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A new RANS-based added turbulence intensity model for wind-farm flow modelling.

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Abstract. This work aims to alleviate the memory requirements of the recent wake engineering model described in Criado Risco et al. [1]. The original model relies on a RANS-based look-up table of three-dimensional velocity deficit and added turbulence intensity fields computed for a stand-alone turbine under a wide variety of conditions. The objective is to develop an alternative to the model of Criado Risco et al. [1], particularly in terms of added turbulence intensity, for which little research has been carried out to date. To achieve this, a one-dimensional analytical expression is fitted to the look-up table and generalized to higher dimensions. The turbulence intensity model is then coupled to a velocity deficit model and implemented in PyWake, an open-source wake engineering software. Overall, the new turbulence intensity model is found to provide a reliable description of the RANS look-up table data while reducing by half the memory requirements of the original model. This conclusion is extended to multiple wake situations, for which this work also establishes a direct link between the adequate superposition method and the definition chosen to describe the added turbulence intensity in the wake.

1. Introduction

When placed in the wake of an upstream rotor, a wind turbine can experience significant power reductions. The modelling of wind turbine wakes is therefore a subject of growing importance, as part of a broader drive to develop renewable energies worldwide. In this context, Criado Risco et al. [1] recently developed a RANS-based surrogate model consisting in a large look-up table (LUT) of three-dimensional velocity deficit and added turbulence intensity (TI) fields computed for a stand-alone turbine under a wide variety of conditions. In particular, ten values of the thrust coefficient (ranging from $C_T = 0.1$ to $C_T = 1.0$) and four ambient turbulent intensity levels: $\mathrm{Ti}_\infty = 0.05, 0.1, 0.2, 0.3$ are considered. The RANS data are generated with the PyWakeEllipSys tool developed at DTU Wind and Energy Systems. In each simulation, the forces are applied through an actuator disc parametrized to represent the 15MW turbine described in van der Laan et al. [2]. The turbine diameter, hub height and constant below rated values are $D = 236$ m, $H = 150$ m, $C_T = 0.8$, $C_P = 0.45$ and $\mathrm{TSR} = 8$. Storing the RANS data generated over a wide set of conditions however comes with significant memory requirements that are expected to impede the use of this wake model in large wind farm applications. Therefore, the present document is based on the Master thesis of Delvaux [3] that proposes a memory efficient alternative to the model of Criado Risco et al. [1] in the form of an analytical expression

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fitted to the RANS look-up table. In particular, emphasis is placed on the modelling of the turbulence intensity. As pointed out in Bastankhah and Porté-Agel [4], the TI level indeed plays a major part in the wake recovery and therefore significantly impacts the annual energy production of a wind farm. Yet, contrary to the wake velocity deficit for which numerous models have been extensively studied, the modelling of the TI level in the wake has only been the topic of very recent research works. From the general definition of the turbulence intensity, *i.e.* Ti $= \sqrt{2/3} \times \sqrt{k}/U$, Criado Risco et al.[1] computes the increase of turbulence intensity past the turbine as

$$\Delta \mathrm{Ti} = \frac{\sqrt{\frac{2}{3}k} - \sqrt{\frac{2}{3}k_\infty}}{U_\infty}, \tag{1}$$

where k and U respectively denote the turbulent kinetic energy (TKE) and the streamwise velocity. The index "∞" refers to the free-stream quantities in both cases. Although this definition ensures that ΔTi remains real for any positive values of k and k_∞, a more common definition of ΔTi relies on the assumption that the TKE produced by the rotor adds up to that of the ambient flow:

$$\Delta \mathrm{Ti} = \sqrt{\frac{2}{3}}\,\frac{\sqrt{k - k_\infty}}{U_\infty}. \tag{2}$$

Besides the single turbine model for ΔTi proposed by Criado Risco et al. [1], analytical relationships have been established and tested against RANS or LES simulations over the past few years. Based on the momentum and turbulent kinetic energy equations, the work of Crespo and Hernandez [8] pioneered the research for an efficient added TI model, proposing distinct relationships for the near and far wake regions. Later, different candidates were proposed by Larsen [9], Xie and Archer [10] or Tian et al [11], among others. Whereas most models essentially focus on the far-wake evolution along the downstream distance of the maximal level of turbulence intensity, Qian and Ishihara [12] established a 3D model designed to remain valid in both the far and the near wake regions. In Qian and Ishihara [12], the three-dimensional added TI field is described with a two-term Gaussian function that allows to capture the increase of turbulence generated at the rotor periphery. A recent experimental study by Lingkan and Buxton [6] also tends to validate the use of a two-term Gaussian function to model the added TI distribution in the vertical plane.

Table 1: Non-exhaustive list of existing superposition methods for the turbulence intensity level in the merged wake of N upstream turbines.

