# The influence of beam kinematic assumptions in a beam contact benchmark

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# Context and applications

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EUROPEAN TRAINING NETWORK

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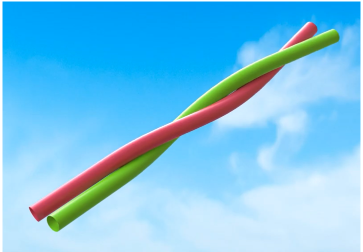
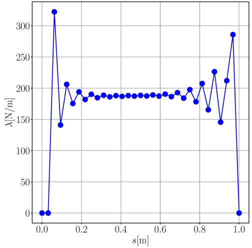
# Motivation for a beam contact benchmark



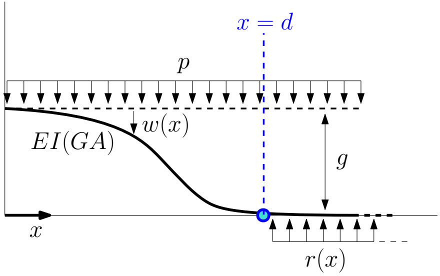



Line contact model between geometrically exact beams (mortar approach)

- Is the observed distributed contact force reasonable?
- What is the influence of the **beam kinematic assumption**? (with or without shear deformation)
- **Benchmarking** with other methods? (point-to-point contact model, penalty formulation)
- Representative test problem with an **analytical solution**?

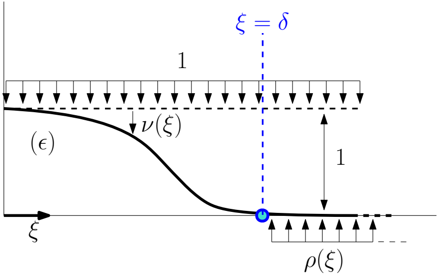
A. Bosten, A. Cosimo, J. Linn and O. Brüls, A mortar formulation for frictionless line-to-line beam contact, *Multibody System Dynamics*, 54:31-52 (2021)

# Proposed test problem




- 2D cantilever beam with infinite length
- Small displacements $\Rightarrow$ linear beam model
- Euler-Bernoulli vs Timoshenko model
- $g$: initial gap w.r.t. rigid support
- $p$: uniformly applied lineic force
- Cross-section deformations are neglected in the contact kinematics

# Adimensionalization




Characteristic lengths and angle :

$$u_1^* = g,\ u_2^* = \left(\frac{gEI}{p}\right)^{1/4},\ \theta^* = \frac{u_1^*}{u_2^*}$$

Change of variables:

$$\xi = \frac{x}{u_2^*},\ \nu = \frac{w}{u_1^*},\ \vartheta = \frac{\theta}{\theta^*},\ \delta = \frac{d}{u_2^*},\ \rho = \frac{r}{p},\ \epsilon^2 = \frac{EI/GA}{{u_2^*}^2}$$

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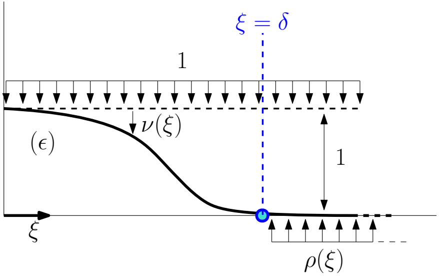


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# Equilibrium and kinematic equations

Euler-Bernoulli and Timoshenko models

$$\begin{aligned} \theta'''(x) - \frac{p - r(x)}{EI} &= 0 \\ \frac{EI}{GA}\theta''(x) - \theta(x) + w'(x) &= 0 \end{aligned}$$

$$\begin{aligned} \vartheta'''(\xi) + \rho(\xi) - 1 &= 0 \\ \epsilon^2\vartheta''(\xi) - \vartheta(\xi) + \nu'(\xi) &= 0 \end{aligned}$$

- In the free region, the reaction is zero: $r(x) = 0$ or $\rho(\xi) = 0$
- In the contact region, the displacement is imposed: $w(x) = g$ or $\nu(\xi) = 1$, $w'(x) = 0$ or $\nu'(\xi) = 0$
- At the transition point $x = d$ or $\xi = \delta$, these equations do not necessarily hold

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# Boundary conditions

Clamp:

$$w(0) = 0, \quad \theta(0) = 0$$

Free end:

$$\theta(x) \to 0 \quad \text{as} \quad x \to +\infty$$

Transition point:

$$w(d) = g, \quad w'(d) = 0, \quad \theta(d) = \theta_d$$

Free boundary problem:

