Collective memories formation and evolution: A geometrical interpretation

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Abstract

To form a collective memory within a group, four conditions must be met: 1) memories must be transmitted through a communication network, 2) they must be shaped by interactions with others, 3) they must converge to a common form within the group, and 4) this representation must stabilize over time, a process we call dynamic stability. While formalizing these conditions mathematically is crucial for defining the formation and evolution of collective memory—a task still to be fully explored—this paper presents a theoretical model to address it. We use concepts from geometry, linear algebra, sociology, and psychology to define the formation, evolution, and dynamic stability of collective memories. Our approach incorporates both individual memories and group interactions, illustrating how collective memory can be modeled by means of an orthogonal matrix and a hypersphere in R^D space, where D represents the number of questions in an evaluation questionnaire.

One of the most fascinating sides of cognitive sciences is the study of memory formation. Notably, one kind of memory of great interest is collective memory because it involves interaction between different types of memory (personal, shared, collaborative, social).

Nowadays, memory is considered as the convergence of individual, social, and shared memories. The concept of collective memory is at the confluence of these types of memories. Collective memory plays a crucial role in shaping, preserving, and serves as a foundation for activating social identities since it significantly affects intergroup dynamics. A mathematical model describing the dynamics of collective memory formation, which take into account the memories of subjects constituting a group and the effects of interactions among them over a significant period of time (e.g., 10 years), could contribute to a better understanding of the complex processes of human collective memory, lead to the better understanding of such a memory formation, and to the development of new insights concerning cognitive performances and interaction.

From an historical point of view, Halbwachs[1] was the first scientist to introduce the concept of collective memory, from a sociological point of view. He stated that “collective memory refers to memories that originate with a groups' shared history, whilst personal memories are memories that belong to the individual, memories of events that the individual directly experienced and always embedded and framed by social relationships with others”. Nevertheless, from a psychological perspective, the definition of collective memory is not well defined and unanimously accepted in the Literature. Astonishingly, it is only for the past two decades or so that psychologists have been concerned with collective memory, an area of interest in the social sciences for several decades.

Before we begin the discussion of this article that will lead to the proposed geometric modeling of collective memory formation, in order to unravel any doubts and misunderstandings we prefer to give the definition of individual (or autobiographical) and collective (shared and/or collaborative) memory right away. Hence, we take as reference the definitions described in[2], specifically: *Individual memory* is the memory of a subject, particularly around the concepts of autobiographical memory and projection into the future. *Collaborative memory* refers to individuals as participants in an interaction. *Shared memory* refers to individuals as members of a group. *Collective memory* refers so strictly to the operations of individual systems of consciousness. Importantly, collective memory is not the memory of a collective, but that of its individual members, either as members of social groups (shared memory) or as participants in social interactions (collaborative memory). In other words, we model collective memories as a result of the dynamics before and after subject interactions.

The collective memory formation process is dynamically complex, we remark that the way memories gradually lose their specificity and become more generalized over time are key themes in understanding the mechanisms of autobiographical memory. Interestingly, Orianne and colleagues[3] argue that the process of semantization extends beyond the individual's consciousness system—where personal memories originate—and involves collaboration between three interconnected systems: The central nervous system, the individual consciousness system, and social system.

It is doubtless that the cognitive characteristics of individual subjects forming a group of arbitrary cardinality[a] as well as the interaction between subjects are two milestones in collective memories formations. The social aspects also play a main role in the formation of a collective memory. Indeed, Lee et al.[4] by investigating the network structure of a data set consisting of reported events by several individuals and how associations connect them, outlined how the cognitive representation of memories and social structure can co-evolve as a contagious process. Nevertheless, since it is also a second-level characteristic of collective memory, this is beyond the scope of our study. This means that we do not directly model this effect.

From a psychological perspective, there are two key trends in defining collective memory. The first emphasizes individual memory, particularly autobiographical memory[5,6], which includes both specific events and broader self-knowledge. This memory is shaped by social interactions and is closely linked to collective memory. The second trend focuses on shared memories within a group, where people may remember the same event differently, depending on their perspective. Individual and collective memories overlap, like two sets in a Venn diagram or DNA strands. Studying collective memory involves understanding how individual memories become shared, how selective remembering leads to forgetting, and how interactions influence future ones. These processes shape both individual and group memory, with communication playing a central role in forming collective memory[7].

