Towards a Gauß-Seidel solver for problems involving line-to-line beam contact

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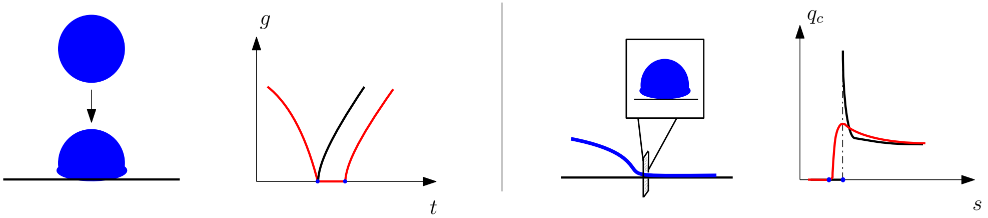
Context

Kinematic hypotheses and discontinuity



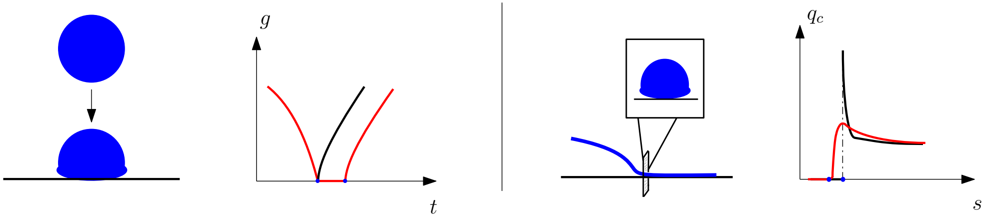
Continuum Mechanics

- Impacts of finite duration, Continuous surface tractions
- Ensured via local deformability

Multibody components

- Reduced kinematics introduces nonsmoothness! $\longrightarrow$ Discontinuities in time & space

Kinematic hypotheses and discontinuity



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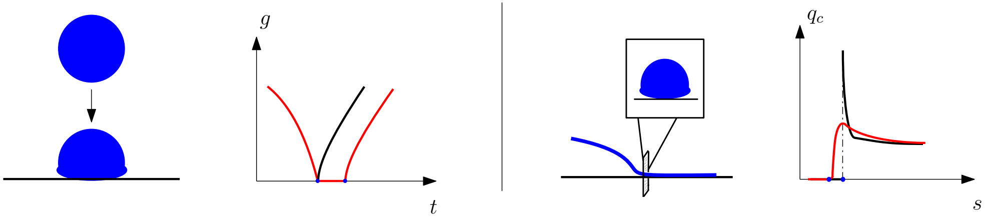

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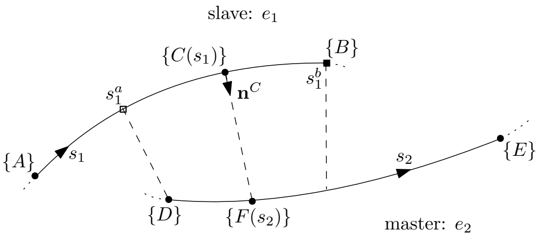
Mortar for beam-to-beam contact

Pointwise expressions for thin circular beam:

$$g = \|\mathbf{x}_{OF} - \mathbf{x}_{OC}\| - 2r$$

$$\mathbf{G}^T = \begin{bmatrix} -\mathbf{M}\mathbf{n}^C \\ \mathbf{M}\mathbf{R}_{OF}^T\mathbf{R}_{OC}\mathbf{n}^C \end{bmatrix}$$



Mortar constraint

$$g^j = \int_{s_a}^{s_b} N^j g \, \mathrm{d}s,$$

Constraint gradient

$$g^j_{\mathbf{q}} = \int_{s_a}^{s_b} N^j \mathbf{G} \, \mathrm{d}s,$$

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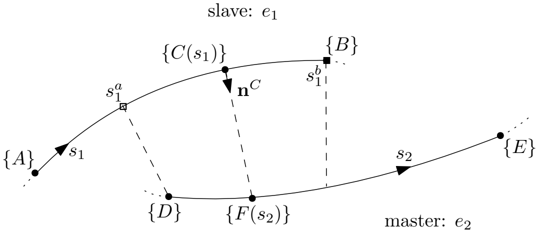

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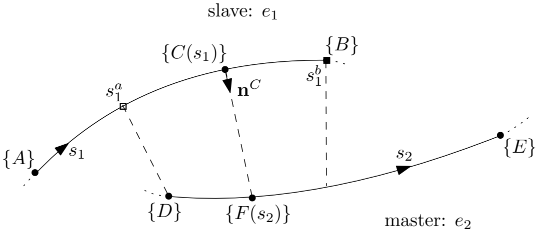

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Dynamic nonsmooth effects

Origins

- Rigid bodies $\rightarrow$ discontinuous velocities & impulsive forces
- Flexible bodies $\rightarrow$ discontinuous velocities
- FE discretization $\rightarrow$ numerical impacts between nodes

Can be confusing for flexible components... they are in the model!

