A Local Frame Approach for Line-to-Line Contact between thin Beams with circular Cross-Sections

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Outline

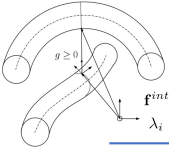
1. Context
2. The Local Frame Approach
3. The Mortar Method
4. Problem Formulation
5. First Results
6. Conclusions and Perspectives

Mesoscopic Cable Modeling

ULiège & ITWM Kaiserslautern

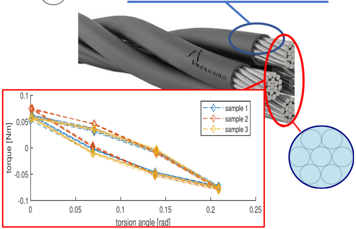
THREAD
EUROPEAN TRAINING NETWORK

Flexible Multibody Contact



$$\mathbf{f}^{int}(q) + \mathbf{f}^{con}(q, \boldsymbol{\lambda}) = \mathbf{f}^{ext}(q, t)$$

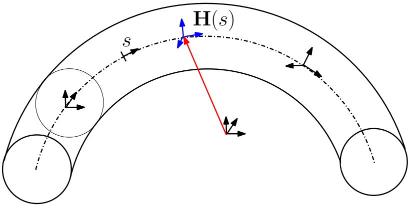
$$\lambda_i \geq 0 \qquad g_i \geq 0 \qquad g_i \lambda_i = 0$$



Constitutive modeling

Geometrically exact beam in $SE(3)$

Simo beam revisited (large rotations, shearing): [*Sonneville* 2013]



- Frames as unknowns:
  $$\begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0} & 1 \end{bmatrix} = \mathbf{H}(s) \in SE(3)$$
- Simple access to strains through the Lie group derivative
- Solve the dynamics in the local frame (reduced non-linearity)

Kinematic equation

$$\mathbf{H}'(s) = \mathbf{H}(s)\tilde{\mathbf{f}} \quad \Longrightarrow \quad \epsilon = \mathbf{f} - \mathbf{f}_0$$

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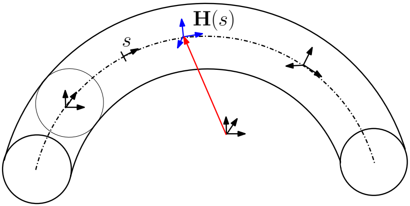

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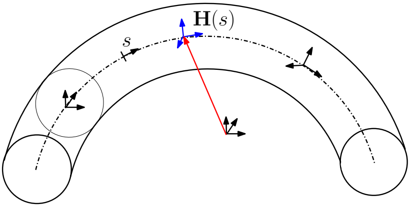

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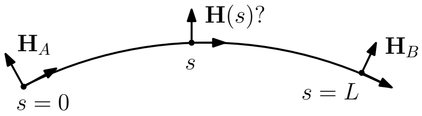
Beam finite element on $SE(3)$

Verify the kinematic equation by construction



- Interpolation formula

$$\mathbf{H}(s) = \mathbf{H}_A \exp(s\tilde{\epsilon}^*)$$

- Discrete deformation

$$\tilde{\epsilon}^* = \frac{\log\left(\mathbf{H}_A^{-1}\mathbf{H}_B\right)}{L}$$

Discretization

- First order interpolation generalized to a non-linear space
- Frame invariant interpolation
- Natural coupling of translation and rotation
- Automatically locking free
- Geometric non-linearities reduce with mesh refinement

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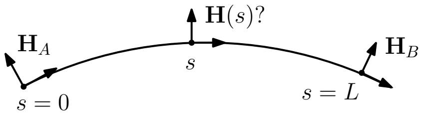

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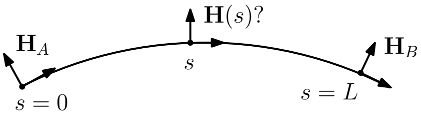

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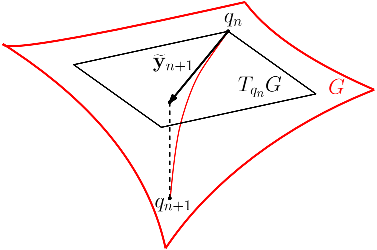
Lie group time integrator

The equations of motion are directly formulated on the Lie group. The numerical solution at every step remains on the manifold. [*Bruls* 2012]



$$q_{n+1} = q_n \circ \exp(\widetilde{\mathbf{y}}_{n+1})$$
$$\mathbf{f}^{int}(q_{n+1}) = \mathbf{f}^{ext}(q_{n+1}, t_{n+1})$$

Integration scheme

- No global parametrization of rotation
- Local parametrization of the manifold at each time step using the exponential map
- Only admissible group operations
- Simple implementation

No tricks! Just the formalism.

