

Numerical Round Robin for Prediction of Dissipation in Lap Joints

L. Salles^(a), C. Swacek^(b), R.M. Lacayo^(c), P. Reuss^(b), M.R.W. Brake^(c), C.W. Schwingshackl^(a)

^(a) Imperial College London, Mechanical Engineering, London, UK

^(b) University of Stuttgart, Institute of Applied and Experimental Mechanics, Stuttgart, Germany

^(c) Sandia National Laboratories¹, Component Science and Mechanics, Albuquerque, NM, USA

ABSTRACT

Joints, interfaces, and frictional contact between two substructures can be modelled as discrete nonlinearities that connect the substructures. Over the past decade, a number of phenomenologically different approaches to modeling and simulating the dynamics of a jointed structure have been proposed. This research focuses on assessing multiple modelling techniques to predict the nonlinear dynamic behaviour of a bolted lap joint, including frequency based sub-structuring methods, harmonic balance methods, discontinuous basis function methods, and high fidelity FEA approaches. The regimes in which each method is best suited are identified, and recommendations are made for how to select a modelling method and for advancing numerical modelling of discrete nonlinearities.

KEYWORDS: Nonlinear damping; Nonlinear vibration; Numerical modeling; Bolted joints; Lap joints

1. INTRODUCTION

The assembly of single components into a more complex structure always leads to the presence of a joint. A wide range of assembly methods are available today, ranging from permanent connections such as welds and adhesives, to separable ones such as bolts, rivets or hooked connections. Depending on the selected joint type it represents a change in the design, and has an impact on the static and dynamic performance of the assembly. For an accurate prediction of the dynamic behaviour of the assembly the special dynamics of the joint of interest must be captured accurately.

One of the most common joint types in today's engineering applications is the bolted joint connection. The combination of a contact surface with a series of bolts to apply the required loading potentially can result in a nonlinear system. The bolts and contact stiffness can lead to a reduction in the global stiffness and relative motion in the contact can add amplitude dependent damping due to friction effects. An analytical model of an assembled structure should take these effects into account to ensure accurate predictions of the dynamic behaviour.

Several different approaches are available to deal with the bolted joint in an analysis. Rigid connections at the joints are the simplest approach, ignoring any possible influence of the joint on the response. Using a set of springs to model the reduced stiffness in the joint [1] can lead to a good agreement of predicted and measured resonance frequencies, but the damping effects due to the nonlinear friction behaviour of the joint are neglected. More advanced approaches not only include the stiffness of the bolt but also the energy dissipation due to the friction between the two contact surfaces [2], using experimental data to update and tune the model. A slightly different approach is the full three dimensional modelling of the contact interface [3] with nonlinear contact elements. It requires input parameters for the contacts, such as friction coefficient and contact stiffness [4], a linear modal model of the different components, and the normal load distribution on the contact surface.

To gain a better understanding of the capabilities of the different approaches to model the nonlinear dynamic behaviour of bolted joints, and in response to a challenge defined during the Third International Workshop on Jointed Structures in Chicago in 2012 [5], the 2014 Sandia Nonlinear Mechanics and Dynamics Summer Research Institute included a round robin numerical modeling challenge. The round robin challenge includes four different methods for the nonlinear response

¹ Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

prediction developed at: Imperial College London, Sandia National Laboratories, and the University of Stuttgart. A straight beam with a simple lap joint with three bolts is investigated (detailed in Section 3) with a transient simulation based on an Iwan model [6] (both in a reduced order and a high fidelity framework), and two different harmonic balance approaches [7, 8]. The resulting findings of the round robin are presented in this paper.

2. APPROACHES TO MODELING FRICTION JOINTS

The Benchmark used for this numerical round-robin is tested with four different approaches. A transient approach based on finite elements and reduced order modeling with discontinuous basis modes is used by the Engineering Sciences Center at Sandia National Laboratories (hereafter referred to as Sandia), while the Vibration University Technology Centre at Imperial College London (hereafter referred to as Imperial) and the Institute of Applied and Experimental Mechanics at the University of Stuttgart (hereafter referred to as Stuttgart) use frequency domain approaches.

