Chapter 19
Comparison of Computational Generalized and Standard Eigenvalue Solutions of Rotating Systems

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Abstract Modal analysis is regularly used to compute natural frequencies and mode shapes of structures via eigenvalue solutions in vibration engineering. In this paper, the eigenvalue problem of a 6 degrees of freedom rotating system with gyroscopic effects, including axial, torsional and lateral motion, is investigated using Timoshenko beam theory. The main focus thereby is the investigation of the computational time and the numerical errors in generalized and standard eigenvalue solutions of rotating systems. The finite element method is employed to compute the global stiffness, mass and gyroscopic matrices of the rotating system. The equations of motion is expressed in the state space form to convert the quadratic eigenvalue problem into the generalized and standard forms. The number of elements in the finite element model was varied to investigate the convergence of the natural frequencies and the computational performance of the two eigenvalue solutions. The numerical analyses show that the standard eigenvalue solution is significantly faster than the generalized one with increasing number of elements and the generalized eigenvalue solution can yield wrong solutions when using higher numbers of elements due to the ill-conditioning phenomenon. In this regard, the standard eigenvalue solution gives more reliable results and uses less computational time than the generalized one.

19.1 Introduction

Dynamic analysis of rotating systems has attracted much interest over the last decades since rotor systems are present in many engineering applications, such as electrical machines, turbo machines, combustion engines and wind turbines. The first known basic rotor model consisting of a flexible rotating shaft, a rigid disk and bearings was defined by Föppl and Jeffcott in 1895 and 1919 respectively, which is known as Jeffcott rotor today [1]. Carrying out modal analysis on the Jeffcott rotor is relatively simple compared to complex rotor systems, because such systems require solution of an eigenvalue problem with large matrices. The finite element and transfer matrix methods [2–6] are widely known methods to compute mass, stiffness, damping and gyroscopic matrices of large scale rotor systems. In the past, transfer matrix method was the preferred method for the solution of large scale rotor dynamics problems, but recently the finite element method has become more widely used because it provides better computational performance compared to the other methods [2, 4].

In large scale eigenvalue problems, matrices could have special properties; they may be symmetric, skew-symmetric, non-symmetric or Hermitian [7]. For example, the gyroscopic matrix for rotating systems is normally skew-symmetric, but the addition of damping leads to the loss of this property [8]. Therefore, the eigenvalue problem of the damped or undamped gyroscopic systems often require special numerical methods for the correct solution such as eigenvalue shifting techniques [9, 10]. Various numerical methods and software tools have been developed to solve large-scale quadratic eigenvalue problems [7, 11–13]. There are also existing studies investigating the solution of non-symmetric eigenvalue problems [14]. The Linear Algebra Package (LAPACK) [15] is one of the most well-known pieces of software used to solve eigenvalue problems including those with non-symmetric matrices. Another important development in the solution of eigenvalue problems has been algorithm reduction techniques, allowing calculation of only certain eigenvalues such as largest or smallest in magnitude [7]. Algorithm reduction is a convenient tool for reducing the computation time.

Eigenvalue problems can be formulated in two forms, (i) the standard eigenvalue problem (SEP) and (ii) the generalized eigenvalue problem (GEP). Inman [16] states that the standard eigenvalue solution is faster than the generalized eigenvalue solution in terms of computational performance. Hereby, it can be inferred that the standard eigenvalue solution is more advantageous than the generalized one in terms of computational time. The matrices can sometimes be badly conditioned, which can lead to wrong solutions to the eigenvalue problem. Kannan et al. [17] emphasize the severity of ill conditioning in

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structural finite element method and state that modelling beam elements with large sections or small lengths can cause these kinds of problems. Although, there are studies about the numerical accuracy of eigenvalue solutions, the numerical accuracy of the generalized and standard eigenvalue problems has not yet been compared.

