LIÈGE université
Sciences Appliquées
Structural & Stochastic Dynamics
Urban & Environmental Engineering

EMI 2025 – Anaheim

UCI Samueli
School of Engineering
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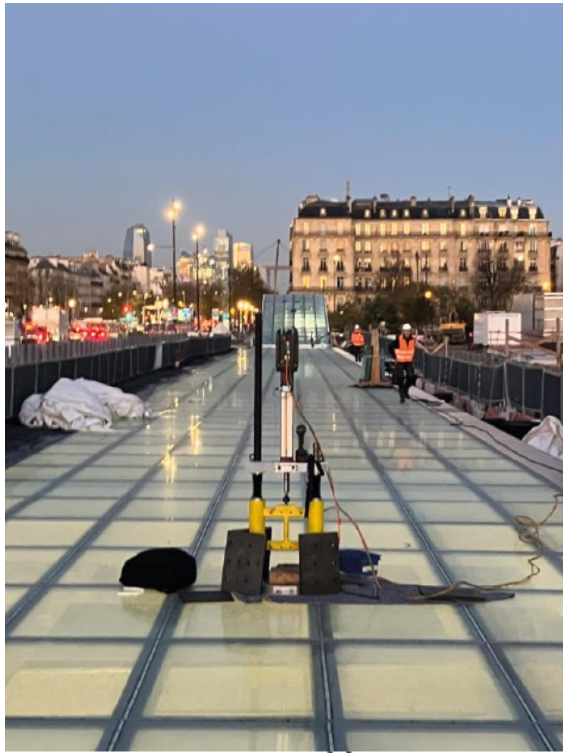
ASCE

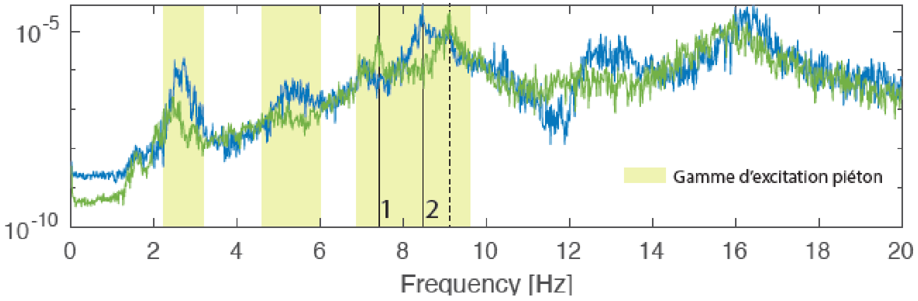
Slow dynamics of the transient response of a structure with tuned-mass damper

Anass Mayou, V. Denoël

Motivation

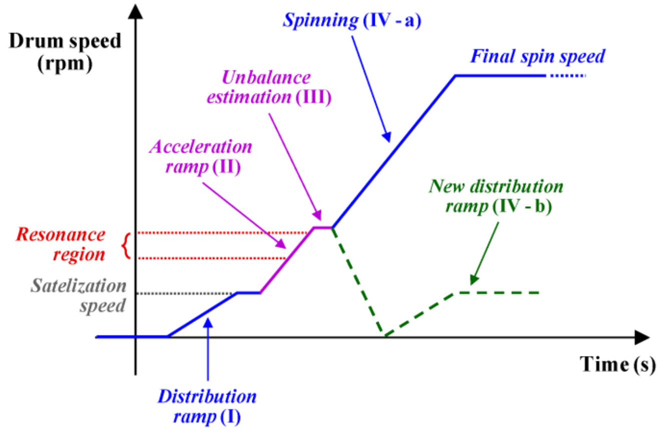
Bridges/footbridges undergoing



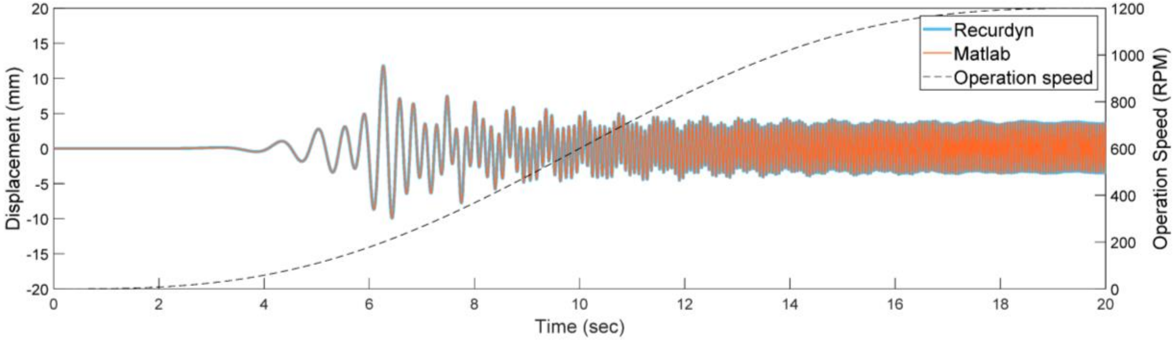


Modal identification of a glass bridge in Porte Maillot (Paris) by Vincent Denoël (ULiège) & V2i

Slabs supporting



An eDrive-Based Estimation Method of the Laundry Unbalance and Laundry Inertia for Washing Machine Applications – Daniele Martinello et al.