State of the art for beam contact

- Point contact
- Node to segment approaches

Correct representation of contact forces?

Integration of contact contributions

- Line to line contact with penalty approach. [*Meier* 2016]

→ Smooth contact forces
→ Dependence of the solution on the penalty parameter

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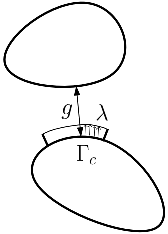
Mortar methods for unilateral contact

Weak enforcement of the non-penetration condition using Lagrange multipliers. [*Popp* 2012]



$$g \geq 0 \quad \lambda \geq 0 \quad \lambda g = 0$$

$$\downarrow$$

$$\int_{\Gamma_c} (\delta\lambda - \lambda) g \, \mathrm{d}\gamma_c \geq 0 \quad \forall \delta\lambda \in \mathcal{M}$$

- Numerical stability and optimal convergence rates if correct discretized spaces are used (**inf-sup** condition)
- Usually combined with active set strategies to deal with the inherent **non-smooth**ness of unilateral contact problems
- Difficulty: Finding the contact region

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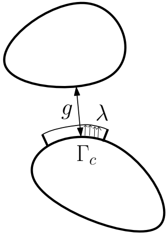

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Contact kinematics and constraint gradient

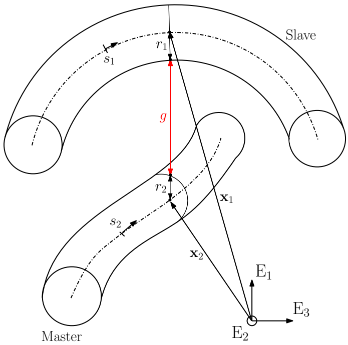
"1" is the slave or non-mortar beam and "2" the master or mortar beam

$$q = \{\mathbf{H}_1, \mathbf{H}_2\}, \qquad \delta\mathbf{h} = \begin{bmatrix} \delta\mathbf{h}_1 \\ \delta\mathbf{h}_2 \end{bmatrix}$$

Circular cross section:

$$g(q) = \|\mathbf{x}_2 - \mathbf{x}_1\| - r_1 - r_2$$

$$\delta g(q) = \mathbf{G}(q)\delta\mathbf{h}, \quad \mathbf{G}^T = \begin{bmatrix} -\mathbf{MN} \\ \mathbf{MR}_2^T\mathbf{R}_1\mathbf{N} \end{bmatrix}$$



Simplified kinematics

- Kinematics entirely written in terms of the centerline
- The constraint gradient only depends on local relative quantities

Continuous formulation

Find $q \in \mathcal{U}$ and $\lambda \in \mathcal{M}$ such that

$$\delta\mathcal{W}_{int}(q, \delta\mathbf{h}) + \delta\mathcal{W}_{con}(q, \lambda, \delta\mathbf{h}) = \delta\mathcal{W}_{ext}(q, \delta\mathbf{h}, t) \quad \forall\delta\mathbf{h} \in \mathcal{V}$$
$$\delta\mathcal{W}_{lag}(q, \delta\lambda) \geq 0 \quad \forall\delta\lambda \in \mathcal{M}$$

where we define

$$\delta\mathcal{W}_{con}(q, \lambda, \delta\mathbf{h}) = \int_{s_1^a}^{s_1^b} (\delta\mathbf{h})^T \mathbf{G}^T \lambda \, \mathrm{d}s_1$$
$$\delta\mathcal{W}_{lag}(q, \delta\lambda) = \int_{s_1^a}^{s_1^b} (\delta\lambda - \lambda) g \, \mathrm{d}s_1$$

Invariance property

- Equilibrium expressed in the local frame
- All contributions only depend on the relative configuration

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Semi discrete formulation

Idea: We use a first order interpolation for the configuration, thus we do the same for the Lagrange multipliers [*Belgacem* 1999]: $\lambda^h = \mathbf{\Phi}(s_1)\boldsymbol{\lambda}$

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Solution

- Solved using Lie group versions of semi-smooth Newton or Augmented Lagrangian methods. [*Cavalieri* 2012]
- The weak constraints are enforced exactly