Method	Rule	Method	Rule
Lin	$\mathrm{Ti} = \mathrm{Ti}_\infty + \sum_i^N (\Delta \mathrm{Ti}_i)$	*Lin-Sqr*	$\mathrm{Ti} = \mathrm{Ti}_\infty + \sqrt{\sum_i^N (\Delta \mathrm{Ti}_i)^2}$
Max	$\mathrm{Ti} = \mathrm{Ti}_\infty + \max_i\{\Delta \mathrm{Ti}_i\}$	*Sqr*	$\mathrm{Ti} = \sqrt{\mathrm{Ti}_\infty^2 + \sum_i^N (\Delta \mathrm{Ti}_i)^2}$

When more than one turbine is considered, the wakes of the upstream turbines merge and lead to an increase of velocity deficit and turbulence intensity. Among the different superposition methods that have been proposed to represent the interactions between the wakes, Table 1 summarizes the TI superposition techniques investigated in Criado Risco et al. [1] (methods *Lin*, *Lin-Sqr* and *Max*). The discussion is further enriched with method *Sqr*, for which Lingkan and Buxton [6] shows promising experimental results using porous discs to represent the turbines. Worth noticing is that method *Sqr* can be seen as the generalization of Eq. 2 to a multiple-wake situation under the assumption that the TKE is additive. Similarly, the linear superposition (method *Lin* in Table 1) reduces to Eq. 1 for a single turbine. Contrary to method *Sqr*, the inflow level of turbulence Ti_∞ is linearly added in method *Lin-Sqr*. In Criado Risco et al. [1], summing the added turbulence intensities in a linear manner (method *Lin*) appears to provide the best agreement with the RANS-generated reference case. Eventually, method *Max* arises from the postulate formulated by Machefaux et al. [7] that only the largest increase of TI is representative of the TI level in the merged wake. In general, this assumption is equivalent to neglecting the effect of all the upstream turbines except the closest one.

The remainder of this paper is structured as follows. In Sec. 2, some general comments are formulated regarding the TI field stored in the LUT. Then, Sec. 3 introduces the methodology followed to establish the one-dimensional $\Delta\mathrm{Ti}$ model and the corresponding results are analysed. The model is finally extended to a three-dimensional space in Sec. 4 and tested in a multiple-wake situation in Sec. 5.

2. Preliminary comments

As stated earlier, Eq. 2 relies on the assumption that the TKE is an additive quantity. Therefore, as this expression is expected to constitute a more physics-based definition of $\Delta\mathrm{Ti}$, it is retained in the remainder of this work. For the sake of consistency, a new look-up table of added TI values is generated following Eq. 2. Note that this choice in no way requires the RANS simulations to be run again, as k and k_∞ are separate outputs of PyWakeEllipSys. Besides relying on a more theoretically grounded description of $\Delta\mathrm{Ti}$, the new LUT of added TI allows to explore the inherent link between the definition of $\Delta\mathrm{Ti}$ and the choice of an appropriate superposition method. This link can be anticipated by re-writing Eq. 1 and Eq. 2 as $\Delta\mathrm{Ti} = \mathrm{Ti}$ - Ti_∞ (linear sum) and $\Delta\mathrm{Ti} = (\mathrm{Ti}^2 - \mathrm{Ti}_\infty^2)^{1/2}$ (quadratic sum), respectively, where Ti denotes the turbulence intensity in the wake (defined with respect to $U = U_\infty$) and Ti_∞ that of the inflow field. In this work, the inflow TI level is defined as $\mathrm{Ti}_\infty = \sqrt{2/3} \times \sqrt{k_\infty}/U_\infty$.

Moreover, it should be stressed that the existing added TI models introduced in Sec. 1 all rely on the streamwise definition of the turbulence intensity: $\mathrm{Ti}=\sigma_u/U$. In this expression, σ_u is the standard deviation of the wind speed fluctuations along the streamwise direction, which is related to the kinetic energy k through the spanwise and vertical components of the speed fluctuations: $k = (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)/2$. This last relationship explicitly shows that both definitions of Ti, *i.e.* in terms of k or σ_u, are equivalent provided that the turbulent fluctuations in the flow are isotropic, *i.e.* $\sigma_u = \sigma_v = \sigma_w$. However, in the wake of a wind turbine, various effects induce heterogeneity, such that the resulting turbulent fluctuations are hardly isotropic, particularly in the near wake region. Still, the Reynolds-average Navier-Stokes equations solved by PyWakeEllipSys rely on the k-ε-f_P closure (see van der Laan et al. [13]) limited by the use of a linear eddy viscosity relationship with which anistropy can not be accurately described. To ensure consistency with the data of the LUT, isotropic turbulence is assumed in the remainder of this work as a basis for developing the new RANS-based added TI model. It should be emphasized that the aim of this work is to develop a model that provides a faithful representation of the RANS data, without further verification of their reliability. Indeed, van der Laan et al. [13] for instance shows that the RANS predictions can overestimate the TI levels in a wake. Therefore, an important asset of this new model is its modularity, thanks to which recalibration of the model against any higher-fidelity database is possible.

3. One-dimensional added turbulence intensity model

This section aims to lay the foundation of the RANS-based TI model by limiting the analysis to the one-dimensional streamwise evolution of the flow. The following development is inspired by the work of Scott et al. [14] in which the notion of increment of eddy viscosity is introduced. The results are then compared to existing TI models and the sensitivity of the derived relationship is evaluated.

