A model- and derivative-free arclength continuation method for tracing bifurcation diagrams in nonlinear systems

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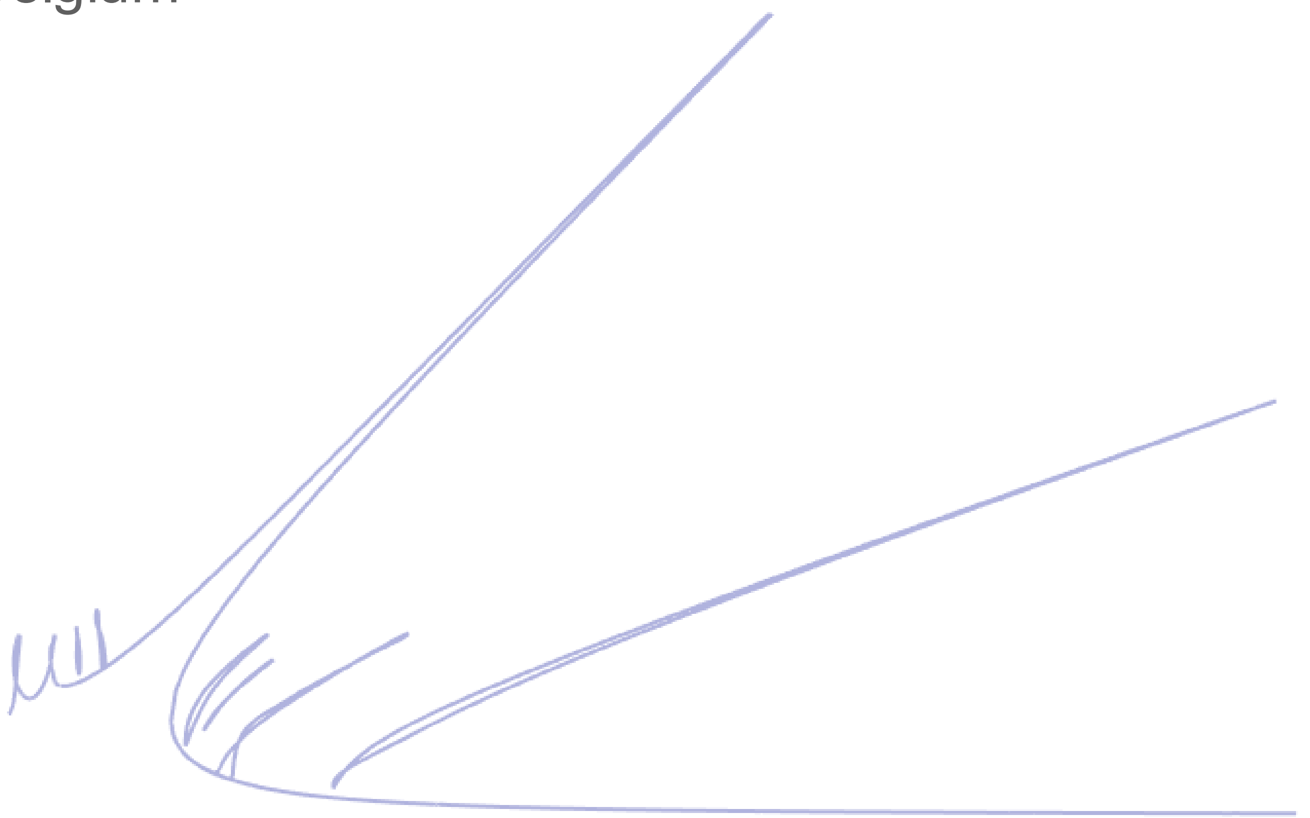
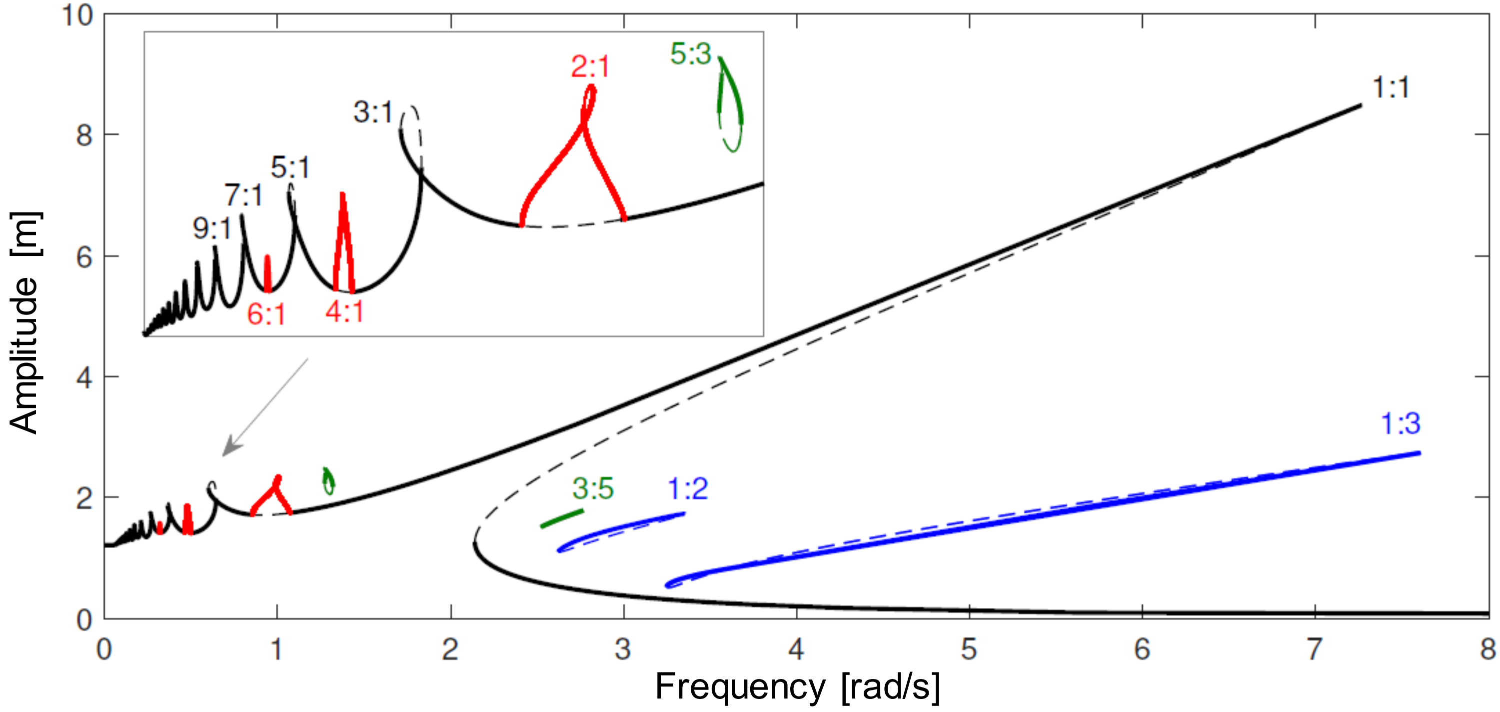


