Simulation of the acoustics of coupled rooms by numerical resolution of a diffusion equation

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Abstract

Over the last few years, some studies showed that the acoustic energy density in closed or semi-closed spaces may be the solution of a diffusion equation. This theory allows non-uniform repartition of energy, and is especially relevant in room acoustics for long rooms or complex spaces such as networks of rooms. In this work, the three-dimensional diffusion equation is solved directly by using a finite-element solver. This approach is used to simulate the acoustics of coupled rooms in terms of spatial variations of intensity levels and sound decay. The obtained results match satisfactorily with a model based on the classical statistical theory of room acoustics, but it allows to perform a finer spatial description of the acoustics of coupled rooms.

1. Introduction

Development of diffuse sound field models in architectural acoustics has attracted considerable developments since Sabine's pioneer works, and has given rise to many analytical and numerical solutions. Among these models, the diffusion model [1] takes into account the non-uniform repartition and decay of the sound energy in a room. Recent studies have shown that this analytical model is especially relevant in architectural acoustics for simple configurations such as long enclosures [2]. For more complex spaces, a numerical approach of the diffusion model has also been investigated using a finite element model (FEM) [3], leading to results in agreement with the analytical approach, both in the stationary and in the time varying state. Among these complex spaces, the coupled rooms have long been studied in architectural acoustics [4], since it represents many interests for acousticians and architects (theatres coupled to the auditorium, churches with chapels, mezzanines and half-open offices for example). However, one may remark that the sound distribution and the energy decay in coupled rooms are still difficult to predict with a good accuracy, particularly for weak coupling. On the other hand, the diffusion model seems well adapted to coupled rooms, since it allows to distribute the sound energy everywhere in both enclosures, regardless the shapes of the enclosures, the source and the receiver locations, and the aperture size. Thus, in this paper we present a numerical application of the diffusion model to coupled room. The analytical model and its numerical implementation will be presented in the next section. Then, numerical simulations will be detailed in sections 3 and 4, both in stationary and impulse states, and compared to a model of coupled rooms developed in the context of statistical theory of room acoustics.

2. Diffusion model and resolution

In a recent paper [1], a model was proposed to simulate the sound fields in rooms with diffusively reflecting boundaries. It was shown that the energy flow per unit surface $\mathbf{J}(\mathbf{r},t)$ in a direction $\mathbf{n}$ and at location $\mathbf{r}$ in the room, may be described by a diffusion gradient equation:

$$\mathbf{J}(\mathbf{r},t) = -D\,\mathbf{grad}(w(\mathbf{r},t)) \tag{1}$$

where $w(\mathbf{r},t)$ is the acoustic energy density and D is a diffusion coefficient which can be written as $D = \frac{\lambda c}{3}$, c being the sound velocity and λ the mean free path of the room (equal to $4V/S$, V being the room volume and S the total area of the surfaces of the room). The energy density in the room, outside the direct field, is then described by a diffusion equation:

$$D\Delta w = \frac{\partial w}{\partial t}. \tag{2}$$

The absorption of acoustic energy at boundaries is taken into account by an exchange coefficient h. For a boundary with absorption coefficient α, it can be shown [2] that the energy flow $\mathbf{J}$ through this surface verifies:

$$\mathbf{J} = -D\frac{\partial w}{\partial \mathbf{n}} = hw, \tag{3}$$

with $h = \frac{c\alpha}{4}$. The symbol $\frac{\partial}{\partial \mathbf{n}}$ denotes the normal derivative to the boundary.

As explained in the introduction section, the diffusion equation (2) is solved numerically by means of a FEM solver [3]. The room volume is modelized by a media with a given diffusion coefficient (see Fig. 1) and meshed with Lagrange linear elements. The walls absorption is taken into account as a Fourier-type boundary conditions as written in equation (3). Each wall can be characterized by a specific exchange coefficient h, i.e. a specific absorption coefficient.

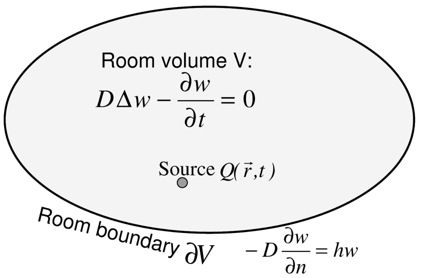

Figure 1: *Diagram of the principle of the numerical approach.*

3. Simulation of stationary response of coupled rooms

Let us consider two rooms (further noted 1 and 2), of respective volumes V_1 and V_2, separated by an aperture which area is S_{12} (Fig. 2). A source providing a stationary acoustic energy is located in room 1. The problem adressed in this section aims at estimating the spatial distribution of acoustic energy in each room.

