Simulation of Multibody Systems with Switching Constraints: Formulation and Time Integration

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EXTENDED ABSTRACT

1 Introduction

In multibody dynamics, it is often assumed that bilateral or two-sided constraints satisfy sufficient continuity and differentiability conditions. This does not necessarily hold, as conditional statements may also trigger abrupt changes in the algebraic constraint expressions. In such situations, the system maintains a constrained state throughout, but the instantaneous switch creates a nonsmooth response with possible velocity jumps. The system dynamics is thus subjected to non-differentiable bilateral constraints. The equations of motion of such multibody systems can be expressed in the form of switched differential-algebraic equations (DAEs), which differ both from classical DAEs and from systems with unilateral constraints. As a result, the mathematical modelling and numerical integration require a specialized treatment. There is a scarcity of scientific research to address the problem of switching bilateral constraints in the field of multibody dynamics. The aim of this work is to present a general modelling procedure and explore possible numerical integration methods for multibody systems with switching bilateral constraints.

2 Method

Let us consider a set of m bilateral constraints $\mathbf{g}(\mathbf{q}(t),t) = \mathbf{0}$ of the form

$$\mathbf{g}(\mathbf{q}(t),t) = \begin{cases} \mathbf{g}_1(\mathbf{q}(t),t), & \text{if } z(\mathbf{q}(t)) < 0 \\ \mathbf{g}_2(\mathbf{q}(t),t), & \text{if } z(\mathbf{q}(t)) \geq 0 \end{cases} \tag{1}$$

where $\mathbf{q}(t) \in \mathbb{R}^n$ is the vector of coordinates and the function $z(\mathbf{q}) : \mathbb{R}^n \to \mathbb{R}$ is the so-called *switching function* and the condition $z(\mathbf{q}(t)) = 0$ defines the *switching surface*. In order to ensure the continuity of the configuration $\mathbf{q}(t)$ at the switching, we also require that the two portions of the constraint space intersect at the switching surface. Introducing the gradients $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{Z}$ of the functions $\mathbf{g}_1$, $\mathbf{g}_2$ and z with respect to $\mathbf{q}$, we also require the matrices $[\mathbf{G}_1;\mathbf{Z}]$ and $[\mathbf{G}_2;\mathbf{Z}]$ to be full rank on the switching surface. However, the gradient $\mathbf{G}_1$, $\mathbf{G}_2$ may differ on the switching surface, which implies that the bilateral constraints are not differentiable, with the consequence that the velocity $\dot{\mathbf{q}}(t)$ can be discontinuous. The mechanical response can thus be affected by impact phenomena. Though not detailed here, this formalism can be extended to systems with multiple switching surfaces.

For almost every time (away from the switching surface), the equations of motion take the standard index-3 DAE form

$$\dot{\mathbf{q}}(t) = \mathbf{v}(t) \tag{2}$$

$$\mathbf{M}(\mathbf{q}(t))\dot{\mathbf{v}}(t) + \mathbf{G}^T(\mathbf{q}(t),t)\boldsymbol{\lambda}(t) = \mathbf{f}(\mathbf{q}(t),\mathbf{v}(t),t) \tag{3}$$

$$\mathbf{g}(\mathbf{q}(t),t) = \mathbf{0} \tag{4}$$

At a switching time t_i, the dynamics can be described by an impact equation of the general form

$$\mathbf{M}(\mathbf{q}(t_i))(\mathbf{v}^+(t_i) - \mathbf{v}^-(t_i)) + \mathbf{R} = \mathbf{0} \tag{5}$$

$$\mathbf{G}^+(\mathbf{q}(t_i),t_i)\mathbf{v}^+(t_i) + \mathbf{g}_t(\mathbf{q}(t_i),t_i) = \mathbf{0} \tag{6}$$

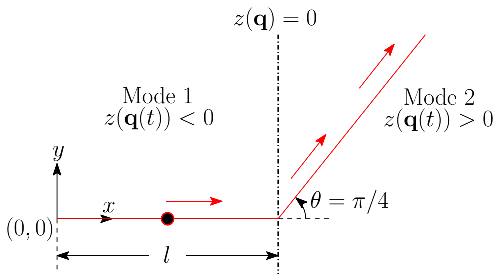
where $\mathbf{R}$ represents the impulse of the constraint reaction force. As the constraint gradient $\mathbf{G}$ is discontinuous at switching, we propose to introduce an intermediate gradient $\mathbf{G}_{\mathscr{E}}$ and model the reaction force as $\mathbf{R} = \mathbf{G}_{\mathscr{E}}^T\boldsymbol{\Lambda}$. The conditions $\mathbf{G}_{\mathscr{E}} = \mathbf{G}_1(\mathbf{q}(t_i),t_i)$ and $\mathbf{G}_{\mathscr{E}} = \mathbf{G}_2(\mathbf{q}(t_i),t_i)$ represent two extreme cases. It can be shown that the particular definition of the intermediate gradient $\mathbf{G}_{\mathscr{E}}$ directly influences the energy behaviour of the system at switching.