3.1. Methodology

As a first step, the focus is placed on establishing a scaling of the evolution of the added TI downstream of a turbine. In Scott et al. [14], it is proposed to decompose the Reynolds stresses $\overline{u'_i u'_j}$ as the sum of some background and wake added components, respectively denoted by $\overline{u'_i u'_j}|_\infty$ and $\Delta\overline{u'_i u'_j}$. By analogy to the Boussinesq approximation applied to both the background and the total flow, Scott et al. [14] defines an increment of eddy viscosity: $\Delta\nu$. The different velocity gradients are then recovered from LES simulations so that the increment of eddy viscosity can be isolated and modelled. Overall, a Rayleigh function of the form $f(\tilde{x}) = (\tilde{x}/\sigma)\exp(-\tilde{x}^2/(2\sigma^2))$, with $\tilde{x} = x/D$ and σ the location of the peak, appears to capture well the behavior of the added eddy viscosity in the wake.

By analogy to the scaling of the eddy viscosity formulated by Davidson [15], *i.e.* $\nu \sim \sqrt{k}\, l_c$, it is assumed in the present work that the increment of eddy viscosity scales as $\Delta\nu \sim \sqrt{\Delta k}\, l_c$. From Eq. 2, a plausible scaling for the added TI therefore reads $\Delta\mathrm{Ti} \sim \Delta\nu/l_c$. In all the expressions above, l_c is the characteristic size of the large eddies in the flow and is expected to monotonically decrease with the downstream distance so that the main behavior is dictated by the evolution of $\Delta\nu$. Consequently, the close link established between $\Delta\mathrm{Ti}$ and $\Delta\nu$ suggests the use of a Rayleigh-like function $f(\tilde{x})$ to describe the evolution of the level of added TI in the wake. Contrary to a Rayleigh function, a Weibull distribution has two parameters so that it can be normalized to reach a unitary maximum value while keeping one tuning parameter free. Denoting m the remaining parameter, it is proposed to write the evolution function $f(\tilde{x})$ as

$$f(\tilde{x}/\tilde{x}_{max}) = \left(\frac{\tilde{x}}{\tilde{x}_{max}}\right)^m \exp\left(m\left(1 - \frac{\tilde{x}}{\tilde{x}_{max}}\right)\right), \tag{3}$$

where $f(\tilde{x} = \tilde{x}_{max}) = 1$. In a sense, replacing a classic Rayleigh function by Eq. 3 allows to account for the monotonic variation of the characteristic size of the large eddies (l_c) when calibrating the free parameter m. Note that alternative scalings, such as the one formulated by Durbin and Pettersson [16] for the eddy viscosity in an axisymmetric self-similar wake, *i.e.* $\nu \sim x^{-1/3}$, were also investigated in the scope of this work. Despite good agreement found in the far wake region, the large discrepancies observed up to five diameters behind the rotor support the need for a more modular function.

Alongside the streamwise evolution of $\Delta\mathrm{Ti}$, its dependency with respect to C_T and Ti_∞ should also be accounted for. Attempts to express the added TI as a product of three independent terms, *i.e.* $\Delta\mathrm{Ti} = A(\mathrm{Ti}_\infty)\,\Phi(C_T)\,f(\tilde{x})$, proved inconclusive as both A and Φ were shown to only partially represent to dependencies on Ti_∞ and C_T, respectively. A more promising approach consists in writing the evolution of the added turbulence intensity under a self-similar form:

$$\Delta\overline{\mathrm{Ti}} = \Delta\overline{\mathrm{Ti}}_{max}\, f(\tilde{x}/\tilde{x}_{max}). \tag{4}$$

In this relationship, $\Delta\overline{\mathrm{Ti}}$ denotes the maximum TI level encountered over a cross-section at location $\tilde{x}$ and $\Delta\overline{\mathrm{Ti}}_{max}$ the maximum value of $\Delta\overline{\mathrm{Ti}}$ over the downstream distance. The downstream distance at which $\Delta\overline{\mathrm{Ti}}_{max}$ is reached is written $\tilde{x}_{max}$. It follows the development of analytical expressions for $\Delta\overline{\mathrm{Ti}}_{max}$ and $\tilde{x}_{max}$ as functions of the free-stream TI and the thrust coefficient of the turbine.

Observations of the look-up table data indicated that the TI level reaches its maximum faster if the wake recovery is enhanced. This is the case for large C_T values as substantial velocity gradients develop between the rotor-encompassed flow and the undisturbed flow. Denoting U_{avg} the disc-averaged streamwise velocity, the position $\tilde{x}_{max}$ is expected to vary as $\tilde{x}_{max} \sim U_{avg}/U_\infty$, where the velocity ratio can be expressed in terms of C_T following the Actuator disc Theory. A faster wake recovery is also achieved for increased level of free-stream TI, for which greater mixing occurs between the wake and the undisturbed flow. In the scope of this work, it is proposed to write $\tilde{x}_{max} \sim 1/\mathrm{Ti}_\infty$, so that the expression of the parameter $\tilde{x}_{max}$ reads

