Higher Order Invariant Manifolds Parametrisation of Geometrically Nonlinear Structures Modelled with Large Finite Element Models

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ABSTRACT

In this contribution we present a method to directly compute asymptotic expansion of invariant manifolds of large finite element models from physical coordinates and their reduced order dynamics on the manifold. We show the accuracy of the reduction on selected models, exhibiting large rotations and internal resonances. The results obtained with the reduction are compared to full-order harmonic balance simulations obtained by continuation of the forced response. We also illustrate the low computational cost of the present implementation for increasing order of the asymptotic expansion and increasing number of degrees of freedom in the structure. The results presented show that the proposed methodology can reproduce extremely accurately the dynamics of the systems with a very low computational cost.

Keywords: Reduced Order Modelling, Invariant Manifold, Large Scale FE models, Direct Parametrisation

INTRODUCTION

Slender structures in large amplitude vibration exhibit geometric nonlinear effects that modify their dynamics. To accurately model structures with complex geometries, large finite element models with a high number of degrees of freedom are often required. Numerically, the geometric nonlinear forces affect all the elements in the structure thus making the simulation of such problems very demanding in terms of computational cost. Reduced order models of geometrically nonlinear structures are then an attractive solution to drastically reduce the size of the problem whilst maintaining the accuracy high.

BACKGROUND

Direct model order reduction methods, such as implicit condensation and quadratic manifold method, give explicit expressions directly in physical coordinates of the reduced models of geometrically nonlinear structures. Conversely, exact reduction methods are not directly applicable to large models because they work in modal coordinates, which are not available for large models. Recent developments by the authors [1-3] have extended the applicability of exact reduction methods to large FE structures, by deriving explicit expressions directly in physical coordinates of the reduced models. A similar work is done in [4], where spectral submanifolds are computed directly in physical coordinates.

METHOD

Mechanical structures in large deformations display geometric nonlinearities in the form of quadratic and cubic restoring forces. When discretised with the finite elements method, the dynamics of such structures can be written as a system of N ordinary differential equations in time, with N number of degrees of freedom:

$$\boldsymbol{M}\,\ddot{\boldsymbol{U}} + \boldsymbol{C}\,\dot{\boldsymbol{U}} + \boldsymbol{K}\,\boldsymbol{U} + \boldsymbol{G}(\boldsymbol{U},\boldsymbol{U}) + \boldsymbol{H}(\boldsymbol{U},\boldsymbol{U},\boldsymbol{U}) = \boldsymbol{F}_{EXT}$$

The idea of using invariant manifolds for model order reduction has been proposed by several authors [5-7] in the context of mechanical systems. The modal coordinates of the system are expressed as a polynomial function of the *master* coordinates $\boldsymbol{z}$, which are tangent to the linear modal coordinates of the nonlinear mode of interest. The reduced dynamical model is also built as a polynomial function of the *master* coordinates in first order form: $\dot{\boldsymbol{z}} = \mathbf{f}(\boldsymbol{z})$. The coefficients of the change of coordinates and reduced dynamics are then found by solving the so-called homological equation for each order of the polynomial expansion. Here, the displacement in physical coordinates is directly expressed as a polynomial function of the *master* coordinates, as $\boldsymbol{U} = \boldsymbol{\Psi}(\boldsymbol{z})$, and the vectors $\boldsymbol{\Psi}_I$ multiplying each monomial $\boldsymbol{z}_I = z_{i_1} z_{i_2} .. z_{i_p}$, are found by solving a $N \times N$ linear system of equations.

RESULTS

In Fig. 1, the forced response and the manifold geometry of a large FE model of a MEMS micromirror are captured by the reduced model up to high rotation angles (12°). The dynamics of this structure is particularly challenging due to the nonlinear coupling of multiple modes and due to strong inertia nonlinearities. The expansion up to order 3 is not accurate enough and a higher order is necessary. In Fig. 2, the 1:3 internal resonance between the second and fourth mode of a microbeam resonator is perfectly captured by the reduced model consisting in two coupled oscillators. In this case the invariant manifold is 4D.

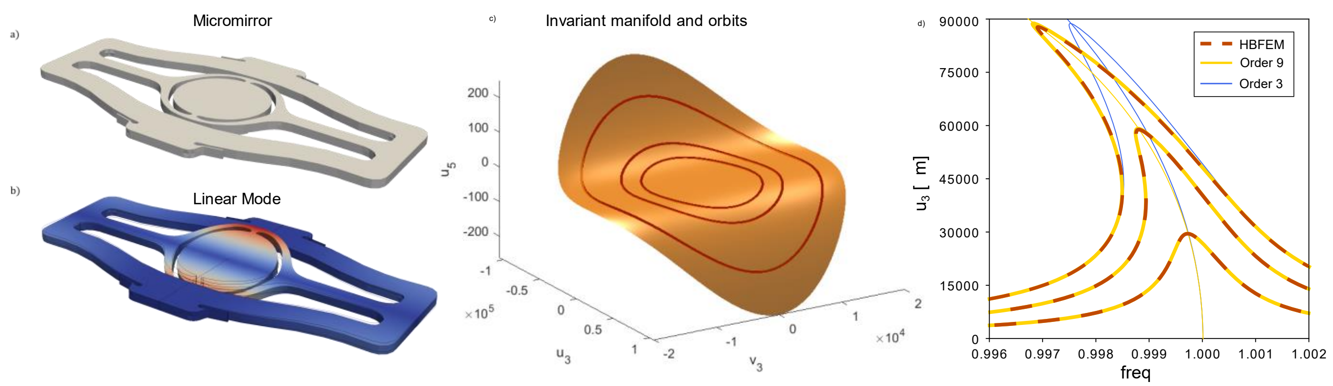

Figure 1: Forced response of a MEMS micromirror in large rotations in the vicinity of the resonance of its third linear mode. a) FE model of the mirror and modeshape. b) Invariant manifold obtained from the parametrisation and orbits of the full model solution. c) FRFs obtained with full and reduced model for different values of the excitation force.

