UNIVERSITY OF LIEGE
Faculty of Sciences
Department of Geography
Geomatics Unit

PRECISION OF SATELLITE POSITIONING

IMPACT OF SATELLITE GEOMETRY

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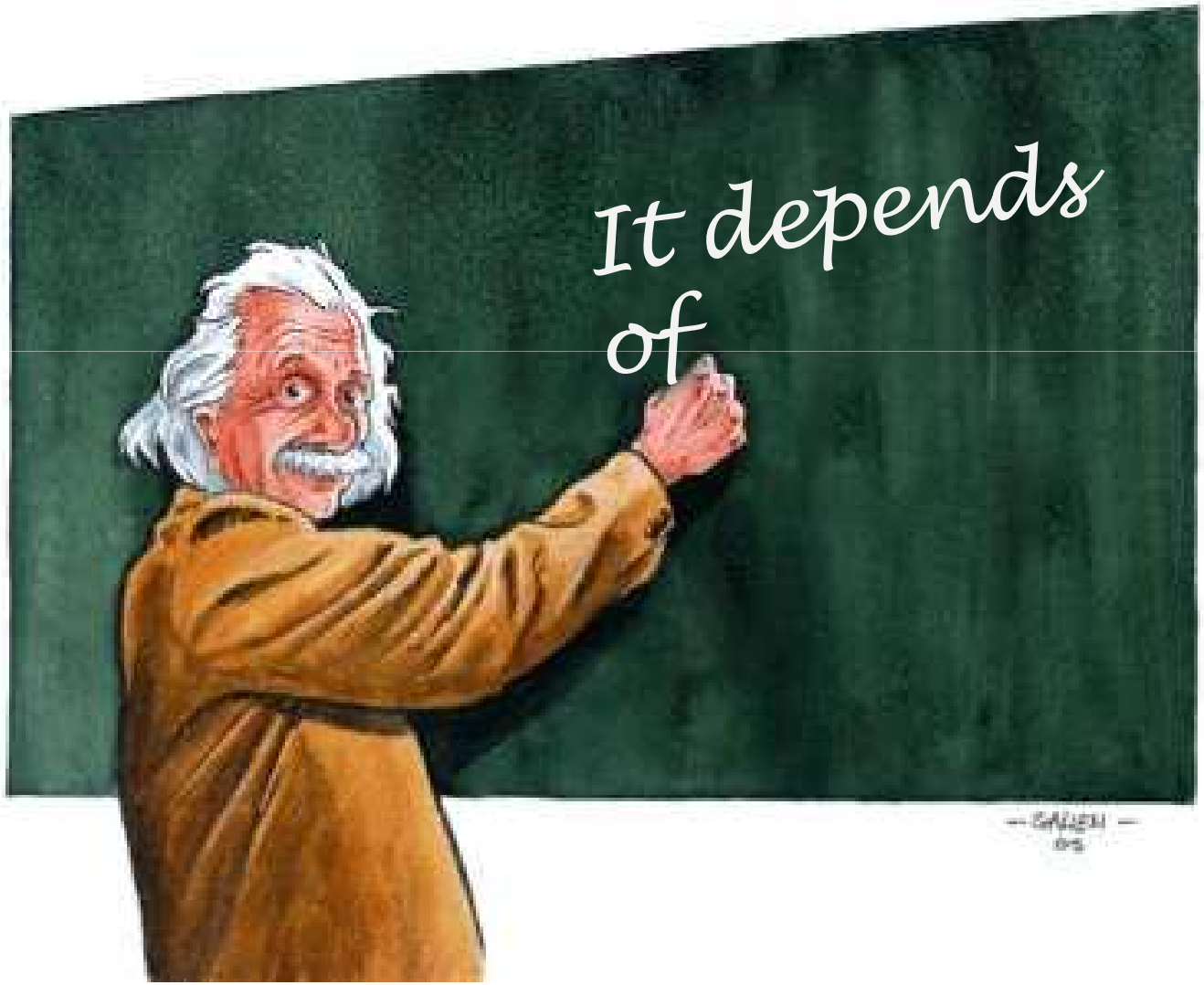
ODISSEA 2009 - December 9th,2009

How precise is satellite positioning?



1. Introduction

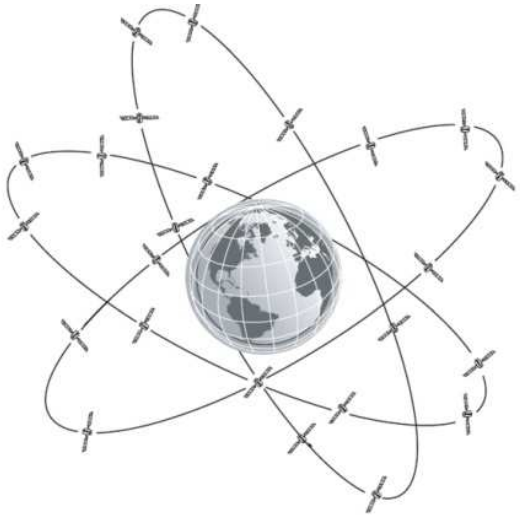
2. Absolute positioning

3. Relative positioning

4. Conclusions and perspectives

Why to focus on satellite geometry?

- GNSS: Global Navigation Satellite System
- Performance levels still rising:
  - ✓ *Precision*
  - ✓ *Spatio-temporal coverage*
- Error sources:
  - ✓ *Ionosphere*
  - ✓ *Troposphere*
  - ✓ *…*
- Amplifying effect of satellite geometry
- Quality indicator of satellite geometry: DOP (*Dilution Of Precision*)



➡ **Objective:** Better understanding of the impact of satellite geometry on the precision of positioning:

- Absolute Positioning (AP) with code measurements (SPSE)
- Relative Positioning (RP) with phase measurements (SE)

1. Introduction

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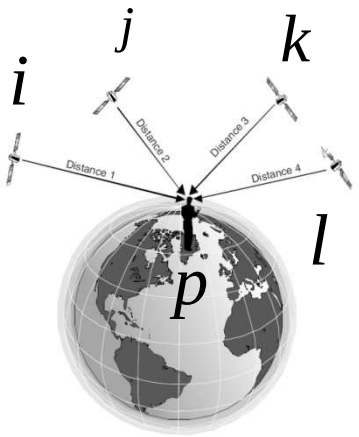
How to calculate a position by means of GNSS?

$$D_p^i = \sqrt{(X^i - X_p)^2 + (Y^i - Y_p)^2 + (Z^i - Z_p)^2}$$

$$P_p^i(t) = D_p^i + T_p^i + I_p^i + M_{p,m}^i + c(\Delta t^i(t_e) - \Delta t_p(t)) + \varepsilon_{p,m}^i$$

