LIÈGE université
Sciences Appliquées
Structural & Stochastic Dynamics
Urban & Environmental Engineering

EACWE 2025
Trondheim

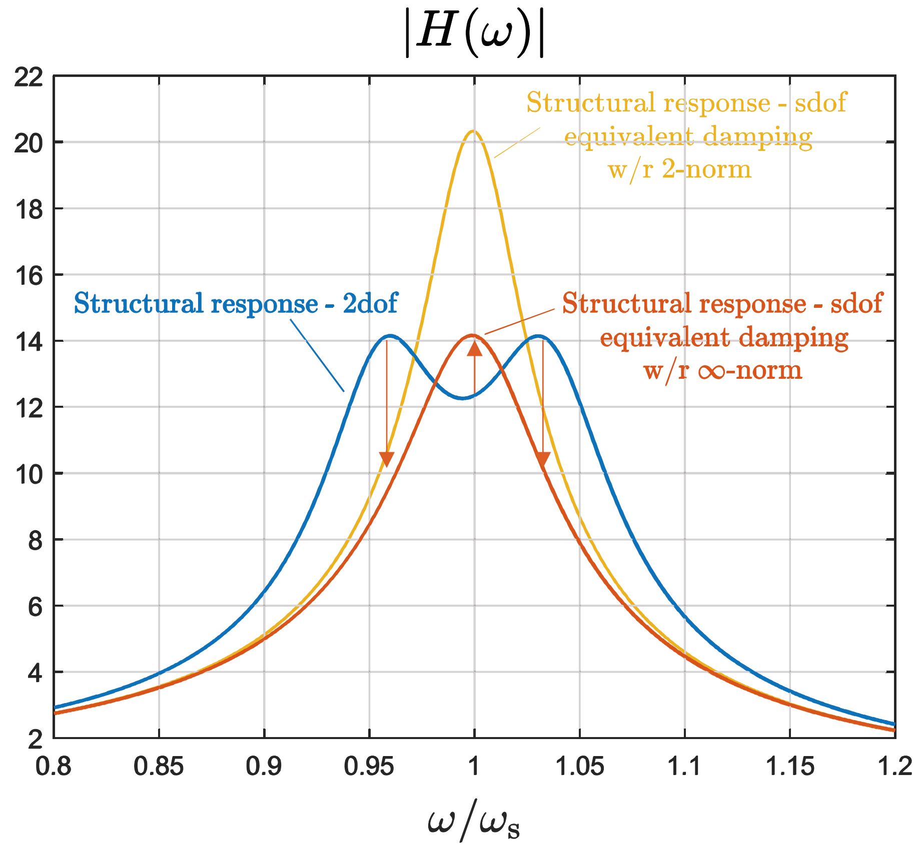
NTNU
Norwegian University of Science and Technology

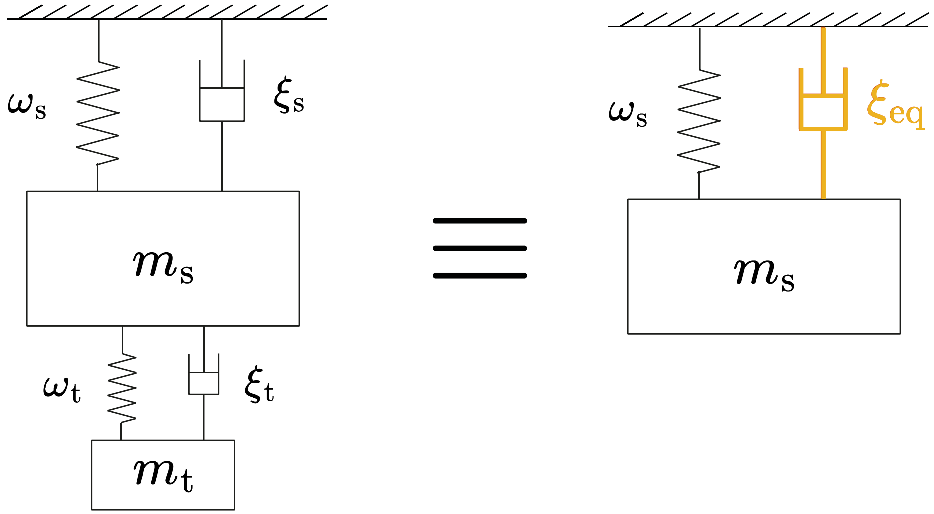
International Association for Wind Engineering

Equivalent damping in structures subjected to vortex induced vibrations and damped with TMDs

Anass Mayou, V. Denoël

Motivation





Strategy 1 : $\dfrac{1}{2\xi_{\mathrm{eq}} k_s} = \max\left(|H_{2\mathrm{dof}}(\omega)|\right)$

Strategy 2 : $\dfrac{\pi}{2\xi_{\mathrm{eq}}} \dfrac{\omega_{\mathrm{s}}}{k_{\mathrm{s}}^2} = \displaystyle\int_{-\infty}^{+\infty} \left|H_{2\mathrm{dof}}(\omega)\right|^2 d\omega$

$$= \frac{\pi}{\mu_t \varphi^2} \frac{\xi_{\mathrm{t}}^2 + (\omega_{\mathrm{t}}/\omega_{\mathrm{s}} - 1)^2 + \mu_{\mathrm{t}}\varphi^2/4}{\xi_{\mathrm{t}}} \frac{\omega_{\mathrm{s}}}{k_{\mathrm{s}}^2}$$

Mayou, A., & Denoël, V. (2022)

? Are these criterias still valid with non-linear systems ?

Multiple Timescales formulation (1)

$$\begin{pmatrix} m_s & 0 \\ 0 & m_t \end{pmatrix} \begin{pmatrix} \ddot{y} \\ \ddot{z} \end{pmatrix} + \begin{pmatrix} c_s + c_t & -c_t \\ -c_t & c_t \end{pmatrix} \begin{pmatrix} \dot{y} \\ \dot{z} \end{pmatrix} + \begin{pmatrix} k_s + k_t & -k_t \\ -k_t & k_t \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$

$$\bar{I}\ddot{q} - \bar{C}\left(1 - \frac{4f^2}{C_{L,0}^2} q^2\right)\dot{q} + \bar{K}\left(q + \frac{\dot{y}}{U}\right) = -\frac{\bar{I}}{\frac{D}{2} + \bar{l}}\ddot{y}$$

Tamura, Y., (1981)

Numerical solver

$y^{ss}(t)$

Full Dynamics

Non-dimensionalisation

Assumptions on orders of magnitude

Truncation at first order

$$\mathcal{M}X'' + \mathcal{C}X' + \mathcal{K}X = 2\varepsilon M_0 \Omega^2 Q \qquad X = \begin{pmatrix} Y \\ Z \end{pmatrix}$$

$$Q'' + 2\varepsilon\bar{\zeta}\left(Q^2 - 1\right)\Omega Q' + \Omega^2 Q = 2\varepsilon\left(A_0 Y'' + A_1 \Omega Y'\right)$$

fluid	structure	TMD
$A_0 = -\dfrac{f}{C_{L,0}} m^* \sim \mathrm{ord}(1)$	$\Omega = \omega_\nu / \omega_s = 1 + \varepsilon\gamma$	$\alpha = \omega_t / \omega_s = 1 + \varepsilon\delta$
$A_1 = -\dfrac{f}{C_{L,0}} S^* \sim \mathrm{ord}(1)$	$\xi_s = \varepsilon^2 \bar{\xi}_s$	$\mu = m_t / m_s = \varepsilon^2 \bar{\mu}_t$
$M_0 = -\dfrac{1}{8\pi^3} \dfrac{\mu_0}{\varepsilon^2} \dfrac{C_{L,0}}{S^2} \sim \mathrm{ord}(1)$		$\xi_t = \varepsilon \bar{\xi}_t$

$$Y_0(\tau, \varepsilon\tau) = R_y(\varepsilon\tau)\, e^{i\phi(\varepsilon\tau)}\, e^{i\tau}$$

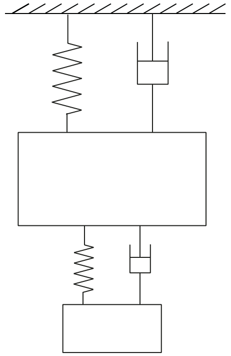
$$Z_0(\tau, \varepsilon\tau) = R_z(\varepsilon\tau)\, e^{i\phi(\varepsilon\tau)}\, e^{i\psi_z(\varepsilon\tau)}\, e^{i\tau}$$

$$Q_0(\tau, \varepsilon\tau) = R_q(\varepsilon\tau)\, e^{i\phi(\varepsilon\tau)}\, e^{i\psi_q(\varepsilon\tau)}\, e^{i\tau}$$

Multiple Timescales

$$R_{y,t}^{ss} = \left(4\frac{\bar{\xi}_t^2}{\bar{\mu}_t} \frac{1}{\sin^2\psi_z}\right)\left(2\frac{M_0}{\bar{\xi}_t}\sin\psi_q\right)\sqrt{1 + \left(4\frac{\bar{\xi}_t^2}{\bar{\mu}_t}\frac{1}{\sin^2\psi_z}\right)\left(\frac{M_0}{\bar{\xi}_t}\right)\left(2A_0\sin^2\psi_q + A_1\sin 2\psi_q\right)}$$