In this paper, a 6 degrees of freedom 1-D rotating Timoshenko beam with gyroscopic effects is investigated using the finite element method. Generalized and standard eigenvalue solutions are sought using the global stiffness, mass and gyroscopic matrices obtained by the finite element method. The main aim of this study is to compare the standard and generalized eigenvalue solutions of rotating systems in terms of their computational performance and numerical accuracy by increasing the number of beam elements in the finite element model. This paper is organised as follows; a short literature review and the main objective of the study are introduced in the first section. Modal analysis of rotating systems is then discussed to provide theoretical background in the second chapter. Numerical analyses carried out on a rotating beam using the standard and generalized eigenvalue solutions are presented in the third chapter, including studies into the convergence and computational effort. Finally, the main findings and concluding remarks are presented in the fourth section.

19.2 Modal Analysis of Rotating Systems

19.2.1 Dynamic Modelling of Rotating Beams with Finite Element Method

Rotating systems can mathematically be represented with mass, stiffness, damping and gyroscopic elements using lumped parameter, continuous system, or finite elements models. The latter discretises the structure into small elements based on the selected mesh sizes [2]. For each element, mass, stiffness, damping and gyroscopic matrices are formulated based on their material and geometry properties. After the computing element matrices, the connectivity between each element is defined by a topology matrix which allows the assembly of the global matrices. Finally, boundary conditions are imposed on the global matrices. Having obtained the updated global mass, stiffness, gyroscopic and damping matrices, the equations of motion for the free response of a six degrees of freedom system can be written as [2];

$$[M]\,\ddot{q}(t) + ([C] + \Omega\,[G])\,\dot{q}(t) + [K]\,q(t) = 0 \tag{19.1}$$

where M, C, G and K represent the mass, damping, gyroscopic and stiffness matrices respectively. Ω is the rotating speed of the rotor, and $q(t)$ is the generalized coordinate for a six degree of freedom (DOF) system;

$$q_i = \left[x_i, y_i, z_i, \theta_{x_i}, \theta_{y_i}, \theta_{z_i}\right] \tag{19.2}$$

The mass, stiffness and gyroscopic matrices of the flexible rotating shaft can be computed using two beam theories: Timoshenko or Euler – Bernoulli. They are used for the finite element formulations of flexible shafts in rotor dynamics. The main difference between the two theories is that shear deflections and rotational inertia effects are taken into account in Timoshenko beam theory [18, 19].

19.2.2 Quadratic Eigenvalue Problem of Gyroscopic Systems

The modal analysis of a rotating system is slightly different from a static one since gyroscopic moments acting on the rotating systems affect their modal parameters in terms of natural frequencies and mode shapes, and causes them to vary with the rotor speed. Similar to the damping terms, the gyroscopic terms introduce a term involving the first derivative of the generalized coordinate to the equation of motion. Therefore, the rotordynamics problem becomes a quadratic eigenvalue problem [20, 21]. In order to obtain the eigenvalues and eigenvectors, quadratic eigenvalue problem can be expressed using the state space representation which yields a first order eigenvalue problem [22–24]. However, this also doubles the sizes of the matrices in the eigenvalue problem, leading to an increase in computational time.

As mentioned in the introduction, there are two forms of eigenvalue problem, known as standard and generalized eigenvalue problem. There is one input matrix in the standard eigenvalue problem, whereas there are two input matrices in generalized eigenvalue problems. In MATLAB, generalized and standard eigenvalue problems can be solved using the `eig` or `eigs` functions. The `eig` function computes all the eigenvalues whereas the `eigs` function computes a subset of eigenvalues such as the smallest or largest eigenvalues, using algorithmic reduction techniques. MATLAB uses LAPACK routines to calculate the eigenvalues; and the eigenvalue solution algorithm is determined based on whether the input matrices are real or complex, symmetric or non-symmetric, Hermitian or non-hermitian; or positive definite [25, 26].

There are currently three rotor dynamics software suites based on the finite element method, written in MATLAB. They are developed by Bucher [27], Genta [28] and Friswell et al. [29], called RotFE, DynRot and Rotordynamics respectively. Among these software, Bucher [27] uses the generalized eigenvalue problem in his RotFE code whereas Friswell et al. [29] use the standard eigenvalue problem in their rotor dynamics software.