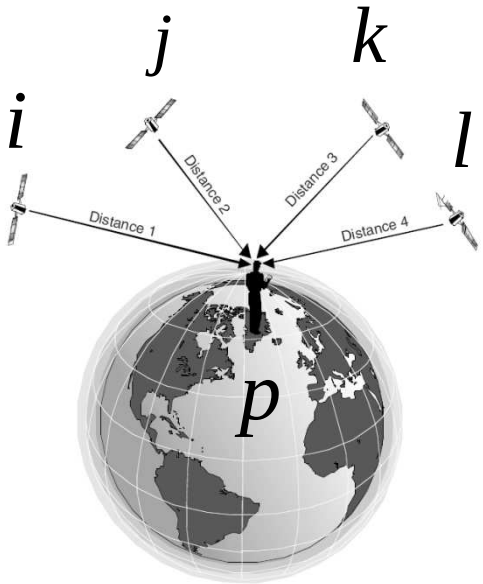


- Tropospheric delay
- Ionospheric delay
- Multipath delay
- Satellite clock error
- Receiver clock error
- Noise

- **The solution is the estimated by means of a least squares adjustment.**

$$P_p^i(t) = f(X_p, Y_p, Z_p, \Delta t_p)$$

- 4 unknown parameters (u = 4)
- Redundant observations (n ≥ 4)



Least squares adjustments

$$A\underline{x} + \underline{W} - \underline{v} = 0$$

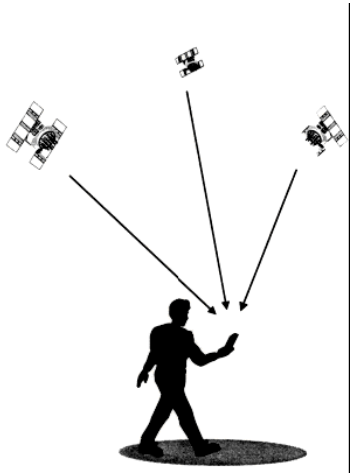
How does a least squares adjustment work?

$$\underline{x} = N^{-1} A^T W$$

- **Design matrix:**

$$A = \underbrace{\begin{bmatrix} -\frac{(X^1 - X_{p,0})}{D^1_{p,0}} & -\frac{(Y^1 - Y_{p,0})}{D^1_{p,0}} & -\frac{(Z^1 - Z_{p,0})}{D^1_{p,0}} & 1 \\ -\frac{(X^2 - X_{p,0})}{D^2_{p,0}} & -\frac{(Y^2 - Y_{p,0})}{D^2_{p,0}} & -\frac{(Z^2 - Z_{p,0})}{D^2_{p,0}} & 1 \\ \dots & \dots & \dots & \dots \\ -\frac{(X^n - X_{p,0})}{D^n_{p,0}} & -\frac{(Y^n - Y_{p,0})}{D^n_{p,0}} & -\frac{(Z^n - Z_{p,0})}{D^n_{p,0}} & 1 \end{bmatrix}}_{u} \Bigg\} n$$

- **Normal equation matrix:** $N = A^T A$

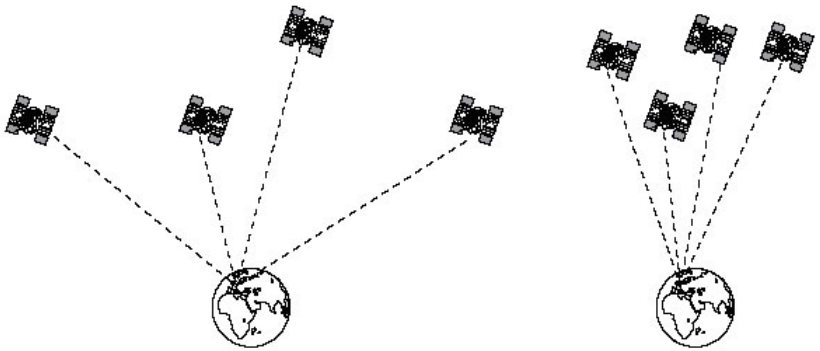


- **The satellite geometry amplifies the impact of errors from observations to adjusted parameters.**

- **Cofactor matrix:** $$Q_{\underline{\hat{x}}} = N^{-1} = \begin{bmatrix} q_{XX} & q_{XY} & q_{XZ} & q_{Xt} \\ q_{YX} & q_{YY} & q_{YZ} & q_{Yt} \\ q_{ZX} & q_{ZY} & q_{ZZ} & q_{Zt} \\ q_{tX} & q_{tY} & q_{tZ} & q_{tt} \end{bmatrix}$$

