ECCOMAS Thematic Conference and IACM Special Interest Conference
Highly Flexible Slender Structures (HFSS 2023)
25–29 September 2023, Rijeka, Croatia

An arbitrary Lagrangian-Eulerian geometrically exact beam formulation applied to reeving systems

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Keywords: Lie group, beam, ALE, multibody system

1. Introduction

Reeving systems generally consist of a cable and one or several pulleys and can be modeled as a multibody system, *i.e.*, as an assembly of rigid or flexible bodies linked by joints. In such applications, a special attention should be paid to the cable model, which must account for two mains aspects: the cable deformation, and frictional contact due to the interaction between the cable and the pulley.

2. Flexible cable model

A cable is a typical example of a highly flexible and slender structure where nonlinearities could arise. Cables are often modeled as beams and most nonlinear finite element beams are formulated based on distinct translation and rotation coordinates. Another approach is to formulate the beam on the special Euclidian group $SE(3)$ where rotations and translations are inherently coupled. This formulation has already been used for beam modeling [1] or more recently for nonsmooth beam-to-beam contact [2]. The advantages of using an $SE(3)$ formulation are diverse. No global set of coordinates is needed since the motion variables are inherent to the mathematical objects used in the formulation. Moreover, with the wisely chosen representation of derivatives, a frame-invariant formulation is obtained. A local parametrization of motion is then applied on motion increments. The continuous equations to be solved in a quasi-static setting are obtained by the variation of the internal potential energy as

$$\delta_{\text{spatial}}(\mathscr{W}_{int}) = \int_0^L \delta_{\text{spatial}}(\boldsymbol{\varepsilon})^T \mathbf{K}\boldsymbol{\varepsilon}\, ds \tag{1}$$

where $\boldsymbol{\varepsilon}$ is the 6×1 strain vector and $\mathbf{K}$ is the 6×6 sectional stiffness matrix.

3. Arbitrary Lagrangian-Eulerian formulation

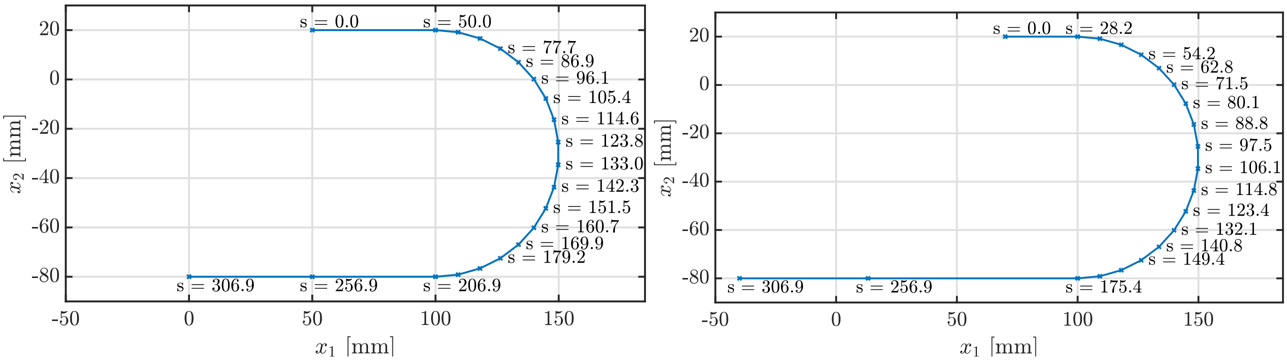
Sometimes, the cable comes into contact with a structure, for instance when it travels around a pulley. In this scenario, there is a need for a fine finite element discretization of this specific portion of the cable in order to accurately represent the frictional contact conditions. Because the cable is moving around the pulley, small elements must often be used along the whole cable length. One remedy to this problem is to use an arbitrary Lagrangian-Eulerian (ALE) formulation of the cable, which permits a dissociation between the mesh and the material points of the cable. In [3], an ALE formulation based on a continuous variational framework is proposed, where a remeshing of the discretized structure provides an optimal energy solution. In multibody dynamics, an ALE formulation was applied in a fully discrete setting in [4], where a joint ALE-ANCF approach for a beam element is developed to model reeving systems. In multibody systems, ideal kinematic joints between different bodies are modeled using bilateral constraints. In this work, the approach proposed by [3] is applied to model cables as Timoshenko beam elements on $SE(3)$ starting from a continuous form of the equations. In this ALE formulation, the connection between the cable and another body such as the pulley can be modelled using bilateral constraints. An equation taking into account the flow of material along the centerline is thus added to Eq. (1):

$$\delta_{\text{material}}(\mathscr{W}_{int}) = \frac{1}{2}\int_0^L \delta_{\text{material}}(F)\left(\boldsymbol{\varepsilon}^T \mathbf{K}\boldsymbol{\varepsilon} - 2\mathbf{f}^T \mathbf{K}\boldsymbol{\varepsilon}\right) ds \tag{2}$$

where F represents the flow of material coordinate and $\mathbf{f}$ is the deformation gradient.

4. Test case

The example of a cable slipping around a fixed pulley illustrates the advantages of an $SE(3)$ beam formulation (with traction, bending, shearing and torsion energy) and a simpler bar formulation (with only traction energy). Firstly, large traction cables might have a non-negligible bending stiffness which cannot be modeled in bar elements. Secondly, the $SE(3)$ formulation can exactly represent the curvature of the beam around the pulley. In Fig. 1, preliminary results are proposed with a unidimensional bar formulation. A displacement of -40 [mm] in x_1 is imposed at the bottom end of the cable, while the other end has an imposed displacement of 20 [mm] in x_1, so that internal forces are developed. The nodes initially present on the pulley are constrained to their initial positions. A flow of the material coordinate s is thus observed. The retrieval of the contact forces between the cable and the pulley is done directly through the evaluation of the constraint reaction forces. Assuming a purely slipping contact, a slipping friction force $T = \mu N$ is then added to every node lying on the pulley, where μ is the friction coefficient and N is the normal contact force.



(a) Initial configuration.

(b) After motion.

Figure 1: ALE simulation of a cable around a pulley using bar elements.

5. Conclusions

In this work, a geometrically exact beam model formulated on $SE(3)$ is developed in an ALE framework. It couples the advantages of using an $SE(3)$ beam, such as the avoidance of a global parametrization of rotations and a local frame formulation, with the perks of the ALE formulation, permitting to model a closed and sliding contact as a bilateral constraint and to directly recover the contact forces. In this scenario, a slipping friction force can be considered in the simulation without further changes in the solver.

Acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860124.
This work has been partly funded by the Robotix Academy project of the Greater Region.

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