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Discrete-time internal model control with disturbance and vibration rejection

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Abstract

This paper presents a novel design methodology for discrete-time internal model control (IMC) used to compute a disturbance filter. The proposed method employs a generalized algorithm for disturbance rejection and for process dynamics compensation. In IMC, the controller is designed based on a model of the process, while ensuring a desired closed loop performance trajectory (for setpoint tracking). However, in some situations, for example poorly damped systems, the open loop poles of the process affect the closed loop disturbance rejection dynamics. The novel design methodology presented is able to compensate both process dynamics and input disturbances. The method is validated both in simulations and in experimental tests on a poorly damped mass–spring–damper testbench.

Keywords

Internal model control, disturbance rejection, diophantine filter, dynamic compensation, vibration control, damping

1. Introduction

Feedback compensators have been recognized as being key elements to guarantee the desired performance of a process. Two main objectives are required: the ability to track a desired setpoint (the *servo* problem) and to efficiently reject disturbances (the *regulatory* problem) (Jiang, 2006; Skogestad and Postlethwaite, 2005). The challenge for the control engineer consists of developing a strategy to deal with these conflicting objectives, which assume some sort of trade-off between fast closed loop dynamics and robustness.

Internal model control (IMC) techniques are model-based control techniques, which are often used in chemical process control, but recently have been utilized in other applications (e.g. mechatronics). These techniques have the potential to achieve good closed loop performance, while taking into account the model structure of the process (e.g. varying time delays and periodic disturbances). Many successful implementations in real life processes have been reported, for example Rivera et al., (1986); Morari and Zafiriou, (1989); Bequette, (2002), to mention a few. It is however observed that, although basic IMC provides adequate suppression of output disturbances, it does a poor job in suppressing input disturbances when the process dynamics are significantly slower than the desired closed loop dynamics (Chien and Fruehauf, 1990; Ho et al., 1994). Consequently, later studies have focused on the search for new filters or/and alternative procedures to improve closed loop bandwidth and robustness, as presented in Campi et al. (1994). The conventional filter was modified to improve input disturbance attenuation on stable plants (Horn et al., 1996), and was subsequently extended for unstable plants (Lee et al., 2000). Some of the ideas of robust control were introduced in Dehghani et al. (2006) where a numerical design based on $H\infty$ ideas was used. Following the same trend, Alcantara et al., (2011a) proposed a simpler IMC-like $H\infty$, which requires less assumptions, thus overcoming some basic limitations of similar approaches. An improvement to the previous method was obtained in Alcantara et al., (2011b) where an analytical solution

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based on H_2 was proposed to achieve a compensator which balances the input/output disturbances rejection performance. One of the limitations of the previous strategy is the difficulty of finding the required weighting design parameters for the case of low-damping systems with more than one resonant frequency (i.e. higher than second order).

In this paper we investigate the potential of the IMC as an integrated solution for effective rejection of disturbances entering at the input of the process. Compared to previous studies, the proposed methodology does not require frequency weights to design the controller. The proposed design methodology has tuning parameters oriented to closed loop performance, making of it a versatile method. It uses concepts of generalized disturbance rejection by means of diophantine equations. Similar concepts are encountered in model-based predictive algorithms (Meadows and Badgwell, 1998; Maciejowski, 2002; Qin and Badgwell, 2002; Camacho and Bordons, 2004), where the authors are well-experienced in developing the extended prediction self-adaptive controller (EPSAC) (De Keyser, 2003). In the EPSAC control formulation, if the main frequency of the disturbance is known, it is possible to model it and to provide this additional information to the controller, obtaining significant improvement in closed loop performance and disturbance rejection capability (De Keyser and Ionescu, 2003).

The performance and efficiency of the proposed strategy is experimentally evaluated, using a mass–spring–damper system with poor damping properties. As such, this specific fourth-order system represents a challenge regarding the design of any regulator (Alcantara et al., 2011b).

The paper is organized as follows. The problem formulation is presented in Section 2. In Section 3 the IMC controller and the new extension of the IMC filter is presented. Next the mass–spring–damper system used as the testbench for the proposed methodology is described in Section 4. The design of the IMC with general disturbance rejection properties is analyzed in Section 5. The results of the IMC controller, along with some implementation aspects followed by the experimental outcomes are presented in Section 6. A conclusion section (7) summarizes the main outcome of this work.

2. Problem formulation

This section illustrates the fundamental difficulties that arise when designing a compensator in order to efficiently reject input disturbances. Although there are other cases in which input disturbance rejection is important, two cases are illustrated here: (1) that of "close to integrating" systems; and (2) that of poorly damped systems.

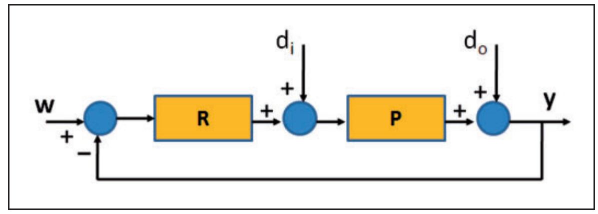

Figure 1. Basic closed loop scheme.

The closed loop control scheme is depicted in Figure 1, where P represents the process, R the compensator, w the reference, d_i the input disturbance and d_o the output disturbance.

2.1. Close to integrating *systems*

There are multiple examples in industry where integrators appear, for example position control in mechatronic systems, level control systems, pulp and paper industry, temperature control in a well isolated reactor etc. *Close to integrating* systems refer to the class of systems with large time constants—in other words, systems with poles very close to the origin in the complex plane. Consider the system presented in (1), with $K = -100$, $\tau_1 = 10$ hours and $\tau_2 = 0.02$ hours

$$P_1(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{1}$$

Two controllers are proposed in this example to show the difficulty of designing a controller which effectively rejects both input and output disturbances, a P controller with gain $K_p = -1.3$ and a PI controller with gains $K_p = -1.3$ and $K_i = -5$, with the PID controller defined as: $K_p + \frac{K_i}{s} + K_d s$. The controllers were tuned to achieve good disturbance rejection using our in-house computer aided design tool (De Keyser and Ionescu, 2006). As observed in the simulation results depicted in Figure 2, the P controller efficiently rejects the output disturbance but it does not reject the input disturbance. On the other hand, the PI controller provides a better input disturbance rejection performance, at the cost of somewhat less output disturbance rejection.