With TMD :



$c_q = \cot\psi_q$
Fluid/Structure

$c_z = \cot\psi_z$
TMD/Structure

$$R^{ss}_{y,\mathrm{t}} = \left(4\frac{\bar{\xi}_\mathrm{t}^2}{\bar{\mu}_\mathrm{t}}(1+c_z^2)\right)\left(2\frac{M_0}{\bar{\xi}_\mathrm{t}}\frac{1}{\sqrt{1+c_q^2}}\right)\sqrt{1+\left(4\frac{\bar{\xi}_\mathrm{t}^2}{\bar{\mu}_\mathrm{t}}(1+c_z^2)\right)\left(2\frac{M_0}{\bar{\xi}_\mathrm{t}}\frac{1}{\sqrt{1+c_q^2}}\right)\frac{A_0+A_1c_q}{\sqrt{1+c_q^2}}}$$

$$c_q = -4\frac{\bar{\xi}_\mathrm{t}^2}{\bar{\mu}_\mathrm{t}}c_z^3 + \left(1-4\frac{\bar{\xi}_\mathrm{t}^2}{\bar{\mu}_\mathrm{t}}\right)c_z$$

$$c_z^7 + a_6c_z^6 + a_5c_z^5 + a_4c_z^4 + a_3c_z^3 + a_2c_z^2 + a_1c_z + a_0 = 0$$

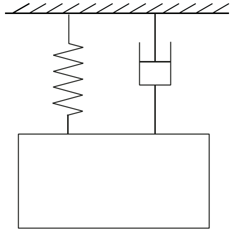
$$D_0^\mathrm{t} = \frac{A_0M_0}{\bar{\xi}_\mathrm{t}^2} \qquad D_1^\mathrm{t} = \frac{A_1M_0}{\bar{\xi}_\mathrm{t}^2}$$

$$\Gamma_0^\mathrm{t} = \frac{\delta-\gamma}{\bar{\xi}_\mathrm{t}} \qquad \Gamma_1^\mathrm{t} = 4\frac{\bar{\xi}_\mathrm{t}^2}{\bar{\mu}_\mathrm{t}}$$

$$\left(\delta_{\mathrm{opt}},\bar{\xi}_{\mathrm{t,opt}}\right) = \left(0,\frac{1}{2}\sqrt{\bar{\mu}_\mathrm{t}}\right)$$

Mayou, A., & Denoël, V. (2022)

Without TMD :



$c = \cot\psi$
Fluid/Structure

$$R^{ss}_{y,0} = \left(2\frac{M_0}{\bar{\xi}_0}\frac{1}{\sqrt{1+c^2}}\right)\sqrt{1+\left(2\frac{M_0}{\bar{\xi}_0}\frac{1}{\sqrt{1+c^2}}\right)\frac{A_0+A_1c}{\sqrt{1+c^2}}}$$

Rigo, F., Andrianne, T., & Denoël, V. (2022)

$$c^3 + \Gamma_0c^2 + \left(1+D_0^0\right)c + \left(\Gamma_0 - D_1^0\right) = 0$$

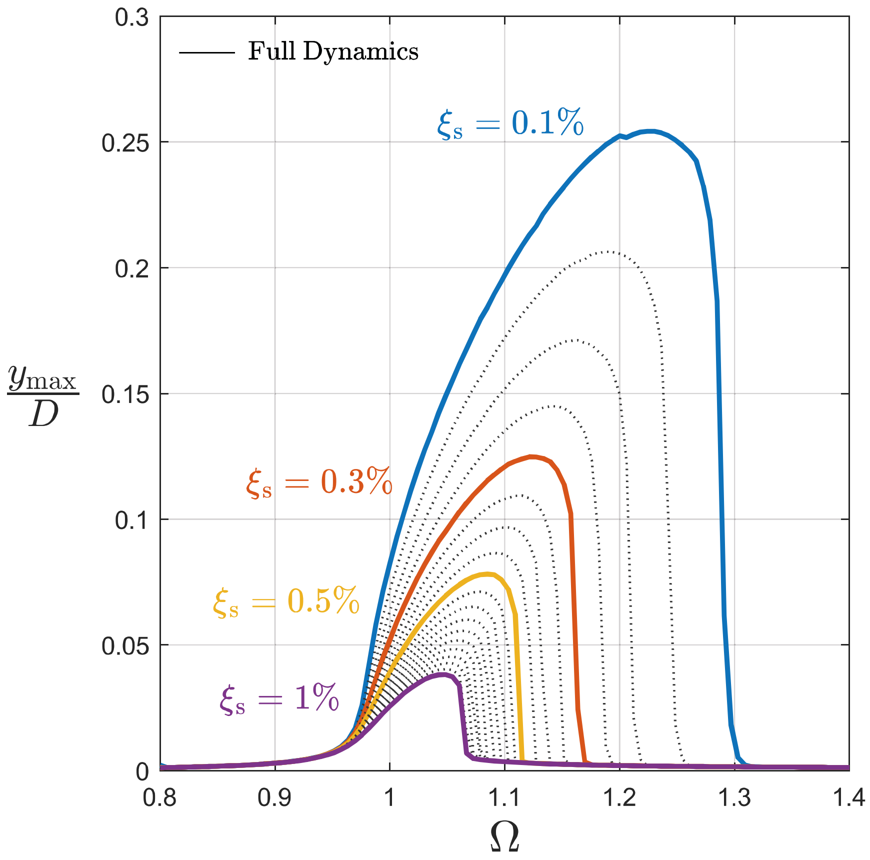
4→3

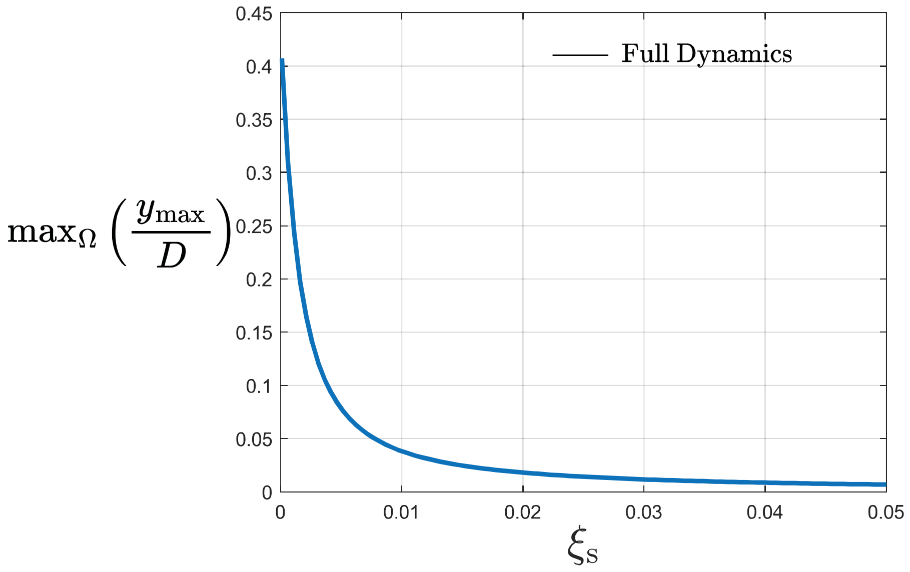
$$D_0^0 = \frac{A_0M_0}{\bar{\xi}_0^2} \qquad D_1^0 = \frac{A_1M_0}{\bar{\xi}_0^2}$$

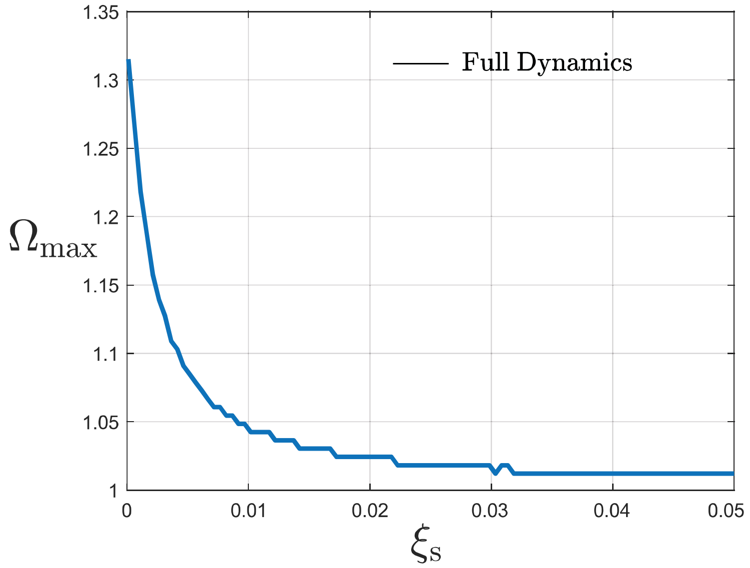
$$\Gamma_0^0 = \frac{\gamma}{\bar{\xi}_0}$$