19.2.2.1 Generalized Eigenvalue Problem of Gyroscopic Systems

The generalized eigenvalue problem of gyroscopic systems [2] can be derived by first expressing the equations of motion from Eq. (19.1) in state space form as shown below:

$$\begin{bmatrix} C+\Omega G & M \\ M & 0 \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{19.3}$$

Then, the equation can be rewritten using state vectors $x = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}$ and $\dot{x} = \begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix}$ as,

$$A\dot{x} + Bx = 0 \tag{19.4}$$

The size of the eigenvalue problem is now 2n as can be seen from the matrix equations. Therefore, 2n eigenvalues and eigenvectors are obtained with the eigenvalue solution. Half of the eigenvalues and eigenvectors are the complex conjugates of the other half. The eigenvalue and eigenvector matrices are denoted by $[\lambda]_{2n\times 2n}$ and $[\phi]_{2n\times 2n}$ respectively where λ is also defined as;

$$\lambda_n = {\omega_n}^2 \tag{19.5}$$

In order to evaluate the accuracy of the generalized eigenvalue solution, parity check can be done with residual value calculation as defined below;

$$residual = B\times\phi + A\times\phi\times\lambda \tag{19.6}$$

After the calculation of the residual values in matrix form, the matrix norm can be computed using the `norm` function in MATLAB. Residual value calculation is a useful tool to quantify the numerical errors.

19.2.2.2 Standard Eigenvalue Problem of Gyroscopic Systems

The vibration problem of gyroscopic systems defined in Eq. (19.1) can be reformulated using inverse matrix operation as below [16];

$$\ddot{q}(t) + M^{-1}(C+\Omega G)\dot{q}(t) + M^{-1}Kq(t) = 0 \tag{19.7}$$

With the state vectors $x = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}$ and $\dot{x} = \begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix}$, standard eigenvalue problem in matrix form is expressed as;

$$\begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}(\Omega G + C) \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} \tag{19.8}$$

where I represents the identity matrix. The matrix equation for the standard eigenvalue problem becomes

$$\dot{x} = Ax \tag{19.9}$$

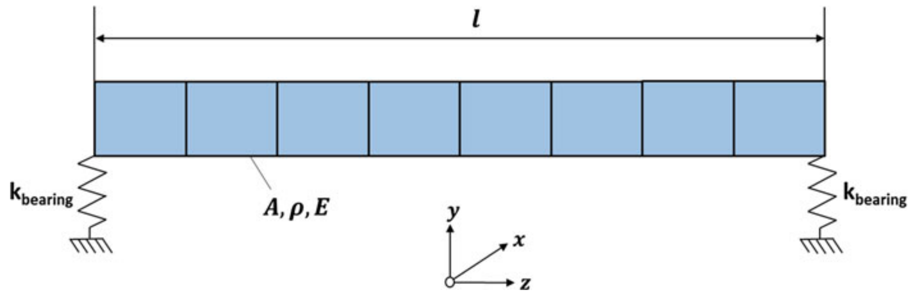

Fig. 19.1 A rotating beam model with finite element model

where A is called state matrix. As with the generalized eigenvalue problem, the size of the standard eigenvalue problem is 2n and half of them are the complex conjugates of the other half, but only one matrix needs to be computed for Eq. (19.9) instead of two for Eq. (19.4). Similar to the parity check of the generalized eigenvalue solution, a residual value formula can be defined for the standard eigenvalue solution as;

$$residual = A \times \phi - \phi \times \lambda \tag{19.10}$$

19.2.3 Modal Assurance Criteria

Independent of the chosen approach to compute the eigenvalue solutions, the Modal Assurance Criteria (MAC) [30] will be used to compare the resulting eigenvectors quantitatively. The formula of the modal assurance criteria is defined as [30, 31];

$$MAC\left(X, A\right) = \frac{\left|\{\phi_X\}_r^T\{\phi_A\}_q\right|^2}{\left(\{\phi_X\}_r^T\{\phi_X\}_r\right)\left(\{\phi_A\}_q^T\{\phi_A\}_q\right)} \tag{19.11}$$

where ϕ_X represents the "x" data set eigenvectors and ϕ_A represents "a" data set eigenvectors. Practically, ϕ_X and ϕ_A can be considered as data sets of experimental and analytical results. In this study, the main aim of the using the MAC is to compare mode shapes of the generalized and standard eigenvalue solutions.