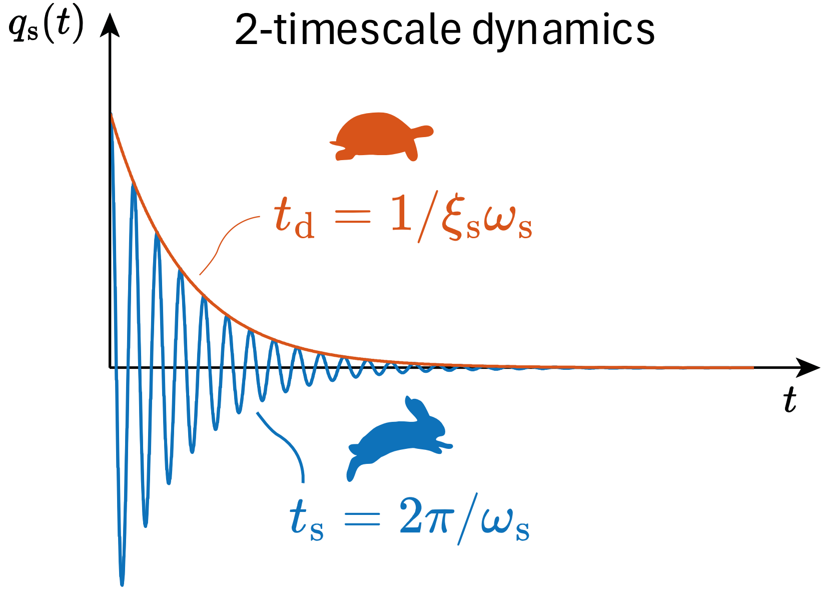


Dynamic Modeling of a Front-Loading Type Washing Machine and Model Reliability Investigation - Jungjoon Park et al.

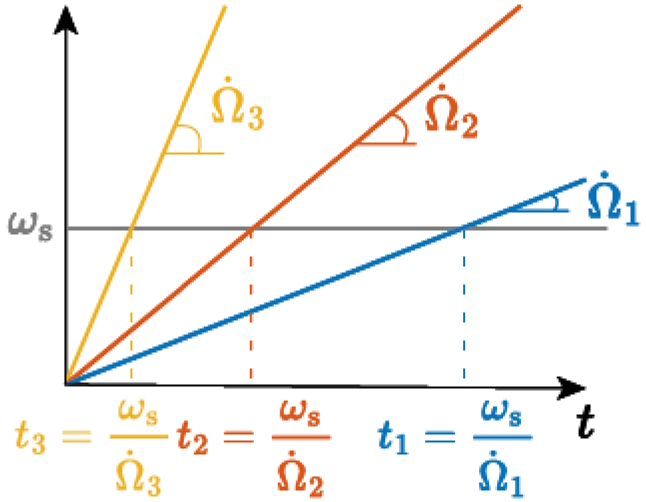
Motivation

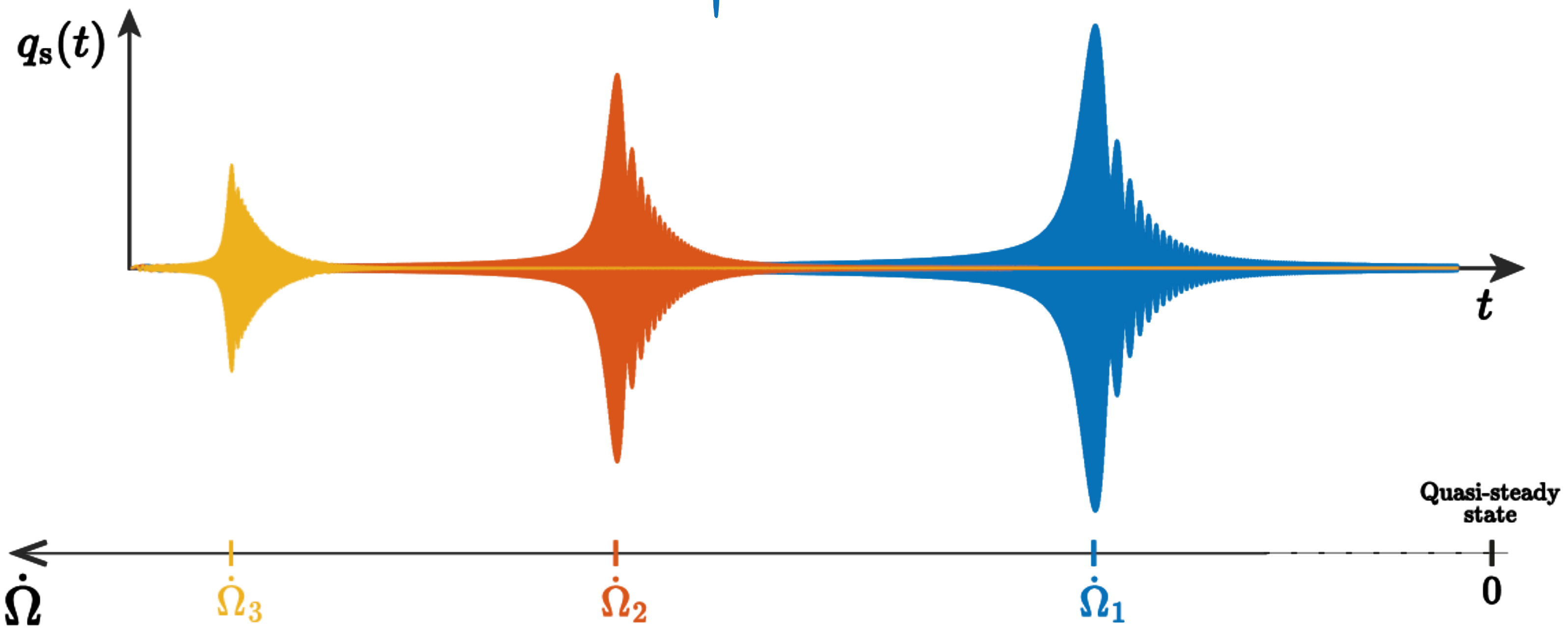
$$m_s\ddot{q}_s + c_s\dot{q}_s + k_s q_s = f_p e^{\frac{1}{2}\dot{\Omega}t^2}$$

$$\omega_{\text{inst}}(t) = \frac{d}{dt}\left(\frac{1}{2}\dot{\Omega}t^2\right) = \dot{\Omega}t$$