- the governing equations are solved analytically on each subdomain that are defined by the limits of the contact region
- Continuity equations are used to localize this transition point

# Euler-Bernoulli solution

The dimensionless free length is

$$\delta = 72^{1/4}$$

The dimensionless shear force is

$$\mathcal{T}(\xi) = \begin{cases} \xi - \delta\left(\frac{1}{2} + \frac{12}{\delta^4}\right) & \text{if } \xi < \delta \\ 0 & \text{if } \xi > \delta \end{cases}$$

A **point contact force** appears due to discontinuity in the shear force

$$f_\delta = \mathcal{T}(\delta^-) - \mathcal{T}(\delta^+) = \frac{1}{3}\delta$$

# Timoshenko solution

The dimensionless free length is obtained by solving the following equation

$$\delta^6 + 6\epsilon\delta^5 + 30\epsilon^2\delta^4 + 72\epsilon^3\delta^3 + 72(\epsilon^4 - 1)\delta^2 - 144\epsilon(\delta + \epsilon) = 0.$$

The shear force is given by

$$\mathcal{T}(\xi) = \begin{cases} \xi - \delta\left(\frac{1}{2} + \frac{12}{\delta^4}\right)\frac{\delta^2}{\delta^2 + 6\epsilon^2} & \text{if } \xi < \delta \\ \frac{\vartheta_\delta}{\epsilon^2} exp\{(\delta - \xi)/\epsilon\} & \text{if } \xi > \delta \end{cases}$$

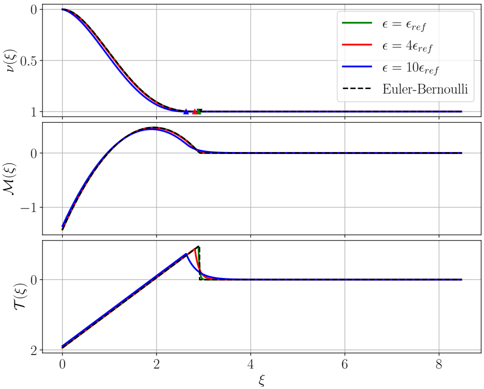
where

$$\vartheta_\delta = \delta\epsilon^2\left(\frac{1}{2} + 6\frac{\epsilon^2}{\delta^2} - \frac{12}{\delta^4}\right)\frac{\delta^2}{\delta^2 + 6\epsilon^2},$$

such that

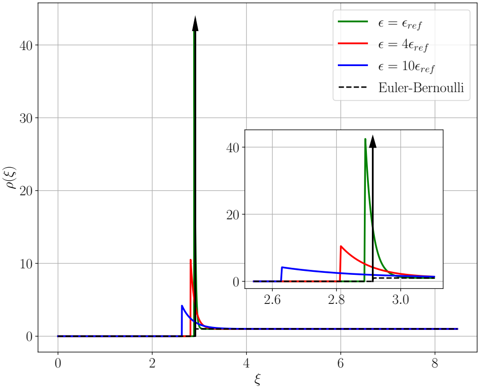
$$\boxed{\mathcal{T}(\delta^-) = \mathcal{T}(\delta^+)}$$

# Analytical solutions




Transverse displacement, bending moment, shear force. The markers denote the first contact point: $\epsilon_{ref} = 0.023$.

# Analytical solutions




Distributed contact force: $\epsilon_{ref} = 0.023$.

# Properties of the analytical solution

**Timoshenko model (with shear deformation)**

- the shear force is continuous
- the distributed contact force is discontinuous & takes large values in a boundary layer of length $\epsilon$

**Euler-Bernoulli model (no shear deformation)**

- the shear force is discontinuous
- the distributed contact force is discontinuous & has an atom (Dirac delta)

$\Rightarrow$ Adding **kinematic restrictions** on the deformation pattern increases the **level of nonsmoothness**