2.2. Poorly damped systems

Poorly damped systems always represent a challenge for control design. Here, we consider a stable second-order system with a pair of poorly damped poles as originally proposed in Alcantara et al. (2011b). The model is given by

$$P_2(s) = \frac{K}{(s/\omega_n)^2 + 2(\xi/\omega_n)s + 1} e^{-sT_d} \tag{2}$$

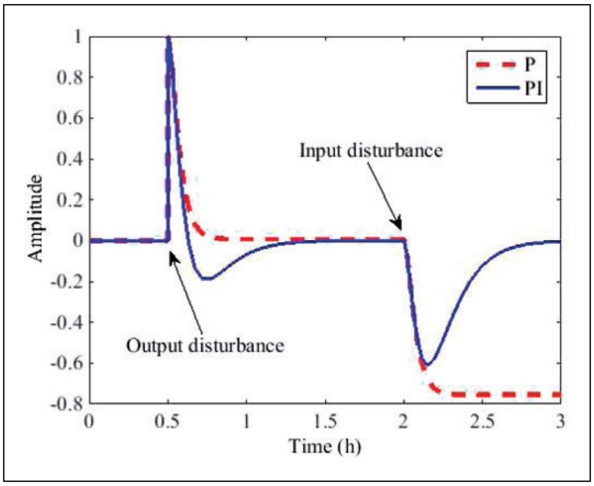

Figure 2. Output and input disturbance rejection for the case of a P and PI controller.

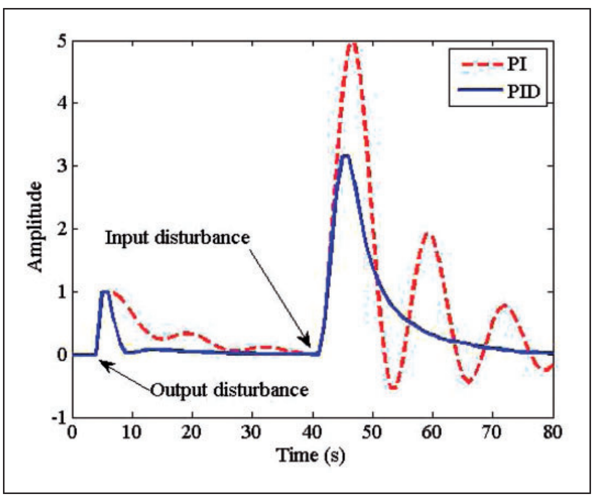

Figure 3. Output and input disturbance rejection for the case of a P and PI controller in a poorly damped system.

where $K = 4$, $T_d = 1$ s, $\omega_n = 0.5$ rad/s and $\xi = 0.25$. In this example two controllers have been considered—a PI with parameters $K_p = 0.043$ and $K_i = 0.024$; and a PID with parameters $K_p = 0.017$, $K_i = 0.022$ and $K_d = 0.42$. The controllers were tuned using the same criteria as in the previous example. The results are depicted in Figure 3. As expected, the PI controller slowly rejects the output disturbance applied at $t = 5$ s and does not reject the oscillations caused by the input disturbance since it does not have the necessary degrees-of-freedom to compensate the complex conjugated poles of the process. On the other hand, the PID controller presents a good performance for both input and output disturbances, as the complex zeros of the controller compensate for the complex poles of the process.

As a preliminary conclusion, one can say that rejecting an input disturbance implies compensating the dynamics of the system. For a poorly damped second-order system, a PID with complex zeros presents a good solution. However, if the system is higher than second order, a higher order compensator should be used. Some methodologies have been proposed as in Alcantara et al. (2011b), although they might be difficult to tune in practice.

In this paper, we propose a discrete-time IMC algorithm with generalized disturbance rejection properties. The methodology allows efficient rejeciton of input disturbances, even for the case of slow processes or systems with very low damping factor. The proposed methodology is evaluated on a fourth-order system with low damping factor, in both simulations and experimental tests.

Note that, for the cases where the resulting tracking performance is not satisfactory, its performance can be increased by introducing a reference prefilter (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 2005). Alternatively, in case of measured disturbances a two-degree-of-freedom topology might be used (Vilanova and Serra, 1997).

3. Internal model control

IMC (Garcia and Morari, 1982; Rivera et al., 1986; Morari and Zafiriou, 1989) is a control strategy which belongs to the class of model-based controllers (Bequette, 2002) and represents a special case of the more generic model-based control strategy depicted in Figure 4. In this section, a brief introduction to IMC is given along with the proposed extension for efficient disturbance rejection.

Some of the basic properties of the IMC controller are listed here:

- If the plant is exactly known, the system behaves as an open loop and the stability issue is trivial.
- Zero steady state error to step input; inherent integral action can be achieved without additional tuning parameters.
- The basic IMC structure has only one tuning parameter and will thus be easy to tune as compared to, for example, the PID structure.

The IMC philosophy relies on the internal model principle, which states that control can be achieved only if the control system encapsulates, either implicitly or explicitly, some representation of the process to be controlled. In particular, if the control scheme has been developed based on an exact model of the process, then perfect control is theoretically possible (Bequette, 2002). The general structure of an IMC controller is depicted in Figure 5.

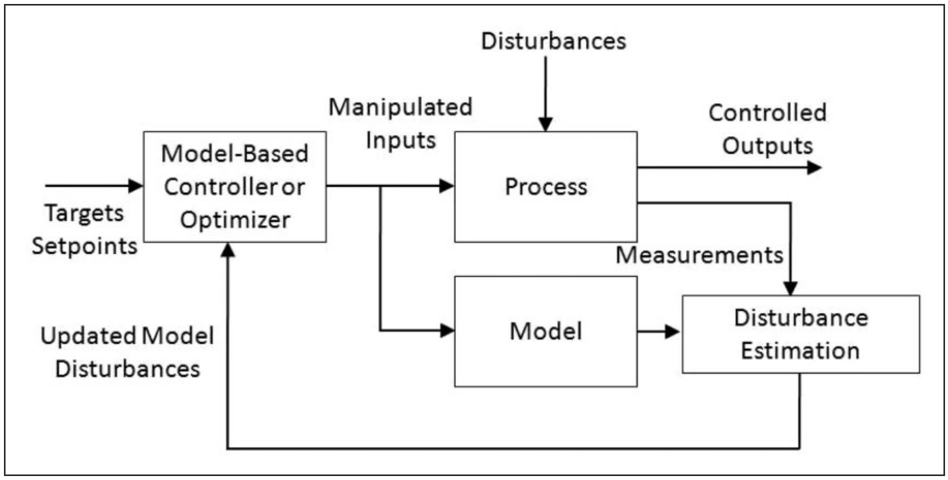

Figure 4. Generic form of the model-based control strategy.

