Stochastic Enhancements in Wake Oscillator Models
— experimental and modeling aspects —

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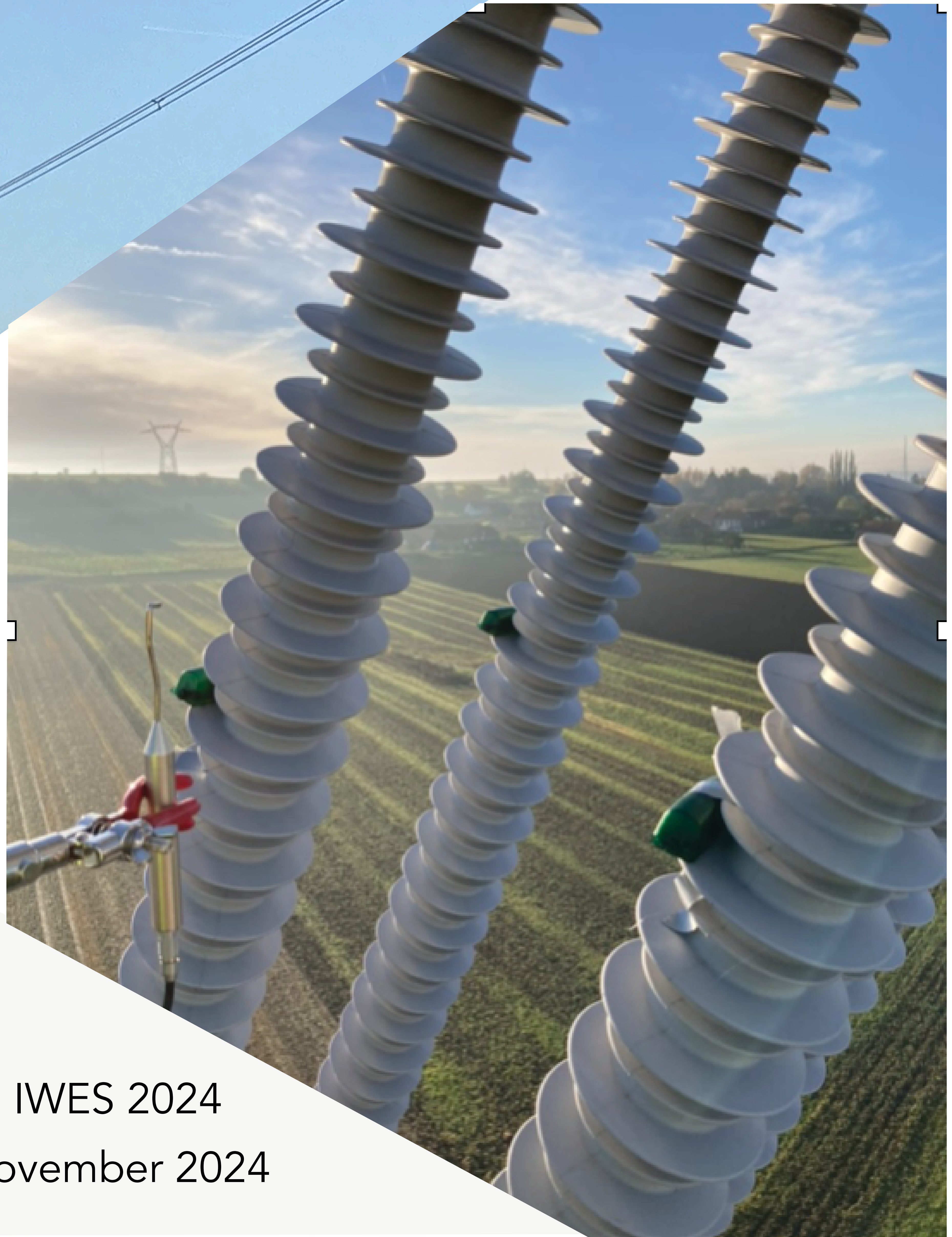
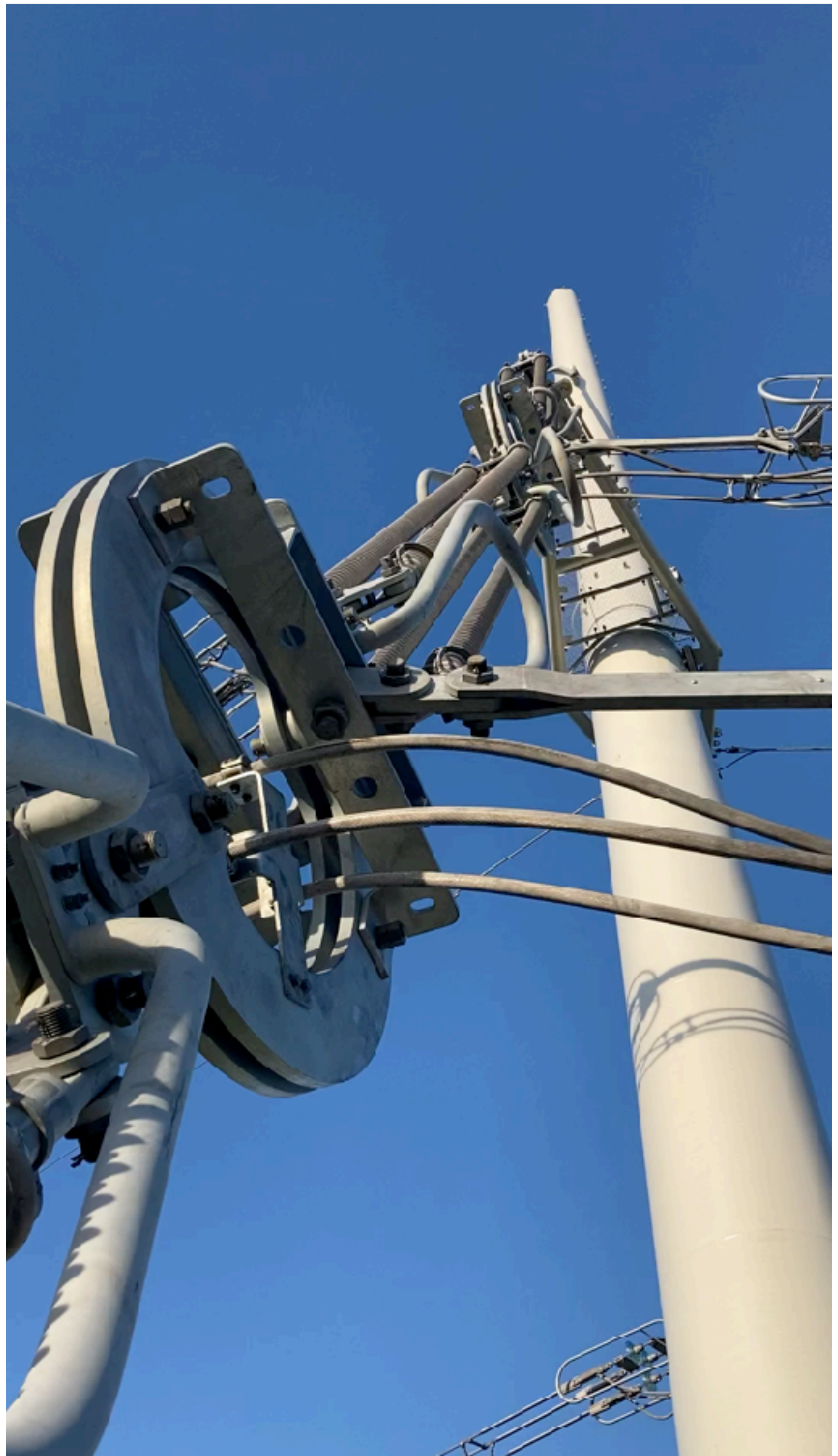
International Wind Engineering Symposium, IWES 2024
Tamkang University, Taipei, Taiwan, 15-16 November 2024





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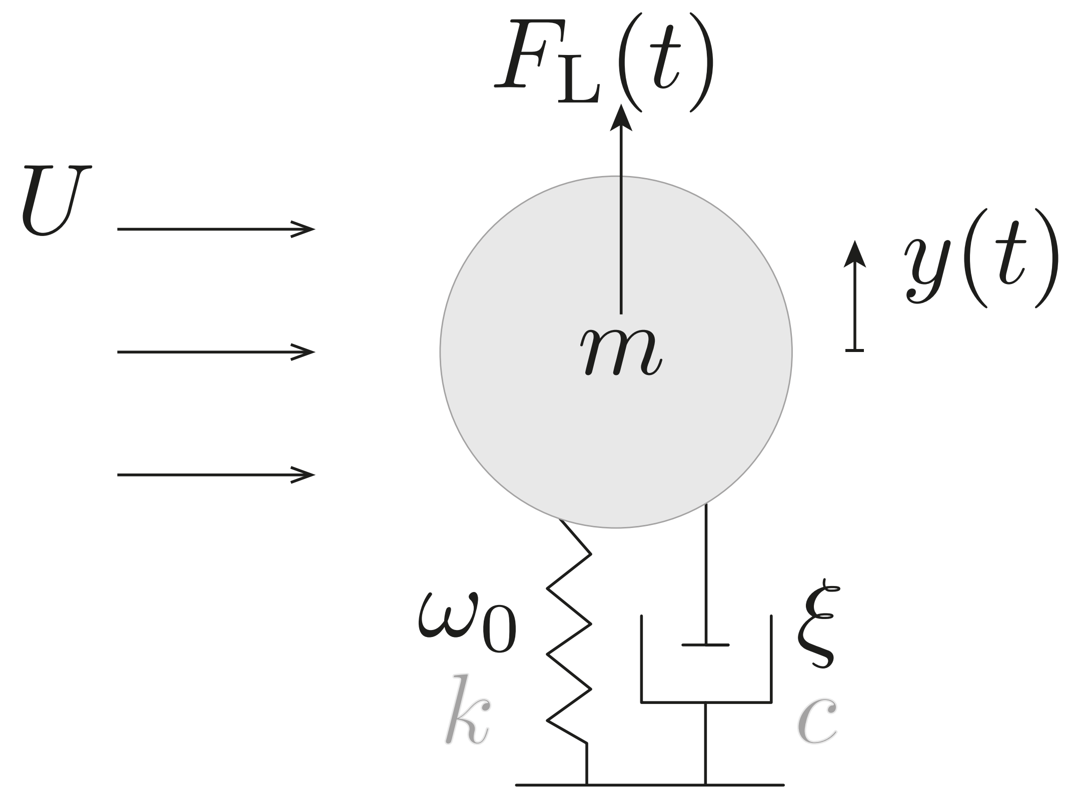
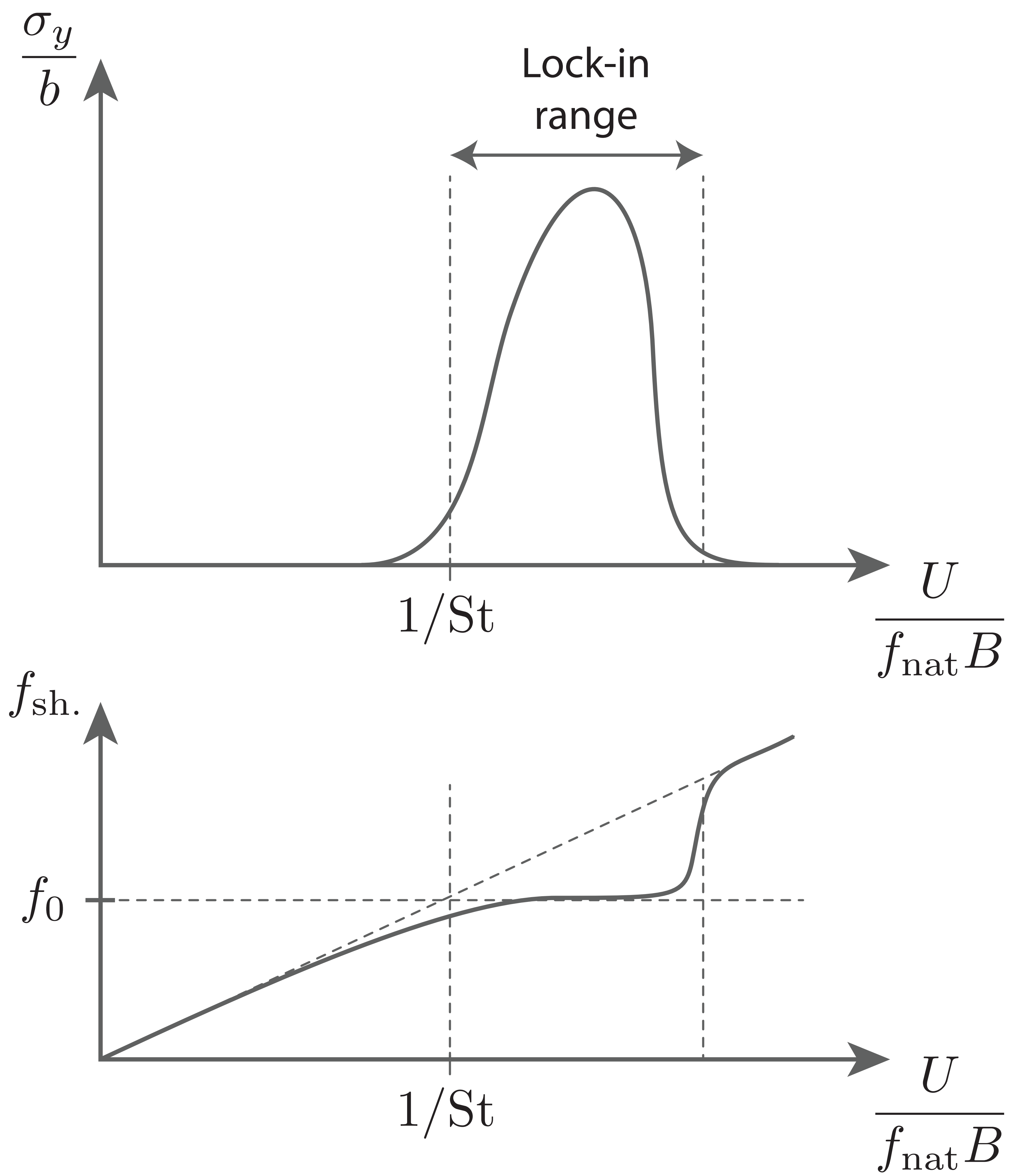
Our ideal(ized) vision of vortex induced vibration

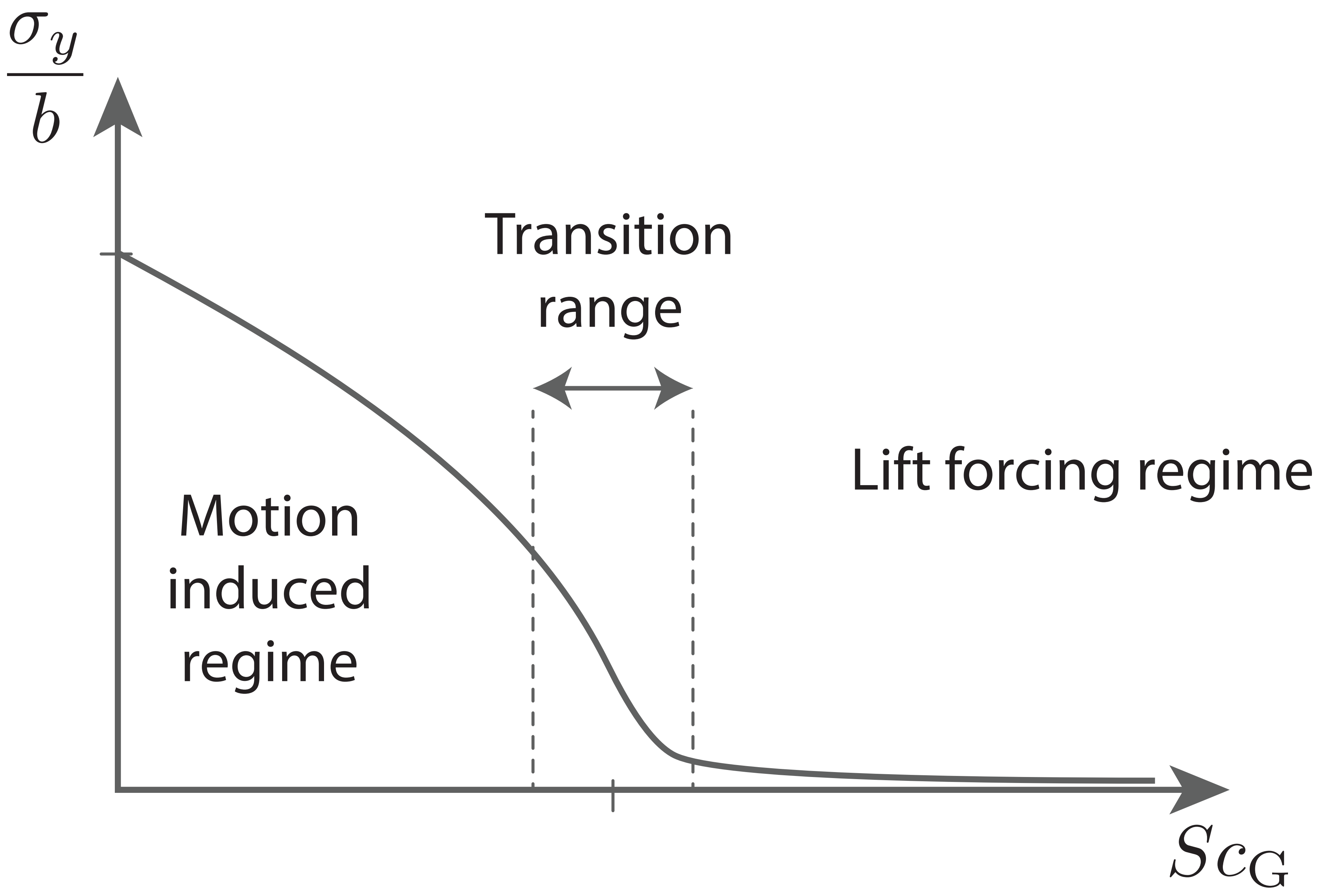
- A lock-in range

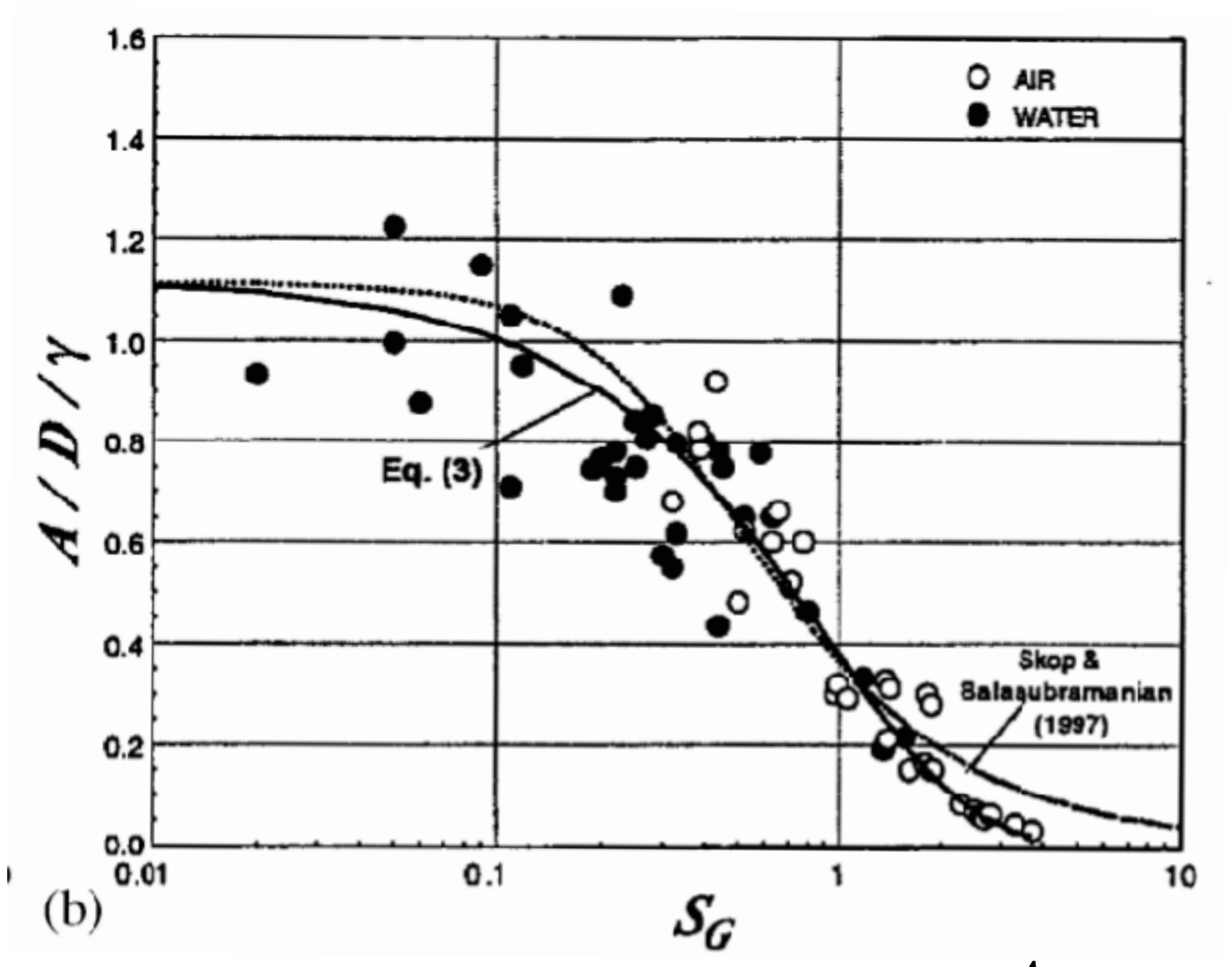




Our ideal(ized) vision of vortex induced vibration

- Influence of mass-damping ratio

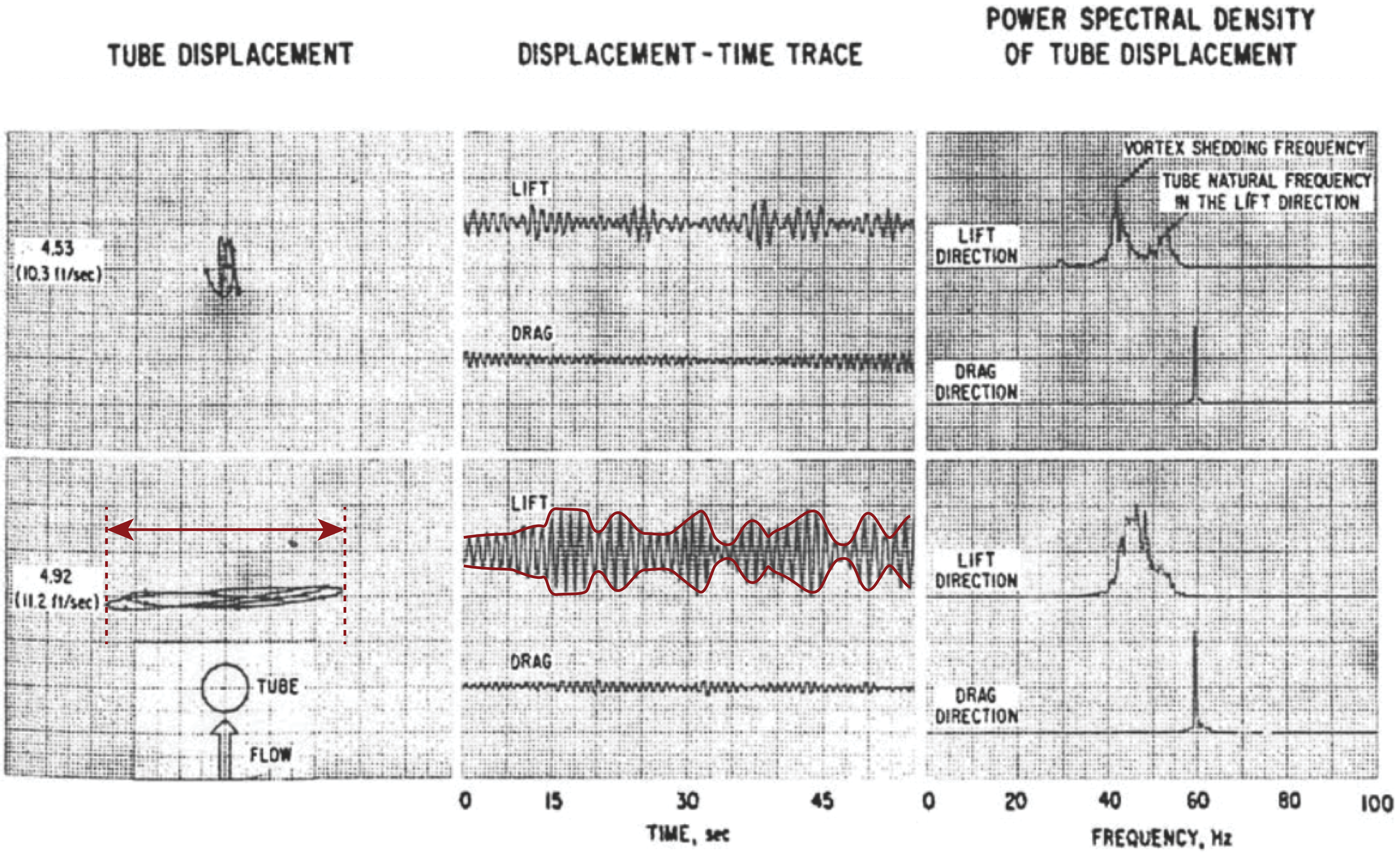




$$\frac{A}{D} \sim \mathrm{e}^{-1.05 S_G}$$

T. Sarpkaya, *A critical review of the intrinsic nature of vortex-induced vibrations*, Journal of Fluids and Structures 19 (2004) 389–447

