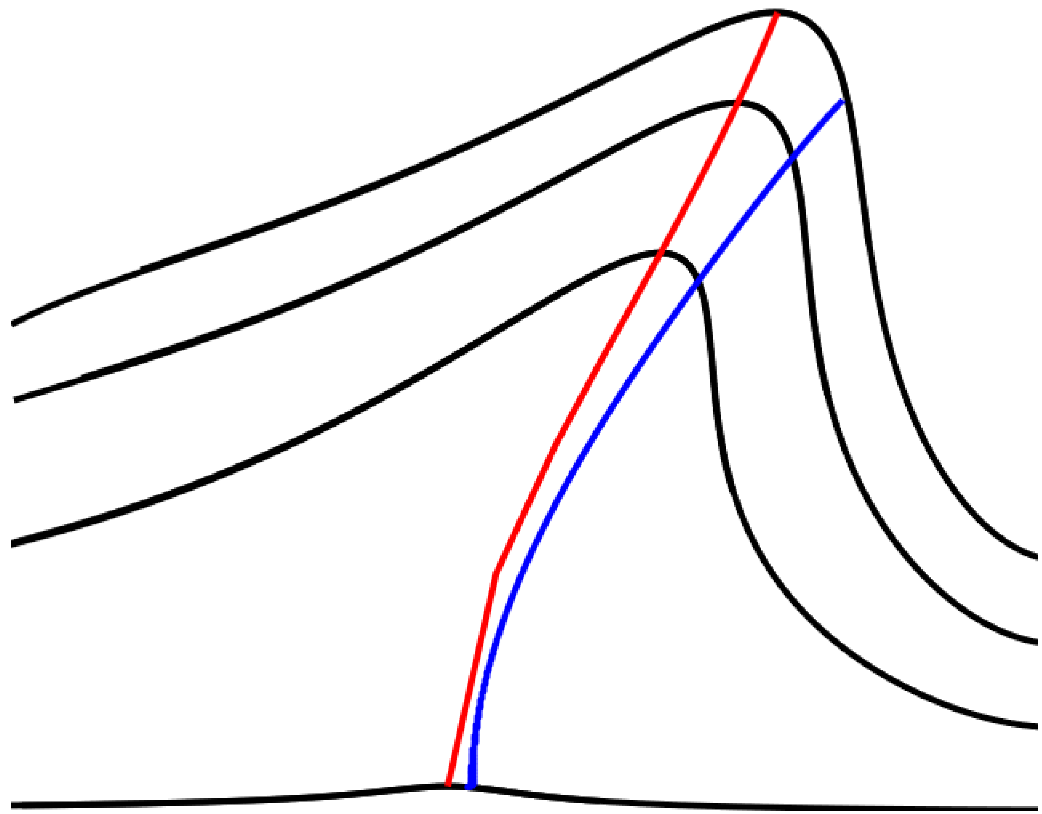
Amplitude Resonance Tracking using Phase-Locked Loops

