.8	199.9	100	10	150	3400.8	199.9
UU93-76-30	N97	25	10	200	3439.1	514.1		10	200	3310.6	235.3	100	10	200	3310.6	235.3
UU93-76-30	N97	25	10	250	3309.3	492.3	100	10	250	3026.6	223.5	100	10	250	3026.6	223.5
UU93-76-30	N97	25	10	300	3042.6	423.9		10	300	2387.0	148.6	100	10	300	2387.0	148.6
UU93-76-30	N97	25	15	50	2527.6	184.5		15	50	3148.5	84.4	100	15	50	3148.5	84.4
UU93-76-30	N97	25	15	100	3116.3	361.3		15	100	3285.6	132.4	100	15	100	3285.6	132.4
UU93-76-30	N97	25	15	150	3243.2	456.8		15	150	3298.4	210.9	100	15	150	3298.4	210.9
UU93-76-30	N97	25	15	200	3460.8	545.9		15	200	3229.2	246.2	100	15	200	3229.2	246.2
UU93-76-30	N97	25	15	250	3305.6	510.9		15	250	3067.3	254.5	100	15	250	3067.3	254.5
UU93-76-30	N97	25	15	300	3096.3	461.2		15	300	2195.2	137.1	100	15	300	2195.2	137.1
UU93-76-30	N97	25	20	50	2852.9	249.6		20	50	3155.6	89.1	100	20	50	3155.6	89.1
UU93-76-30	N97	25	20	100	3051.1	367.3		20	100	3311.9	161.6	100	20	100	3311.9	161.6
UU93-76-30	N97	25	20	150	3284.9	478.4	100	20	150	3285.4	226.9	100	20	150	3285.4	226.9
UU93-76-30	N97	25	20	200	3526.7	587.8		20	200	3109.4	248.4	100	20	200	3109.4	248.4
UU93-76-30	N97	25	20	250	3246.5	510.1		20	250	3098.2	280.1	100	20	250	3098.2	280.1
UU93-76-30	N97	25	20	300	2822.9	393.1		20	300	2064.9	131.4	100	20	300	2064.9	131.4
UU93-76-30	N97	25	25	50	2709.5	235.9		25	50	3080.8	83.5	100	25	50	3080.8	83.5
UU93-76-30	N97	25	25	100	3287.9	441.8		25	100	3376.9	190.0	100	25	100	3376.9	190.0
UU93-76-30	N97	25	25	150	3105.7	451.6		25	150	3369.6	262.4	100	25	150	3369.6	262.4
UU93-76-30	N97	25	25	200	3533.1	612.7		25	200	3072.4	261.1	100	25	200	3072.4	261.1
UU93-76-30	N97	25	25	250	3323.5	562.8		25	250	3063.5	297.9	100	25	250	3063.5	297.9
UU93-76-30	N97	25	25	300	2625.5	351.8		25	300	2466.5	205.0	100	25	300	2466.5	205.0
UU93-76-30	N97	25	30	50	2691.6	246.0	100	30	50	3028.9	98.4	100	30	50	3028.9	98.4
UU93-76-30	N97	25	30	100	3040.8	403.1		30	100	3379.0	217.2	100	30	100	3379.0	217.2
UU93-76-30	N97	25	30	150	3342.4	543.6		30	150	3293.8	272.4	100	30	150	3293.8	272.4
UU93-76-30	N97	25	30	200	3407.1	594.2		30	200	3247.3	325.3	100	30	200	3247.3	325.3
UU93-76-30	N97	25	30	250	3145.7	520.5		30	250	2994.9	302.9	100	30	250	2994.9	302.9
UU93-76-30	N97	25	30	300	3101.7	516.5		30	300	2416.0	213.0	100	30	300	2416.0	213.0
UU93-76-30	N97	25	40	50	2720.4	249.5		40	50	3048.1	143.2	100	40	50	3048.1	143.2
UU93-76-30	N97	25	40	100	2970.1	419.7		40	100	3095.8	217.0	100	40	100	3095.8	217.0
UU93-76-30	N97	25	40	150	3432.4	626.4		40	150	3389.8	340.5	100	40	150	3389.8	340.5
UU93-76-30	N97	25	40	200	3473.1	672.9		40	200	3183.3	370.6	100	40	200	3183.3	370.6
UU93-76-30	N97	25	50	50	2698.2	281.2	100	50	50	3302.5	180.1	100	50	50	3302.5	180.1
UU93-76-30	N97	25	50	100	3160.2	506.6		50	100	3158.8	255.6	100	50	100	3158.8	255.6
UU93-76-30	N97	25	50	150	3106.1	553.1		50	150	3313.9	369.3	100	50	150	3313.9	369.3

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