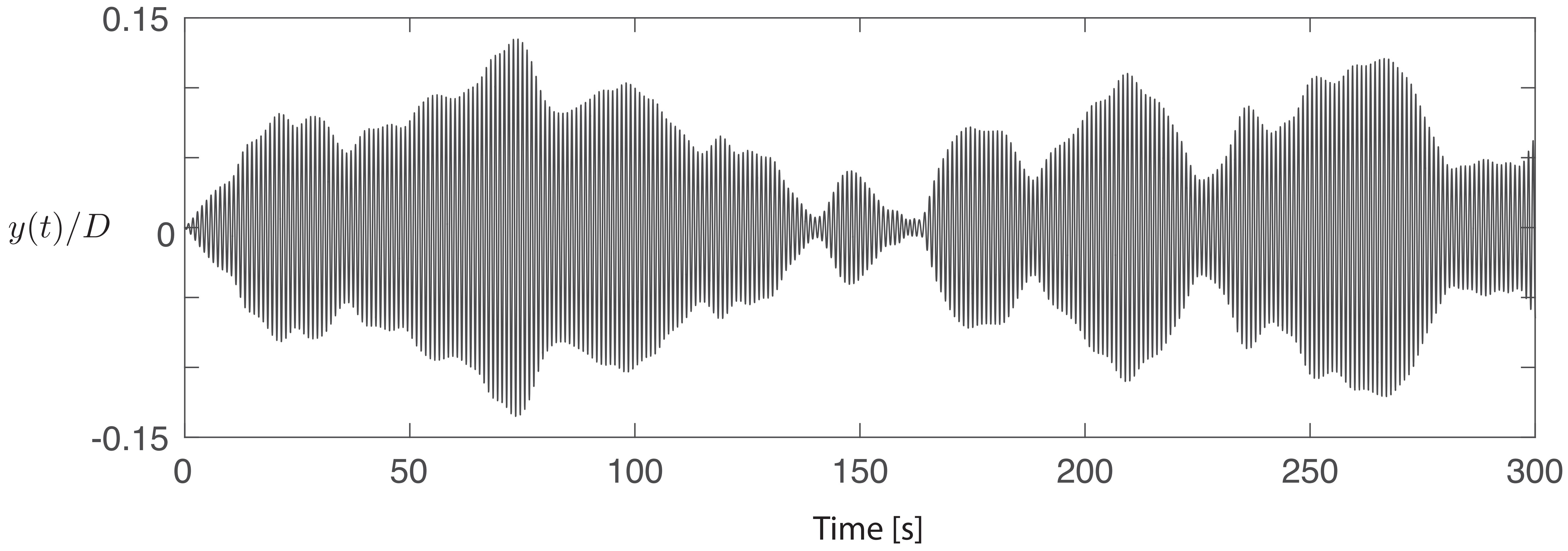
Our ideal(ized) vision of vortex induced vibration

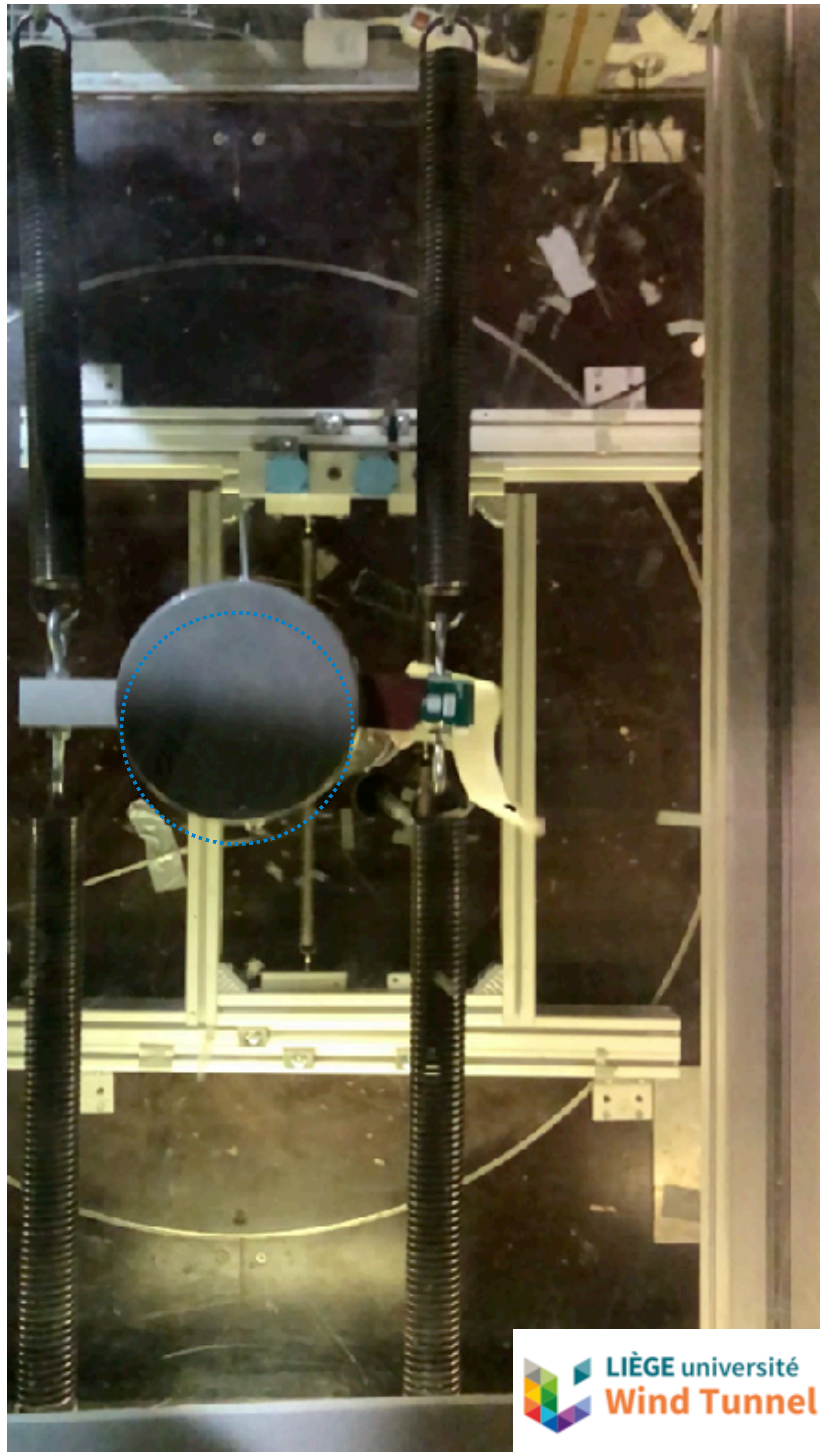


Chen, S.S., Jendrzejczyk, J.A., 1979. Dynamic response of a circular cylinder subjected to liquid cross flow. Journal of Pressure Vessel Technology 101, 106–112.

Our ideal(ized) vision of vortex induced vibration

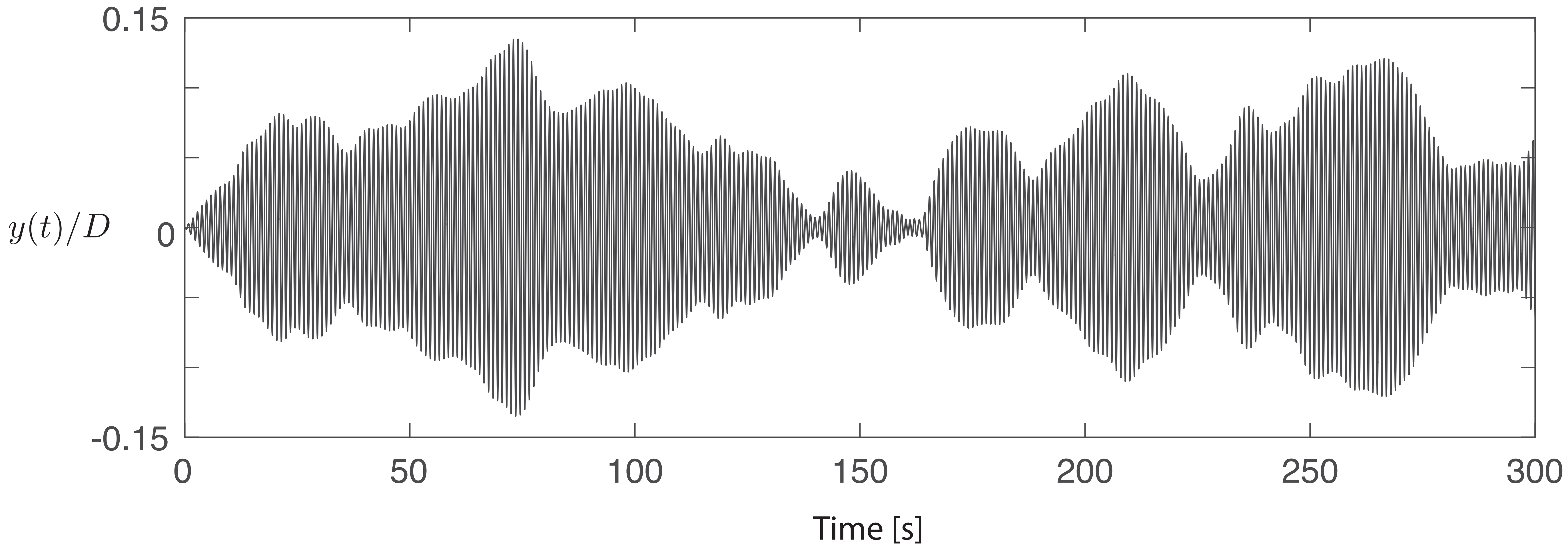
- Example of cross-flow amplitude of motion in lock-in regime

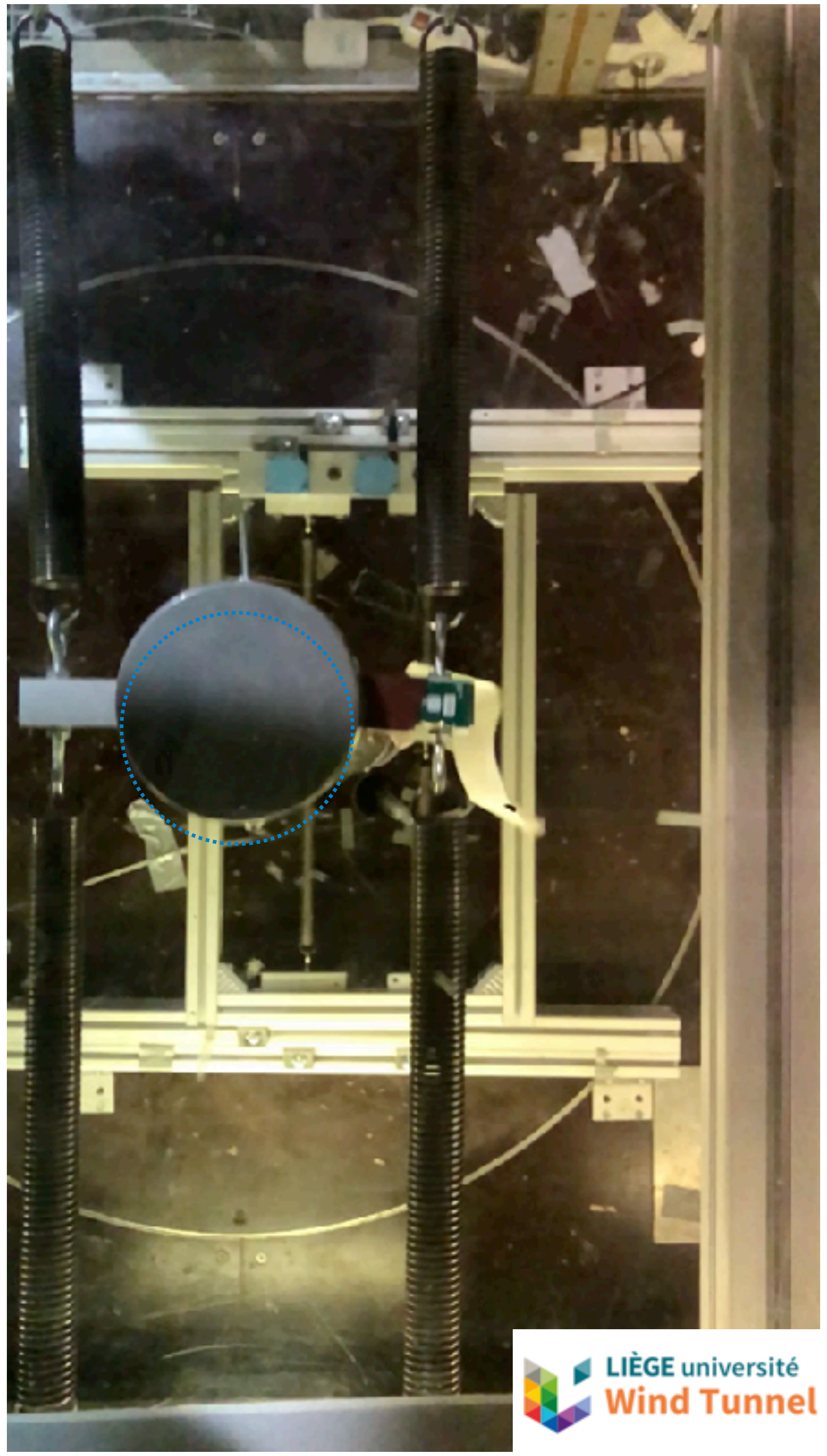




Motivation

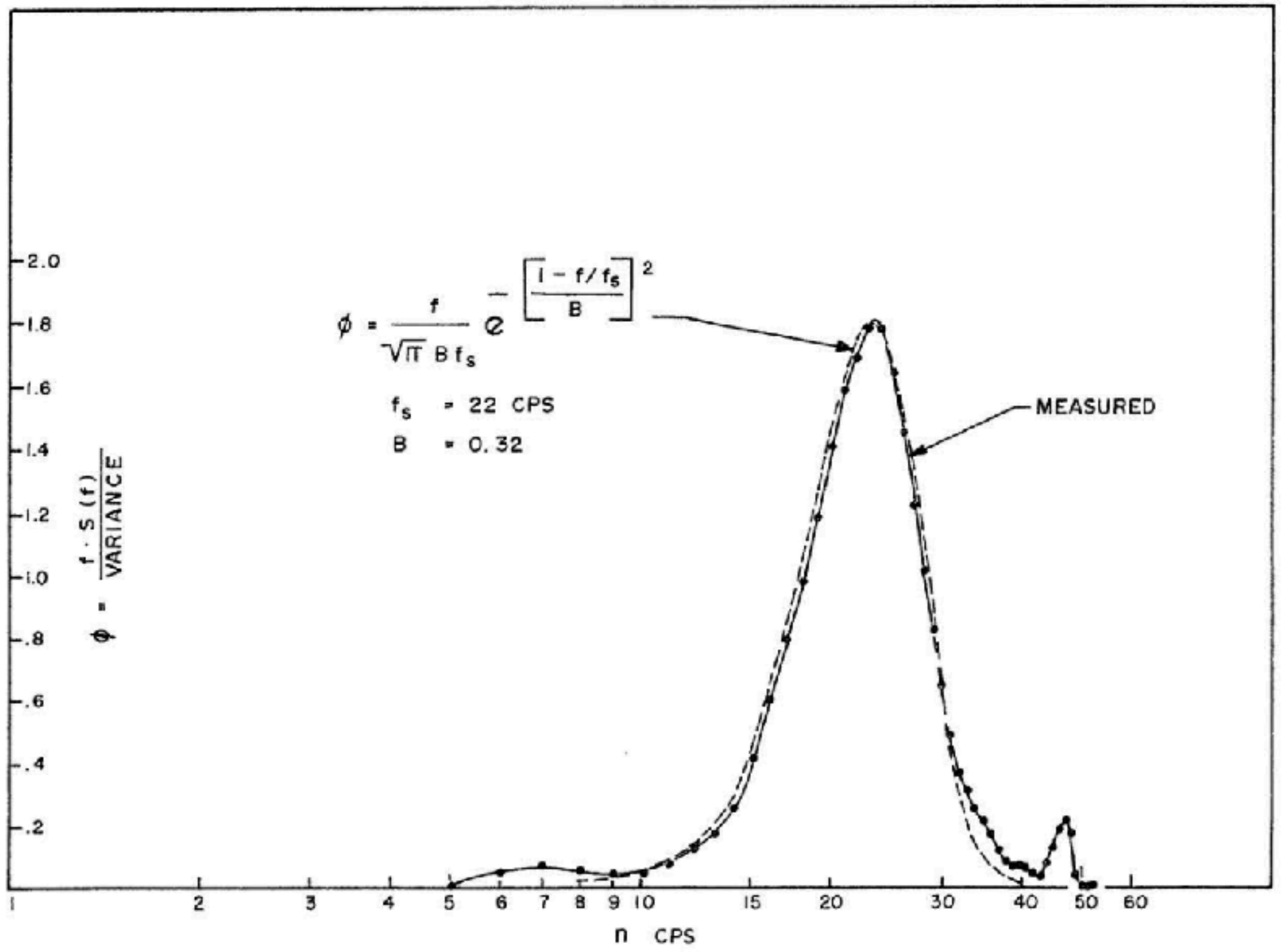
- Example of cross-flow amplitude of motion in lock-in regime

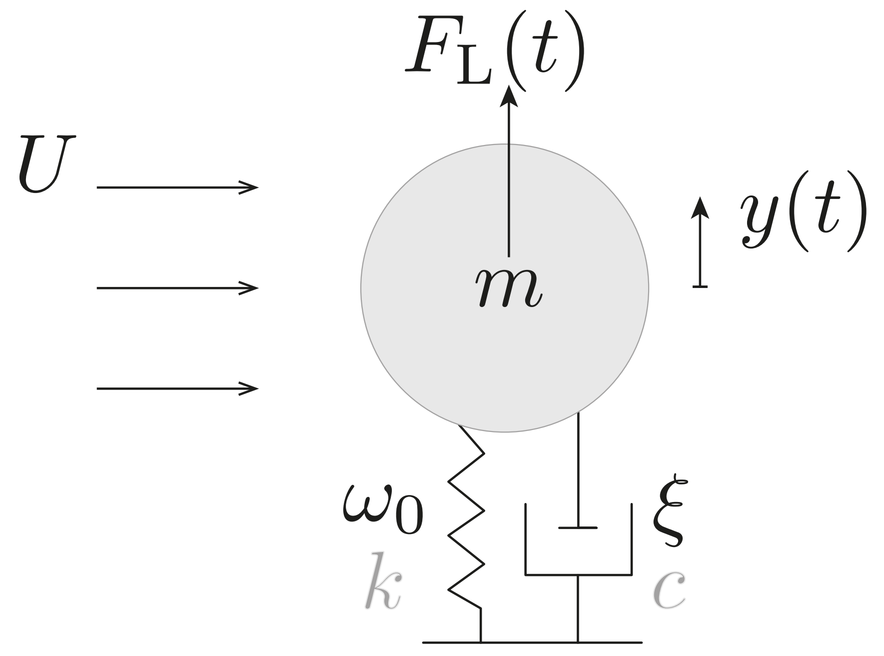




- The amplitude of vibration influences fatigue lifetime
- Amplitude fluctuates in a **random manner**

- The spectral model, Vickery & Clark (1972), Vickery & Basu (1983)