Using classical DAE time integration schemes, a numerical approximation can be obtained in the smooth phase of motion, but it may not correctly capture the discontinuous behaviour at the switching surfaces. In contrast, the dynamic response can be evaluated based on methods from nonsmooth dynamics such as event-driven and time-stepping schemes.

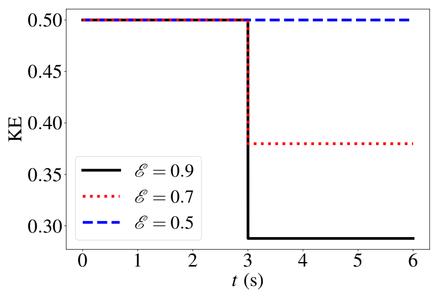
In this work, the numerical solution of dynamic systems with switching constraints will be evaluated using the nonsmooth generalized-α scheme (NSGA) [1]. This time-stepping scheme is based on the reformulation of the dynamic equilibrium as an equality of differential measures as

$$\mathrm{M}(\mathbf{q}(t))\mathrm{d}\mathbf{v} - \mathbf{G}_{\mathscr{E}}^T\mathrm{d}\mathbf{i} = \mathbf{f}(\mathbf{q}(t),\mathbf{v}(t),t)\mathrm{d}t \tag{7}$$

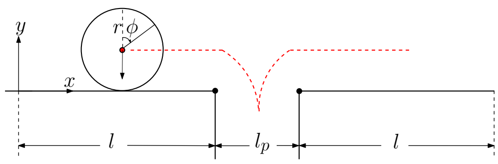
where d$\mathbf{v}$ is the differential measure associated with the velocity, d$\mathbf{i}$ is the impulse measure of the constraint reaction forces, and dt is the Lebesgue measure.



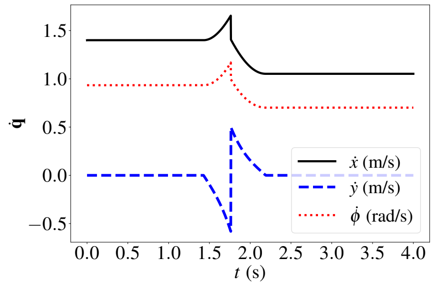
(a) Material point on a nonsmooth track.



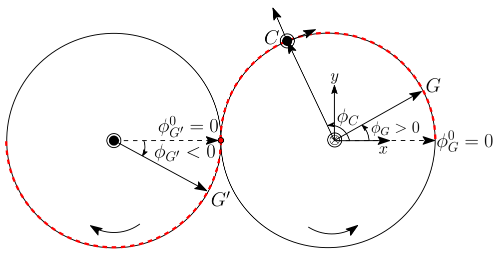
(b) Nonsmooth track: kinetic energy for different choices of the intermediate gradient.



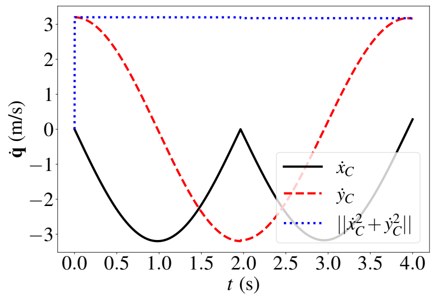
(c) Wheel rolling across a pothole.



(d) Pothole: linear velocities $\dot{x}(t)$, $\dot{y}(t)$ and angular velocity $\dot{\phi}(t)$.



(e) Simplified braiding machine: the carrier C is transferred between two horn gears G and G'.



(f) Braiding machine: carrier velocity components.

Figure 1: Dynamic systems with switching bilateral constraints.

3 Results

The proposed modelling framework and time integration procedure is applied to several examples shown in Figure 1. The numerical results illustrate its ability to describe the transition between modes with discontinuous dynamic phenomena. In particular, Figure 1b shows that, depending on the choice of the intermediate gradient $\mathbf{G}_{\mathscr{E}}$, the impact at the switching can be conservative or dissipative.

4 Conclusions

Multibody systems with switching bilateral constraints may exhibit a nonsmooth dynamic response with discontinuities at the velocity level. Standard index-3 DAE formulations and DAE time integration schemes cannot be used in this context. Instead, the problem can be solved using formulations and time integration methods for nonsmooth dynamic systems.

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References

[1] A. Cosimo, J. Galvez, Javier, F.J. Cavalieri, A. Cardona, and O. Brüls. A robust nonsmooth generalized-α scheme for flexible systems with impacts. Multibody System Dynamics, 48(2):127-149, 2020.