UU93-76-30	3C90	25	5	50	3117.3	214.9	60	5	50	3454.5	118.9	100	5	50	3607.0	114.1
UU93-76-30	3C90	25	5	100	3343.6	330.4	60	5	100	3597.1	195.5	100	5	100	3621.0	184.8
UU93-76-30	3C90	25	5	150	3358.8	390.2	60	5	150	3489.2	243.9	100	5	150	3398.8	226.1
UU93-76-30	3C90	25	5	200	3318.5	399.2	60	5	200	3373.1	260.8	100	5	200	3183.3	232.8
UU93-76-30	3C90	25	5	250	3159.5	370.1	60	5	250	3152.1	249.1	100	5	250	2840.2	206.3
UU93-76-30	3C90	25	5	300	2891.7	313.5	60	5	300	2789.8	211.1	100	5	300	2331.7	145.6
UU93-76-30	3C90	25	10	50	3085.4	227.5	60	10	50	3461.4	136.9	100	10	50	3607.2	135.6
UU93-76-30	3C90	25	10	100	3324.3	364.0	60	10	100	3562.8	220.9	100	10	100	3588.8	218.9
UU93-76-30	3C90	25	10	150	3384.2	416.7	60	10	150	3503.8	270.0	100	10	150	3394.1	253.8
UU93-76-30	3C90	25	10	200	3335.0	427.3	60	10	200	3395.0	287.3	100	10	200	3148.3	258.0
UU93-76-30	3C90	25	10	250	3145.5	387.4	60	10	250	3082.2	261.5	100	10	250	2787.6	218.3
UU93-76-30	3C90	25	10	300	2887.5	330.1	60	10	300	2702.2	214.6	100	10	300	2282.1	152.4
UU93-76-30	3C90	25	15	50	2947.0	229.7	60	15	50	3147.2	124.6					
UU93-76-30	3C90	25	15	100	3210.2	349.8	60	15	100	3494.7	236.4	100	15	100	3325.1	208.8
UU93-76-30	3C90	25	15	150	3277.3	409.9	60	15	150	3446.9	283.2	100	15	150	3448.2	289.7
UU93-76-30	3C90	25	15	200	3201.9	417.7	60	15	200	3430.3	315.4	100	15	200	3145.0	280.3
UU93-76-30	3C90	25	15	250	2916.3	350.4	60	15	250	2884.0	251.0	100	15	250	2644.5	223.0
UU93-76-30	3C90	25	15	300	2864.4	346.0	60	15	300	2816.6	257.3	100	15	300	2190.8	157.3
UU93-76-30	3C90	25	20	50	2861.3	235.9	60	20	50	3437.4	180.1					
UU93-76-30	3C90	25	20	100	3309.3	400.4	60	20	100	3480.2	250.2	100	20	100	3561.9	268.1
UU93-76-30	3C90	25	20	150	3281.5	448.5	60	20	150	3253.1	278.2	100	20	150	3277.3	290.9
UU93-76-30	3C90	25	20	200	3201.9	443.8	60	20	200	3237.1	311.3	100	20	200	2882.2	264.5
UU93-76-30	3C90	25	20	250	2945.6	385.5	60	20	250	2894.1	275.3	100	20	250	2579.8	241.4
UU93-76-30	3C90	25	20	300	2697.5	331.6	60	20	300	2413.9	202.8	100	20	300	2131.6	171.7
UU93-76-30	3C90	25	25	50	2968.3	242.3	60	25	50	3185.8	160.2					
UU93-76-30	3C90	25	25	100	3205.8	396.8	60	25	100	3081.6	224.6	100	25	100	3515.1	306.4
UU93-76-30	3C90	25	25	150	3348.7	480.5	60	25	150	3356.3	323.4	100	25	150	3351.6	351.0
UU93-76-30	3C90	25	25	200	3235.1	479.4	60	25	200	3328.0	369.0	100	25	200	3047.4	327.4
UU93-76-30	3C90	25	25	250	2874.4	388.8	60	25	250	2863.1	292.9	100	25	250	2600.7	257.0
UU93-76-30	3C90	25	25	300	2877.0	395.1	60	25	300	2611.9	257.3	100	25	300	2092.7	174.1
UU93-76-30	3C90	25	30	50	2767.1	238.6	60	30	50	3208.3	190.3					
UU93-76-30	3C90	25	30	100	3173.1	409.3	60	30	100	3430.1	297.4	100	30	100	3330.5	281.9
UU93-76-30	3C90	25	30	150	3268.3	500.3	60	30	150	3466.9	379.8	100	30	150	3348.3	373.8
UU93-76-30	3C90	25	30	200	3246.8	513.2	60	30	200	3288.8	388.2	100	30	200	3149.3	382.7
UU93-76-30	3C90	25	30	250	3003.1	442.4	60	30	250	3107.8	373.4	100	30	250	2536.6	263.9