What to do?

- Regularize (penalty) & keep classical mathematical tools
- Mathematical framework for nonsmooth systems $\rightarrow$ new class of time integration schemes

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Discontinuous velocities

Kinematics

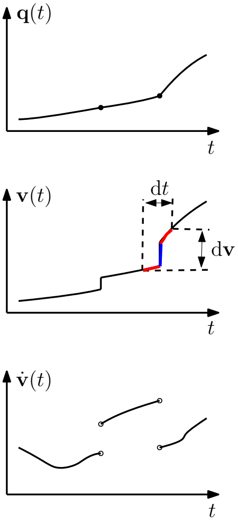
- $\mathbf{q}$ is continuous
- $\mathbf{v}$ has finite jumps
- $\dot{\mathbf{v}}$ defined for smooth motion

Substite time derivatives by differential measures

$$\mathrm{d}\mathbf{v} = \dot{\mathbf{v}}\,\mathrm{d}t + \sum_i \left[\mathbf{v}^+(t_i) - \mathbf{v}^-(t_i)\right]\delta_{t_i},$$

such that

$$\int_{(t_1,t_2]} \mathrm{d}\mathbf{v} = \mathbf{v}^+(t_2) - \mathbf{v}^+(t_1).$$



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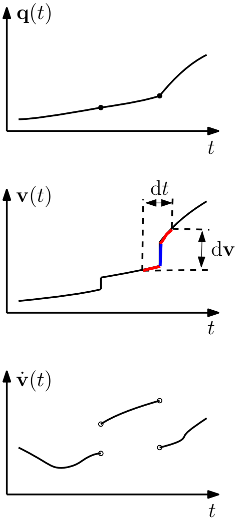

Contact forces

Similarly, we introduce the impulse measure of the contact reaction

$$\mathrm{d}\mathbf{i} = \boldsymbol{\lambda}\,\mathrm{d}t + \sum_i \mathbf{p}_i \delta_{t_i},$$

- $\boldsymbol{\lambda}$ is the vector of smooth Lagrange multipliers
- $\mathbf{p}_i$ is the impulse producing the jump at instant t_i

Equations of motion

$$\dot{\mathbf{q}} = \mathbf{v},$$
$$\mathbf{M}(\mathbf{q})\mathrm{d}\mathbf{v} - \mathbf{g}_{\mathbf{q}}^{T}\mathrm{d}\mathbf{i} = \mathbf{f}(\mathbf{q}, \mathbf{v}, t)\,\mathrm{d}t,$$
$$\text{if } g^j(\mathbf{q}) \leq 0 \quad \text{then } 0 \leq g_{\mathbf{q}}^j\mathbf{v} + \cancel{e^j g_{\mathbf{q}}^j \mathbf{v}^-} \perp \mathrm{d}i^j.$$

- Unified equations for the smooth motion and impacts
- Impact law = Constraint at velocity level $\rightarrow$ numerical drift
- $e = 0$ for flexible bodies

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NSGA: Equations of motion

- GGL-type formulation $\rightarrow$ position & velocity constraints
- $(\tilde{\mathbf{q}}, \tilde{\mathbf{v}})$ selected as solution of the contact free problem

$$\mathbf{M}(\widetilde{\mathbf{q}})\dot{\widetilde{\mathbf{v}}} = \mathbf{f}(\widetilde{\mathbf{q}}, \widetilde{\mathbf{v}}, t),$$
$$\mathrm{d}\mathbf{v} = \mathrm{d}\mathbf{w} + \dot{\widetilde{\mathbf{v}}}\mathrm{d}t,$$