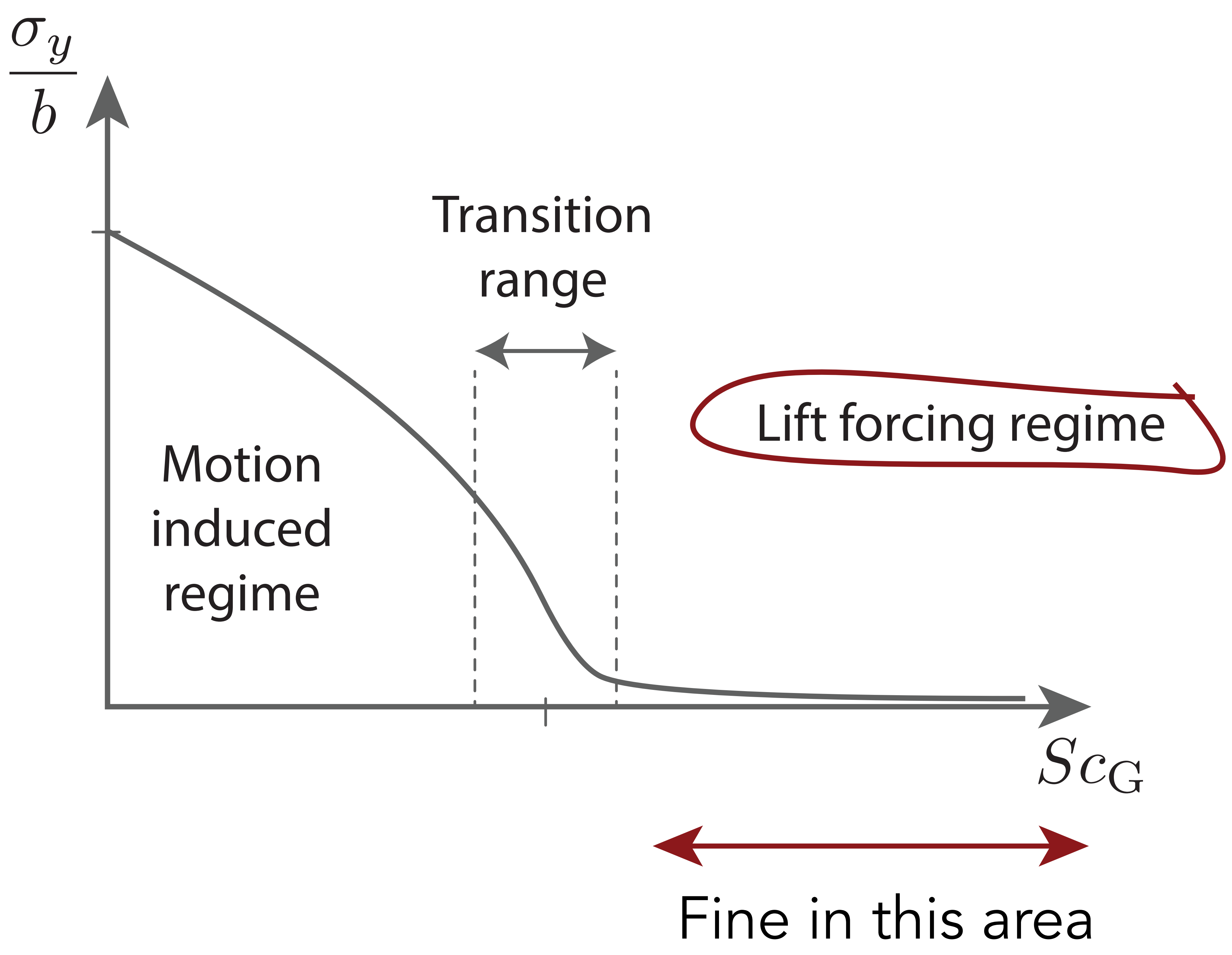
$$\ddot{y}(t) + 2\omega_0\xi\,\dot{y}(t) + \omega_0^2\,y(t) = \frac{F_\mathrm{L}(t)}{m}$$

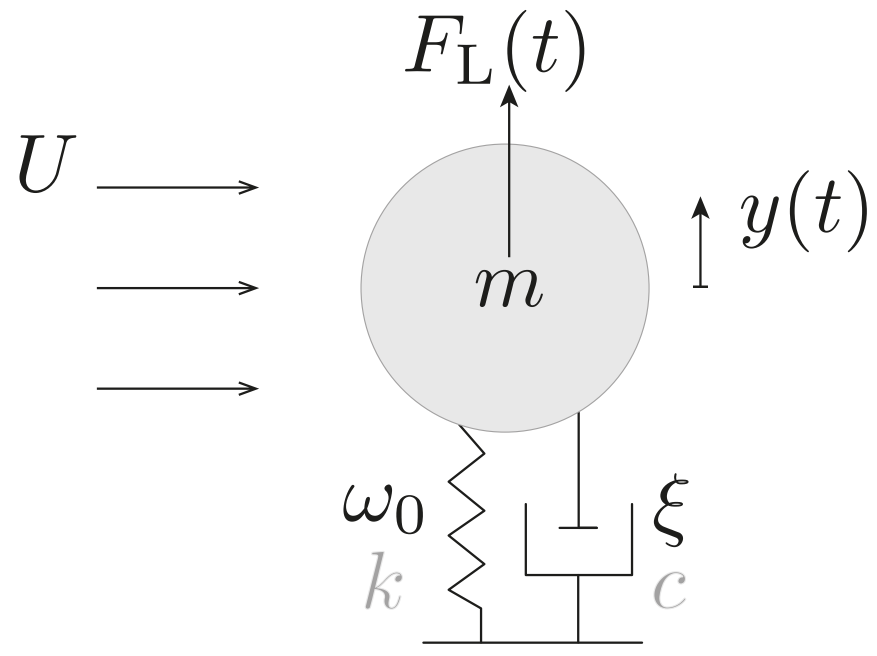
- Experimental evidence that the lift force on a fixed (stationary) cylinder is a narrow band random process
- They suggest a bell-shape PSD for $F_L(t)$
- So the structural response can be computed …
- … OK, … as long as amplitude remains limited

Vickery, B. J., & Clark, A. W. (1972). Lift or across-wind response of tapered stacks. *Journal of the Structural Division*, *98*(1), 1-20.
Vickery, B. J., and R. I. Basu. "Across-wind vibrations of structures of circular cross-section. Part I. Development of a mathematical model for two-dimensional conditions." *Journal of Wind Engineering and Industrial Aerodynamics* 12.1 (1983): 49-73

The fixed cylinder

- The spectral model, Vickery & Clark (1972), Vickery & Basu (1983)





$$\ddot{y}(t) + 2\omega_0\xi\,\dot{y}(t) + \omega_0^2\,y(t) = \frac{F_L(t)}{m}$$

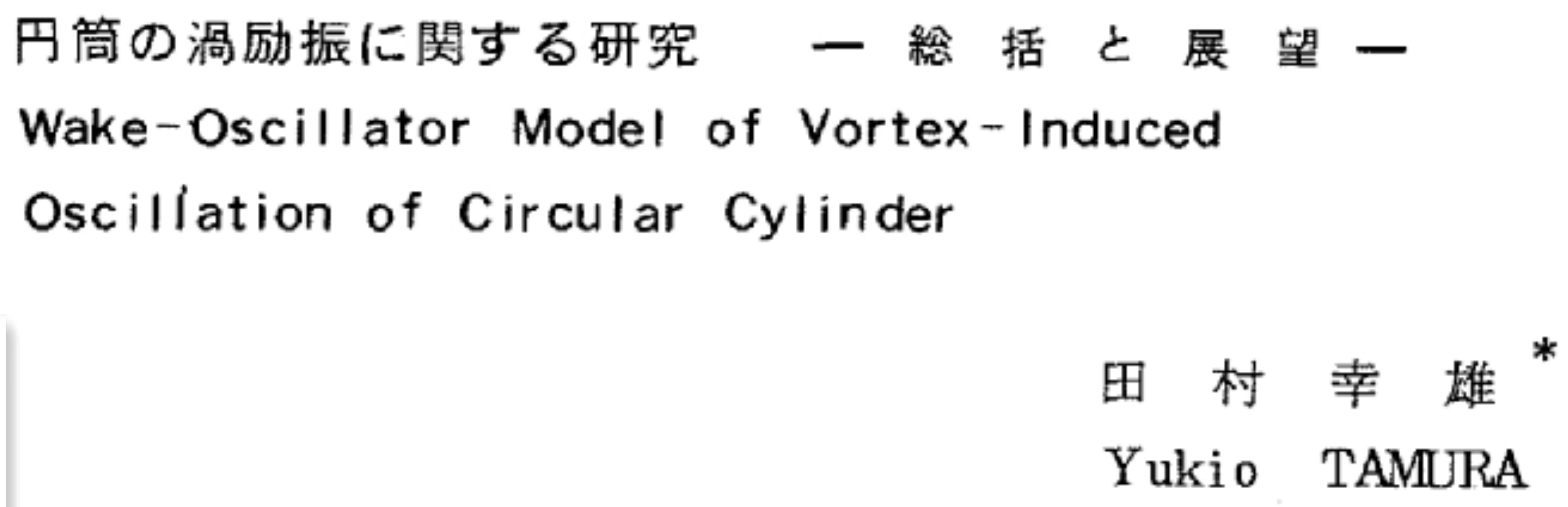
- Some attempts at expanding to large amplitudes, but without a real physical justification

$$F_L(t) = \frac{1}{2}\rho U^2 B C_L(t) + k_a\left(\dot{y}(t) - G\dot{y}^3(t)\right)$$

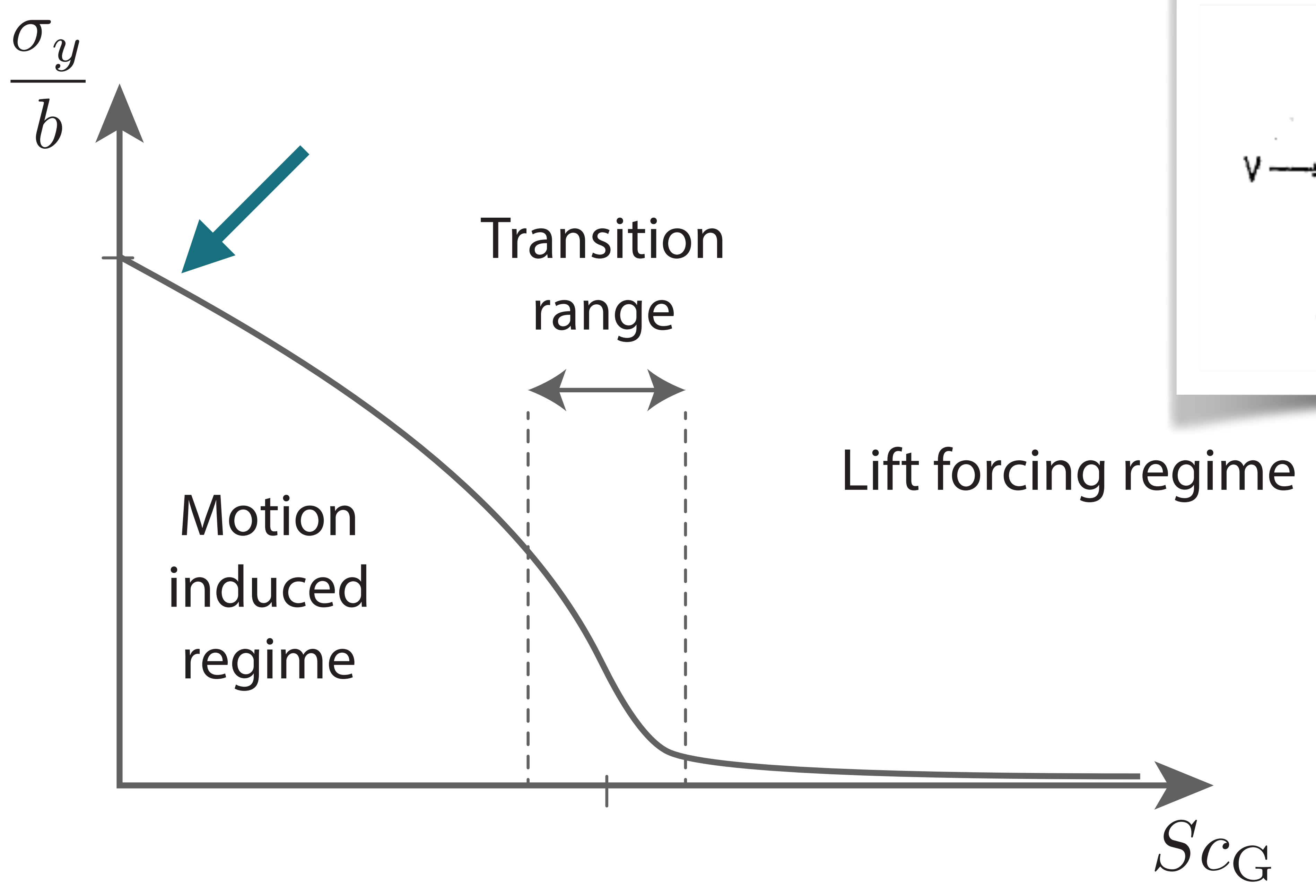
Vickery, B. J., & Clark, A. W. (1972). Lift or across-wind response of tapered stacks. *Journal of the Structural Division*, *98*(1), 1-20.
Vickery, B. J., and R. I. Basu. "Across-wind vibrations of structures of circular cross-section. Part I. Development of a mathematical model for two-dimensional conditions." *Journal of Wind Engineering and Industrial Aerodynamics* 12.1 (1983): 49-73

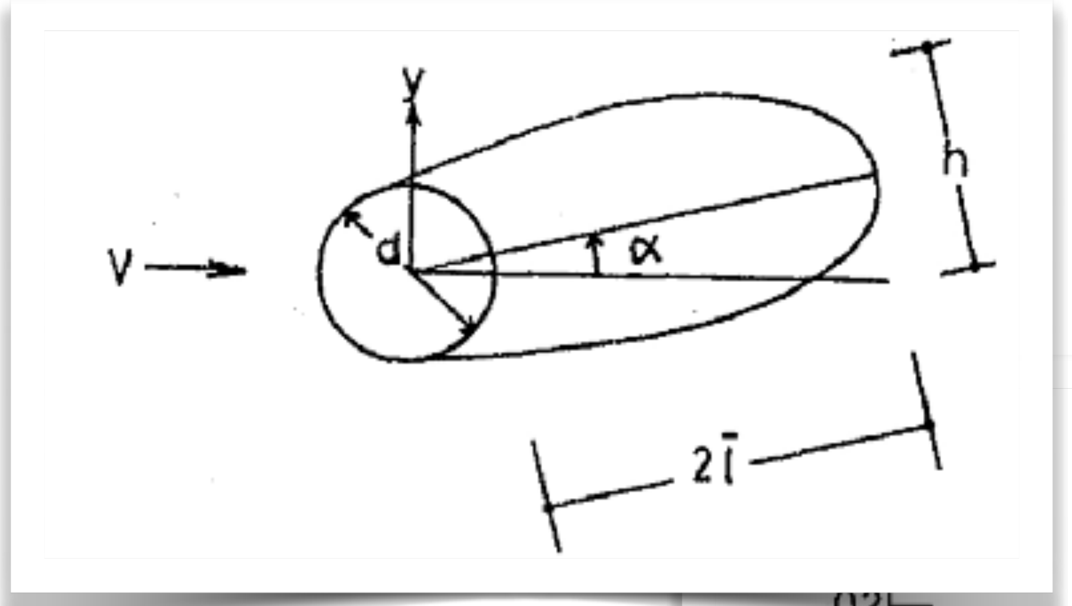
The cylinder with larger amplitude

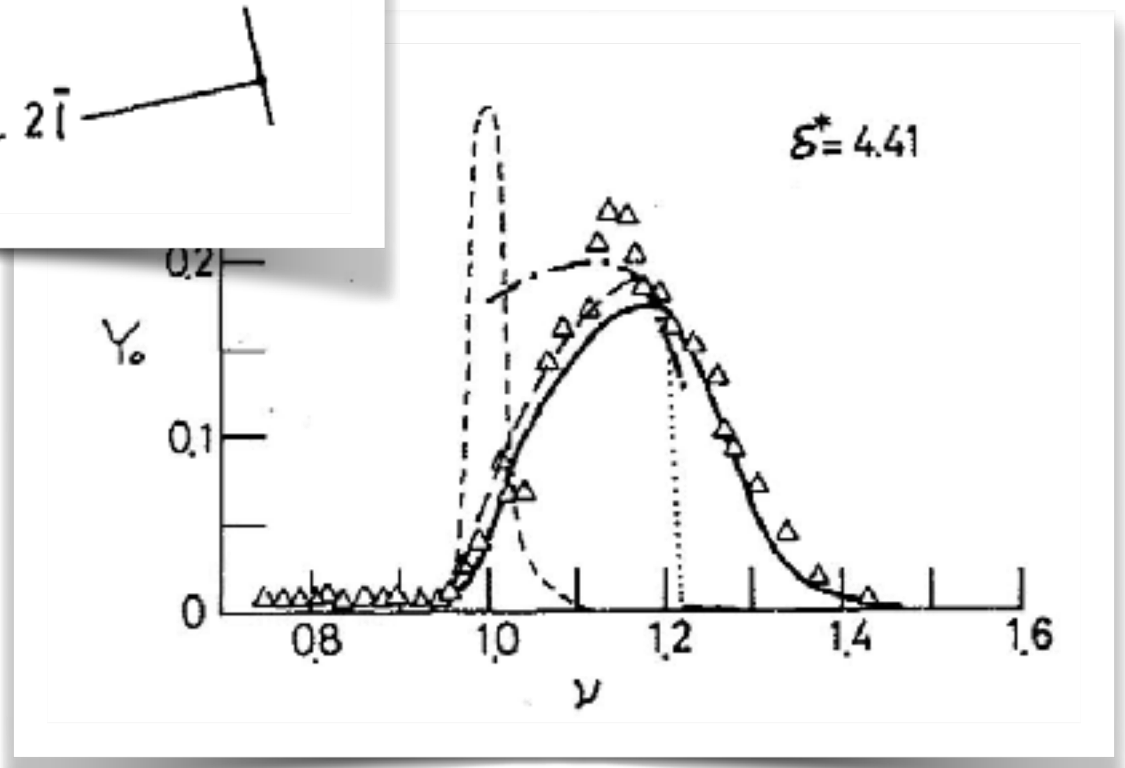
- Wake-Oscillator model(s), Tamura-Matsui (1980)

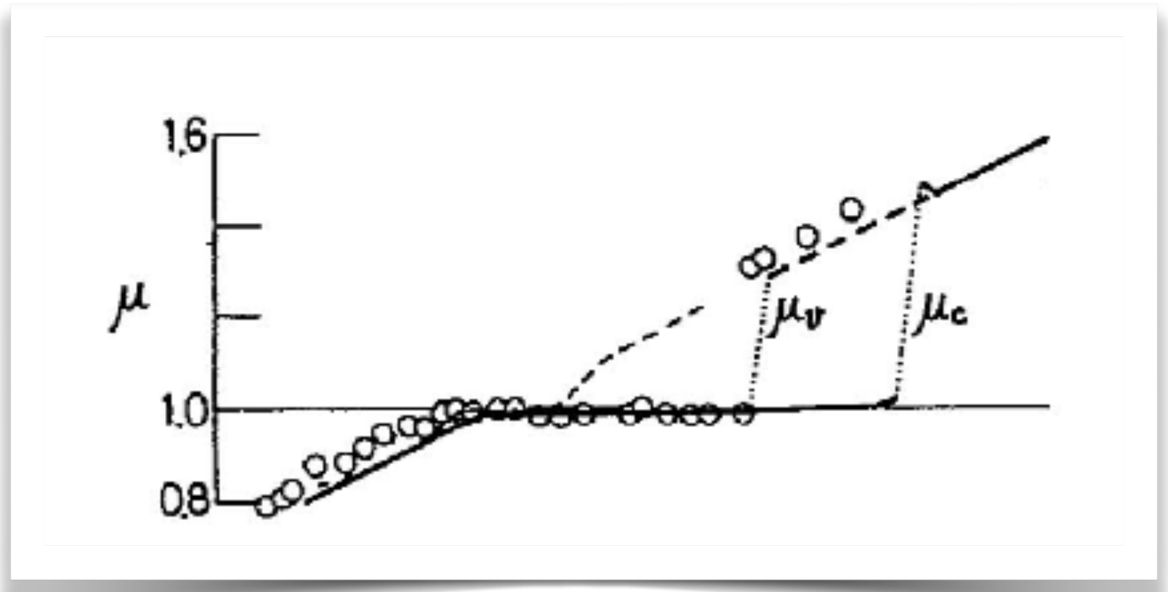

円筒の渦励振に関する研究　— 総 括 と 展 望 —
Wake-Oscillator Model of Vortex-Induced Oscillation of Circular Cylinder

田 村 幸 雄 *
Yukio TAMURA









Nonlinearity here is in the fluid equation

Model is built from first principles of mechanics !

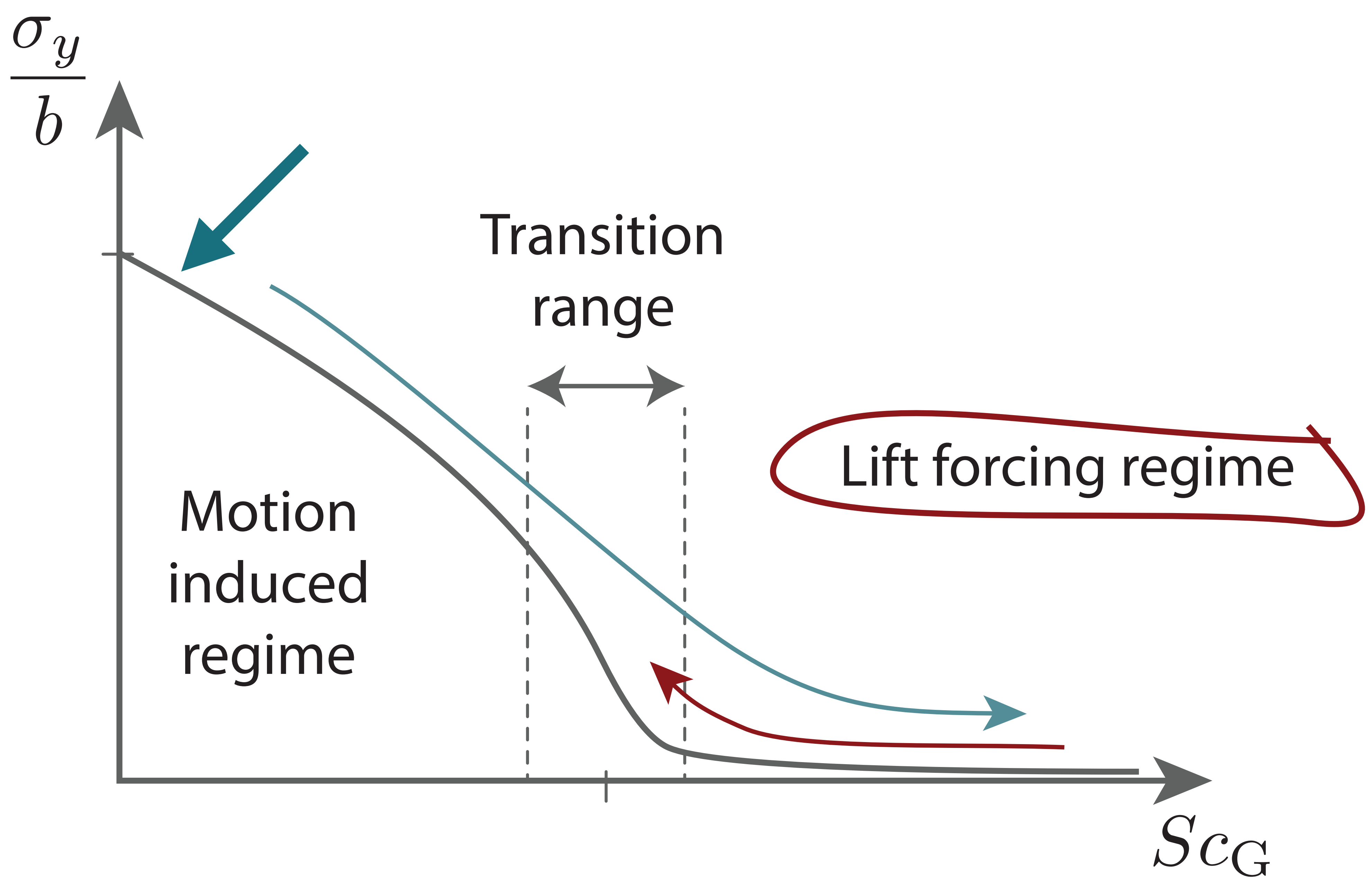
S: $\ddot{y}(t) + 2\omega_0\xi\,\dot{y}(t) + \omega_0^2\,y(t) = \dfrac{F_L(t)}{m}$

F: $\ddot{F}_L(t) + f_1\left(\dot{F}_L, F_L(t)\right) = f_2\left(\ddot{y}(t), \dot{y}(t)\right)$

Tamura, Y., and G. Matsui. "Wake-oscillator model of vortex-induced oscillation of circular cylinder." *Wind Engineering*. Pergamon, 1980. 1085-1094.