UU93-76-30	3C90	25	30	300	2410.0	287.6	60	30	300	2507.6	256.1	100	30	300	1759.3	132.3
UU93-76-30	3C90	25	40	50	2778.5	295.2	60	40	50	3417.3	274.7	100	40	50	3565.3	274.0
UU93-76-30	3C90	25	40	100	2954.7	394.0	60	40	100	3276.6	320.0	100	40	100	3020.7	277.5
UU93-76-30	3C90	25	40	150	3450.0	600.7	60	40	150	3362.3	409.6	100	40	150	3055.2	360.0
UU93-76-30	3C90	25	40	200	3212.9	554.9	60	40	200	3256.6	435.9	100	40	200	2658.9	306.4
UU93-76-30	3C90	25	50	50	3026.0	366.6	60	50	50	3136.7	237.2	100	50	50	3410.6	308.7
UU93-76-30	3C90	25	50	100	3313.4	555.3	60	50	100	3214.0	357.3	100	50	100	3731.7	500.3
UU93-76-30	3C90	25	50	150	3622.8	766.0	60	50	150	3166.5	410.8	100	50	150	2993.8	393.9

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
Torus	3C90	25	10	20	2470.0	126.5
Torus	3C90	25	10	50	2908.5	277.4
Torus	3C90	25	10	100	3299.0	425.0
Torus	3C90	25	10	200	3790.6	669.2
Torus	3C90	25	10	300	3556.3	680.5
Torus	3C90	25	10	400	1065.7	59.3
Torus	3C90	25	20	20	2374.7	129.1
Torus	3C90	25	20	50	2697.9	250.6
Torus	3C90	25	20	100	3295.1	466.2
Torus	3C90	25	20	200	3776.1	738.7
Torus	3C90	25	20	300	3592.1	765.1
Torus	3C90	25	50	20	2371.9	175.3
Torus	3C90	25	50	50	2860.4	359.2
Torus	3C90	25	50	100	3301.4	583.0
Torus	3C90	25	70	20	2593.4	208.0
Torus	3C90	25	70	50	2959.6	434.5
Torus	3C90	25	70	100	3369.8	605.6
Torus	3C90	25	100	20	2571.4	249.2
Torus	3C90	25	100	50	2976.5	470.9
Torus	3C90	25	200	20	2683.1	291.1
Torus	3C90	25	200	50	3007.5	528.2
Torus	3C90	25	200	100	3316.5	890.3
Torus	3C90	25	500	20	2857.1	695.5
Torus	3C90	25	500	50	3066.0	1339.3
Torus	3C90	25	1000	20	2399.0	2838.7
TX13-7-5	3C90	30	5	50	4094.1	331.5
TX13-7-5	3C90	30	5	100	4392.5	452.6
TX13-7-5	3C90	30	5	150	4623.9	583.4
TX13-7-5	3C90	30	5	200	4917.9	764.6
TX13-7-5	3C90	30	5	250	4857.7	832.4
TX13-7-5	3C90	30	5	300	4604.0	822.8
TX13-7-5	3C90	30	10	50	3729.8	230.6
TX13-7-5	3C90	30	10	100	4348.8	488.8
TX13-7-5	3C90	30	10	150	4493.5	606.7
TX13-7-5	3C90	30	10	200	4935.3	816.3
TX13-7-5	3C90	30	10	250	4855.6	877.6
TX13-7-5	3C90	30	10	300	4604.0	822.8
TX13-7-5	3C90	30	15	50	2726.3	119.0
TX13-7-5	3C90	30	15	100	3172.2	263.4
TX13-7-5	3C90	30	15	150	4643.1	676.8
TX13-7-5	3C90	30	15	200	4588.6	727.3
TX13-7-5	3C90	30	15	250	4614.6	831.3
TX13-7-5	3C90	30	15	300	4011.1	723.2
TX13-7-5	3C90	30	20	50	2679.9	96.4
TX13-7-5	3C90	30	20	100	3579.6	306.4
TX13-7-5	3C90	30	20	150	4090.9	515.6
TX13-7-5	3C90	30	20	200	4307.9	708.4
TX13-7-5	3C90	30	20	250	4593.6	826.6
TX13-7-5	3C90	30	20	300	4548.7	976.5
TX13-7-5	3C90	30	25	50	3161.7	220.9
TX13-7-5	3C90	30	25	100	3132.3	285.5
TX13-7-5	3C90	30	25	150	4813.4	825.8
TX13-7-5	3C90	30	25	200	4580.3	832.4
TX13-7-5	3C90	30	25	250	4489.0	806.1
TX13-7-5	3C90	30	25	300	4283.2	858.2
TX13-7-5	3C90	30	30	50	2704.8	130.6
TX13-7-5	3C90	30	30	100	4046.7	499.7
TX13-7-5	3C90	30	30	150	4422.9	680.1
TX13-7-5	3C90	30	30	200	4917.5	976.6
TX13-7-5	3C90	30	30	250	4221.1	776.0
TX13-7-5	3C90	30	30	300	3977.6	809.6
TX13-7-5	3C90	30	40	50	2696.9	119.6
