Towards a quasi-static ALE-frictional formulation for cable-actuated multibody systems

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EXTENDED ABSTRACT

1 Introduction

Cable actuation is exploited in various application fields, for instance, in soft robotics or for deployable space structures. The cable is commonly guided by reeving it around a pulley and passing it through a sleeve or a structural element. In such scenarios, the cable is thus prone to permanent contact and friction around the pulley and inside the sleeve, while a large portion of the cable span is contact-free. When working with a finite element description of the cable, small elements are then needed in the contact region. In order to avoid dealing with an unnecessary fine mesh discretization of the whole cable span, an arbitrary Lagrangian-Eulerian (ALE) description can be considered, also sometimes referred to as variable-length beams, axially-moving beams or sliding beams. For frictional contact, the main advantage of an ALE formulation is that the spatial position of the cable nodes in the contact patch is fixed on the interacting body, and a flow of material is permitted at these nodes, like in an Eulerian description of the motion, while the other nodes can follow a classical Lagrangian description of the motion. Such formulations have been proposed, for example, without friction in [1] or with friction in [2] or [3].

The objective of this work is to model frictional contact between a cable and any other rigid body in a quasi-static setting. The friction description derives from an augmented Lagrangian formulation, avoiding both the need for a regularization of Coulomb's law, as usually done in compliant models, and the *ad hoc* implementation of the stick-slip transition depending on the considered example. For example, the formulation aims to model the frictional interactions between a cable modeled as a beam and a pulley using a quasi-static ALE formulation. If the contact is frictionless, the cable slips around the pulley, which is not driven by the cable. However, when friction is considered, the cable can stick to the pulley, in which case the sticking constraint involves the motion of the pulley, the spatial motion of the cable nodes and the cable material flow, or slip on the pulley, in which case a tangential force proportional to the normal force drives the pulley. It should be noted that some regions of the contact patch can be in a stick state while others can slip.

2 Methodology

First, the quasi-static ALE cable model proposed in [4] is extended to consider a geometrically exact beam on the Lie group $SE(3)$. Hence, curvature and bending stiffness are taken into account. For each node, the spatial coordinates are described by a frame $\mathbf{H}(s)$ and the material coordinate by the centerline coordinate s. The ALE equilibrium equations for the beam are derived from the virtual work principle as

$$\delta\mathscr{W} = \delta_{\mathbf{H}}\mathscr{W} + \delta_{\mathbf{s}}\mathscr{W} = 0 \tag{1}$$

where $\mathscr{W}$ is the total potential energy, $\delta_{\mathbf{H}}(\bullet)$ is the variation with respect to the spatial coordinates while keeping the material coordinate fixed, and $\delta_{\mathbf{s}}(\bullet)$ is the variation with respect to the material coordinate while keeping the spatial coordinates fixed.

Next, friction is modeled using Coulomb's law. Assuming that the contact remains closed, the frictional contact conditions are

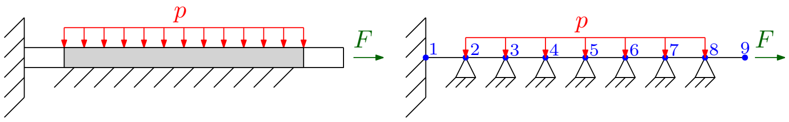
$$|v_T| \geq 0, \quad |\lambda_T| \leq \mu\lambda_N, \quad |v_T|\,(|\lambda_T| - \mu\lambda_N) = 0 \tag{2}$$

with the slip rule $|v_T|\lambda_T = -|\lambda_T|v_T$, where v_T is the relative slip between the cable and the other body, λ_N and λ_T are the Lagrange multipliers related to the normal and tangential components of the contact force, respectively, and μ is the friction coefficient. In this model, frictional effects are only considered in the cable tangent direction and v_T and λ_T are thus scalar fields. In the context of an ALE formulation, it is important to notice that unlike traditional frictional formulations, both the relative slip v_T and the tangential component of the contact force λ_T are involved in the material part of the equilibrium equations, i.e., the equations for the material coordinate s, because the spatial position of the nodes in the contact patch is fixed.