University of Liège, Belgium



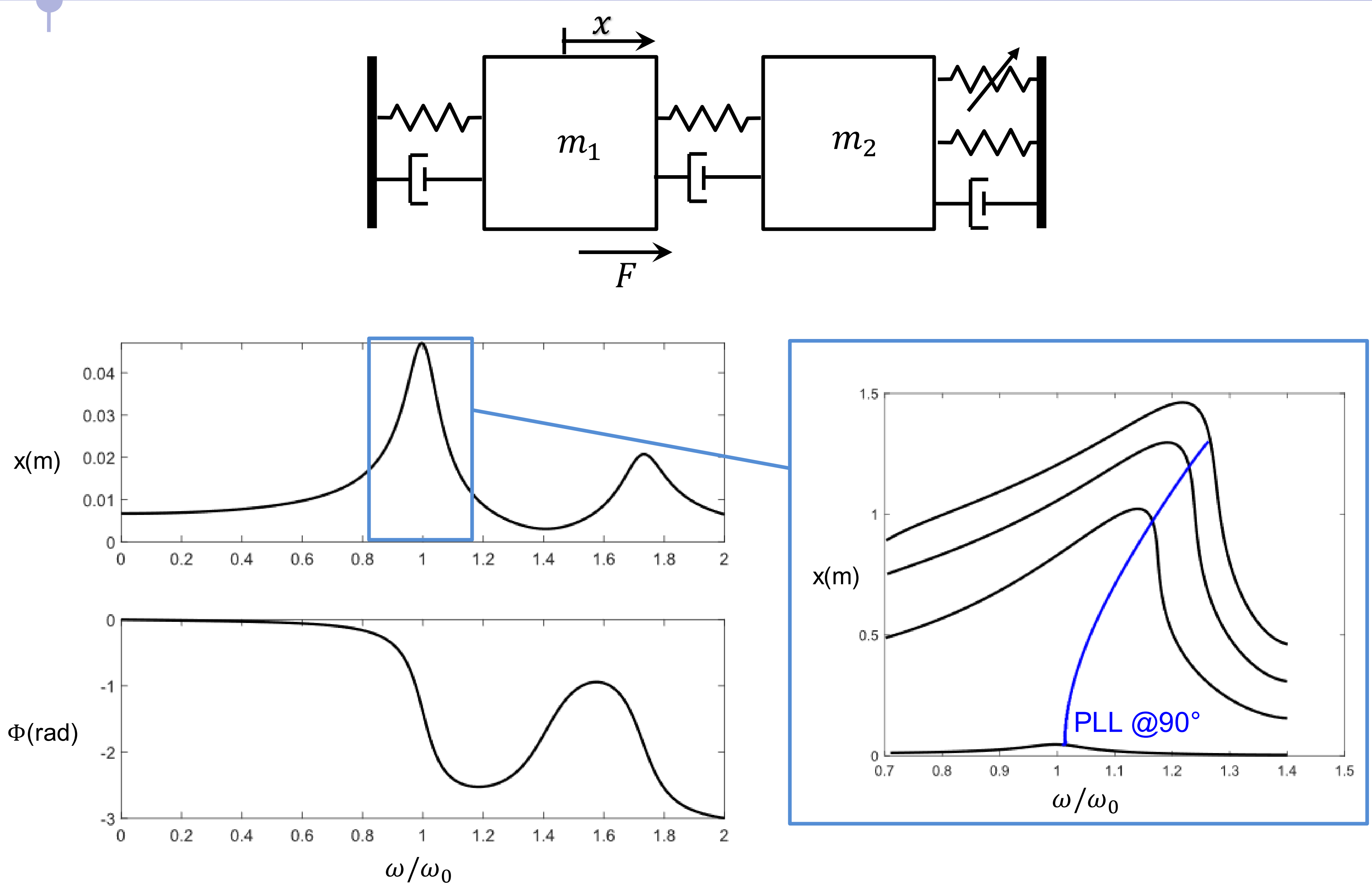
Francois Winand

Christophe Collette

Gaëtan Kerschen

XCON 2025

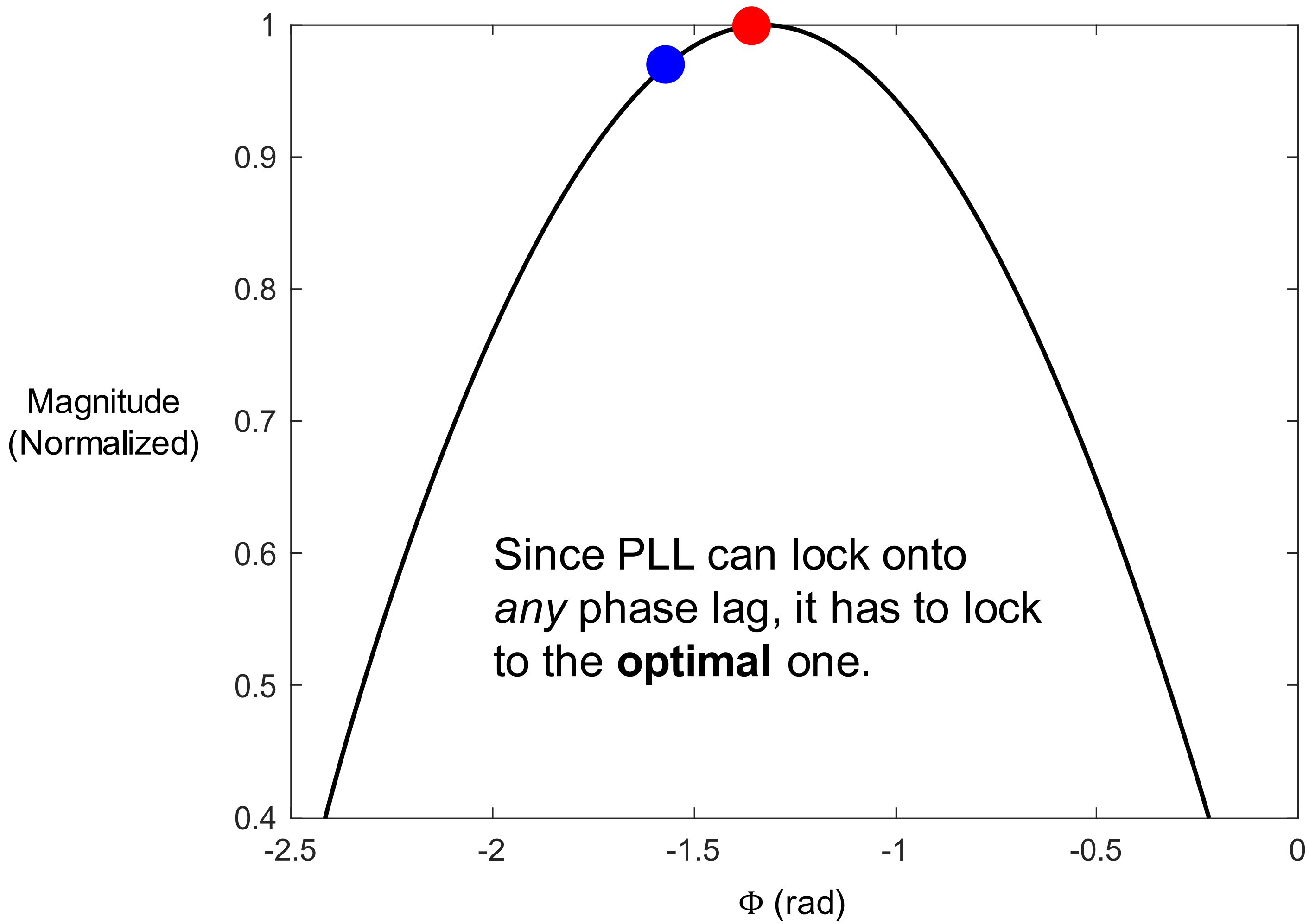
Phase quadrature may not correspond to amplitude resonance



A linear system with relatively closely-spaced modes

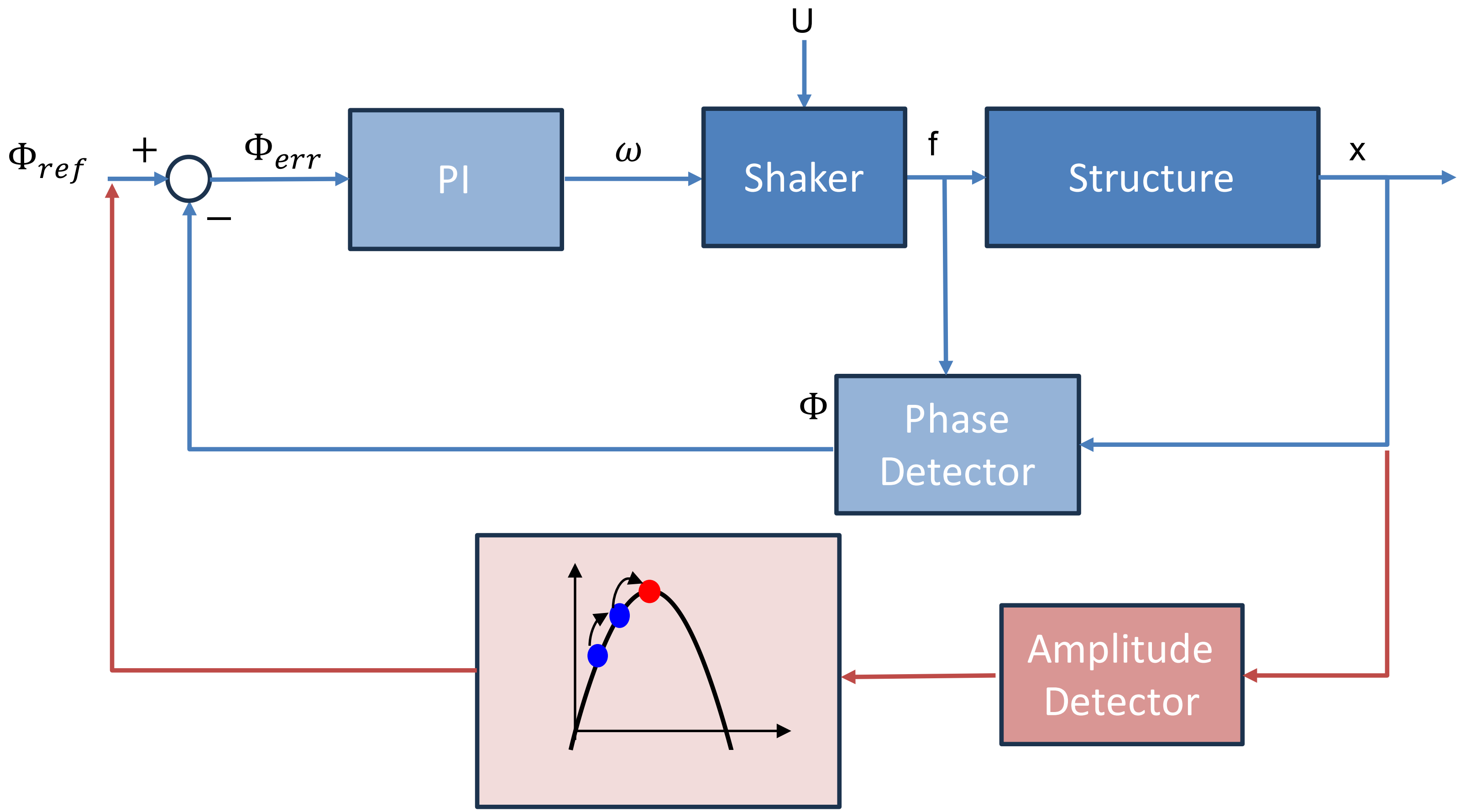
Finding amplitude resonance relies on an optimization process

Goal : Driving the PLL to maximum amplitude (from 🔵 to 🔴)



New method: extremum seeking-based PLL

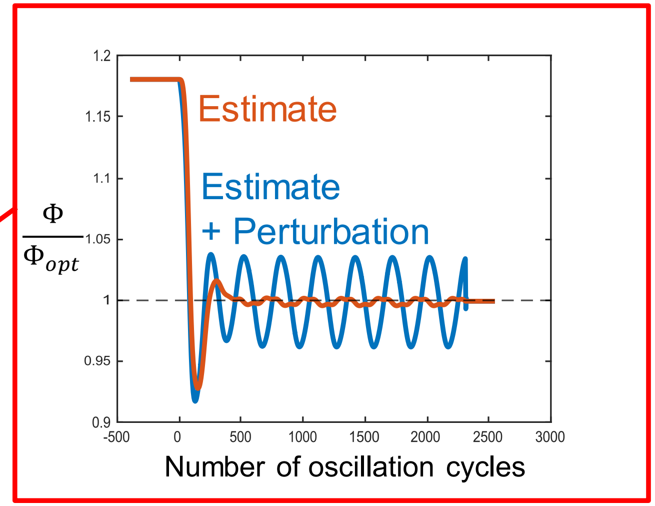
Adapt Φ_{ref} until amplitude is maximized