$$\sigma_{POS} = DOP\ \sigma_P$$

Dilution Of Precision

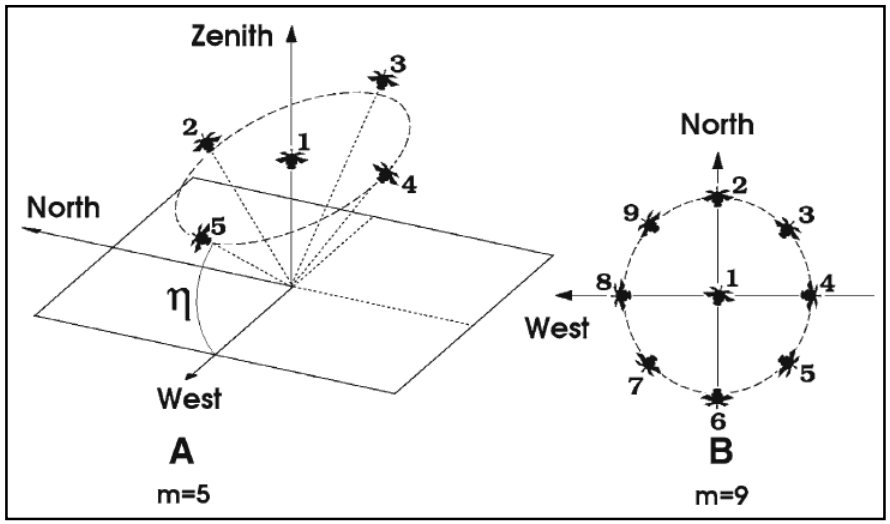


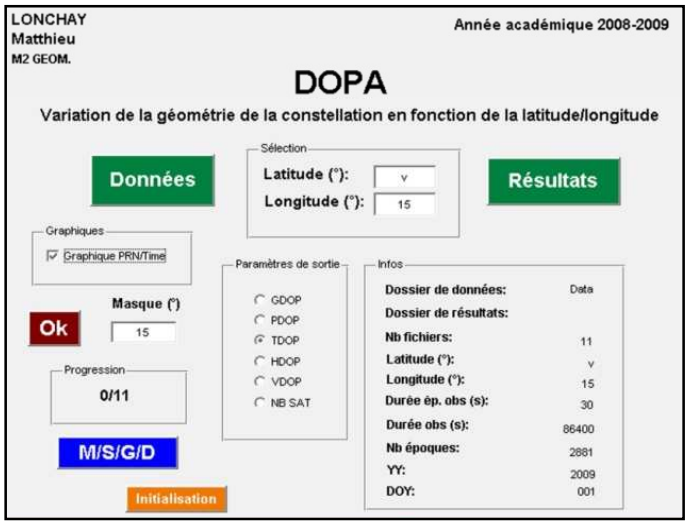
Bad DOP value

Good DOP value

From symmetric to real satellite distribution…

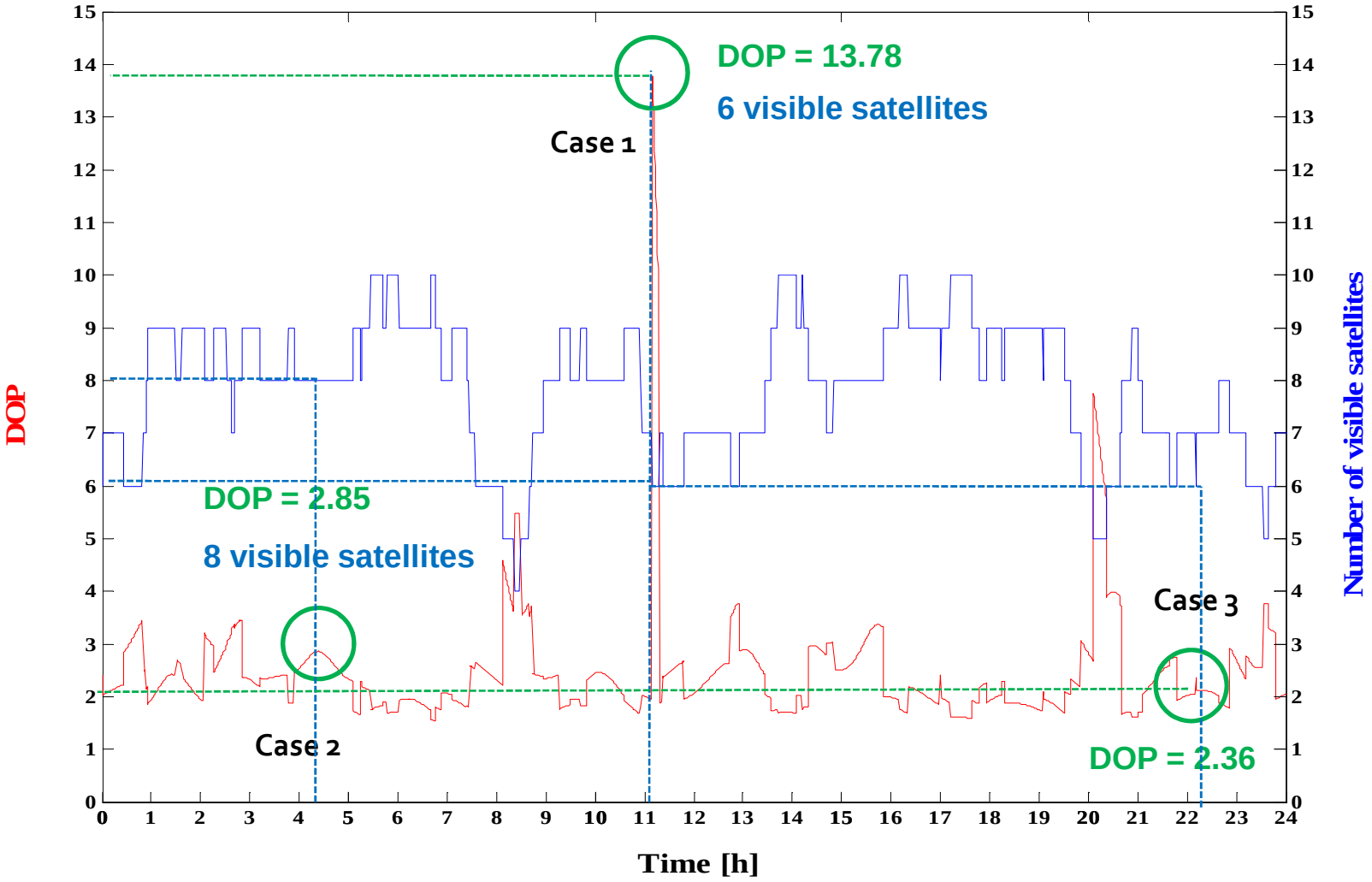
- GPS satellites
- Highly symmetric satellite distribution ➟ analytical form of DOP





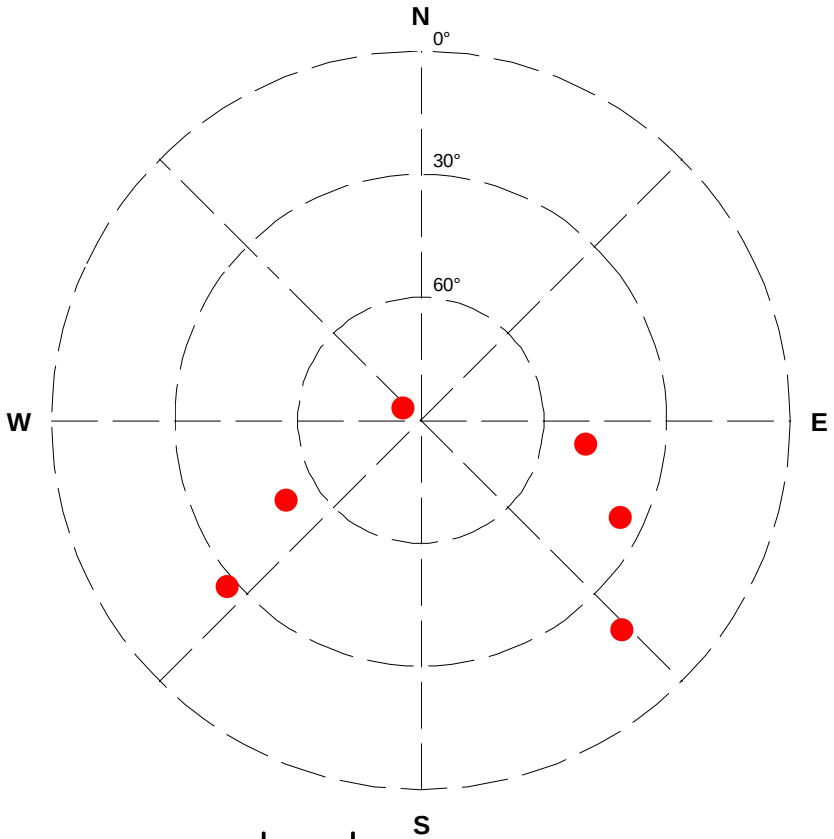
- Real GPS constellation ➟ spatio-temporal variability of DOP

The satellite sky distribution influences the DOP.