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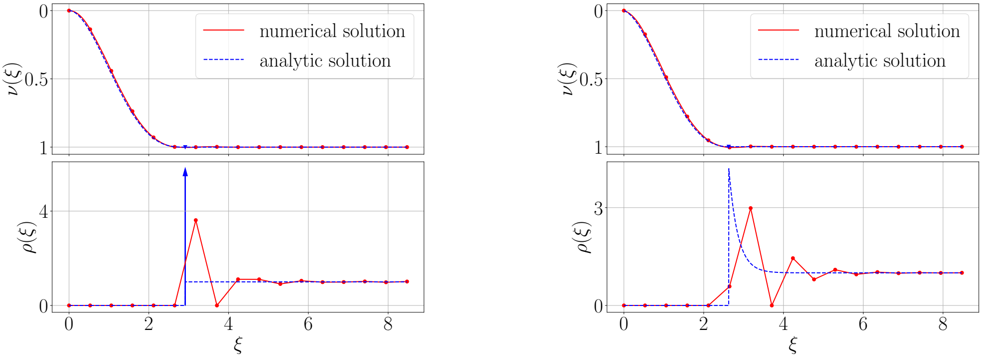
# Comparison with a mortar numerical solution




Left: Euler-Bernoulli. Right: Timoshenko with $\epsilon = 10\epsilon_{ref}$ and $h = 0.53 > \epsilon$.

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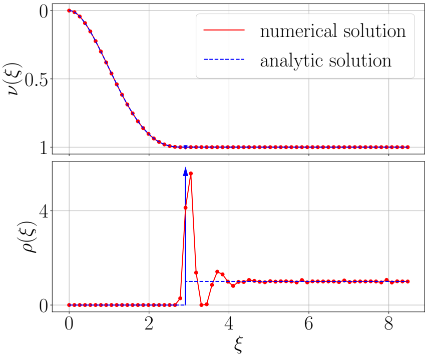
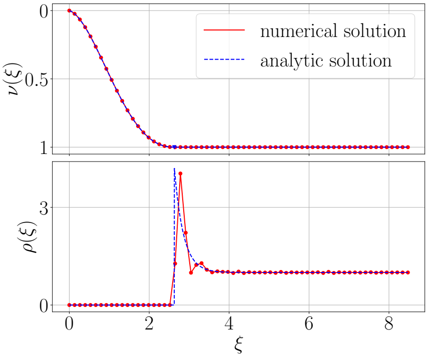
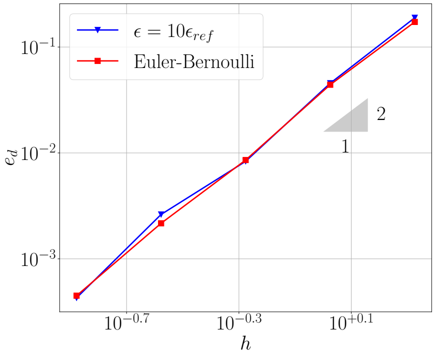
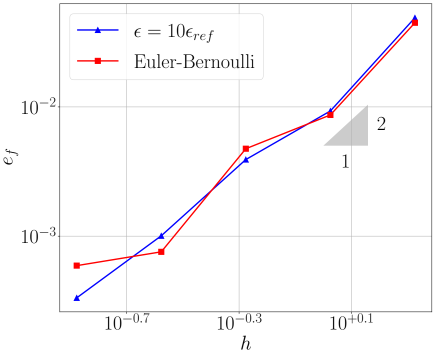



Left: Euler-Bernoulli. Right: Timoshenko with $\epsilon = 10\epsilon_{ref}$ and $h = 0.13 < \epsilon$.

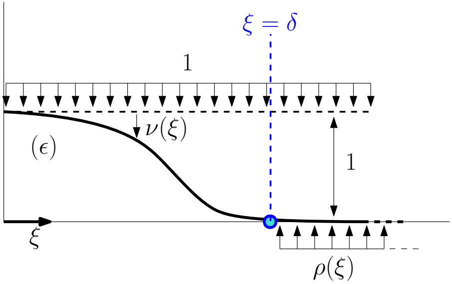
# Comparison with a mortar numerical solution

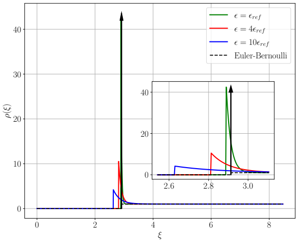



Left: Error on the vertical displacement. Right: Error on the integrated contact force.

# Conclusion




A **benchmark for beam contact problems** has been proposed and an **analytical solution** has been presented.

The analytical solution reveals the **significant influence of the kinematic assumptions of the beam model** on the solution **at the contact transition**.

Compared to the Euler-Bernoulli model, the **Timoshenko model** leads to **smoother solutions** which are easier to capture by numerical schemes.

# Thank you for your attention!

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