This raises the question of whether individuals remember better in groups[8]. While individual memories form the foundation for collective memory through interaction, it's also important to examine how group processes affect later individual memory[9]. Research has shown that group collaboration can reduce individual recall and lead to memory errors, though it may also improve memory under certain conditions[10]. Group recall creates a new, stable representation of the material that differs from individual memory. The stability of collective memories, as opposed to individual ones, may be a key distinction. Mutlutürk and colleagues[11] found that transformative events can alter collective memory structures, while sociopolitical identity plays a role in the stability of these memories.

The modeling of collective memory formation is still largely unexplored, especially in terms of individual memories, collaboration, and the dynamic stability of shared memory from a mathematical perspective. We define stability as "dynamic stability" in this context, meaning small disturbances don't drastically affect the system. However, in psychology, stability can change over time. Collective memories tied to specific events may remain stable for varying periods and can fade as time passes, even if a strong collective memory persists throughout an individual's life. This paper aims to provide a geometric interpretation of collective memory formation. Mathematical modeling of memory is growing due to the need for models that explain

[a] Cardinality is defined as the number of elements in a set or other grouping.

memory formation and address memory-related diseases, involving fields like neurology, psychology, imaging, mathematics, and computation. To date, various models have been proposed, each of which have strengths and weaknesses, for a review see[12]. Differently from other kind of memories, collective memories are shared representations of a group's past based on a common identity[13]. Although cognitive and emotional elements play a role in their formation, human interactions with other people or with cultural artifacts are the main setting for it. Their intervention in the formation, preservation, and expression of social identities has a profound effect on relations between groups. That is one of the reasons for which a modeling approach would be desirable.

Collective memory refers to the shared recollections of individuals within various groups, from families to nations, and shapes their sense of identity. Unlike formal history, collective memory may not align with factual events. While psychologists have focused on individual memory, and computational scientists on modeling processes, collective memory itself remains underexplored. Roediger III[14] identifies three key aspects: it serves as a knowledge base that evolves over time, reflects a group's identity tied to its origin story, and is dynamic, with debates on how the past should be remembered. Studying and modeling collective memory can reveal how different groups understand and interpret history, highlighting both shared and divergent perspectives, such as the differing memories of WWII held by the Flemish and Walloons.

Results

We are going to explain the results of our geometric model on the formation of collective memory (see Section *Material and methods* for details) by means of synthetic data, pointing out the methodology for their collection.

As commented in the Remark at the end of Subsection *Construction of a set of vectors*, in general the interaction matrix does not have any special structure, meaning that there is no collective memory formation. From the geometric point of view, this means that the norms of the progressively computed vectors do not stabilize. The result is as shown in Figure 1, where the generated synthetic data do not show such a stability condition.

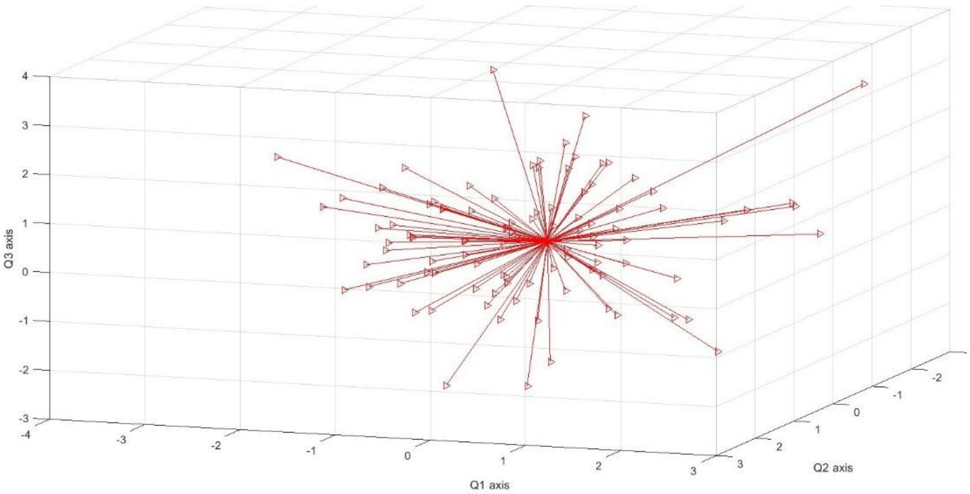

Figure 1. Lack of formation of a collective memory.

Differently, we wish now to show how the formation of a collective memory could be investigated with the proposed approach. In this case, we have utilized *ad hoc* functions, which are well-known in cognitive science[12], to generate appropriate synthetic data.