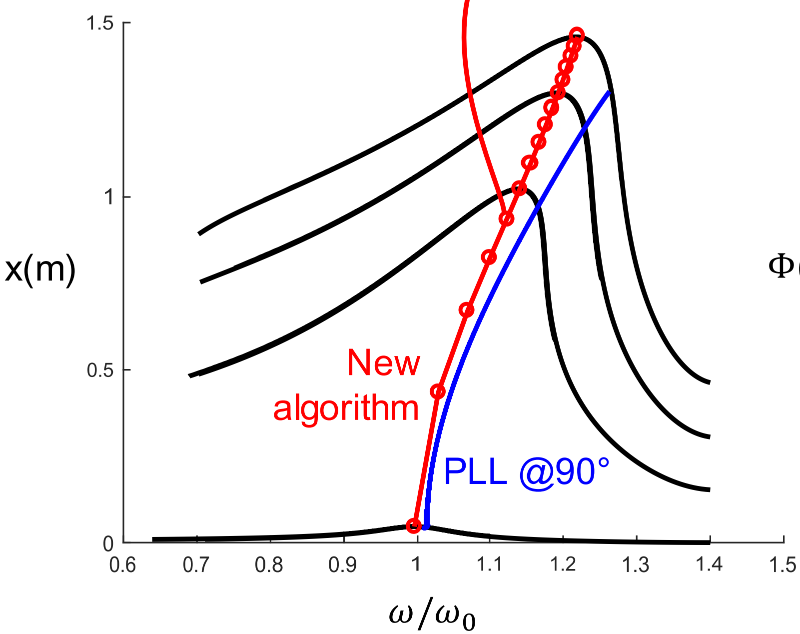


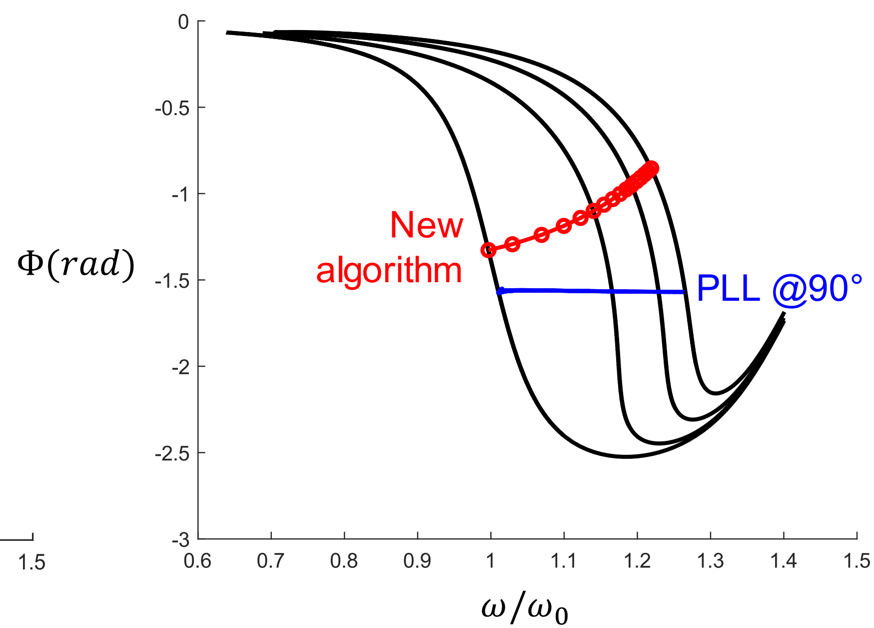
Extremum Seeking Control

An *amplitude backbone curve* is obtained using continuation

Optimization for each level







Conclusion

- Classical PLL is a well-established and effective technique but can fail to follow the maximum amplitude of the resonance peak.
- The combination of PLL with extremem seeking control resolves this issue.
- Good performance was achieved.
- Increased testing duration and extra complexity.

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Derivative-Free Arclength Control-Based Continuation

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Alexandre Spits

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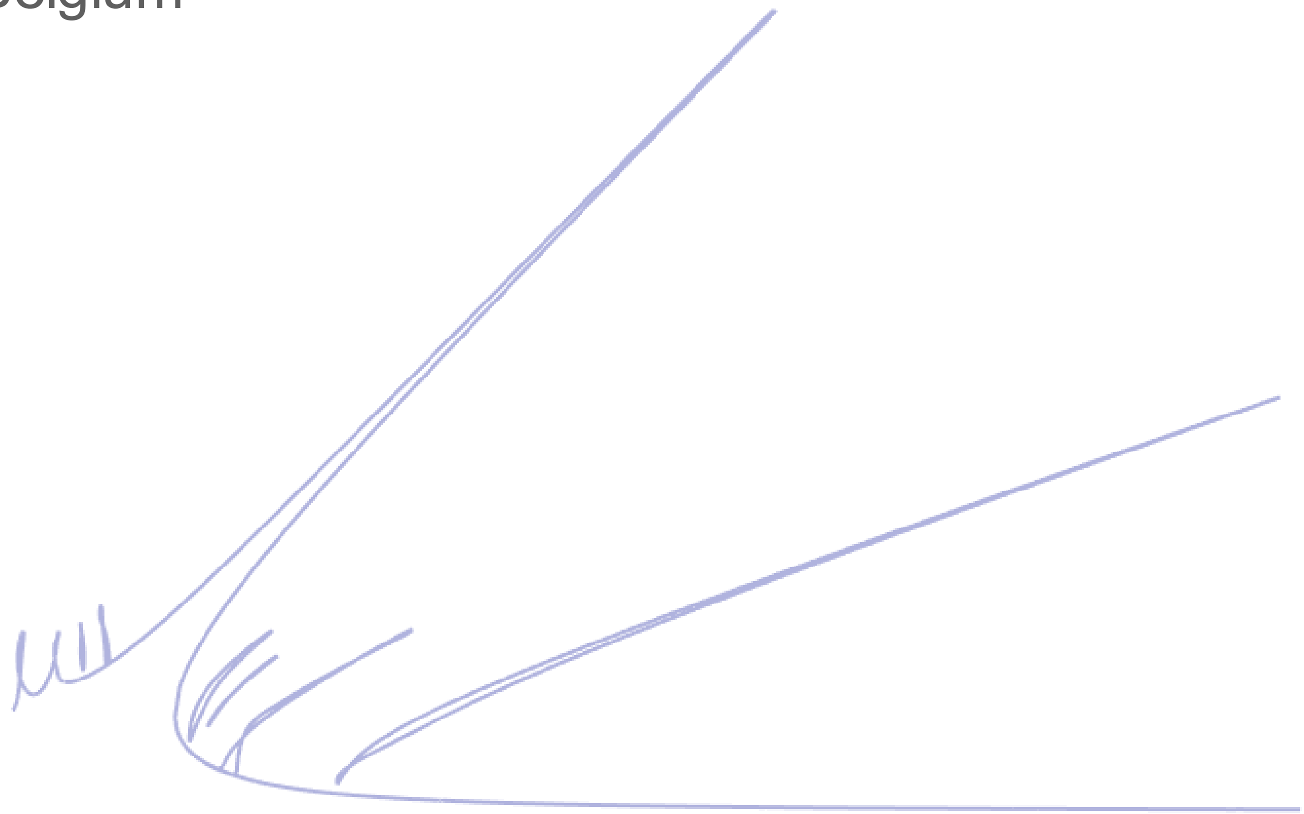
Gaëtan Kerschen