Structure – without TMD

Full Dynamics

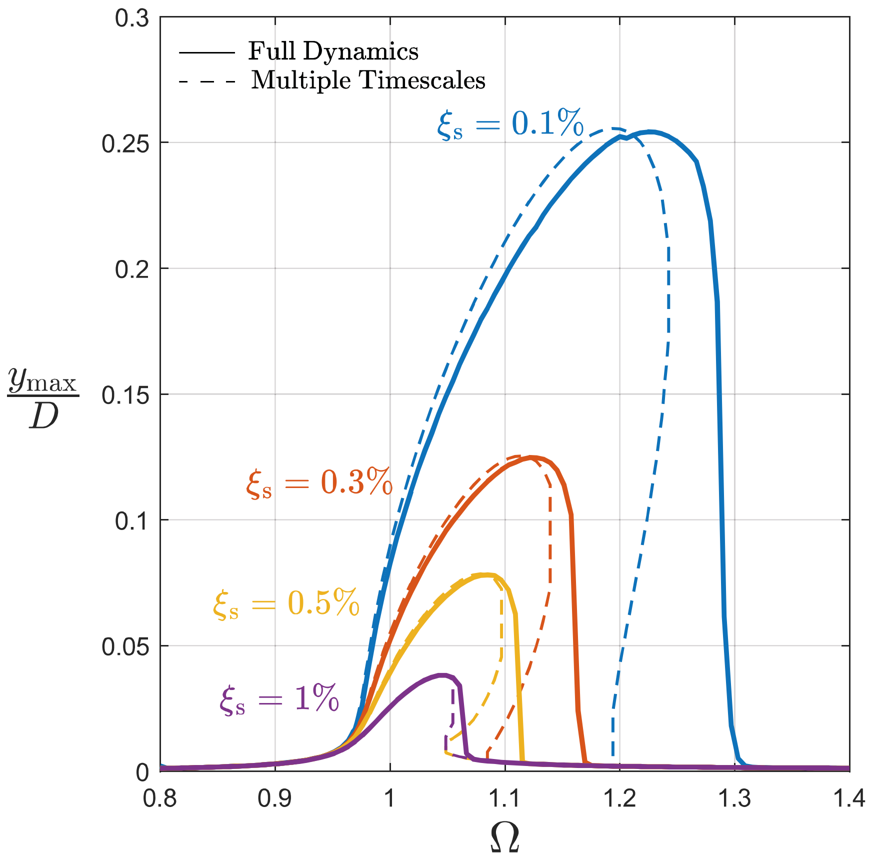


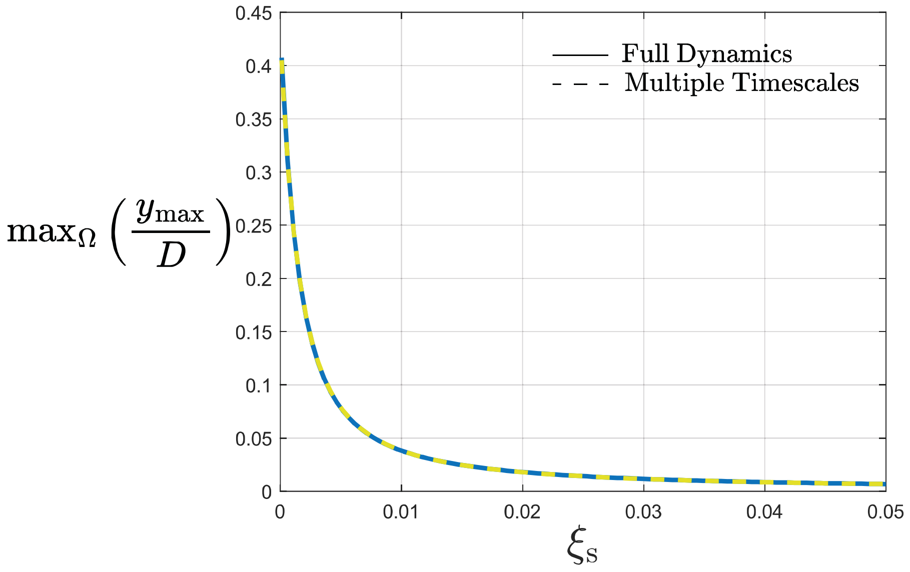


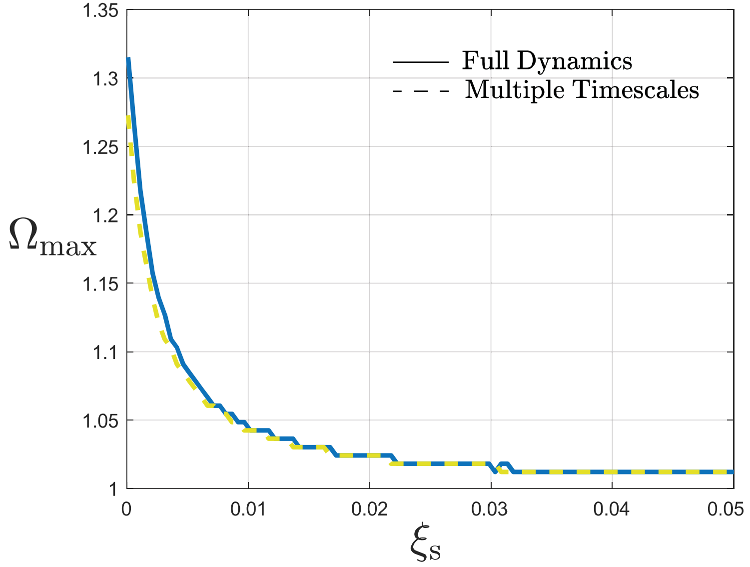


Structure – without TMD

Full Dynamics vs Multiple Timescales

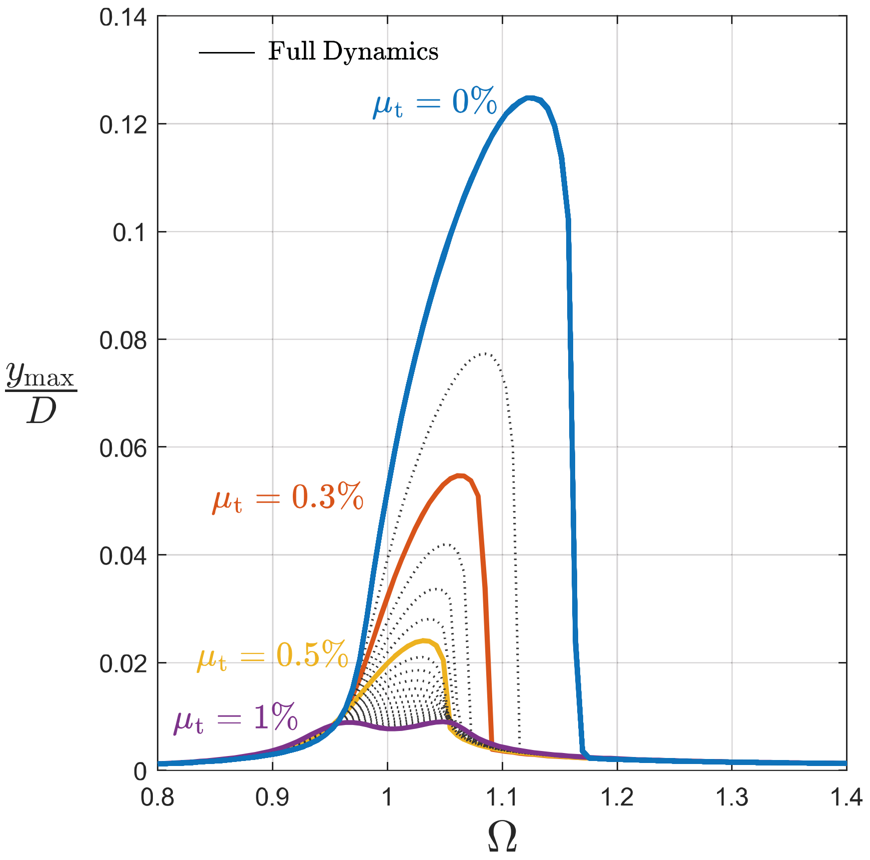


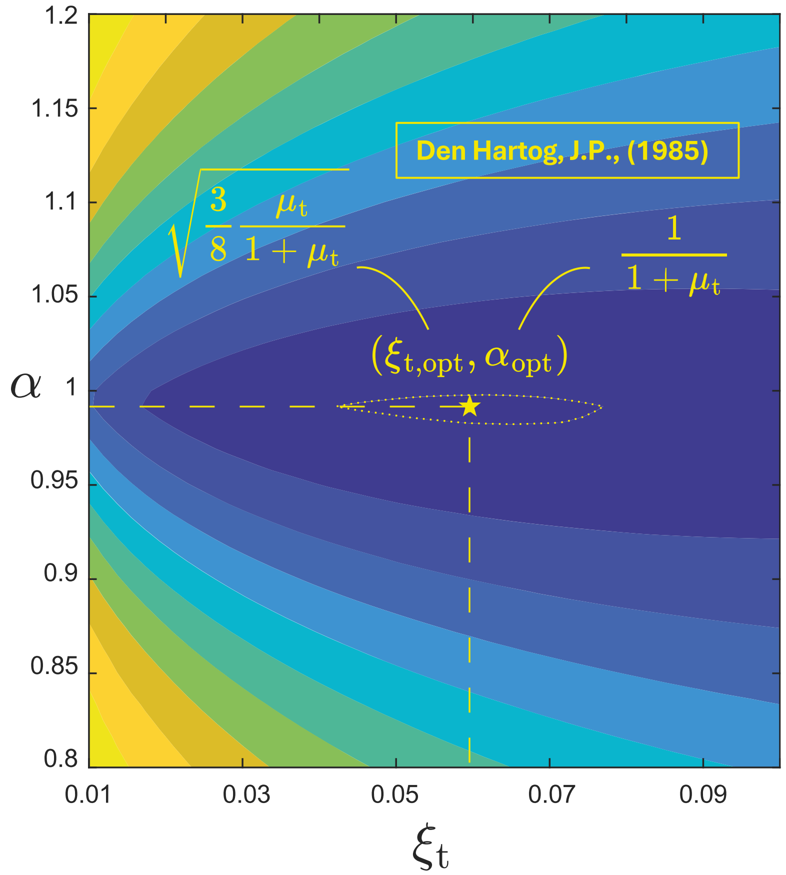




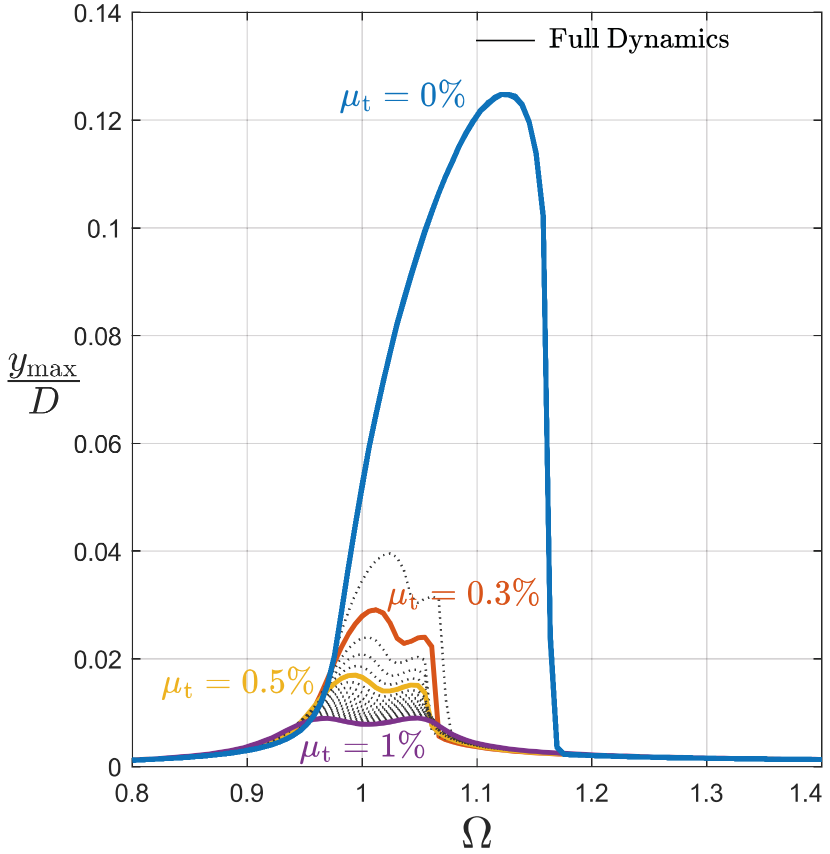