3.1. Statistical theory model

The classical statistical analysis theory for reverberant rooms can provide estimates of the difference between the acoustic energies located in the two coupled rooms. It distinguishes between the mean energy densities E_1 and E_2 equally distributed in the respective rooms 1 (source room) and 2. This analysis is then not able to describe the graduate change of energy at the coupling area, but provides coarse estimates than can provide reference values for validating the results issued from the diffusion model.

In the context of this theory, it can be shown that the ratio between E_2 and E_1 can be simply written as [4]:

$$\frac{E_2}{E_1} = \frac{S_{12}}{S_{12} + A_2} = k, \tag{4}$$

or equivalently, in terms of intensity levels:

$$L_{I1} - L_{I2} = -10\log(k). \tag{5}$$

The term A_2 represents the absorption area of the surfaces of room 2. The term k is usually called the coupling factor from room 2 to room 1, and depends both on the absorption coefficients of all the surfaces of room 2 and on the coupling area. When the coupling factor gets small with respect to 1 ($S_{12} \ll A_2$), the drop-off of energy density entering room 2 is enhanced.

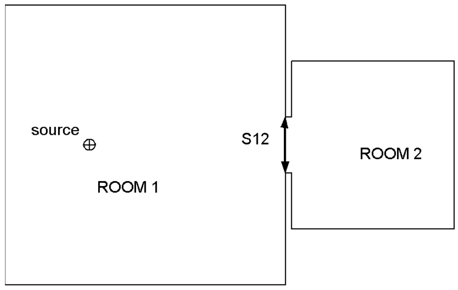

Figure 2: *Diagram of two coupled rooms.*

3.2. Results and discussion

The physical problem investigated here concerns the excitation of coupled rooms with a stationary source. Hence, the stationary diffusion equation:

$$D\Delta w = 0, \tag{6}$$

corresponding to the equation (2) without the time-dependent term, is solved numerically in a three-dimensional medium which shape represents the volume defined by the two coupled rooms. For each room, the diffusion coefficient is calculated by using the mean free path λ equal to the one of the individual room. This involves the assumption that the mean free path of each room is not significantly influenced by the coupling aperture, i.e. S_{12} is small with respect to the area of the surfaces of the room. The results presented in this study concern only the diffuse sound field and do not take into account the contribution of the direct field.

An example of calculation results is presented for two loosely coupled identical rooms of dimensions $(10 \times 10 \times 3)$ m^3 with uniform absorption coefficient $\alpha = 0.1$. The coupling surface S_{12} is 6 m^2 and the coupling factor k is 0.16. A point source is located at point ($-2; 0$ m) in the (X, Y) plane, at 1.5 m height: It retrieves at its location an intensity level of 60 dB. The spatial distribution of intensity level in an horizontal plane at 1 m height, obtained from the numerical resolution of equation (6), is depicted in Fig. 3. As opposed to the statistical theory, the diffusion model restitutes spatial variations of the sound level over the rooms. An area of higher level of the diffuse sound field is noticeable around the source location, and the level is rapidly changing around the coupling ara.

To better observe the level variations, Fig. 4 plots the sound level variations along two lines at $Y = 0$ and $Y = 3$ m (top curves), and along two lines in the orthog-

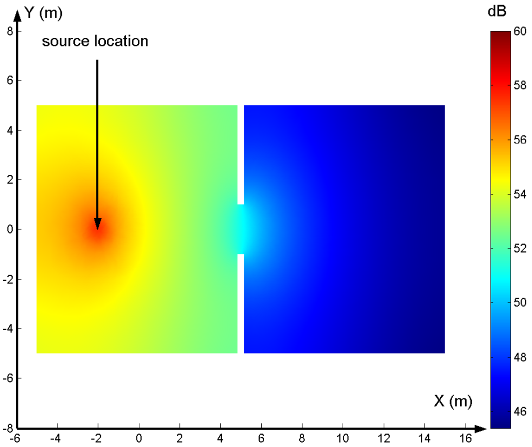

Figure 3: *Intensity level map in dB at 1 m height for two coupled rooms (k=0.16) with uniform absorption α=0.1.*

