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Nonlinear Dynamics of Anchoring Elements for Submerged Floating Tunnels subject to Hydrodynamic Loads

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1. Introduction

Seabed-anchored Submerged Floating Tunnels (SFTs) are interesting modular structures which are deemed to be a valuable option for crossing deep and long waterways, such as sea straits, fjords, bays and alpine lakes. Due to their inherent flexibility, SFTs are prone to the effect of dynamic loadings, such as earthquakes, waves, currents and traffic. Among all of them, hydrodynamic loadings are deemed to be the most critical for serviceability and fatigue life assessments of the whole SFT, because of their consistent and cyclic nature. While the global hydrodynamic response of the SFT can be studied by resorting to several modeling strategies, the local dynamic response of the anchoring elements is inherently hard to capture with standard finite element models, because their natural frequencies are well-separated from those of the global vibration modes.
The present work briefly reviews the main findings of the research activity recently carried out by the authors on this topic [1, 2]. A reduced-order cross-sectional model of the SFT is formulated and the coupled system of equations governing its dynamic response is derived. Different loading conditions are then identified as critical and studied one at time. The multiple time-scales perturbation method is applied to derive approximate closed-form solutions of the steady-state vibration amplitude of the anchoring elements. As an application example, a parametric analysis in the drag coefficient of the anchor is carried out, providing useful considerations aimed at supporting the preliminary (conceptual) design phase of SFTs.

2. Cross-sectional model

The cross-section of the tube is modeled as a two-dofs oscillator, possessing mass and stiffness which are representative of the same modal quantities describing the SFT global vibration mode of interest. The kinematics of the tube cross-section is modeled with two degrees of freedom, namely the vertical and horizontal displacements, which are respectively denoted by the symbols $w_T(t)$ and $v_T(t)$, while its torsional behavior is disregarded. The anchoring elements are modeled according to the small-sag cable theory, accounting for geometrical nonlinearities, supports motion, added mass and damping effects to the presence of the water. Each anchoring element is then described as a single-dof system, by adopting a standard Galerkin discretization procedure. Numerical inspection of the dimensionless coefficients of the equation of motion performed with reference to hollow-core circular cross-section tethers reveal that (1) the dynamic coupling is one-way only. i.e. the bare tunnel has an influence on the dynamics of the mooring system and not vice-versa, and that (2) the in-plane and out-of plane response of the SFT are almost decoupled. Exploiting the cross-section's symmetry, coupled in-plane vibrations are controlled by the following system of dimensionlesss equations of motion:

$$\begin{cases} \ddot{w}_T + 2\xi_T \omega_T \dot{w}_T + \omega_T^2 w_T = F_T(\tau) + T(\tau)\sin\theta \\ \ddot{z} + 2\xi^s \omega \dot{z} + \omega^2 z + \nu z^3 + 3\beta z^2 + 3\tilde{\beta} z^2 w_T \sin\theta - \eta^* w_T z + \ddot{w}_T(\zeta^* + \alpha^*) = F(\tau) \end{cases} \tag{1}$$

where τ is the dimensionless time variable, T is the dimensionless dynamic tension of the anchor, ω (ω_T), ξ^s (ξ_T), and $F(\tau)$ ($F_T(\tau)$) are the dimensionless natural circular frequency, damping coefficient and generalized external force of the anchoring element (tube cross-section), respectively. Moreover, θ is the inclination angle of the tether, while β, $\tilde{\beta}$, ν are coefficients governing geometrical nonlinearities, η^* is related to the parametric excitation term, while α^* and ζ^* are related to the inertial support motions.
Three different loading conditions have been identified as critical: the Loading Conditions 1 (LC1) and 2 (LC2) consider an hydrodynamic forcing acting on the tunnel which is expressed as an harmonic function