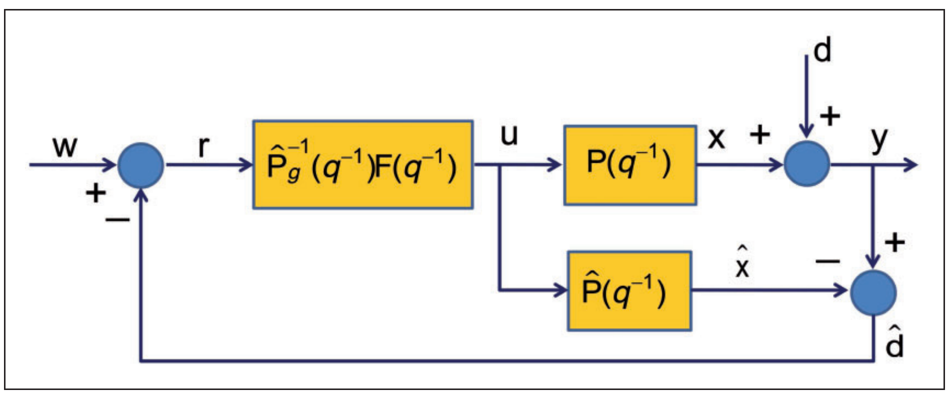

Figure 5. Schematic overview of the IMC structure.

In this figure, d is an unknown disturbance affecting the system. The manipulated input u is introduced to both the process and its model. The process output, y, is compared with the output of the model $\hat{x}$, resulting in a signal $\hat{d}$. If the process is well known then a perfect estimation of the disturbances will be reached. It is important in IMC control to avoid an unstable or non-causal compensator transfer function when designing the controller by adding a filter $F(q^{-1})$ to make the compensator proper (Bequette, 2002), and to the model into "invertible" (P_g) and "noninvertible" (P_b) transfer functions, such that $P = P_g P_b$.

3.1. Principles of the IMC formulation

In a discrete-time formulation, the objective of a model-based controller is to use the process input sequence at each sampling instant, calculating an inverse function which intends to compensate for the process dynamics. Usually, the process model is a nonlinear dynamic relationship between the process output y and the manipulated process input u (i.e. $y(t) = f[y(t-1), \ldots, u(t-1), \ldots])$. However, IMC assumes that a linear approximation of the process dynamics is available.

A closed loop control scheme is depicted in Figure 1, where R denotes the controller and P the process. Within the IMC context, the controller is designed based on compensating the process dynamics while ensuring a desired closed loop performance trajectory.

In order to make use of the inverse of the process to compensate dynamics, one must split the process into an invertible (good) part and a noninvertible (bad) part. This implies that, if the process has nonminimum phase dynamics or time delays in the transfer function, they are part of the noninvertible (bad) part of the process, i.e. $B_b(q^{-1})$. The remaining, invertible (good), part of the process is then denoted by $B_g(q^{-1})$ and the inverse is a causal and stable transfer function with $B(q^{-1}) = B_g(q^{-1})B_b(q^{-1})$.

A *"basic" IMC filter*, designed to follow step changes in the setpoint w and to reject step disturbances d at the output of the process, is given by

$$F(q^{-1}) = \frac{(1+a)^n}{(1+a\cdot q^{-1})^n} \tag{3}$$

with steady state gain $F(1)=1$ and a a design parameter defined as $a=-e^{-T_s/\lambda}$ where T_s is the sampling period. The (negative) values of this design parameter are in the range $0 \ll |a| < 1$ and it is related to the closed loop speed: if λ is bigger, $|a|$ is closer to 1, and the settling time will be bigger.

An *"extended" IMC filter*, designed to follow a ramp setpoint and to reject ramp disturbances at the output of the process or step disturbance at the input of the process, is given by

$$F(q^{-1}) = \frac{(1+a)^n}{(1+a\cdot q^{-1})^n}\cdot(1-f+fq^{-1}) \tag{4}$$

with

$$f = \frac{na}{1+a} - (b_1 + 2b_2 + 3b_3 \ldots + mb_m) \tag{5}$$

and the parameters b_i the coefficients of the bad part of the process

$$\begin{cases} B_b(q^{-1}) = 0 + b_1q^{-1} + b_2q^{-2} + \cdots + b_mq^{-m} \\ \sum_{i=1}^{m} b_i = 1 = B_b(1) \end{cases} \tag{6}$$

3.2. Proposed extension

The idea introduced in this paper is based on modeling principles borrowed from model-based predictive algorithms (Maciejowski, 2002; De Keyser, 2003; Camacho and Bordons, 2004). In the model-based predictive control formulation (De Keyser and Ionescu, 2003), if the main frequency of the disturbance is known, it is possible to model it and provide this additional information to the controller.

For instance, if a repetitive (e.g. sinusoidal) disturbance is present in the system, this can be modeled and used as a filter in the IMC algorithm. This can be best viewed in Figure 6, where the nominal closed loop can be defined, as in the next equation, based on the IMC control structure (Figure 3) and on the fact that $B(q^{-1}) = B_g(q^{-1})B_b(q^{-1})$

$$\begin{aligned} y(t) &= B_b(q^{-1})F(q^{-1})w(t) \\ &+ \left[1 - B_b(q^{-1})F(q^{-1})\right]\frac{C(q^{-1})}{D(q^{-1})}h(t) \end{aligned} \tag{7}$$

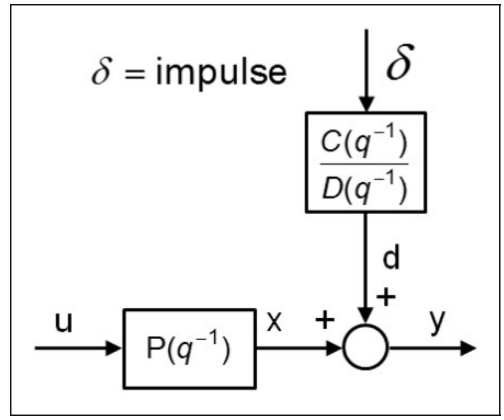

Figure 6. Model of the disturbance effect.

where $h(t)$ is a discrete-time unit impulse. The disturbance model can be defined as a function of the type and location of the disturbance

- step at process output $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\cdots}{1-q^{-1}}$
- step at process input $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\cdots}{(1-q^{-1})A(q^{-1})}$
- ramp at process output $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\cdots}{(1-q^{-1})^2}$
- periodic/repetitive $\frac{C(q^{-1})}{D(q^{-1})} = \frac{\cdots}{(1-e^{j\omega T_s}q^{-1})(1-e^{-j\omega T_s}q^{-1})}$

However, other kinds of dynamics than repetitive disturbances can be filtered. For instance, for input disturbance, the process dynamics will appear in the output. These can also be filtered if the disturbance filter is designed as proposed in the remainder of this section.