Structure – with TMD

Full Dynamics vs Multiple Timescales





Optimal tuning conditions

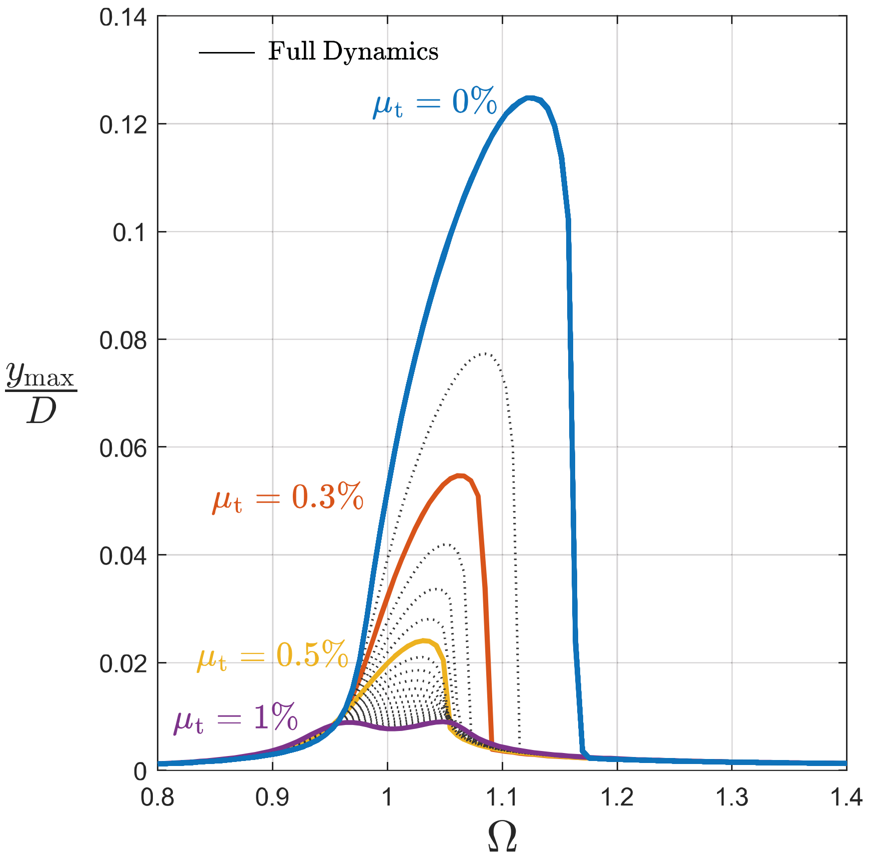


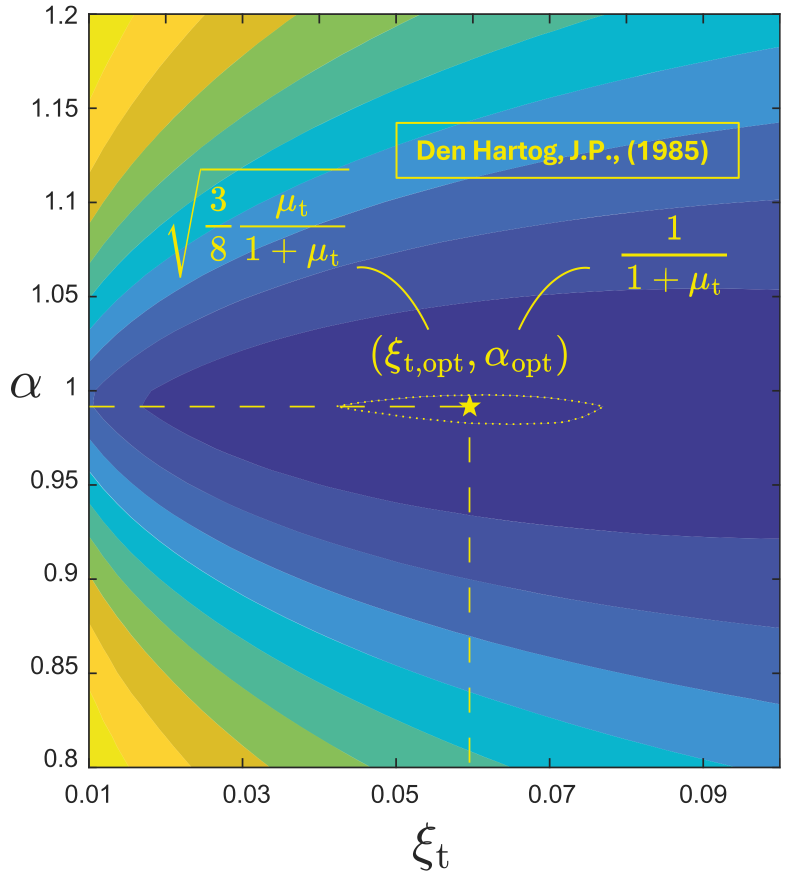
$$\mathcal{M}X'' + \mathcal{C}X' + \mathcal{K}X = 2\varepsilon M_0 \Omega^2 Q$$

$$Q_0(\tau, \varepsilon\tau) = R_q(\varepsilon\tau)\, e^{i\phi(\varepsilon\tau)}\, e^{i\psi_q(\varepsilon\tau)}\, e^{i\tau}$$

