Influence of geometry in the case of relative positioning with GNSS

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Introduction

- Ionosphere remains the main error source for GNSS relative positioning. A software developed at RMI allows to compute the **positioning error only due to the ionosphere.**
- **Bad geometric conditions** can affect this positioning error and acts as an amplification factor of the double-difference *a priori* precision (σ_{DD})

$$\sigma_{rel.\,pos} = RPDOP \cdot \sigma_{DD}$$

OUR GOALS :
1. Detect and **filter out** bad geometric conditions by using geometric index (threshold) during quiet ionospheric periods
2. Use cleaned dataset to build a **service** providing informations about current ionospheric conditions over the Belgian dense network (Active Geodetic Network, or AGN : 66 dual-frequency GPS stations)

1. Methodology

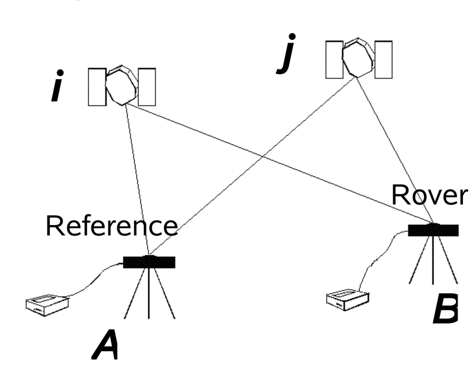
Objective = compute the **positioning error** only due to the **ionosphere** for a given baseline

→ ***SoDIPE-RTK*** ≡ *Software for Determining Ionospheric Positioning Error on Real-Time Kinematic*

a) Building ionospheric residual term in **double differences** (DD) of phase measurements:

$$\phi^{ij}_{AB,k} = (\phi^i_{A,k} - \phi^i_{B,k}) - (\phi^j_{A,k} - \phi^j_{B,k})$$
$$= D^{ij}_{AB} - I^{ij}_{AB,k} + T^{ij}_{AB} + M^{ij}_{AB,k} + \lambda_k N^{ij}_{AB} + \varepsilon^{ij}_{AB,k}$$

geometry, ionosphere, troposphere, multipath, ambiguity, noise



Use of the « **Geometric-Free** » combination to isolate the ionospheric effect. Then, neglecting multipath and noise, we obtain:

$$\phi^{ij}_{AB,GF} = \phi^{ij}_{AB,L1} - \phi^{ij}_{AB,L2}$$
$$= 40.3\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) STEC^{ij}_{AB} + \lambda_k N^{ij}_{AB}$$

If ambiguities are fixed, we get ionospheric residual term on each carrier:

$$I^{ij}_{AB,k} = 40.3 \frac{STEC^{ij}_{AB}}{f_k^2}$$

b) Positioning error only due to the ionosphere

Least-squares adjustment of all DD in view at the epoch considered:

$$\underline{l} = A\,\underline{x} \iff \underline{x} = (A^T P A)^{-1} A^T P \underline{l}$$

with
- *A* the design matrix
- *l* the observations vector
- *P* the weight matrix
- *x* the unknowns vector

N = normal matrix

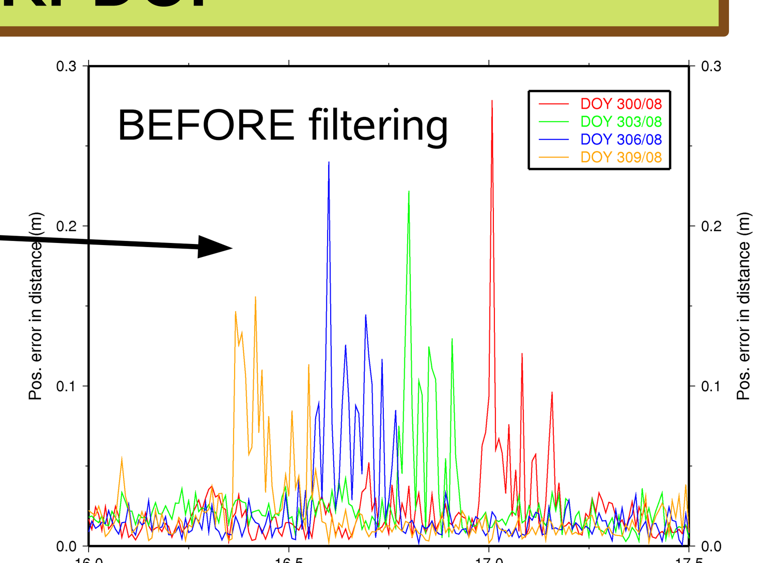
$$l_k(t) = -I^{ij}_{AB,k}(t)$$

Pos. error only due to the ionosphere in topocentric coordinates: **(ΔN,ΔE,ΔH)**

And also in terms of distance: **ΔD** with $\Delta D = \sqrt{\Delta N^2 + \Delta E^2 + \Delta H^2}$