Journées annuelles du GdR EX-MODELI
2025

A Duffing oscillator exhibits complex resonant dynamics

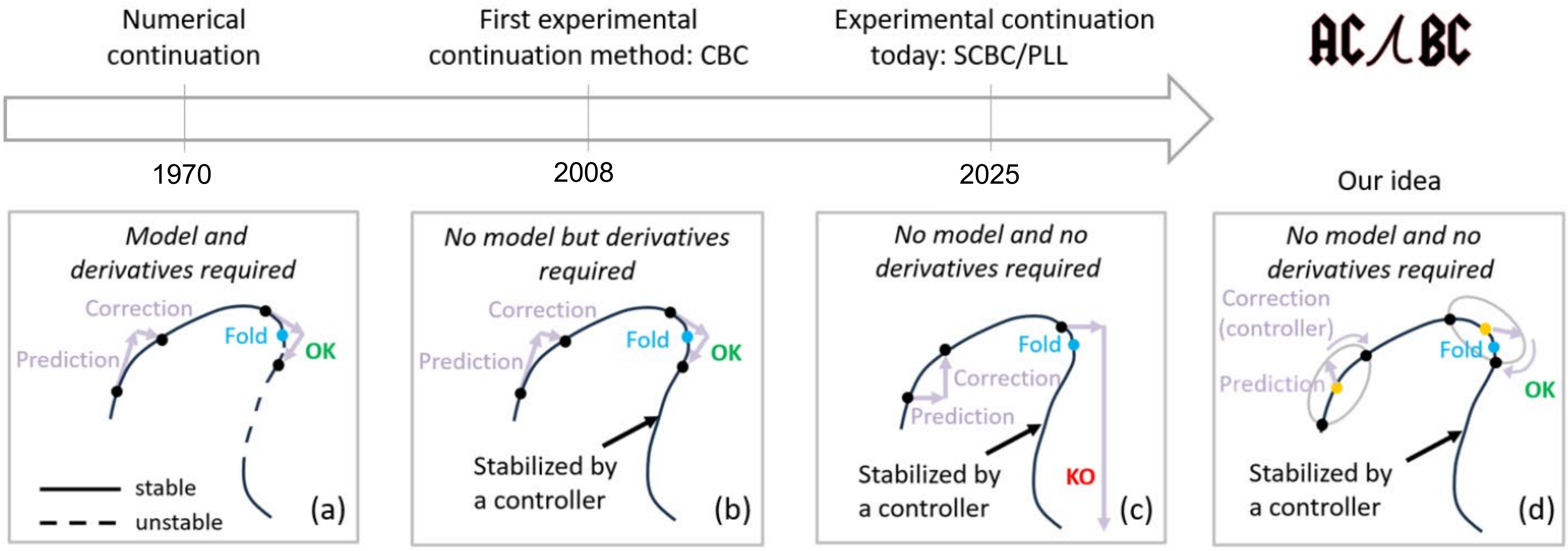
$$\ddot{x} + 0.05\,\dot{x} + x + x^3 = 3\sin\omega t$$



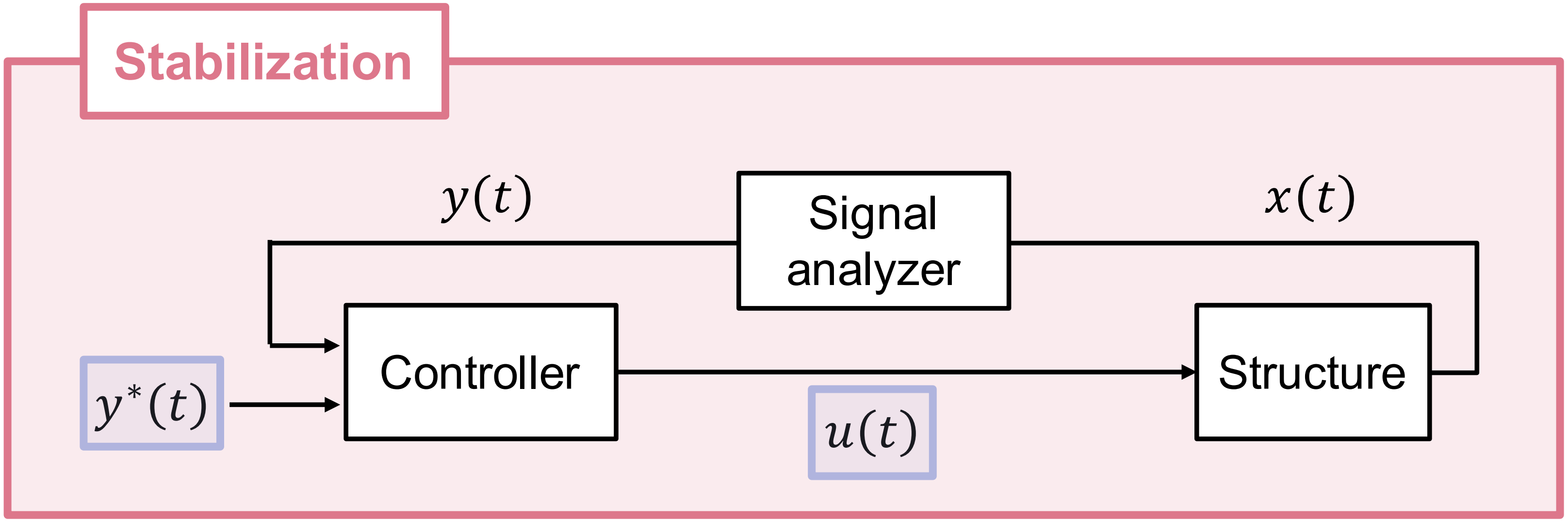
Can we experimentally identify this bifurcation diagram in a model-less manner?