TX13-7-5	3C90	30	40	100	3370.0	418.3
TX13-7-5	3C90	30	40	150	4365.6	796.6
TX13-7-5	3C90	30	40	200	4548.9	885.7
TX13-7-5	3C90	30	50	50	2711.2	155.6
TX13-7-5	3C90	30	50	100	3105.6	374.2
TX13-7-5	3C90	30	50	150	3635.3	563.2

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
TX51-32-19	3C90	30	5	50	3110.0	204.1	60	5	50	3569.0	128.0	100	5	50	3445.2	77.0
TX51-32-19	3C90	30	5	100	3455.3	354.5	60	5	100	3759.3	227.9	100	5	100	3636.8	154.3
TX51-32-19	3C90	30	5	150	3643.4	454.3	60	5	150	3845.6	309.7	100	5	150	3726.4	244.5
TX51-32-19	3C90	30	5	200	3766.1	545.9	60	5	200	3823.1	371.8	100	5	200	3699.4	319.1
TX51-32-19	3C90	30	5	250	3669.6	578.4	60	5	250	3584.4	387.8	100	5	250	3546.0	347.1
TX51-32-19	3C90	30	5	300	3415.3	527.0	60	5	300	3098.0	322.7	100	5	300	2285.1	151.5
TX51-32-19	3C90	30	10	50	3128.9	219.6	60	10	50	3554.8	144.8	100	10	50	3506.6	80.3
TX51-32-19	3C90	30	10	100	3459.0	367.7	60	10	100	3730.3	245.2	100	10	100	3609.4	174.4
TX51-32-19	3C90	30	10	150	3659.6	494.3	60	10	150	3825.5	333.1	100	10	150	3712.2	277.8
TX51-32-19	3C90	30	10	200	3772.0	584.4	60	10	200	3817.4	401.0	100	10	200	3759.5	360.4
TX51-32-19	3C90	30	10	250	3684.8	604.6	60	10	250	3740.4	456.9	100	10	250	3381.0	334.1
TX51-32-19	3C90	30	10	300	3203.4	479.2	60	10	300	3462.6	433.8	100	10	300	1525.4	69.3
TX51-32-19	3C90	30	15	50	3055.5	211.4	60	15	50	3391.3	153.0					
TX51-32-19	3C90	30	15	100	3424.2	378.3	60	15	100	3562.6	253.6	100	15	100	3493.7	167.4
TX51-32-19	3C90	30	15	150	3618.2	501.8	60	15	150	3928.3	366.9	100	15	150	3653.3	284.2
TX51-32-19	3C90	30	15	200	3760.3	603.5	60	15	200	3556.2	365.4	100	15	200	3334.4	295.1
TX51-32-19	3C90	30	15	250	3567.4	589.2	60	15	250	3740.4	484.4	100	15	250	2475.4	179.8
TX51-32-19	3C90	30	15	300	3463.1	585.3	60	15	300	3411.9	450.3	100	15	300	624.5	12.0
TX51-32-19	3C90	30	20	50	3067.0	224.6	60	20	50	3395.1	182.4					
TX51-32-19	3C90	30	20	100	3372.6	389.3	60	20	100	3593.6	276.8	100	20	100	3535.5	192.9
TX51-32-19	3C90	30	20	150	3615.4	527.2	60	20	150	3695.6	341.3	100	20	150	3440.3	266.2
TX51-32-19	3C90	30	20	200	3722.4	624.6	60	20	200	3322.5	331.9	100	20	200	3380.5	341.2
TX51-32-19	3C90	30	20	250	3597.2	629.2	60	20	250	3765.6	542.5	100	20	250	2399.6	184.8
TX51-32-19	3C90	30	20	300	2668.8	352.3	60	20	300	3363.1	467.6	100	20	300	329.6	3.0
TX51-32-19	3C90	30	25	50	3092.4	251.8	60	25	50	3351.3	181.1					
TX51-32-19	3C90	30	25	100	3390.1	403.7	60	25	100	3619.1	288.7	100	25	100	3670.1	237.6
TX51-32-19	3C90	30	25	150	3600.3	542.7	60	25	150	3886.0	404.9	100	25	150	3747.3	364.7
TX51-32-19	3C90	30	25	200	3532.8	577.3	60	25	200	3687.5	433.7	100	25	200	3589.4	414.7
TX51-32-19	3C90	30	25	250	3504.6	646.9	60	25	250	3785.0	556.9	100	25	250	2550.7	230.1
TX51-32-19	3C90	30	25	300	3429.8	620.4	60	25	300	3494.6	492.2	100	25	300	480.8	8.4
TX51-32-19	3C90	30	30	50	3105.0	267.8	60	30	50	3303.2	185.0					
TX51-32-19	3C90	30	30	100	3388.5	423.2	60	30	100	3596.0	308.5	100	30	100	3430.0	208.7