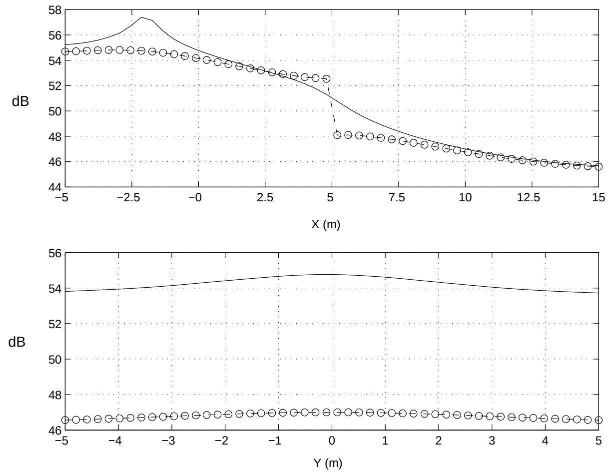

Figure 4: *Sections of the intensity level map of Fig. 3. Top: Intensity level in dB for $Y = 0$ (solid line) and $Y = 3$ (○). Bottom: Level in dB for $X = 0$ (solid line) and $X = 10$ (○).*

onal direction, at $X = 0$ and $X = 10$ m (bottom curves). The following observations holds for all the simulations carried out in this study : two area of sharp variation (top of Fig. 4, $Y = 0$) of the diffuse sound field are observed, one around the source, and the other one in the vicinity of the coupling area, where occurs the graduate change of energy between the rooms. Conversely the level variation for $Y = 3$ m is weaker (about 2 dB for each room), with a 4 dB jump at the wall. In the same way, the level exhibits low variations in the Y direction (about 1 dB, see bottom of Fig. 4). For comparison with results given by the statistical theory, which restitutes uniform levels for each room, a difference between two average values of the sound level in the rooms is provided by calculating the level difference between the two curves of the bottom of Fig. 4. This difference is about 8 dB for the case considered here. The result given by the statistical theory, calculated from $-10\log(k)$ (equation (5)) with k=0.16, is very close to 8 dB. Both models are then in good agreement.

Additive calculations show that the level difference of level between both rooms does not depend on the absorption coefficient of the source room. Some systematic simulations have assessed the agreement of the diffusion model with the statistical theory model, when the following set of parameters is varied: Neighbouring room absorption, volume, and size of the coupling area S_{12}. In all cases both methods have given very similar results, with differences not exceeding 2 dB.

4. Simulation of sound decay in coupled rooms

4.1. Statistical theory model

The sound decay is studied for the same general configuration than in section 3 (see Fig. 2). By using a time-varying formulation of sound energy in two coupled rooms within the theory classical statistical analysis [4], it can be shown that the mean energies densities E_1 and E_2 are:

$$E_1 = E_{11}\exp(-2\delta_I t) + E_{21}\exp(-2\delta_{II} t) \quad (7)$$

$$E_2 = E_{12}\exp(-2\delta_I t) + E_{22}\exp(-2\delta_{II} t), \quad (8)$$

where the ratios between the initial values E_{ii} ($i = 1, 2$) of each decay function are functions of the parameters of the coupling. The damping constants $\delta_{I,II}$ can be written:

$$\delta_{I,II} = \frac{(\delta_1 + \delta_2)}{2} \mp \sqrt{\frac{(\delta_1 + \delta_2)^2}{4} + \kappa^2 \delta_1 \delta_2}. \quad (9)$$

δ_i ($i = 1, 2$) are the damping constants $c(S_{12} + A_i)/8V_i$, where A_i is the absorption area of the surfaces of each room i: they correspond to the decays of each room as if they were uncoupled. The term κ is equal to $\sqrt{S_{12}^2/((S_{12} + A_1)(S_{12} + A_2)}$: it is the mean coupling factor. In both rooms, the reverberation ends with the exponential function having the smaller value of δ. For weakly coupled rooms (i.e. $\kappa^2 \ll 1$), the damping constants $\delta_{I,II}$ are very close to $\delta_{1,2}$.

