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Cumulant Equation Method for Vortex-Induced Vibrations under Turbulent Oncoming Flows

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1 BACKGROUND

Vortex-induced vibrations (VIV) have long presented challenges to engineers. It is usually modelled with the spectral approach developed by Vickery and coworkers, or with the wake-oscillator approach initially promoted by the works of Hartlen and Currie. In this work a randomized version of the latter is considered, aiming at studying in an alternative manner the influence of turbulence of the oncoming wind on the structure.

Despite extensive experimental studies on turbulence's impact on VIV, no clear consensus has emerged. Some studies suggest that turbulence can completely suppress VIV by reducing synchronization, while others report minimal effects on response amplitude. These conflicting results highlight the complexity of the problem. Additionally, in contrast with the linear stochastic dynamics problem, the time varying length of the lamina leads the system to be non-linear, and the responses to be non-Gaussian. The stochastic analysis of such systems is particularly difficult because of these "non's".

Based on Itô's formula, Grigoriu proposed the moment equation method to calculate the statistical response of linear systems under random excitations. With this method, the statistical moments of any order of the response can be directly obtained by solving a system of linear differential equations with high efficiency. Although this method has been applied in several wind engineering problems, it has been rarely reported for non-linear non-Gaussian vibration analysis. Particularly, the coupling of nonlinear wake-oscillators, i.e., Van der Pol oscillator, renders the system unclosed so additional tactic should be introduced to make the equation solvable. This paper aims to present a cumulant equation method to investigate the influence of turbulence on vortex induced vibrations. This study is regarded as subsequent research by the authors in order to generalize the previous linear/Gaussian case to a non-linear/non-Gaussian case.

2 METHODOLOGIES

2.1 Nonlinear Stochastic Wake-oscillator Model

The dimensionless equations of motion of a 2D cylinder coupled with the nonlinear wake-oscillator proposed by Tamura under white noise excitations following the quasi-steady theory reads

$$Y'' + \left[2\xi_0 + \frac{n\nu}{S^*}(f_m + C_D + C_L')\right]Y' + Y = \frac{f_m n\nu^2}{S^{*2}}\alpha - \frac{n\nu^2}{S^{*2}}C_L - \frac{n\nu^2}{S^{*2}}(C_D + C_L')\frac{w}{U}$$

$$\alpha'' - 2\beta\nu\left[1 - \frac{4f_m^2}{C_{L0}^2}\alpha^2\right]\alpha' + \nu^2\alpha = \lambda Y'' + \nu S^* Y'$$

with the dimensionless parameters defined as usual in the literature. The damping term $\alpha^2\alpha'$ renders the system non-linear, so the responses are non-Gaussian despite Gaussian excitations.

2.2 Differential Equations Satisfied by Statistical Moments

Based on Itô's lemma, the moments equation is derived from first-order differential equations

$$\mathrm{dE}[\xi]/\mathrm{d}t = \sum_{i}^{4} \mathrm{E}[g_i(\partial\xi/\,\partial X_i)] + (1/2)\sum_{i,j}^{4} \mathrm{E}\left[h_i h_j\left(\partial^2\xi/\,\partial X_i\,\partial X_j\right)\right]$$

where $\mathrm{E}[\xi] = \mathrm{E}[Y^{e_1}Y'^{e_2}\alpha^{e_3}\alpha'^{e_4}]$ denotes the statistical moments. Due to the non-linear damping, moments of higher orders render the system unclosed, which means that calculating the moments of a certain order $s = \sum_1^4 e_i = l$ relies on the moments of higher orders $s = l + 2$ that have not yet been determined, when the approach consists in solving in increasing order. The assembled moments equation can be expressed as

$$\dot{\mathbf{m}}_{s\le l} = \mathbf{P}_0\mathbf{m}_{s\le l} + \mathbf{P}_1\mathbf{m}_{s=l+1} + \mathbf{P}_2\mathbf{m}_{s=l+2} + \mathbf{Q}$$

in which $\mathbf{m}_{s\le l}$ is the vector of all the unknown moments of the order s less than or equal to l; $\mathbf{m}_{s=l+1}$, and $\mathbf{m}_{s=l+2}$ are the vectors of all the unknown moments of higher orders.

2.3 Cumulant Neglect Closure

To close the moments equation, the cumulant-neglect closure method is employed by assuming that cumulants above a prescribed closure level are negligible. Therefore, higher-order moments can be transformed as $\mathbf{m}_{s=l+1} = \mathbf{F}_1(\mathbf{m}_{s\le l})$ in which $\mathbf{F}_1(\cdot)$ is a non-linear function. The moments equation can be finally rewritten as

$$\dot{\mathbf{m}}_{s\le l} = \mathbf{P}_0\mathbf{m}_{s\le l} + \mathbf{P}_1\mathbf{F}_1(\mathbf{m}_{s\le l}) + \mathbf{P}_2\mathbf{F}_2(\mathbf{m}_{s\le l}) + \mathbf{Q}$$

which is closed and solvable. The above formulas are the cumulant equation method (CEM).

3 NUMERICAL APPLICATIONS

To illustrate the reliability, the proposed CEM is validated by comparison with Monte Carlo Method (MCM) on a simple Van der Pol oscillator under external Gaussian white noise. Fig. 1 shows the RMS of displacement responses given by the CEM with closure level $l = 2,4,6,8$ and the MCS with 10^5 samples. Results of CEM with closure level higher than 4 agree well with MCM. Besides, the computing efficiency of CEM (7.8 s) is much higher than that of MCM (10 h).

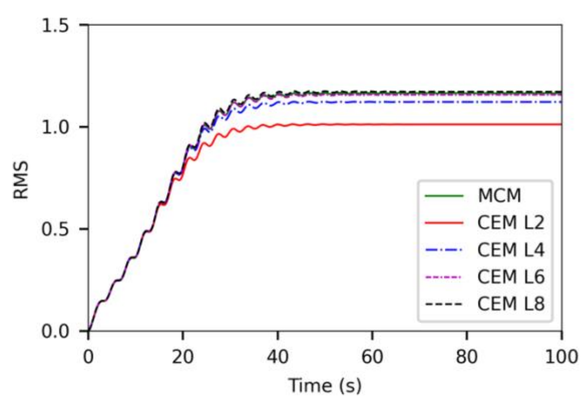

Fig. 1. RMS responses of Van der Pol oscillator