Models at extremities of the spectrum

- Wake-Oscillator model, Tamura-Matsui (1980)
- The spectral model, Vickery & Clark (1972), Vickery & Basu (1983)

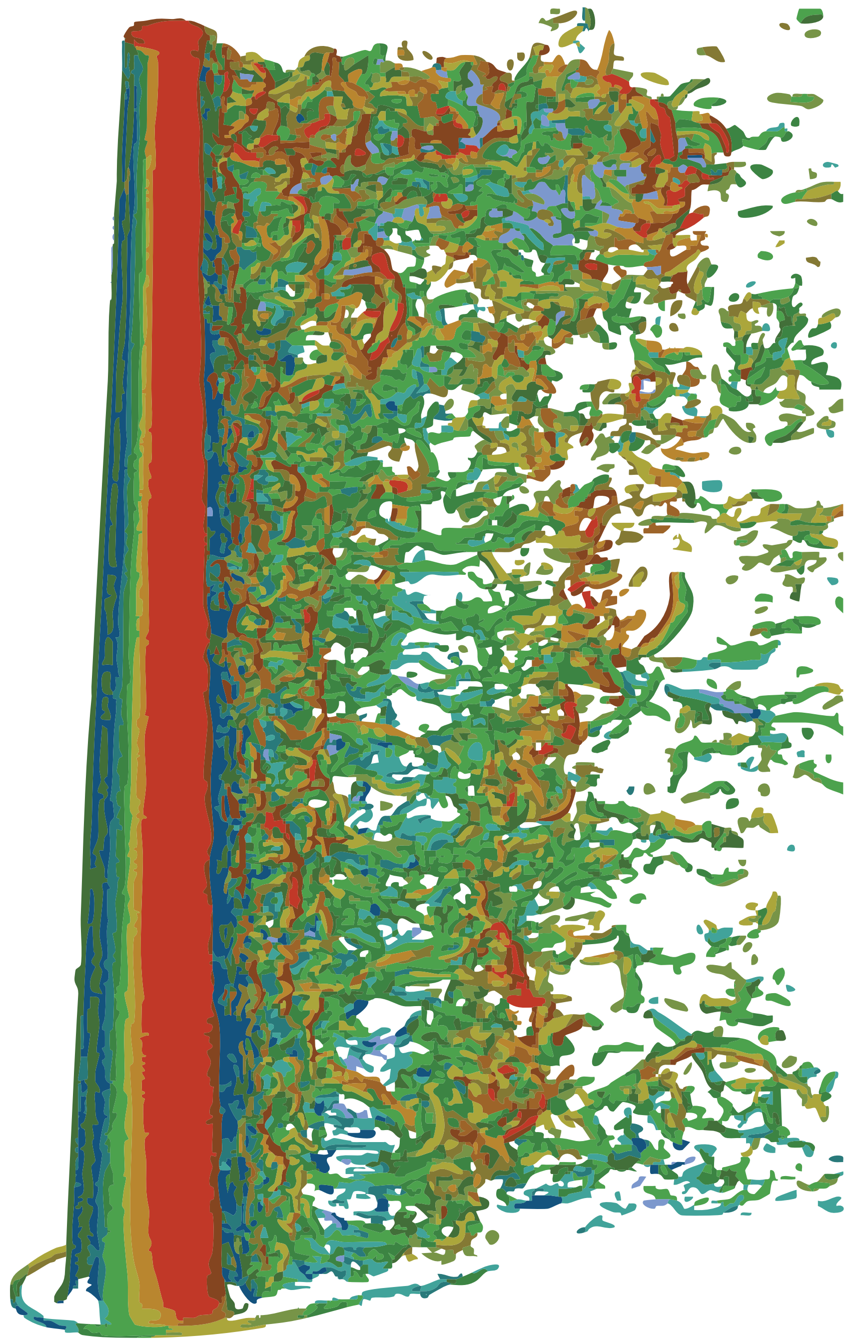


- Two models - don't really talk to each other
- What about transition ?
- Spectral model is great, but limited to small amplitudes.
- Wake-Oscillator model is great, but what if small amplitudes ? Stochastic nature of the response ?

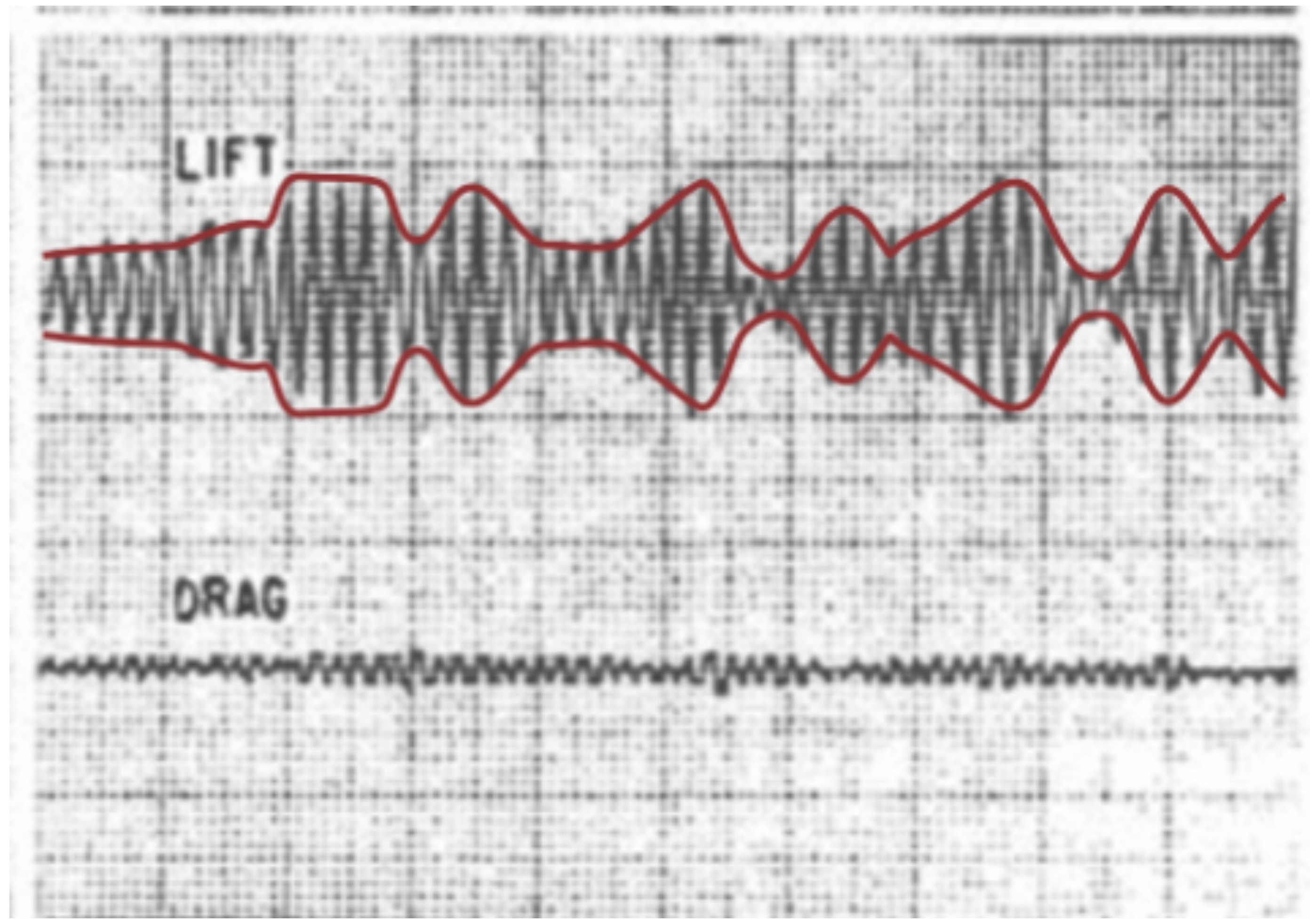
- Start with the wake-oscillator approach, and focus on small amplitude of motion

Sources of randomness

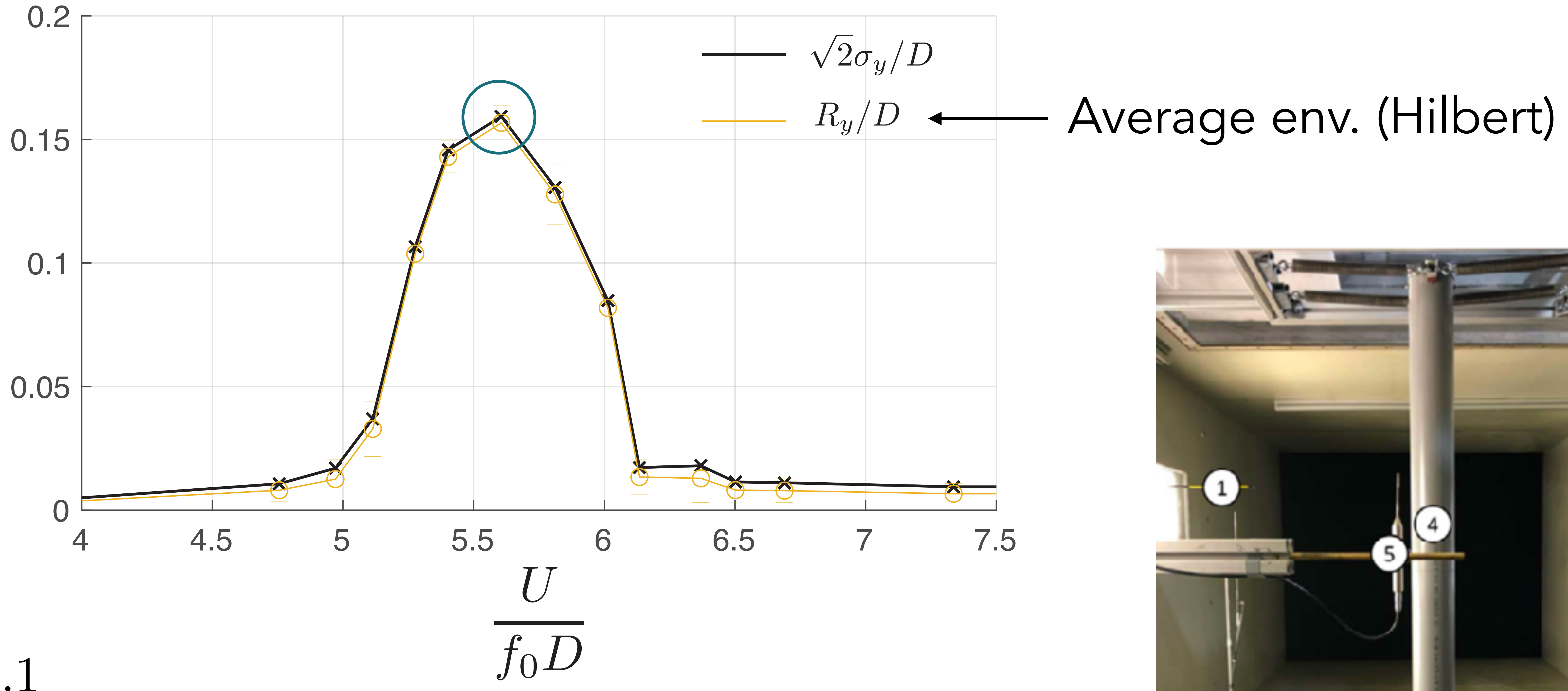
- Turbulence generated by the cylinder (wake)
- Turbulence of the atmospheric boundary layer

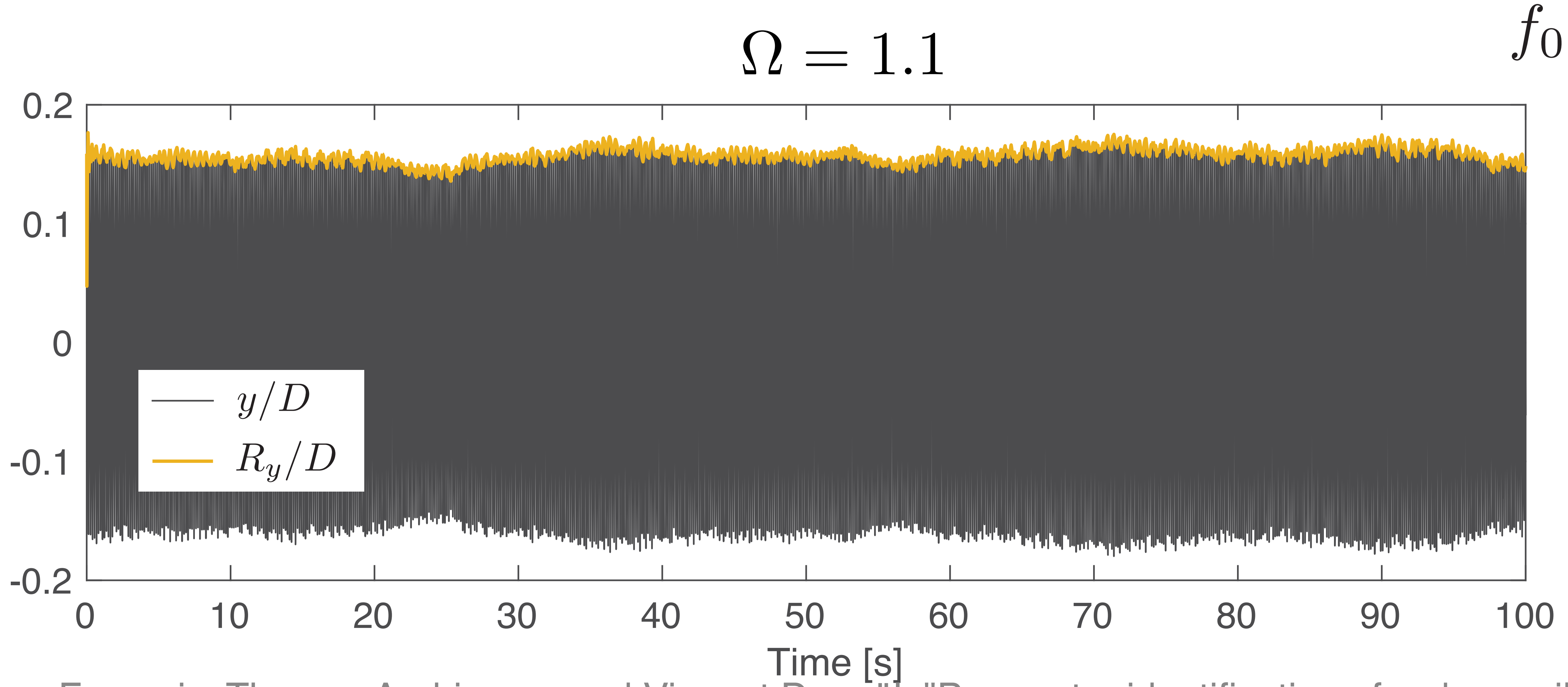


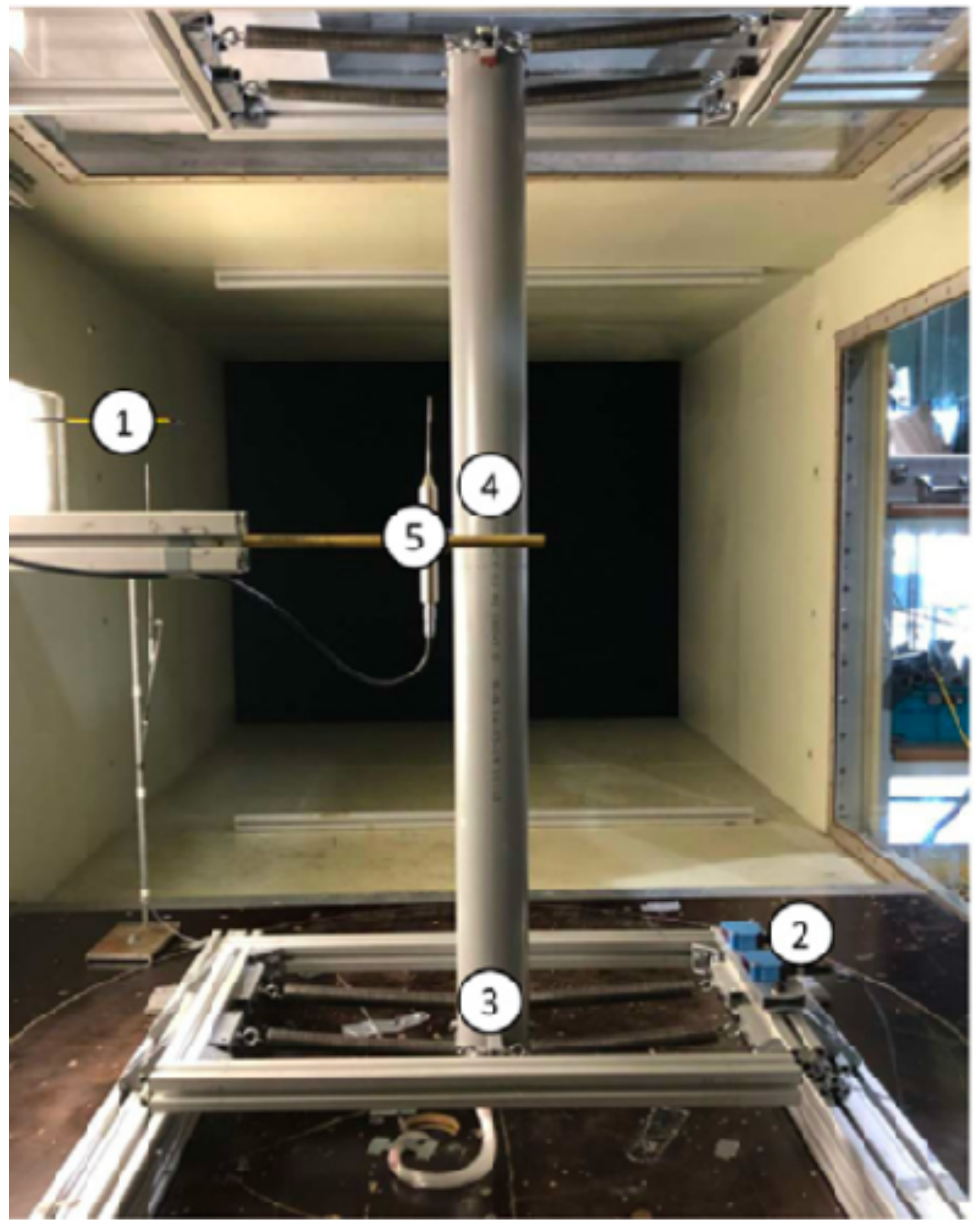
Karim Loueslati, EuroCFD (2020)



Example - Spring-mounted cylinder







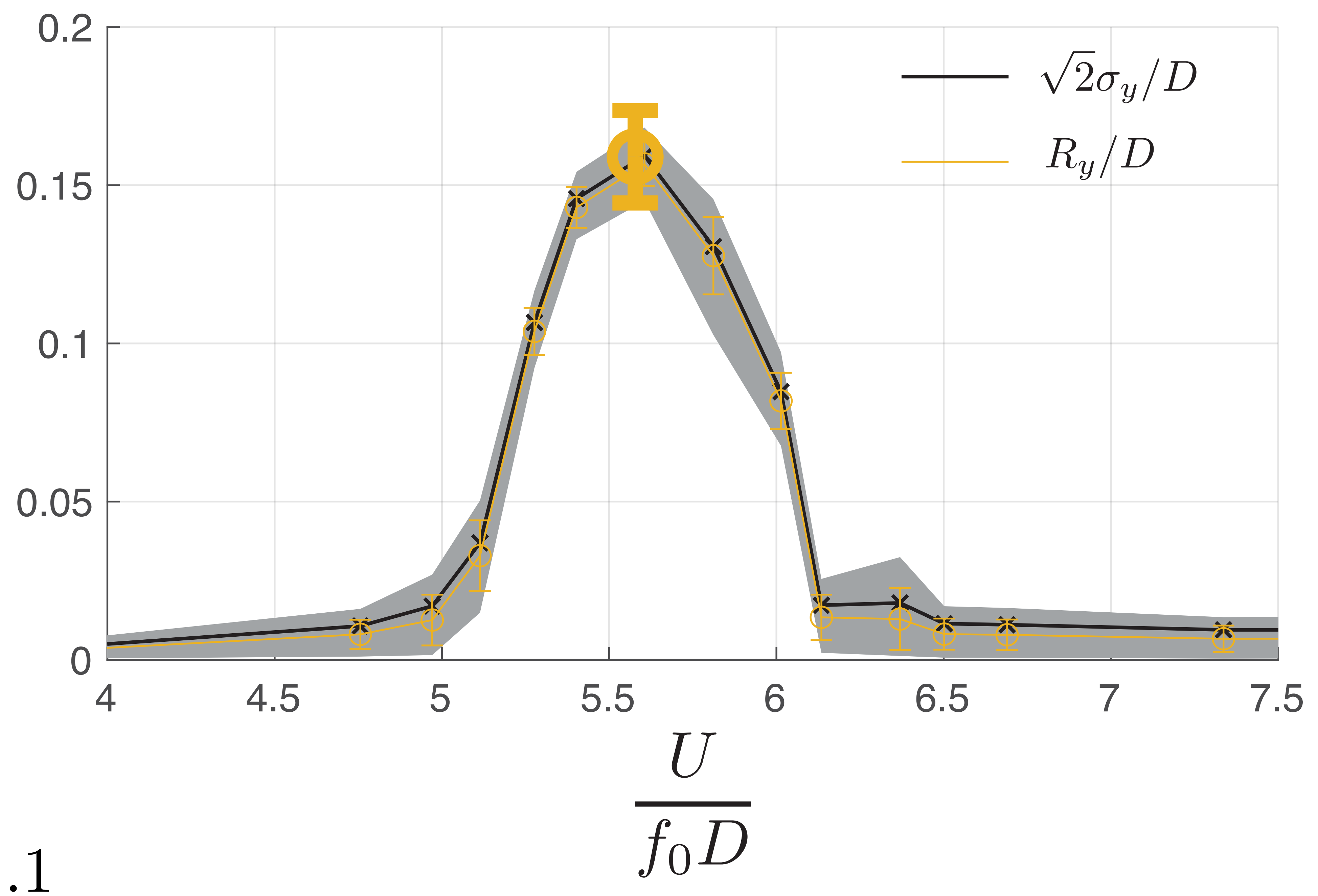
- Low turbulence oncoming flow
- Nat. Freq.: 7.2 Hz, D=100 mm
- U = [3.1; 11.7] m/s

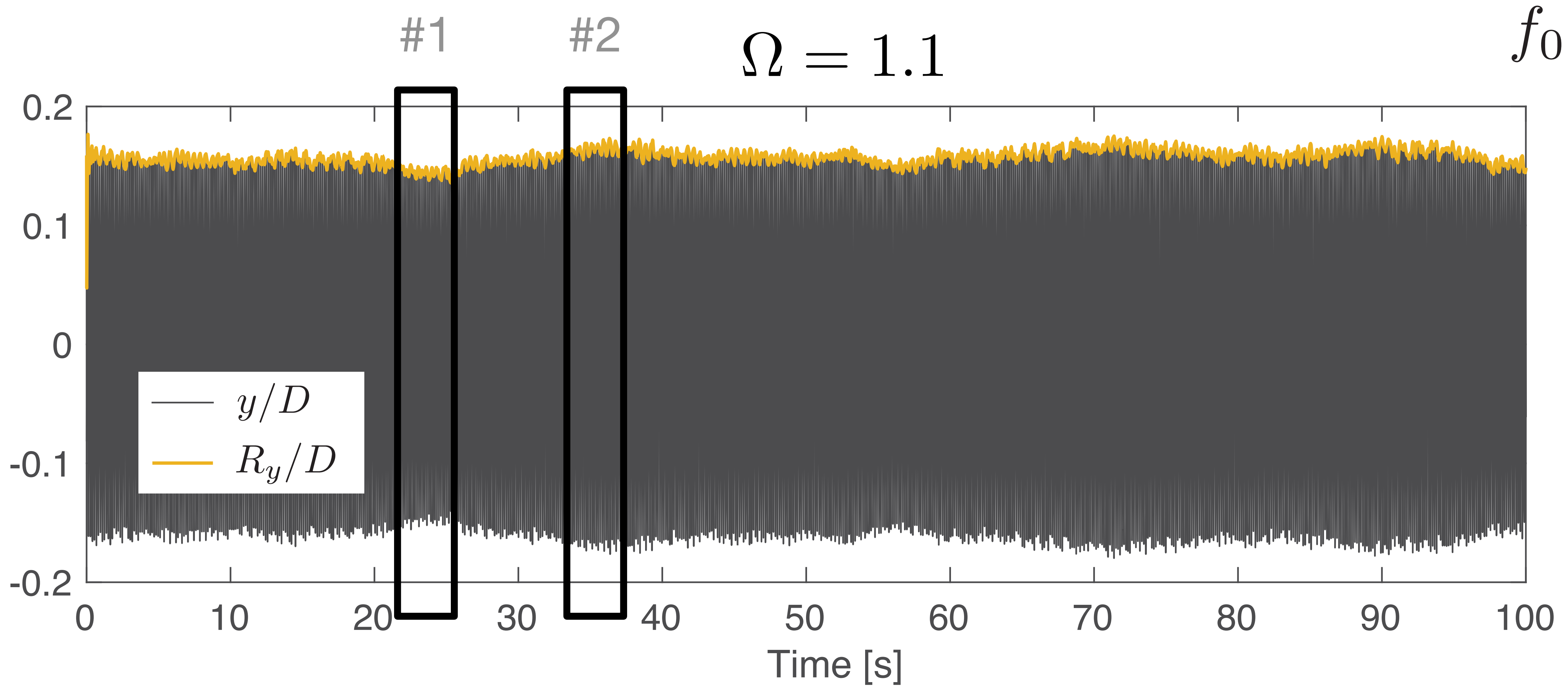
Rigo, François, Thomas Andrianne, and Vincent Denoël. "Parameter identification of wake-oscillator from wind tunnel data." *Journal of Fluids and Structures* 109 (2022): 103474.