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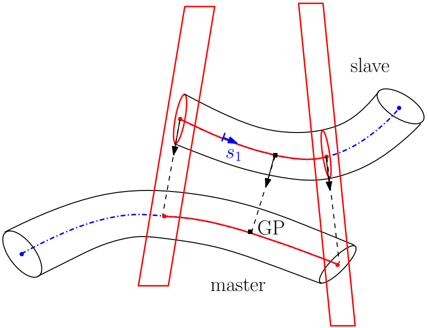
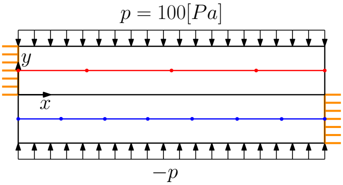
Simple computation of contact patch

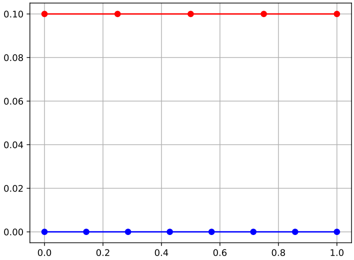
Intersection between the slave cross section plane and the master centerline

- Appropriate for thin beams with circular cross sections

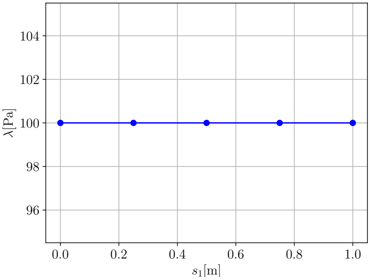


Patch test: Two clamped beams





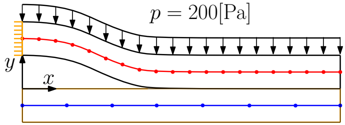
centerlines

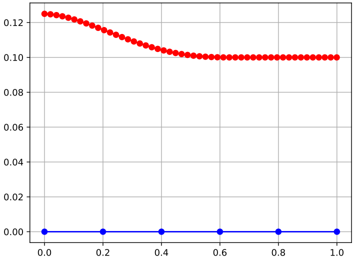


Lagrange multiplier

- Correct representation of constant distributed pressure field.

Cantilever: Non-smooth contact pressures





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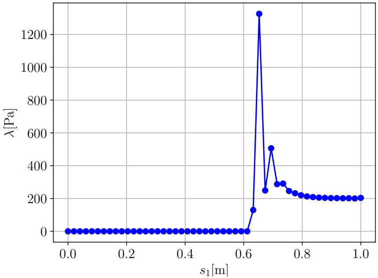
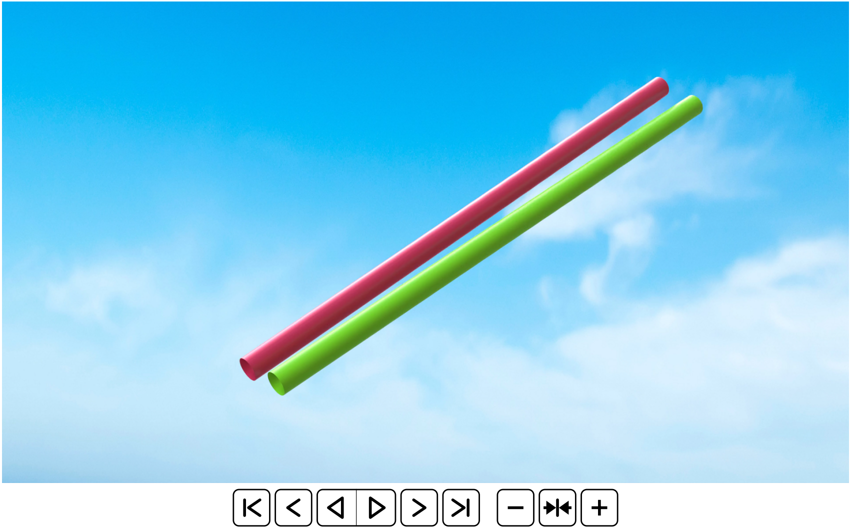


Lagrange multiplier

- Imposed kink in the curvature produces impulse-like contact force at the contact interface. [Civilek 1975]

Flying helix spaghetti



Conclusion, perspectives and open questions

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- In $SE(3)$ the contact formulation for beams has interesting properties
- The Mortar method allows own to correctly represent line contact geometries and an accurate computation of contact pressures.
- Both are combined with the aim to construct a robust method

Perspectives

- Use it to simulate realistic cable assemblies
- Extend the formulation to friction

Open Question

- Is it possible to derive a unified model that is efficient for both point and line contact?

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References 1

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