G. Raze, G. Abeloos, and G. Kerschen, Nonlinear Dynamics, 2024

Historical perspective



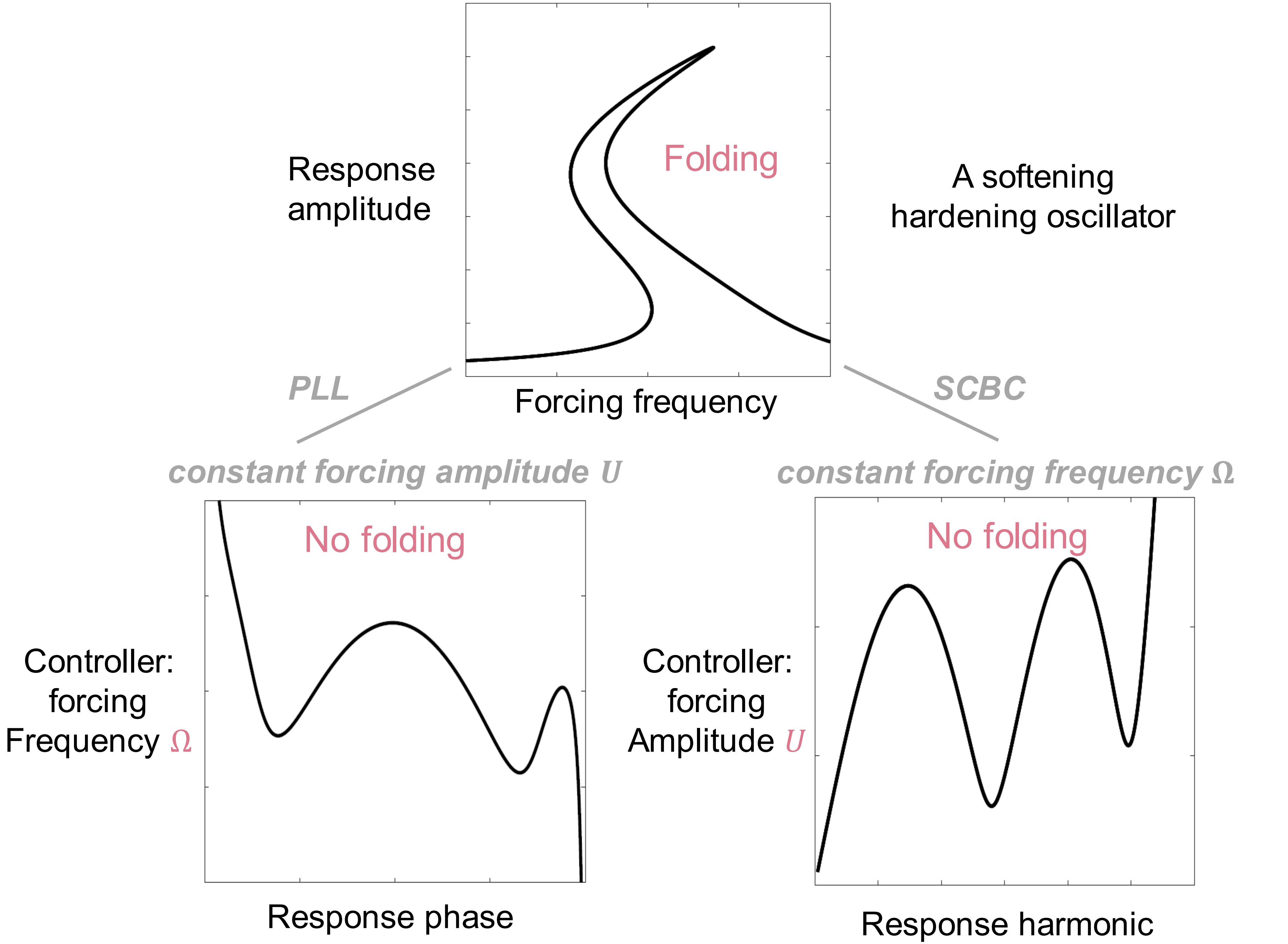
PLL and SCBC identify responses under harmonic forcing