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} - \mathbf{g}_{\mathbf{q}}^{T}\boldsymbol{\mu} = \mathbf{M}(\mathbf{q})\mathbf{v},$$
$$\mathbf{0} \leq \mathbf{g}^{\mathcal{U}} \perp \boldsymbol{\mu} \geq \mathbf{0},$$

$$\mathbf{M}(\mathbf{q})\mathrm{d}\mathbf{w} - \mathbf{g}_{\mathbf{q}}^{T}\mathrm{d}\mathbf{i} = \mathbf{f}^{*}(\mathbf{q}, \mathbf{v}, \widetilde{\mathbf{q}}, \widetilde{\mathbf{v}}, \dot{\widetilde{\mathbf{v}}}, t)\,\mathrm{d}t,$$
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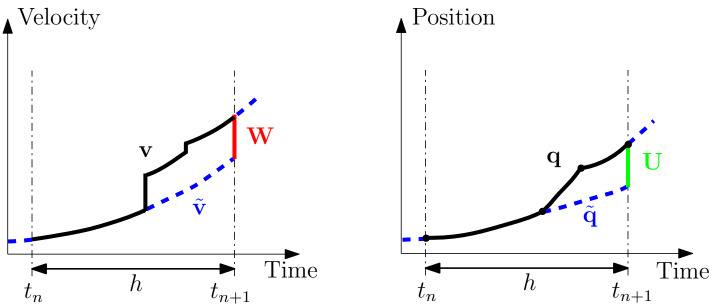
NSGA: One time step



$$\mathbf{v}(t) = \widetilde{\mathbf{v}}(t_n; t) + \mathbf{W}(t_n; t) \to \boldsymbol{\Lambda}$$
$$\mathbf{q}(t) = \widetilde{\mathbf{q}}(t_n; t) + \mathbf{U}(t_n; t) \to \boldsymbol{\nu}$$

- Smooth predictions $\tilde{\mathbf{q}}$ and $\widetilde{\mathbf{v}}$ integrated with generalized-α formulae

NSGA: Subproblems

Smooth prediction

$$\mathbf{M}(\widetilde{\mathbf{q}})\dot{\widetilde{\mathbf{v}}} - \mathbf{f}(\widetilde{\mathbf{q}}, \widetilde{\mathbf{v}}, t) = \mathbf{0}$$

Position correction

$$\mathbf{M}(\widetilde{\mathbf{q}})\mathbf{U} - h^2\mathbf{f}^p(\mathbf{q}, \widetilde{\mathbf{q}}, \mathbf{v}, \widetilde{\mathbf{v}}, t) - \mathbf{g}_{\mathbf{q}}^T\nu = \mathbf{0}$$
$$0 \leq g^j(\mathbf{q}) \perp \nu^j \geq 0$$

Velocity jump

$$\mathbf{M}(\mathbf{q})\mathbf{W} - h\mathbf{f}^*(\mathbf{q}, \widetilde{\mathbf{q}}, \mathbf{v}, \widetilde{\mathbf{v}}, \dot{\widetilde{\mathbf{v}}}, t) - \mathbf{g}_{\mathbf{q}}^T\Lambda = \mathbf{0}$$
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Contact solver for the position problem

Augmented Lagrangian can be used

$$\mathcal{L}^{\text{con}}(q,\boldsymbol{\nu}) = \sum_{j\in\mathcal{C}} \left(\frac{p_p}{2}\left(g^j\right)^2 - k_p \lambda^j g^j - \frac{1}{2p_p}\text{dist}^2(\xi^j, \mathbb{R}^+) \right)$$

(Semismooth) Newton updates: direct inversion of linear system

$$\mathbf{S}^p \Delta\mathbf{x}^p = -\mathbf{r}^p \quad \text{with} \quad \Delta\mathbf{x}^p = \begin{bmatrix} \Delta\mathbf{U}_{k+1} \\ \Delta\boldsymbol{\nu}^{\mathcal{A}}_{k+1} \\ \Delta\boldsymbol{\nu}^{\bar{\mathcal{A}}}_{k+1} \end{bmatrix}$$

Iteration matrix has to be full rank

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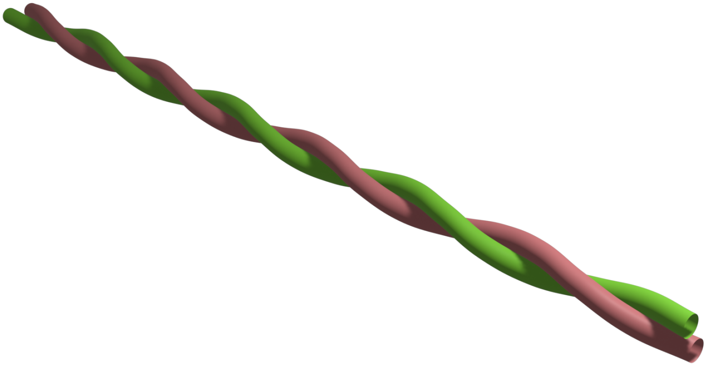
Dynamic twisting mortar (Odin)