19.3 Numerical Analysis of a Rotating Beam

A rotating beam was modelled using the finite element method with Timoshenko beam elements, each with 6 DOF. The beam is supported by bearings at each end as shown in Fig. 19.1.

In Fig. 19.1, $k_{bearing}$ represents bearing stiffness, and A, ρ, E and l represent cross sectional area, mass density, Young's modulus and length of the beam. Bearing elements of the rotating beam have stiffness in lateral and axial directions as defined below;

$$k_{bearing} = \begin{bmatrix} k_{xx} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{Q_{xx}} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{Q_{yy}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19.12}$$

No damping was included in the rotating beam or the bearings to enhance the effect of the gyroscopic matrix. The global mass, stiffness and gyroscopic matrices of the rotating beam were computed via a finite element code written in MATLAB. Generalized and standard eigenvalue solutions of the rotating beam were found for an increasing number of beam elements,

Table 19.1 Parameters of the rotating beam model

Parameter	Value
Length of the beam [m]	5
Cross sectional area of the beam [m^2]	0.0314
Density of the beam [kg/m^3]	7810
Young modulus of the beam [GPa]	211
Shear modulus of the beam [GPa]	81.2
Rotating beam speed [rpm]	8000
Bearing Stiffness k_{xx} [N/m]	10^8
Bearing Stiffness k_{yy} [N/m]	10^8
Bearing Stiffness k_{zz} [N/m]	10^8
Bearing Stiffness $k_{Q_{xx}}$ [Nm/rad]	10^6
Bearing Stiffness $k_{Q_{yy}}$ [Nm/rad]	10^6