Sequential continuation

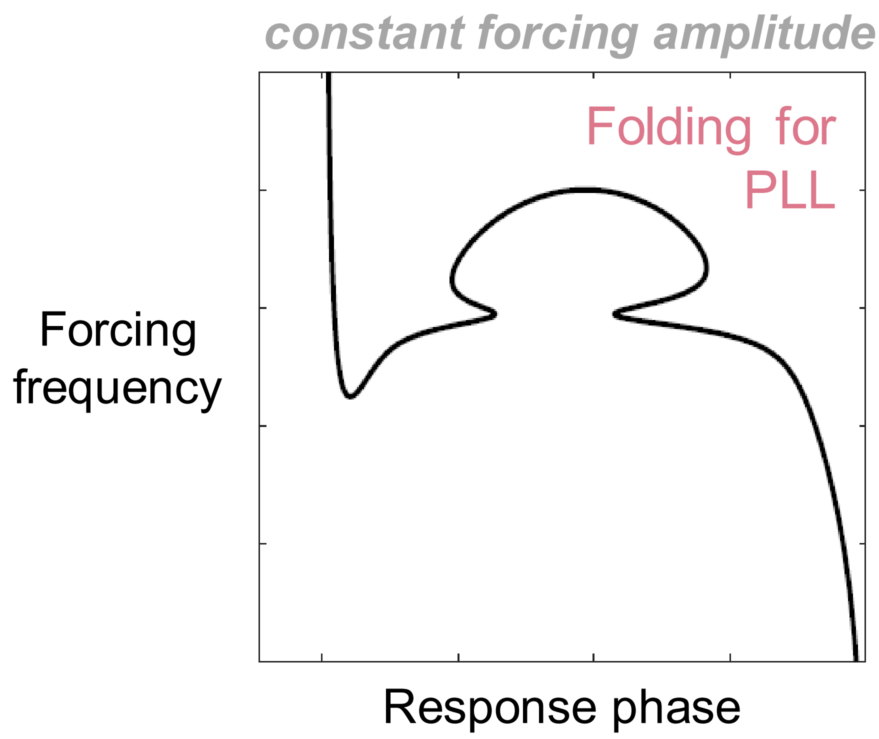
$$u(t) = U \sin(\Omega t)$$

⟶ PLL and SCBC fix Ω or U and adjust the other

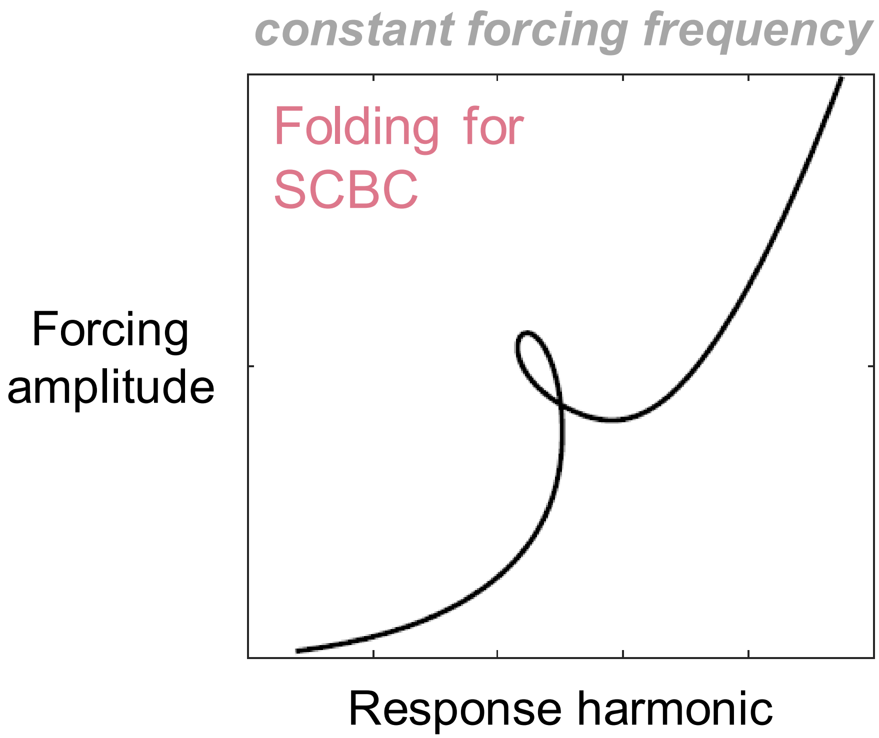

Response amplitude
Folding
A softening hardening oscillator
Forcing frequency
PLL
SCBC
constant forcing amplitude U
constant forcing frequency Ω
No folding
Controller: forcing Frequency Ω
Response phase
No folding
Controller: forcing Amplitude U
Response harmonic