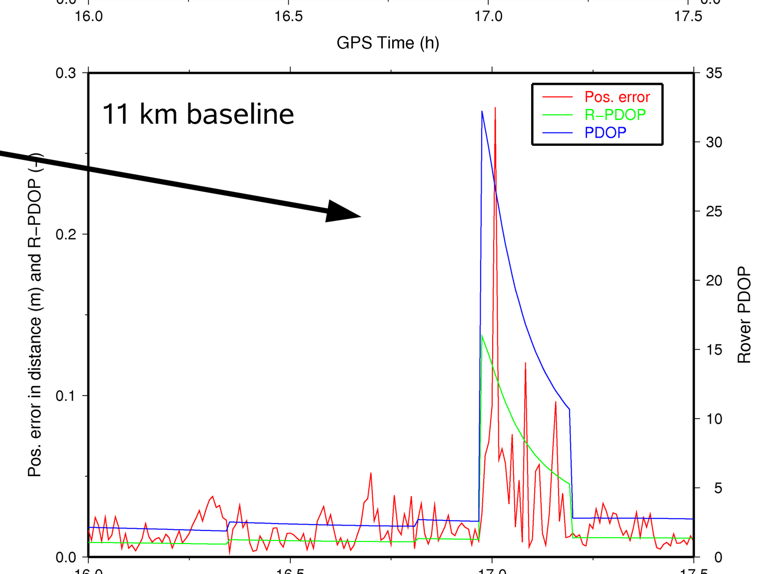
2. ΔD, PDOP and RPDOP

Problem: During very quiet ionospheric conditions, peaks (outliers) not due to ionosphere are present in temporal series. Cases study is shown for DOY 300-309 in 2008.



Clue: There is a identical temporal lag between peaks corresponding to consecutive days → as GPS constellation repeats itself in less than 24h, geometry is supposed to be the cause.

Confirmation: For our cases study, peaks in ΔD are correlated to large values in both RPDOP and PDOP at the rover station. DOY 309/08 pictured.



Investigation: Both PDOP and RPDOP seem to be good indicators of geometric peaks in ΔD. In this case of relative positioning, is RPDOP more reliable than PDOP to identify those peaks and why?

PDOP

- **Absolute** positioning
- *A* contains geometric information about "**one-way**" measurements. All "one-way" have the same precision → weight matrix is identity matrix

$$Q = N^{-1} = (A^T A)^{-1}$$
$$PDOP = \sqrt{q_{xx} + q_{yy} + q_{zz}}$$

- **Easy** geometric interpretation: *A* contains non differenced measurements
- Bad geometric coverages identified easily: PDOP is a **unique** value for given station and satellite geometry

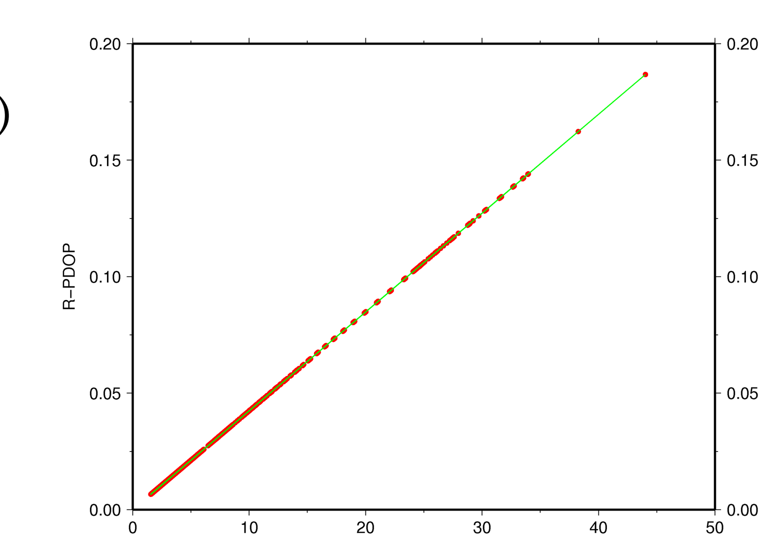
RPDOP

- **Relative** positioning
- A_R contains geometric information about all **DD** in view. Moreover, correlation between DD → weight matrix different from identity matrix

$$Q_R = N_R^{-1} = (A_R^T P A_R)^{-1}$$
$$R-PDOP = \sqrt{q_{R,xx} + q_{R,yy} + q_{R,zz}}$$

- **Difficult** geometric interpretation: A_R contains differenced measurements
- For a given baseline and a given geometry, **different RPDOP values** due to:
  - Different manners to form independent DD
  - P matrix changes with the number of available DD

Considering all AGN baselines (about 160), 11 quiet days in terms of ionospheric conditions (300-310/08) and all measurement epochs (→ ~ 4.10⁶ points), we can observe that **PDOP** computed at rover station and **RPDOP** are **perfectly correlated** *(r=1)*



We can chose either PDOP or RPDOP to isolate geometric peaks. We chose **PDOP** because of the advantages described above (unique value, ease in the interpretation)

3. Setting PDOP threshold

- Same amount of data as previously
- Statistical correlation between large PDOP and large ΔD : r ~ 0.35 → **poor correlation**