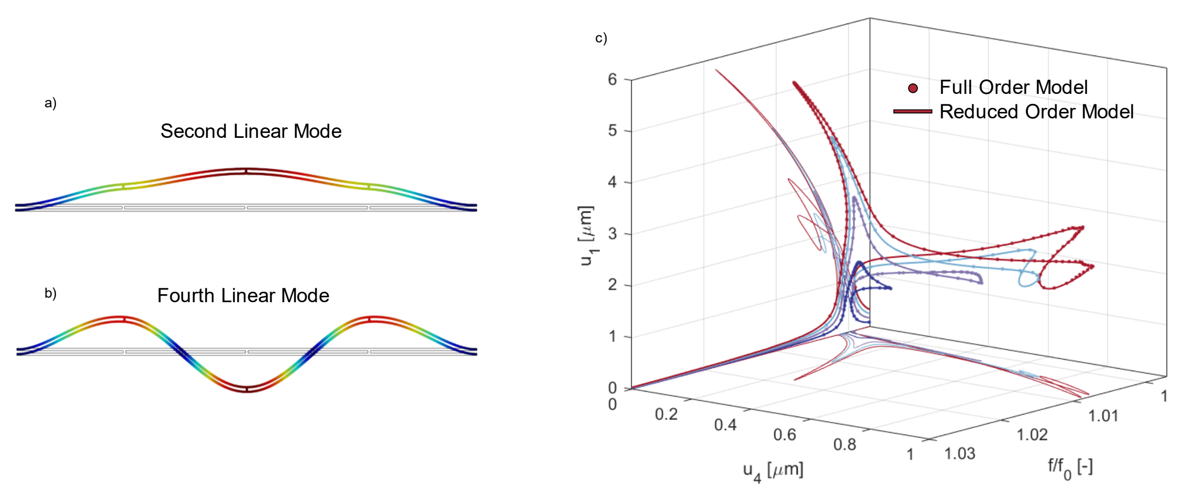

Figure 2: Forced response of a microbeam in the vicinity of the internal resonance between its first and third mode. a) Second modeshape. b) Fourth modeshape. c) FRFs obtained with full and reduced model at order 9 for different values of the excitation force.

COMPUTATIONAL COST

The cost of model-order reduction is composed of an offline phase to compute the invariant manifold and the reduced dynamics, and an online phase to solve the dynamics of the reduced model itself. The latter is negligible because, regardless of the size of the original system, the reduced model is either a single oscillator or two coupled oscillators in case an internal resonance occurs. In Fig. 3, the offline computational cost to build the model is reported as a function of the order of the expansion and of the degrees of freedom of the original FE model. It is shown that higher orders of expansion are attainable at a very low computational cost, which scales quadratically with the order of expansion p. Moreover, the computational cost of large-scale FE models' reduction is also extremely low, not only as compared to full order HB simulations but also as compared to other model reduction techniques.

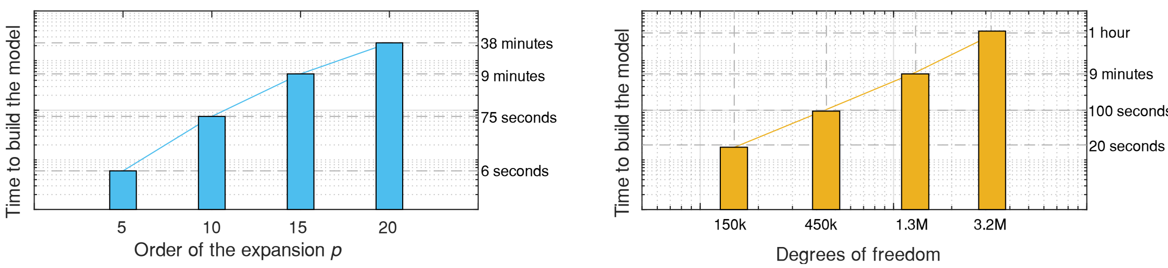

Figure 2: Computational time to build a single oscillator ROM from a 3483 DoFs FE model for increasing order of expansion (left). Computational time to build a single oscillator ROM of order 3 for increasing size of the FE model (right).

CONCLUSION

Nonlinear modes have become an essential tool for engineers in that they provide crucial insights on the dynamics of structures. The computation of nonlinear modes and their associated invariant manifolds for model order reduction is of utmost importance for modern industry. In this respect, the extension of nonlinear modes calculations to large scale finite element models is a critical issue that will allow the use of this tool in industrial scale problems. The method proposed here is based on a classical asymptotic expansion of invariant manifolds, but its computation is adapted to the case of FE models with a high number of degrees of freedom. The method is a cutting-edge development that will enable a better understanding of the nonlinear dynamics of real-world structures exhibiting large displacements and rotations. The rigourous mathematical foundation of the method coupled with a thoughtful implementation make it extremely accurate and fast, thus making it the ideal candidate for reduced order modelling of geometrically nonlinear structures in an industrial framework.

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