Effective disturbance rejection can be achieved by including the dynamics of the disturbance signal (polynomial $D(q^{-1})$) in the filter $F(q^{-1}) = \frac{F_N(q^{-1})}{F_D(q^{-1})}$. Hence, the bad poles of the disturbance dynamic response become zeros in the controller. In this way, perfect compensation can be achieved (if a perfect model is available). Of course, in reality, we always have modeling errors. Following this reasoning, the numerator of $[1 - B_b(q^{-1})F(q^{-1})]$ has to explicitly include $D(q^{-1})$, which employs the following equivalence

$$F_D(q^{-1}) - B_b(q^{-1})F_N(q^{-1}) = D(q^{-1})Q(q^{-1}) \tag{8}$$

Given $D(q^{-1})$, $F_D(q^{-1})$, and $B_b(q^{-1})$, one needs to find the polynomials $F_N(q^{-1})$ and $Q(q^{-1})$ using concepts of generalized disturbance rejection via the diophantine equation

$$B_b(q^{-1})F_N(q^{-1}) + D(q^{-1})Q(q^{-1}) = F_D(q^{-1}) \tag{9}$$

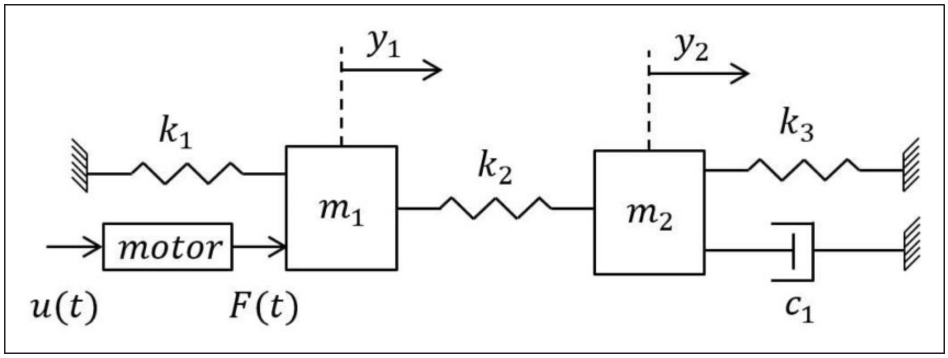

Figure 7. Schematic representation of the mass–spring–damper system.

Knowing that the controller $R(q^{-1})$ for an IMC structure (Figure 3) is given by

$$R(q^{-1}) = \frac{A(q^{-1})F(q^{-1})}{B_g(q^{-1}) - F(q^{-1})B(q^{-1})} \tag{10}$$

it can be seen that the poles $D(q^{-1})$ of the disturbance filter are then indeed poles of the controller.

4. Process description

The mass–spring–damper system used in this paper (see Figure 7) is an electromechanical system with two movable masses $m1$ and $m2$ (the third mass in the picture is fixed for this experiment), three springs with spring constants $k1$, $k2$, and $k3$, a damper with damping constant $c1$, and a motor which drives the system. The input of the system is the voltage to the motor $u(t)$ and the outputs are the mass displacements $y_1(t)$ and $y_2(t)$ expressed in centimeters. Therefore a complete model of the electromechanical plant should describe the dynamics from $u(t)$ to $y_1(t)$ and from $u(t)$ to $y_2(t)$. The (fast) dynamics of the electrical motor can be neglected; hence, the motor can be represented by a pure static gain $F(t) = Ku(t)$, with $F(t)$ the force on the 1st mass and the gain of the motor $K = 3.5$ N/V. The parameters of the set-up are: $m_1 = 1.85\,\text{Kg}$, $m_2 = 1.35\,\text{Kg}$, $k_1 = k_2 = 800\,N/m$, $k_3 = 450\,N/m$, and $c_1 = 9\,N/(m/s)$.

This system has two eigen-frequencies $\omega_1 = 20.8\,\text{rad}/s$ and $\omega_2 = 39.1\,\text{rad}/s$, and damping factors $\zeta_1 = 0.08$ and $\zeta_2 = 0.08$.

To derive the physical model of the mass–spring–damper system (Figure 7), the first step is to derive the transfer functions from the motor to masses $m1$ and $m2$ (cm/Volts). Thus, we derive the two differential equations that describe the mass–spring–damper systems

$$F(t) = m_1\ddot{y}_1 + k_2y_1 + k_1y_1 - k_2y_2 \tag{11}$$

$$0 = m_2\ddot{y}_2 + k_2y_2 + k_3y_2 - k_2y_1 + c_1\dot{y}_2 \tag{12}$$

with $F(t) = K\,u(t)$

Taking the Laplace transform of equations (11) and (12) the following equations are obtained

$$F(s) = m_1s^2\,Y_1(s) + k_2\,Y_1(s) + k_1\,Y_1(s) - k_2\,Y_2(s) \tag{13}$$

$$0 = m_2s^2\,Y_2(s) + k_2\,Y_2(s) + k_3\,Y_2(s) - k_2\,Y_1(s) + c_1s\,Y_2(s) \tag{14}$$

where $Y_1(s)$ is the Laplace transform of $y_1(t)$, $Y_2(s)$ is the Laplace transform of $y_2(t)$ and $U(s)$ is the Laplace transform of $u(t)$.

Solving equations (13) and (14) for $Y_1(s)$ and $Y_2(s)$, and by inserting the numerical values for all parameters, the following transfer functions are obtained

$$tf_{1st\_dof}(s) = \frac{350(1.35s^2 + 9s + 1250)}{den} \tag{15}$$

and

$$tf_{2nd\_dof}(s) = \frac{280{,}000}{den} \tag{16}$$

with $den = 2.498s^4 + 16.65s^3 + 4473s^2 + 14{,}400s + 1{,}360{,}000$ and the step responses of the corresponding transfer functions are depicted in Figure 8.

To identify models for the electromechanical plant from input-output data, a parametric identification method (the prediction error method—PEM) was considered. A sinesweep signal with an amplitude between -1 V and 1 V which can be seen in Figure 9(a) is used for identification. From the mathematical model we have the resonant frequencies of the system: a peak at 20 rad/s and a peak at 37 rad/s (Figure 9(b)). Therefore, in order to perform identification, the initial frequency for the sinesweep is taken at 5 rad/s and the frequency at target time 15 s is taken to be 100 rad/s (linear

