Generation of anticoherent spin states

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Motivations

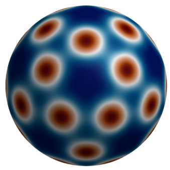
Anticoherent spin states are optimal for rotation and magnetic field sensing[1] and draw increasing attention due to their non-classical properties[2]. However, until now, no general protocols existed to generate these states experimentally.

Anticoherent spin states

An anticoherent state to order t is an isotropic state for which the moments of $\mathbf{J}\cdot\mathbf{n}$ up to order t are independent of the direction $\mathbf{n}$, i.e.

$$\mathrm{Tr}[\rho(\mathbf{J}\cdot\mathbf{n})^k] \neq f(\mathbf{n}) \quad \text{for } k = 1, 2, \ldots, t$$

We can quantify the degree of anticoherence of any spin state $|\psi\rangle$ through the measure of anticoherence[3]



5-anticoherent state

$$\mathcal{A}_t^{\mathrm{Bures}} = 1 - \sqrt{\frac{\sqrt{t+1} - \sum_{i=1}^{t+1}\sqrt{\lambda_i}}{\sqrt{t+1}-1}}$$

where the λ_i are the Schmidt coefficients of $|\psi\rangle$. This measure can serve as a figure of merit for numerical optimisation.

Any spin state can be expanded in the multipolar basis associated to irreducible representation of SU(2)

$$\rho = \sum_{L=0}^{2j}\sum_{M=-L}^{M}\rho_{LM}T_{LM}.$$

In this manner, a state is anticoherent of order t iff

$$\rho_{LM} = 0 \text{ for } L = 1, 2, \ldots, t \text{ and } M = -L, \ldots, L$$

Hence, in order to generate anticoherent states, the goal is to completely depopulate the lower state multipoles ρ_{LM}.

Pulse-based protocol

Our protocol consists of a sequence of n_C cycles in which we apply first a rotation followed by a squeezing[4]

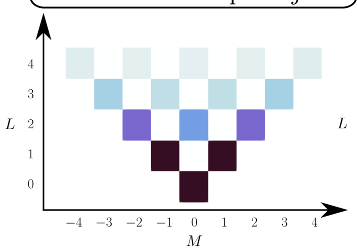
$$\Pi_y(\theta) = e^{-iJ_y\theta} \qquad \Pi_{z2}(\eta) = e^{-iJ_z^2\eta}$$

such that the final state is

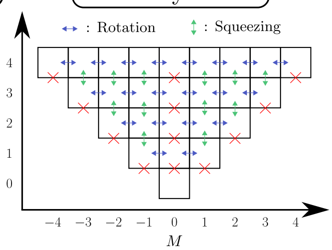
$$|\psi_{n_C}\rangle = \left(\prod_{i=1}^{n_C}\Pi_{z2}(\eta_i)\Pi_y(\theta_i)\right)|\psi_0\rangle$$

where $\theta_1 = 0$ and the initial state is $|\psi_0\rangle = e^{-iJ_x\pi/2}|0\rangle$ where $|0\rangle$ is the coherent state along the z axis.

Initial state multipoles $j = 2$



Pulses dynamics



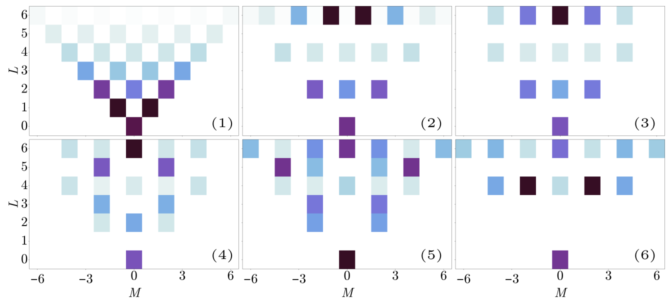
From a physical point of view, our protocol exploits the fact that the rotations (resp. squeezings) only couple multipoles with the same L (resp. M). Moreover, during a squeezing along z, the transitions for $M = 0$ are forbidden.

For different spin numbers j and anticoherence order t, we use the Nelder-Mead algorithm to find the $2n_C$ parameters θ_i and η_i that maximise $\mathcal{A}_t^{\mathrm{Bures}}$.

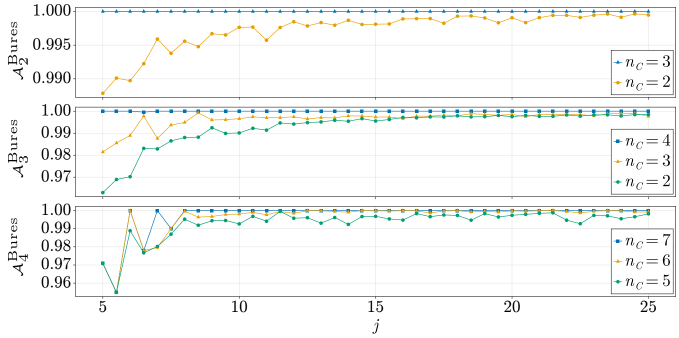
Numerical results

For all spin $j \leq 24$, the optimised parameters are able to generate the highest order anticoherent state allowed.

State multipoles under a sequence of pulses $j, t = 3$



Achieved anticoherence for a fixed number of cycles n_C

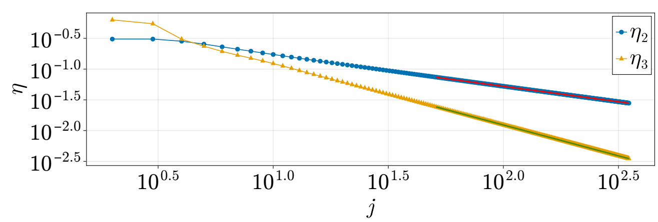


For each anticoherence order ($t = 2$, 3 and 4), there is a threshold number of cycles above which the state is perfectly anticoherent (up to numerical errors).

For $j = 24$, we obtain a 9-anticoherent state, the maximal order for this j, verifying $\mathcal{A}_9^{\mathrm{Bures}} > 0.99$ with $n_C = 14$.

High-dimensional systems

We observe that, for integer j, in order to generate an anticoherent state of order 2, we can take $\eta_1 = \frac{\pi}{2}$, $\theta_2 = -\frac{\pi}{4j}$ and $\theta_3 = \frac{\pi}{2}$ and only optimise over η_2 and η_3.



It allows us to obtain the scaling laws $\eta_2 = \frac{3}{4\sqrt{2j}}$ and $\eta_3 = \frac{5}{4j}$ that we can exploit to generate anticoherent states with $j = 5000$ verifying $1 - \mathcal{A}_2^{\mathrm{Bures}} < 10^{-3}$.

Perspectives

- Test the protocol in experiments with spin or Bose-Einstein condensate systems.
- Use anticoherent states for improved gyroscopy or magnetometry applications in the lab.

References

1. J. Martin, S. Weigert and O. Giraud, Quantum **4**, 285 (2022)
2. M. Rudzinski, A. Burchardt and K. Zyczkowski, Quantum **8**, 1234 (2024)
3. D. Baguette and J. Martin, Phys. Rev. A **96**, 032304 (2017)
4. S. C. Carrasco *et al.*, Phys. Rev. Applied **17**, 064050 (2022)