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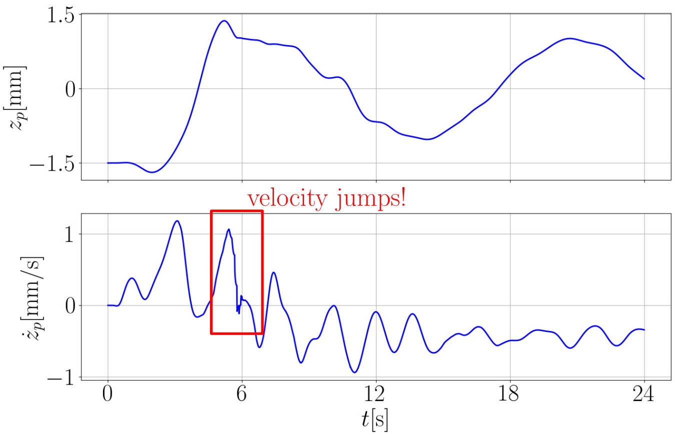

z component of position and velocity of the top beam mid point.

Towards a Gauß-Seidel solver

Monolithic augmented Lagrangian + semismooth Newton

- A lot of weight on the direct solver
  - Overconstrainment in systems with densely packed fibers $\longrightarrow$ singular Jacobian matrix

Hybrid Gauß-Seidel solver (SoBogus)

- Replace the direct solver by an iterative (GS) technique
- Interpreted as solving *one contact at the time*
  - Focus on robust solution for the local contact problem!

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GS approach for the position subproblem

Linearized problem in the contact space

$$\mathbf{g}_{k+1} = \mathbf{D}_k \boldsymbol{\nu}_{k+1} + \mathbf{b}_k$$
$$\mathbf{0} \leq \mathbf{g}_{k+1} \perp \boldsymbol{\nu}_{k+1} \geq \mathbf{0}$$

Delassus operator & load vector

$$\mathbf{D}(\mathbf{q}, \tilde{\mathbf{q}}, \mathbf{v}, \tilde{\mathbf{v}}, t) = \mathbf{g}_{\mathbf{q}} \mathbf{S}^{-1} \mathbf{g}_{\mathbf{q}}^T \qquad \mathbf{b}(\mathbf{q}, \tilde{\mathbf{q}}, \mathbf{v}, \tilde{\mathbf{v}}, t) = -\mathbf{g}_{\mathbf{q}} \mathbf{S}^{-1} \mathbf{R} + \mathbf{g}$$

where we define

$$\mathbf{R}(\mathbf{q}, \widetilde{\mathbf{q}}, \mathbf{v}, \widetilde{\mathbf{v}}, t) = \mathbf{M}(\widetilde{\mathbf{q}})\mathbf{U} - h^2 \mathbf{f}^p(\mathbf{q}, \widetilde{\mathbf{q}}, \mathbf{v}, \widetilde{\mathbf{v}}, t)$$
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Splitting of the Delassus operator + Forward substitution
$\rightarrow$ One contact problem

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Local contact problem to be solved $\forall j$ [1]
$\rightarrow$ Find (g, ν) s.t.

$$g = D\nu + \bar{b}$$
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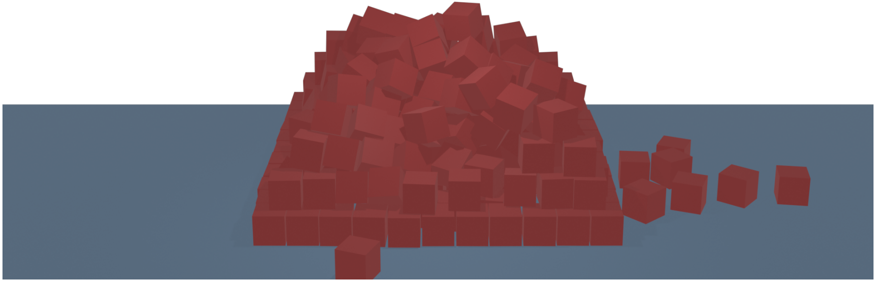
Example with rigid bodies and friction



Conclusion and perspectives

- Need to handle discontinuities in the model for flexible components
- This can be done with mortar and the NSGA
- Presence of convergence issues in Newton scheme related to the direct solver

- Trade convergence speed for robustness using an iterative solver (GS)
- Proven effective for frictional rigid body contact $\longrightarrow$ to be extended to beam-to-beam contact
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