TX51-32-19	3C90	30	30	150	3485.5	532.2	60	30	150	3646.1	389.9	100	30	150	3470.4	315.7
TX51-32-19	3C90	30	30	200	3586.8	622.0	60	30	200	3902.8	530.7	100	30	200	3691.3	460.8
TX51-32-19	3C90	30	30	250	3271.5	544.1	60	30	250	3748.9	584.8	100	30	250	2976.0	330.4
TX51-32-19	3C90	30	30	300	3669.8	700.1	60	30	300	3682.4	533.0	100	30	300	937.8	32.5
TX51-32-19	3C90	30	40	50	3097.2	295.4	60	40	50	3424.7	218.3	100	40	50	3496.2	140.6
TX51-32-19	3C90	30	40	100	3422.0	472.0	60	40	100	3502.1	327.6	100	40	100	3534.0	251.2
TX51-32-19	3C90	30	40	150	3624.5	619.3	60	40	150	3722.5	444.4	100	40	150	3652.4	412.4
TX51-32-19	3C90	30	40	200	3771.1	742.9	60	40	200	3560.6	484.6	100	40	200	3249.1	401.0
TX51-32-19	3C90	30	50	50	3010.3	301.7	60	50	50	3329.5	222.6	100	50	50	3495.8	167.8
TX51-32-19	3C90	30	50	100	3335.3	500.6	60	50	100	3509.7	349.6	100	50	100	3611.6	292.8
TX51-32-19	3C90	30	50	150	3562.3	660.7	60	50	150	3768.6	528.6	100	50	150	3673.2	455.6
TX51-32-19	3C90						60	100	50	3292.9	208.2					
TX51-32-19	3C90						60	100	100	3716.8	421.9					

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
E43-10-28	3C95	30	5	50	1646.1	25.2	60	5	50	1685.4	21.5	100	5	50	1761.9	15.1
E43-10-28	3C95	30	5	100	1700.7	44.2	60	5	100	1735.2	36.2	100	5	100	1771.2	34.0
E43-10-28	3C95	30	5	150	1732.5	56.4	60	5	150	1769.3	50.2	100	5	150	1778.7	50.0
E43-10-28	3C95	30	5	200	1723.0	70.0	60	5	200	1760.0	66.5	100	5	200	1736.9	65.8
E43-10-28	3C95	30	5	250	1701.5	76.5	60	5	250	1722.6	72.0	100	5	250	1622.1	65.8
E43-10-28	3C95	30	5	300	1644.3	78.0	60	5	300	1632.8	73.2					
E43-10-28	3C95	30	10	50	1646.2	30.1	60	10	50	1687.4	25.0					
E43-10-28	3C95	30	10	100	1687.0	47.5	60	10	100	1741.1	42.3	100	10	100	1759.3	41.1
E43-10-28	3C95	30	10	150	1731.9	61.1	60	10	150	1780.1	55.8	100	10	150	1775.9	57.2
E43-10-28	3C95	30	10	200	1716.4	71.5	60	10	200	1783.7	67.5	100	10	200	1760.3	70.9
E43-10-28	3C95	30	10	250	1699.9	78.5	60	10	250	1709.3	71.9	100	10	250	1285.5	43.0
E43-10-28	3C95	30	10	300	1603.1	74.8	60	10	300	1581.4	66.9					
							60	15	100	1689.2	37.5					
E43-10-28	3C95	30	15	150	1594.4	51.8	60	15	150	1730.3	56.7					
E43-10-28	3C95	30	15	200	1660.2	67.6	60	15	200	1707.6	66.9					
E43-10-28	3C95	30	15	250	1675.7	74.9	60	15	250	1679.5	71.9					
E43-10-28	3C95	30	15	300	1603.1	79.8	60	15	300	1588.7	71.4					
							60	20	100	1693.3	41.2					
E43-10-28	3C95	30	20	150	1735.7	66.4	60	20	150	1748.9	60.7					
E43-10-28	3C95	30	20	200	1635.2	76.6	60	20	200	1770.9	77.1					
E43-10-28	3C95	30	20	250	1696.2	86.3	60	20	250	1731.6	83.1					
E43-10-28	3C95	30	20	300	1393.3	61.7	60	20	300	1592.4	75.3					
							60	25	100	1740.9	44.1					
E43-10-28	3C95	30	25	150	1647.7	63.6	60	25	150	1723.2	65.9					
E43-10-28	3C95	30	25	200	1682.7	82.9	60	25	200	1726.4	77.4					
E43-10-28	3C95	30	25	250	1641.6	88.3	60	25	250	1594.4	72.2					
E43-10-28	3C95	30	25	300	1421.2	69.0	60	25	300	1667.5	87.7					
							60	30	100	1680.8	49.4					