Structure – with TMD

Full Dynamics vs Multiple Timescales





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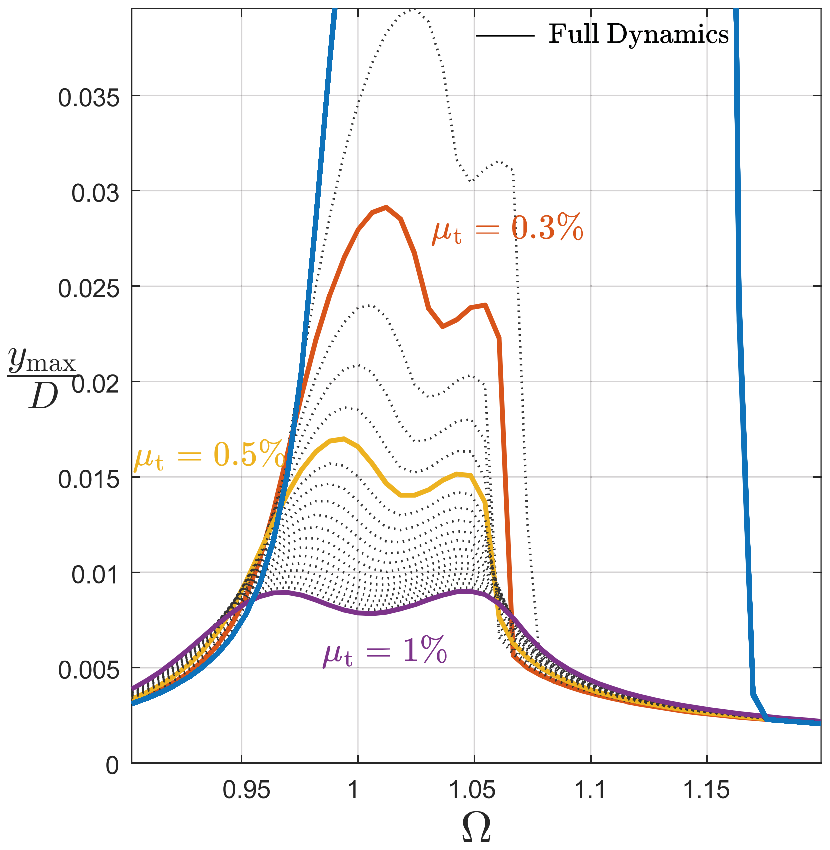

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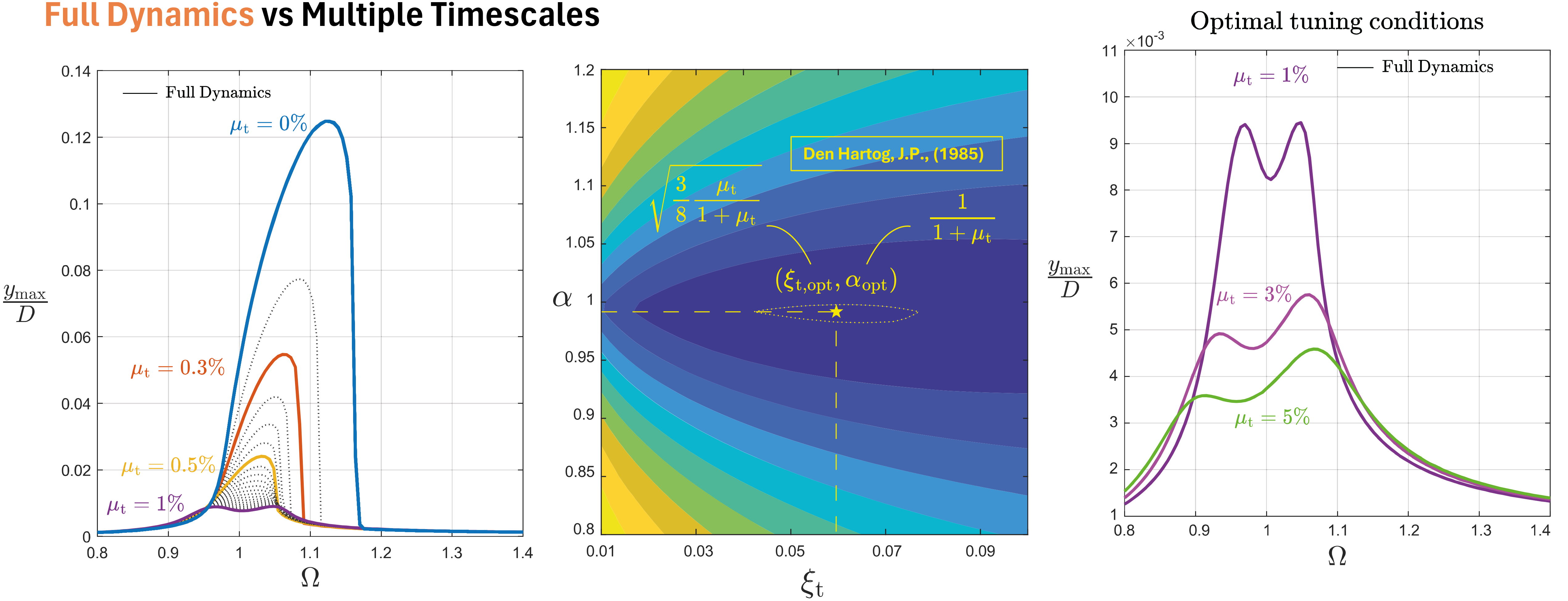
Structure – with TMD

Full Dynamics vs Multiple Timescales



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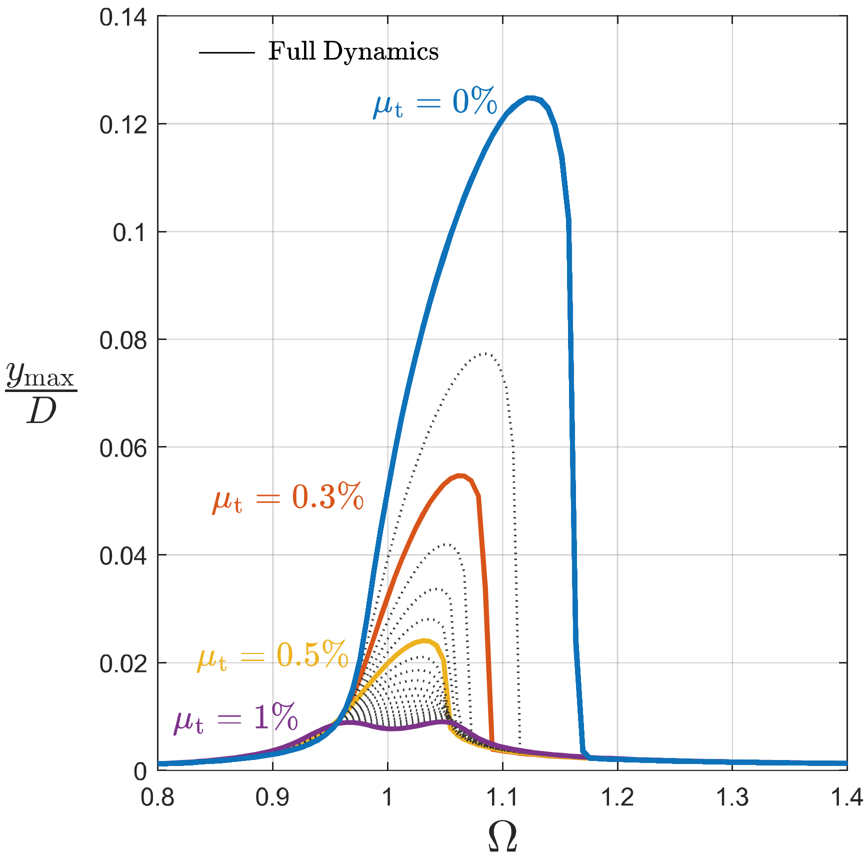
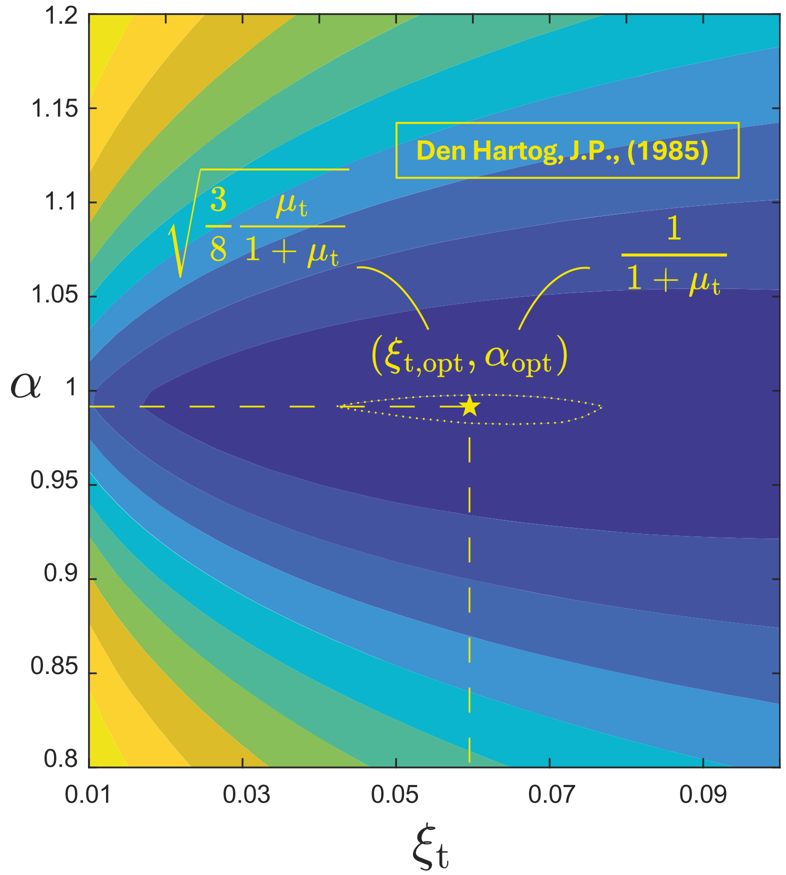
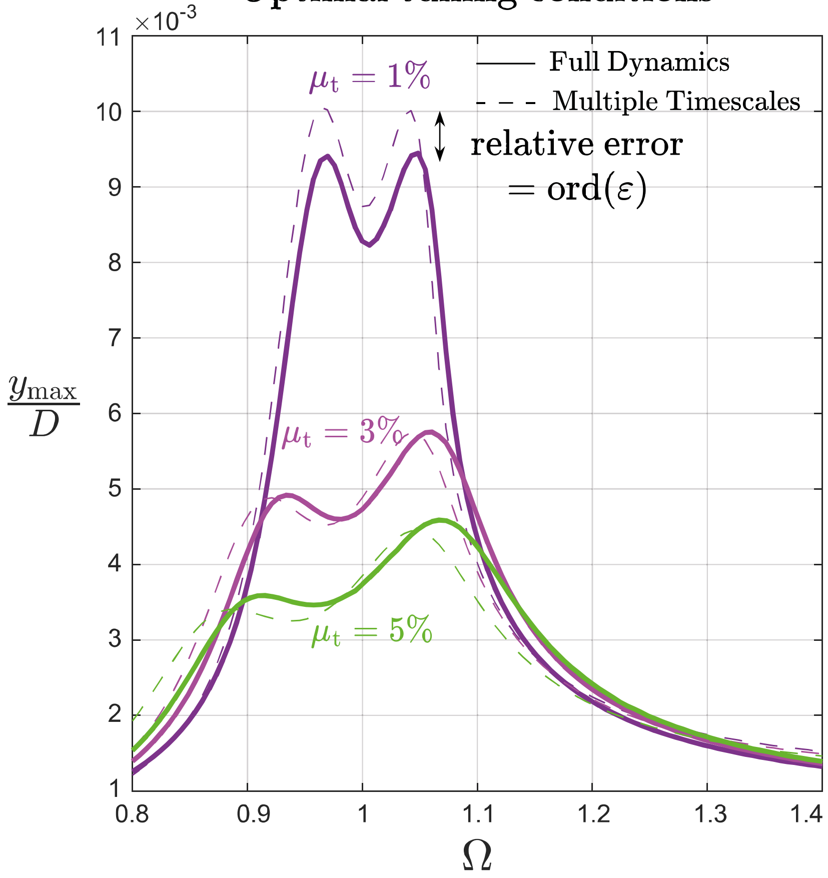
Full Dynamics vs Multiple Timescales