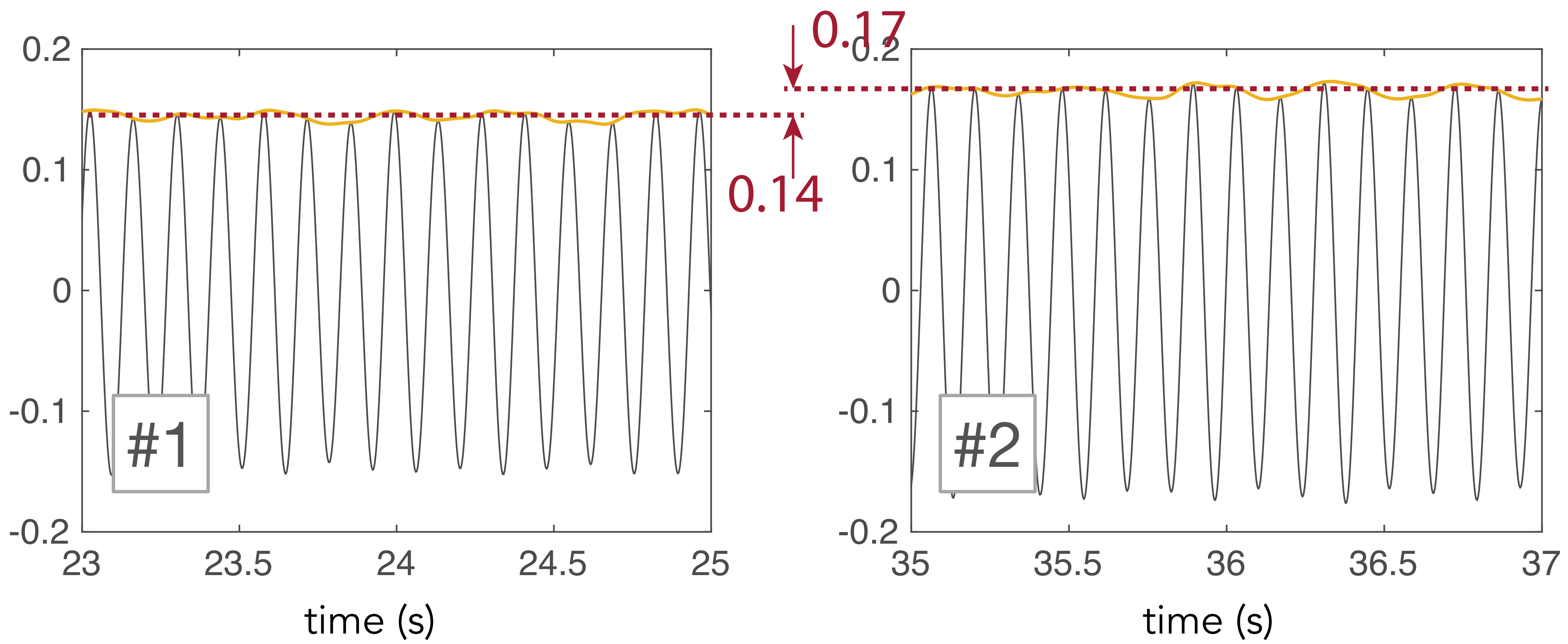
Example - Spring-mounted cylinder

The envelope is slowly changing with time

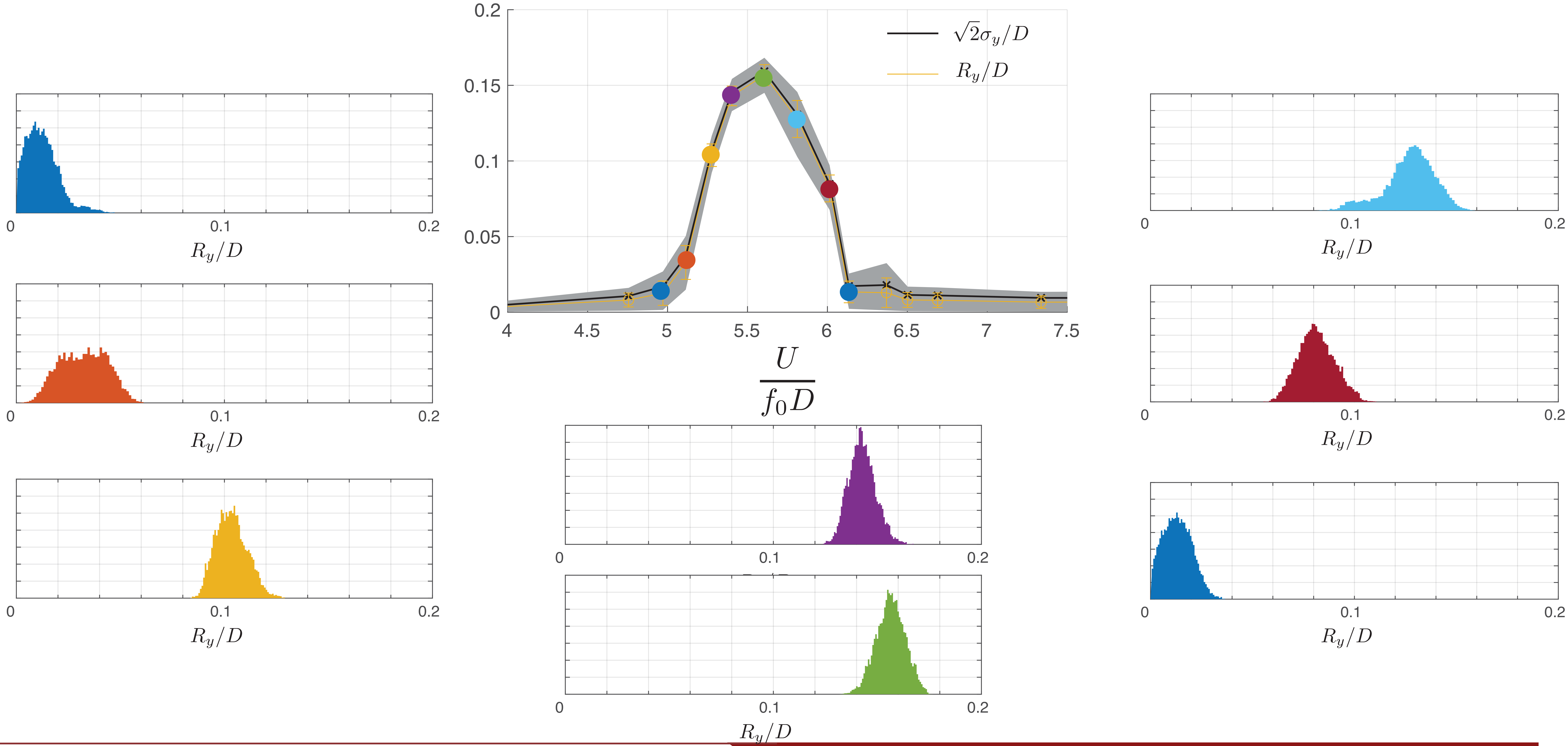
Tip: Analyze the data with the slow envelope, so that you can compute its statistics (not possible with y-RMS)







Spring-mounted cylinder - Statistics of slow envelope



Uniform oncoming flow — Small amplitude & Turbulent wake

$$Y'' + \left(2\eta + n(f+C_D)\frac{\Omega}{S^\star}\right)Y' + Y = -\frac{fn\Omega^2}{S^{\star 2}}\alpha$$

$$\alpha'' - 2\zeta\Omega\left(1 - \frac{4f^2}{C_{L0}^2}\alpha^2\right)\alpha' + \Omega^2\alpha = -m^\star Y'' - \Omega S^\star Y'$$

⇨

$$\cancel{y'' + 2(\xi_s + \xi_a)y' + y = 2\varepsilon\mathcal{M}\Omega^2 q}$$

$$q'' + \underline{\varepsilon\Omega(q^2-1)q'} + \Omega^2 q = \cancel{2\varepsilon(\mathcal{A}_0 y'' + \mathcal{A}_1 y')}$$

Nonlinear terms

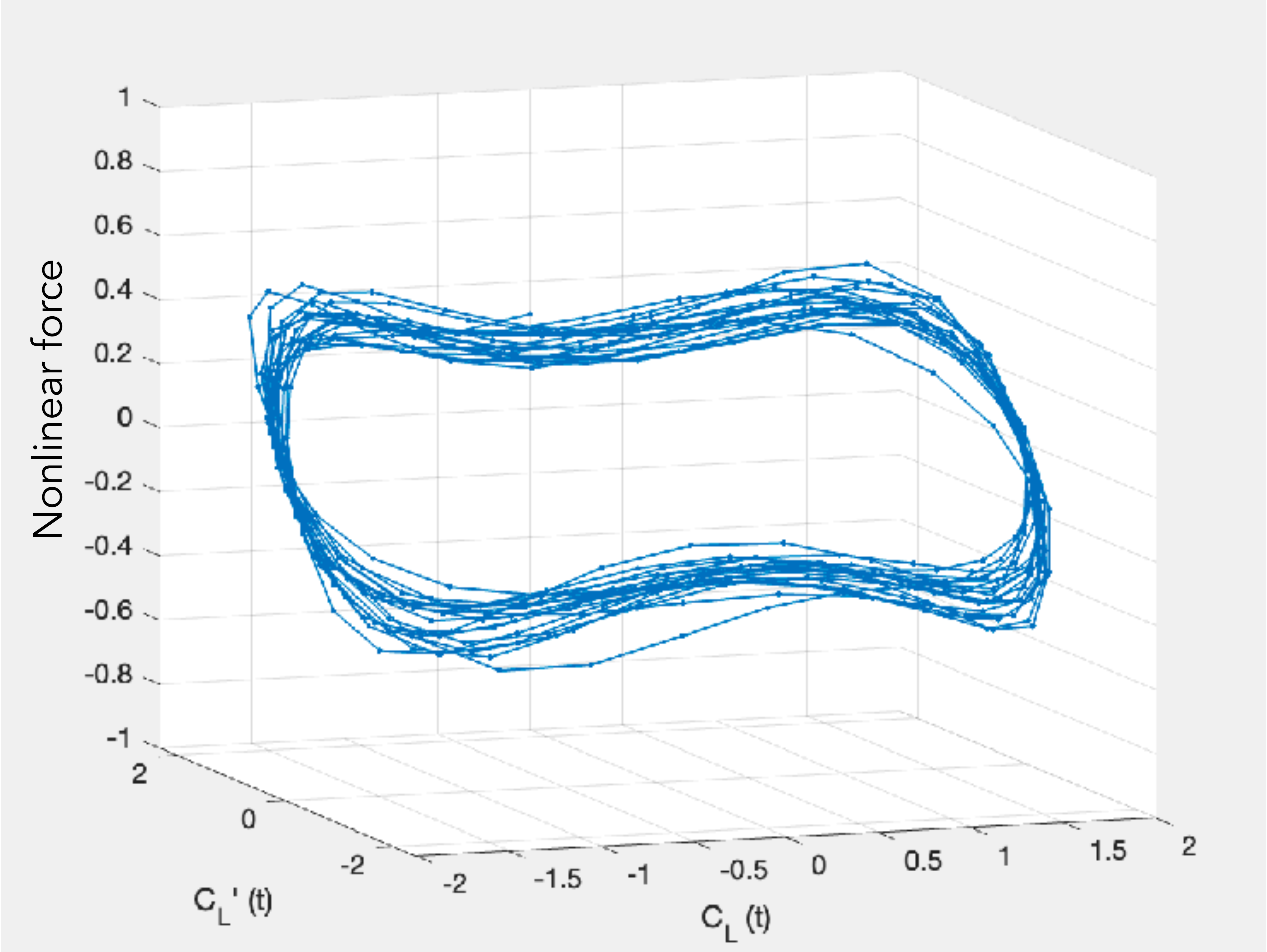
$$\ddot{q} + q = F(q,\dot{q}) = \dot{q}(\alpha q^2 + \underline{\beta q\dot{q}} + \gamma\dot{q}^2 + \delta)\boxed{+\eta(t)}$$

Useless

- Fix the structural motion : **small motion amplitude**
- Take advantage of this study to generalize the nonlinear restoring force
- Add a random perturbation : **turbulent wake**

Generalized wake model

$$\ddot{q} + q = F(q, \dot{q}) = \dot{q}(\alpha q^2 + \cancel{\beta q \dot{q}} + \gamma \dot{q}^2 + \delta)$$



- Measure the lift force, $q(t)$, on a fixed cylinder
- Be accurate and bandpass filter
- Differentiate twice, $\ddot{q}(t)$
- Compute the (measured) value of $\ddot{q}(t) + q(t)$
- Plot $\ddot{q}(t) + q(t)$ as a function of $q(t)$ and $\dot{q}(t)$
- Find parameters in $F(q, \dot{q})$ which minimize distance with these trajectories

$$\bar{\alpha} = -0.09, \quad \bar{\gamma} = 0.009, \quad \bar{\delta} = +0.063$$

Not too far from Tamura's model

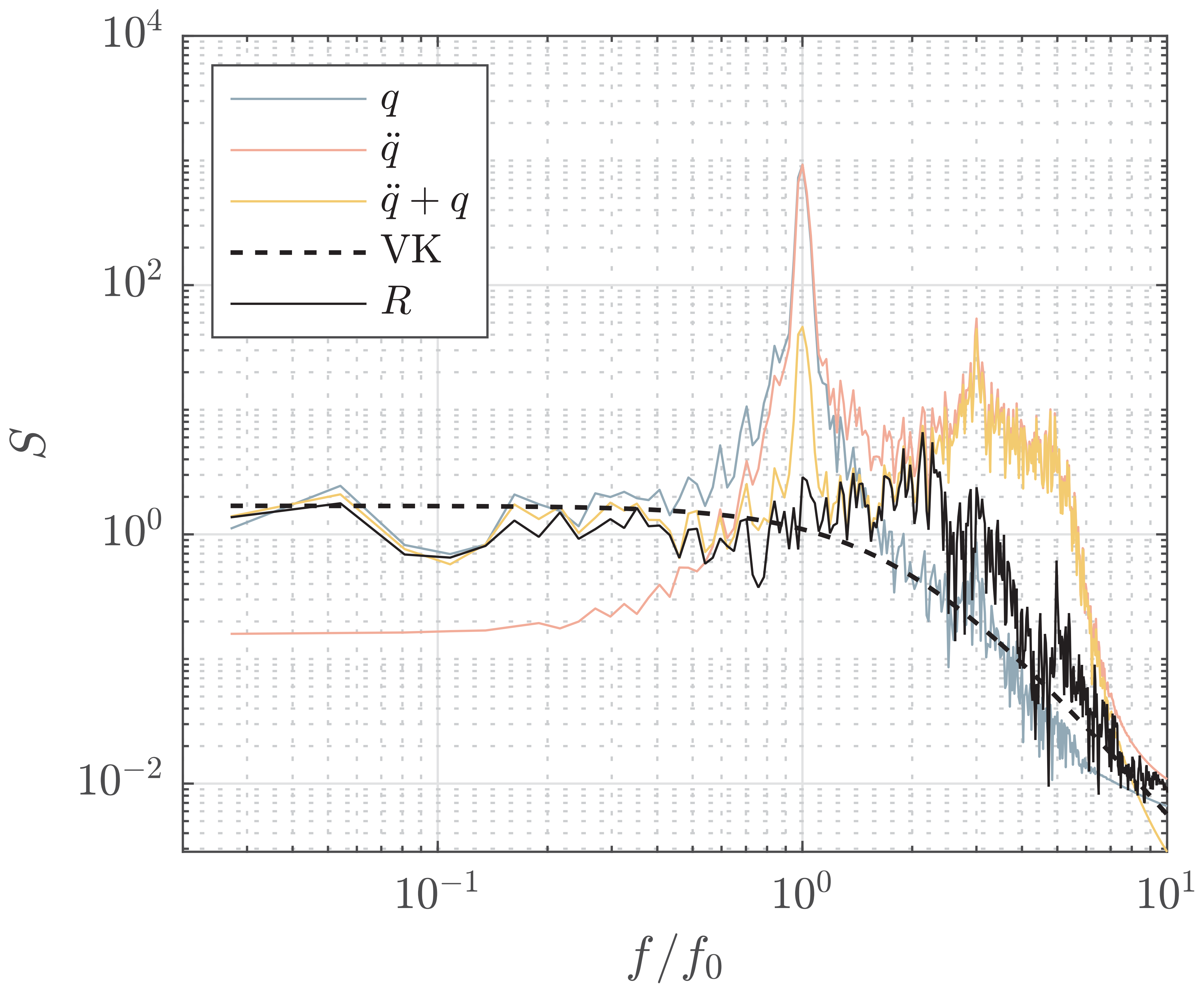
$$\bar{\alpha} = -0.076, \quad \bar{\gamma} = 0, \quad \bar{\delta} = +0.076$$

Rigo, F., Andrianne, T., & Denoël, V. (2022). Generalized lift force model under vortex shedding. *Journal of Fluids and Structures*.