E43-10-28	3C95	30	30	150	1677.8	73.3	60	30	150	1764.5	71.5					
E43-10-28	3C95	30	30	200	1581.8	75.9	60	30	200	1679.2	76.9					
E43-10-28	3C95	30	30	250	1624.0	94.3	60	30	250	1546.4	72.6					
E43-10-28	3C95	30	30	300	1550.1	88.7	60	30	300	1632.8	89.3					
E43-10-28	3C95	30	40	150	1693.8	87.0	60	40	150	1728.5	78.4					
E43-10-28	3C95	30	50	150	1687.9	100.0	60	50	150	1721.6	86.0					

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
ETD39-20-13	3C95	30	5	100	2471.5	52.7	60	5	100	2406.6	53.6	100	5	50	2477.2	23.7
ETD39-20-13	3C95	30	5	150	2516.6	81.6						100	5	100	2482.9	59.3
ETD39-20-13	3C95	30	5	200	2429.3	98.6	60	5	200	2461.0	98.1	100	5	150	2516.9	92.1
ETD39-20-13	3C95	30	5	250	2257.2	103.9	60	5	250	2345.9	116.7	100	5	200	2473.9	117.5
ETD39-20-13	3C95	30	5	300	2184.2	109.4	60	5	300	2210.4	114.8	100	5	250	2127.1	110.3
ETD39-20-13	3C95	30	10	100	2395.7	64.3						100	5	300	1088.9	31.8
ETD39-20-13	3C95	30	10	150	2450.7	89.3	60	10	100	2439.6	58.9	100	10	50	2418.3	35.8
ETD39-20-13	3C95	30	10	200	2395.2	107.3						100	10	100	2497.3	78.4
ETD39-20-13	3C95	30	10	250	2189.0	100.8	60	10	200	2444.4	104.7	100	10	150	2511.7	104.4
ETD39-20-13	3C95	30	10	300	1865.6	82.9	60	10	250	2422.9	121.1	100	10	200	2358.4	117.9
							60	10	300	2158.4	108.1	100	10	250	1822.6	81.6
												100	10	300	1803.4	90.7
												100	15	100	2275.1	65.2
												100	15	150	2400.4	99.2
												100	15	200	2448.4	135.8
												100	15	250	2318.5	146.1
												100	15	300	2088.4	127.2
							60	20	100	2380.6	59.9	100	20	100	2369.9	69.1
												100	20	150	2494.5	121.8
												100	20	200	2347.9	139.8
							60	20	250	2251.3	116.4	100	20	250	2372.9	159.2
							60	20	300	2183.4	125.9	100	20	300	2119.2	136.9
												100	25	50	2154.5	53.4
												100	25	100	2392.2	89.8
												100	25	150	2432.9	119.7
												100	25	200	2154.3	127.6
												100	25	250	2220.2	154.7
												100	25	300	2109.5	150.6
							60	30	100	2222.6	69.4	100	30	50	2375.6	63.8
												100	30	100	2351.6	86.9
												100	30	150	2324.1	115.8
							60	30	200	2370.0	121.9	100	30	200	2260.6	144.8
							60	30	250	2139.1	120.9	100	30	250	2547.8	209.2
							60	30	300	1843.8	95.7	100	30	300	2049.9	144.6
ETD39-20-13	3C95	30	40	50	2292.1	67.9										
ETD39-20-13	3C95	30	40	100	2336.7	111.5										
ETD39-20-13	3C95	30	40	150	2358.7	139.6	60	40	150	2344.1	109.6					
ETD39-20-13	3C95	30	40	200	2411.3	169.8										
ETD39-20-13	3C95	30	50	50	2190.6	75.1										
ETD39-20-13	3C95	30	50	100	2377.0	121.8	60	50	100	2370.9	94.0	100	50	100	2064.4	103.4
ETD39-20-13	3C95	30	50	150	2344.5	145.1	60	50	150	2330.3	123.1					
												100	50	200	2113.8	177.9
												100	50	300	1787.2	145.1
							60	100	100	2426.9	63.1					
							60	100	150	2471.3	127.0	100	100	100	2119.6	101.4
							60	100	200	2346.6	105.0					

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
ETD34-17-11	3C97	25	5	50	2596.0	62.9	60	5	100	2799.8	91.8	100	5	50	2730.6	67.2