Optimal tuning conditions



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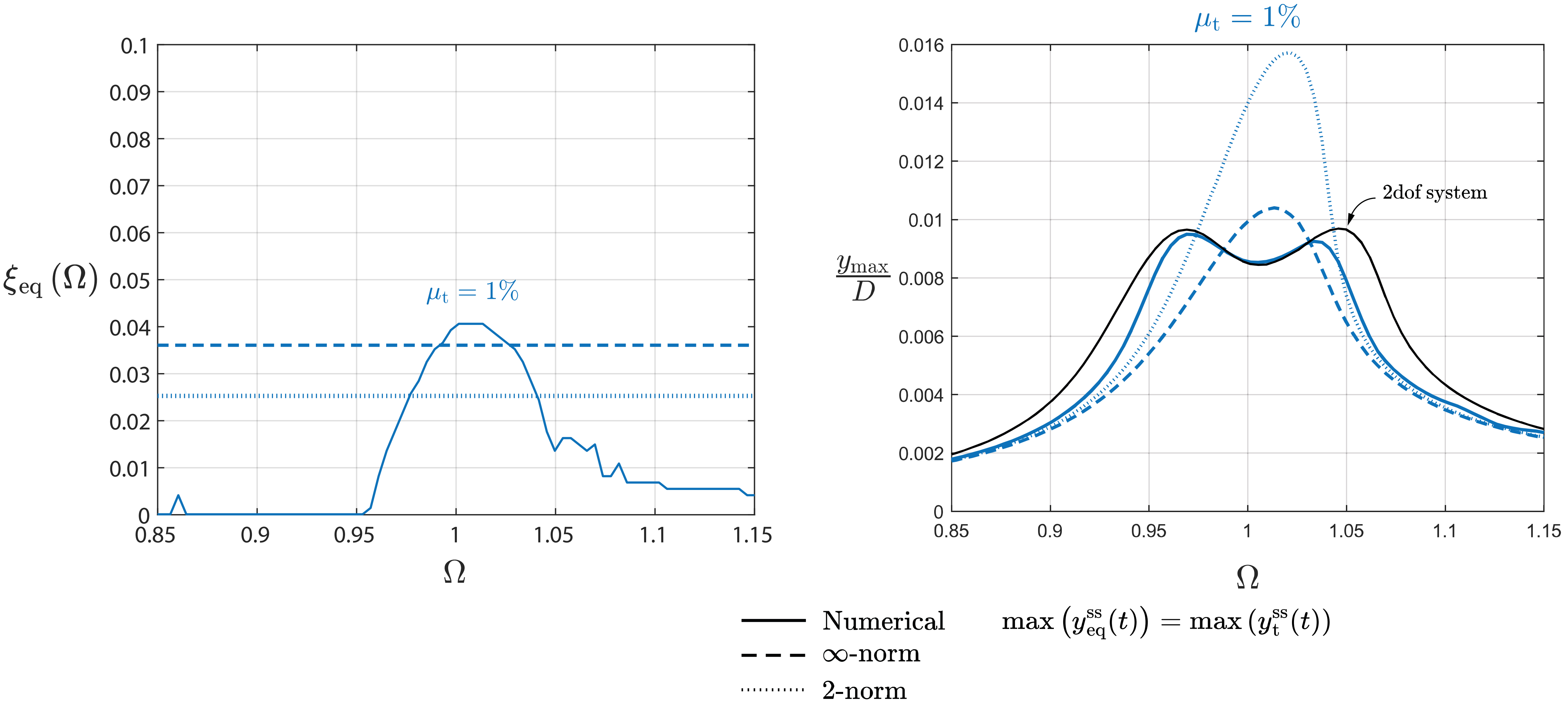
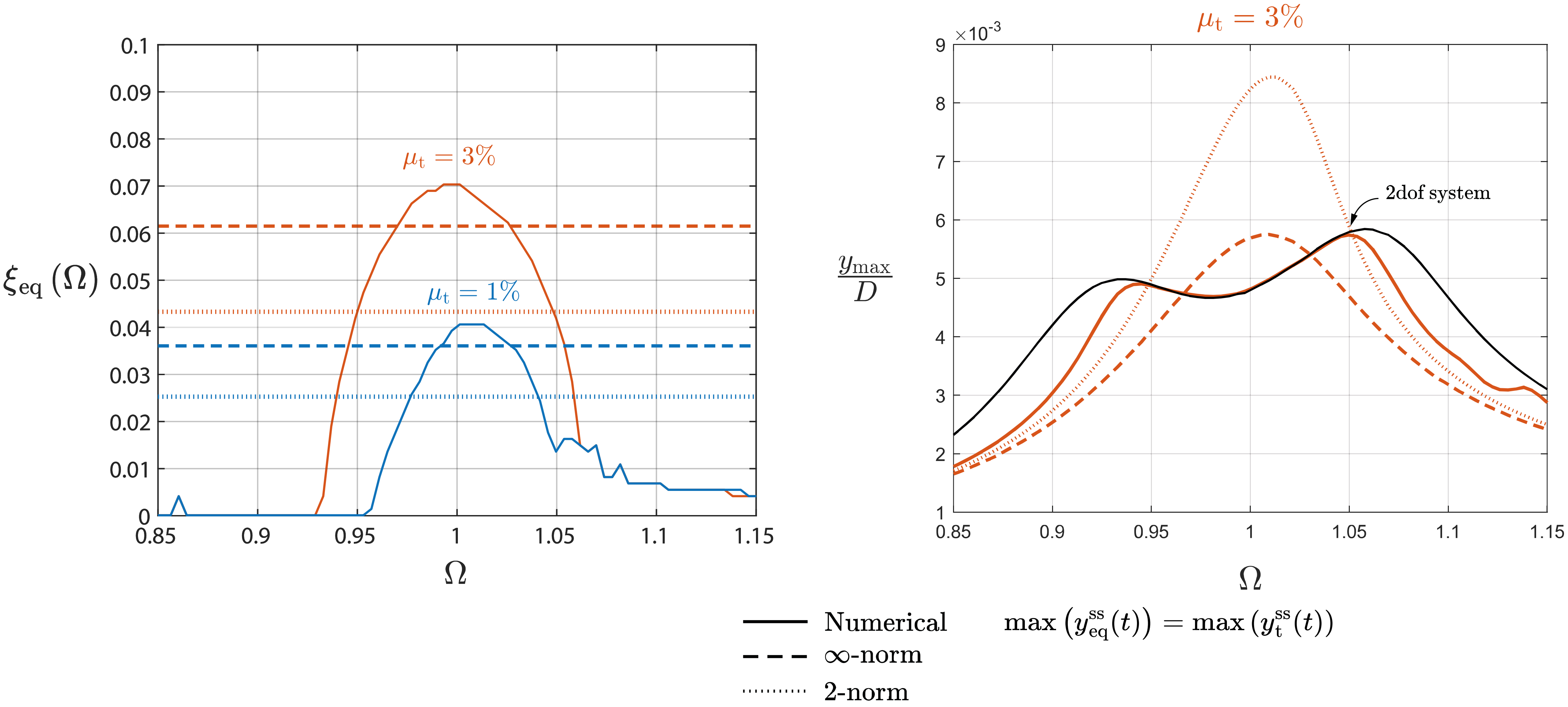
Equivalent damping {Structure + TMD} vs VIV