$$\omega_{\text{inst}}(t) = \dot{\Omega}t$$



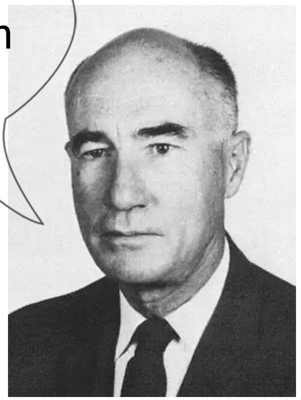


Motivation

Optimal TMD design in steady-state

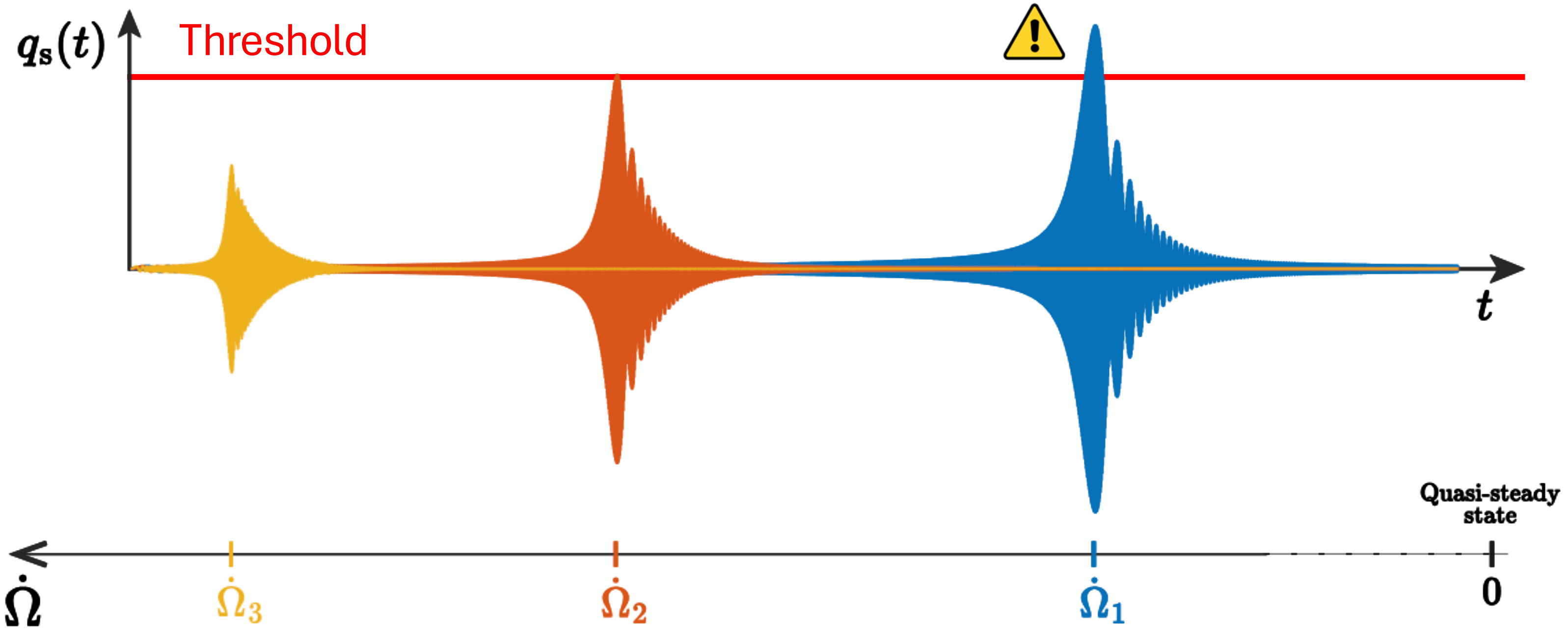
Den Hartog (1956)	$f_{opt} = \frac{1}{1+\mu}$ $\zeta_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)}}$
Warburton (1982)	$f_{opt} = \frac{\sqrt{1-\mu/2}}{1+\mu}$ $\zeta_{d_{opt}} = \sqrt{\frac{\mu(1-\mu/4)}{4(1+\mu)(1-\mu/2)}}$
Sadek et al. (1997)	$f_{opt} = \frac{1}{1+\mu}\left(1-\zeta\sqrt{\frac{\mu}{1+\mu}}\right)$

Let's see if it's a good idea in transient !



J.P. Den Hartog

- Adjust loading parameters ;
- Modify structural properties ;
- Add a TMD



Structure - without TMD - transient regime

Multiple Timescales formulation

$$\ddot{q}_{\mathrm{s}} + 2\xi_{\mathrm{s}}\omega_{\mathrm{s}}\dot{q}_{\mathrm{s}} + \omega_{\mathrm{s}}^2 q_{\mathrm{s}} = \frac{f_{\mathrm{p}}}{m_{\mathrm{s}}} \mathrm{e}^{\frac{\mathrm{i}}{2}\dot{\Omega}t^2}$$

$\xi_{\mathrm{s}} = \varepsilon\bar{\xi}_{\mathrm{s}}$ $f_{\mathrm{p}} = \varepsilon\bar{f}_{\mathrm{p}}$ $\dot{\Omega} = \varepsilon^2\kappa\omega_{\mathrm{s}}^2$

$x_{\mathrm{s}} = \frac{q_{\mathrm{s}}}{q_{\mathrm{s}}^*}$ $\tau = \frac{t - t_{\mathrm{r}}}{t_{\mathrm{s}}}$

