A Test-Case on Continuation Methods for Bladed-Disk Vibration with Contact and Friction

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Abstract Bladed-disks in turbo-machines experience harsh operating conditions and undergo high vibration amplitudes if not properly damped. Friction at the blade-to-blade or blade-to-disk interfaces plays a key role in dampening the high amplitudes. Due to the inherent complexity of these structures and non-linearities introduced by the friction joints, accurate response prediction becomes very difficult. There are variety of methods in the literature to predict non-linear vibration due to contact friction. However, their application to the bladed-disks remains limited. Furthermore, there are not many 3D realistic test-cases in the open literature for testing those methods and serve as a benchmark. A bladed-disk representative of a real turbine is presented as an open numerical test-case for the research community. It is characterized by a blade root joint and a shroud joint. The bladed-disk sector is meshed in different ways along with component mode synthesis (CMS) model order reduction for onward non-linear computations. The steady-state solution is obtained by multi-Harmonic Balance method and then continuation method is employed to predict the non-linear frequency response. Thus, it can serve as a case for testing previous and new methods as well as a benchmark for comparative studies.

Keywords Bladed-disk · Cyclic symmetry · Nonlinear forced response · MHBM · Continuation method

27.1 Introduction

Turbo-machine components design is a great challenge in the presence of multitude of problems. High cycle fatigue of the rotating blades is one of those problems and requires that accurate modelling of the forced response is done including the damping from all the possible sources [1]. Contact friction at the blade-roots or shrouds (or under-platform dampers) provides some damping but it is highly non-linear. Different solution approaches in the literature have been presented in a period of over two decades, comprising of friction models as well as iterative solutions, for example [2–4].

In spite of various advanced models and methods developed for prediction of friction damping, their application to the bladed-disks remains limited due to their inherent complexity and large number of degrees of freedom (DOF). On the other hand, access to a real geometry with some experimental data also remains a big concern due to confidentiality and competitiveness of the industries. Sometimes, choosing a particular model is based on current practices of the institutions or as a call of faith, instead of the comparative knowledge basis. Therefore, a test-case geometry of blade assembly for the FUTURE project (2009–2012) [4] is used in this study and open-access modules are developed ranging from building the system matrices, model order reduction, harmonic balance method with continuation steps to penalty based friction models with a possibility to include fretting and wear. Such a modular framework might already be in a more advanced form with

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some industries and research institutions, but falls under confidentiality agreements. As a baseline work, modules on state-of-the-art methods have been developed and tested on simple cases.

27.2 Methodology and Analysis

The approach to solution of the aforementioned problem is followed using coupled static and dynamic multi-harmonic balance method [5]. Due to cyclic nature of the structure and excitation encountered by a bladed-disk, only a sector is considered for modelling using cyclic symmetry boundary conditions, as shown in Fig. 27.1a. The system matrices of the sector, discretized with finite elements (FE), undergoing subsequent refining for convergence of static and modal solutions up to a desired frequency range. The FE model is too large for non-linear response solution that it needs to be reduced. A Craig-Bampton reduction is applied to retain the interface, internal (for response) and cyclic DOFs and the modes of interest. The equation of motion of a single sector whose DOFs are condensed at the interface and response DOFs is

$$\bar{M}\ddot{u} + \bar{C}\dot{u} + \bar{K}u - \bar{f}_{\text{ext}} + \bar{f}_{NL}(u, \dot{u}) = 0 \tag{27.1}$$

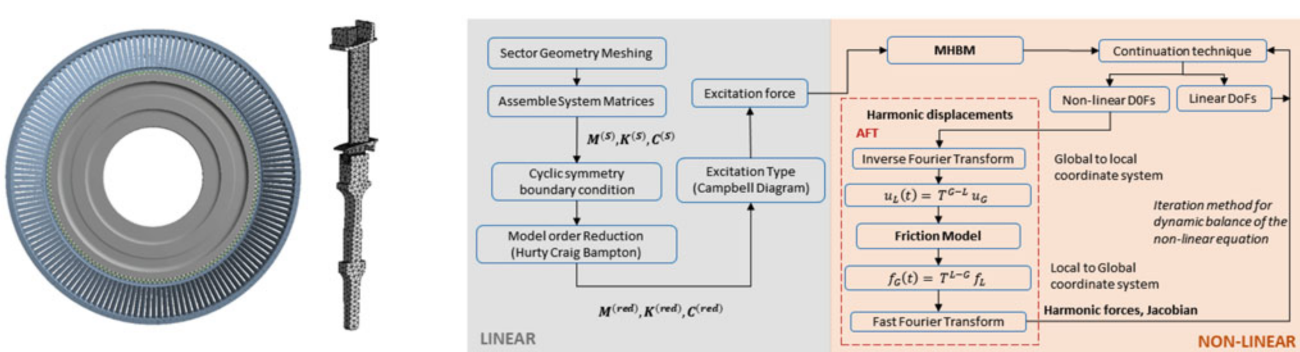
where $\bar{M}$, $\bar{C}$, $\bar{K}$ are the reduced mass, Rayleigh damping and prestressed stiffness matrices due to rotation, respectively, $\bar{f}_{\text{ext}}$ is an external excitation harmonic force and $\bar{f}_{NL}(u, \dot{u})$ is a non-linear force due to contact friction. The reduced matrices are subsequently used in multi-harmonic balance method (MHBM) in which N nonlinear differential equations are converted to $N(2N_H + 1)$ nonlinear algebraic equations:

$$(-h^2\omega^2\bar{M} + ih\omega\bar{C} + \bar{K})\hat{u}^{(h)} = \hat{f}^{(h)}_{\text{ext}} - \hat{f}^{(h)}_{NL}(\hat{u}) \tag{27.2}$$

with $h = 0, \ldots, N_H$ and at $h = 0$, only $\bar{K}$ is considered for static balance. The non-linear forces are computed using a friction element in an alternate frequency time domain (AFT) fashion. The friction element is based on 3D Jenkins element with coupled tangential motion and variable normal load. The friction force is calculated in terms of contact parameters such as a tangential stiffness K_t, a normal stiffness K_n and a coefficient of friction μ. The Jacobian needed for Newton iteration within the continuation iteration is also computed analytically to save computational time and better convergence [3, 6]. The solution in the frequency domain is tracked by a parametric continuation procedure, such as pseudo arc-length [7]. A value for frequency ω_i is chosen and the residual of equation Eq. (27.1) is linearized based on the previous amplitude solution $\tilde{u_i}$

$$\left[\frac{\partial \tilde{R}(\tilde{u_i}; \omega_k)}{\partial \tilde{u_i}}\right]\delta\tilde{u}^{\text{pred}} = \tilde{R}(\tilde{u_i}) \tag{27.3}$$

where $\delta\tilde{u}^{\text{pred}} = \tilde{u}^{\text{pred}}_{i+1} - \tilde{u_i}$ is the amplitude predictor. The solution of Eq. (27.3) must be updated by an additional pseudo arc-length constraint, known as the corrector phase $C(\tilde{u}^{corr}_{i+1}, \omega_{i+1}) - ||\delta\tilde{u}^{\text{pred}}|| = 0$. A flow-chart of the above methodology with appropriate transformations is shown in Fig. 27.1b.



(a) (b)

Fig. 27.1 (**a**) A bladed-disk with a meshed sector. (**b**) Flow chart for the framework

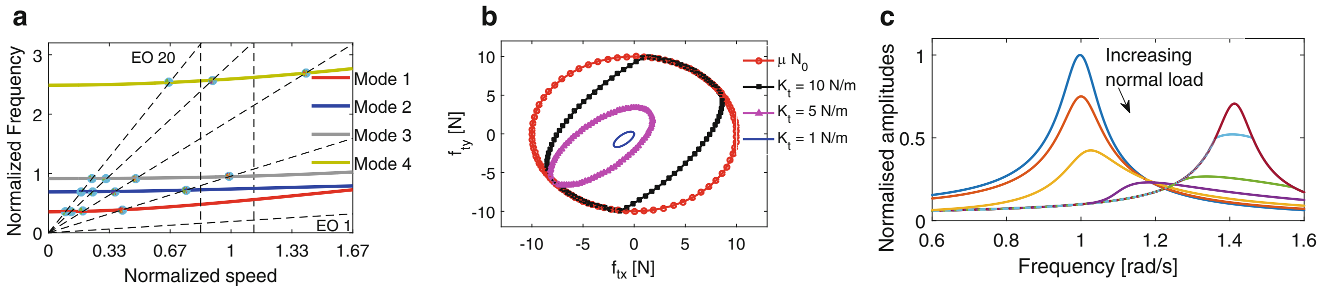


Fig. 27.2 (**a**) Campbell Diagram at ND = 5, (**b**) coupled tangential contact forces (x and y direction) over one period of oscillation and (**c**) first harmonic response of a mass and slider using pseudo arc-length continuation

27.2.1 Framework

The reduced system matrices are obtained in AMfe, an open source finite element library, are exported in an MHBM continuation code based on C++. This solves Eq. (27.3) iteratively using Trilinos library [8] and evaluates the nonlinear friction forces. The framework is modular with separate modules for friction model, continuation method, model order reduction, etc. The test-case blade assembly is used for verification of the solver and will be available as a benchmark. The framework and the geometry are available in open GIT repository: http://github.com/jenovencio/NDCSI4-2018.

27.2.2 Preliminary Results

During testing of the modules, critical resonances of the cyclic sector are obtained by the Campbell diagram, see Fig. 27.2a. This is helpful in selecting the nodal diameter modes to be retained in the basis for non-linear iterative solution. The evolution of the tangential frictional forces is plotted in Fig. 27.2b with coupled behavior and Coulomb limit prescribed by a circle (for isotropic friction coefficient). Finally, the pseudo-arc length continuation scheme is plotted in Fig. 27.2c for a simple mass and slider with free and stick response and slipping in between for varying normal loads.

27.3 Conclusion

This work has aimed to develop individual modules for the bladed-disks subject to nonlinear friction forces with an open-access for all. All the modules have been individually tested on simple cases. This work can be used by the research community to compare different methods available in the research spectrum, along with their own work, against different parameters of interest. The present framework has two different platforms Python (AMfe library) and C++ (interface code and Trilinos library) which needs to be converted into one platform. Necessary interfacing between coding environments and inclusion of more models will be done in the future.

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