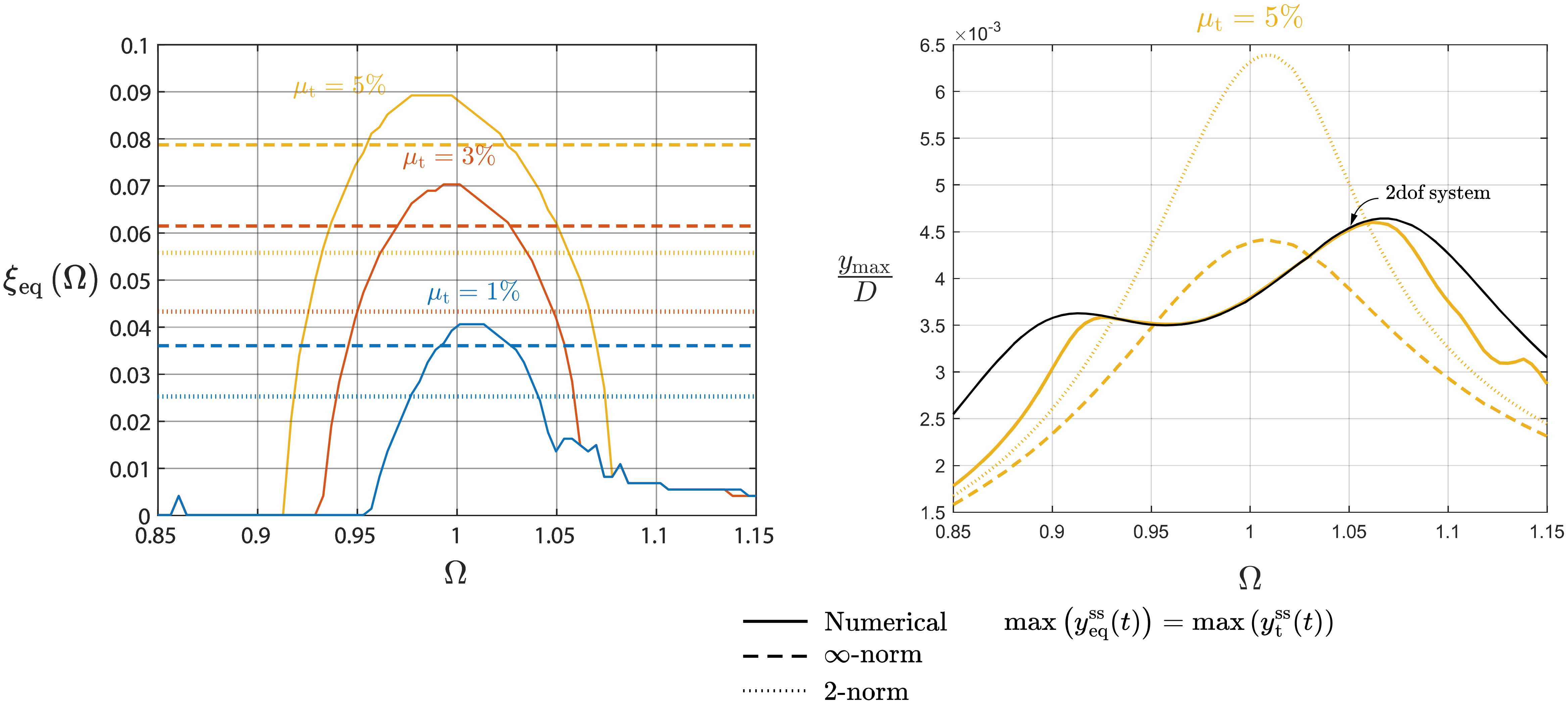


Equivalent damping {Structure + TMD} vs VIV

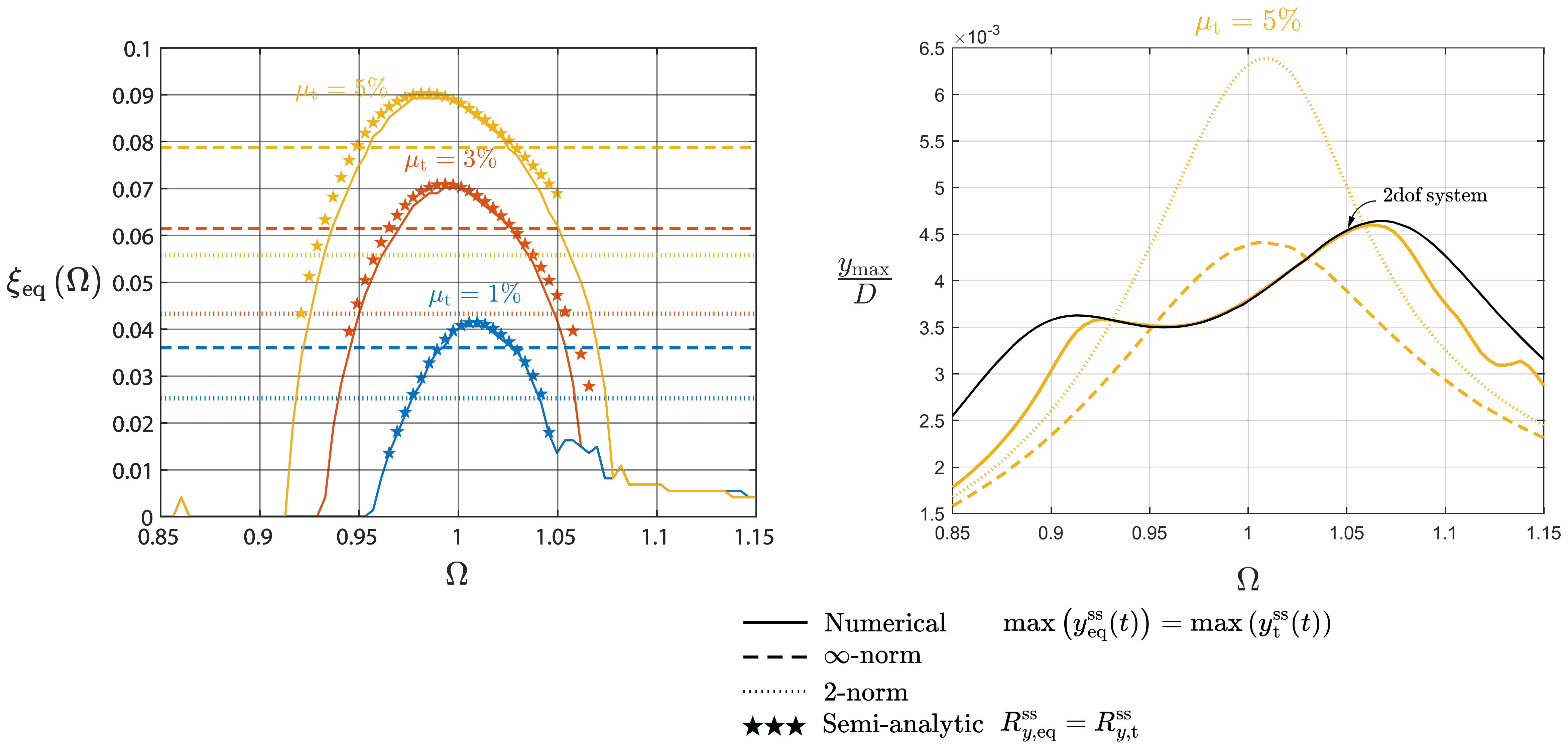


$$\max\left(y_{eq}^{ss}(t)\right) = \max\left(y_{t}^{ss}(t)\right)$$

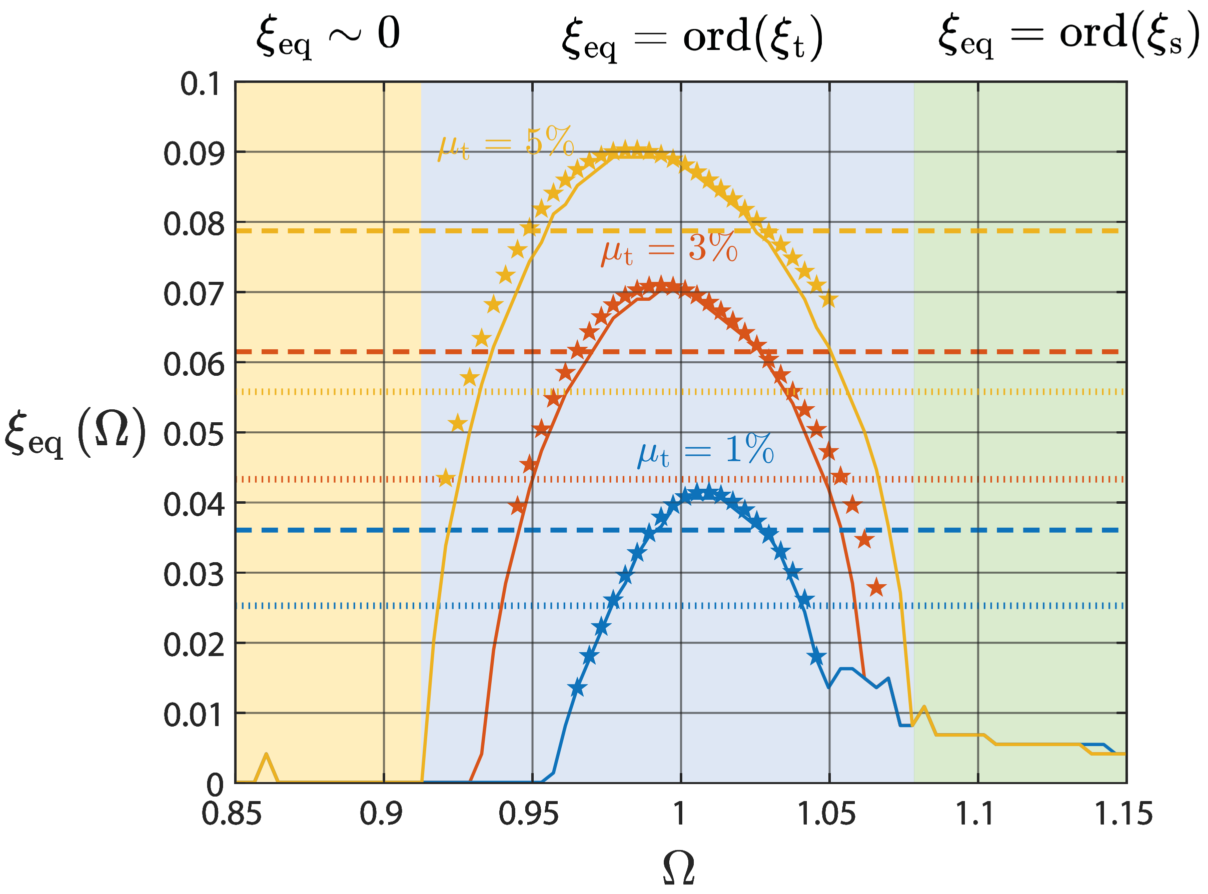
Equivalent damping {Structure + TMD} vs VIV

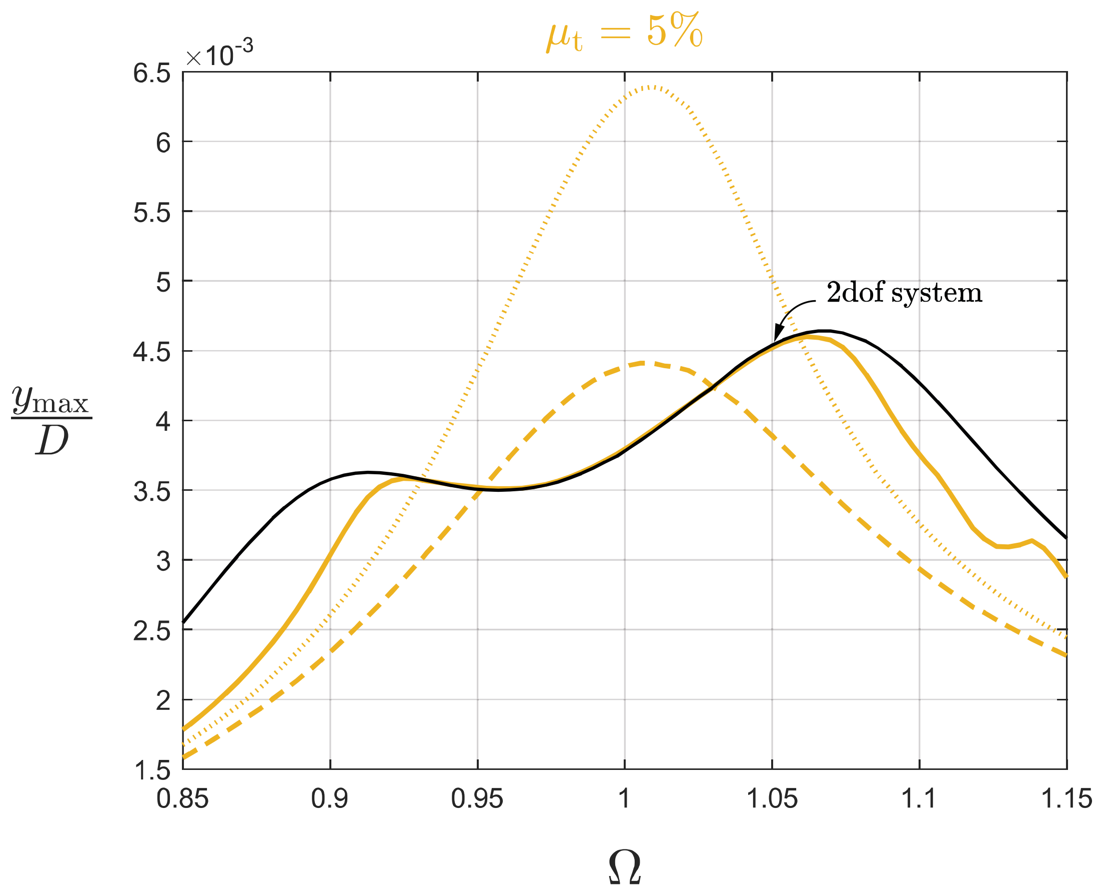


Equivalent damping {Structure + TMD} vs VIV



Equivalent damping {Structure + TMD} vs VIV





Numerical $\max\left(y_{eq}^{ss}(t)\right) = \max\left(y_t^{ss}(t)\right)$

