# Reformulating the Simultaneous Multi-Scale Method with the H-Phi Thin-Shell Model for Efficient Stacked HTS Coil Simulations


Louis Denis, Benoît Vanderheyden, Christophe Geuzaine

University of Liège, Belgium



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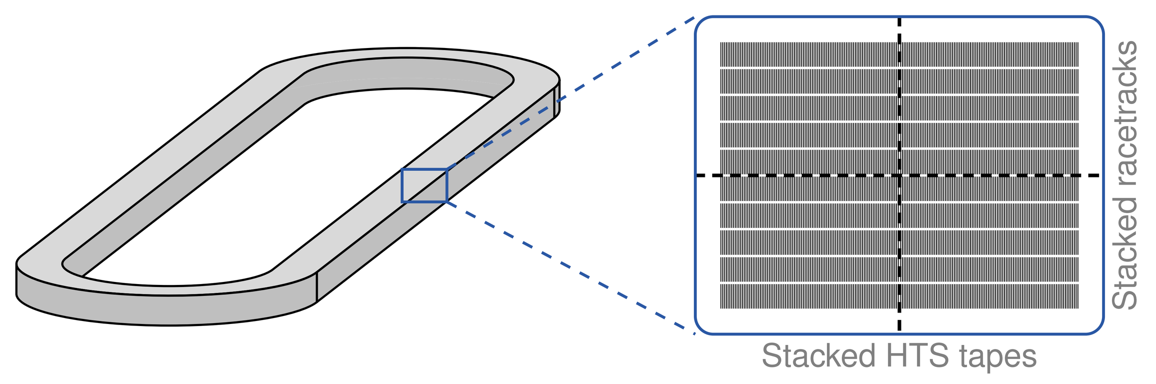
## Motivation

- Long-term goal: efficient 3-D simulations of large-scale HTS magnets




- Present contribution: develop efficient $\boldsymbol{H}$-$\phi$ homogenization methods

  $\Rightarrow$ **scalar unknowns** outside the HTS
- Among others, Simultaneous Multi-Scale (SMS) method: [1] for full-$\boldsymbol{H}$ and $\boldsymbol{T}$-$\boldsymbol{A}$ formulations. $\boldsymbol{T}$-$\boldsymbol{A}$ enhanced in [2]
- Today: SMS extension to $\boldsymbol{H}$-$\phi$ TS [3]

  $\Rightarrow$ $\boldsymbol{H}$-$\phi$ SMSTS

## Finite-element formulations

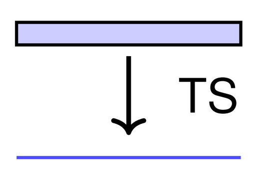
Three coupled subproblems, with **magnetic field decomposition**: [5]

$$\boldsymbol{H} = \boldsymbol{H}_r + \boldsymbol{H}_s$$

- $\boldsymbol{J}_s \Rightarrow \boldsymbol{H}_s$ with **curl** $\boldsymbol{H}_s = \boldsymbol{J}_s$

  Gauging: tree co-tree technique
- $\boldsymbol{H}_s \Rightarrow \boldsymbol{H}_r$

  TS formulation, see [3]




- $\boldsymbol{H}_r \Rightarrow \boldsymbol{J}_s$

  Linear interpolation via *extrusion* operator

Coupled formulations require **flexible** software

$\Rightarrow$ **GetDP** [6]