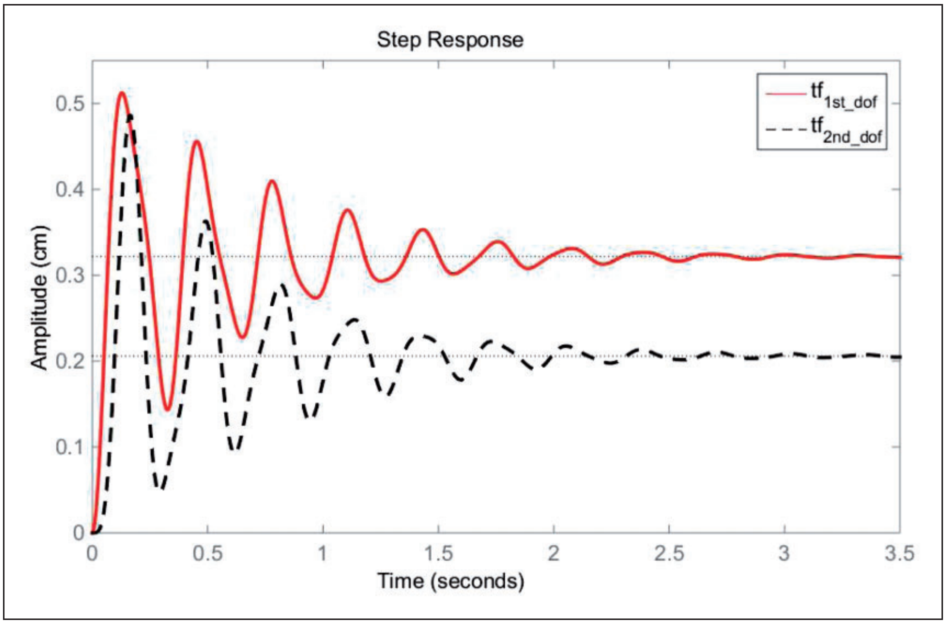

Figure 8. Step responses of the continuous transfer functions for the two masses.

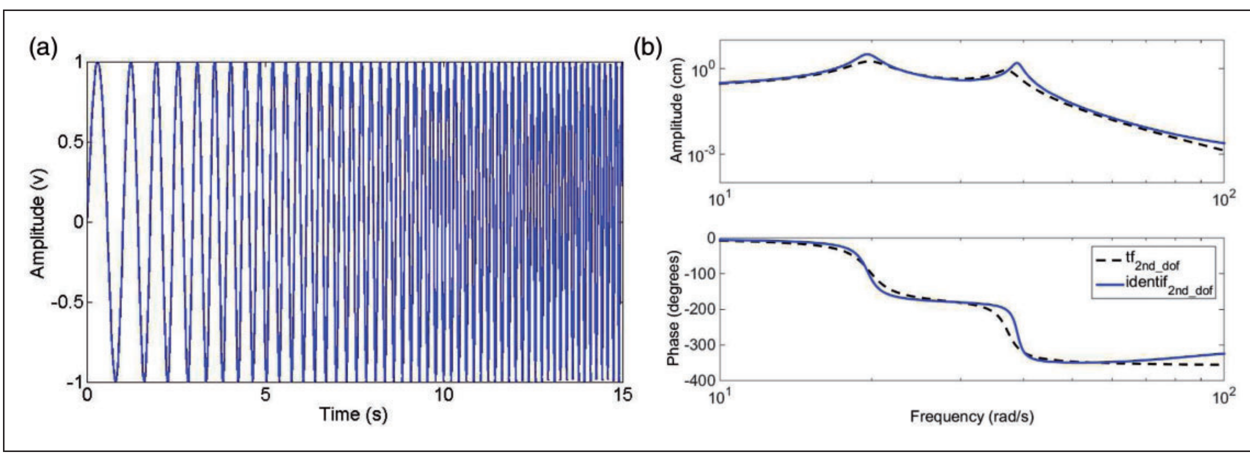

Figure 9. (a) The generated sinesweep signal; (b) Bode plot for transfer function of the 2nd mass.

distribution of the frequencies). The Bode plots of the theoretical and the identified transfer functions for the second mass (tf_{2nddof}) are represented in Figure 9(b).

5. Numerical example

This section describes the necessary steps for IMC controller design. The objective is to control the position of the second mass of the mass–spring–damper system described in the previous section.

In order to implement the transfer function of the process in real time, we need to find its equivalent in discrete time. A suitable sampling period T_s of 10 ms has been chosen. The discrete-time equivalent transfer function expressed in q^{-1} is given by

$$P(q^{-1}) = \frac{10^{-5}(4.58q^{-1} + 49.26q^{-2} + 48.61q^{-3} + 4.40q^{-4})}{1 - 3.76q^{-1} + 5.46q^{-2} - 3.63q^{-3} + 0.93q^{-4}} \tag{17}$$

Next, we split $B(q^{-1})$ in a good part and a bad part defined as in

$$\begin{aligned} B_g(q^{-1}) &= 10^{-3} * (0.97 + 0.09q^{-1}) \\ B_b(q^{-1}) &= 0.0472q^{-1} + 0.5027q^{-2} + 0.4501q^{-3} \end{aligned} \tag{18}$$

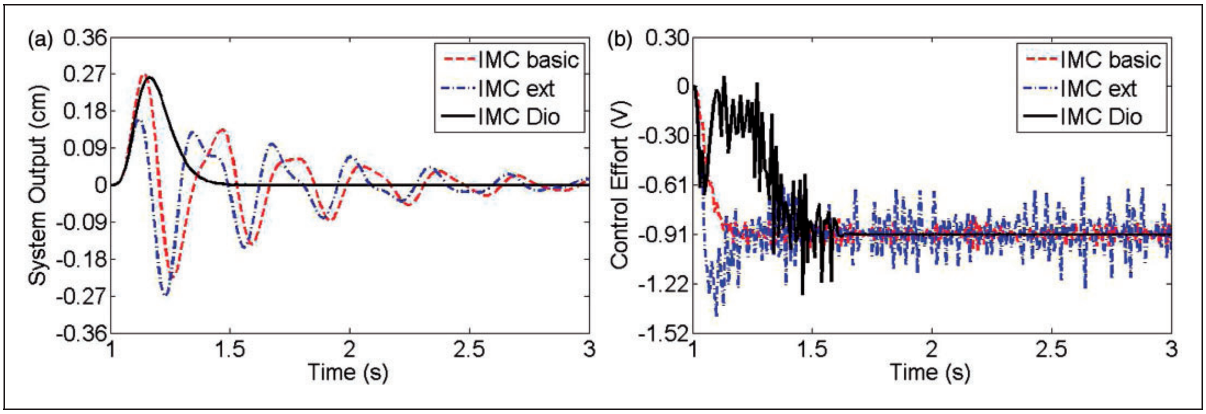

Figure 10. Comparison between the IMC controllers (basic, extended, and diophantine) for the simulation test: (a) Disturbance rejection; (b) Control effort.