Let us consider the following questionnaire, based on three factual questions concerning the November 13, 2015, attacks in Paris: 1) How many innocent people died in the terrorist attacks on November 13, 2015? (right answer: 131); 2) How many locations were affected by the bombings and shootings? (right answer: 8); 3) How many terrorists were responsible for the Bataclan massacre? (right answer: 3). Suppose that the questionnaire is submitted to 100 different subjects. Each subject answers without any external hints, without being able to choose among multiple answers, and without ever knowing which is the right answer. We assume that the questionnaire will be submitted 4 times in 10 years, without any specific cyclicity (n=0, n=6 months, n=6 years, n=10 years). In order to avoid bias, the questions may be shuffled, and/or the text may be slightly changed. To further reduce the bias effect other questions of different type may be added. However, the 3 questions are always of factual type.

At time t_0, the subjects have not interacted with them yet, so we simulate the scores given to the 100 answers by random generating them for each one of the three questions, with minimal constraints to mimic real answers. The data are then processed in the following way: We compute the difference between the received scores and the true answers. If the score corresponding to an answer is greater than the correct answer, then we associate it with a positive discrepancy. Conversely, if the score is lower than the correct answer, then we associate a negative discrepancy. In this way, we obtain a matrix $\boldsymbol{M} = [m_{kj}]$, whose size is 3x100, and whose elements are the positive or negative discrepancies between the given answer and the right answer. For example, if the right answer to question #3 is "3", and $m_{3,50} = +5$, this means that the answer given by subject #50 to question #3 is "8", since the discrepancy is "8-3", i.e. "+5". Then, we take the average score for each one of the three lines (corresponding to the three questions) of the matrix. So, the information is compacted in the vector $\boldsymbol{x}(0)$. Hence, suppose that we obtain: $\boldsymbol{x}(0) = [-3.0600, -2.4000, -2.0600]^T$

At time n=1, i.e. 6 months later, we repeat the same procedure. The only difference is that in this case the subjects interacted among them. In this way we obtain vector $\boldsymbol{x}(1)$. We assume that in this case, the entries change as follows: $\boldsymbol{x}(1) = [-5.6505, -2.7056, -2.0518]^T$.

In a similar way, we assume that after 6 years (n=2) and 10 years (n=3) from the first submission of the questionnaire the vectors $\boldsymbol{x}(2)$, and $\boldsymbol{x}(3)$, respectively, are characterized by the following entries:

$\boldsymbol{x}(2) = [-5.0765, -3.3582, 2.6451]^T$ and $\boldsymbol{x}(3) = [-2.1669, -6.2671, 0.2637]^T$.

Now, assume that the mutual interactions among the involved subjects during these first intervals of time allow to deduce an interaction matrix. As commented above, it must be a sum between an orthogonal matrix and a time-depending matrix converging to the null matrix. So, as a possible choice, we use the following:

$$\boldsymbol{A}(t_n) = \begin{bmatrix} \frac{1}{3} + e^{-\alpha(t_n)t_n} & \frac{2}{3} + e^{-\beta(t_n)t_n} & \frac{2}{3} + \frac{1}{\gamma t^4} \\ \frac{2}{3} + \frac{1}{\delta t^5} & \frac{1}{3} + e^{-\epsilon(t_n)t_n} & -\frac{2}{3} + e^{-\epsilon(t_n)t_n} \\ -\frac{2}{3} + e^{-\theta(t_n)t_n} & \frac{2}{3} + \frac{1}{\gamma t^7} & \frac{1}{3} + e^{-\mu(t_n)t_n} \end{bmatrix},$$

where, $\alpha(t_n), \beta(t_n), \gamma(t_n), \delta(t_n), \varepsilon(t_n), \epsilon(t_n), \theta(t_n), \vartheta(t_n), \mu(t_n)$, are time-dependent positive constants. Basically, this represents the general structure expected for the entries in case of collective memory formation[12].

Note that, as n increases, then the matrix $\boldsymbol{A}(t_n)$ tends to an orthogonal matrix $\boldsymbol{H}$, which does not depend on time.

$$\boldsymbol{H} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

In our case, the longitudinal simulation lasts 10 years. Hence, if we repeat the procedure until the final submission at $t_n = 10$ years. The emerging (nearly) orthogonal matrix is:

$$
\boldsymbol{H}^{*} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} + \frac{1}{4053} & \frac{2}{3} \\ \frac{2}{3} + \frac{1}{17013} & \frac{1}{3} & -\frac{2}{3} + \frac{1}{8106} \\ -\frac{2}{3} & \frac{2}{3} + \frac{1}{21873} & -\frac{1}{3} \end{bmatrix}
$$

The evolution is shown in Figure 2 - Left, where the cone spans the region on the sphere that contains the convergent vectors. The width of the cone depends on how vivid the memory of the event is. A thinner cone would represent a higher vivid collective memory (see Figure 2 - Right).