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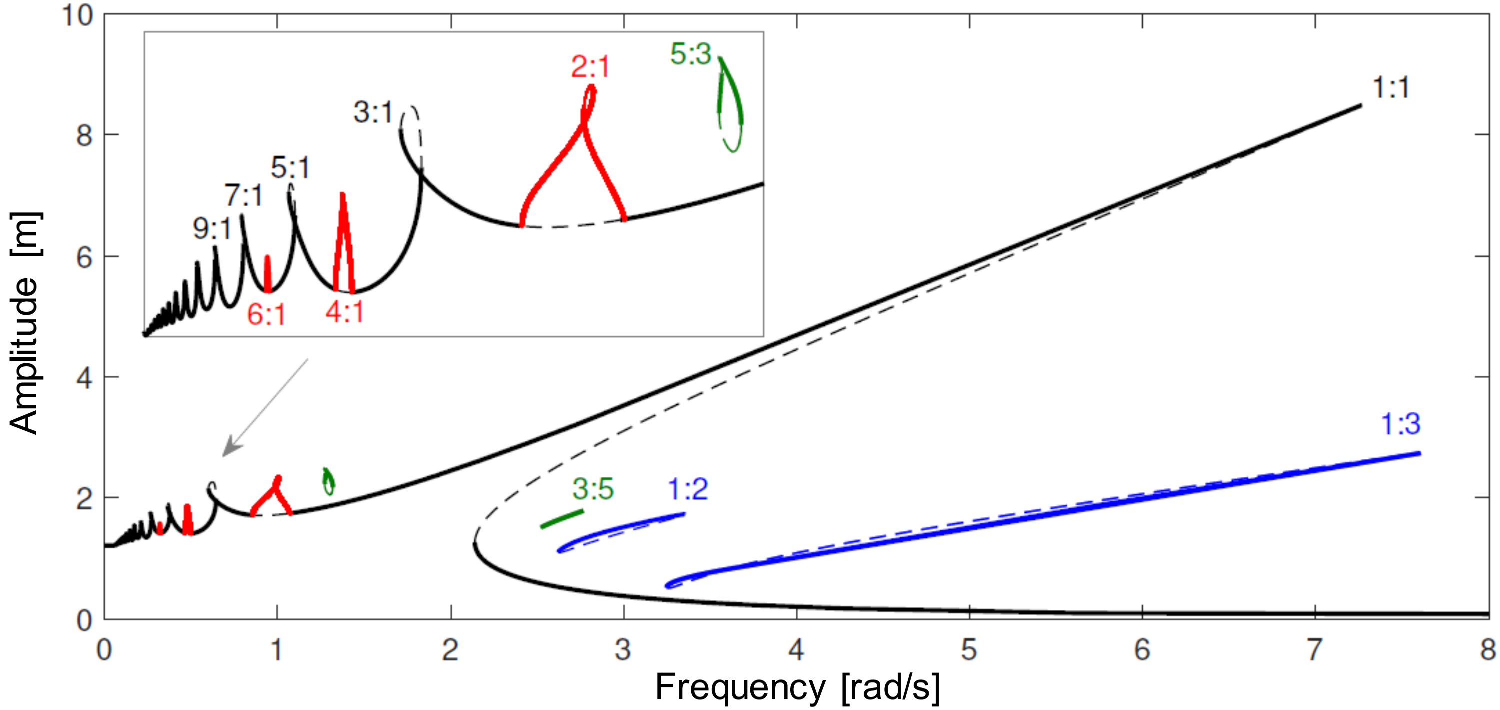
A Duffing oscillator exhibits complex resonant dynamics

$$\ddot{x} + \dot{x} + 0.05\,x + x^3 = 3\sin\omega t$$

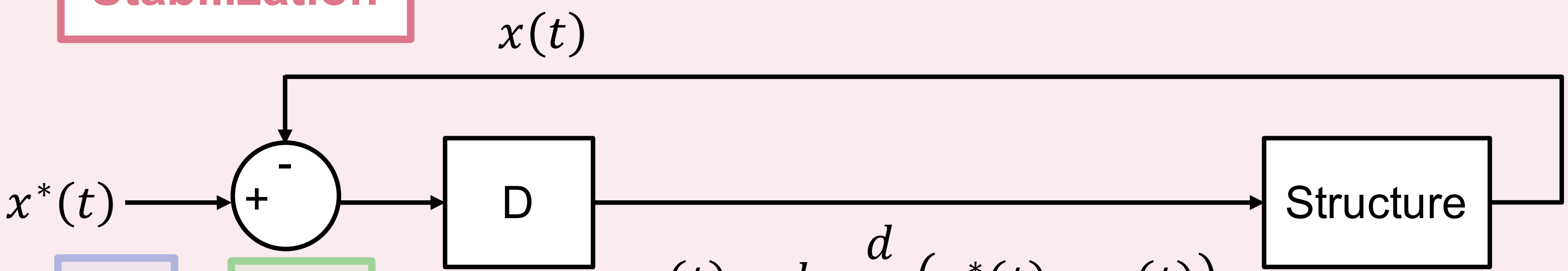


Can we experimentally identify this bifurcation diagram in a model-less manner?