The basic filter used to make the transfer function of the controller semi-proper is given by

$$F_{\text{basic}}(q^{-1}) = \frac{(1+a)^n}{(1+aq^{-1})^n} = \frac{0.027}{1-2.1q^{-1}+1.47q^{-2}-0.34q^{-3}} \tag{19}$$

The order of the basic filter is $n = 3$, and the value of the parameter a is taken to be -0.7. The disturbance applied to the system is a step disturbance at the input of the process, and the simulation (Figure 10) shows that the disturbance is not efficiently rejected when the basic filter is used. For the extended IMC controller, the order of the filter is equal to 4, and $a = -0.7$, thus the extended filter is given by the following transfer function

$$F_{\text{ext}}(q^{-1}) = \frac{0.10-0.09q^{-1}}{1-2.8q^{-1}+2.94q^{-2}-1.37q^{-3}+0.24q^{-4}} \tag{20}$$

From the simulation results (Figure 10), we notice that, even if the extended filter is used, the disturbance is not efficiently rejected. Therefore, we conclude that the dynamics of the poorly damped system play an important role and therefore need to be taken into account when the filter is designed. Hence, the next step is to propose a filter based on the diophantine equation (D-IMC). If one envisages *to compensate the dynamics of the system*, then *the disturbance model should include the denominator of the transfer function of the system* such that $\frac{C(q^{-1})}{D(q^{-1})} = \frac{1}{(1-q^{-1})A(q^{-1})}$. In this case, the denominator of the disturbance model transfer function is given by

$$D(q^{-1}) = 1-4.76q^{-1}+9.22q^{-2}-9.10q^{-3} + 4.57^{-4}-0.93q^{-5} \tag{21}$$

As already mentioned in Subsection 3.2, the idea is to remove the disturbance modes from the closed loop by including the polynomial $D(q^{-1})$ in the controller.

To obtain a proper/semi-proper transfer function of the controller, the filter order of the proposed D-IMC needs to be minimally $n = 7$. The parameter a was set to the same value as for the basic and extended filter (-0.7). The denominator F_D of the filter is given by

$$F_D(q^{-1}) = 1-4.9q^{-1}+10.29q^{-2}-12q^{-3}+8.4q^{-4} - 3.52q^{-5}+0.82q^{-6}-0.08q^{-7} \tag{22}$$

Based on the diophantine equation from (9), we have a matrix form

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.04 & 0 & 0 & 0 & 0 & -4.76 & 1 & 0 \\ 0.5 & 0.04 & 0 & 0 & 0 & 9.22 & -4.76 & 1 \\ 0.45 & 0.5 & 0.04 & 0 & 0 & -9.10 & 9.22 & -4.76 \\ 0 & 0.45 & 0.5 & 0.04 & 0 & 4.57 & -9.10 & 9.22 \\ 0 & 0 & 0.45 & 0.5 & 0.04 & -0.93 & 4.57 & -9.10 \\ 0 & 0 & 0 & 0.4 & 0.5 & 0 & -0.93 & 4.57 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0 & -0.93 \end{bmatrix} \begin{bmatrix} f_0^N \\ f_1^N \\ f_2^N \\ f_3^N \\ f_4^N \\ q_0 \\ q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4.9 \\ 10.29 \\ -12 \\ 8.4 \\ -3.52 \\ 0.82 \\ -0.08 \end{bmatrix} \tag{23}$$

from where the unknown coefficients can be determined

$$\begin{cases} f_0^N = 0.30 \\ f_1^N = -1.19 \\ f_2^N = 1.76 \\ f_3^N = -1.17 \\ f_4^N = 0.29 \\ q_0 = 1.00 \\ q_1 = -0.15 \\ q_2 = 0.23 \end{cases} \quad (24)$$

Thus, the filter obtained by employing the diophantine equation is

$$F(q^{-1}) = \frac{0.30 - 1.19q^{-1} + 1.76q^{-2} - 1.17q^{-3} + 0.29q^{-4}}{1 - 4.9q^{-1} + 10.29q^{-2} - 12q^{-3} + 8.4q^{-4} - 3.52q^{-5} + 0.82q^{-6} - 0.08q^{-7}} \quad (25)$$

The resulting transfer functions of the controllers R computed with (10) are given by

$$R_{\text{basic}}(q^{-1}) = \frac{27.82 - 104.6q^{-1} + 152.1q^{-2} - 101.2q^{-3} + 26.02q^{-4}}{1 - 2q^{-1} + 1.24q^{-2} - 0.20q^{-3} - 0.03q^{-4}}$$

$$R_{\text{ext}}(q^{-1}) = \frac{106.3 - 497.7q^{-1} + 949.6q^{-2} - 922.2q^{-3} + 455.7q^{-4} - 91.62q^{-5}}{1 - 2.70q^{-1} + 2.61q^{-2} - 1.07q^{-3} + 0.14q^{-4} + 0.02q^{-5}}$$

$$R_{\text{dio}}(q^{-1}) = \frac{317.3 - 2427q^{-1} + 8195q^{-2} - 1.59e04q^{-3} + 1.96e04q^{-4} - 1.55e04q^{-5} + 7801q^{-6} - 2255q^{-7} + 287.9q^{-8}}{1 - 4.81q^{-1} + 9.69q^{-2} - 10.6q^{-3} + 6.93q^{-4} - 2.93q^{-5} + 0.82q^{-6} - 0.09q^{-7} - 0.02q^{-8}} \quad (26)$$

In order to reject the disturbance and to compensate the dynamics of the plant, the zeros of $(1 - B_b(q^{-1})F(q^{-1}))$ should cancel the poles of the disturbance model. The poles of the disturbance model D are

$$\begin{cases} 0.9154 + 0.3572i \\ 0.9154 - 0.3572i \\ 1.0000 \\ 0.9652 + 0.1931i \\ 0.9652 - 0.1931i \end{cases} \quad (27)$$

For the D-IMC filter the zeros of $(1 - B_b(q^{-1})F(q^{-1}))$ are

$$\begin{cases} 0.0767 + 0.4753i \\ 0.0767 - 0.4753i \\ 0.9154 + 0.3572i \\ 0.9154 - 0.3572i \\ 1.0000 \\ 0.9652 + 0.1931i \\ 0.9652 - 0.1931i \end{cases} \quad (28)$$

The simulation results for disturbance rejection using the proposed D-IMC filter and the corresponding controller are illustrated in Figure 10. One may notice that the basic and the extended IMC controllers cannot efficiently reject the disturbance. However, this is achieved with the IMC design based on diophantine equations for the cases where the disturbance signal can be identified or the disturbance is periodic. This approach can also be used to compensate for special dynamics in the system.

6. Experimental results and discussion

As observed from the simulation results presented in the previous section, the IMC with the filter based on diophantine equations outperforms IMC both with basic and extended filters. Consequently, this design was utilized on the real-time system. For experimental validation we considered the real-time mass–spring–damper system presented in Figure 7, i.e. the ECP 210 rectilinear plant with a Labview Real-Time interface. The controller is downloaded and run in real-time from an FPGA system integrated within the National Instruments framework. The current implementation allows working with an integer precision of 16 bits, which results in quantization errors with respect to the values of control effort. The value of the step disturbance applied at the input of the process is 0.9V and was implemented in software to ensure repeatability.