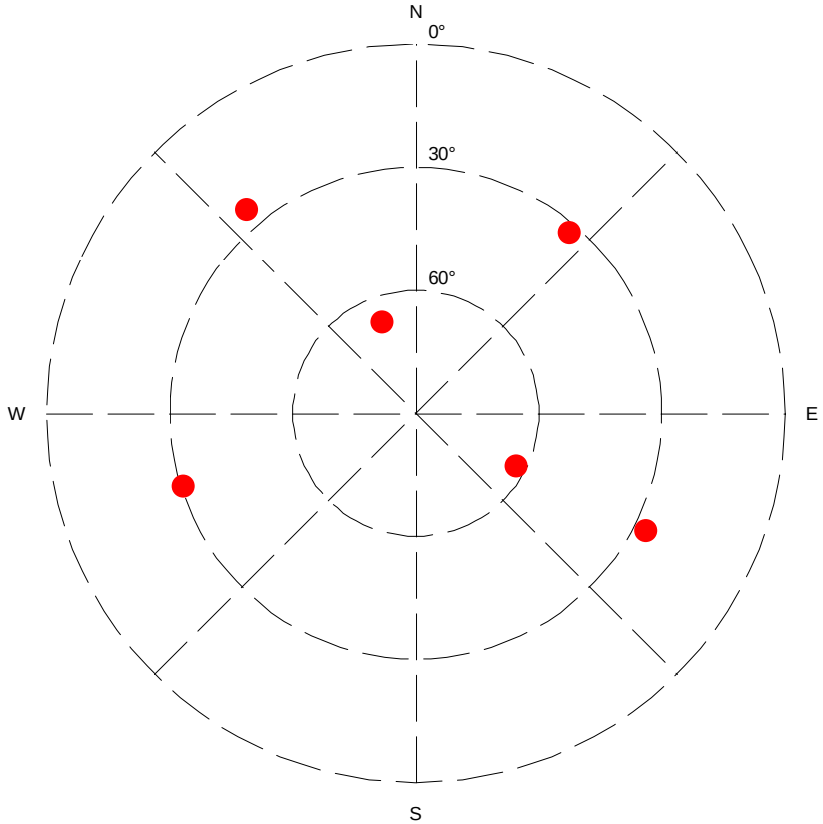
- **The satellite sky distribution influences the DOP.**

Case 1



$$|N| = 0.04$$

Case 3



$$|N| = 3.47$$

- **High DOP values are related to singular states of N matrix.**

$$|N| = 0$$

$$Q_{\underline{\hat{x}}} = N^{-1} = (A^T A)^{-1}$$

$$\longrightarrow DOP$$

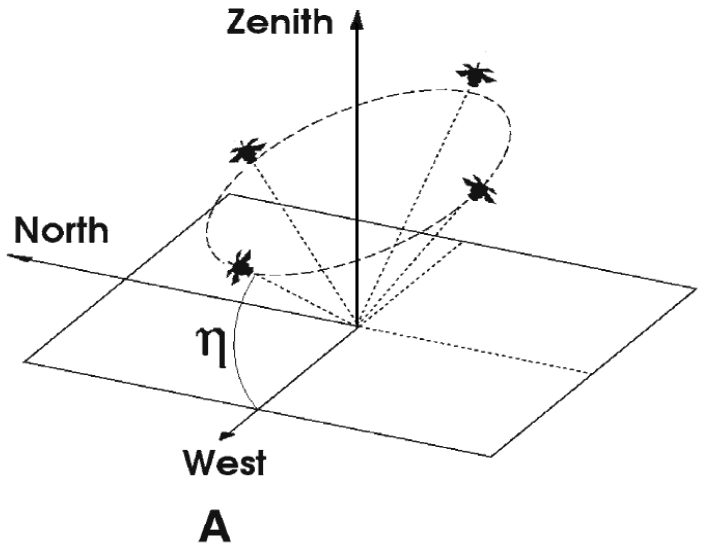
N is singular/invertible

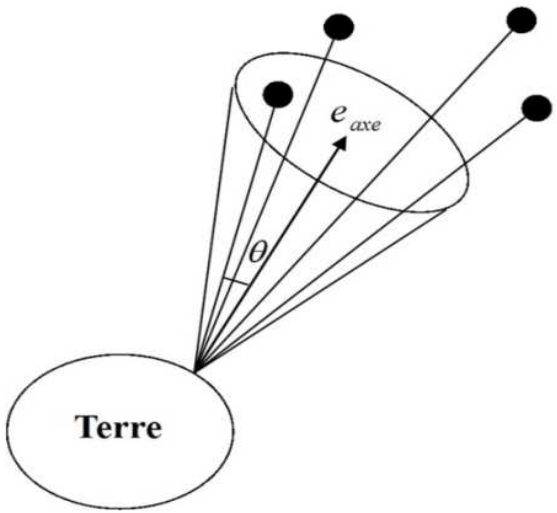
Linear relation between rows of N matrix ⟷ **Linear relation between rows of design matrix A**

A conical satellite sky distribution involves a singular state of the N matrix.

- Topocentric form of the design matrix:

$$A = \begin{bmatrix} -\cos\eta^1 \sin\chi^1 & -\cos\eta^1 \cos\chi^1 & -\sin\eta^1 & 1 \\ -\cos\eta^2 \sin\chi^2 & -\cos\eta^2 \cos\chi^2 & -\sin\eta^2 & 1 \\ \dots & \dots & \dots & \dots \\ -\cos\eta^n \sin\chi^n & -\cos\eta^n \cos\chi^n & -\sin\eta^n & 1 \end{bmatrix}$$

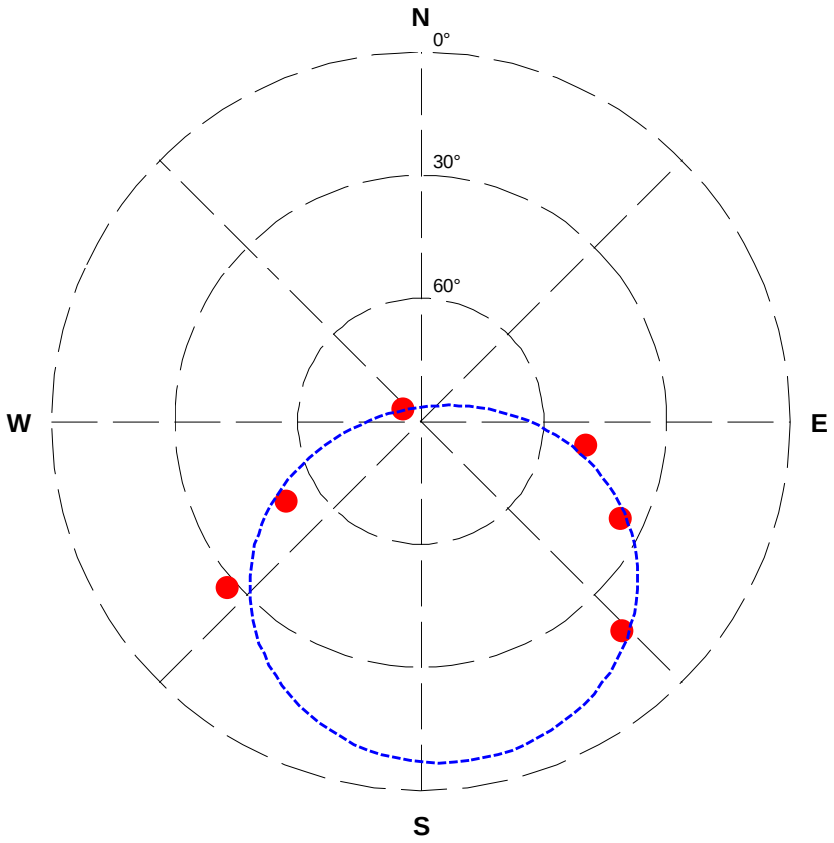




➡ Least squares adjustment of 3D cone on real satellite sky distributions.