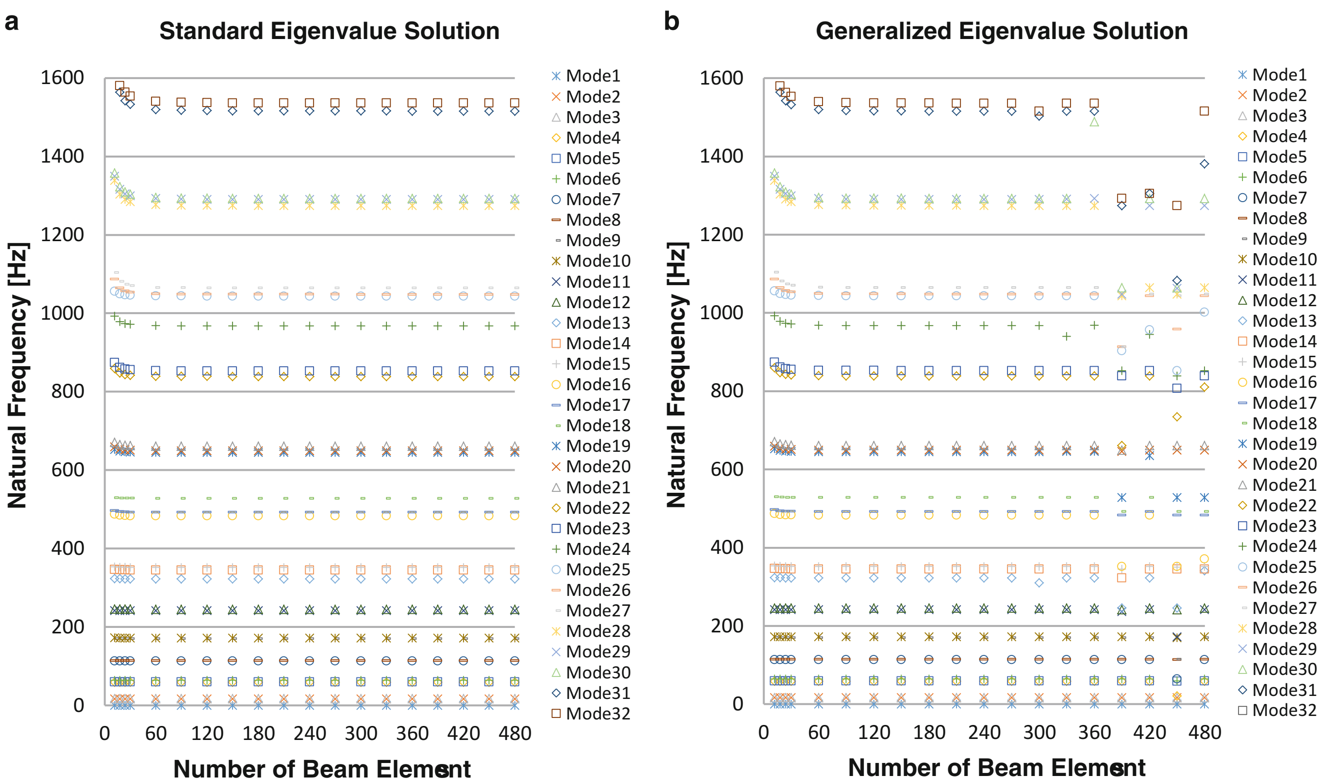

Fig. 19.2 Natural frequency comparison of (**a**) standard eigenvalue solution and (**b**) generalized eigenvalue solution

ranging from 6 to 480. For each number of beam elements, the natural frequencies, mode shapes and computation time were calculated using the two methods. The parameters of the rotating beam with its bearings are shown in Table 19.1.

19.3.1 Eigenvalue Solution Results

The finite element convergence studies were done by increasing the number of beam elements from 6 to 480 for both standard and generalized eigenvalue solutions in MATLAB. Eigenvalues and eigenvectors were computed using standard and generalized eigenvalue solutions. Natural frequency and mode shape comparison of the two methods are shown for the first 32 modes in Figs. 19.2 and 19.3 respectively.

Natural frequencies obtained from standard and generalized eigenvalue solutions were plotted with respect to the number of beam elements as seen in Fig. 19.2. It is clearly seen that generalized eigenvalue solution does not give stable results after 300 beam elements while standard eigenvalue solution gives the correct and stable results for each number of beam