The resulting equations are solved using an augmented Lagrangian approach with an implicit Newton scheme as proposed in [5].

3 Preliminary results and discussion

To illustrate the formulation, a simple example is first proposed, but more complex examples will also be investigated. It consists of a cable modeled as a beam with a Young's modulus $E = 3$ [GPa] and a Poisson's ratio $\nu = 0.3$ [-]. The beam is 1 meter long and has a circular cross-section of radius $r = 2$ [mm]. The left end node is clamped, whereas a axial traction force, increasing linearly from $F = 0$ [N] to $F = 4500$ [N] is applied to the right end node. The beam is sliding inside a fixed sleeve which exerts



(a) Continuous problem. (b) Discrete finite element model.

Figure 1: Continuous and discretized descriptions of a beam sliding inside a fixed sleeve under axial loading.

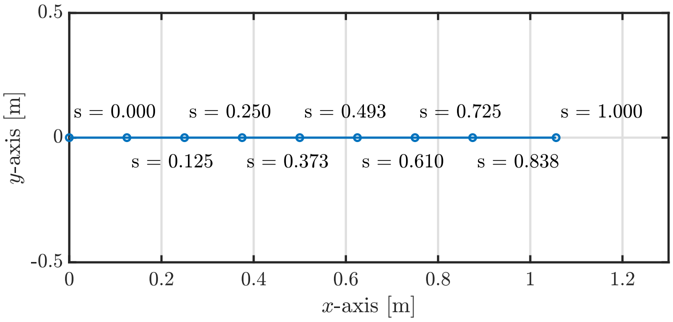


Figure 2: Final load step. $F = 4500$ [N] and $p = 22500$ [N/m].

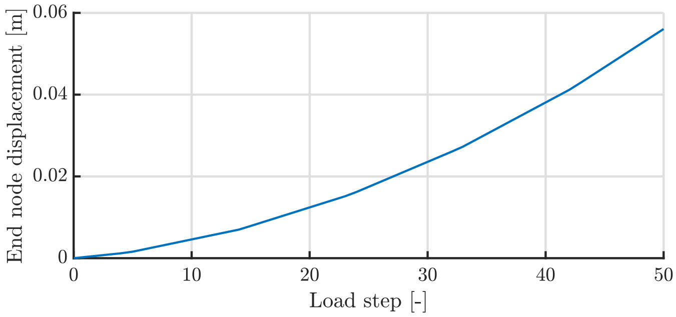


Figure 3: End node displacement as a function of the load step.

a normal distributed force per unit (spatial) length $p = 22500$ [N/m], as depicted in Fig. 1a. Dry friction is present between the cable and the sleeve with a Coulomb friction coefficient fixed to $\mu = 0.3$ [-].

Numerically, the beam is discretized using 8 beam finite elements and the loading F is incrementally applied over 50 load steps to the end node. No-flow boundary conditions are applied. To model the sleeve, the nodes in the contact region (2 to 8) are spatially fixed, while the flow of material is allowed through them. The distributed loading p is constantly applied to each of these inside nodes. The numerical model is illustrated in Fig. 1b.

In Fig. 2, the solution after 50 load steps is shown. The blue circles represent the nodes of the mesh. It can be observed that they remain spatially fixed, apart from the right end node. There is a flow of material for nodes 4 to 8, indicating slip, while there is no flow of material at nodes 2 and 3, meaning that they are still in sticking contact. It is important to notice that the solution is history-dependent due to the hysteresis phenomenon so that the final equilibrium position could be different if another loading path was chosen. This result aligns with the expected behavior, where slipping sequentially begins near the applied force and progresses towards the fixed node. Fig. 3 illustrates the displacement of the end node as a function of the load step. Contrary to the frictionless case where the displacement increases linearly, it is shown that the displacement of the end node follows a nonlinear evolution.

4 Conclusion

In this work, an ALE cable model with friction is proposed and preliminary results are presented. The approach is based on the virtual work principle for obtaining the equilibrium equations and dry friction is modeled without any regularization using an augmented Lagrangian approach. This represents a first step for the development of a general framework enabling the frictional quasi-static simulation of any cable-actuated multibody system including sleeves and pulleys.

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