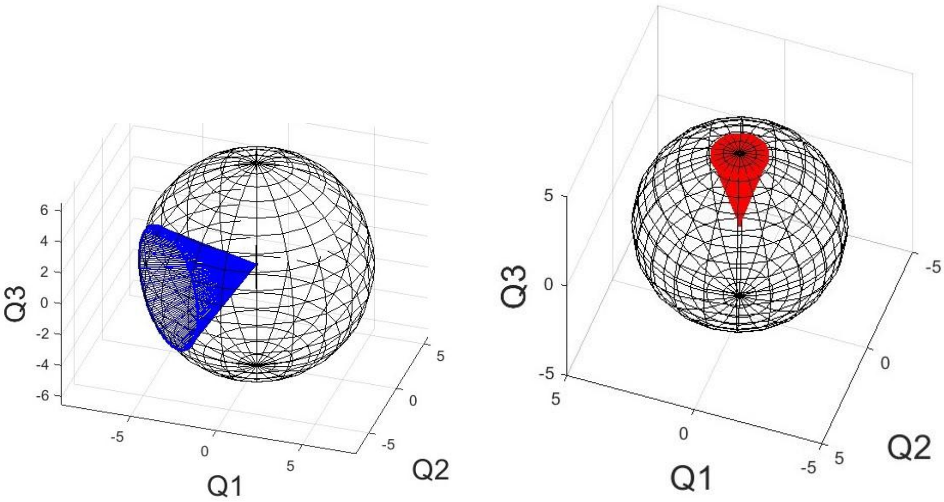

Figure 2- Left: The evolution and formation of the sphere representing the collective memory in the analyzed case. -Right: A thinner cone would represent a higher vivid collective memory. To better show the cone(s) we have simulated 200 questionnaire administrations.

Remark. We expect that, the thinner the cone, the quicker a collective memory appears.

We also investigated the role of the cardinality of the sample of subjects. In this case for each questionnaire administration, we observed how the subjects' memory performance was arranged in the three-dimensional space of the questions under consideration. In essence, the performance is represented by points in the space of the questions, and the coordinates represent the deviations between the given and the correct answers. The intensity of collective memory is linked to the projections of the subjects' memory performance on the sphere representing the collective memory. Figure 3 shows the results.

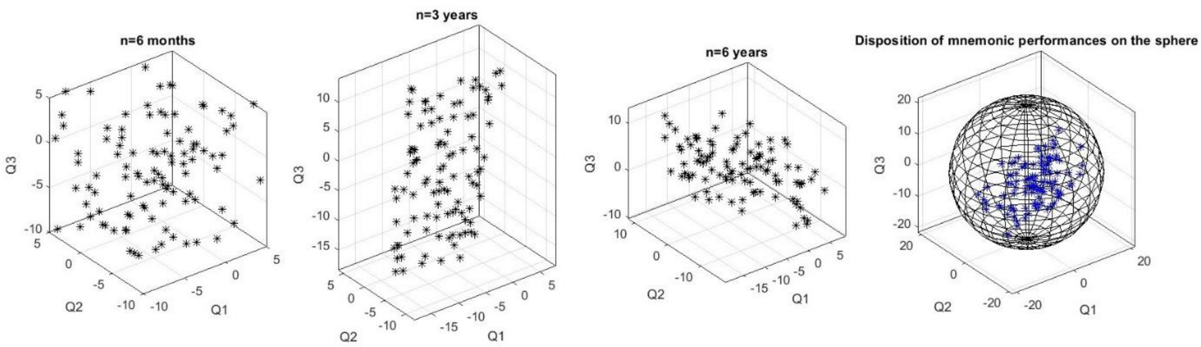

Figure 3. The figure shows the dispersion of subjects' memory performance at different times of questionnaire administration in the three-dimensional space of the analyzed questions. When the collective memory is formed such memory performances, which are the

blue points in $\boldsymbol{R}^3$, can be viewed as projections of such points on the sphere representing the collective memory. The denser the disposition of mnemonic performances on the sphere's surface, the more vivid the collective memory is among the subjects.

