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On the Evaluation of the Tangential Slip Increment in Quasi-static Beam-to-Beam Contact Problems

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1. Introduction

Frictional contact models between beams are relevant for the numerical simulation of beam assemblies in various applications such as wiring harnesses, cable bundles or textile manufacturing processes. An essential ingredient of the contact model is the kinematic description of the relative motions of the material particles in the contact zone. In particular, frictional contact models require a quantification of the relative slip motion between the two bodies. This work addresses the definition of the tangential slip increment between geometrically exact beams using the special Euclidean group formalism. This geometric setting guarantees the frame invariance of the results and enables an accurate treatment of complex contact behaviours, such as rolling without slipping interactions.

2. Contact model

In contact mechanics, the normal and tangential contact forces λ_N and $\boldsymbol{\lambda}_T$ can be expressed as inclusions

$$\lambda_N \in \partial \psi_{\mathbb{R}^+}(g_N), \quad \boldsymbol{\lambda}_T \in \partial \psi^*_{\mathscr{C}(\lambda_N)}(\mathbf{u}_T) \tag{1}$$

where g_N is the *normal gap*, $\mathbf{u}_T$ is the *tangential slip velocity*, $\psi_{\mathbb{R}^+}$ is the indicator function of the set of positive real numbers and $\mathscr{C}(\lambda_N)$ is the Coulomb disk defined as

$$\mathscr{C}(\lambda_N) = \{\boldsymbol{\lambda}_T \in \mathbb{R}^2 : \|\boldsymbol{\lambda}_T\| \leq \mu\, \lambda_N\} \tag{2}$$

In a quasi-static analysis, this contact model can still be considered provided that the slip velocity $\mathbf{u}_T$ is substituted by a *tangential slip increment* $\Delta\mathbf{g}_T(t-\Delta t, t)$ over the pseudo-time interval Δt evaluated as

$$\Delta\mathbf{g}_T(t-\Delta t, t) = \int_{t-\Delta t}^{t} \mathbf{u}_T(\tau)\, \mathbf{d}\tau \tag{3}$$

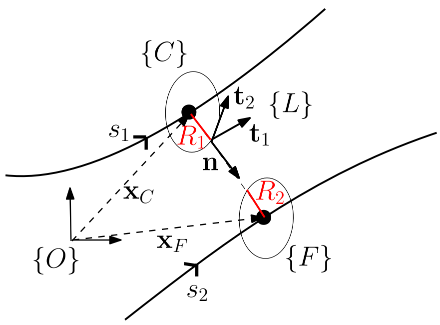

Figure 1: Kinematics of the beam-to-beam contact

3. Contact kinematics

Let us consider two beams with circular cross-sections of radii R_1 and R_2 (see Fig. 1). At each point of the centerline, the position and orientation of the cross-section shall be treated as an element of the special Euclidean group $SE(3)$ [1]. Furthermore, we assume that the deformation of the cross-section as well as the shear deformation can be neglected in the contact kinematics. One of the beam is qualified as the slave and the other one as the master. For the slave beam, the centerline is parameterized using the material coordinate s_1, the section-attached frame at point s_1 is denoted as $\{C\}$ (with origin C), and the first axis $\mathbf{e}_{C1}$ is pointing in the tangential direction to the slave centerline. For a given point s_1 on the slave beam, an orthogonal projection is applied to define the potential contact section on the master beam, which is represented by the frame $\{F\}$ with origin F at the material coordinate s_2. By construction, this frame $\{F\}$ is such that the vector joining C and F is orthogonal to $\mathbf{e}_{C1}$ and the first axis $\mathbf{e}_{F1}$ is pointing in the tangent direction to the master centerline.

On each beam, one additional frame is defined at the potential contact point on the external contour of the cross-section. On the slave beam, the frame $\{L\}$ at the potential contact point is defined such that (i) the normal vector $\mathbf{n} = \mathbf{e}_{L1}$ is along the directed line joining C to F, (ii) the tangent vector $\mathbf{t}_1 = \mathbf{e}_{L2}$ is equal to $\mathbf{e}_{C1}$ and the tangent vector $\mathbf{t}_2 = \mathbf{e}_{L3}$ completes the orthonormal basis, (iii) the origin L is localized on the external contour of the slave cross-section (i.e., at a distance R_1 of C) and on the directed line joining C to F. On the master beam, the frame $\{K\}$ at the potential contact point is defined such that its three axes coincide with $\mathbf{n}$, $\mathbf{t}_1$ and $\mathbf{t}_2$ and its origin K is localized on the external contour of the master cross-section (i.e., at a distance R_2 of F) and on the directed line joining F to C.

The relative configuration is then obtained as a 4×4 matrix $\mathbf{H}_{LK} \in SE(3)$ and the *normal gap* is simply

$$g_N = [\mathbf{H}_{LK}]_{11} \tag{4}$$

From now, the frames $\{L\}$ and $\{K\}$ are considered as material frames attached to the material points on the contour of the cross-sections. The relative velocity is represented by a 6×1 twist vector $\mathbf{v}_{LK}^K$ with components in the frame $\{K\}$. The *tangential slip velocity* is then obtained as a two dimensional vector

$$\mathbf{u}_T = \begin{bmatrix} [\mathbf{v}_{LK}^K]_2 \\ [\mathbf{v}_{LK}^K]_3 \end{bmatrix} \tag{5}$$

4. Tangential slip increment

For quasi-static problems, we propose to define the tangential slip increment by assuming a constant relative velocity over the pseudo-time interval Δt. The 6×1 *relative motion increment* is then introduced as

$$\Delta\mathbf{y}_{LK}^K(t-\Delta t, t) = \int_{t-\Delta t}^{t} \mathbf{v}_{LK}^K(\tau)\,\mathbf{d}\tau = \mathbf{v}_{LK}^K(t)\,\Delta t \tag{6}$$

This motion increment is related to the change of $\mathbf{H}_{LK}$ by the finite difference formula

$$\Delta\mathbf{y}_{LK}^K(t-\Delta t, t) \simeq \log(\mathbf{H}_{LK}^{-1}(t-\Delta t)\,\mathbf{H}_{LK}(t)) \tag{7}$$

where log is the logarithm map on $SE(3)$. Then, the *tangential slip increment* is obtained from the components

$$\Delta\mathbf{g}_T = \begin{bmatrix} [\Delta\mathbf{y}_{LK}^K]_2 \\ [\Delta\mathbf{y}_{LK}^K]_3 \end{bmatrix} \tag{8}$$

which can be interpreted as a geometrically consistent finite difference approximation of $\mathbf{u}_T$ scaled by Δt.

To the best of our knowledge, the definition of the finite slip increment $\Delta\mathbf{g}_T$ based on a finite difference on $SE(3)$ has not been presented in the literature. In the presentation, the formulation will be illustrated using quasi-static simulations of beam-to-beam frictional contact problems.

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