$$\tilde{x}_{max} = \frac{\sqrt{1 - C_T}}{\psi \, \mathrm{Ti}_\infty}, \tag{5}$$

where ψ is a tuning parameter. Contrary to $\tilde{x}_{max}$, the turbulence level $\Delta\overline{\mathrm{Ti}}$ is expected to be an increasing function of the velocity gradients. For the sake of simplicity, the linear relationship

$$\Delta\overline{\mathrm{Ti}}_{max} = \lambda \, C_T \tag{6}$$

is assumed to complete the new TI model given by Eq. 4. Note that an alternative scaling inspired by Larsen [9], *i.e.* $\Delta\overline{\mathrm{Ti}}_{max} \sim \sqrt{1 - \sqrt{1 - C_T}}$ was also investigated but did not lead to any improvements of the final model. In addition, analysis of values of $\Delta\overline{\mathrm{Ti}}_{max}$ retrieved from the RANS look-up table indicated only little variation with Ti_∞, the influence of which is therefore neglected in Eq. 6. This simplification however appeared to be no longer acceptable if $\Delta\mathrm{Ti}$ was defined following Eq. 1, thus requiring a more sophisticated expression than Eq. 6 for $\Delta\overline{\mathrm{Ti}}_{max}$. This observation can be easily explained under the assumption of additive TKE made to derive Eq. 2. Indeed, for a given increase of free-stream TKE (k_∞), or equivalently of Ti_∞, the level of TKE in the wake (k) increases by approximately the same amount so that the difference $\Delta k = k - k_\infty$ is unchanged. As a result, $\Delta\overline{\mathrm{Ti}}_{max}$ defined according to Eq. 2 remains constant in the event of variations of free-stream TI levels.

3.2. Results and sensitivity analysis

This section aims at evaluating the performances of the new one-dimensional TI model developed in Sec. 3.1. In order to determine the values of the scaling parameters ψ and λ respectively introduced in Eq. 5 and Eq. 6, calibration is performed over the set $\{C_T\} \times \{\mathrm{Ti}_\infty\} = [0.1; 0.8] \times [0.05; 0.3]$. Each combination of C_T and Ti_∞ is tested so that the error between the results of Eq. 5 and Eq. 6, and the corresponding quantities recovered from the RANS look-up table appears to be minimized for $\psi = 2.03$ and $\lambda = 0.175$. The peak location and amplitude retrieved from the RANS look-up table are denoted $\tilde{x}^*_{max}$ and $\Delta\overline{\mathrm{Ti}}^*_{max}$ in the remainder of this work. It is interesting to note that this optimization process has been carried out on a more general form of Eq. 5, *i.e.* $\tilde{x}_{max} = (1 - C_T)^\phi \, \mathrm{Ti}^\theta_\infty / \psi$, for which the optimal values of the two additional parameters were found to be $\phi = 0.485$ and $\theta = -1.04$. This encouraging observation strongly supports the development of Eq. 5. Moreover, an empirical function f^* for the streamwise evolution of TI can be retrieved from the turbulence intensity field stored in the LUT and compared to the predictions of Eq. 3. The absolute error between f^* and f (Eq. 3) is integrated over a downstream distance of $20D$ for each combination of C_T and Ti_∞. Contrary to the two previous tuning parameters for which a unique value has been set, the decision is made to store the optimal values of the m parameter in a small look-up table as a function of C_T and Ti_∞ (see Appendix). This choice is motivated by the high sensitivity observed in the model with respect to the parameter m.

Fig. 1a and Fig. 1b compare the evolution of the added TI level predicted by the model and the corresponding RANS values. The performance of the new model is also compared to that of state-of-the-art counterparts. Under a free-stream turbulence intensity level of 5%, excellent agreement is shown in Fig. 1a in both the near and far wake regions, significantly outperforming the predictions of the current TI models for all the tested C_T values. As turbines located in the wakes of upstream turbines are subject to increased level of free-stream turbulence, it is crucial to ensure the reliability of the model for large Ti_∞ values. In Fig. 1b, good agreement is observed for $\mathrm{Ti}_\infty = 0.20$, albeit with a larger discrepancy between the new model and the RANS results for $\tilde{x} > 5$ when the turbine operates at $C_T = 0.8$. In the context of wakes superposition, the slight deterioration of the model's performances in the far wake region is however expected to be negligible compared to the influence of the most upstream turbines on their downstream counterparts.

