Equivalent damping in structures subjected to vortex induced vibrations and damped with tuned mass dampers

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SUMMARY:
This paper discusses methods for deriving equivalent damping for vortex-induced vibrations. Here, equivalence is defined as the structural damping ratio that would produce the same maximum displacement as a structure equipped with a Tuned Mass Damper. Given that the wind loads resulting from VIV are neither perfectly narrowband nor wide band, the equivalence presented in this paper is unique. Based on Tamura's 2-D wake oscillator model, the three governing differential equations of the damped wake oscillator are analyzed and illustrated numerically.

Keywords: multiple scales, equivalence,

1. CONTEXT

Vortex-Induced Vibration (VIV) poses significant challenges in structural engineering, particularly for slender structures like bridges and towers. Effective control methods are crucial for maintaining structural integrity. There are several ways to model VIV, spanning from the simplest passive harmonic loading, to more complex wake-oscillator models **Hartlen1970577**, passing through nonlinear models and their statistical equivalent forms **Simiu1996**. Based on physical concepts and principles of mechanics, Tamura and Matsui's original 2-D wake oscillator model **TAMURA19801085** is able to capture many of the features of the problem, among which synchronization. It treats the structure as a single-degree-of-freedom oscillator, and the wake (lamina) as another degree-of-freedom, emphasizing the intricate dynamics essential for predicting lock-in conditions. Although their model has been refined multiple times to account for multi-degree-of-freedom systems, or specific sheared wind profiles or aerodynamic cross sections, the original version is studied in this work, in view of providing a simple understanding of the response of structures equipped with tuned mass dampers (TMDs).

Very few experimental studies have been reported on the use of tuned mass dampers for VIV. However, doughnut and tuned liquid dampers are regularly installed on tall chimneys to tame their daily vibrations **kareem1999mitigation**; **Rahman2024**. Also Gerges and Vickery **Gerges20031069** demonstrated the substantial response reduction achieved by TMDs during wind tunnel tests, suggesting that these systems offer improved equivalent damping compared to linear designs. Otherwise, many authors have focused on the wind response to VIV of structures equipped with TMDs, by means of numerical analysis **Andersen2001217**; **Yu2023**; **Ricciardelli20011539**.

2. METHODS AND MODEL

2.1. Governing equations

A structure damped with a TMD is represented with two degrees-of-freedom, y and z, representing the motion of the structure and the motion of the tuned mass in a fixed reference frame. The governing equations read

$$\begin{pmatrix} m & 0 \\ 0 & m_\mathrm{T} \end{pmatrix}\begin{pmatrix} \ddot{y} \\ \ddot{z} \end{pmatrix}+\begin{pmatrix} c+c_\mathrm{T} & -c_\mathrm{T} \\ -c_\mathrm{T} & c_\mathrm{T} \end{pmatrix}\begin{pmatrix} \dot{y} \\ \dot{z} \end{pmatrix}+\begin{pmatrix} k+k_\mathrm{T} & -k_\mathrm{T} \\ -k_\mathrm{T} & k_\mathrm{T} \end{pmatrix}\begin{pmatrix} y \\ z \end{pmatrix}=\begin{pmatrix} F_\mathrm{L} \\ 0 \end{pmatrix} \tag{1}$$

where m and m_T are the structural mass and the tuned mass, c and c_T are the structural viscosity and the viscosity of the damper, k and k_T are the structural stiffness and the spring stiffness of the damper, and F_L represents the lift force resulting from vortex shedding. The mass ratio $\mu = m_\mathrm{T}/m$ is usually fixed by the designer to values of about 1% to 3%, and the other two damper-specific parameters are adjusted to provide optimal performances **Fujino1993833**; **Mayou2022** They are usually expressed as a function of the frequency ratio $\beta = \omega_\mathrm{T}/\omega = \sqrt{(k_\mathrm{T}/m_\mathrm{T})\,/\,(k/m)}$, and the damping ratio $\xi_\mathrm{T} = c_\mathrm{T}/\left(2\sqrt{k_\mathrm{T}m_\mathrm{T}}\right)$. Concerning the structural parameters, m also represents the generalized mass, a quantity that is usually known from a prior modal analysis, together with the circular frequency ω, so that $k = m\omega^2$. In Tamura and Matsui's model, the total structural damping ratio ξ is a combination of the inherent structural damping $\eta = c/\left(2\sqrt{km}\right)$ and an aerodynamic damping proportional to the wind velocity: $\xi = 2\eta + n(f+C_D)\frac{\Omega}{S^\star}$. The lift force is expressed by means of the angular position of the lamina, which oscillates in the wake of the cylinder, following

$$\alpha'' - 2\zeta\Omega\left(1-\frac{4f^2}{C_{L0}^2}\alpha^2\right)\alpha' + \Omega^2\alpha = -m^\star Y'' - \Omega S^\star Y' \tag{2}$$

where $Y = y/D$ is the dimensionless crossflow displacement obtained by normalizing through the crossflow dimension D, and C_D, C_{L0}, n, $S^\star$, $m^\star$, ζ and f are model parameters. The reduced wind velocity $\Omega = \frac{VS^\star}{\omega D}$, is the control parameter in VIV analyses. The reader can refer to **TAMURA19801085** for detailed explanations on these parameters. In the following illustrations, they are chosen as indicated in Table 1.