Limitation of existing derivative-free approaches

Merging of an isola with a resonance peak

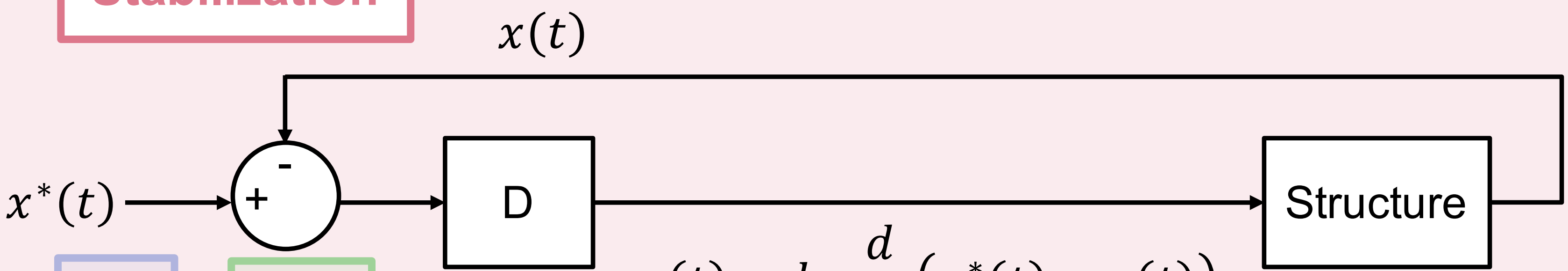


3:1 superharmonic resonance



ACBC identifies responses under harmonic forcing

Stabilization



$= x_f^*(t) + x_{nf}^*(t)$

$$u(t) = k_d \frac{d}{dt}\left(x^*(t) - x(t)\right)$$

$$= u_f(t) + u_{nf}(t)$$

Non-invasive if $u_{nf}(t) = 0$ → $= p\sin(\omega t + \phi)$

Arclength continuation

$$u_f(t) = p\sin(\omega t + \phi)$$

→ Modifies $x_f^*(t)$ until $p = p^*$

Adaptive filters

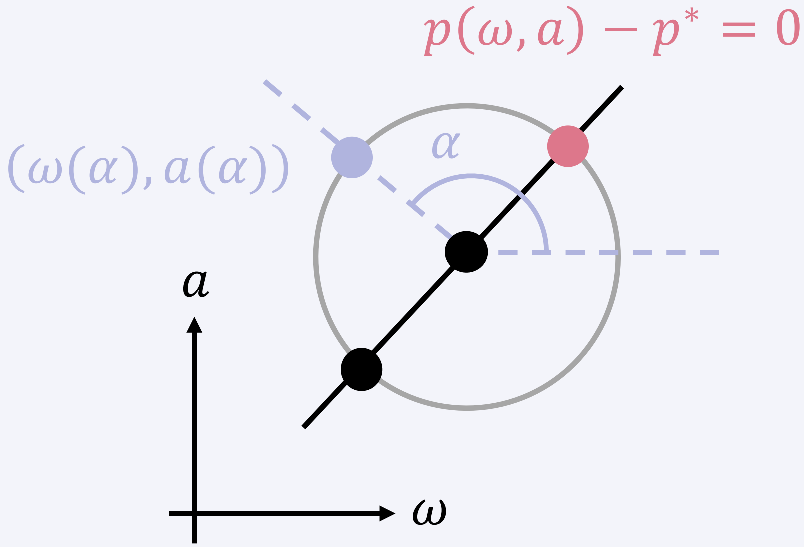
$$u_{nf}(t) = 0$$

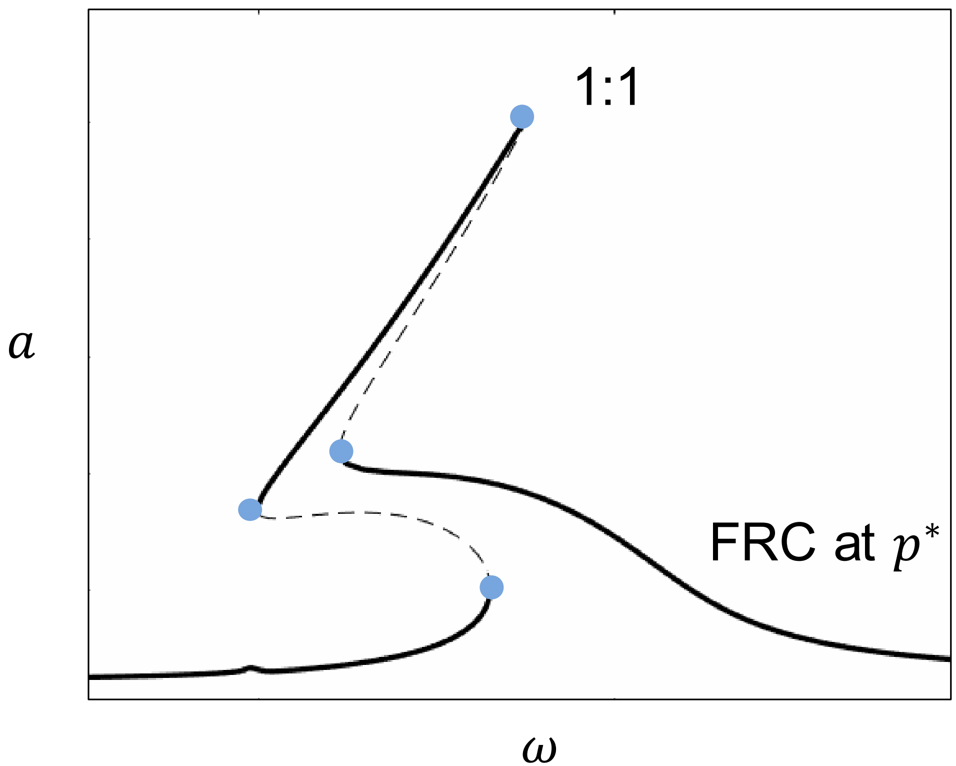
→ Enforce $x_{nf}^*(t) = x_{nf}(t)$

ACBC goes through bifurcations in a derivative-free manner

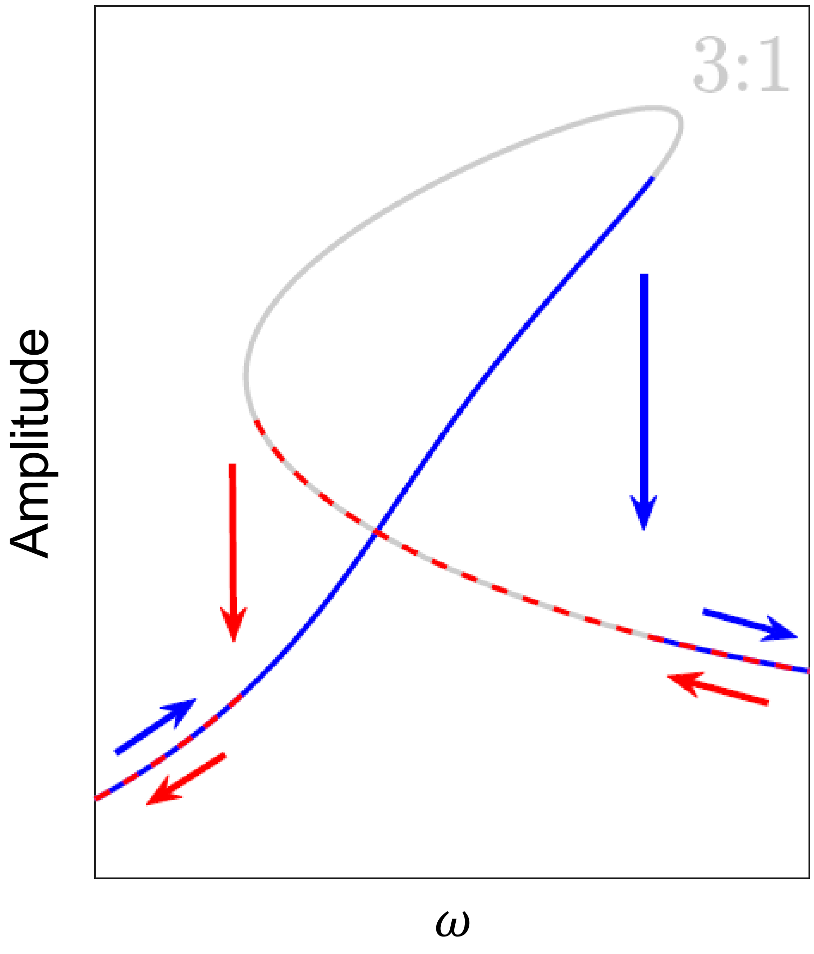
Arclength continuation

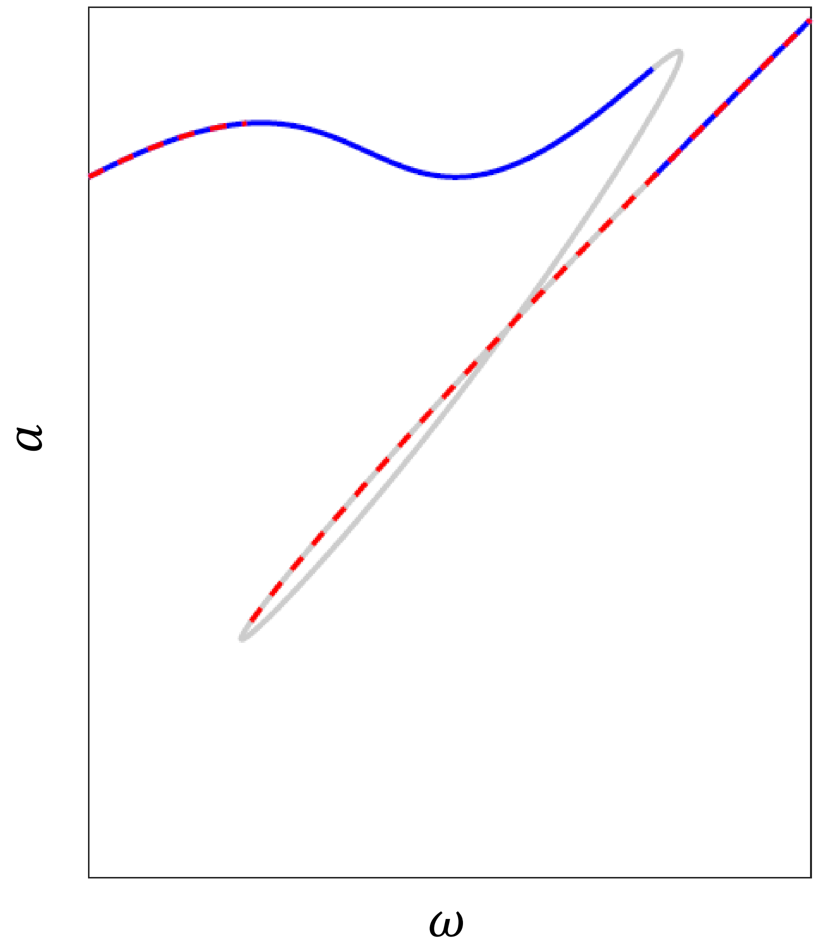
→ Modifies $x_f^*(t) = a \sin(\omega t)$