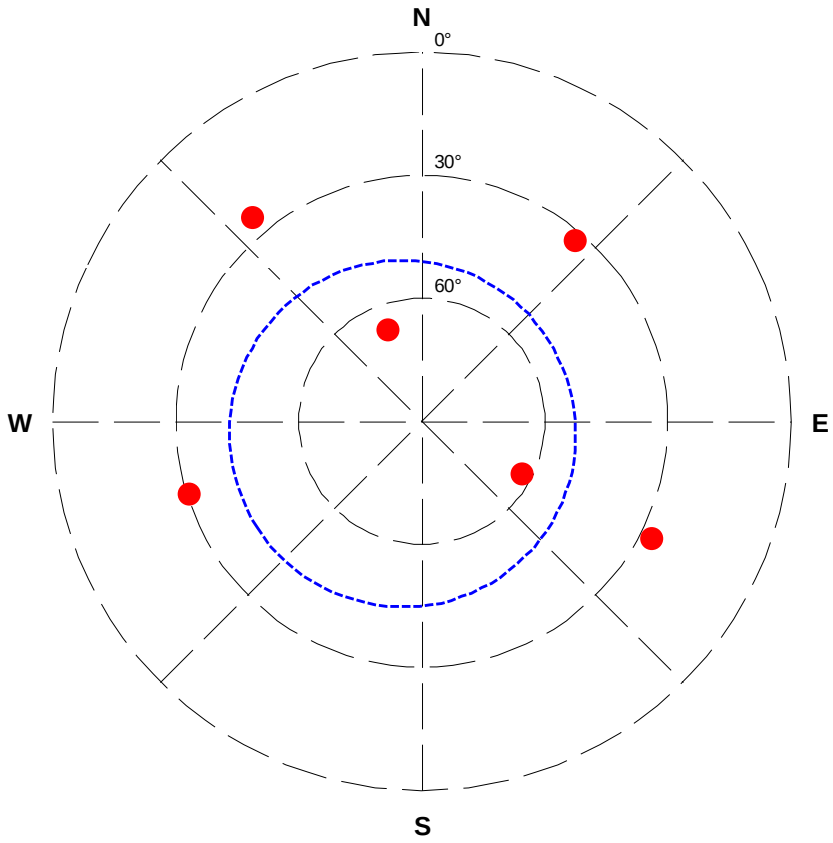
- **A conical satellite sky distribution involves high DOP values.**

Case 1



DOP = 13.78

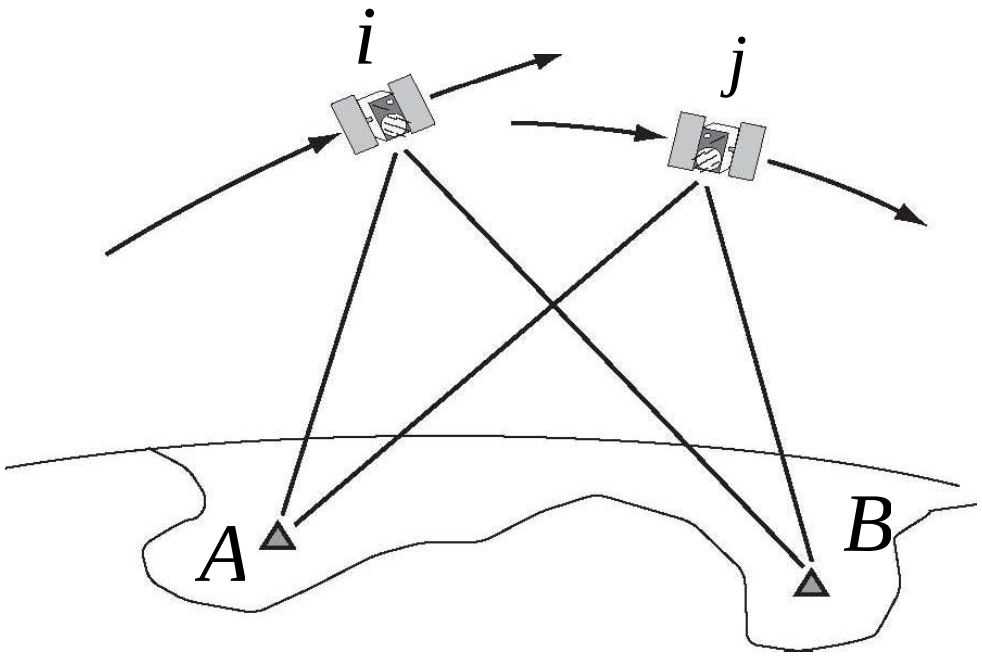
Case 3



DOP = 2.36

What are the general principles of relative positioning?

$$P_{AB}^{ij}(t) = P_{AB}^{i} - P_{AB}^{j} = D_{AB}^{ij} + T_{AB}^{ij} + I_{AB}^{ij} + M_{AB,m}^{ij} + \varepsilon_{AB}^{ij}$$



$$P_{AB}^{ij}(t) = f(X_B, Y_B, Z_B)$$

- 3 unknown parameters (u = 3)
- Redundant observations (n ≥ 3)