Analysis of the time evolution of the output of the system presented in Figure 11(a) suggests that the results obtained in simulation (section 5) concur with the real-time experiments illustrated in this section. We may conclude that the disturbance is efficiently rejected and the dynamics of the system are well compensated if an adequate filter is used.

Figure 11(a) depicts the results obtained after implementing the IMC designs with basic, extended, and diophantine filters. The results confirm that the diophantine filter is able to efficiently reject the disturbance. The control effort obtained during this experiment is depicted in Figure 11(b). The oscillations

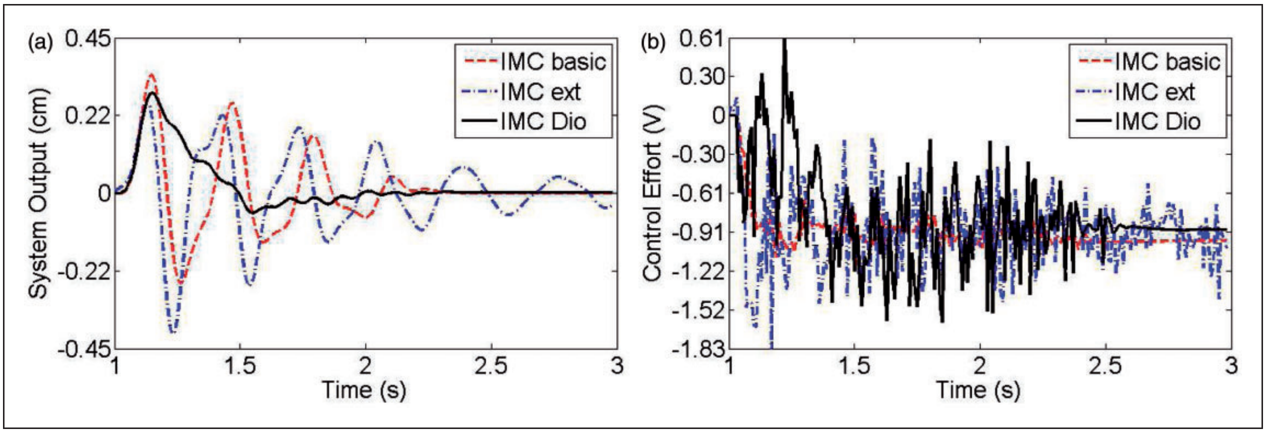

Figure 11. Comparison between the IMC designs (basic, extended, and diophantine) for the real-time experiment: (a) Disturbance rejection; (b) Control effort.

Table 1. Performance index for output error in the different control strategies.

Controller	*ISE*	*IAE*	*ITAE*
IMC nom	2.243	12.773	18.756
IMC ext	3.033	18.156	30.746
IMC dio 7	**1.165**	**7.452**	**9.9853**

Table 2. Performance index for control effort in the different control strategies.

Controller	*ISE*	*IAE*	*ITAE*
IMC nom	169.49	180.19	368.32
IMC ext	190.51	184.16	369.45
IMC dio 7	**159.47**	**164.19**	**339.99**

of the control action are induced by the input disturbance signal, the effort to compensate for the dynamic of the system, and also by the quantization error effect.

To further prove the effectiveness of the proposed methodology, the performance of the controllers is evaluated using the well known performance indices: *integral of the square of the error* (ISE), *integral of the absolute magnitude of the error* (IAE), and *integral of time multiplied by the absolute value of error* (ITAE) with their mathematical definition given by the equations

$$ISE = \int error^2 \,\mathrm{d}t \quad IAE = \int |error| \,\mathrm{d}t \qquad ITAE = \int t|error| \,\mathrm{d}t \tag{29}$$

The results are summarized in Tables 1 and 2 for the output error and control effort, respectively. The errors were calculated with respect to zero for both output and control effort. From these values, the conclusion can be summarized as twofold: the IMC design with the diophantine filter gives the best results both in terms of disturbance rejection and minimal control effort. Since the controllers are designed in discrete-time, is not necessary to have a proper/semi-proper transfer function of the controller. Therefore, different orders can be chosen for the IMC filters, but this analysis is out of the scope of this paper. Even so, we have to take into account that if the sampling time is small, then the discrete-time is closer to its continuous-time equivalent.

In order to test the robustness of the designed controllers, a variation of ±25% of the parameter k_2 in the nominal transfer function of the process (16) have been investigated. We choose to alter k_2 (representing the spring between masses) because it affects the numerator as well as the denominator of the process transfer function (hence a strong effect). The new transfer functions of the process are described by

$$process_{(-25\%)} = \frac{210{,}000}{\left\{\begin{matrix}2.498s^4 + 16.65s^3 + 3833s^2 + 12{,}600s \\ +1{,}110{,}000\end{matrix}\right\}} \tag{30}$$

$$process_{(+25\%)} = \frac{350{,}000}{\left\{\begin{matrix}2.498s^4 + 16.65s^3 + 5113s^2 + 16{,}200s \\ +1{,}610{,}000\end{matrix}\right\}} \tag{31}$$

The variation of the magnitude and phase of the nominal process compared with the process ±25% variation is represented in the Bode plot from

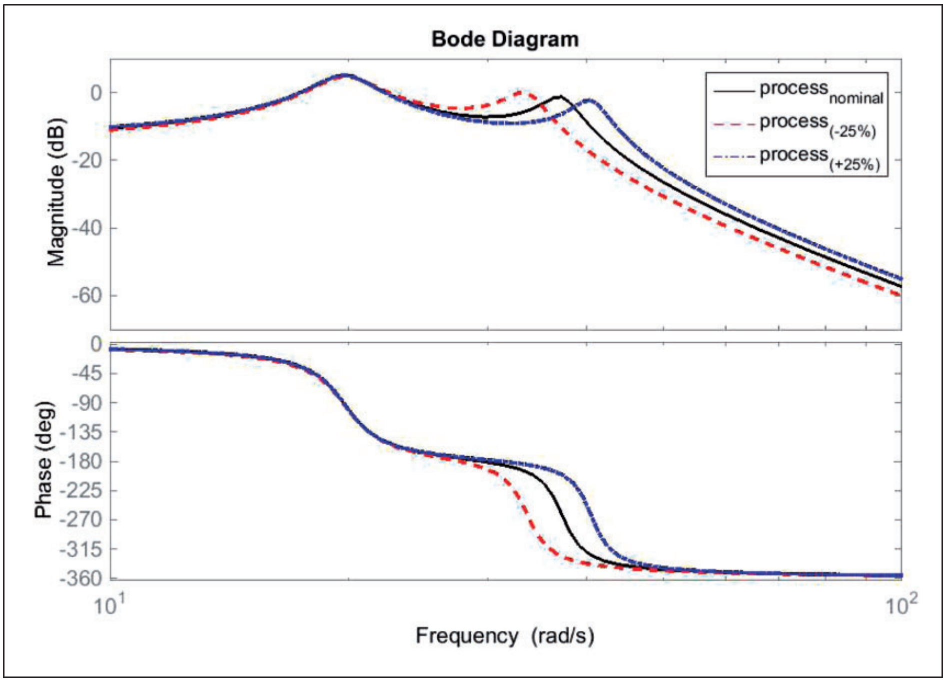

Figure 12. Bode plot for the nominal process and for the process with ±25% variation of k_2.