G. Raze, G. Abeloos, and G. Kerschen, Nonlinear Dynamics, 2024

ACBC identifies responses under harmonic forcing

Stabilization



$= x_f^*(t) + x_{nf}^*(t)$

$$u(t) = k_d \frac{d}{dt}\left(x^*(t) - x(t)\right)$$
$$= u_f(t) + u_{nf}(t)$$

Non-invasive if $u_{nf}(t) = 0$ ⟶ $= p\sin(\omega t + \phi)$

Arclength continuation

$$u_f(t) = p\sin(\omega t + \phi)$$

⟶ Modifies $x_f^*(t)$ until $p = p^*$

Adaptive filters

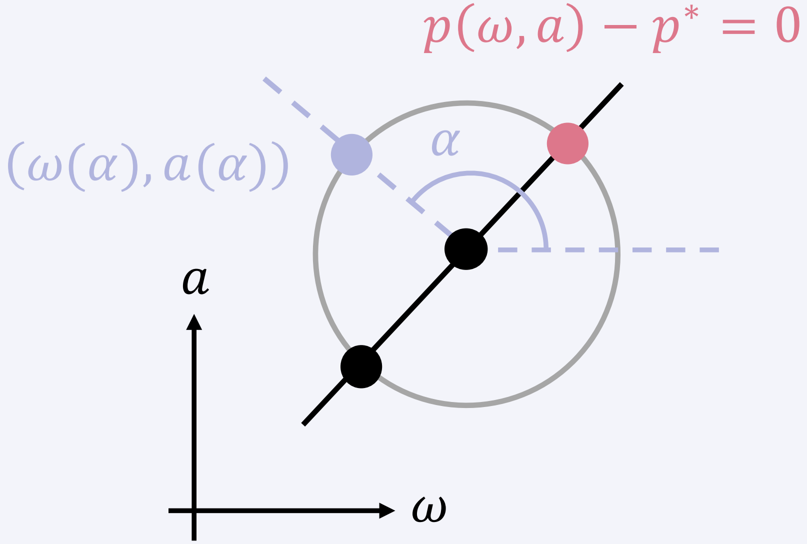
$$u_{nf}(t) = 0$$

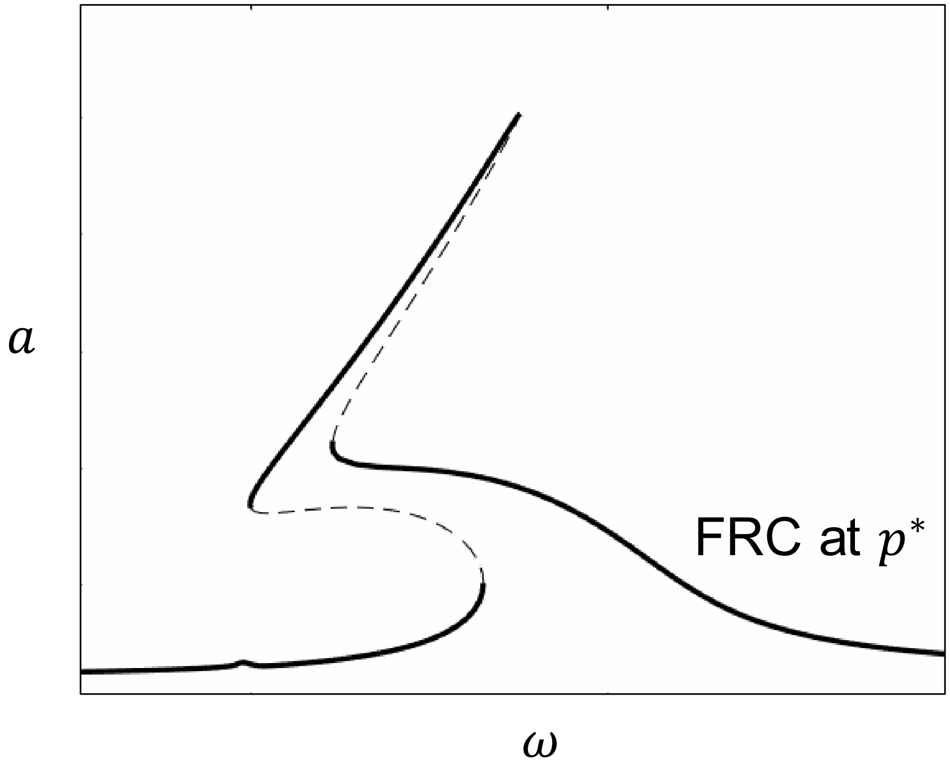
⟶ Enforce $x_{nf}^*(t) = x_{nf}(t)$

ACBC goes through bifurcations in a derivative-free manner

Arclength continuation

→ Modifies $x_f^*(t) = a \sin(\omega t)$





Unlike ACBC, x-ACBC uncovers secondary resonances

$$u(t) = u_f(t) + u_{nf,r}(t) + u_{nf,nr}(t) = p\sin(\omega t + \phi)$$

Control of the resonant harmonic component

$$u_{nf,r}(t) = 0$$

→ Enforce $x^*_{nf,r}(t) = x_{nf,r}(t)$ and reach the desired branch

Arclength continuation

$$u_f(t) = p\sin(\omega t + \phi)$$

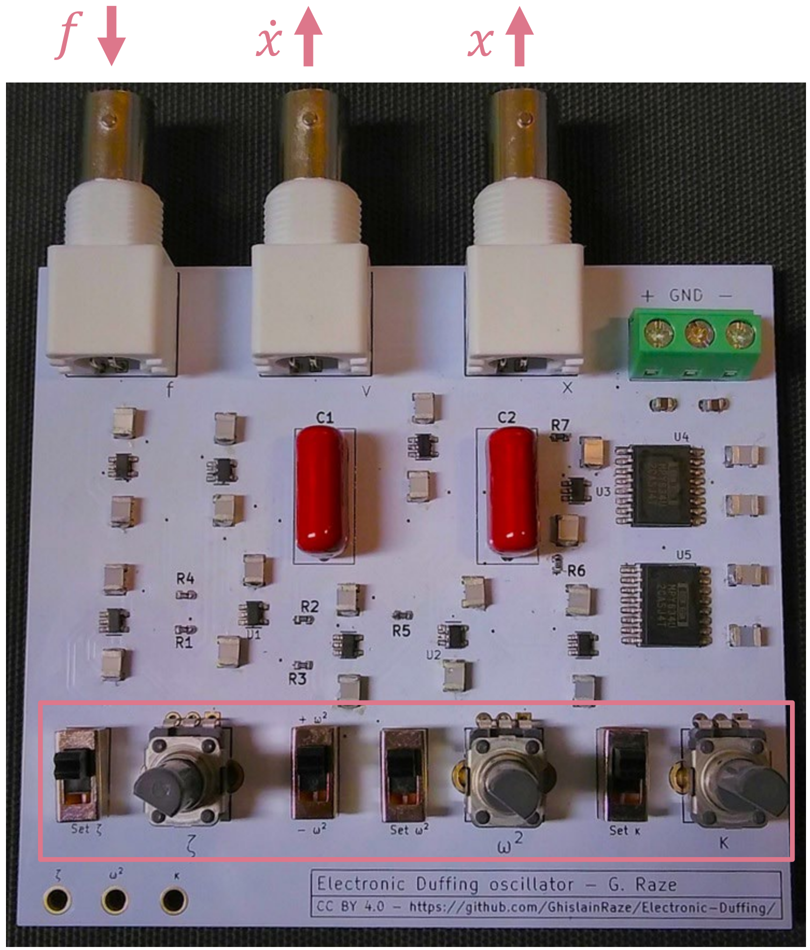
→ Modifies $x^*_f(t)$ until $p = p^*$

Adaptive filters

$$u_{nf,nr}(t) = 0$$

→ Enforce $x^*_{nf,nr}(t) = x_{nf,nr}(t)$

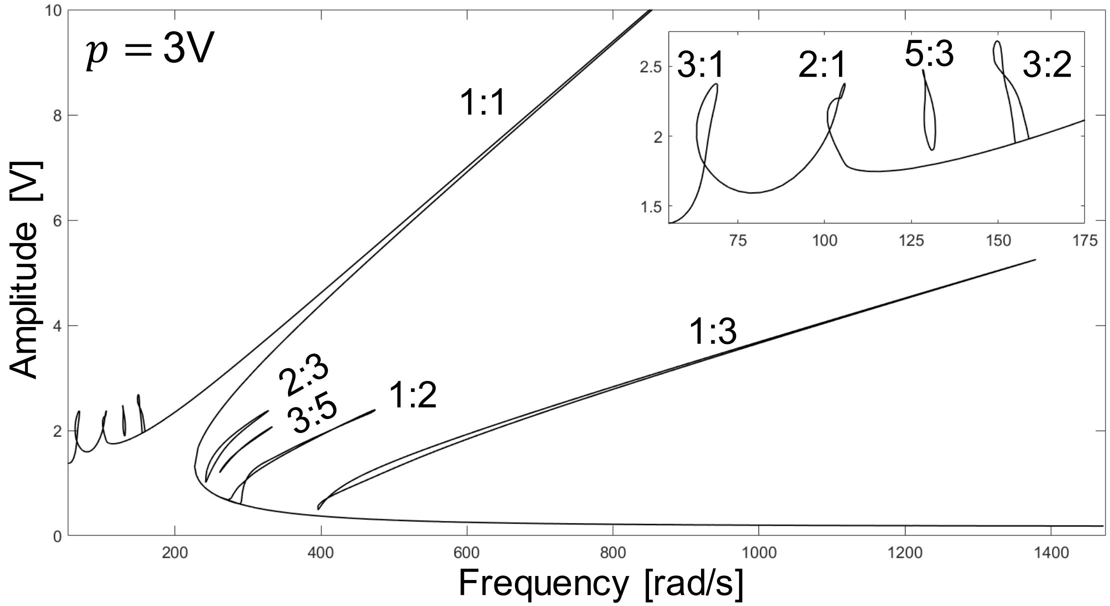
Experimental validation on an electronic Duffing oscillator



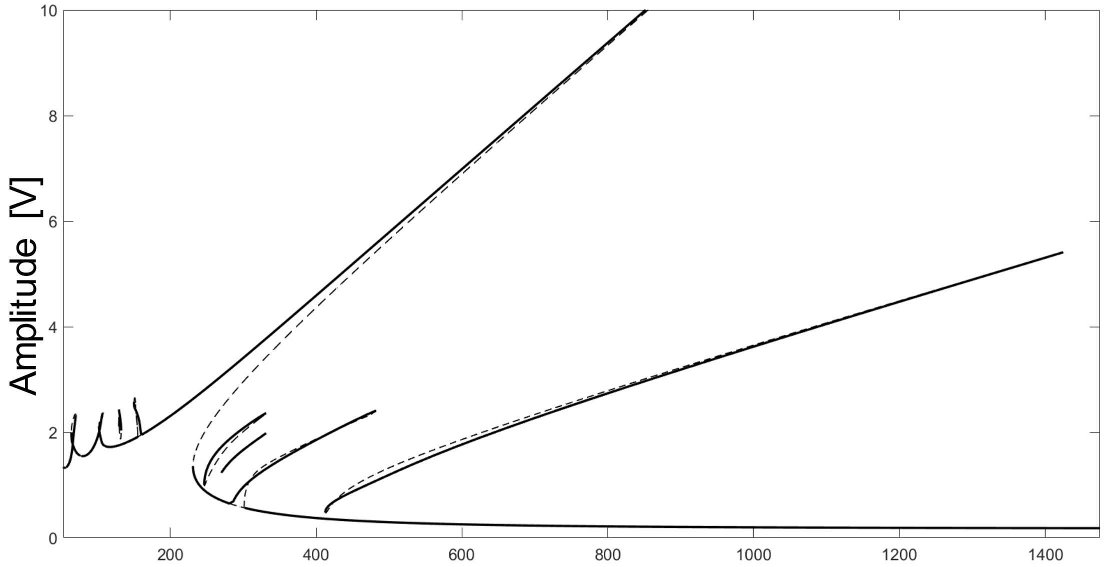
$$10^{-4}\,\ddot{x} + 1.49\ \times 10^{-4}\,\dot{x} + 1.75\,x + 0.99\,x^3 = p\sin\omega t = f(t)$$

Raze, G. An electronic Duffing oscillator (Github)

ACBC fully identifies the complex resonant behavior



Experimental (x-ACBC)