ACBC fails for secondary resonances





Unlike ACBC, x-ACBC uncovers secondary resonances

$$u(t) = u_f(t) + u_{nf,r}(t) + u_{nf,nr}(t) = p \sin(\omega t + \phi)$$

Control of the resonant harmonic component

$$u_{nf,r}(t) = 0$$

→ Enforces $x^*_{nf,r}(t) = x_{nf,r}(t)$ and reaches the desired branch

Arclength continuation

$$u_f(t) = p \sin(\omega t + \phi)$$

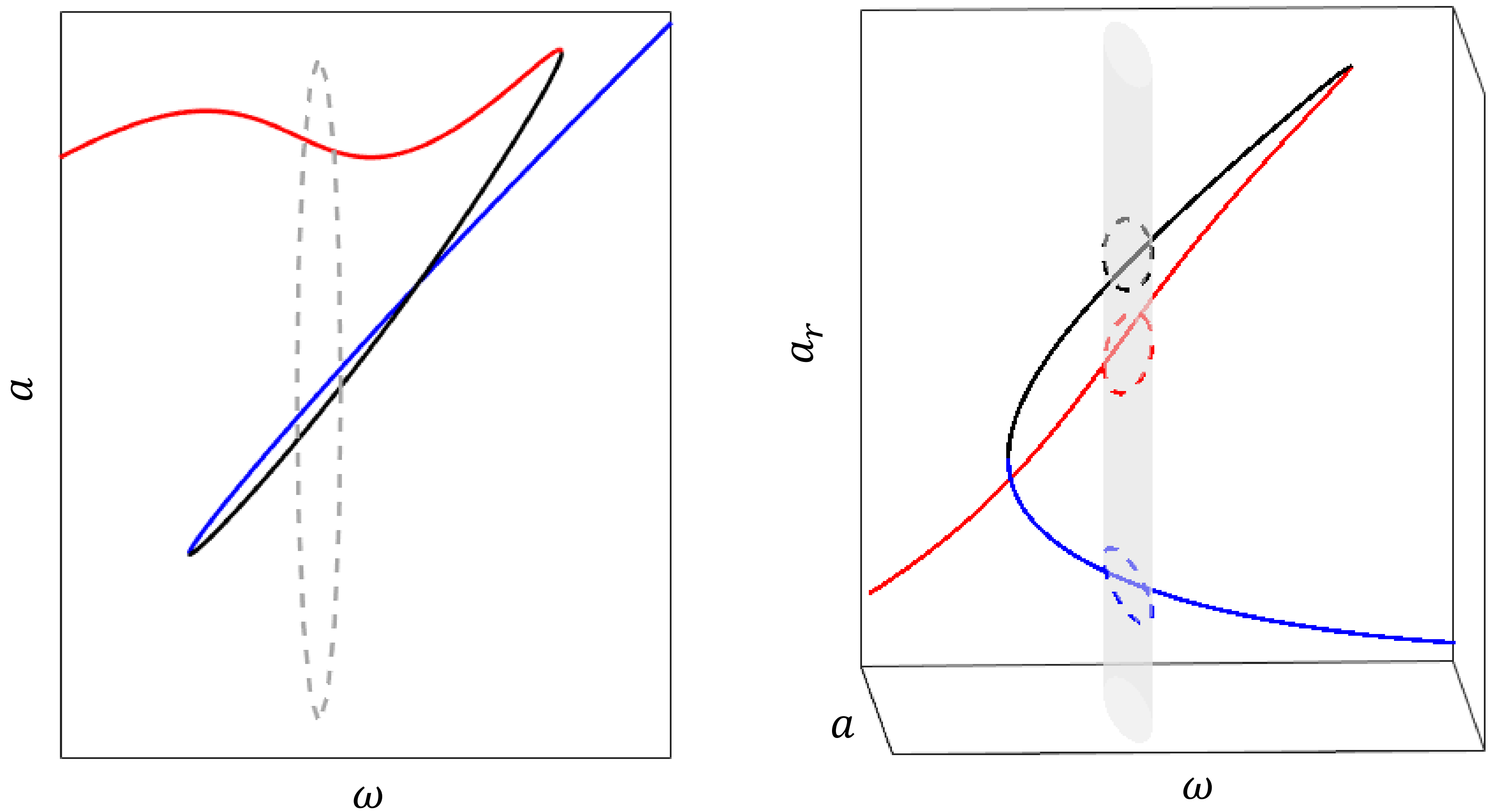