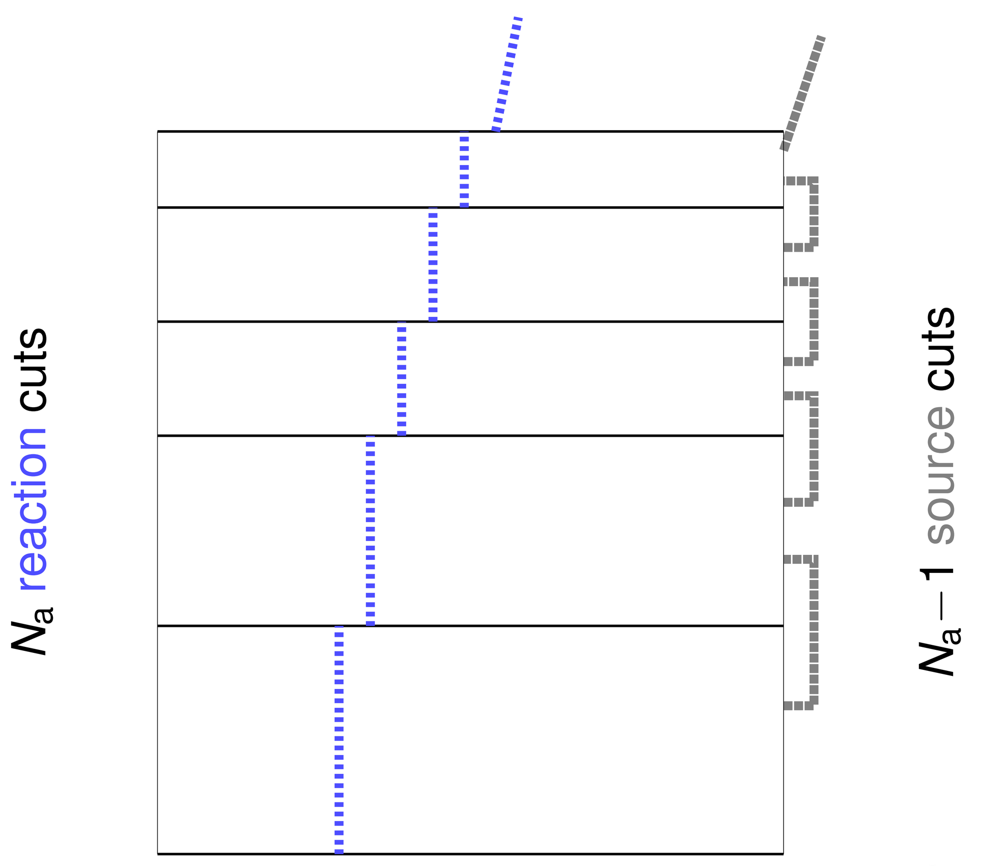
- **Open-source** models

Life-HTS

## Cohomology

- Magnetic field decomposition:

  $\Rightarrow$ Two sets of cohomology basis functions




$\Rightarrow$ Advanced cohomology solver

Gmsh

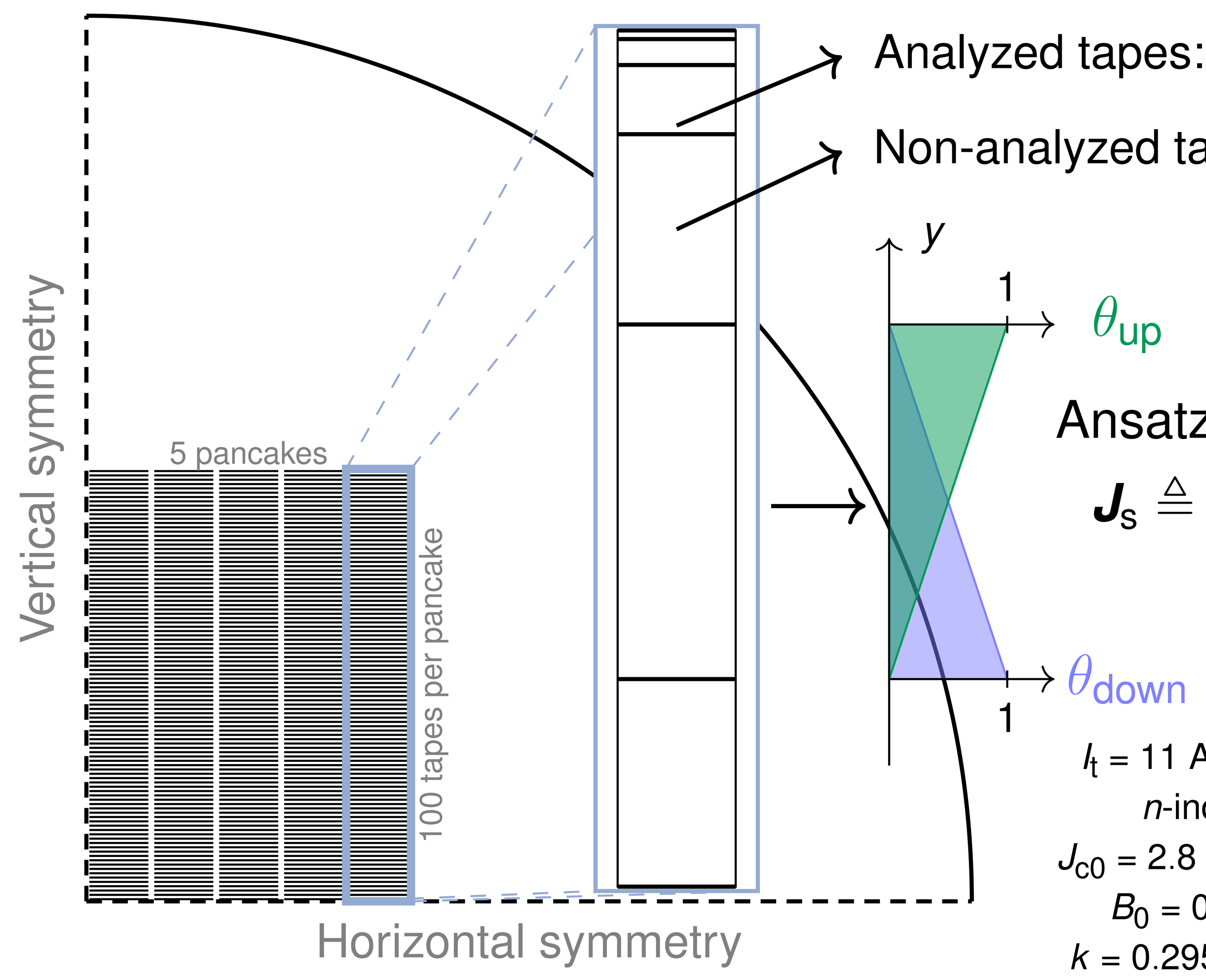
## Review of H-Phi homogenization

Presentation by B. Vanderheyden

**Today at 15:00** - Archive Hall

## Numerical approach and benchmark problem

Benchmark from [1,2,4]




Analyzed tapes: thin shell $\boldsymbol{H}$-$\phi$ [3]

Non-analyzed tapes $\rightarrow$ bulk source domains

Ansatz:

$$\boldsymbol{J}_s \triangleq f_{SC}\left[\theta_{up}(y)\ \boldsymbol{J}_{up} + \theta_{down}(y)\ \boldsymbol{J}_{down}\right]$$

$$f_{SC} = \frac{d_{SC}}{d_{turn}}$$

$I_t$ = 11 A, $f$ = 50 Hz

$n$-index: 38

$J_{c0} = 2.8 \times 10^{10}$ A/m$^2$

$B_0$ = 0.04265 T

$k$ = 0.29515, $\alpha$ = 0.7

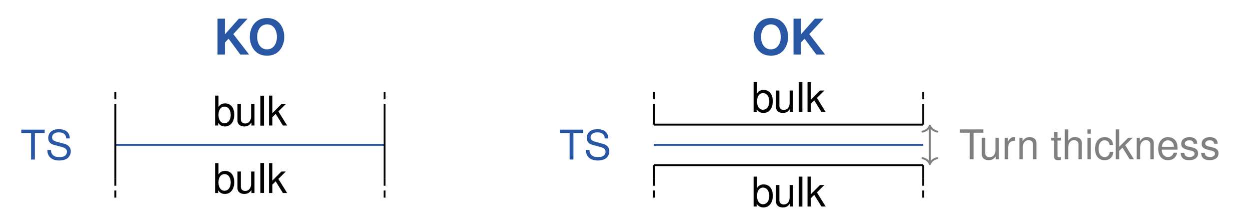
$$J_c(\boldsymbol{B}) = \frac{J_{c0}}{\left(1 + \frac{\sqrt{k^2B_t^2+B_n^2}}{B_0}\right)^{\alpha}}$$

$$\rho(\boldsymbol{J}, \boldsymbol{B}) = \frac{E_c}{J_c(\boldsymbol{B})}\left(\frac{\|\boldsymbol{J}\|}{J_c(\boldsymbol{B})}\right)^{n-1}$$

## Mesoscopic model of analyzed tapes

Caution:

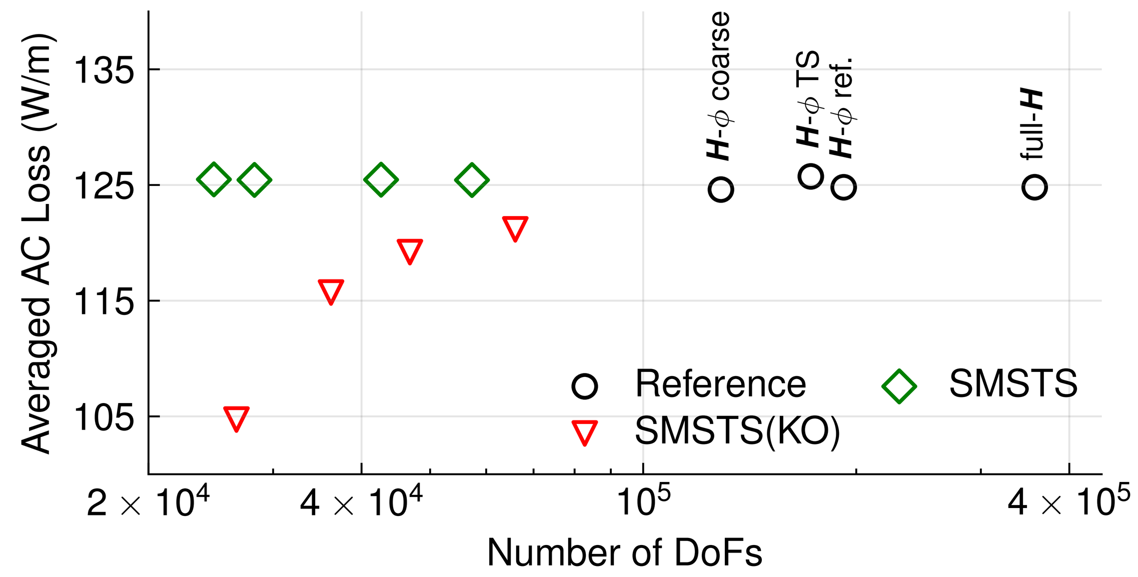
Analyzed tapes should consider its neighbouring insulating layers

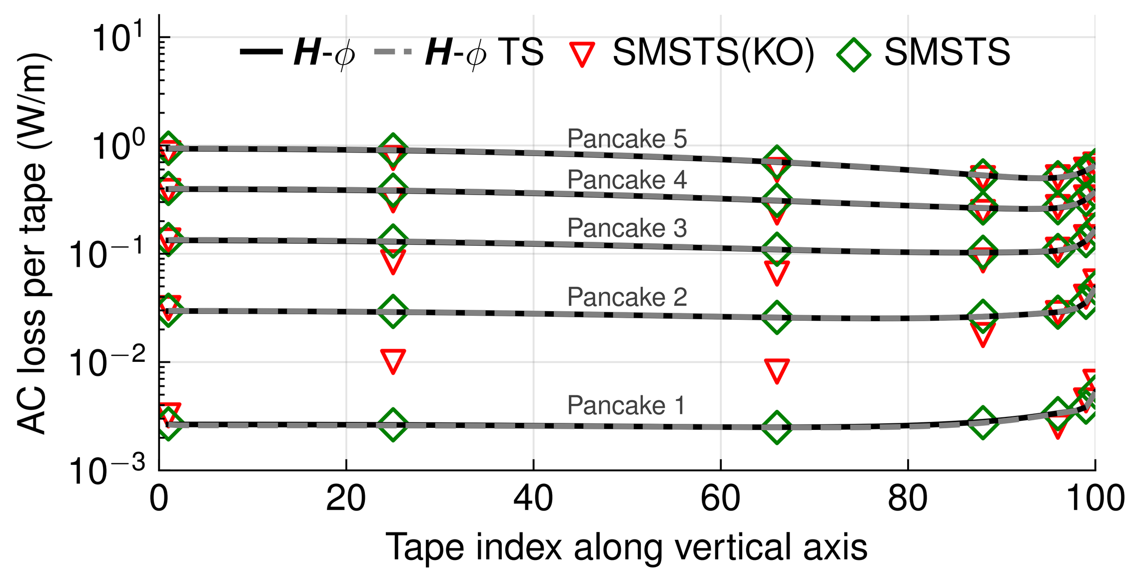



- Principal suspect: incorrect orientation of field *concentration* and anisotropic $j_c$
- Multi-scale model requires consistent *meso-scale submodel*
- Similar observation in [1]