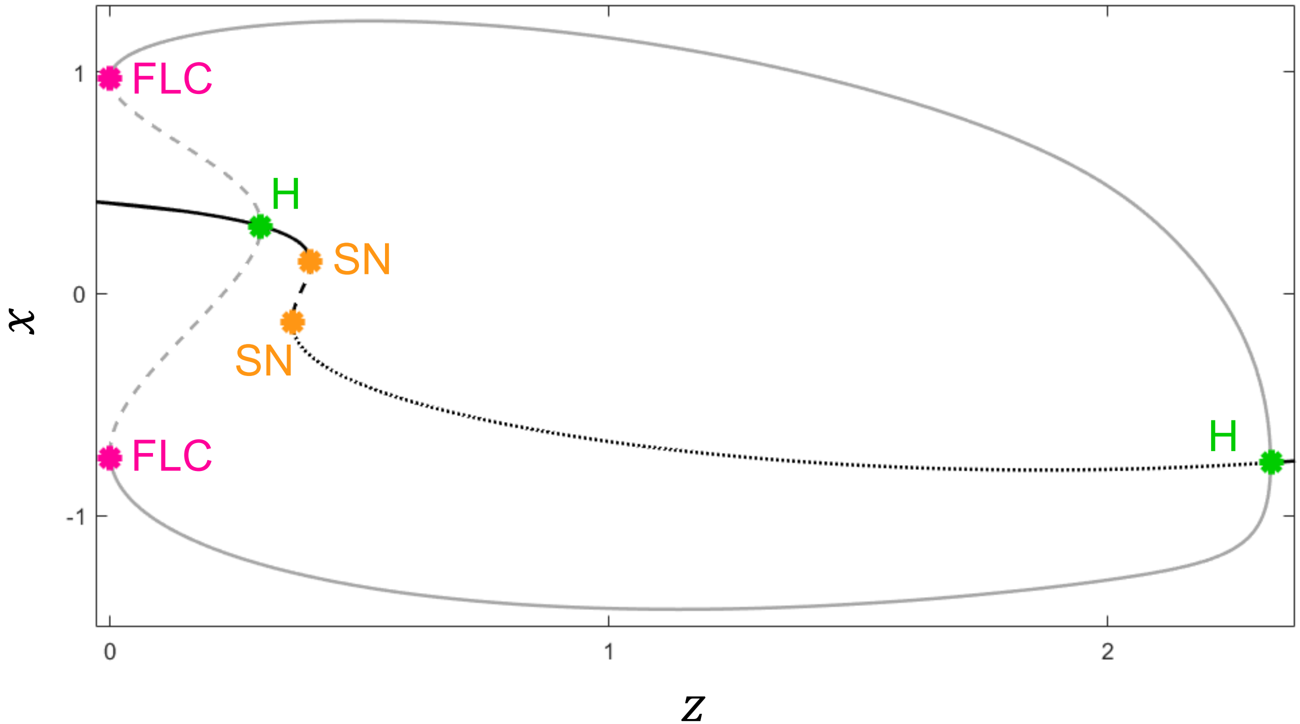
$\omega_0 = 132$ [rad/s]

Numerical (HB + pseudo-arclength continuation)

Conclusions

- A first: x-ACBC can identify all resonances of a Duffing oscillator including its subharmonic and ultrasubharmonic resonances.

- ACBC can also be applied to autonomous systems to uncover fixed points and limit cycles.



M. Saggio, V. Jirsa, et al., eLife, 2020

Machine learning for experimental continuation

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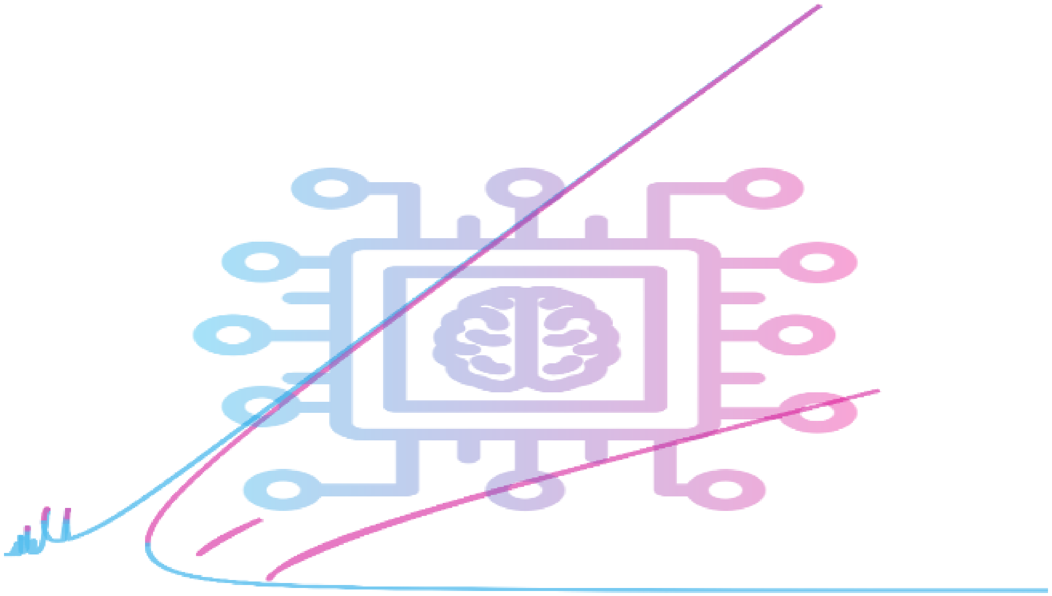
XCON

Grégoire Bourdouch,
Pierre Geurts,
Gaëtan Kerschen



Can machine learning bridge the gap?