depending upon a detuning (mistuning) parameter δ, Quasi-Simple-Resonant and Quasi-Parametric-Resonant with the anchor, respectively, i.e.: $F_T(\tau) = f\,\sin[\omega(1+\delta\varepsilon)\tau]$ for LC1, and $F_T(\tau) = f\,\sin[\omega(2+\delta\varepsilon)\tau]$ for LC2, where ε is a small dimensionless ordering variable. Finally, the Loading Condition 3 (LC3) considers an hydrodynamic force acting on the anchor, Quasi-Resonant with the anchor itself, i.e. $F(\tau) = \tilde{f}\,\sin[\omega(1+\delta\varepsilon)\tau]$. For each loading condition, the equations of motion of the system have been attacked according to an application of the Multiple Times-scales perturbation method, and closed-form expressions for the steady-state vibration amplitude a of the anchoring elements have been derived.

3. Parametric analyses

After a numerical validation of the closed-form expressions has been performed, extensive parametric analyses in the space of technically relevant parameters of the mooring system have carried out.
As an application example, the influence of the drag coefficient C_d of the tether on its dimensionless vibration amplitude a is depicted in Fig. 1, for Loading Conditions 1 and 2.

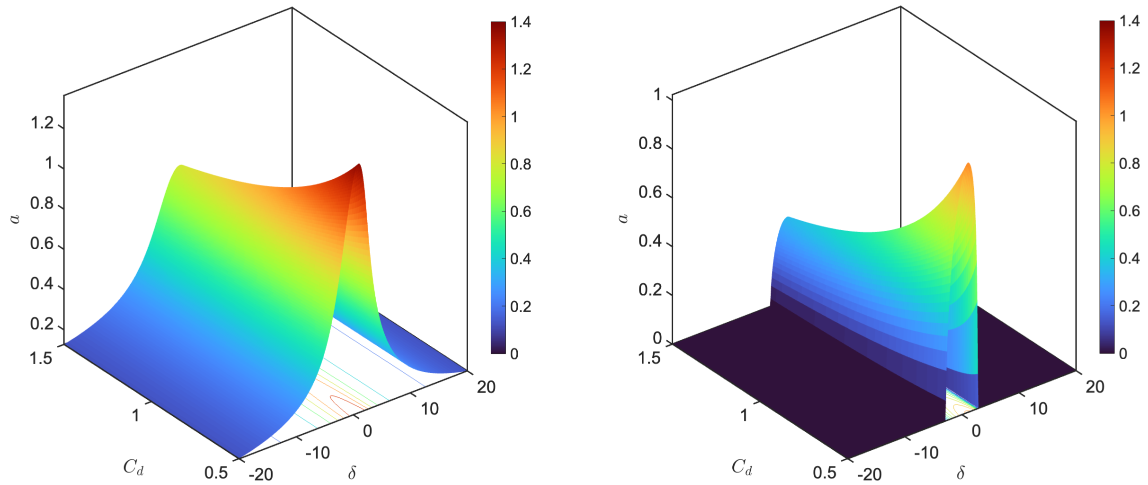

Figure 1: Dimensionless steady-state vibration amplitudes a of the anchor predicted by the perturbation solutions, as a function of the detuning parameter δ and of the drag coefficient C_d, for LC1 (left) and LC2 (right).

Fig. 1 clearly shows that the dynamic response of the anchor, subject to loading scenario 1 can be assimilated to that of a Duffing oscillator with hardening behavior. On the contrary, loading scenario 2 delivers a Mathieu-Duffing type response, with hardening behavior as well. The drag coefficient negatively correlates with the amplitude of oscillations. In particular, the nonlinear increase of vibration amplitude observed passing for high values to low values of C_d is faster for LC2, compared to LC1. This highlights the sensitivity of the Mathieu-Duffing equation to the damping of the system.

4. Conclusions

This work dealt with the complete characterization of the nonlinear dynamic response of anchoring elements for SFTs considering the coupling with the tube and hydrodynamic loading conditions. The derived approximate closed-form solutions allowed for an extreme reduction of the computational burden encountered with standard time-domain numerical simulations, serving as a fundamental tool in the optimization of the mooring system's parameters for the preliminary (conceptual) design phase of seabed-anchored SFTs.

References

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