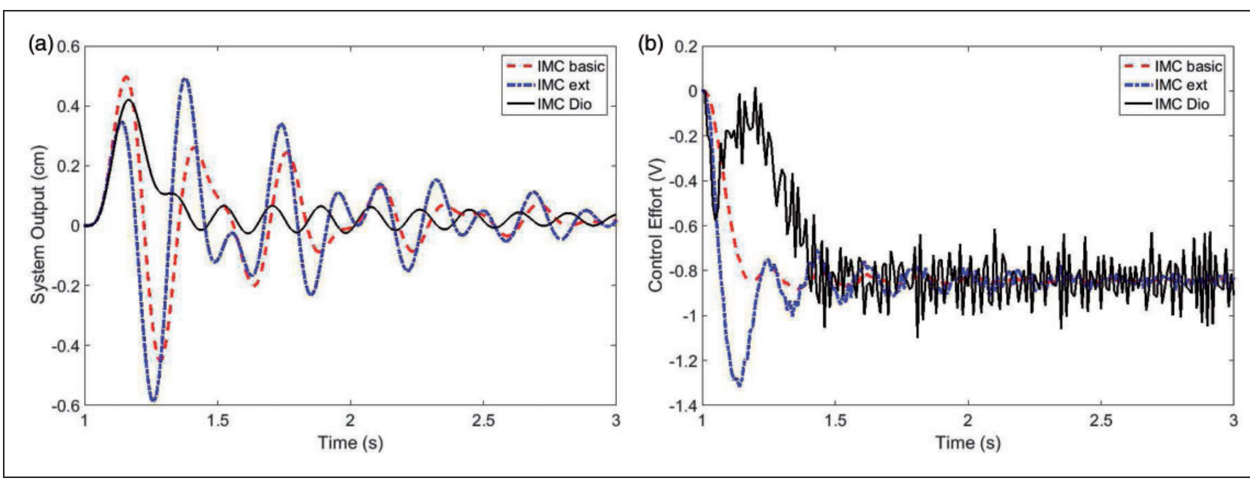

Figure 13. Robustness analysis of the IMC controllers when k_2 is varying with −25% ($k_2 = 600$): (a) Disturbance rejection; (b) Control effort.

Figure 6. As can be noticed from the figure, the resonant frequencies of the system have also been changed, thus the second peak is at 33 rad/s and 41 rad/s respectively compared with the nominal process where the peak is at 37 rad/s. As can be observed from the definitions of the new transfer functions, the process has a big variation in the gain as well as in the pole location. The output of the system and the corresponding control effort are illustrated in Figures 13 and 14. From the simulation results it can be concluded that the proposed controller is stable, but the performances are suboptimal due to modeling error. This is due to the fact that the IMC controller cannot perfectly compensate the pole of the system and thus the oscillation behavior of the output signal. Obviously, if we put in the effort to achieve a good approximation of the model, the results are drastically improved (see Figure 10).

7. Conclusions

A novel methodology for discrete-time internal model control incorporating disturbance rejection properties has been proposed in this paper. The proposed strategy makes use of diophantine equations to effectively reject input/output disturbance. Based on seminal ideas from model-based predictive control, the diophantine

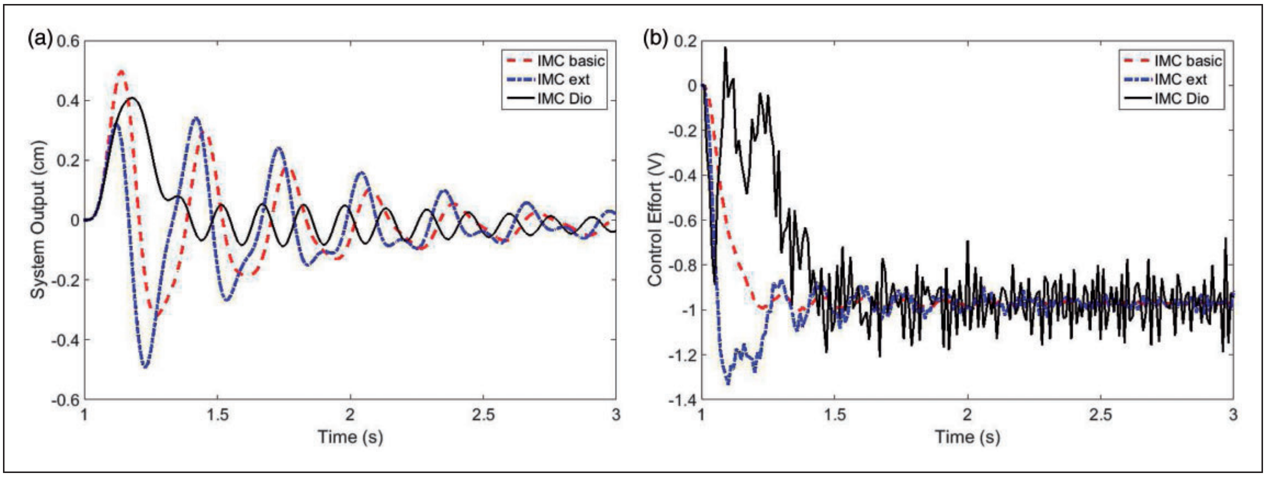

Figure 14. Robustness analysis of the IMC controllers when k_2 is varying with +25% ($k_2 = 1000$): (a) Disturbance rejection; (b) Control effort.

equation can also be used in the internal model controller to effectively compensate the dynamics of the process, allowing a fast disturbance rejection even for poorly damped processes. The controller is designed based on the model of the process while ensuring a desired setpoint tracking. The load disturbances entering the process are challenging problems from a control engineering point of view due to the dynamics of the process itself. The performance of the proposed method is compared against basic and extended IMC filters. The obtained results indicate that the proposed methodology provides good performance in both disturbance rejection and robustness.

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