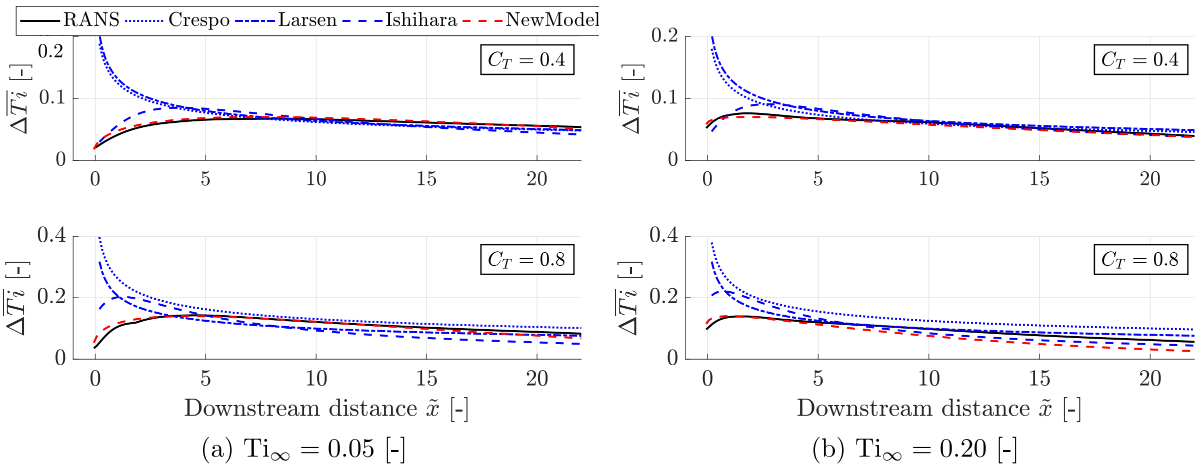

Figure 1: Streamwise evolution of the added turbulence intensity for two values of C_T and Ti_∞. The predictions of the new model are compared to the RANS reference and to the models of Crespo and Hernandez [8], Larsen [9] and Qian and Ishihara [12]. The scale of the y-axis is adapted between the upper and lower figures for readability.

In order to assess the robustness of the new model, a sensitivity analysis is discussed. The analytical approach followed below aims at breaking down the different source terms that contribute to the absolute error, denoted by $d(\Delta\overline{\mathrm{Ti}})$, between the model predictions and the corresponding RANS results. Introducing $d(\Delta\overline{\mathrm{Ti}}_{max})$ and $d(\tilde{x}_{max})$ the absolute modelling errors on $\tilde{x}_{max}$ (Eq. 5) and $\Delta\overline{\mathrm{Ti}}_{max}$ (Eq. 6) respectively, the global absolute error can be approximated by the first-order truncated development of Eq. 4:

$$d(\Delta\overline{\mathrm{Ti}}) \simeq \frac{\partial \Delta\overline{\mathrm{Ti}}}{\partial \Delta\overline{\mathrm{Ti}}_{max}} d(\Delta\overline{\mathrm{Ti}}_{max}) + \frac{\partial \Delta\overline{\mathrm{Ti}}}{\partial f} d(f) \quad \text{with} \quad d(f) \simeq \frac{\partial f}{\partial \tilde{x}_{max}} d(\tilde{x}_{max}) + \varepsilon_f. \tag{7}$$

In Eq. 7, the term ε_f is computed from the RANS look-up table data and accounts for the residual error between the evolution function $f(\tilde{x}/\tilde{x}^*_{max})$ (Eq. 3) and the empirical evolution function f^*. The sensitivity factors in Eq. 7 can be further developed given Eq. 4 and Eq. 3 so that Eq. 7 can be re-written as

$$d^r(\Delta\overline{\mathrm{Ti}}) = \underbrace{d^r(\Delta\overline{\mathrm{Ti}}_{max})}_{T_1} + \underbrace{m\left(\frac{\tilde{x}}{\tilde{x}^*_{max}} - 1\right)\frac{f}{f^*}\, d^r(\tilde{x}_{max})}_{T_2} + \underbrace{\varepsilon^r_f}_{T_3}, \tag{8}$$

where the notation $d^r(.)$ refers to the relative errors with respect to the RANS values. The total relative error $d^r(\Delta\overline{\mathrm{Ti}})$ computed from Eq. 8 is represented against the streamwise distance in Fig. 2 under $C_T = 0.8$ and $\mathrm{Ti}_\infty = 0.05$. The contribution of the three right-hand side terms in Eq. 8, respectively denoted T_1, T_2 and T_3, is also shown in Fig. 2.

Overall, the error appears to be mainly driven by T_3 in the very near and far wake region - that is, where the evolution of added TI most differs from the assumed Weibull shape. One can also notice that little error is obtained between $5D$ and $15D$, partially due to a compensation of the three error terms. It is important to bare in mind that Eq. 8 provides an analytical estimate of how a modelling error made in Eq. 3, Eq. 5 or Eq. 6 impacts the differences observed between the new TI model and the RANS LUT. Therefore, as a result of the first order limited development in Eq. 7, there exists a discrepancy between the relative errors predicted by the analytical analysis and the corresponding empirical errors. The measured relative error is shown with black markers in Fig. 2. Good agreement with respect to the predicted error is observed overall, with the exception of the very near wake where the importance of higher-order terms, omitted in Eq. 7, becomes significant. The analytical sensitivity analysis also allows to quantify the impact of a slight variation of the parameters ψ, λ and m, motivating the use of the small look-up table for m.