Some additional comments

The case above is not dependent on the type of question asked. If functional or symbolic questions had been chosen, a different approach would be required. These questions introduce subjectivity in responses, as each person answers based on their own life experience, making the measurement scale not absolute. This subjectivity adds statistical noise, which may reduce the quality of observed effects and should be considered. In such cases, normalizing x-vectors is necessary. Despite the lack of an absolute reference or "right answer," it is still possible to investigate relationships between variables. For instance, it still makes sense to examine if people with high scores on one question tend to have high scores on a different question. This can be done through regression analysis, which models the relationship between dependent and independent variables. Regression allows prediction or explanation of one variable's variation based on another. As for score ranges, there's no mathematical difference between a 1-5 scale and a -2 to 2 scale; most p-values, R^2, β, etc., will be the same. We plan to normalize scores when dealing with functional or symbolic questions. The main difference lies in interpreting the scores, which combines psychological, clinical, and statistical perspectives.

Discussion

This paper proposes a novel mathematical interpretation of collective memory formation. We consider the questionnaire as the practical tool to obtain psychometric data. The reference space is R^D, where D is the number of questions composing the questionnaire. Such a questionnaire is proposed at N different times along the longitudinal study to investigate collective memory formation. In our proposal the interaction between the subjects, which is a milestone in collective memory formation, together with the individual memories, results in matrix, which is represented by the sum of a time-invariant orthogonal matrix and a time-dependent matrix which is regarded as a sort of "noise" in the process of collective memory formation. As collective memory begins to form, that is, when mnemonic convergence becomes stable in a certain temporal surround, then the entries of the time-dependent matrix vanish. Hence, the orthogonal matrix is the matrix responsible for the formation of the collective memory. This is equivalent to stating that the collective memory is geometrically represented as a hypersphere Σ in R^D. Our proposal aims to provide a mathematical approach for investigating collective memory formation and could be exploited on experimental data for a better understanding of the whole process. This could also open the door to applications to clinical research, not only for the diagnosis of early stages brain diseases, but also for the natural history of diseases and the longitudinal follow-up of patients. Indeed, some subjects in the study group could show significant statistical deviations from the mean responses, or a lack of correlation between the responses given at different times. Mathematically, this means that the entries of the matrix $\mathbf{U}(\mathrm{n})$ do not tend to zero as fast as they would if such subjects were excluded from the group, so providing an indicator of possible problems at the neurological level, such as MCI (mild cognitive impairment), or worse amnesia, or even an onset of Alzheimer's disease. Lastly, we would like to point out that our proposal is not intended to replace, but to corroborate the clinical approach. For example, once a given time n* is found for a collective memory formation, then this would bypass any subjective, and perhaps not totally uniform, interpretations expressed by psychologists and psychiatrists in charge of analyzing the responses given by the subjects who make up the survey group. Lastly, there is also an economic reason: understanding when collective memory is formed would avoid further administration of questionnaires and thus save money and time in processing the results of such questionnaires.

Material and methods

Methods of investigations

Due to the complex nature of collective memory, which intertwines individual, shared, collaborative, and social memories, finding a single method to investigate it comprehensively is challenging. Depending on the study's goal, different methods may be more effective. Three main approaches to studying collective memory are: 1) theoretical and methodological approaches, 2) empirical research methods, and 3) interdisciplinary tools.

Theoretical approaches include functional, phenomenological, and post-structural perspectives, while empirical methods are used in fields like sociology, political science, and psychology[15]. Interdisciplinary tools involve psychometric data, word mining, and computer science. For studying collective memory using the model in this article, interdisciplinary methods, such as functional imaging, cluster analysis, and surveys, are the most suitable, as they contribute to a dynamic, cross-disciplinary understanding of collective memory.

Gagnepain et al.[16] used functional imaging to study the medial prefrontal cortex, a key area for social cognition and memory. They found that the structure of collective memory more accurately predicted individual memory arrangements than models based on contextual or semantic memory. Social memory, beyond individual experience, can shape personal memories and link people's recollections across time and space. Geana et al.[17,18] applied complex network theory to study collective memory formation in 192 Princeton University students. They grouped the students into twelve 16-member networks and examined how network structure influenced memory formation. They found that clustering within networks significantly impacted collective memory development, while network reachability did not. The study also showed that an individual's position in the network and the timing of their conversations affected their contribution to collective memory. CREDOC surveys offer another way to study collective memory by exploring the connection between personal experience and collective interpretation. These surveys also examine how media influence collective memory formation. Pierre Nora argues that media discourse creates memorial events by linking current events to memory[19], while Luhmann suggests that mass media shape social memory and create "latent everyday culture"[20]. Mass media, including print, radio, TV, the Internet, and social networks, maintain social memory through two main dimensions: semantic securitization (labeling events) and critical awareness, such as fostering vigilance and challenging societal systems.Finally, a popular and widespread tool of investigation to study collective memory is the questionnaire. A plethora of questionnaires have been proposed to this aim (see for example[21]). When using the questionnaire as a survey tool, different kinds of questions can be chosen.1) Factual Questions: These establish basic facts and review concepts, such as "who," "what," "where," and "when." Examples include: "Can you name precisely the different locations where the November 13, 2015 attacks took place? "[3]. These questions often have one correct answer.2) Functional Questions: These assess the frequency of information sharing or personal searches, such as how often someone discusses or thinks about an event. For example: "I follow sources related to the event to learn more." Participants rate frequency on a scale from 1 (Never) to 5 (Very Frequently)[21]. 3) Symbolic Questions: These evaluate the importance or meaning attributed to an event compared to others. For example, a survey might ask, "Which terrorist attacks have impacted you most since 2000?" or "What do you think caused the 13/11 attacks?" Responses are scored on a scale from 1 (strongly disagree) to 5 (strongly agree).These types of questions help form collective memory, as demonstrated in studies like the CREDOC survey on the memory of the November 13 attacks[3,22].In this article, we've only used factual questions to make our modeling exercise more readable and educational. But all three types of question could be treated using the same model.