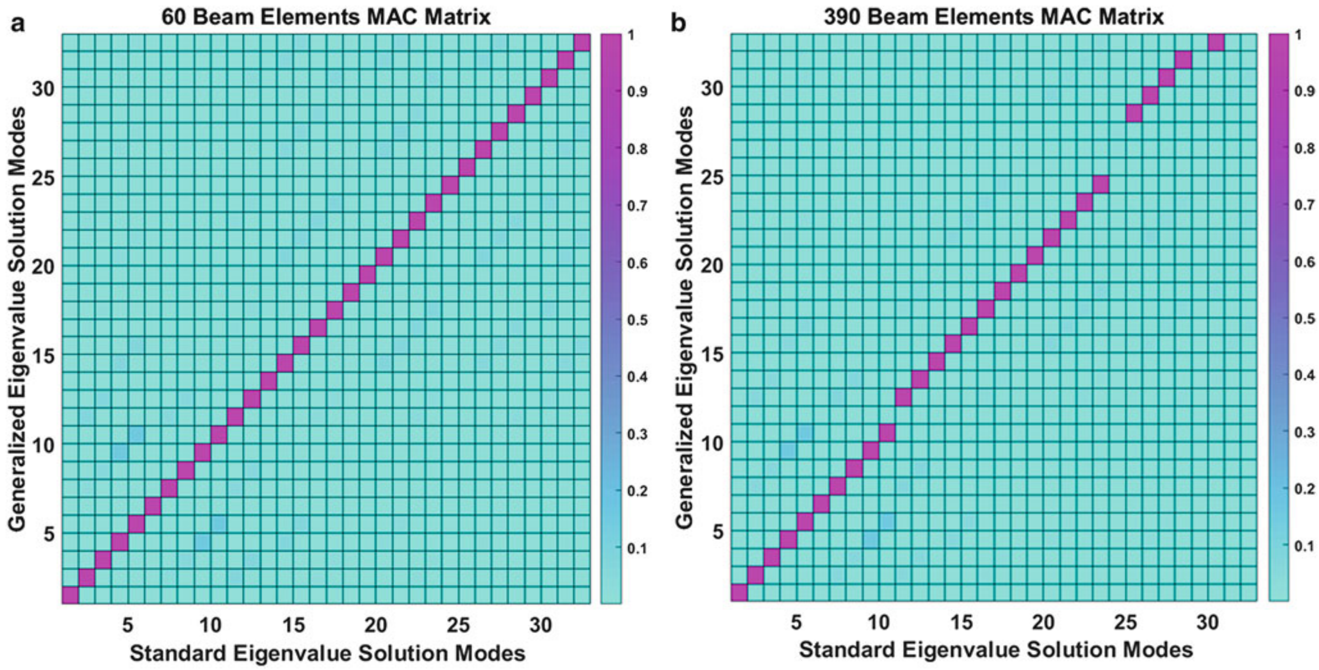

Fig. 19.3 MAC comparison of standard and generalized eigenvalue solution, (**a**) 60 beam elements, (**b**) 390 beam elements

Table 19.2 Norm of the residual values of generalized and standard eigenvalue solutions

Eigenvalue solution method	60 beam elements	390 beam elements
Generalized eigenvalue solution	0.0043	0.2901
Standard eigenvalue solution	0.0000246	0.0030

elements. In particular, for the generalized eigenvalue solution, the natural frequencies were wrongly calculated to be 236 Hz for the 11th mode, 902 Hz for the 25th mode, 912 Hz for the 26th mode and 912 Hz for the 27th mode when using 390 beam elements.

The MAC values of the two eigenvalue solutions are shown for 60 and 390 beam elements in Fig. 19.3. As can be expected from the natural frequency plot in Fig. 19.2, generalized and standard eigenvalue solutions gives fully consistent mode shapes for 60 beam elements. On the other hand, distorted correlation between the standard and generalized eigenvalue solutions for 390 beam elements is observed due to the wrongly calculated 11th, 25th, 26th and 27th modes by the generalized eigenvalue solution. These modes were also determined as torsional modes.

To check the quality of the eigenvalue computation and identify the source of the observed discrepancies, the norm of the residual values, which were defined in Sect. 19.2.2, were calculated for both standard and generalized eigenvalue solutions for 60 and 390 beam elements. The resulting norm of the residuals in Table 19.2 show significant residual differences between the standard and generalized eigenvalue solutions, where the latter leads to high residual values for both the 60 and 390 beam element case. Particularly very large norm of the residual values were observed for the 390 element beam.

As described in reference [17], ill-conditioning can lead wrong results in structural finite element models. In our specific rotating system problem with gyroscopic effects, this phenomenon occurred in the generalized eigenvalue solution, but it did not happen in the standard eigenvalue solution. Based on these findings, it may be concluded that generalized eigenvalue solution is not the best tool for the modal analysis of rotating systems due to its increased sensitivity towards ill-conditioning which can be introduced by the gyroscopic matrix for larger systems. Standard eigenvalue solution on the other hand gives correct and stable results and can be recommended for the analysis of rotating structures.

19.3.2 Computational Cost

A final comparison of the two eigenvalue solution approaches is based on their computational performance since any finite element analysis needs to find the right balance between accuracy and computational cost. A standard desktop PC with an