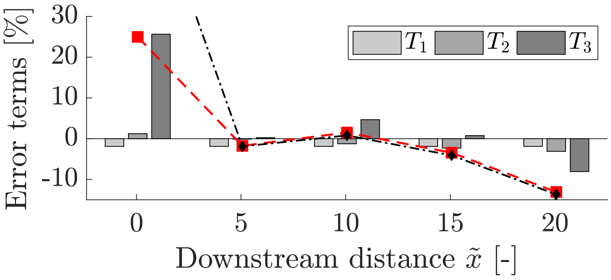

Figure 2: Streamwise evolution of the predicted (red) and measured (black) total relative error $d^r(\Delta\overline{\mathrm{Ti}})$. The three RHS terms in Eq. 8 are denoted T_1, T_2 and T_3 respectively.

4. Generalisation of the model to higher dimensions

4.1. Methodology

In this section, the one-dimensional added TI model developed in Sec. 3.1 is generalized to a three-dimensional space under the axisymmetric wake assumption. Contrary to the velocity deficit profile, for which the largest deficit is usually observed along the axis of the rotor (except in the near wake region), the presence of significant velocity gradients at the rotor periphery leads to increased turbulence intensity levels at the edge of the wake. Therefore, following the approach presented in Lingkan and Buxton [6], a two-term Gaussian function is used to represent the added turbulence intensity profile at any location downstream of the rotor. Assuming that both Gaussian terms have an amplitude $C(x)$ and a standard deviation $\sigma(x)$, the analytical form of the axisymmetric profile reads:

$$\Delta\mathrm{Ti}(x,r) = C\left(\exp\left(-\frac{(r-r_c)^2}{2\,\sigma^2}\right) + \exp\left(-\frac{(r+r_c)^2}{2\,\sigma^2}\right)\right). \tag{9}$$

In this expression, the radial distance is denoted r and the position of the center-line of each Gaussian term (r_c) is considered to coincide with the rotor edge, *i.e.* $r_c = D/2$. The wake half-width is defined as $r_{1/2} = \sigma\sqrt{2\ln(2)} + r_c$ and approximately corresponds to the radial distance at which the level of added TI is equal to $C/2$. Note that the amplitude C of each of the two Gaussian terms in Eq. 9 is linked to the amplitude $\Delta\overline{\mathrm{Ti}}$ modelled in Eq. 4 through $C = \Delta\overline{\mathrm{Ti}}/(1+\exp(-2r_c^2/\sigma^2))$.

In order to complete the three-dimensional model, the standard deviation $\sigma(x)$ is assumed to linearly increase with the downstream distance (see Lingkan and Buxton [6]). Therefore, the linear relationship developed by Bastankhah and Porté-Agel [4] for the standard deviation of the Gaussian velocity deficit is used to express $\sigma(x)$. In Bastankhah and Porté-Agel [4], the wake growth is represented by a wake expansion coefficient k_w, the value of which is found to be function of the inflow TI level (Niayifar and Porté-Agel [5]). Following Niayifar and Porté-Agel [5], this coefficient writes $k_w = \alpha_1 \mathrm{Ti}_\infty + \alpha_2$, where $\alpha_1 = 0.384$ and $\alpha_2 = 0.0037$ are two parameters that can be re-calibrated to better represent the TI field stored in the RANS look-up table. For that purpose, the parameters C and σ of the two-term Gaussian function defined in Eq. 9 are fitted to the horizontal $\Delta\mathrm{Ti}$ profile recovered from the RANS LUT at hub height. The resulting quantities are denoted C^* and σ^*, and a reference wake half-width is naturally defined as $r^*_{1/2} = \sigma^*\sqrt{2\ln(2)} + r_c$. This process leads to a two-term RANS fitted Gaussian function that can be evaluated at any downstream location $\tilde{x}$ and for any of the C_T

and Ti_∞ values of interest. Eventually, the values of the parameters α_1 and α_2 that dictate the wake growth are calibrated against the RANS fitted standard deviation σ^* over the set $\{\tilde{x}\} \times \{C_T\} \times \{\mathrm{Ti}_\infty\} = [0; \tilde{x}_f] \times [0.1; 0.8] \times [0.05; 0.3]$. The value $\tilde{x}_f$ is arbitrarily chosen as the location above which the amplitude of the added TI drops below 0.1%. The calibration results in $\alpha_1 = 0.248$ and $\alpha_2 = 0.0114$. As expected, those values differ from the ones of Niayifar and Porté-Agel [5] recovered from LES simulations performed over a smaller range of conditions.

With the aim of fully substituting the RANS look-up table model of Criado Risco et al. [1] for a memory efficient alternative, the new TI model developed here is combined with the classical Gaussian velocity deficit model of Bastankhah and Porté-Agel [4]. A methodology similar to the one described above is followed to re-calibrate the two parameters of a wake expansion coefficient k'_w defined with respect to the Gaussian velocity deficit. The two resulting parameters are denoted α'_1 and α'_2 and are equal to 0.264 and 0.0126, respectively. Even though those values are close to the ones of α_1 and α_2 found for k_w, this supports the observation made by Lingkan and Buxton [6] that the wake expansion coefficient varies depending on whether the velocity deficit or the TI level is used to characterized the wake growth.