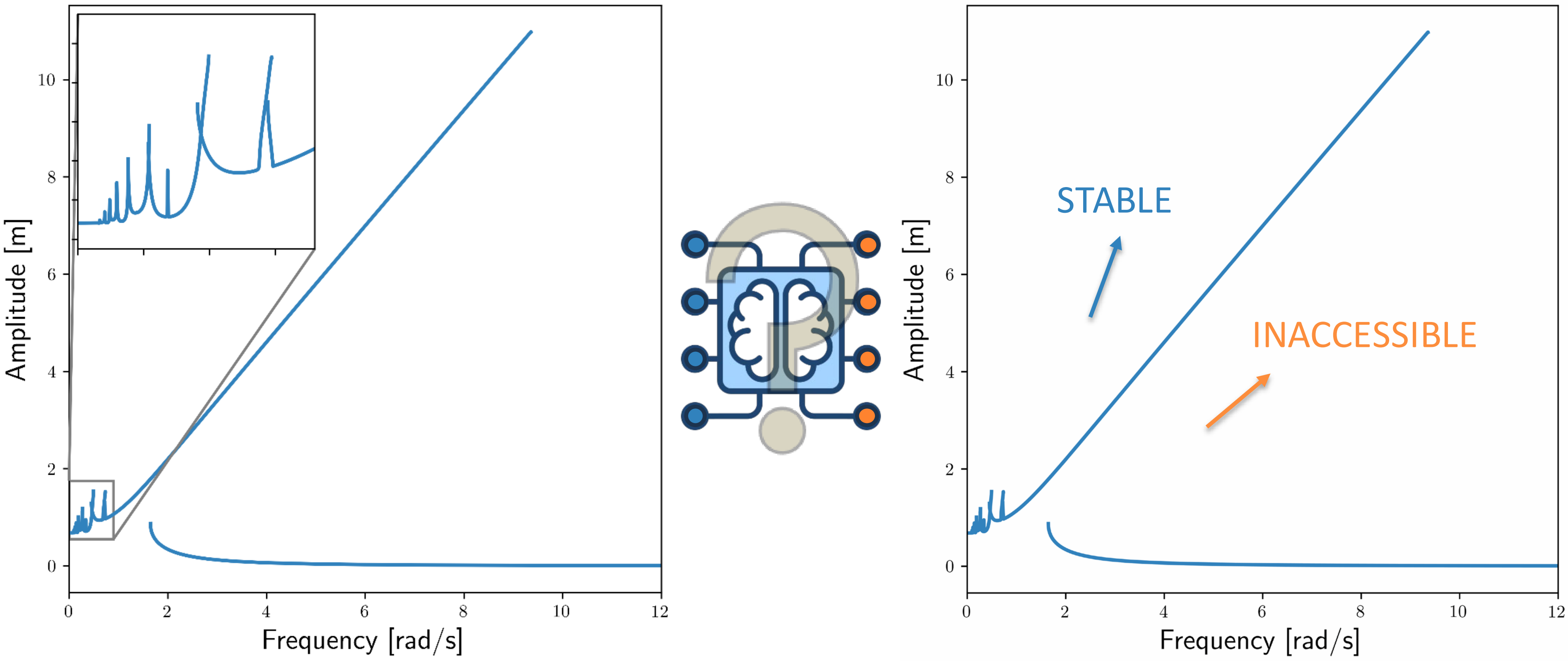
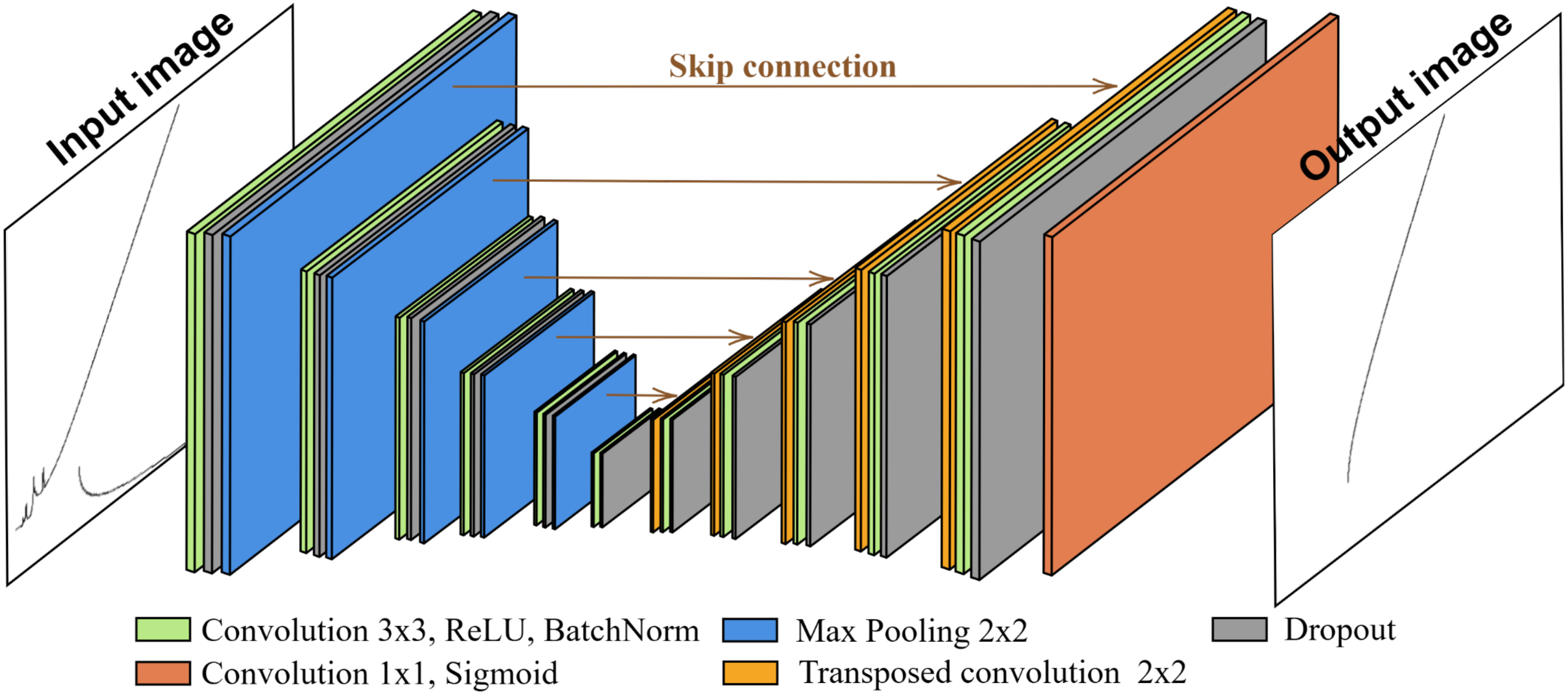