As one easily can guess, not every memory and interaction result in a collective memory. Interactions play a fundamental role in collective memory formation since the autobiographical memory of a subject could be influenced by the autobiographical memories of other subjects, which interact, in any way, with the subject itself[23,24]. Hence, the construction of a mathematical model describing such a kind of memory should consider the interaction among the subjects. Due to such characteristics, geometry, linear algebra, sociology and psychology represent a suitable way to model the formation of collective memory.

Main issues to comply for a geometric model of the collective memory formation

In Subsection *A geometrical theory of collective memory formation*, we present a geometrical approach to describe the formation of collective memory, which is thought to be as the memory of individuals, either as members of social groups (shared memory) or as participants in social interactions (collaborative memory), and consequently propose an answer to the above questions. As far as we know, this topic is still uninvestigated from a mathematical point of view.

In view of our purpose, we detail three main issues that should be considered in the model.

Mnemonic convergence

One challenge is the lack of a method to evaluate how strong and stable a collective memory is over time. Memory stability, which influences how long a memory persists without retrieval, determines the likelihood of forgetting—higher stability means less chance of forgetting. Stability (S) and Retrievability (R) are part of the Two-Component Model of long-term memory, where stability increases with each review, slowing memory interference and preventing forgetting[12]. This principle can also be applied to artificial neural networks to mitigate catastrophic forgetting. Another key feature of collective memory formation is "mnemonic convergence," which refers to the overlap in community members' memories[18]. It is influenced by individual information-processing and the structure of conversational social networks. This information is incorporated into the mnemonic evaluation questionnaire, the higher the scores the better the mnemonic performance. Research shows that memories become more similar after networked conversations, and larger, more dispersed networks tend to show greater convergence. The size and structure of the group influence how quickly collective memory forms, and its temporal stability is crucial[18,25], but these conditions have not yet been mathematically defined.

False memories

As outlined in[26], previous studies demonstrated the positive and negative effects of collaboration on memory (both veridical and false recall) and suggestibility in face-to-face contexts. False memories are particularly interesting. In psychology, a mental experience that is recalled as factual but is either wholly false or substantially different from what actually happened is referred to as a false memory (see for example[27–30]). Put differently, a false memory could be a distorted recollection of an actual event, or it could be a completely imagined fabrication. A very famous examples of false memory are the so called "Mandela effect", in which a lot of people thought that Nelson Mandela had died in prison in the 1980s when, in fact, he had died in 2013, as well as the clock at Bologna Centrale railway station (see the study led by de Vito et al.[31]). False memories are a particular case of collective memory, hence a model explaining collective memory formation should explain also such a case.

Divergent memories

A related issue is the difficulty in capturing differences in retrieval processes among individuals, particularly in visualizing these differences geometrically. An example is the divergent collective memories of WWII in Belgium, where Flemish and Walloon communities have distinct perceptions. Belgium consists of two linguistic groups: Flemish, who speak Dutch and live in the north, and Walloons, who speak French and live in the south, including Brussels. Flanders associates with collaboration, while Wallonia with resistance. A study by De Guissmé and colleagues[32] found that these stereotypes—Flemish collaboration and Walloon resistance—remain strong in both communities' collective memories, despite the more nuanced historical reality, and have resurfaced in political debates on the Belgian linguistic conflict.

A geometrical theory of collective memory formation

After having resumed the main psychological features leading to the formation of a collective memory, we wish to propose a mathematical interpretation of the whole process.