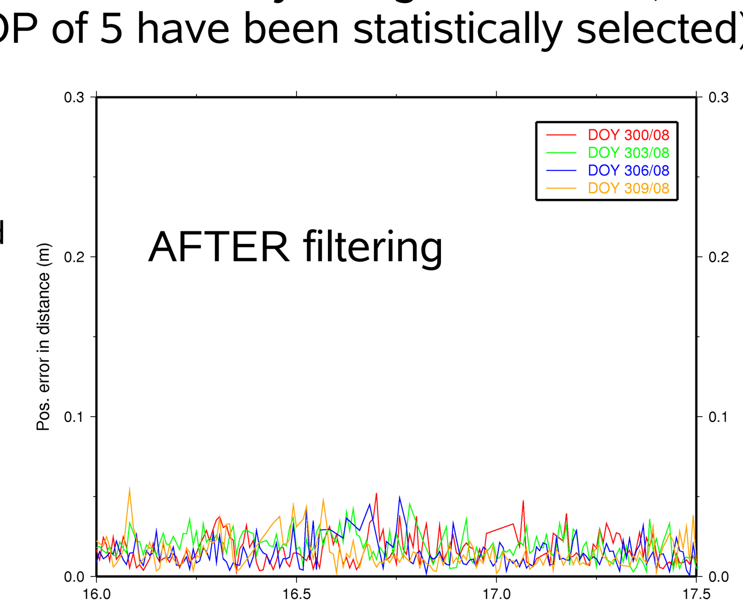
In practice, a large PDOP value does not induce automatically a large ΔD value (see table below where thresholds of 5 cm and PDOP of 5 have been statistically selected)

PDOP	ΔD < 5 cm	ΔD ≥ 5 cm
< 5	92.2%	6%
≥ 5	1%	0.8%

6%: ΔD outliers due to multipath or residual cycle slips

0.8%: ΔD outliers due to bad geometric conditions (least frequent than other outliers)

However, ΔD values relative to these 6% outliers are not very large (generally < 10 cm) in comparison with ΔD values relative to large PDOP (up to several meters) → we prefer not to filter them out because of their relatively small value



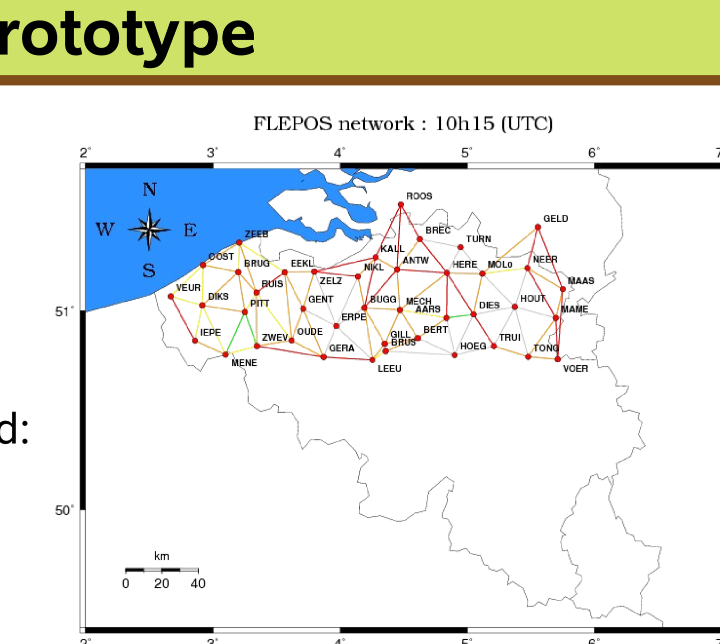
Criterion implemented:

ΔD is filtered out if (PDOP > 5) && (ΔD > 5 cm)

Applying filter to the same cases study as previously (DOY 309/08, 11 km baseline): geometric peaks have been correctly filtered out.

4. Web service prototype

- All FLEPOS (northern part of Belgium) baselines (~100) → **dense spatial information**
- **A posteriori** method (data of the day before)
- Computation of ΔD variability every 15 min using filtered data and assignment of a **colour code** according to the variability degree and threats encountered for relative positioning user on the field:
  - **Green** : no ionospheric threat
  - **Yellow** : minor iono threats
  - **Orange** : major iono theats
  - **Red** : extreme iono conditions



Example : during the occurrence of a medium-scale TID

5. Conclusions and future work

1. We have developed a software allowing the computation of positioning error only due to the ionosphere. This term (called ΔD) present outliers due to bad geometric conditions, which can be easily identified during quiet ionospheric conditions.
2. PDOP at the rover station has been computed and chosen to isolate bad geometric coverage (perfect correlation with relative PDOP, or RPDOP).
3. Thresholds of 5 for PDOP and 5 cm for ΔD have been proposed to filter out "geometric outliers" in ΔD data set.
4. Cleaned data are used in a web service allowing to inform users about ionospheric conditions over a given area (at this moment, Flanders) every 15 min for the previous day. A real-time method and an extension to the whole Belgium is under investigation.