ETD34-17-11	3C97	25	5	100	2729.2	93.2						100	5	100	2903.6	111.2
ETD34-17-11	3C97	25	5	150	2828.0	137.0						100	5	200	2980.1	190.4
ETD34-17-11	3C97	25	5	200	2862.7	167.7	100	5	250	2871.8	199.3					
ETD34-17-11	3C97	25	5	250	2831.8	190.2	100	5	300	2033.1	109.3					
ETD34-17-11	3C97	25	5	300	2767.4	194.8	60	5	300	2567.8	165.3	100	5	300	2033.1	109.3
ETD34-17-11	3C97	25	10	50	2629.9	68.5	60	10	100	2744.2	98.3	100	10	50	2870.8	68.0
ETD34-17-11	3C97	25	10	100	2774.3	118.6						100	10	100	2924.7	116.3
ETD34-17-11	3C97	25	10	150	2839.4	149.5						100	10	150	3072.0	177.8
ETD34-17-11	3C97	25	10	200	2868.7	184.2	60	10	200	2904.8	175.6	100	10	250	2869.5	216.9
ETD34-17-11	3C97	25	10	250	2816.8	198.2	60	10	250	2813.9	192.9					
ETD34-17-11	3C97	25	10	300	2684.7	206.0	60	10	300	2564.4	176.2					
ETD34-17-11	3C97	25	15	50	2401.7	60.3	60	15	100	2635.4	103.3	100	15	100	2869.5	216.9
ETD34-17-11	3C97	25	15	100	2612.4	99.9										
ETD34-17-11	3C97	25	15	150	2752.2	143.5										
ETD34-17-11	3C97	25	15	200	2818.6	184.1	60	15	250	2750.6	190.1					
ETD34-17-11	3C97	25	15	250	2691.9	196.5	60	15	300	2432.0	165.5					
ETD34-17-11	3C97	25	15	300	2696.6	216.3	60	15	300	2432.0	165.5	100	15	300	2434.4	170.4
ETD34-17-11	3C97	25	20	50	2511.8	59.6										
ETD34-17-11	3C97	25	20	100	2653.9	108.8										
ETD34-17-11	3C97	25	20	150	2710.4	147.7	60	20	100	2645.9	107.3	100	20	100	2869.5	216.9
ETD34-17-11	3C97	25	20	200	2836.1	201.6	60	20	200	2895.8	202.6					
ETD34-17-11	3C97	25	20	250	2702.7	205.4	60	20	250	2796.0	216.1					
ETD34-17-11	3C97	25	20	300	2631.7	211.2	60	20	300	2634.8	207.8	100	20	250	2869.5	216.9
ETD34-17-11	3C97	25	25	50	2492.2	65.9										
ETD34-17-11	3C97	25	25	100	2680.2	113.7	60	25	100	2680.4	117.9					
ETD34-17-11	3C97	25	25	150	2787.0	174.8										
ETD34-17-11	3C97	25	25	200	2770.8	204.2						60	25	200	2964.6	214.1
ETD34-17-11	3C97	25	25	250	2737.6	225.7	60	25	250	2830.6	228.4	100	25	250	2869.5	216.9
ETD34-17-11	3C97	25	25	300	2592.1	224.0	60	25	300	2650.2	225.6					
ETD34-17-11	3C97	25	30	50	2648.0	93.3	60	30	100	2757.8	132.9					
ETD34-17-11	3C97	25	30	100	2611.8	116.3										
ETD34-17-11	3C97	25	30	150	2750.3	175.4						60	30	200	2788.9	213.8
ETD34-17-11	3C97	25	30	200	2871.2	231.0	60	30	250	2856.7	257.0	100	30	250	2869.5	216.9
ETD34-17-11	3C97	25	30	250	2810.5	257.1	60	30	300	2588.2	224.3					
ETD34-17-11	3C97	25	30	300	2185.5	163.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	50	2369.7	91.2										
ETD34-17-11	3C97	25	40	100	2503.3	124.2						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	150	2839.2	203.3										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.2	203.3						60	40	100	2503.3	124.2
ETD34-17-11	3C97	25	40	200	2835.1	251.6										
ETD34-17-11	3C97	25	40	200	2835.1	251.6	60	40	100	2503.3	124.2					
ETD34-17-11	3C97	25	40	100	2503.3	124.2										
ETD34-17-11	3C97	25	40	150	2839.											