$\bar{\xi}_{\mathrm{s}} = \mathrm{ord}(1)$

$\bar{f}_{\mathrm{p}} = \mathrm{ord}(1)$

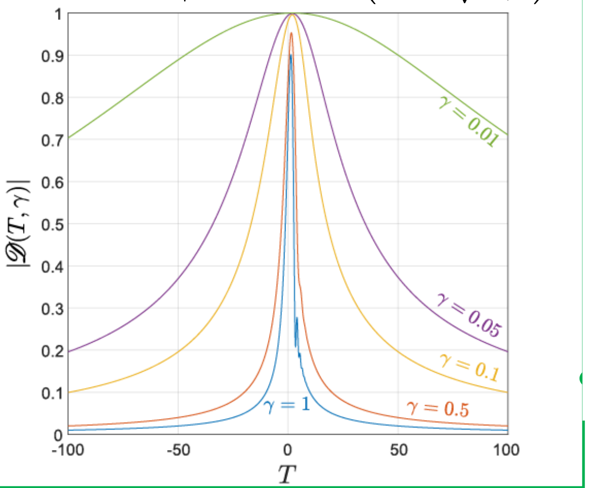
$\kappa = \mathrm{ord}(1)$

Numerical solver

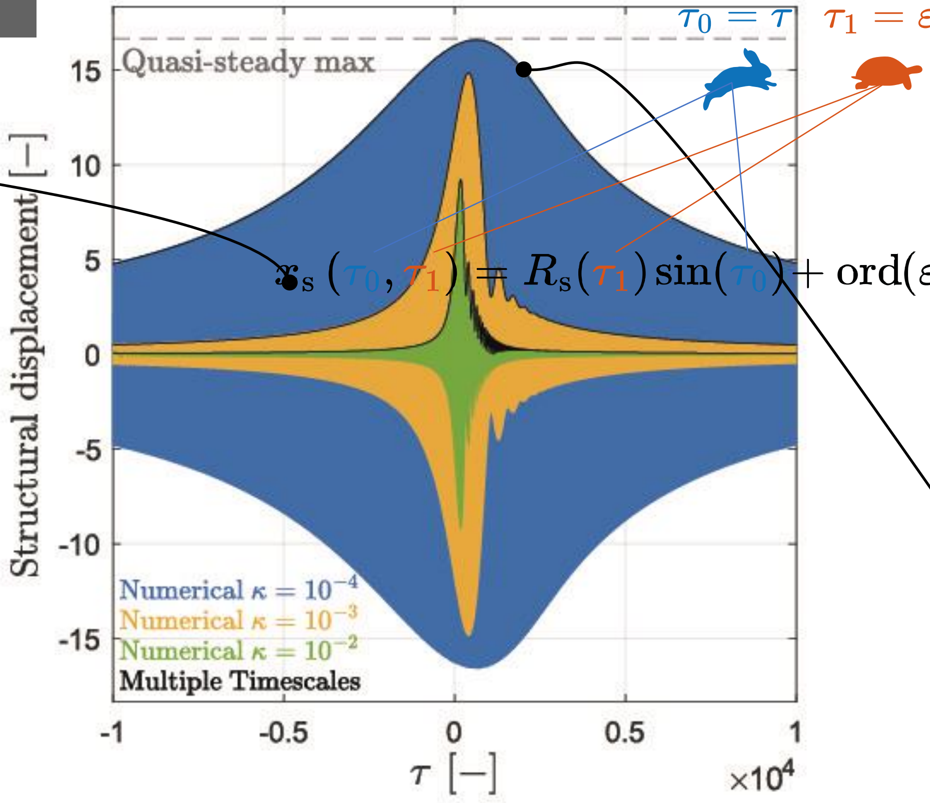
$q_{\mathrm{s}}(t)$

Full Dynamics

$$\mathscr{D}(T;\gamma) := \sqrt{\frac{\mathrm{i}\pi}{2\gamma}\mathrm{e}^{\frac{\mathrm{i}}{2\gamma}}}\,\mathrm{erfc}\left(\mathrm{e}^{-\frac{\mathrm{i}\pi}{4}}\frac{\mathrm{i}-\gamma T}{\sqrt{2\gamma}}\right)$$



$\tau_0 = \tau$ $\tau_1 = \varepsilon\tau$



$$x_{\mathrm{s}}(\tau_0, \tau_1) = R_{\mathrm{s}}(\tau_1)\sin(\tau_0) + \mathrm{ord}(\varepsilon)$$

- Non-dimensionalisation
- Assumptions on orders of magnitude
- Truncation at first order

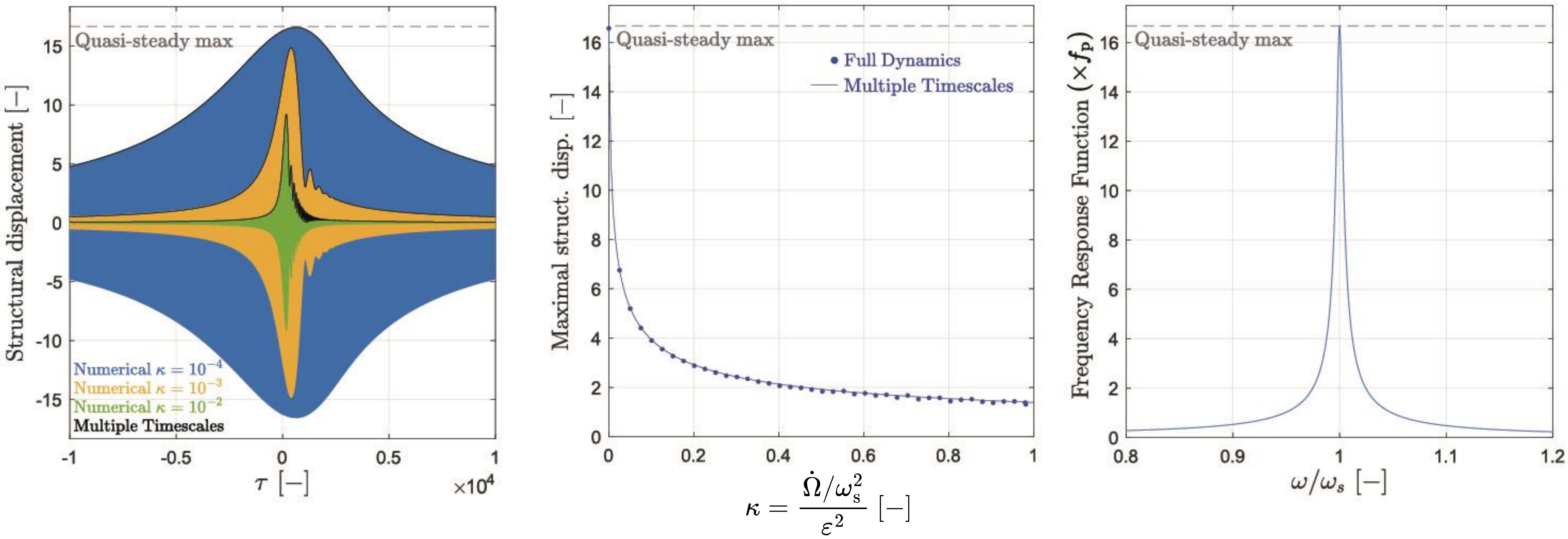
$$R_{\mathrm{s}}' + \bar{\xi}_{\mathrm{s}}R_{\mathrm{s}} = -\frac{\mathrm{i}}{2}\mathrm{e}^{\frac{\mathrm{i}}{2}\frac{1}{\varepsilon^2\kappa}}\mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\tau_1^2}$$

$$R_{\mathrm{s}}(\tau_1) = C\mathrm{e}^{-\bar{\xi}_{\mathrm{s}}\tau_1}\mathscr{D}\left(\bar{\xi}_{\mathrm{s}}\tau_1, \frac{\kappa}{\bar{\xi}_{\mathrm{s}}^2}\right)$$