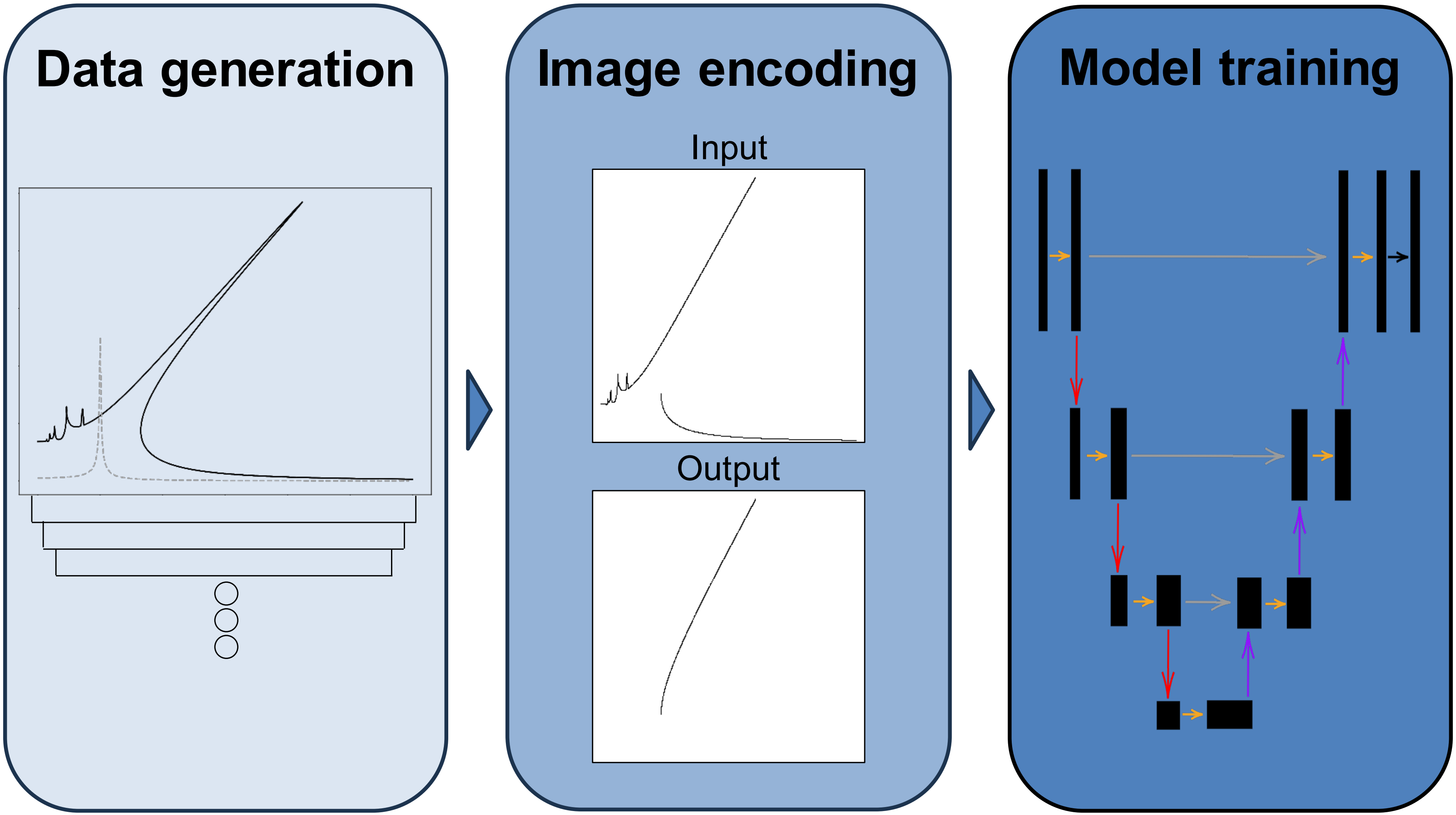
Image-based machine learning approach

U-Net architecture

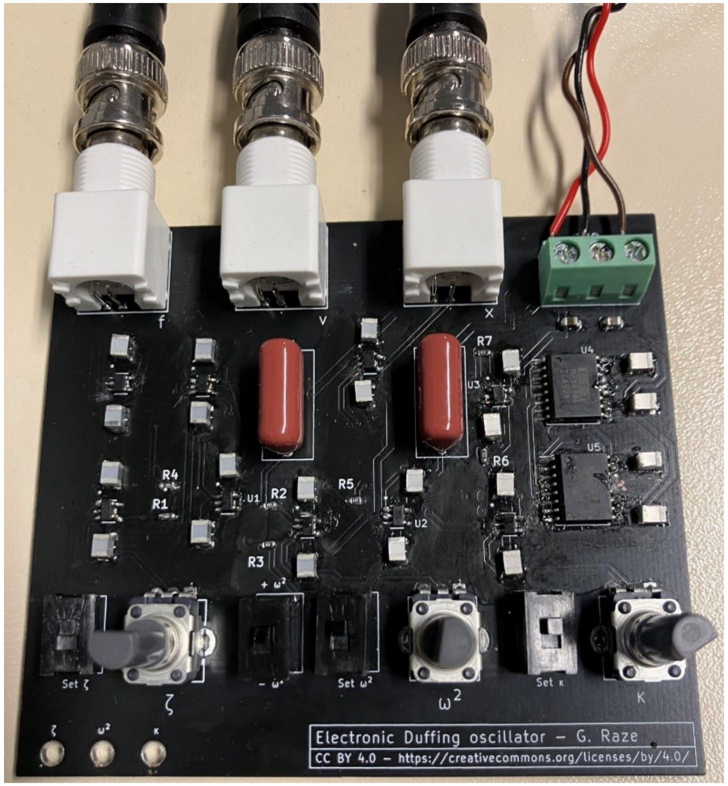


[U-Net architecture (adapted from Ronneberger et al., 2015)]

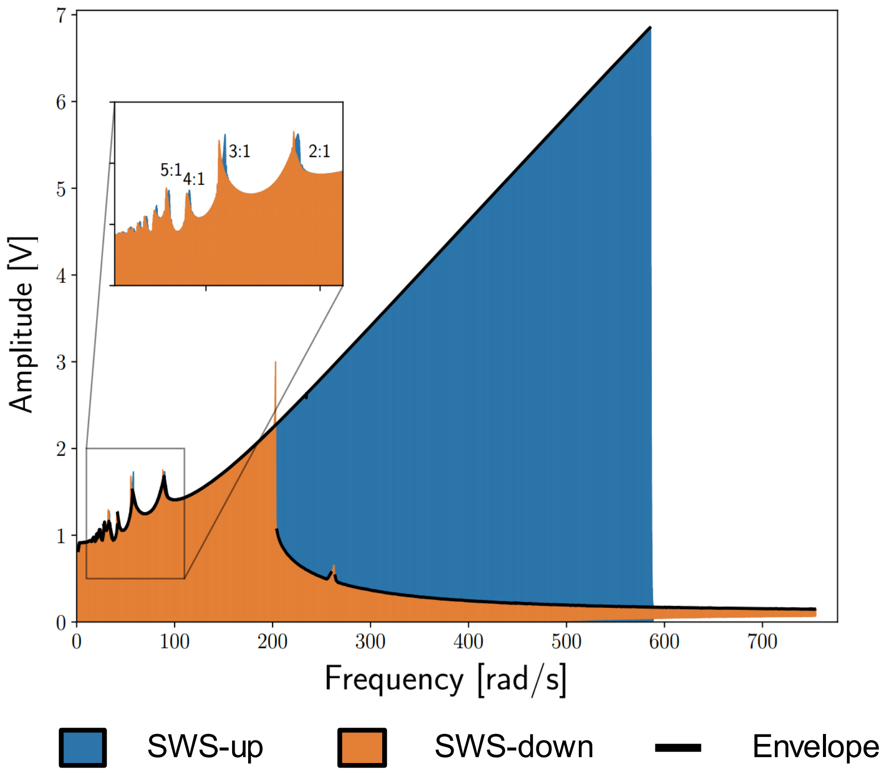
Data-driven framework



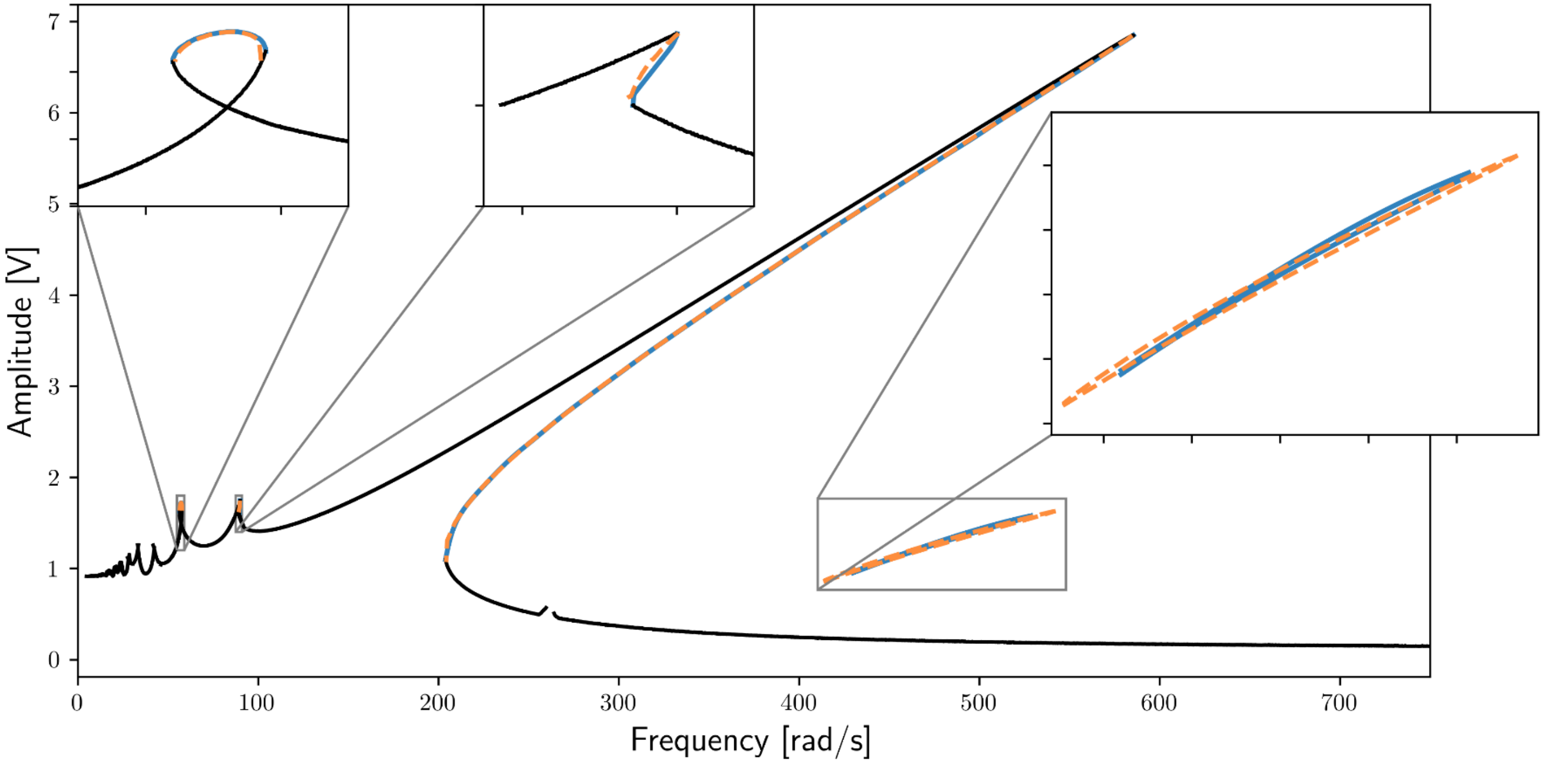
Experimental validation on an electronic Duffing



[G. Raze, "An electronic Duffing oscillator"]



Performance on the experimental Duffing



The ML results closely match the x-ACBC measurements!

In summary

- **Goal**: Predict unstable FRC branches from sine sweep experimental tests.
- **Method**: ML framework using HB-generated data → binary images (stable → unstable).
- **Result**: CNN reconstructs full FRCs including secondary resonances.
- **Validated** on an experimental electronic Duffing.

Thank you for your attention,

I welcome your questions!