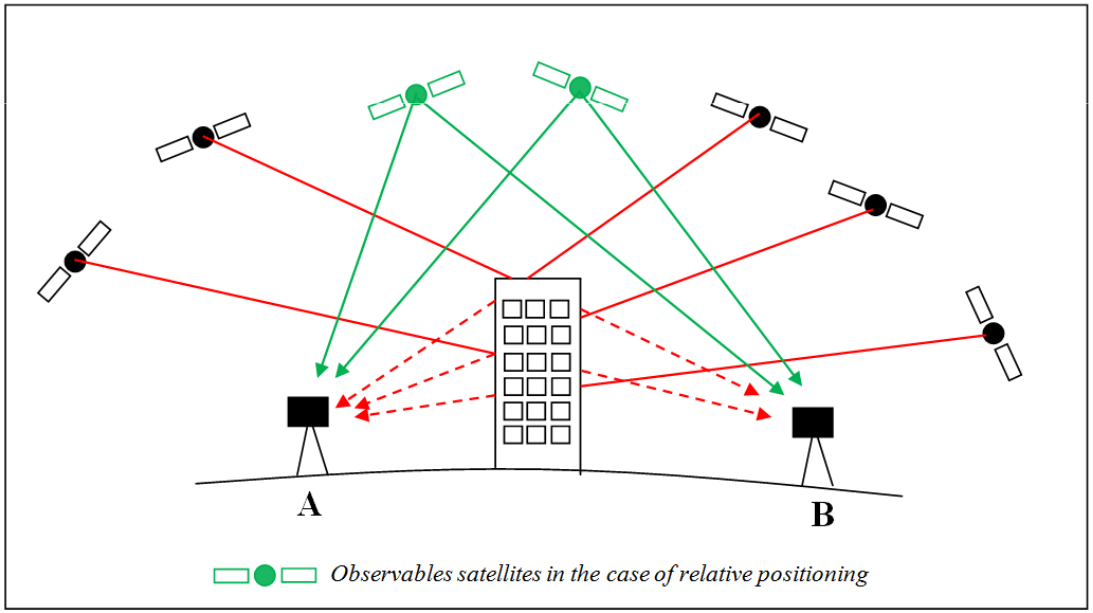
→ **Least squares adjustment**

$$Q_{\underline{\hat{x}}} = N^{-1} = (A^T A)^{-1}$$

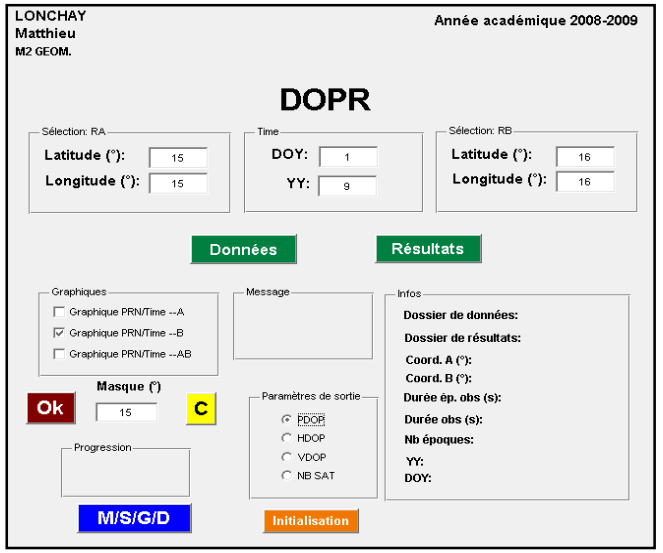
$$\rightarrow RDOP$$

From Absolute DOP value to Relative DOP value…

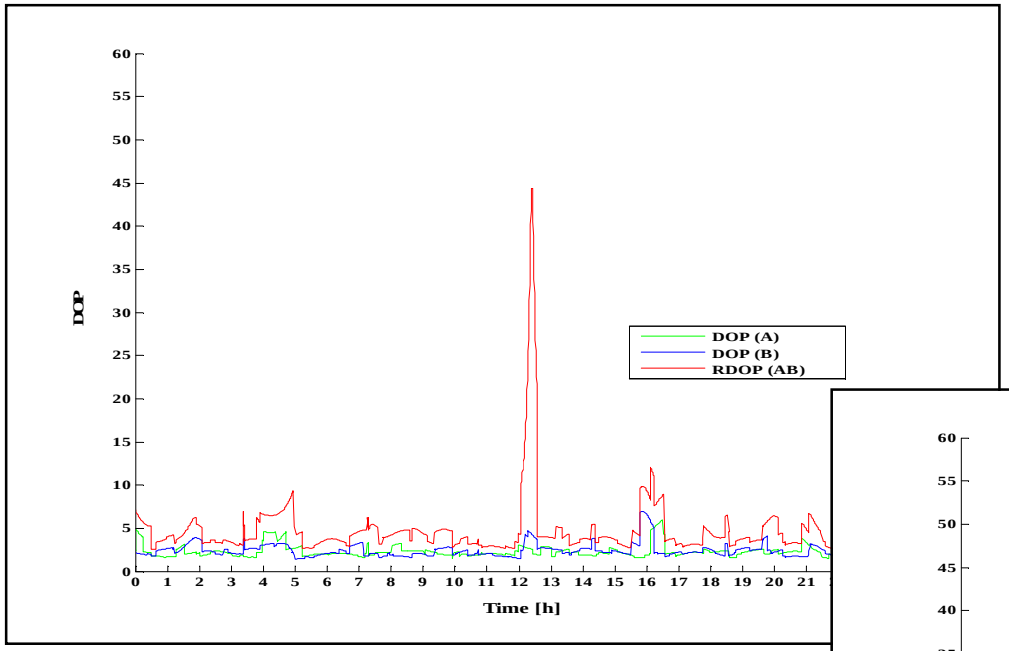
- GPS satellites
- Is there a correlation between DOP and RDOP values?