Multiple Timescales

Structure - without TMD - transient regime

Validation of the asymptotic approach



Multiple Timescales formulation

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{p}$$

Numerical solver

$q_{\mathrm{s}}(t)$

Full Dynamics

Non-dimensionalisation

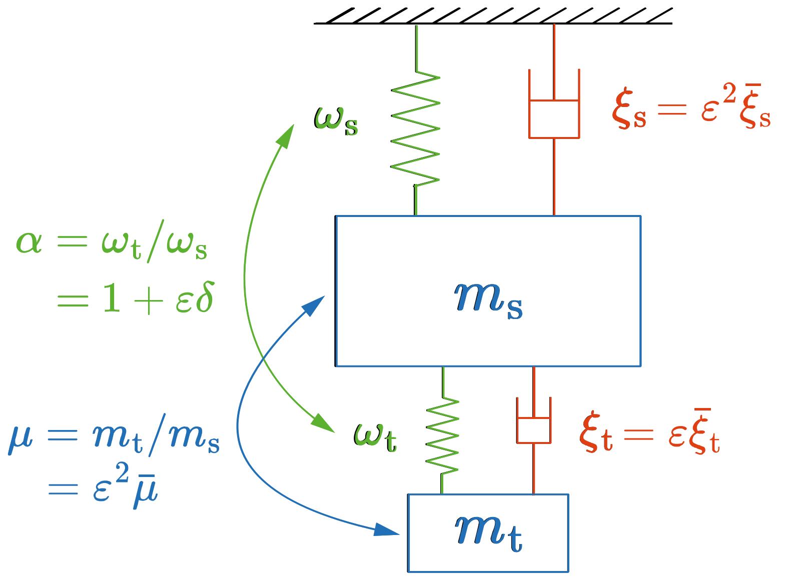
Assumptions on orders of magnitude

Truncation at first order

$$R_{\mathrm{s}}'' + \left(\bar{\xi}_{\mathrm{t}} - \mathrm{i}\delta\right) R_{\mathrm{s}}' + \frac{1}{4} R_{\mathrm{s}} = \frac{1}{2}\left(\kappa\tau_1 - \delta - \mathrm{i}\bar{\xi}_{\mathrm{t}}\right) \mathrm{e}^{\frac{\mathrm{i}}{2}\frac{1}{\varepsilon^2\kappa}} \mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\tau_1^2}$$

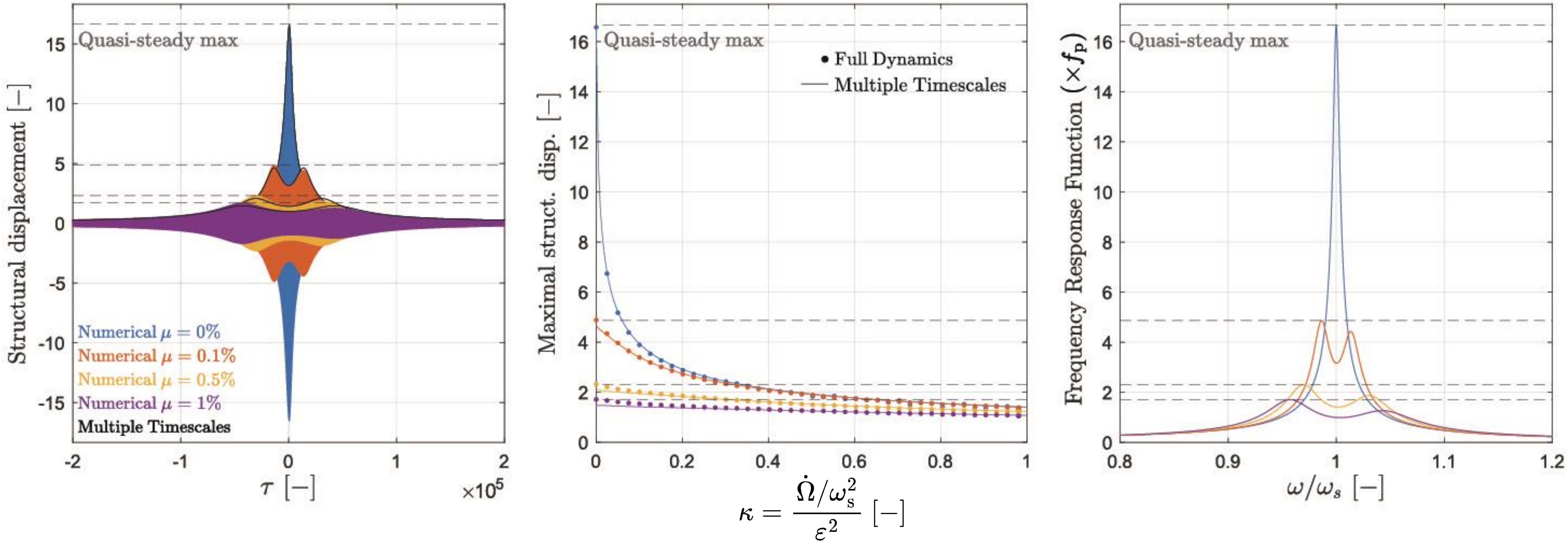
$$R_{\mathrm{s}}\left(\tau_1\right) = \sum_{i=1}^{2} C_i \mathrm{e}^{-\lambda_i\tau_1} \mathscr{D}\left(\lambda_i\tau_1, \frac{\kappa}{\lambda_i^2}\right)$$