Random wake model

$$\ddot{q} + q = F(q, \dot{q}) = \dot{q}(\alpha q^2 + \beta q \dot{q} + \gamma \dot{q}^2 + \delta) + \eta(t) \quad \longrightarrow \quad \eta(t) = \ddot{q}(t) + q(t) - F(q(t), \dot{q}(t))$$

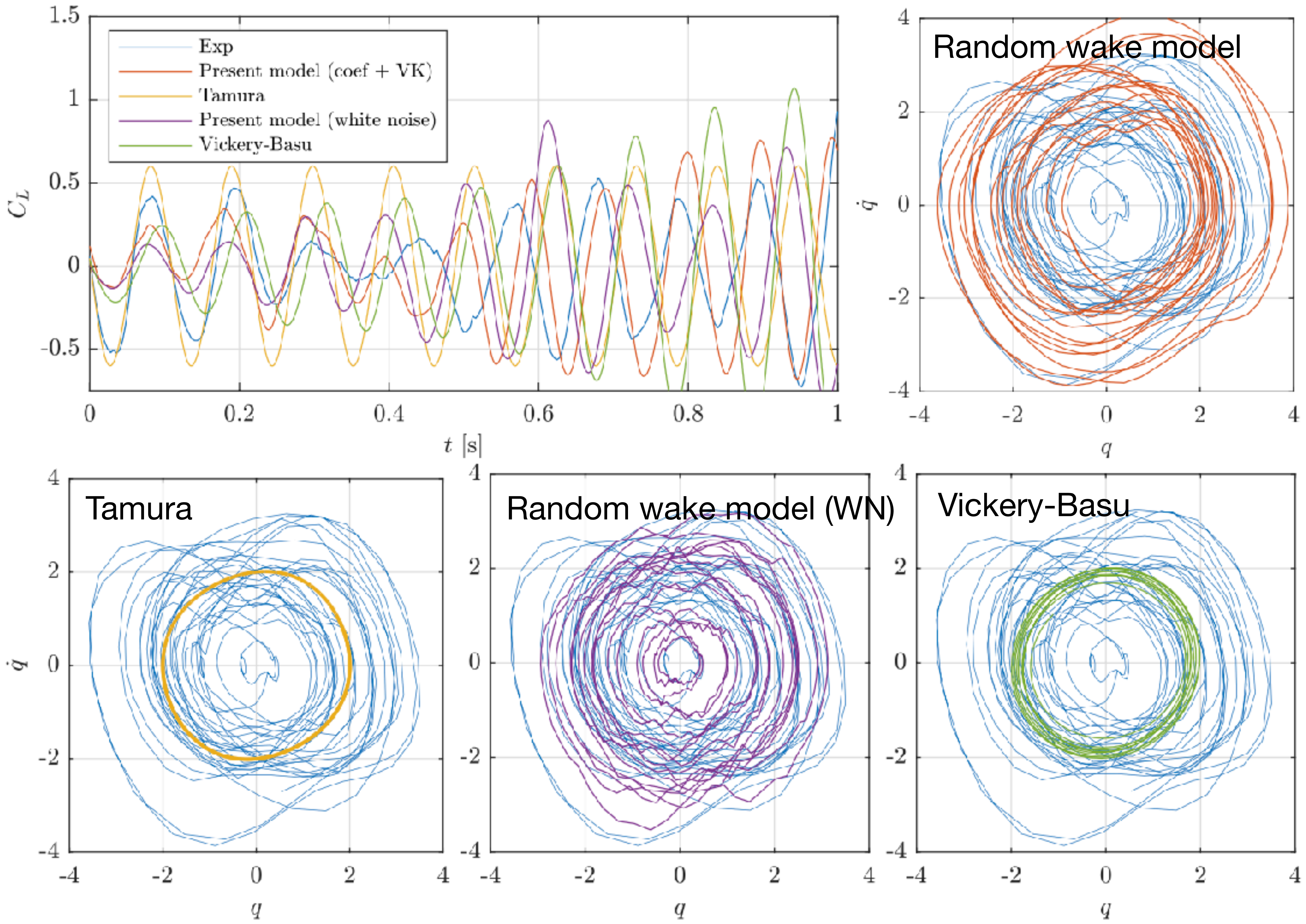
- Model the remainder as a Gaussian noise



von Karman-type spectrum works well :

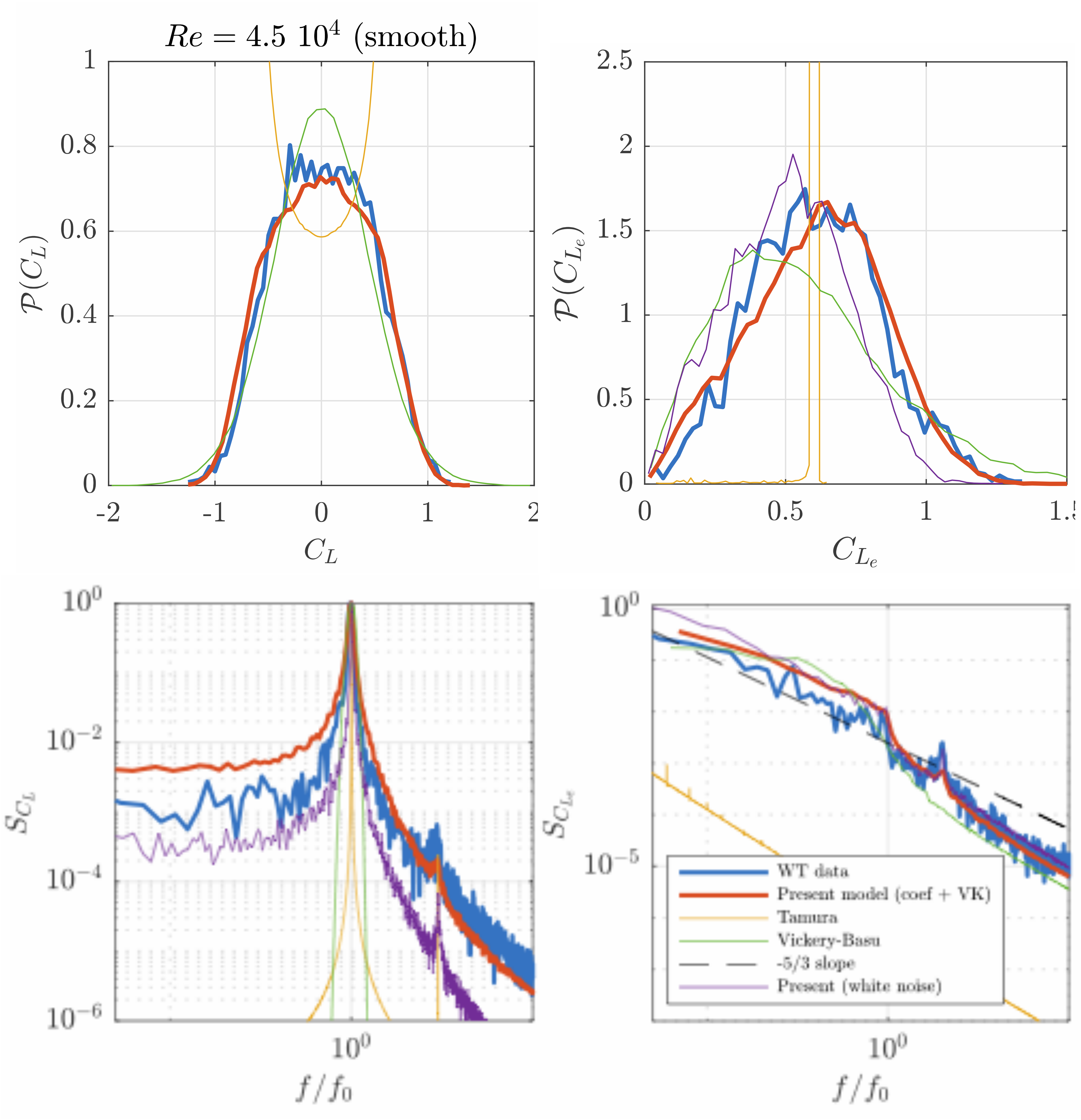
$$\Phi_\eta(\omega) = \sigma_\eta^2 \frac{2L_\eta}{\pi U_\infty} \frac{1}{\left(1 + \left(1.339 L_\eta \frac{\omega}{U_\infty}\right)^2\right)^{\frac{5}{6}}}$$

Random wake model



- Proposed model - matches experimental data
- Tamura's original wake-oscillator is deterministic
- Vickery-Basu random too, but less fluctuations here

Random wake model



- Proposed model - Non gaussian lift, while Vickery-Basu is Gaussian
- Nice fit of the PSD of lift force in the low frequency content

We have a wake oscillator that does the “same” job as the spectral model for the fixed cylinder

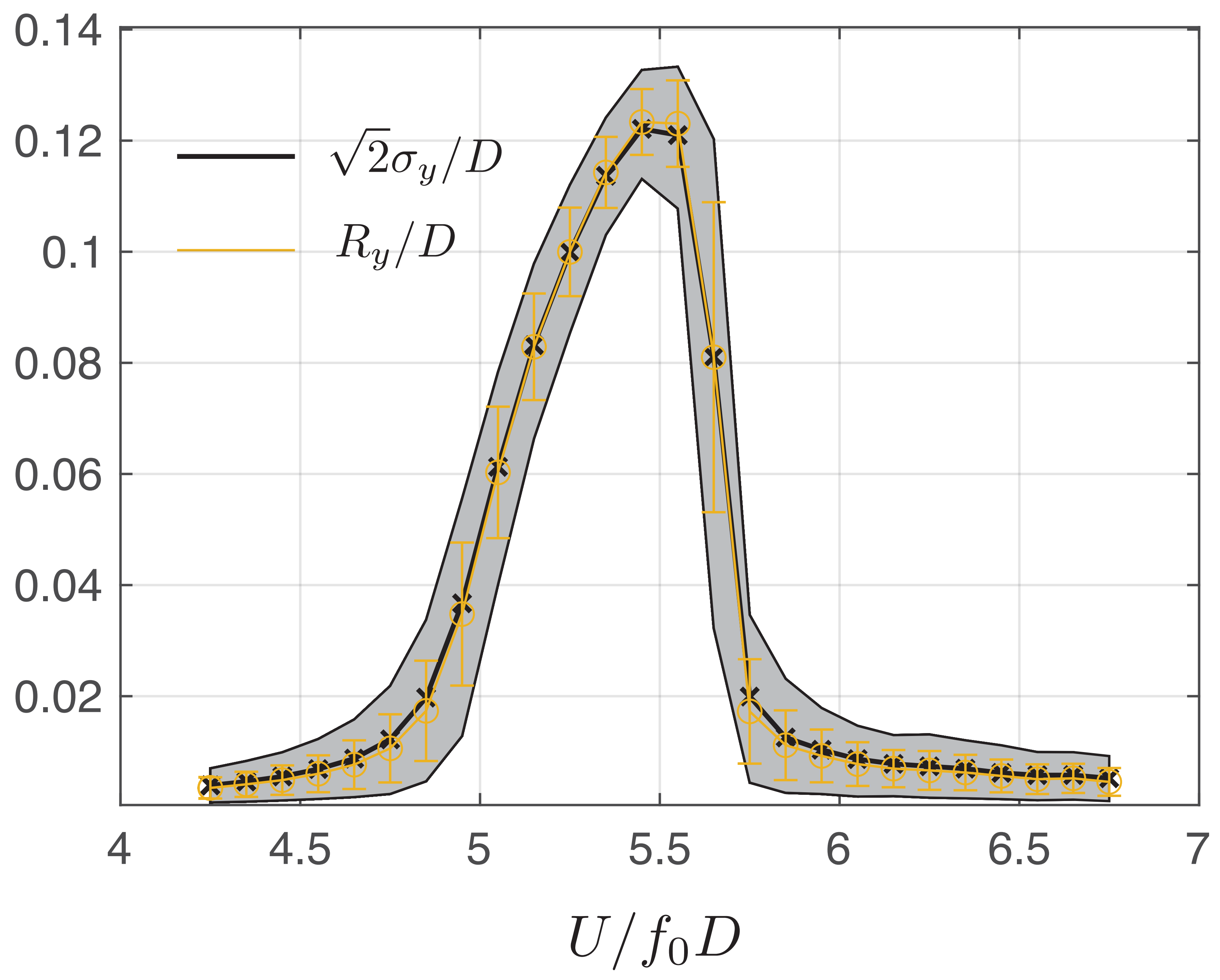
The random wake oscillator model

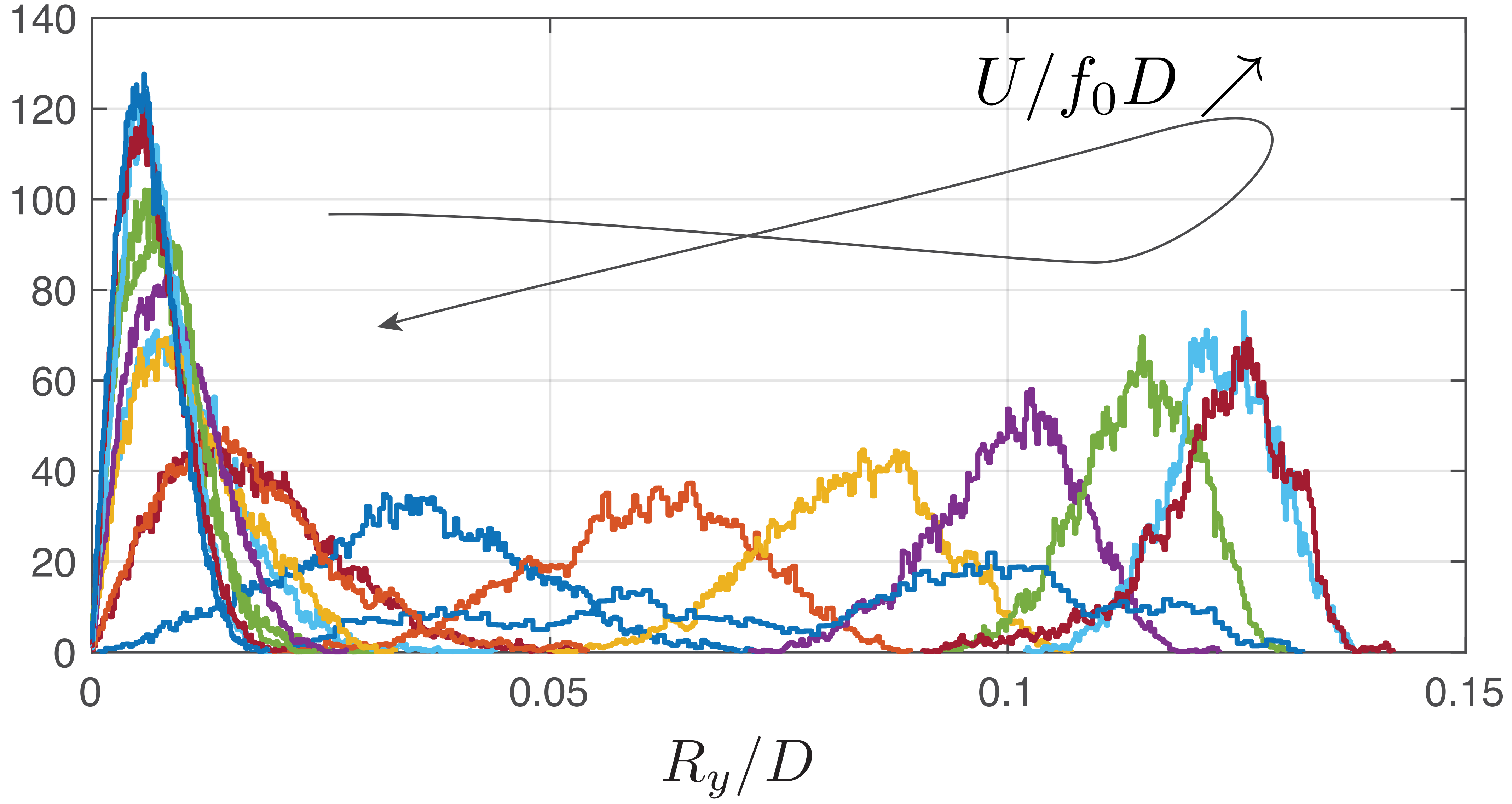
$$y'' + 2(\xi_s + \xi_a)y' + y = 2\varepsilon\mathcal{M}\Omega^2\, q \boxed{+w(t)}$$

$$q'' + \varepsilon\Omega(\alpha q^2 + \gamma q'^2 + \delta)q' + \Omega^2 q = 2\varepsilon(\mathcal{A}_0 y'' + \mathcal{A}_1 y') + \eta$$

$$\boxed{\Omega \to \Omega + u(t)}$$

- At this stage, simulate with Monte Carlo simulations
- Do statistics of the slow envelope
- Notice possible addition of two components of turbulence in the oncoming flow





The random wake oscillator model

$$y'' + 2(\xi_s + \xi_a)y' + y = 2\varepsilon \mathcal{M}\Omega^2 q \boxed{+w(t)}$$

$$q'' + \varepsilon\Omega(\alpha q^2 + \gamma q'^2 + \delta)q' + \Omega^2 q = 2\varepsilon(\mathcal{A}_0 y'' + \mathcal{A}_1 y') + \eta$$

$$\boxed{\Omega \to \Omega + u(t)}$$

- At this stage, simulate with Monte Carlo simulations
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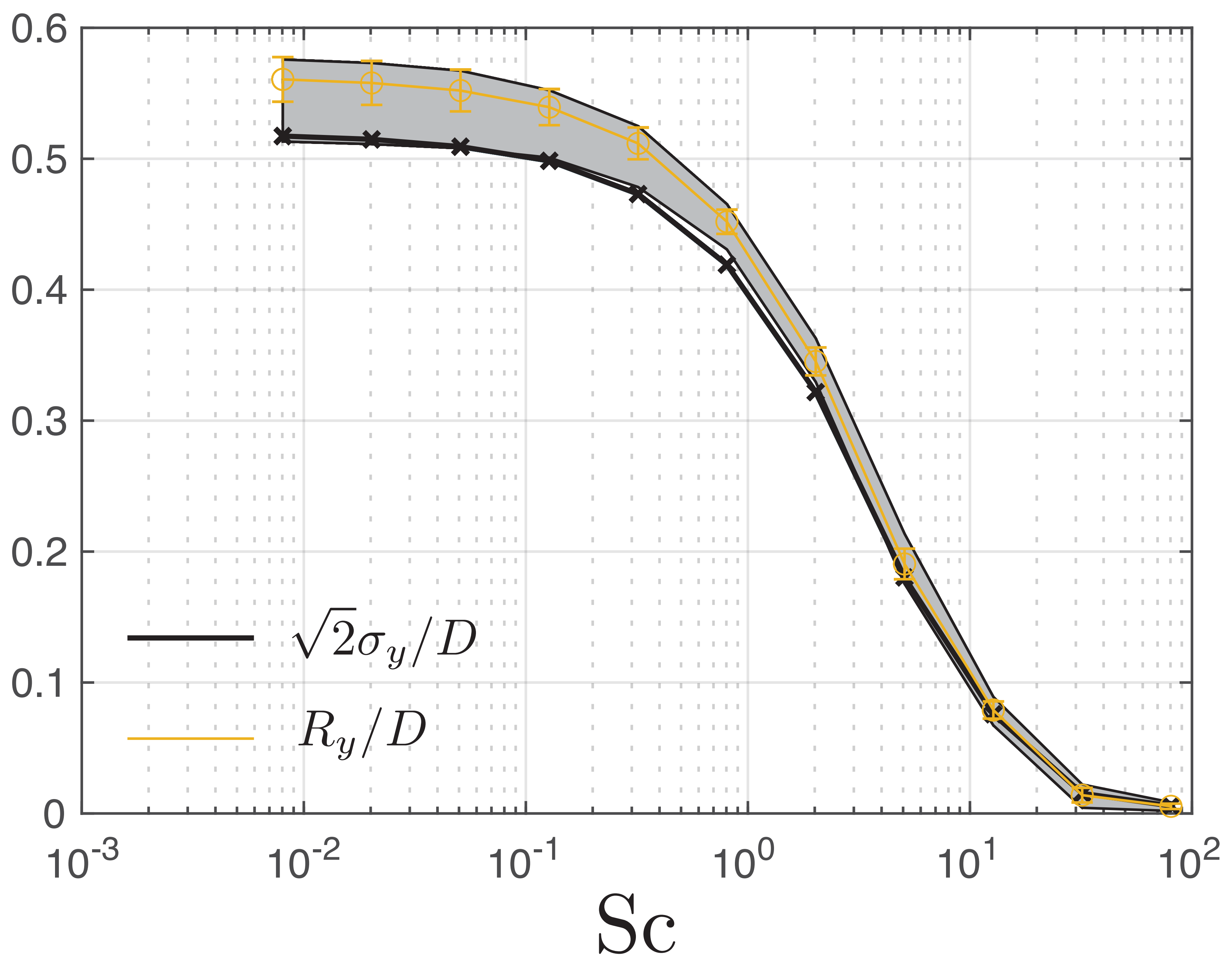
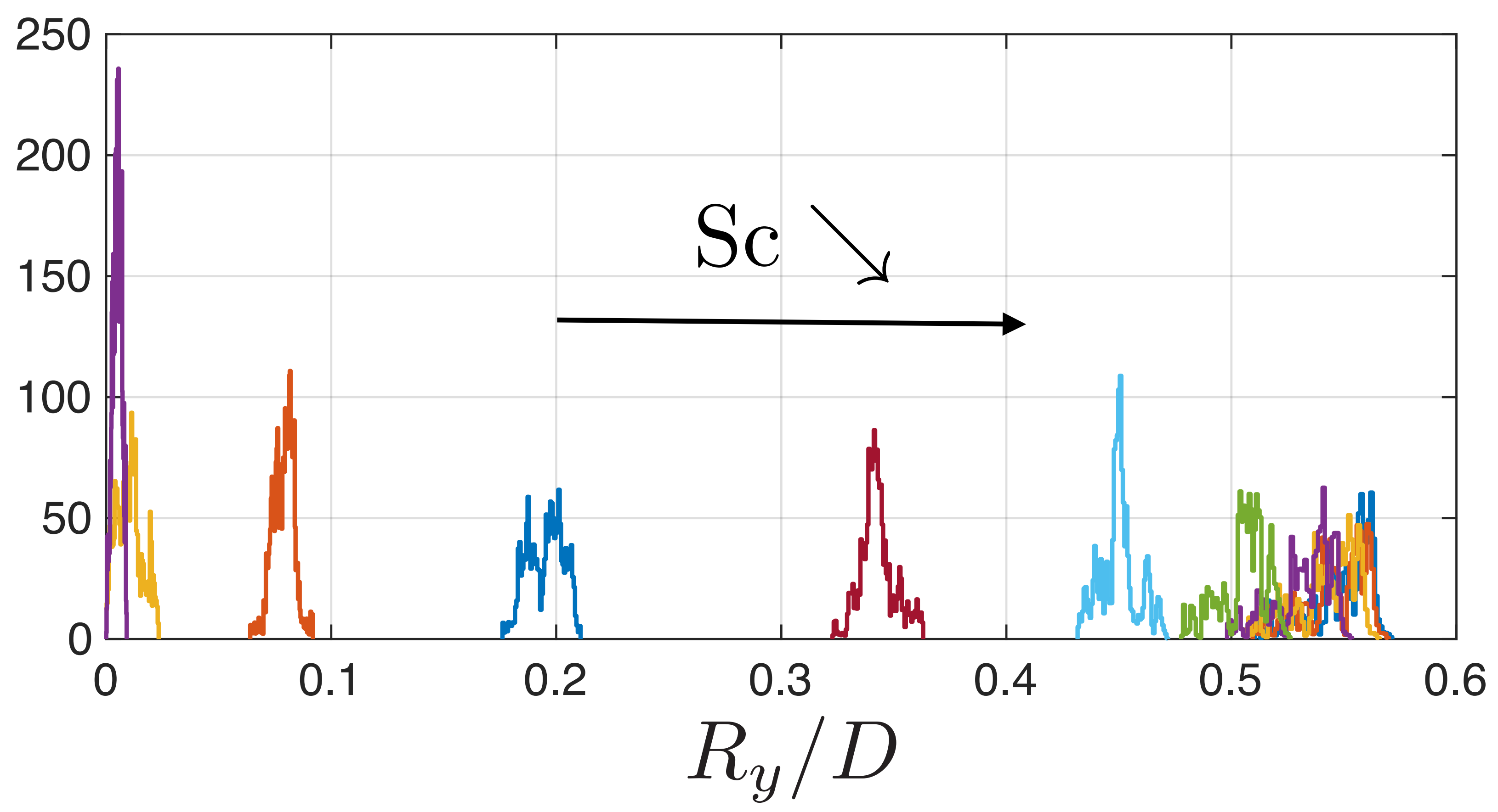


- Response amplitude vs. Scruton looks fine
- Larger fluctuations for smaller Scruton (?)