As a first step, we consider the following assumptions:

1) Depending on the time the questionnaire is submitted, the topology of the questions can be slightly changed, without losing the general meaning, as well as their order of occurrence can be shuffled. In this way, possible biases are avoided[33].
2) The process of remembering an event strongly depends on individual memories, as well as on the mutual interactions occurring among the involved subjects[26].

Our approach keeps into account both the memories of each single subject involved in the process, and the role of interaction among the subjects belonging to a group, a nation, etc.

The memory related to a single subject is modeled as a vector, while the interaction is modeled as a matrix. This reflects a consolidated idea[18,34–37] in different areas of research.

We mimic a longitudinal study, meaning that we imagine a questionnaire, consisting of *D* questions[21], and submitted at $N + 1$ different times t_n, with $n \in \{0,1,2,3,\dots,N\}$, to the subjects involved in the study, where $n = 0$ is when the questionnaire is submitted for the first time, so *before* any interaction among the subjects takes place.

Furthermore, we provide a condition on the matrix representing the interactions among subjects for a collective memory to form and to consolidate.

Construction of a set of vectors

At the starting time t_0, after selecting a sample of *S* subjects that have been involved in the event under investigation, each one of them is tested through a questionnaire consisting of D suitable chosen questions. Each answer receives a score, ranging from 0 to a (real) number λ, which depends on the chosen test[38]. After appropriate reprocessing (e.g., normalization in case of functional or symbolic questions), the score assigned to a single response to a given question can be positive, negative, or zero.

This leads to the construction of a vector $\boldsymbol{x}(0) = [x_1(0), x_2(0), \dots, x_D(0)]^T$, where, for each $k \in \{1,\dots,D\}$, $x_k(0)$ is the average of the scores received by the S subjects to the k-th question.

The same is repeated at different times $t_1. t_2, \dots t_N$, so forming the vectors $\boldsymbol{x}(1), \boldsymbol{x}(2), \dots, \boldsymbol{x}(N)$ as above

We assume that, for any $n \in \{0,\dots,N-1\}$ the vector $\boldsymbol{x}(n+1)$ results from $\boldsymbol{x}(n)$ thanks to the mutual interactions among the involved subjects during the interval of time $\Delta_n = t_{n+1} - t_n$. Such interactions are condensed as entries of a matrix $\boldsymbol{A}(n+1) = \left[a_{ij}(n+1)\right]$, $i, j = 1,\dots,D$, so that $\boldsymbol{x}(n+1) = \boldsymbol{A}(n+1)\boldsymbol{x}(n), n \in \{0,\dots,N-1\}$. This means that, for each $k \in \{1,\dots,D\}$, the score $x_k(n+1)$ can be computed as follows:

$$x_k(n+1) = a_{k1}(n+1)x_1(n) + a_{k2}(n+1)x_2(n) + \cdots + a_{kD}(n+1)x_D(n),$$

namely, it is a weighted sum of all the scores received at time t_n, with weights provided by the entries $a_{kj}(n+1)$ forming the *k-th* row of $\boldsymbol{A}(n+1)$, each one representing the influence on the score of the *j-th* question at time t_n.

Remark. The interaction matrix $\boldsymbol{A}(1)$ should result by a deep analysis of the social, and psychological interactions among the involved subjects occurring during the first interval $\Delta_1 = t_1 - t_0$. Then, based on $\boldsymbol{A}(1)$, the structure of the generic matrix $\boldsymbol{A}(n+1)$ should be deduced and tested on a few successive intervals, so to reach, in a limited number of steps, the functional dependency on time of each entry. In general, we cannot expect any special behavior for these entries, whose ranges should be mutually independent, without any convergence property. Differently, the phenomenon of formation of a collective memory should reflect in the existence of some particular link among such matrices, which is precisely what we wish now to discuss.

Conditions for the formation of collective memory and its geometric interpretation

As a consequence of the above remark, we expect that, when a collective memory forms, some geometric invariant characterizes the vectors obtained at different testing times. In particular, we focus on their Euclidean norm $\|x(n)\| = \sqrt{x_1(n)^2 + x_2(n)^2 + \cdots + x_D(n)^2}$, and we assume that

$$\|\boldsymbol{x}(n+1)\| = \|x(n)\| + \varepsilon(t_n, \alpha_1, \alpha_2, \dots, \alpha_K) \quad \forall n \in \{0, 1, 2, \dots, N\}.$$

[1]

Here, $\varepsilon(\mathrm{t_n}, \alpha_1, \alpha_2, \dots, \alpha_K)$ is a suitable function depending on the time, and on some neuropsychological variables $\alpha_1, \alpha_2, \dots, \alpha_K$, such as the cognitive, emotional or stress level. However, it is reasonable to guess that, for healthy people, $\alpha_1, \alpha_2, \ldots, \alpha_K$ do not have a critical impact on the analysis, so, at a first order, we can approximate $\varepsilon(t_n, \alpha_1, \alpha_2, \ldots, \alpha_K)$ just as a time-dependent function.