Multiple Timescales



Structure - with TMD - transient regime

Validation of the asymptotic approach



Transient Amplification of the Damped System

$$\mu = m_\mathrm{t}/m_\mathrm{s} = \varepsilon^2 \bar{\mu}$$
$$\alpha = \omega_\mathrm{t}/\omega_\mathrm{s} = 1 + \varepsilon\delta$$
$$\xi_\mathrm{t} = \varepsilon \bar{\xi}_\mathrm{t}$$
$$\dot{\Omega} = \varepsilon^2 \kappa \omega_\mathrm{s}^2$$

$$R_\mathrm{s}'' + \left(\bar{\xi}_\mathrm{t} - \mathrm{i}\delta\right) R_\mathrm{s}' + \frac{1}{4}\bar{\mu} R_\mathrm{s} = \frac{1}{2}\left(\kappa\tau_1 - \delta - \mathrm{i}\bar{\xi}_\mathrm{t}\right) \mathrm{e}^{\frac{\mathrm{i}}{2}\frac{1}{\varepsilon^2\kappa}} \mathrm{e}^{\frac{\mathrm{i}}{2}\kappa\tau_1^2}$$

$$\mathcal{G}_\xi = \frac{\xi_\mathrm{t}}{\sqrt{\mu}} \qquad \mathcal{G}_\delta = \frac{\alpha - 1}{\sqrt{\mu}} \qquad \mathcal{G}_\kappa = \frac{\dot{\Omega}/\omega_\mathrm{s}^2}{\mu}$$