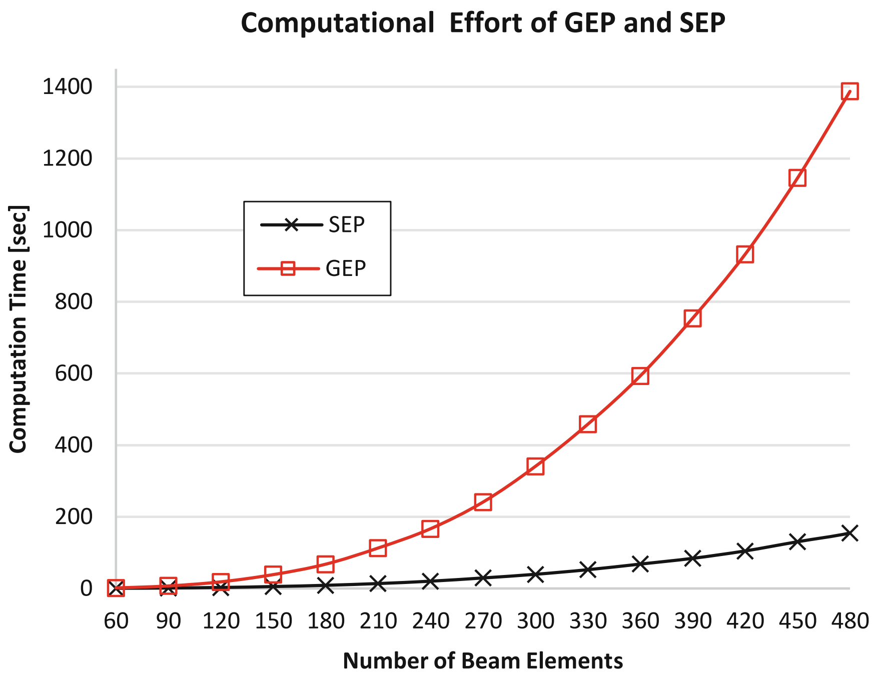

Fig. 19.4 Computational cost of generalized (GEP) and standard (SEP) eigenvalue problem

Intel® Core™ i7-6700 processor which has 3.40 GHz CPU and 16 GB random access memory (RAM) was used to compute the previously presented eigenvalues and eigenvectors in MATLAB R2016a.

In Fig. 19.4, it is clearly seen that the computation time difference between standard and generalized eigenvalue solutions increases with the number of beam elements. At lower numbers of beam elements, their computational performance is close to each other. However, there is a large computation time difference between them at higher mesh densities. It is numerically proven that the standard eigenvalue solution is faster than the generalized one. Based on the results in Fig. 19.4, the standard eigenvalues solution can be recommended for the fast computation of rotating systems with gyroscopic effects.

19.4 Conclusion

An evaluation of the accuracy and computational efficiency of the standard and generalized eigenvalue solutions for rotating beam elements with gyroscopic effects was presented. A finite element code written in MATLAB was used for this purpose. Natural frequencies and mode shapes were obtained using both eigenvalue solution methods. Then, the accuracy of the two methods was investigated using their eigenvalues, eigenvectors and residual values.

With an increase in the number of beam elements (>300), the generalized eigenvalue solution started to introduce inaccurate eigenvalues and eigenvectors whereas the standard eigenvalue solution provided consistent results. High residual values were also computed with generalized eigenvalue solutions while low residual values were obtained from standard eigenvalue solution. This suggests that the generalized eigenvalue solution may not be reliable when using higher number of beam elements. Moreover, this calls into question the reliability of LAPACK, the numerical solver employed by MATLAB for eigenvalue problems. It was also found that the standard eigenvalue solution is significantly faster than the generalized eigenvalue solution for higher numbers of elements.

Based on these findings, the standard eigenvalue solution is recommended for modal analysis of rotating systems with gyroscopic effects, and particular care is required when using the generalized eigenvalue solution with fine meshes. It is recommended that the residual values should be checked for the eigenvalue solutions to ensure the problem has been solved to the required level of accuracy. It can be concluded that the standard eigenvalue solution gives more reliable results than generalized eigenvalue solution and it is faster than the generalized eigenvalue solution.