∞-norm

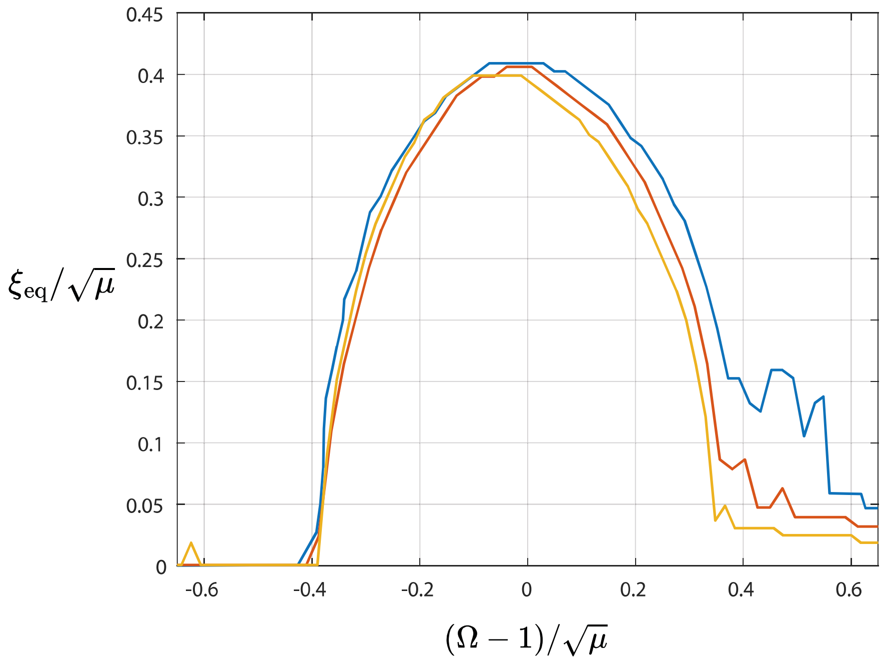
2-norm

★★★ Semi-analytic $R_{y,eq}^{ss} = R_{y,t}^{ss}$

Concluding remarks

- **Multiple timescale** asymptotic method
  - predicts structural dynamics without/with TMD
  - is **accurate** and **computationally efficient**

- **Equivalent damping**
  - enables **universal curves** for the equivalent damping for practitioners



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Thank you – Takk