## Results

- AC Losses




- Comparison metrics

$$P = \frac{2}{T}\int_{T/2}^{T}\int_{\Omega_c} \boldsymbol{E}\cdot\boldsymbol{J}\, d\Omega_c\, dt$$

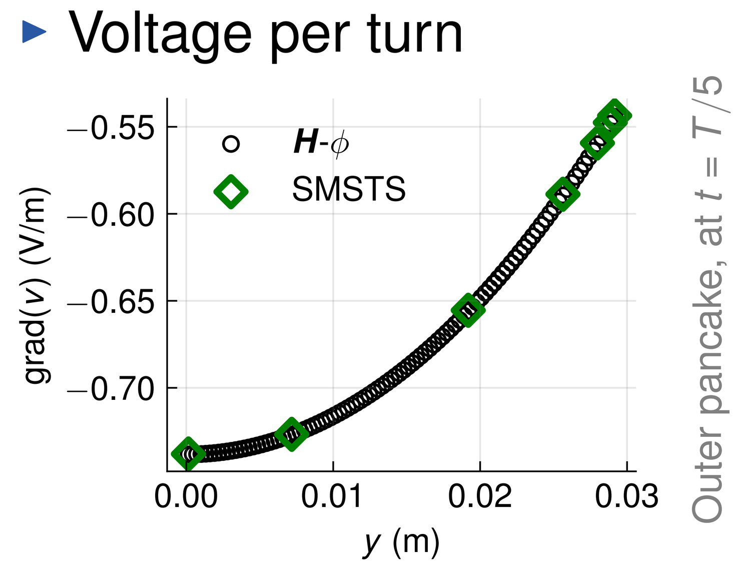
$$R^2 = 1 - \frac{\int_0^T\int_{\Omega_c}(\boldsymbol{J}-\boldsymbol{J}_{H\text{-}\phi})^2\, d\Omega_c\, dt}{\int_0^T\int_{\Omega_c}(\bar{\boldsymbol{J}}_{H\text{-}\phi}-\boldsymbol{J}_{H\text{-}\phi})^2\, d\Omega_c\, dt}$$

- Numerical performance

| Model | Dofs | $P$ (W/m) | Error($P$) | $R^2$ | Wall time (min) |
|---|---|---|---|---|---|
| Ref. $\boldsymbol{H}$-$\phi$ | 192k | 124.8 | / | / | 90+99=189 |
| full-$\boldsymbol{H}$ | 357k | 124.8 | 0.003% | 0.998 | 28+180=208 |
| $\boldsymbol{H}$-$\phi$ TS | 173k | 125.7 | 0.75% | 0.985 | 82+89=171 |
| SMSTS(KO) | 26.6k | 104.7 | -16.1% | 0.806 | 6.2+8.1=14.3 |
| SMSTS | 24.7k | 125.5 | 0.55% | 0.978 | 7.2+8.4=15.6 |

PETSc-MUMPS solver, 8 cores. 600 time steps. Assembly + solve.

- Voltage per turn




## Conclusions

- Successful SMS extension to $\boldsymbol{H}$-$\phi$ TS formulation
- Global accuracy: error on AC losses $< 1\%$
- Local accuracy: $R^2 > 0.97$ in $\boldsymbol{J}$ distribution
- Speed-up factor $\approx$ 12 w.r.t. full $\boldsymbol{H}$-$\phi$ TS model

## Outlook

- 3-D extension
- Coupling with thermal physics

## References


[1] Berrospe *et al.*, 2021, *SUST*, *Advanced electromagnetic modeling of large-scale high-temperature superconductor systems based on H and T-A formulations*.

[2] Wang *et al.*, 2022, *Cryogenics*, *An efficient HTS electromagnetic model combining thin-strip, homogeneous and multi-scale methods by T-A formulation*.

[3] B. de Sousa Alves *et al.*, 2022, *SUST*, *Thin-shell approach for modeling superconducting tapes in the H-$\phi$ finite-element formulation*.

[4] L. Quéval *et al.*, 2016, *SUST*, *Numerical models for ac loss calculation in large-scale applications of HTS coated conductors*.

[5] P. Dular, 1994, *Modélisation du champ magnétique et des courants induits dans les systèmes tridimensionnels non-linéaires*, Ph.D. dissertation, ULiège.

[6] P. Dular and C. Geuzaine, *A general environment for the treatment of discrete problems and its application to the finite element method*, https://getdp.info.