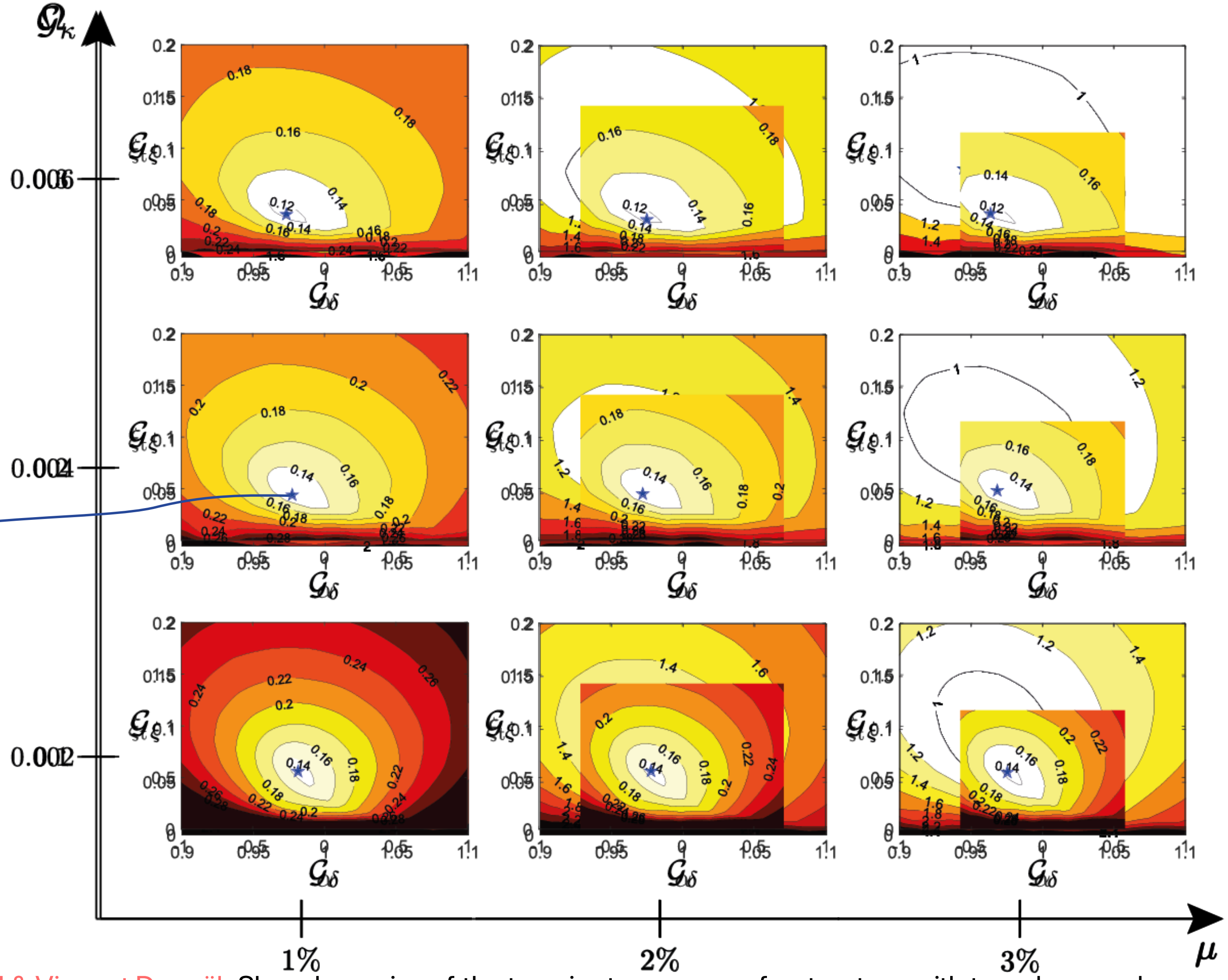
Our small number

$$R_\mathrm{s} \times \sqrt{\mu}$$

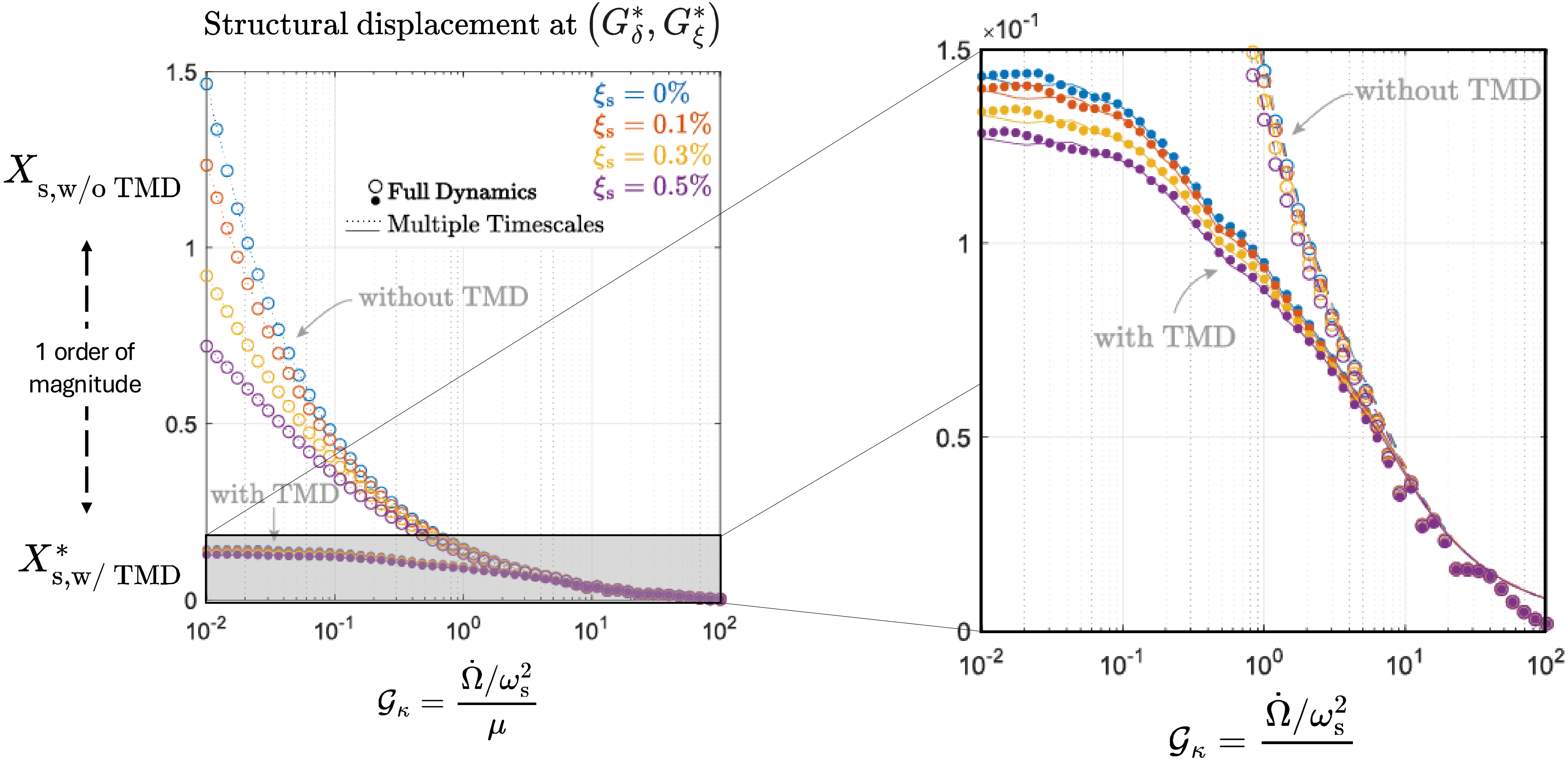
Dimensionless groups

4 → 3

$X^*_{\mathrm{s,w/\ TMD}}$ located at (G^*_δ, G^*_ξ)

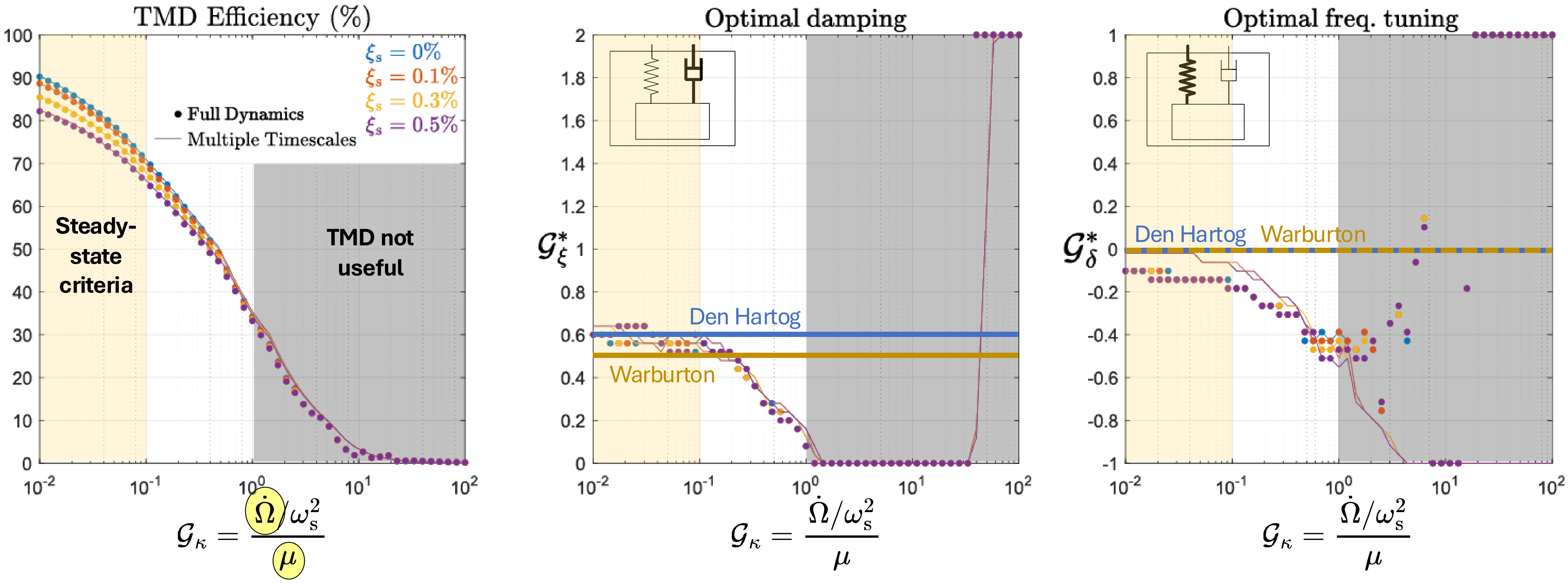


Optimal tuning of the TMD



Efficiency/Usefulness of the TMD

$$\left(1 - \frac{X^*_{\mathrm{s,w/\ TMD}}}{X_{\mathrm{s,w/o\ TMD}}}\right) \times 100\%$$