Observables satellites in the case of relative positioning

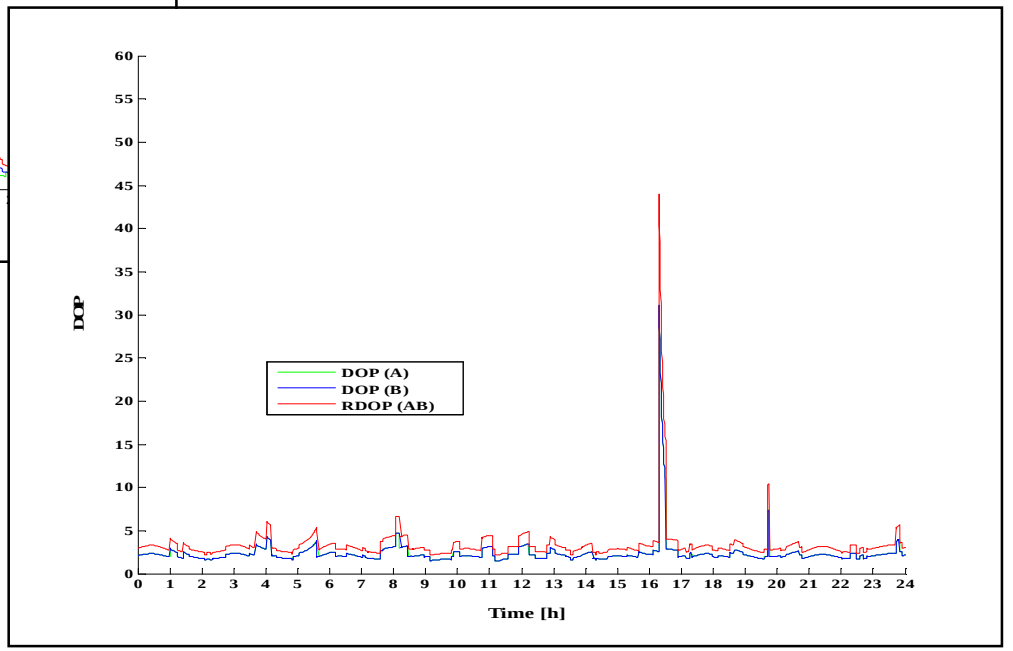


From Absolute DOP value to Relative DOP value…

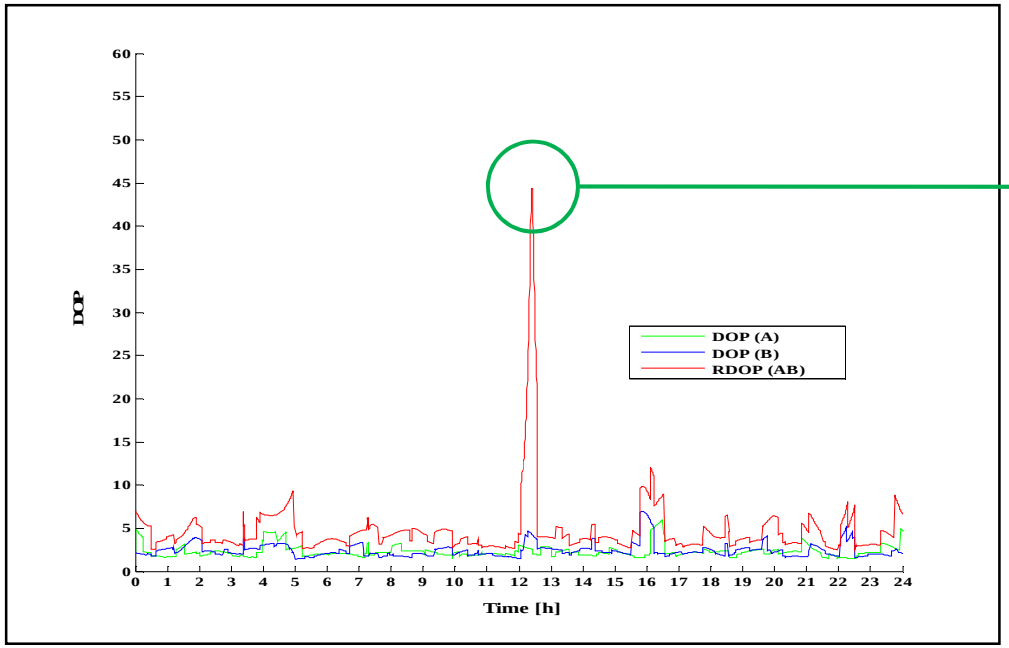


Very long baseline (≈ 1600 km)

Short baseline (≈ 20 km)

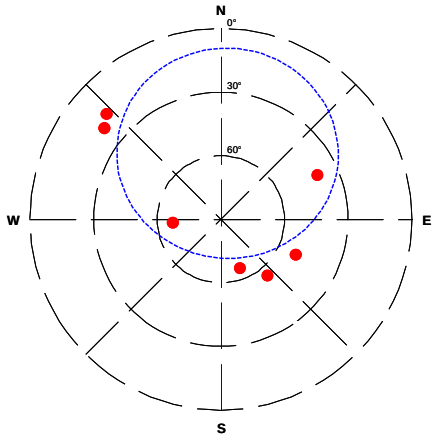


Correlation between DOP and RDOP evolves according to the baseline length.

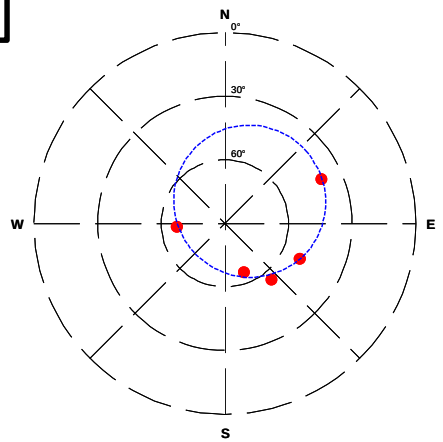


Very long baseline (≈ 1600 km)