FERROMAGNETIC MATERIALS CHARACTERIZATION

Geometry	Material	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r	T [°C]	f [kHz]	b [mT]	μ'_r	μ''_r
P70-14.5	N22	30	5	50	3241.6	40.3	60	5	50	3158.1	52.7	100	5	50	3501.0	62.9
P70-14.5	N22	30	5	100	3377.9	86.1	60	5	100	3231.2	103.9	100	5	100	3506.6	144.4
P70-14.5	N22	30	5	150	3423.0	130.6	60	5	150	3312.5	157.8	100	5	150	3509.8	201.7
P70-14.5	N22	30	5	200	3438.9	170.5	60	5	200	3298.6	193.9	100	5	200	3307.0	214.6
P70-14.5	N22	30	5	250	3263.6	194.8	60	5	250	3194.6	214.4					
P70-14.5	N22	30	5	300	2903.6	167.3										
P70-14.5	N22	30	10	50	3225.5	43.8	60	10	50	3164.0	54.2	100	10	50	3465.9	73.2
P70-14.5	N22	30	10	100	3367.1	89.6	60	10	100	3228.2	117.9	100	10	100	3496.7	165.2
P70-14.5	N22	30	10	150	3415.5	147.9	60	10	150	3299.1	175.9	100	10	150	3495.2	227.5
P70-14.5	N22	30	10	200	3444.9	190.9	60	10	200	3297.4	215.5	100	10	200	3290.5	234.8
P70-14.5	N22	30	10	250	3277.1	201.2	60	10	250	3243.8	226.2					
P70-14.5	N22	30	10	300	2855.5	166.8										
P70-14.5	N22	30	15	50	3201.8	45.9	60	15	50	3040.9	55.1					
P70-14.5	N22	30	15	100	3366.0	100.8	60	15	100	3183.4	124.3					
P70-14.5	N22	30	15	150	3399.2	154.3	60	15	150	3290.9	186.8					
P70-14.5	N22	30	15	200	3417.0	195.7	60	15	200	3291.2	228.6	100	15	200	3378.0	261.7
P70-14.5	N22	30	15	250	3306.0	217.3	60	15	250	3201.3	233.5					
P70-14.5	N22	30	15	300	2936.1	175.5										
P70-14.5	N22	30	20	50	3150.6	52.1	60	20	50	3110.6	63.2					
P70-14.5	N22	30	20	100	3356.0	112.0	60	20	100	3208.7	138.8					
P70-14.5	N22	30	20	150	3406.7	165.8	60	20	150	3270.9	198.2					
P70-14.5	N22	30	20	200	3431.6	213.8	60	20	200	3334.6	249.3	100	20	200	3401.7	283.3
P70-14.5	N22	30	20	250	3224.6	224.4	60	20	250	3032.4	216.5	100	20	230	1897.8	92.5
P70-14.5	N22	30	20	300	3094.1	202.9										
P70-14.5	N22	30	25	50	3227.4	63.7	60	25	50	3116.4	76.2					
P70-14.5	N22	30	25	100	3404.0	119.3	60	25	100	3243.5	153.8					
P70-14.5	N22	30	25	150	3369.3	176.3	60	25	150	3329.4	220.8					
P70-14.5	N22	30	25	200	3426.7	225.9	60	25	200	3344.7	265.7	100	25	200	3401.2	298.5
P70-14.5	N22	30	25	250	3274.3	241.3	60	25	250	3199.5	257.1					
P70-14.5	N22	30	25	300	3002.3	203.3										
P70-14.5	N22	30	30	50	3199.9	62.4	60	30	50	3071.2	85.3					
P70-14.5	N22	30	30	100	3391.2	130.9	60	30	100	3162.2	156.2					
P70-14.5	N22	30	30	150	3389.6	187.1	60	30	150	3252.6	232.2					
P70-14.5	N22	30	30	200	3438.6	241.1	60	30	200	3297.5	277.9	100	30	200	3445.5	326.3
P70-14.5	N22	30	30	250	3153.4	240.5	60	30	250	3138.0	267.5					
P70-14.5	N22	30	30	300	3120.9	187.4										
P70-14.5	N22	30	40	100	3100.7	139.8										
P70-14.5	N22	30	40	150	3316.1	216.4										
P70-14.5	N22	30	40	200	3378.7	272.8										
P70-14.5	N22						60	40	250	3156.9	297.0					
P70-14.5	N22	30	50	100	3218.3	154.1										
P70-14.5	N22	30	50	150	3345.2	243.8	60	50	250	3222.7	340.2					
P70-14.5	N22															

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