Fast vs. Slow dynamics

Fast dynamics

$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\,\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\,\Omega\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

→ Multiple timescales

Slow dynamics

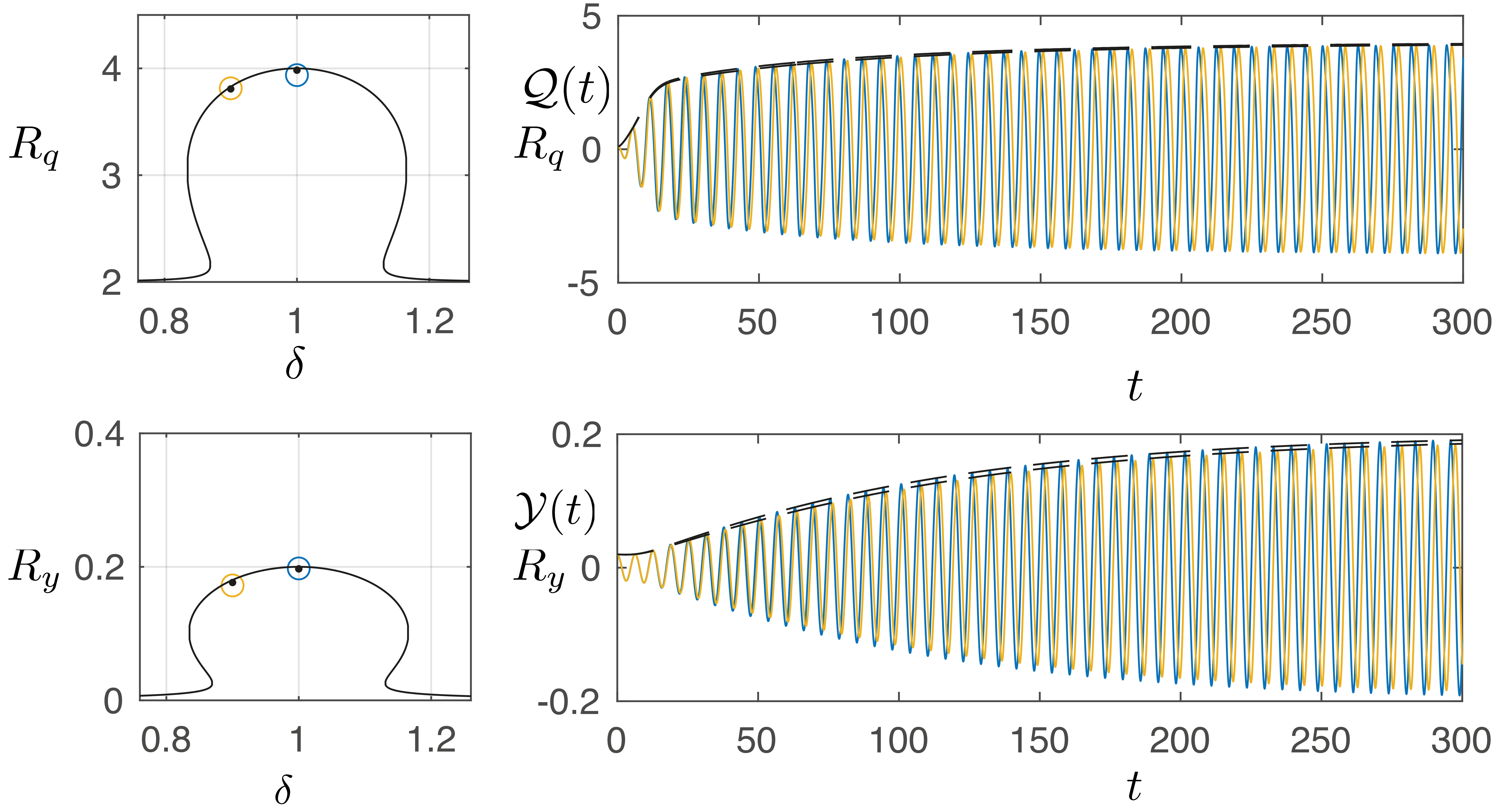
$$R_q' = \mathcal{A}_0 R_y \sin\psi - \frac{1}{8}R_q^3 + \frac{1}{2}R_q$$

$$R_y' = \mathcal{M}_0 R_q \sin\psi - \xi_0 R_y$$

$$\psi' = \left(\mathcal{A}_0\frac{R_y}{R_q} + \mathcal{M}_0\frac{R_q}{R_y}\right)\cos\psi + \delta$$

Wake (lift) :

Cylinder :



$\varepsilon = 0.05, \mathcal{A}_0 = 1,$

$\mathcal{M}_0 = 0.9, \xi_0 = 0.6$

Denoël, V. (2020). Derivation of a slow phase model of vortex-induced vibrations for smooth and turbulent oncoming flows. *Journal of Fluids and Structures*, *99*, 103145.

Fast vs. Slow dynamics

Fast dynamics

$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\,\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\,\Omega\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

Slow dynamics

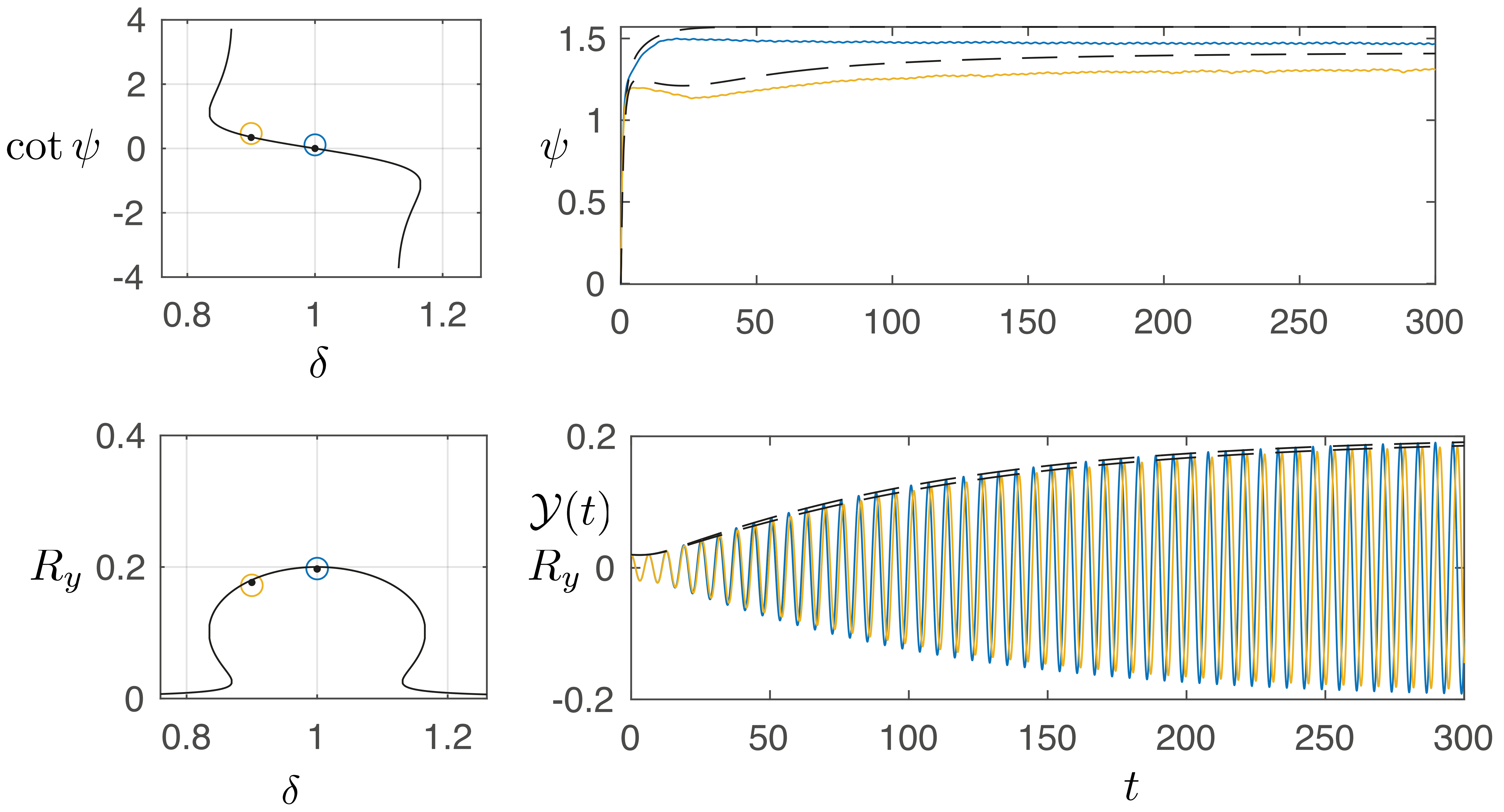
$$R_q' = \mathcal{A}_0 R_y \sin\psi - \frac{1}{8}R_q^3 + \frac{1}{2}R_q$$

$$R_y' = \mathcal{M}_0 R_q \sin\psi - \xi_0 R_y$$

$$\psi' = \left(\mathcal{A}_0\frac{R_y}{R_q} + \mathcal{M}_0\frac{R_q}{R_y}\right)\cos\psi + \delta$$

Phase :

Cylinder :



$\varepsilon = 0.05, \mathcal{A}_0 = 1,$

$\mathcal{M}_0 = 0.9, \xi_0 = 0.6$

Denoël, V. (2020). Derivation of a slow phase model of vortex-induced vibrations for smooth and turbulent oncoming flows. *Journal of Fluids and Structures*, *99*, 103145.

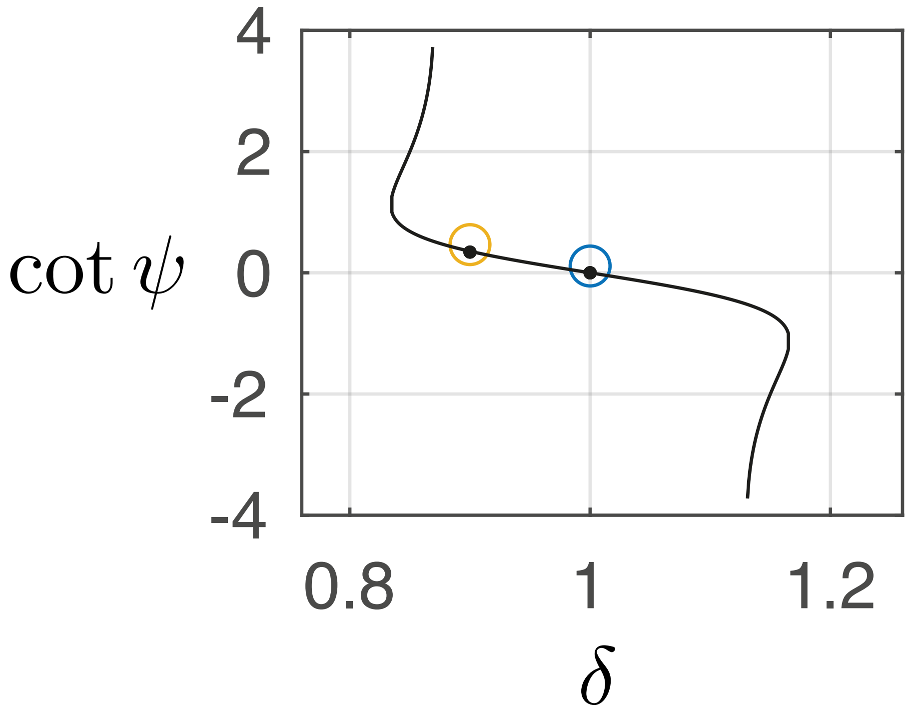
Steady state solutions (deterministic)

Fast dynamics

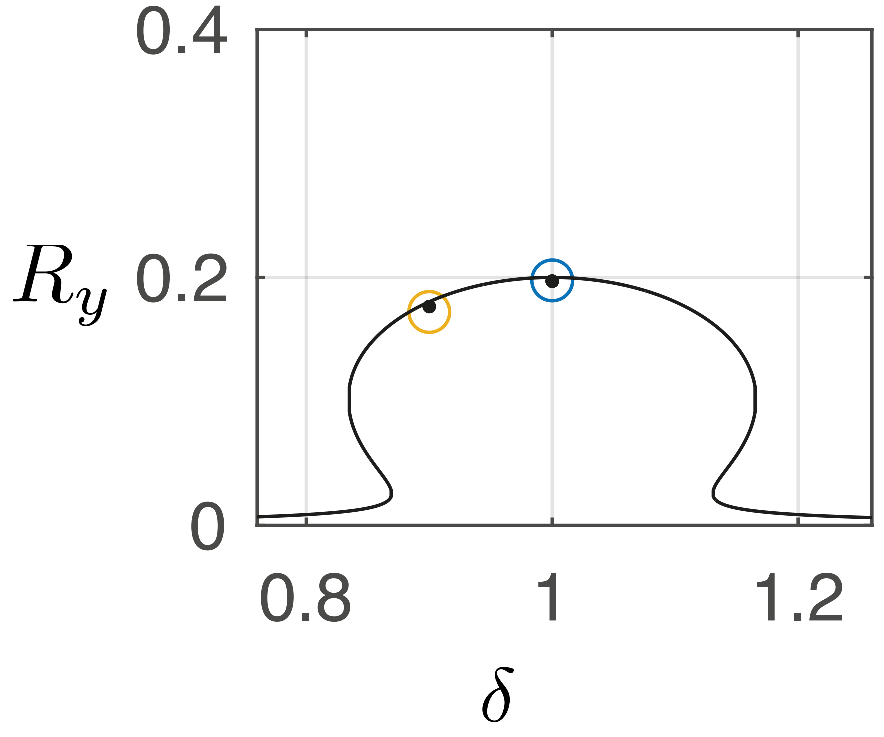
$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\,\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\,\Omega\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

Phase :



Cylinder :



Slow dynamics

$$R_q' = \mathcal{A}_0 R_y \sin\psi - \frac{1}{8}R_q^3 + \frac{1}{2}R_q$$

$$R_y' = \mathcal{M}_0 R_q \sin\psi - \xi_0 R_y$$

$$\psi' = \left(\mathcal{A}_0\frac{R_y}{R_q} + \mathcal{M}_0\frac{R_q}{R_y}\right)\cos\psi + \delta$$

Slow phase dynamics :

$$\psi' = \xi_0\cot^3\psi + \delta\cot^2\psi + \left(\xi_0 + \mathcal{A}_0\frac{\mathcal{M}_0}{\xi_0}\right)\cot\psi + \delta = 0$$

Two randomized wake-oscillators

A simple wake-oscillator model :

Facchinetti-de Langre-Biolley (2004)

$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\,\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\,\Omega\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

OPTION 1.

Smooth flow, turbulence <u>in the wake</u> :

Add perturbation in the fluid equation

$$\mathcal{Y}'' + 2\left(\xi_\mathrm{s} + \xi_\mathrm{a}\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\Omega^2\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\Omega\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}'' + \eta$$

OPTION 2.

Turbulent <u>oncoming flow</u> :

Replace U by $U + u(t)$

$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\left(1 + I_u\mathcal{U}\right)\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\left(1 + I_u\mathcal{U}\right)^2\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\Omega\left(1 + I_u\mathcal{U}\right)\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\left(1 + I_u\mathcal{U}\right)^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

Denoël, V. (2020). Derivation of a slow phase model of vortex-induced vibrations for smooth and turbulent oncoming flows. *Journal of Fluids and Structures*, *99*, 103145.

Option 2. Turbulent oncoming flow

Randomized wake-oscillator model :

$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\left(1 + I_u\mathcal{U}\right)\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\left(1 + I_u\mathcal{U}\right)^2\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\Omega\left(1 + I_u\mathcal{U}\right)\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\left(1 + I_u\mathcal{U}\right)^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

Slow equations :

$$R_q' = \mathcal{A}_0 R_y \sin\psi - \frac{1}{8}R_q^3 + \frac{1}{2}R_q$$

$$R_y' = \mathcal{M}_0 R_q \sin\psi - \xi_0 R_y$$

$$\psi' = \left(\mathcal{A}_0\frac{R_y}{R_q} + \mathcal{M}_0\frac{R_q}{R_y}\right)\cos\psi + \xi_0\delta + \mathcal{I}_0\mathcal{U}$$

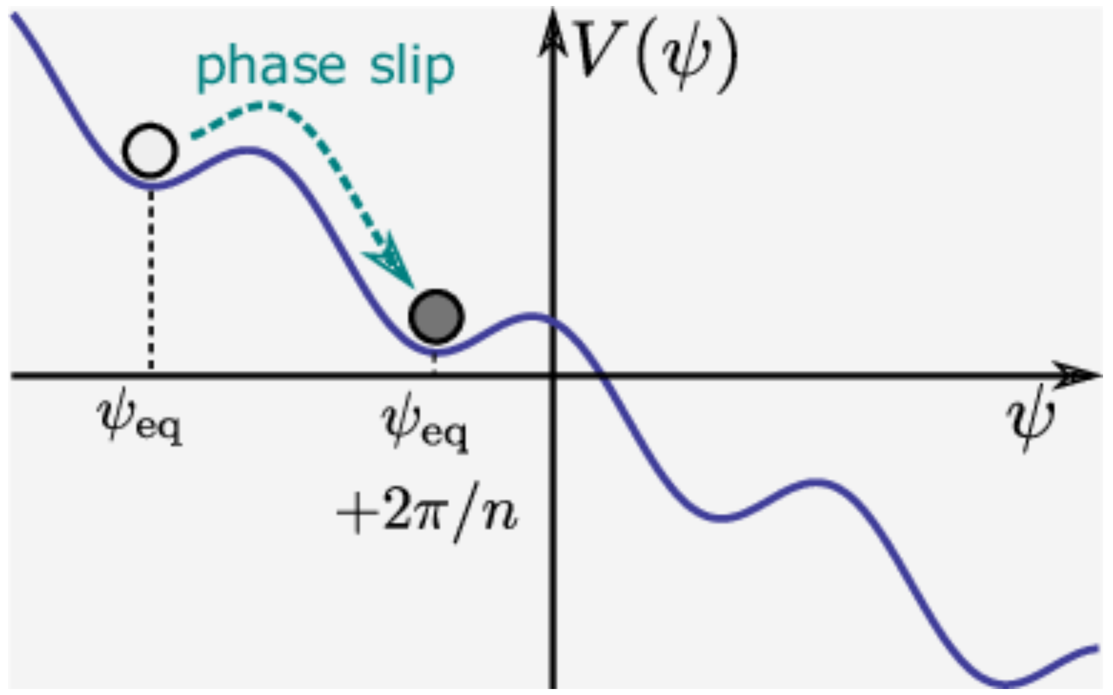
- stochastic differential equation
- steady-state = on a statistical sense

$$I_u = \varepsilon\mathcal{I}_0$$

Describe the perturbed dynamics with the slow phase variable only.
Resolve the perturbations transverse to the limit cycle with the help of a perturbation technique.

$$\psi' = \frac{\mathcal{A}_0\mathcal{M}_0}{\xi_0}\sin\psi\cos\psi + \xi_0\cot\psi + \xi_0\delta + \mathcal{I}_0\mathcal{U}$$

Phase slips accumulation



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Turbulent oncoming flow - Simulations of the model

Randomized wake-oscillator model :

$$\mathcal{Y}'' + 2\left(\xi_s + \xi_a\left(1 + I_u\mathcal{U}\right)\right)\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\left(1 + I_u\mathcal{U}\right)^2\mathcal{Q}$$

$$\mathcal{Q}'' + \varepsilon\Omega\left(1 + I_u\mathcal{U}\right)\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\left(1 + I_u\mathcal{U}\right)^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''$$

Slow equations :

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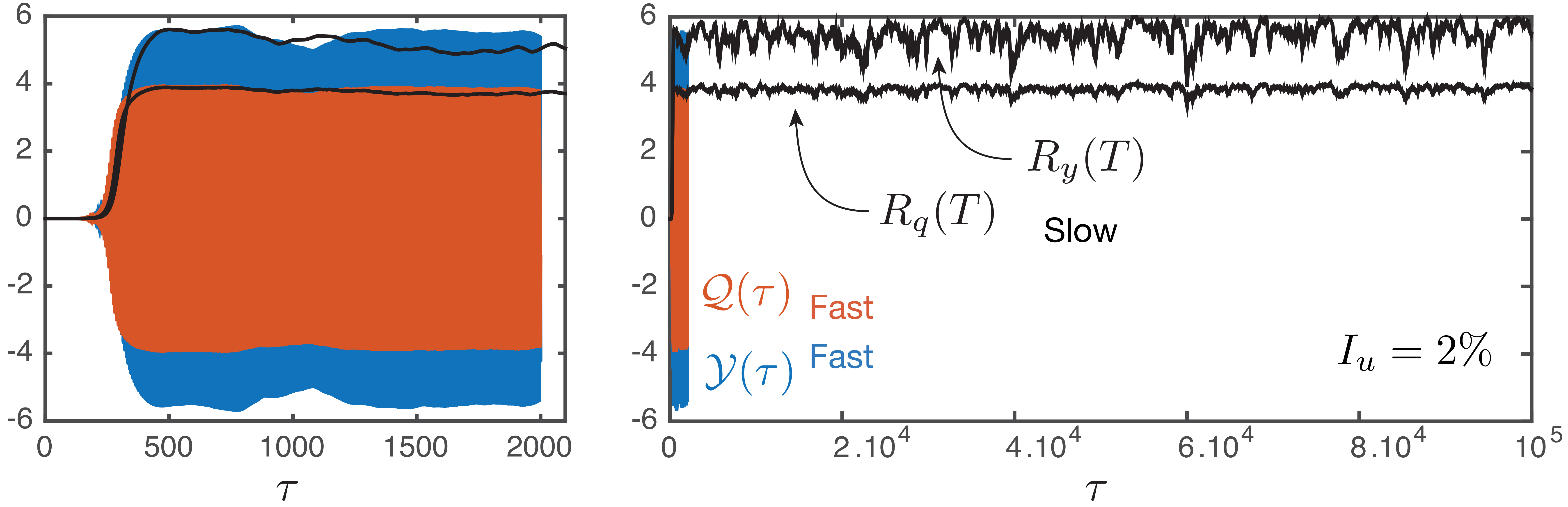
- stochastic differential equation
- steady-state = on a statistical sense

Small turbulence intensity

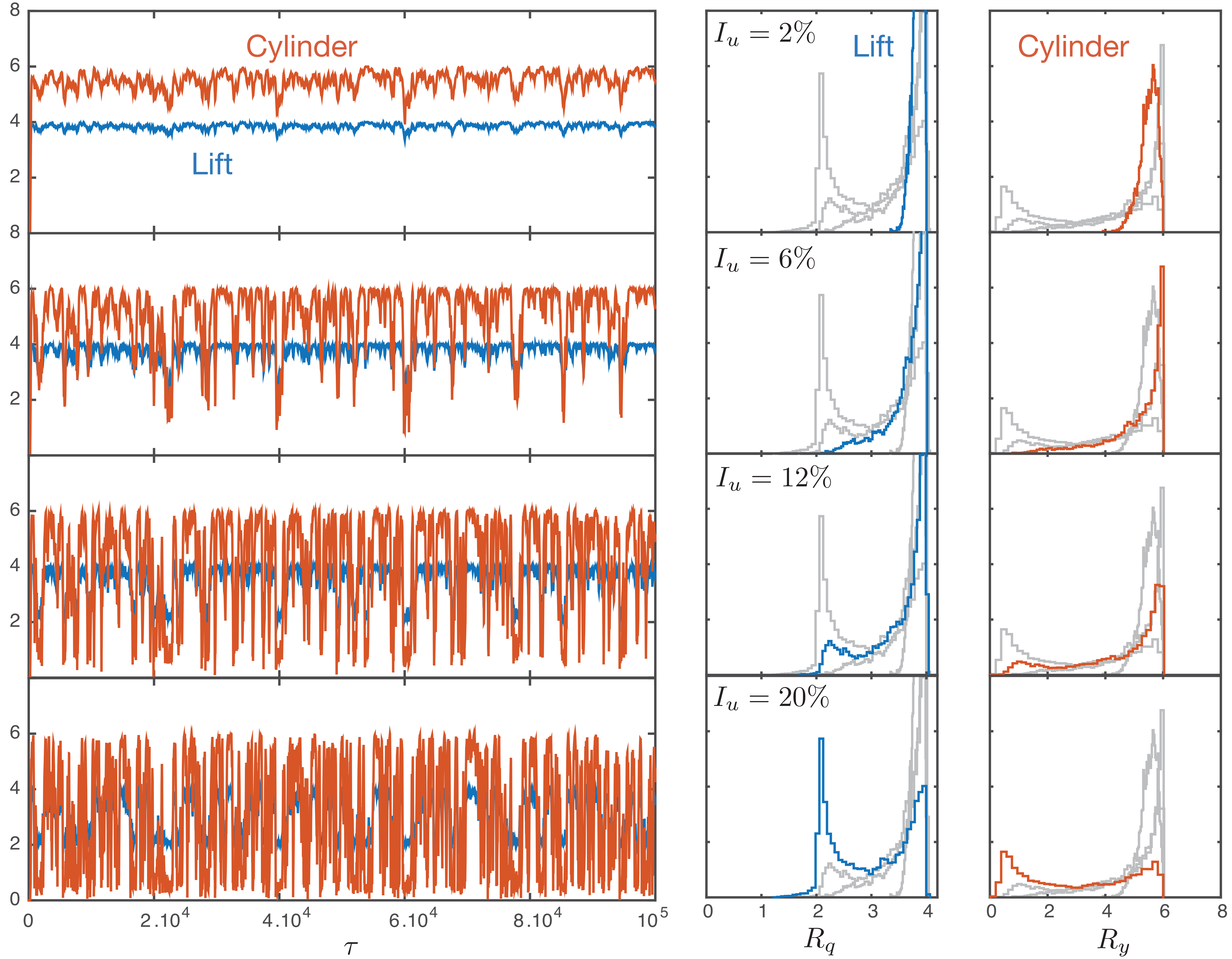


$\varepsilon = 0.05, \mathcal{A}_0 = 1,$
$\mathcal{M}_0 = 0.9, \xi_0 = 0.6$
$\alpha = 0.002,$
$\Omega = 1(\delta = 0)$