Since the presence of a collective memory implies a kind of uniform responses of the subjects to the questionnaires at different times, then a progressive stabilization must be reached for the norm of the computed vectors. We assume that the condition for the formation of the collective memory is:

$$\lim_{n\to+\infty} \varepsilon(\mathrm{t_n}) = 0.$$

[2]

On the applicative side, this means that, after a suitable number of repetitions of the process, we can neglect small discrepancies among the norms of the vector collected at different times. Namely, $n^* < N$ exists, such that

$$\|\boldsymbol{x}(n+1)\| \cong \|\boldsymbol{x}(n)\| \quad \forall n \geq n^*,$$

[3]

that is, for $t_n \geq t_{n^*}$ we can consider the norm of $\boldsymbol{x}(n)$ nearly constant. This reflects on the interaction matrix $\boldsymbol{A}(n)$, which can be decomposed as

$$\boldsymbol{A}(n) = \boldsymbol{H} + \boldsymbol{U}(n),$$

[4]

where, $\boldsymbol{H} = [h_{kj}]$ is a square orthogonal matrix, independent of time. Here $\mathbf{U}(\mathrm{n}) = [u_{kj}(n)]$ is a *discrepancy matrix*, representing "noise" in the process of collective memory formation, and such that

$$\lim_{n\to+\infty} \boldsymbol{U}(n) = \boldsymbol{0}$$

[5]

being $\boldsymbol{0}$ the null matrix. Therefore, $a_{kj}(n)$ converge to the fixed values h_{kj}, that is, from the applicative point of view $\boldsymbol{U}(n)$ can be neglected $\forall n \geq n^*$, and $\boldsymbol{A}(n) = [a_{ij}(n)]$ can be identified with $\boldsymbol{H} = [h_{kj}]$, so preserving the vector norms (see Box). We observe that the entries of $\boldsymbol{U}(n)$ are time-dependent decreasing functions, which is a usual approach employed in cognitive psychology to explain forgetting[12]. As a result, the dynamics of interaction tends to confine all the $\boldsymbol{x}$-vectors on a single hypersphere Σ of R^D, that, consequently, provides a geometrical representation of the collective memory.

Analysis of the geometric structure of a collective memory

As $n \to \infty$, the portion of the hypersphere Σ spanned by the vectors $\boldsymbol{x}(n)$, $n \geq n^*$, relates to the diffusion of the collective memory, which, in turn, depends on the number of involved subjects. Specifically, the intensity of collective memory is linked to the projections of the subjects' memory performance on the surface of the sphere. The broader the area, the more blurred is the obtained collective memory. This assumption lies on the idea that a larger number of subjects contributing to the same collective memory reasonably provides several vectors that, although having the same norms, could have largely different entries, so leading to a great angular separation among the different vectors.

However, it can also happen that many subjects provide a short angular separation, when the memory of the event is very vivid among all participants. In this case, not only the norm of the vectors stabilizes, but even the single scores to each question remain approximatively unchanged along the different questionnaire submissions. This is mainly possible when n^* is small. On the contrary, in general, when n^* is sufficiently large, we expect that, although a collective memory still forms, it becomes less lucid, meaning that the angular separation among the different vectors increases.

It is worth commenting also on the possibility that a significant number of subjects, belonging to a subset of all the involved subjects, individually develops a memory of the event that differs from the actual collective memory. In this case we refer to false memory (see Subsection *False memories*). If such a subset is large, then a "false" collective memory of the event is formed, since these false memories impact on the intensity (or strongness) of the collective memory formation (as in the case of the well-known “Mandela effect” or the clock at Bologna Centrale railway station). Once a false collective memory is formed is not possible to distinguish it from a true collective memory: Indeed, both generate a hypersphere in the space R^D.

The same phenomenon occurs for divergent collective memories (see, for example, Subsection *Divergent memories* for the different collective memories regarding WW II in Belgium, depending on if habitants are Flemish or Walloons). In fact, even this case leads to different disjoint hyperspheres. From a psychological and social point of view this represents the irreconcilability of the two divergent representations of the past, preventing Belgian citizens from developing a strong national identity and a clear sense of common belonging. It is a dynamic situation, meaning that it could be removed after some time (e.g., after several generations) leading thus to a collective memory represented as a single hypersphere.

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Data availability statement

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

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