2.2. Dimensionless equations

The nonlinearity in the wake equation is responsible for a limit cycle and, when coupling with the structural motion takes place ($m^\star \neq 0$ or $\Omega \neq 0$), this limit cycle can include the structural degree(s)-of-freedom. In the absence of additional loading, this phenomenon is semantically not a synchronization **Denoel2020**. The set of equations (1)-(2) is cast into a dimensionless formulation, with dimensionless time $\tau = \omega t$, similar to that employed in **Rigo2022**

$$\begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix}\begin{pmatrix} Y'' \\ Z'' \end{pmatrix}+2\begin{pmatrix} \xi+\beta\mu\xi_t & -\beta\mu\xi_t \\ -\beta\mu\xi_t & \beta\mu\xi_t \end{pmatrix}\begin{pmatrix} Y' \\ Z' \end{pmatrix}+\begin{pmatrix} 1+\beta^2\mu & -\beta^2\mu \\ -\beta^2\mu & \beta^2\mu \end{pmatrix}\begin{pmatrix} Y \\ Z \end{pmatrix}=\begin{pmatrix} -\frac{fn\Omega^2}{S^{\star 2}}\alpha \\ 0 \end{pmatrix} \tag{3}$$

so that a multiple scale analysis of the resulting set of equations can be derived. These two governing equations are combined with (2) to form the simplest version of the problem, expressed as a function of the states $Y = y/D$, $Z = z/D$ and α. The full version of the article will develop the multiple timescale analysis of the problem, and analyze the amplitude of the structural response in the entire lock-in range.

3. DEFINITION OF EQUIVALENT DAMPING RATIO

Designers often encounter challenges when simulating the response of damped structures to vortex-induced vibrations, since it requires establishment and solution of a set of equations similar to (2)-(3). Accurately capturing these dynamics can be complex and resource-intensive for practitioners. A practical and straightforward approach would be to employ an equivalent damping value, which allows for a simplified analysis of the system's behavior under VIV conditions. This equivalent damping can be defined as the H_∞ norm of the frequency response function (FRF), effectively consolidating the dynamic effects into a single parameter.

The equivalent damping $\xi_{\text{eq}}^{\text{FRF}}$ can be expressed mathematically as $\|H(j\omega)\|_\infty := 1/\left(2\xi_{\text{eq}}^{\text{FRF}}\right)$, which is engages a simpler approach for practitioners. This formulation enables designers to access the response of the damped structure without resolving the specific characteristics of the wind loading. In the article, we show that this approach is not accurate, sometimes significantly unconservative. Alternatively, a more rigorous definition of the equivalent damping ξ_{eq} is given in this paper as the value of total structural damping ξ to be chosen in the analysis of the undamped system ($\mu = 0$), so that the maximum structural response in this case corresponds to the baseline results of the 3-DOF problem:

$$x^{\text{3-DOF}}(\xi, \Omega) = x^{\text{2-DOF}}\left(\xi_{\text{eq}}, \Omega\right). \tag{4}$$

4. ILLUSTRATIONS AND DISCUSSION

Table 1 provides the numerical values chosen for the illustrations of Test Case #1. They correspond to the values adjusted in **TAMURA19801085** to the experimental results of Feng. Additionally, the mass ratio μ of the TMD is varied across several values, while its relative frequency ratio β and damping ratio ξ_{T} are fixed to 0.99 and 5% irrespective of the mass ratio (this choice is made to make Test Case #1 simple, but is in fact suboptimal with respect to several existing tuning strategies **Fujino1993833**).

Figure 1-a shows the maximum structural displacement Y_{max} obtained in Test Case #1 while varying the total structural damping ξ from 10^{-3} to 10^{-1}. These responses have been obtained by simulating the 3-DOF problem with $\mu = 0$, which degenerates into the 2-DOF problem (without TMD). These values of the structural response will serve as a reference to define the equivalent damping ξ_{eq}. Figure 1-b shows the maximum structural response for various values of the mass ratio μ. It is noted that under nearly ideal tuning conditions, the TMD demonstrates remarkable efficiency, likely surpassing that observed with other VIV loading models **Andersen2001217**. This is understood as a strength of the Tamura model in effectively capturing the rapid variations of Y_{max} with respect to Sc (more detailed illustrations will follow in the full paper).