→ Modifies $x^*_f(t)$ until $p = p^*$

Adaptive filters

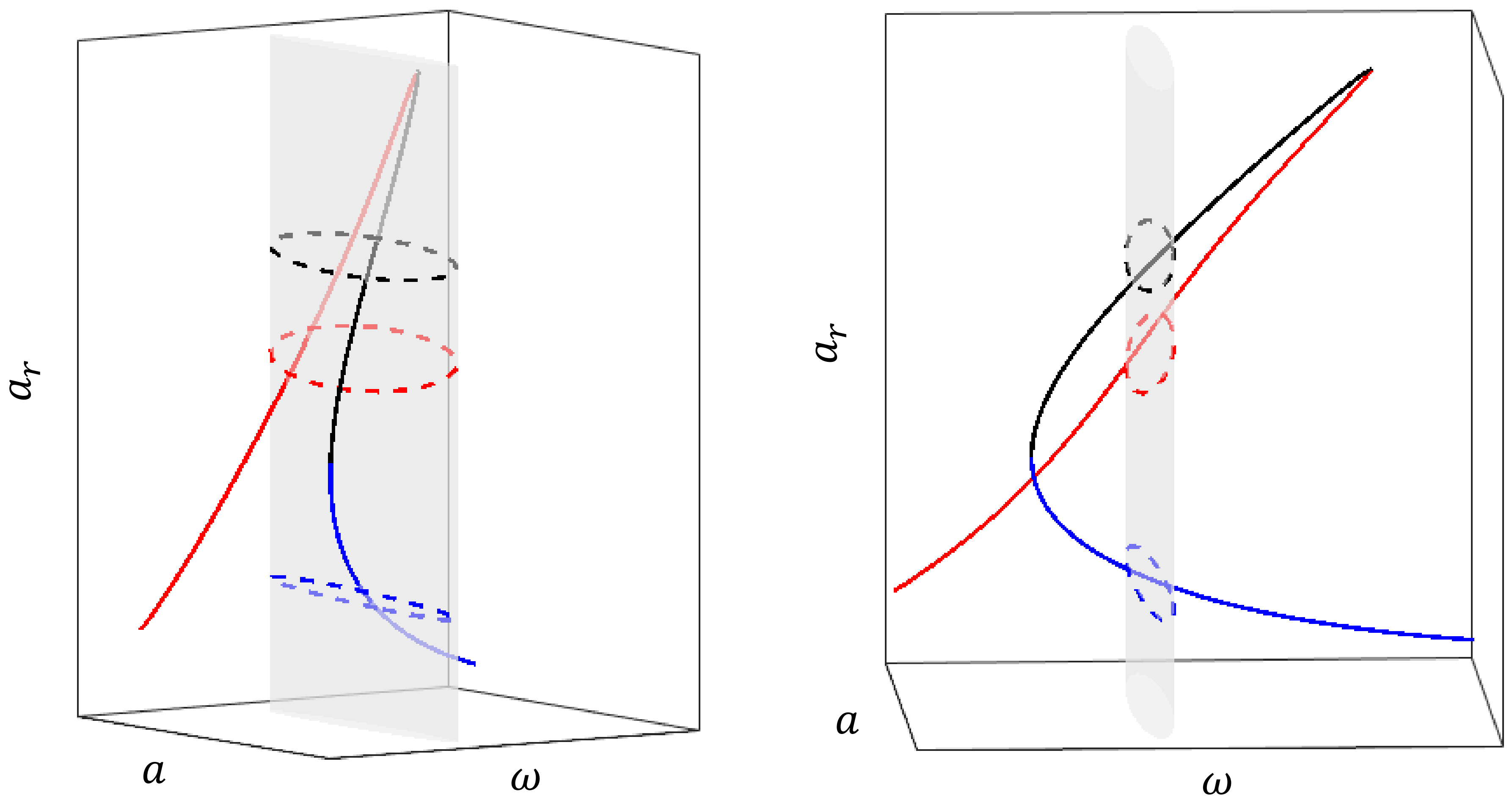
$$u_{nf,nr}(t) = 0$$

→ Enforce $x^*_{nf,nr}(t) = x_{nf,nr}(t)$

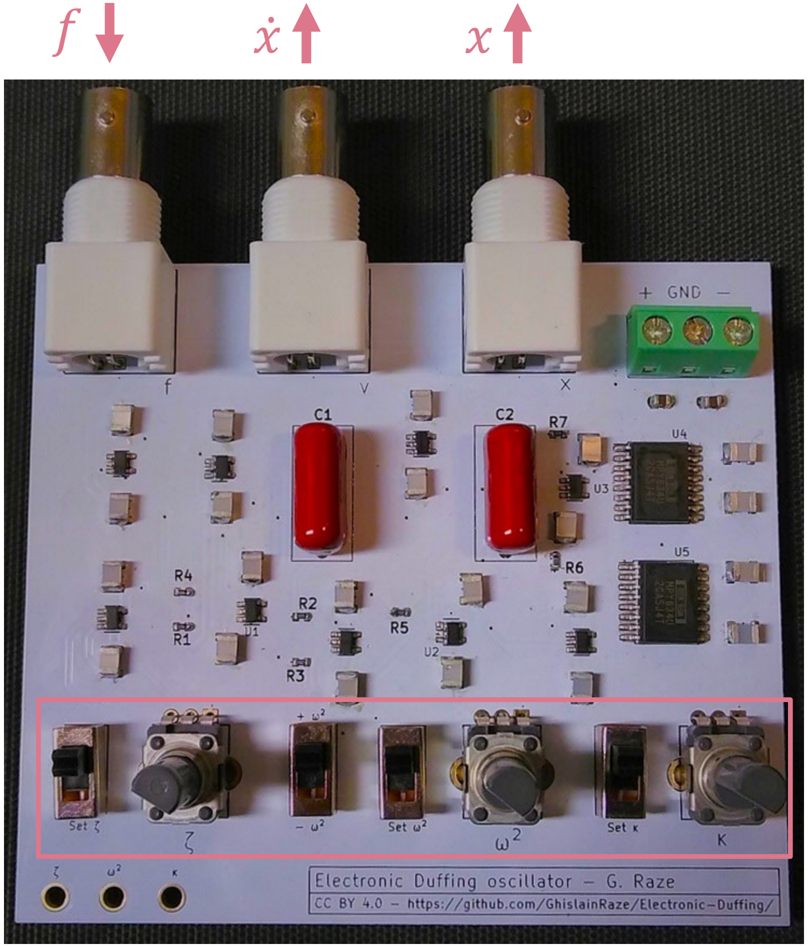
Influence of the resonant non-fundamental harmonic



Influence of the resonant non-fundamental harmonic



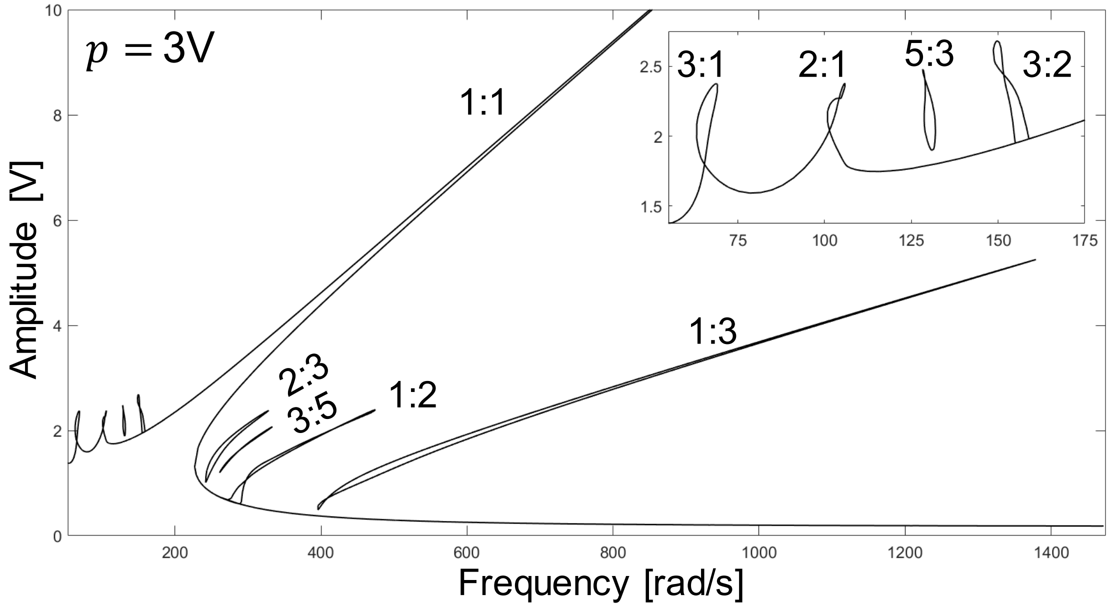
Experimental validation on an electronic Duffing oscillator