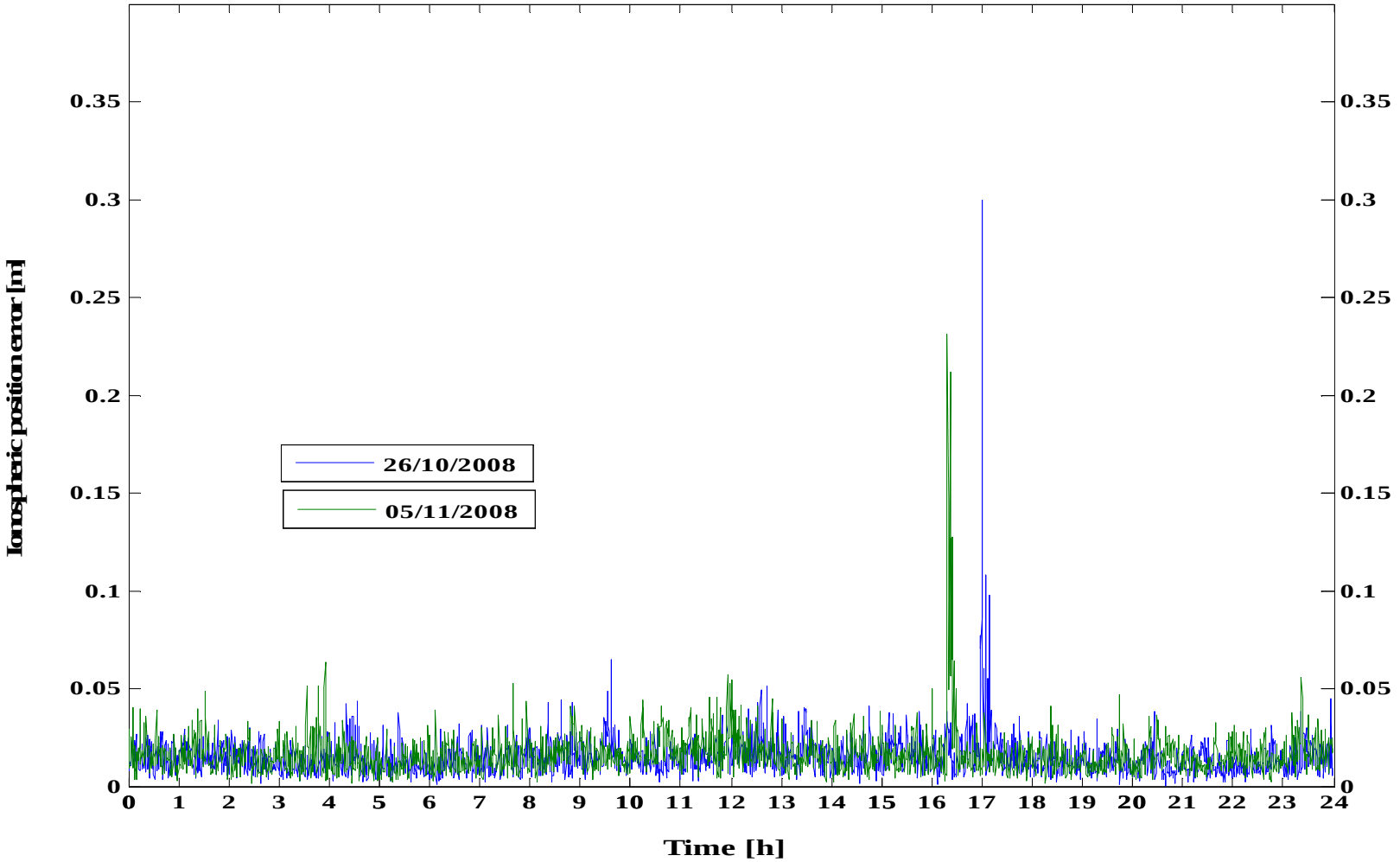
B



AB

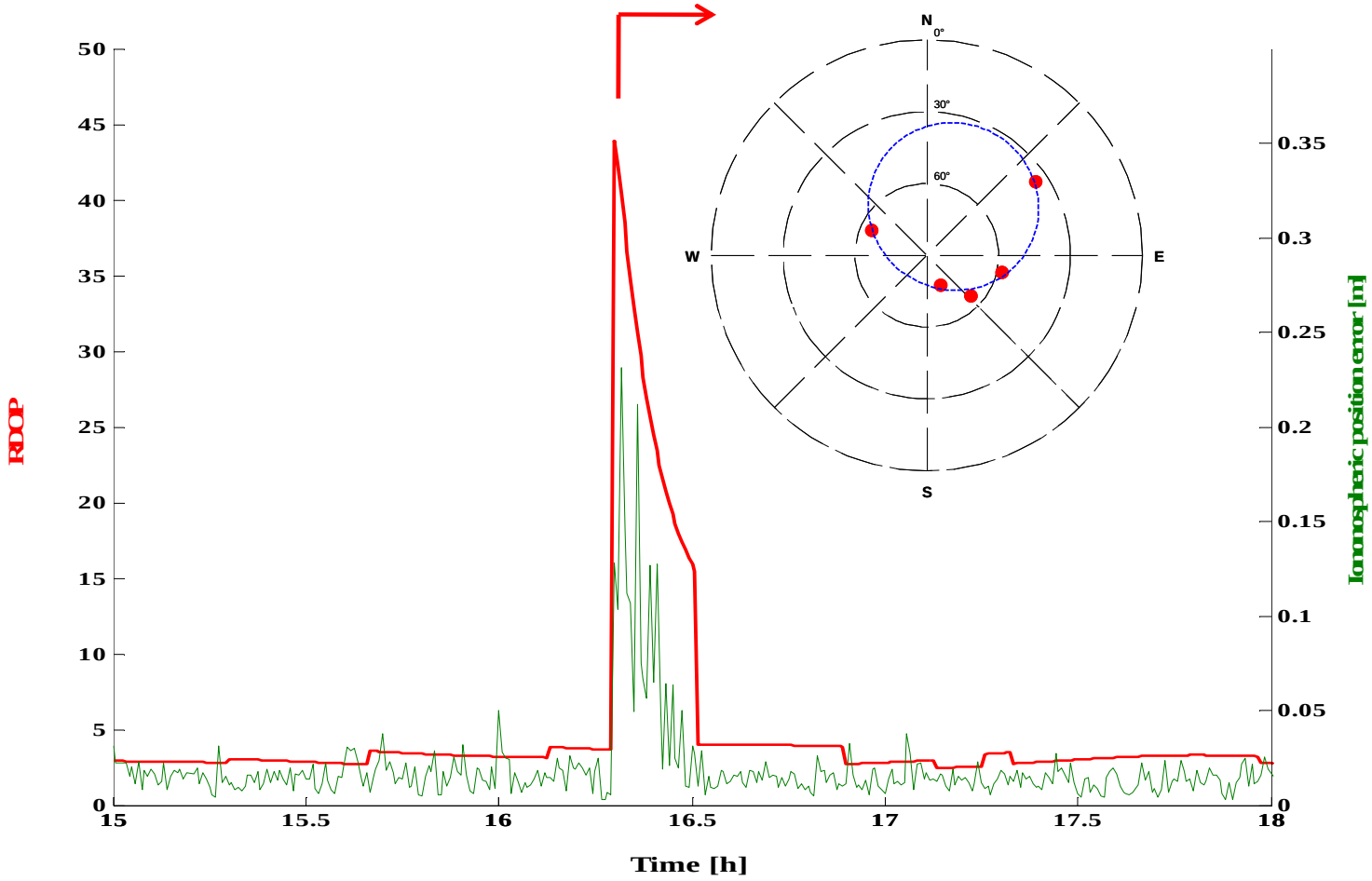


SoDIPE-RTK computes the ionospheric position error.



$(Station\,A\,(GILL) \rightarrow Station\,B\,(LEEU), M15°, 26/10/2008 - 05/11/2008)$

- **Satellite geometry is the origin of high position errors in SoDIPE-RTK results.**



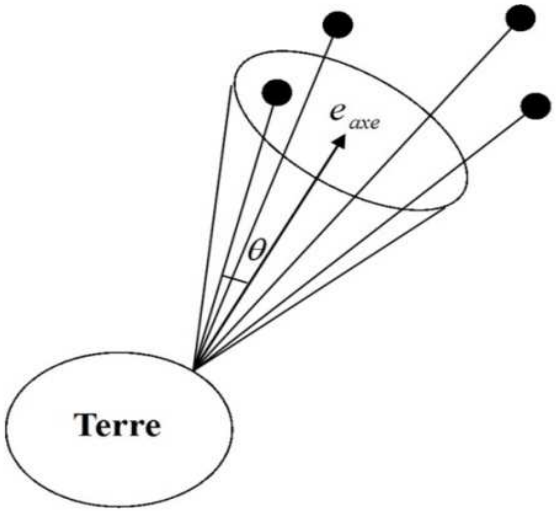
$(Station\,A\,(GILL) \rightarrow Station\,B\,(LEEU), M15°, 26/10/2008-05/11/2008)$

Absolute Positioning

- GPS constellation offers a very good coverage in time and space.
- Identification of factors influencing the quality of the satellite geometry.
- Identification of critical geometry leading the N matrix to a singular state and DOP to high values: the conical satellite sky distribution.

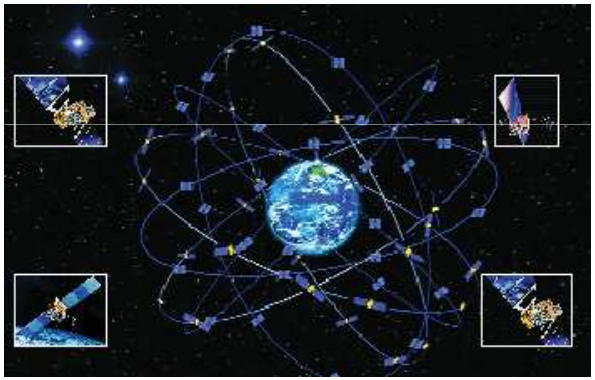
Relative Positioning

- Correlation between DOP/RDOP
- SoDIPE-RTK



Perspectives

- Actuality: "Is GPS satellite failure imminent?"
- GPS, Galileo, GLONASS, SBAS: interoperability between GNSS



- Identification of all satellite geometries leading N matrix to a singular state.
- Study of the link between DOP values and volume of polyedra built on unit vector pointing from receiver to satellites.

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