The equivalent damping ξ_{eq} is defined independently for each value of the reduced wind velocity Ω, as the value of the structural damping ξ (in 1-a) that provides the same Y_{max} as the considered case. This yields the results shown in Figure 1-d. Since the equivalent damping is determined independently for each wind velocity, the primary result of this study is $\xi_{\text{eq}}(\Omega)$. This definition yields results that vary significantly with wind velocity, reaching up to 7.5% for a TMD mass ratio of only 1.5%. However, to provide a conservative estimate of the equivalent damping ratio across the entire range of wind velocities within the lock-in range, one should consider the minimum value, which is approximately 1.5% in this case. These values are visually compared to the equivalent damping ξ_{eq}^{FRF}, derived solely from the infinite norm of the FRF. While for lower mass ratios, i.e. smaller equivalent damping, ξ_{eq}^{FRF} tends to be an upper bound of $\xi_{eq}(\Omega)$ across the lock-in range, for larger mass ratios such as $\mu = 1.5\%$, ξ_{eq}^{FRF} only agrees well on average with $\xi_{eq}(\Omega)$, yielding therefore to significant over- and under-estimations in the structural response.

Table 1. Numerical values used for the illustrations. Additional test cases will be included in full paper.

Param.:	Ω	μ	β	η	ξ_T	f	C_{L0}	C_D	n	$m^\star$	$S^\star$	ζ
Case #1 :	$\{\cdots\}$	$\{\cdots\}$	0.99	0.00181	0.05	1.16	0.4	1.2	0.00257	0.625	$2\pi\cdot 0.2$	0.038

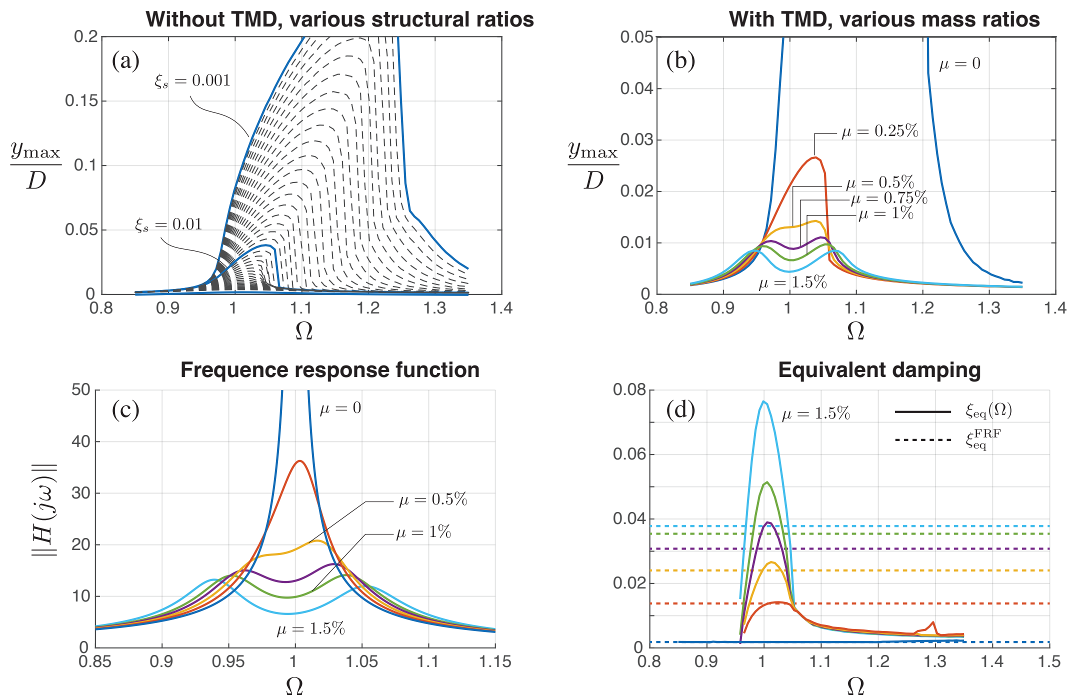

Figure 1. (a) Structural response obtained without TMD and for various structural damping ratios, (b) Structural response obtained with variousTMDs and with $\xi_s = 0.0018$, (c) FRFs of the damped structure for various mass ratios, (d) equivalent damping: solid lines=current definition; dashed lines, definition based on H_∞ norm of the FRF.