4.2. Two identically damped rooms

The diffusion equation (2) is solved numerically for a point source providing a supply of energy at time t=0. The sound decays can then be retrieved at any location in the room. It is first noticed that, for a given room, the sound decay remains almost identical for any point in this room. Fig. 5 presents the decays obtained for two

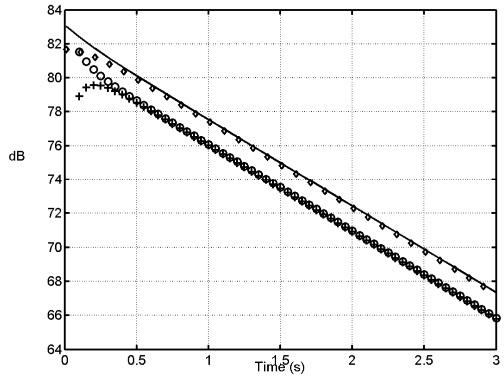

Figure 5: *Sound decays in two coupled rooms with uniform absorption ($\alpha = 0.01$). Solid line and $\diamond$: rooms 1 and 2 with statistical theory, $\circ$ and + : rooms 1 and 2 with diffusion model.*

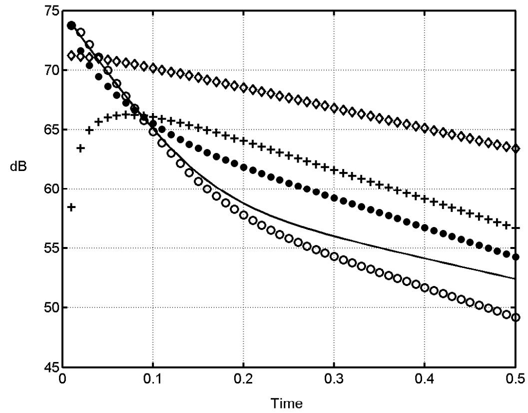

Figure 6: *Sound decays in two coupled rooms with absorption (α=0 .2 and 0.02). Solid line and $\diamond$: rooms 1 and 2 with statistical theory, $\circ$ and + : rooms 1 and 2 with diffusion model, $\bullet$: decay at coupling aperture .*

highly reverberant rooms with uniform absorption coefficient ($\alpha = 0.01$), volumes V_1 and V_2 of respectively 150 and 100 m^3, and with a coupling aperture S_{12} of 3.75 m^2. The mean coupling coefficient is high in this case (κ=0.69). One can see that the decay is described by the lower damping constant for both rooms. The damping constant given by the statistical model (eqs. (7-8)) and the diffusion model are in good agreement. The slight shift between the curves lies in the problem of determining an identical initial energy located in the source room for both models, as the statistical model imposes a uniform initial energy and the diffusion model modelizes a more realistic point source. This question is currently being addressed.

4.3. Highly damped room coupled with a reverberant room

In the case of a room with high absorption coupled with a reverberant room, a phenomenon of "double-decay" can appear in the source room: the reverberation with short decay is first heared, and then the longer reverberation of the neighbouring room appears. This situation is simulated with two rooms of volume 125 m^3, respective absorption 0.2 and 0.02 for rooms 1 and 2, and a coupling area of 2.5 m^2. In this case the damping constants $\delta_{I,II}$ are close to the damping constants of each room taken individually ($\kappa = 0.17$). Fig. 6 shows the obtained decays for each room and both models. The double-decay in room 1 appears clearly, the longer reverberation of room 2 appearing clearly after $t = 0.2$ s. The tendency is very similarly restituted by both models. A slight difference lies in the damping constant in room 2: the estimated decay is slightly faster with diffusion theory. Moreover the difference of levels in both rooms at a given time is about 3 dB lower for the diffusion model. The decay at the coupling area, as given by the diffusion model, is also plotted, to show that the graduate change in decay behaviour between both rooms can be investigated, as opposed to the statistical theory which provides identical decays all over each room.

5. Conclusions

The numerical resolution of a diffusion equation has been used to predict the acoustics of coupled rooms, in terms of stationary response and sound decays. The obtained results match satisfactorily with a model based on the classical statistical theory. The advantage of the presented method is that it is able to provide a finer description of the spatial variation of intensity level and sound decay. Experimental measurements are being conducted to validate the predictions of the diffusion model.

6. References

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