4.2. Results

The new three-dimensional model for $\Delta\mathrm{Ti}$ is evaluated by comparison to the RANS data and to the corresponding RANS fitted profile under operating conditions $C_T = 0.8$ and $\mathrm{Ti}_\infty = 0.1$. In Fig. 3, a cut at hub-height shows good agreement between the modelled profile and the two-term fitted Gaussian, the largest discrepancy generally occurring along the wake axis. A

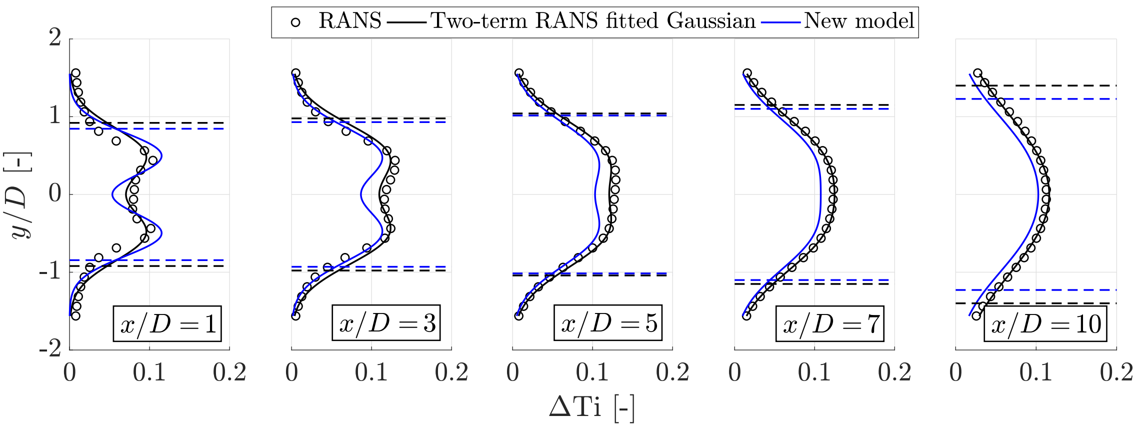

Figure 3: Streamwise evolution of the added TI profile at hub height for $\mathrm{Ti}_\infty = 0.1$ and $C_T = 0.8$. The predictions of the new model are compared to the RANS fitted profile. The wake half-widths $r_{1/2}$ and $r^*_{1/2}$ are represented as dashed lines in the figure.

sensitivity analysis, not detailed here for conciseness, pointed out the substantial dependence of the centerline value on the modelling of σ. Similar conclusions were drawn for all the other conditions for which the wake model has been derived. Therefore, the flexibility of the model as well as its ability to represent the mechanisms of production, radial diffusion and dissipation of turbulence supports the choice of a two-term Gaussian function for future research works. However, it is good to bear in mind that the axisymmetric wake assumption made earlier is a substantial simplification of the problem. Indeed, the presence of the ground, the hub interference and the non-uniform boundary layer profile were observed to impact the distribution of TI over the rotor section. Approaches proposed by Qian and Ishihara [12] and Tian et al. [11] suggest the use of an additional term in Eq. 9 to address the vertical dependence.

5. Multiple-wake configuration

The objective of this last section is twofold. First, it aims at providing a qualitative overview of the performances of the new model used as an alternative to the RANS look-up table model of Criado Risco et al. [1] in a multiple-wake configuration. The second goal is to identify and motivate the choice of a relevant superposition method for the added turbulent intensity. In the scope of this work, the new TI model is implemented in the wake engineering open-source software PyWake. The analysis described below is limited to the academic case of a one-dimensional array of five wind turbines positioned $5D$ apart and operating at $C_T = 0.8$ with conditions $U_\infty = 10\,\mathrm{m/s}$ and $\mathrm{Ti}_\infty = 0.05$. The results of a RANS simulation performed over the whole column of turbines is used as a reference. In the following discussion, the LUT model of Criado Risco et al. [1] is denoted "Model 3" whereas the new ΔTi model coupled to the re-calibrated Bastankhah and Porté-Agel deficit model is refered to as "Model 1". An in-between solution, termed "Model 2", consists of a combination of the new added TI model developed in this work and the velocity deficits stored in the RANS LUT of Criado Risco et al. [1].