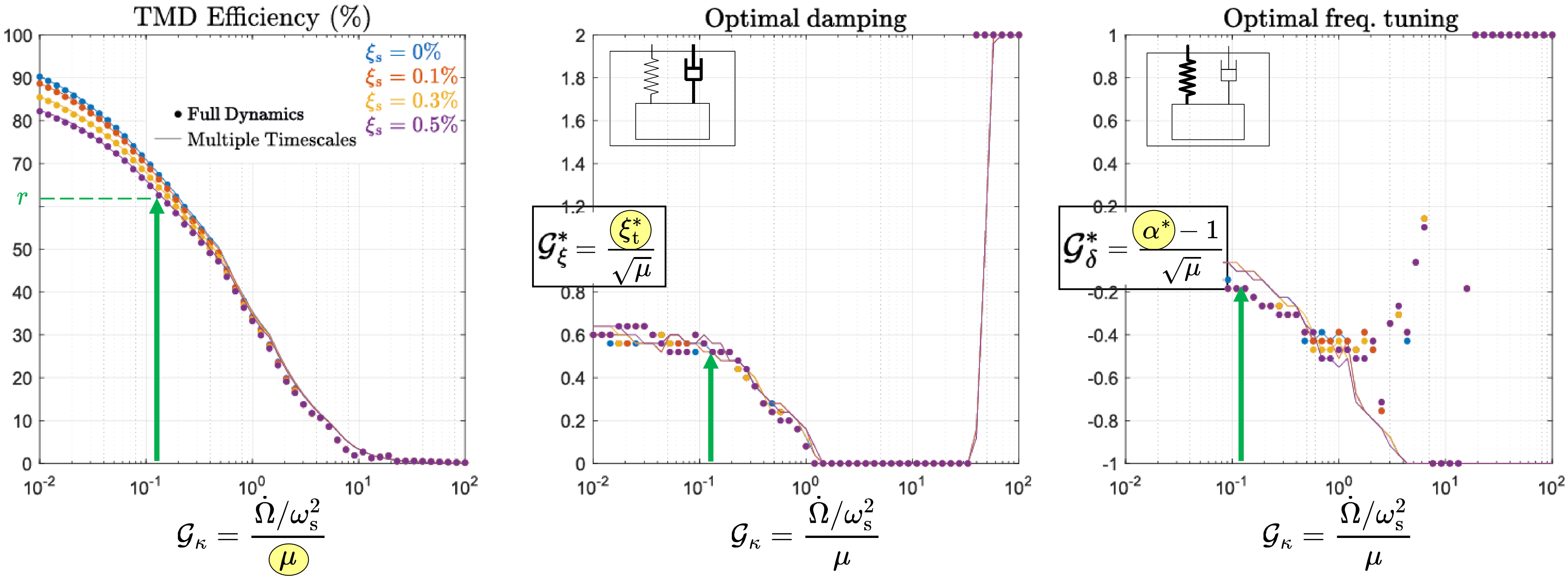
Procedure of design

Given a structure (m_s, ω_s, ξ_s) subjected to sine-sweep loading of chirp rate $\dot{\Omega}$:

1. Fix a reduction factor r

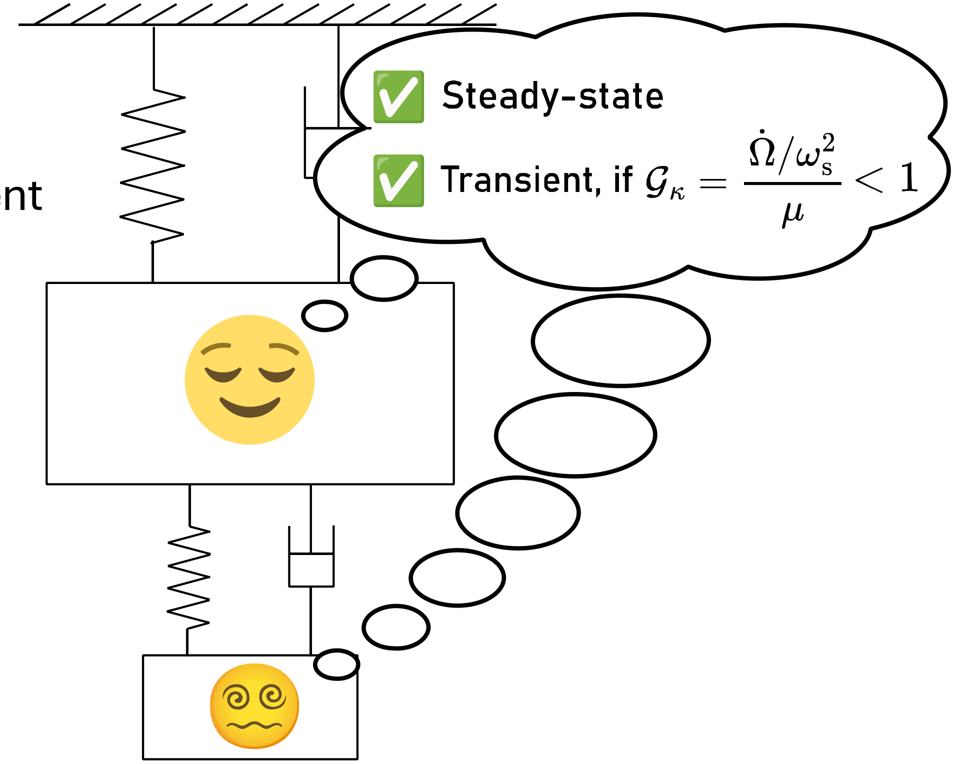
2. Extract mass ratio μ to reach r

3. Deduce the optimal damping and frequency tuning (α^*, ξ_t^*)



Conclusion

- **Multiple timescale** asymptotic method
  - predicts structural slow dynamics
  - is **accurate** and **computationally efficient**
  - reveals **the governing dimensionless groups** of the response at leading
- **Universal solution**
  - enabling a **simple and rapid procedure** for the design and assessment of TMDs in transient regime.



Thank you for your attention

Anaheim