



Improved meteoroid trajectory and speed reconstruction with BRAMS: pre- t_0 phase technique and uncertainty quantification

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This study presents a significant advancement in reconstructing meteoroid trajectories and speeds using the Belgian RADio Meteor Stations (BRAMS) forward scatter radio network. We introduce an improved method based on a novel extension of the pre- t_0 phase technique, initially developed for backscatter radars, and adapt it for continuous wave forward scatter systems. This approach leverages phase information recorded before the meteoroid reaches the specular reflection point t_0 to enhance speed estimations. Furthermore, we combine this newly determined pre- t_0 speed with time of flight measurements to reduce uncertainties in the reconstructed meteoroid paths and velocities. The robustness of our method is assessed using Markov Chain Monte Carlo techniques and validated against optical observations from the CAMS-BeNeLux network.

Measurement uncertainties

A critical aspect of reliable trajectory reconstruction is the accurate characterization of measurement uncertainties, particularly for the times of flight (Δt) between receiving stations. The uncertainty $\sigma_{\Delta t}$ is closely tied to the uncertainty in determining the specular timing t_0 at each station. We developed a method to determine the uncertainty σ_{t_0} as a function of two parameters: the rise time of the meteor amplitude curve (t_{rise}) and the signal-to-noise ratio (SNR).

To derive this relationship, we performed a series of Direct Monte Carlo (DMC) simulations. For each combination of t_{rise} and SNR, ideal meteor echoes were generated using the Cornu Spiral model, including some diffusion. For each SNR value, a large number of noisy clones of the ideal echo were created by adding Gaussian noise. The t_0 values were extracted from these noisy echoes using the same post-processing chain as real observations, and the statistical spread σ_{t_0} was computed.

Solver improvement

Building on the measured uncertainties, we integrate them directly into the trajectory reconstruction process by redefining the cost function used by the solver:

$$L = (1 - w)L_{\text{tof}} + wL_{\text{pt0}},$$

where w is a weight parameter balancing the influence of time of flight (L_{tof}) and pre- t_0 speed (L_{pt0}) measurements:

$$L_{\text{tof}} = \sum_{i=1}^n \left(\frac{\Delta t_{i,\text{obs}} - \Delta t_{i,\text{model}}}{\sigma_{\Delta t_{i,\text{obs}}}} \right)^2 ,$$

$$L_{\text{pt0}} = \sum_{j=1}^m \left(\frac{v_{j,\text{pt0}} - v_{j,\text{model}}}{\sigma_{v_{j,\text{pt0}}}} \right)^2 .$$

Optimizing this cost function across different values of w leads to the creation of a Pareto front representing the trade-off between minimizing the two components. The optimal solution is chosen at the "knee" of the curve, corresponding to the maximum curvature point.

Validation against optical observations

The reconstructed trajectories and speeds are compared to CAMS-BeNeLux optical data, which shows good agreement when a combination of time of flight and pre- t_0 information is used. The differences are of the order of 5 % on the speed and 2-4° on the inclination.

Uncertainty propagation

To accurately quantify uncertainties on the reconstructed parameters, we employ a Markov Chain Monte Carlo approach. Assuming independent, Gaussian-distributed errors and uniform priors, the cost function L is proportional to the logarithm of the posterior probability. Thus, minimizing L is equivalent to maximizing the posterior. We use a Single Component Adaptive Metropolis-Hastings algorithm to efficiently explore the parameter space as well as to determine uncertainties and correlations.