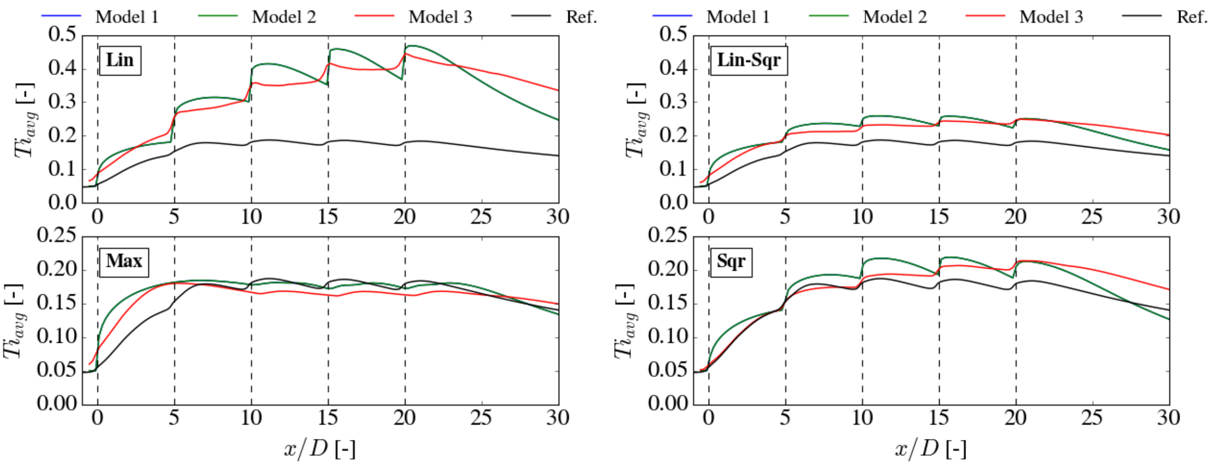

Figure 4: Comparison of the four turbulence superposition methods listed in Table 1 in terms of the disc-averaged values of the wake turbulence intensity. The velocity deficits are linearly superposed. The position of each of the five turbines in the array is indicated by a black vertical dotted line. The vertical scale is adapted for readability in *Max* and *Sqr*.

For each of the three models, the added turbulence intensities are superimposed following method *Lin*, *Lin-Sqr*, *Max* or *Sqr* introduced in Table 1. The resulting disc-averaged TI level is represented against the downstream distance in Fig. 4. A linear superposition method for the velocity deficit is used in each case, following from the recommendation of Criado Risco et al. [1]. Still, note that this choice as well as that of a specific velocity deficit model have no impact on the Ti field as long as C_T remains constant. For this reason, Model 1 and Model 2 can be seen to overlap in Fig. 4. From the same figure, it is clear that the highest error with respect to the RANS reference is obtained with method *Lin*. This discrepancy can be halved by selecting the superposition method *Lin-Sqr*. Contrary to the results of Criado Risco et al. [1], a significantly closer match is however observed with method *Sqr*. As mentioned in Sec. 2, the work of Criado Risco et al. [1] and the present study use Eq. 1 and Eq. 2, respectively. This observation therefore seems to support the idea introduced in Sec 1 of the additive properties of the TKE. More importantly, it points out that the choice of a suitable superposition method for ΔTi is essentially governed by the expression, either Eq. 1 or Eq. 2 used to defined it. Eventually, the smallest discrepancy is achieved when superposing the added TI according to method Max. This conclusion aligns with that of Lingkan and Buxton [6], in which they show that the most upstream turbine has a dominant effect for spacing as large as $5D$.

In addition to explicitly assessing the superposition methods for ΔTi, Fig. 4 indicates a close correspondence between Model 2 and Model 3, in particular in the bottom most plots (methods *Max* and *Sqr*). This tends to validate the new added TI model as a promising alternative to the RANS look-up table model employed in multiple-wake conditions. However, Fig. 5 shows that substituting the RANS-generated velocity deficits for the predictions of the re-calibrated Bastankhah and Porté-Agel model (*i.e.* Model 1) leads to successive under and over-predictions of the disc-averaged velocity. Therefore, Model 2 is retained as a good comprise that allows to replace the TI field stored in the LUT by a calibrated analytical relationship, thus reducing by half the memory requirements of the original Criado Risco et al. [1] model.

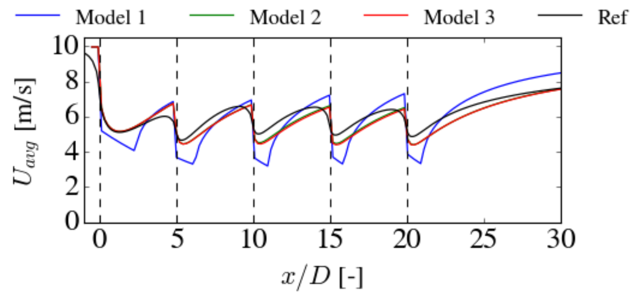

Figure 5: Streamwise evolution of the disc-averaged velocity as predicted by the three models. The velocity deficits and the ΔTi fields are superposed linearly and according to method *Sqr*, respectively.

6. Conclusion

This work proposes a new wake model for the added turbulence intensity as an alternative to the RANS look-up table model of Criado Risco et al.[1]. The reliability of the model has been observed over a large range of C_T and Ti_∞ values, enabling the model to be applied in merged-wake situations. Eventually, the new TI model has been found to effectively halve the memory requirements of the initial RANS look-up table approach. Besides, this work also sheds light on the inherent link between the definition of ΔTi and the choice of a suitable superposition method.

Appendix - Table for the m parameter

C_T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\mathrm{Ti}_\infty = 0.05$	0.2695	0.3015	0.3350	0.3595	0.3785	0.2800	0.3170	0.2930
$\mathrm{Ti}_\infty = 0.1$	0.0820	0.1080	0.1295	0.1450	0.1720	0.1805	0.1895	0.1505
$\mathrm{Ti}_\infty = 0.2$	0.0485	0.0550	0.0650	0.0740	0.0835	0.0845	0.0935	0.1030
$\mathrm{Ti}_\infty = 0.3$	0.0505	0.0485	0.0530	0.0580	0.0625	0.0670	0.0715	0.0760

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