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References

1. Nelson, F.C.: Rotor dynamics without equations. Int. J. COMADEM. **10**(3), 2–10 (2007)
2. Friswell, M.I.: Dynamics of Rotating Machines. Cambridge University Press, Cambridge (2010)
3. Muszynska, A.: Rotordynamics. CRC Press, Boca Raton (2005)
4. Rao, J.S.: History of Rotating Machinery Dynamics. Springer Science & Business Media, Dordrecht
5. Genta, G.: Dynamics of Rotating Systems. Springer Science & Business Media, New York (2007)
6. Yamamoto, T., Ishida, Y.: Linear and Nonlinear Rotordynamics: a Modern Treatment with Applications. A Wiley-Interscience publication, Wiley, Hoboken (2001)
7. Golub, G.H., Van der Vorst, H.A.: Eigenvalue computation in the 20th century. J. Comput. Appl. Mathods. **123**(1), 35–65 (2000)
8. Zheng, Z., Ren, G., Williams, F.W.: The eigenvalue problem for damped gyroscopic systems. Int. J. Mech. Sci. **39**(6), 741–750 (1997)
9. Ferng, W.R., Lin, W.-W., Wang, C.-S.: Numerical algorithms for undamped gyroscopic systems. Comput. Math. Appl. **37**(1), 49–66 (1999)
10. Qian, J., Lin, W.W.: A numerical method for quadratic eigenvalue problems of gyroscopic systems. J. Sound Vib. **306**(1–2), 284–296 (2007)
11. Kerner, W.: Large-scale complex eigenvalue problems. J. Comput. Phys. **85**(1), 1–85 (1989)
12. Kressner, D.: Numerical Methods for General and Structured Eigenvalue Problems. Springer-Verlag Berlin Heidelberg, Berlin (2005)
13. Saad, Y.: Numerical Methods for Large Eigenvalue Problems: Revised Edition. SIAM, Philadelphia (2011)
14. Bai, Z.: Progress in the numerical solution of the nonsymmetric eigenvalue problem. Numer. Linear. Algebra. Appl. **2**(3), 219–234 (1995)
15. Anderson, E., et al.: LAPACK: A portable linear algebra library for high-performance computers. In: Proceedings of the 1990 ACM/IEEE Conference on Supercomputing, pp. 2–11, New York, USA (1990)
16. Inman, D.J.: Engineering Vibration, 4rth edn. Pearson, Prentice Hall, London (2008)
17. Kannan, R., Hendry, S., Higham, N.J., Tisseur, F.: Detecting the causes of ill-conditioning in structural finite element models. Comput. Struct. **133**, 79–89 (2014)
18. Thomas, D.L., Wilson, J.M., Wilson, R.R.: Timoshenko beam finite elements. J. Sound Vib. **31**(3), 315–330 (1973)
19. Nelson, H.D.: A finite rotating shaft element using Timoshenko beam theory. J. Mech. Des. **102**(4), 793 (1980)
20. Tisseur, F., Meerbergen, K.: The quadratic eigenvalue problem. SIAM Rev. **43**(2), 235–286 (2001)
21. Berhanu, M.: The Polynomial Eigenvalue Problem, Ph.D. Thesis, University of Manchester, Manchester, UK (2006)
22. Gutiérrez-Wing, E.S., Ewins D.J.: Modal characterisation of rotating machines. Proceedings of the 19th International Modal Analysis Conference, Orlando, Florida (2001)
23. Lee, C.: A complex modal testing theory for rotating machinery. Mech. Syst. Signal Process. **5**(2), 119–137 (1991)
24. Bucher, I., Ewins, D.J.: Modal analysis and testing of rotating structures. Philos. Trans. R. Soc. Lond. A Math. Phys. Eng. Sci. **359**(1778), 61–96 (2001)
25. Anderson, E., et al.: LAPACK Usersguide: Release 1.0. Argonne National Lab, Chicago, Illinois, USA (1992)
26. Moler, C.: "Matlab incorporates LAPACK," Increasing the speed and capabilities of matrix computation. MATLAB News & Notes–Winter (2000)
27. Bucher, I.: RotFE 2.1 The finite element rotor analysis package. Faculty of mechanical Engineering, Technion, Haifa, Israel (2000)
28. Genta, G., Bassani, D., Delprete, C.: DYNROT: A Matlab Toolbox for Rotordynamics Analysis, Polytechnic University of Turin, Turin, Italy (1994)
29. Friswell, M.I., Penny, J.E.T., Garvey, S.D., Lees, A.W.: Dynamics of Rotating Machines Rotordynamics Software Manual, pp. 1–19. Cambridge University Press, Cambridge (2010)
30. Pastor, M., Binda, M., Harčarik, T.: Modal assurance criterion. Procedia Eng. **48**, 543–548 (2012)
31. Ewins, D.J.: Modal Testing: Theory, Practice and Application, Research Studies Press, Baldock, (2000)