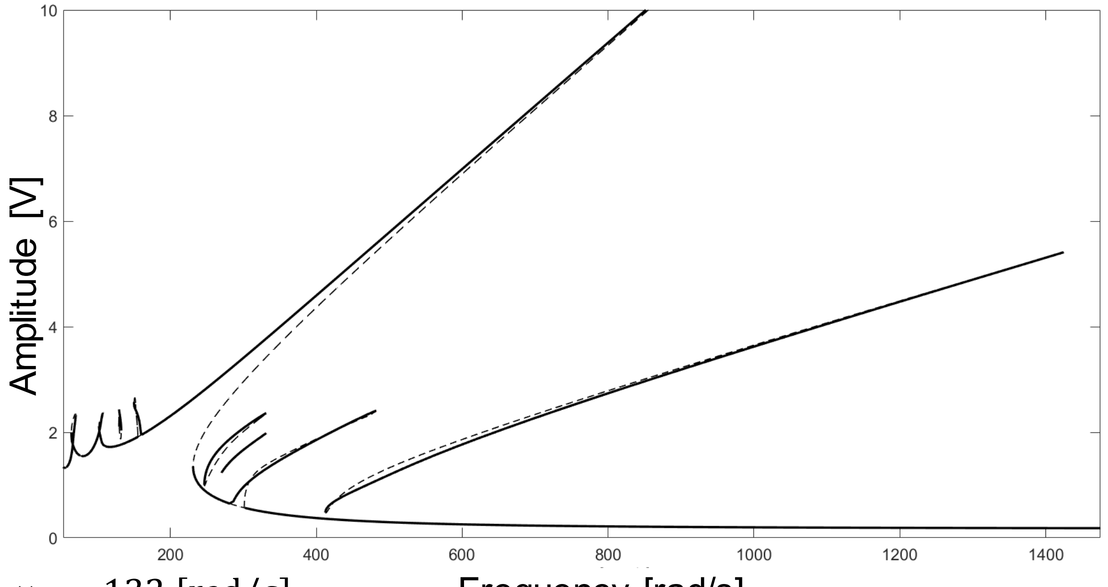
$$10^{-4}\,\ddot{x} + 1.49\,\times 10^{-4}\,\dot{x} + 1.75\,x + 0.99\,x^3 = p\sin\omega t = f(t)$$

Raze, G.: An electronic Duffing oscillator (2024). https://github.com/GhislainRaze/Electronic-Duffing

ACBC fully identifies the complex resonant behavior



Experimental
(x-ACBC)

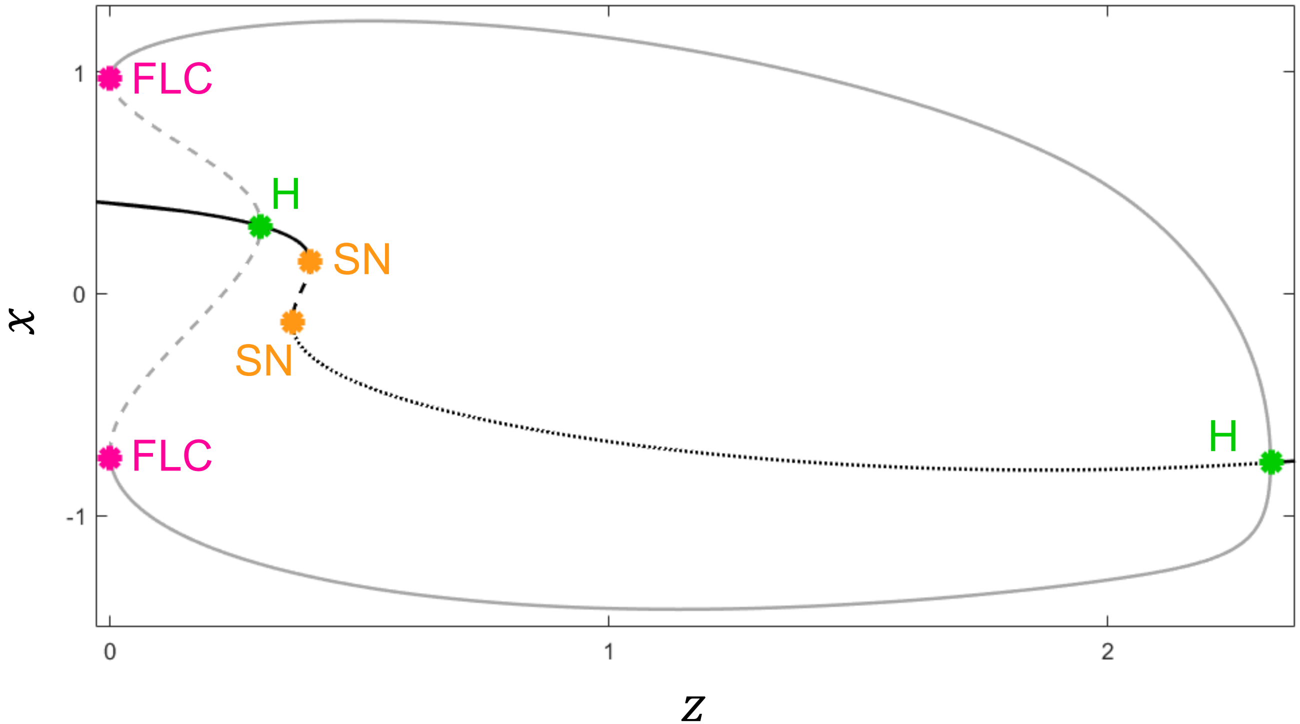


Numerical
(HB + pseudo-arclength continuation)

$\omega_0 = 132$ [rad/s]

Conclusions

- A first: x-ACBC can identify all resonances of a Duffing oscillator including its subharmonic and ultrasubharmonic resonances.
- ACBC can also be applied to autonomous systems to uncover fixed points and limit cycles.



M. Saggio, V. Jirsa, et al., eLife, 2020

Thank you for your attention!

Stay tuned, an article is on the way!

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