Turbulent oncoming flow - Simulations of the model

Influence of I_u

$\Omega = 1.03\ (\delta = 0.6)$

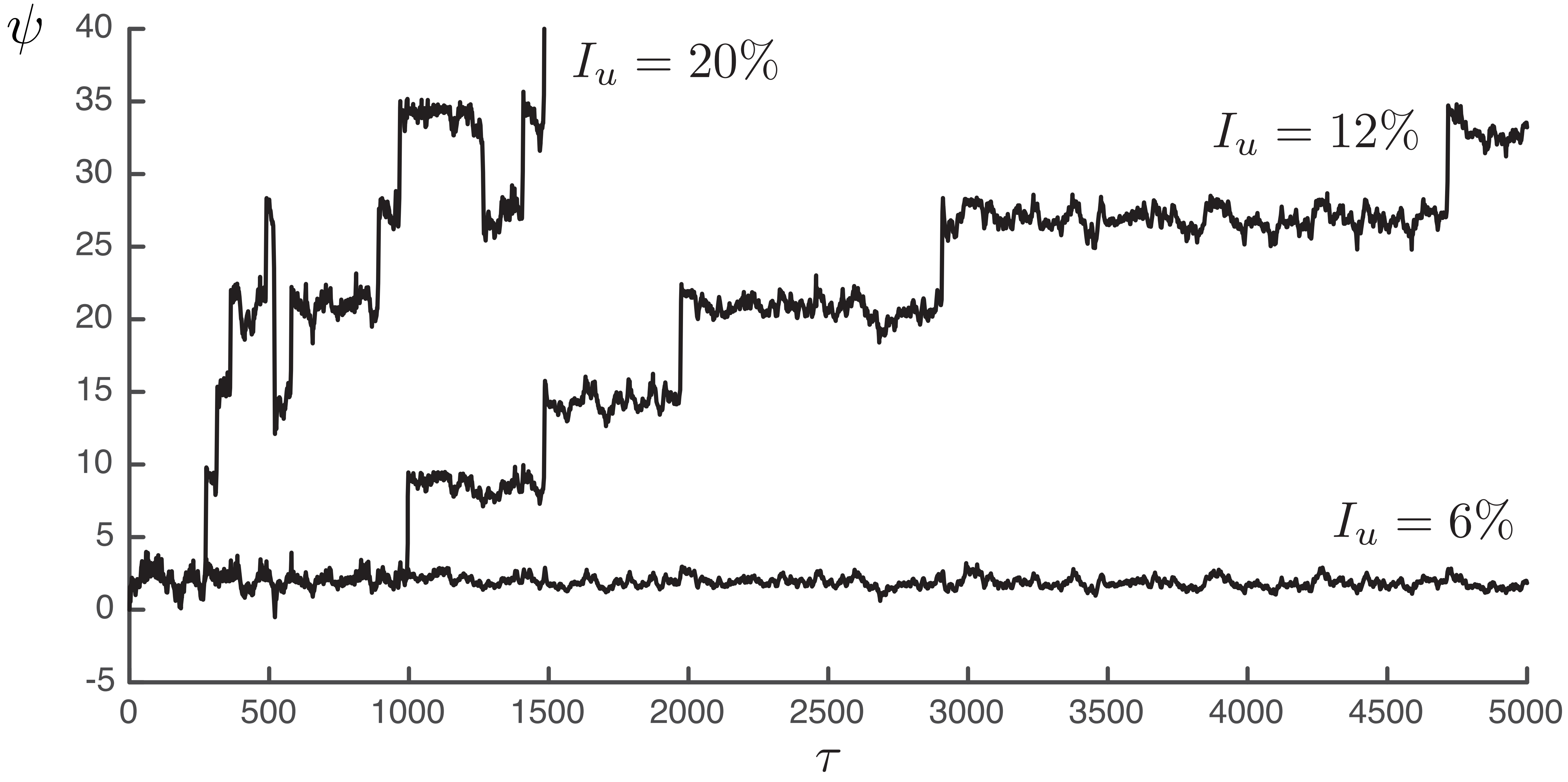


$\varepsilon = 0.05, \mathcal{A}_0 = 1,$

$\mathcal{M}_0 = 0.9, \xi_0 = 0.6$

$\alpha = 0.002,$

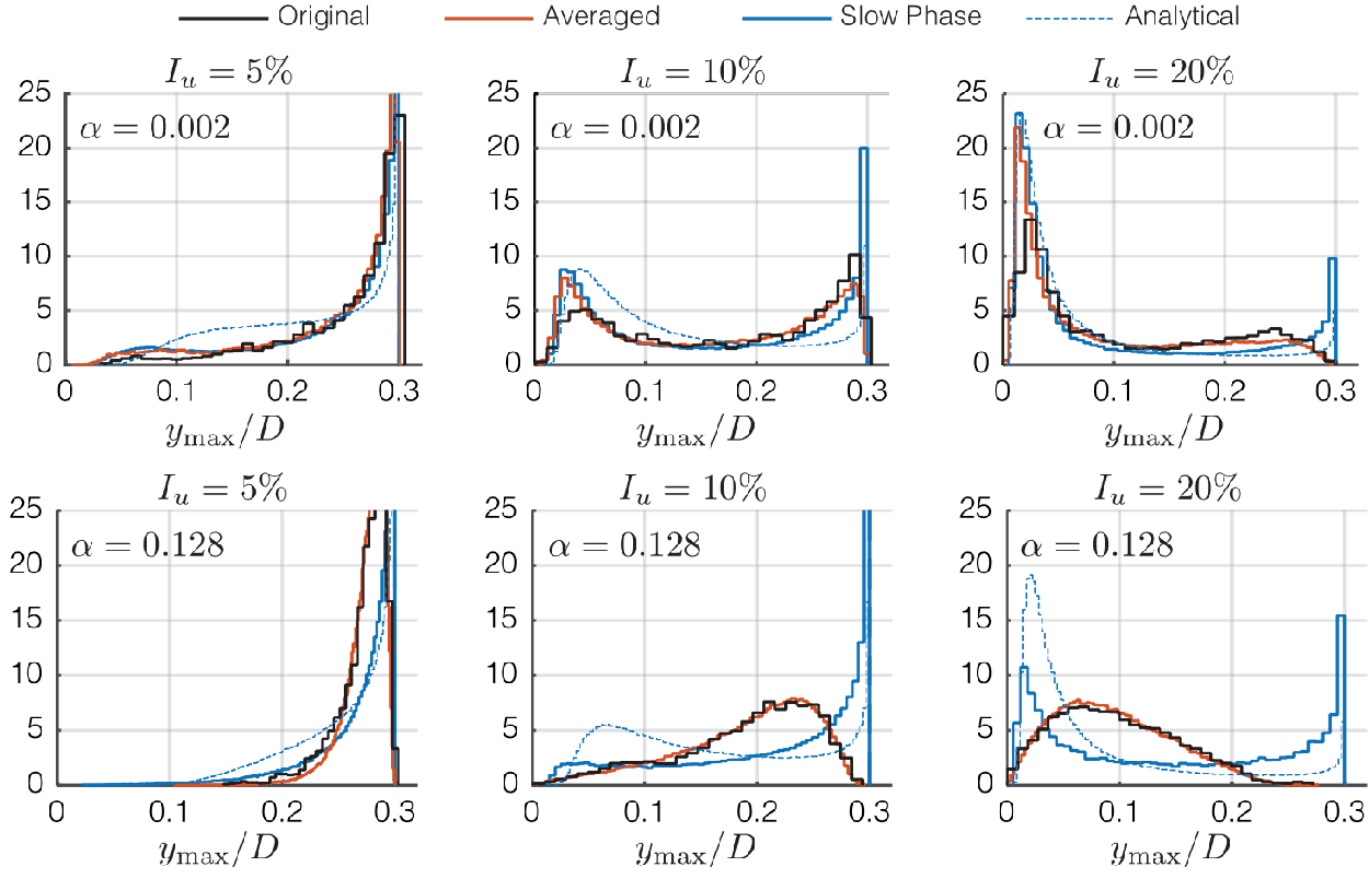
Slow phase — Phase slip

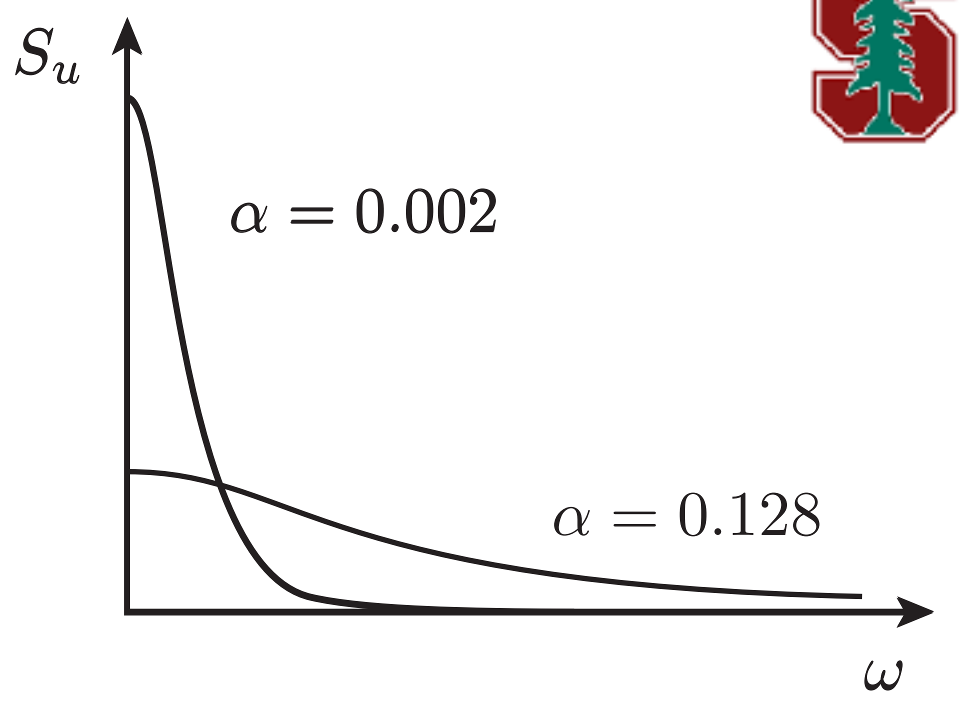


When turbulence intensity increases, more often, the phase is further away from the worst situation $\pm\frac{\pi}{2} + k\pi$.

The system is less often in a "synchronization" mode

Turbulent oncoming flow, some results





Very Low freq. turbulence
(Large-scale turbulence)

Low frequency turbulence

$\varepsilon = 0.05, \mathcal{A}_0 = 1,$
$\mathcal{M}_0 = 0.9, \xi_0 = 0.6$
$\alpha = 0.002,$
$\Omega = 1(\delta = 0)$

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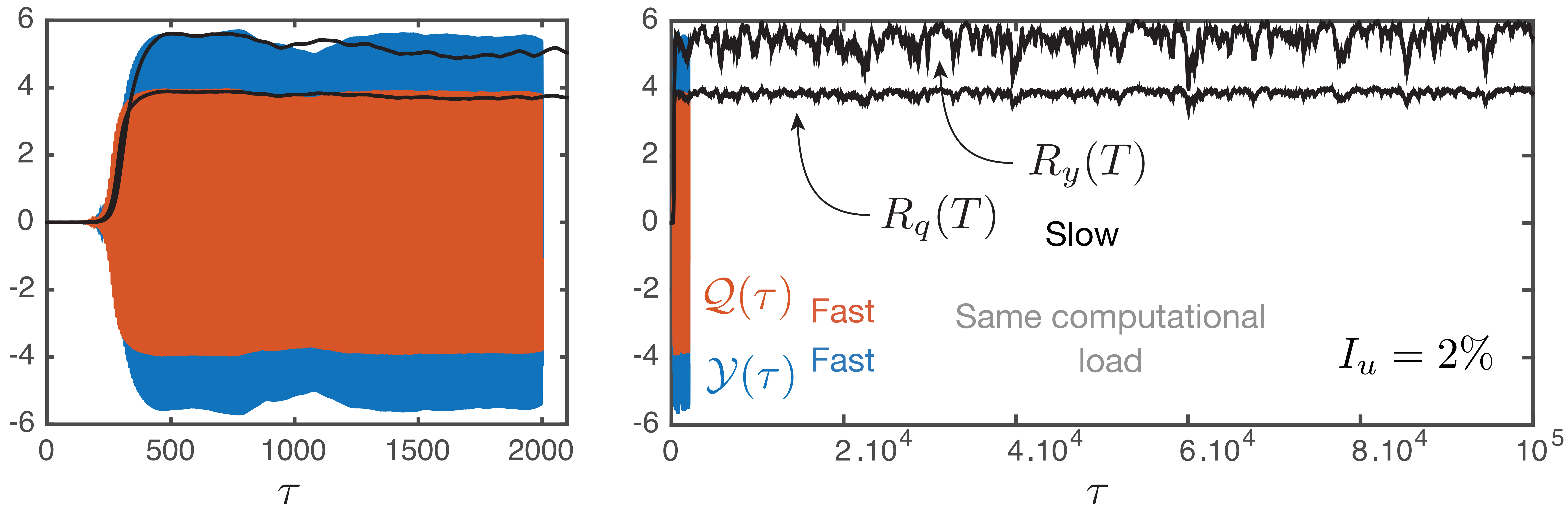
Take-home messages

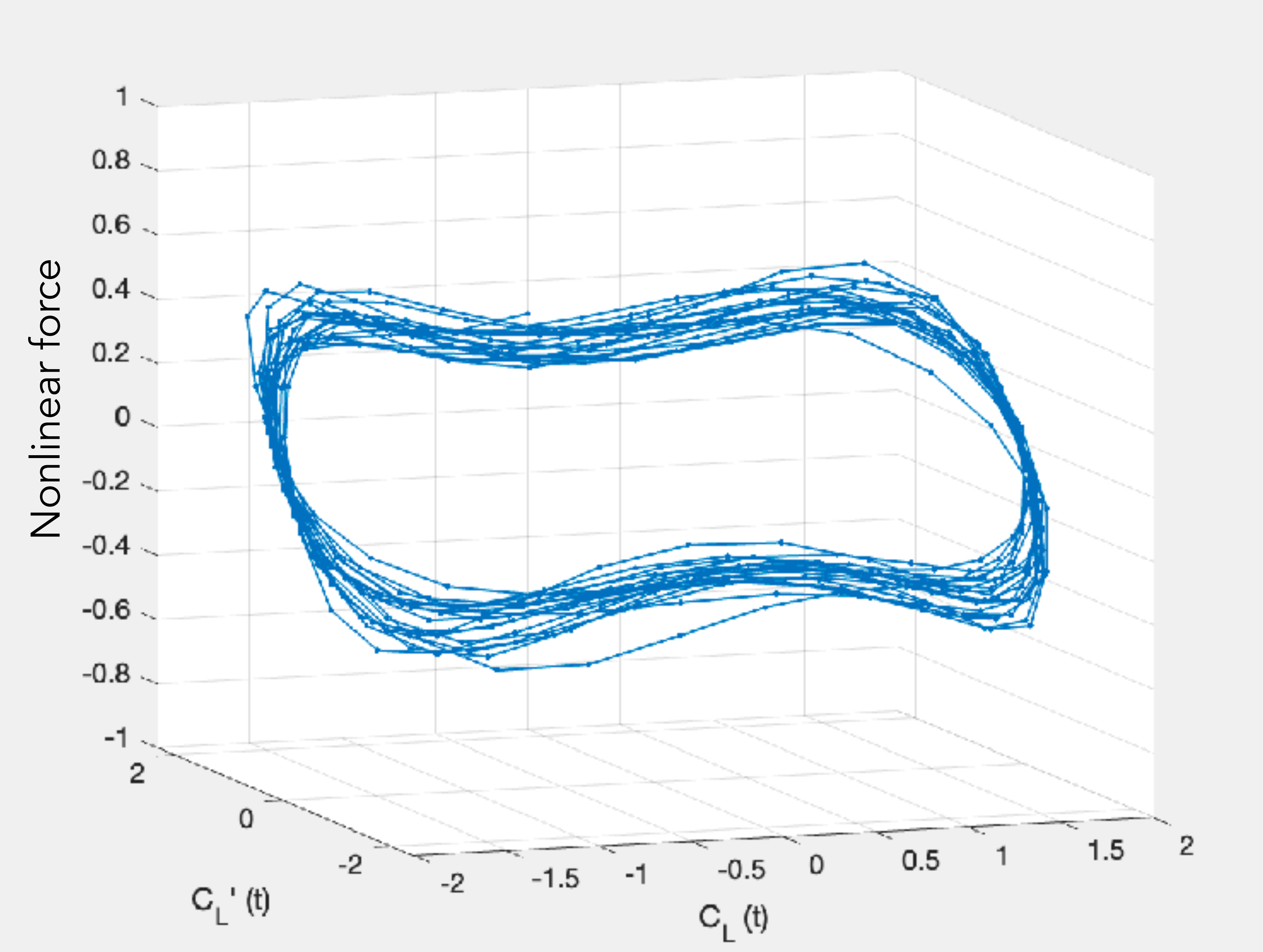
- 1. Use the slow envelope, not the standard deviation
  - It is more educating since the envelope is a process, the STD is a (just) a scalar
  - Think of times scales. Run longer at no cost: fast dynamics are not resolved
  - If possible do the same with the slow phase



Take-home messages

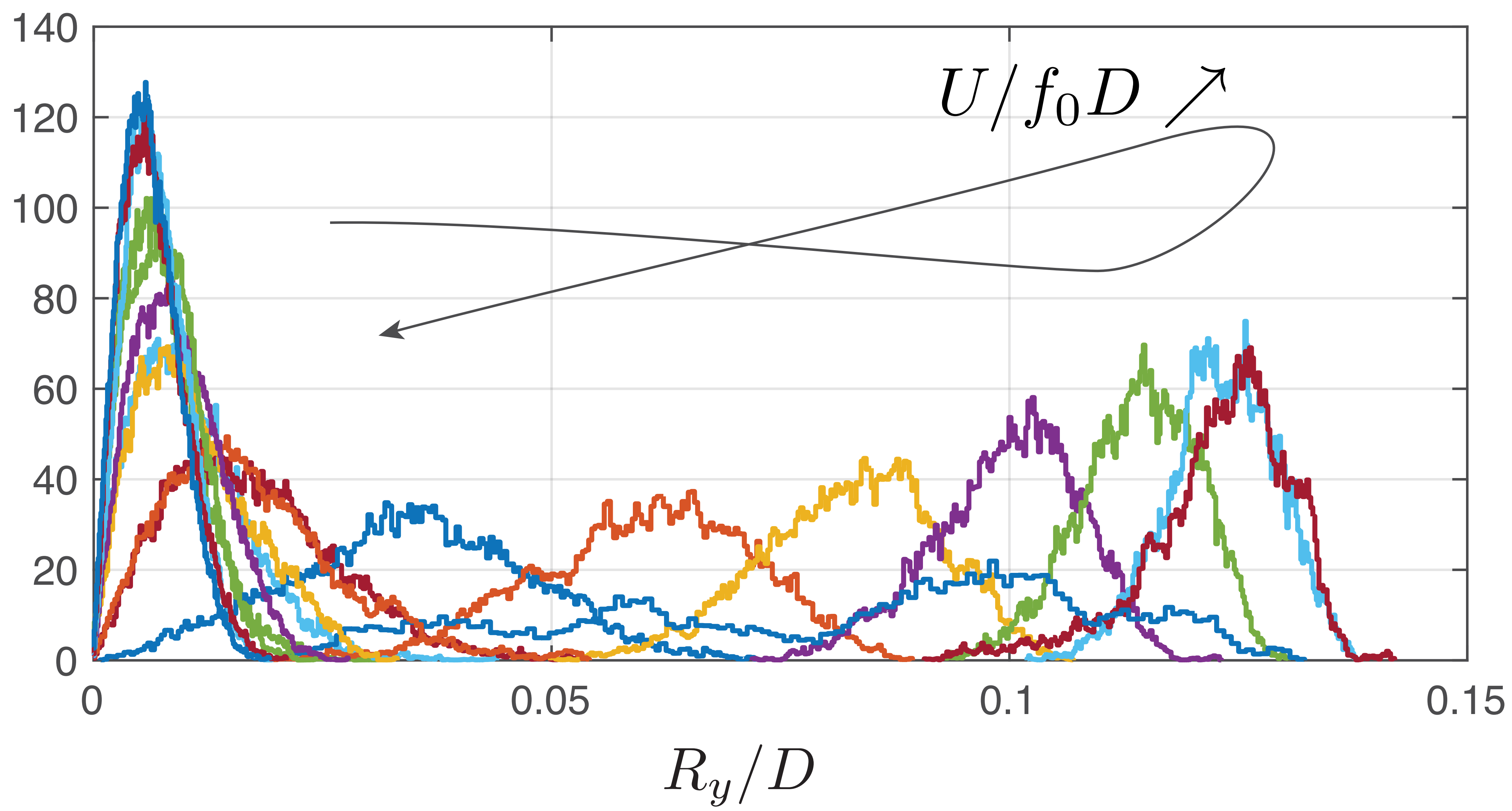
- 2. Random wake model, identified from experimental data
  - Similar to Tamura's model
  - Can be adapted to other sections too - it is a methodological approach
  - Builds on a deterministic nonlinear model + additive stochastic perturbations

$$\ddot{q} + q = F(q, \dot{q}) = \dot{q}(\alpha q^2 + \beta q \dot{q} + \gamma \dot{q}^2 + \delta)$$



Take-home messages

- 3. Randomizing the wake-oscillator models
  - Wake turbulence
  - ABL-type flow - turbulence in oncoming flow



Stochastic Enhancements in Wake Oscillator Models — experimental and modeling aspects —

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