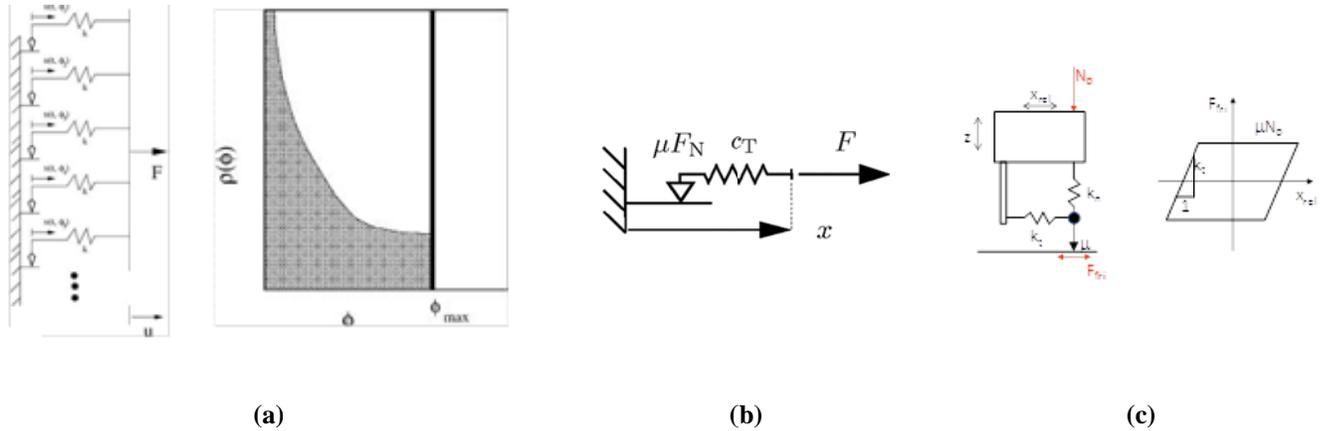


Figure 1 Friction elements a) Iwan model (Sandia), b) Jenkins Model (Stuttgart) and 3D element (Imperial)

There are several notable differences between the different approaches. Qualitatively, the responses for the methods developed at Sandia are obtained in the time domain; whereas, Stuttgart and Imperial College use harmonic balance methods (HBMs) to obtain responses in the frequency domain. The Sandia transient methods are versatile in that they are developed for arbitrary excitations and nonlinear constitutive models, but at the cost of intensive computation and inaccurate frequency-space representation by Fourier transform (since a nonlinear decaying response changes the frequency behaviour). The Stuttgart and Imperial HBM are computationally efficient, but are limited somewhat to harmonic input excitations. The HBM approaches differ mainly in the use of the reduction technique (Craig Bampton versus Hybrid Method), the implementation of the nonlinear forces (linearization versus alternating frequency time procedure), and the HBM (single versus multi harmonic).

2.1. Iwan Model

The four parameter Iwan model [9], is used to represent the physical mechanisms observed in measurements of the dynamic behavior of joints. The Iwan model (see Figure 1.a) is based on a distribution of friction sliders (Jenkins elements) that approximate the response of a jointed surface in both the micro-slip and macro-slip regimes. The four parameters that characterize the Iwan model are the tangential stiffness in the micro-slip regime, the insipient force for macro-slip, and the power law slope and exponent for the dissipation characteristics of the system as a function of excitation amplitude.

The approach taken at Sandia National Laboratories for modeling systems with frictional interfaces [6] divides the jointed surfaces into a series of contact patches (these can be as few as one patch per surface, or as many as are computationally feasible). Each of the nodes within a contact patch is rigidly connected to a new node that represents the degrees of freedom for that contact patch. This new node, in turn, is connected to the corresponding node for the contact patch on the opposite

surface using an Iwan element. This modeling approach replaces the kinematics of the adjacent interfacial surfaces with a nonlinear constitutive model. The four parameters for this model generally must be determined from representative experimental data; however, a number of uncertainty approaches [10] exist for when there is an insufficient amount of data [11].

Two of the methods used by the research groups at Sandia National Laboratories include high fidelity modeling using SIERRA [12] (a massively parallel finite element program), and reduced order modeling using the method of discontinuous basis functions [13, 14]. The framework for the discontinuous basis function method is established by augmenting the linear mode shapes of a system composed of linear substructures defined by the Craig-Bampton method [15] with a series of discontinuous basis functions that are smooth everywhere except at the location of the discrete nonlinearity (i.e. joint). The mathematical definition of the discontinuous basis functions is based on the research of [16], and inclusion of the discontinuous basis functions with the linear basis functions enhances the convergence properties of the system. The resulting set of nonlinear, coupled equations of motion are integrated directly in time to calculate the transient dynamic response of the system.

2.2. Stuttgart Approach (Harmonic Balance method)

The Stuttgart approach [7] relies on a node-to-node contact model with friction in the tangential direction and a nonlinear contact law in the normal direction. For the vibration analysis a linearization based on the HBM is performed and the nonlinear response is calculated by an iterative solution technique. The finite element model is reduced to the nonlinear nodes and the excitation nodes using the Craig-Bampton method [15]. The equation of motion becomes

$$\begin{bmatrix} \mathbf{M}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}^{(1)} \\ \dot{\mathbf{x}}^{(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}^{(1)} \\ \dot{\mathbf{x}}^{(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{K}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{x}^{(1)} \\ \mathbf{x}^{(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_T^{(1)} \\ \mathbf{B}_T^{(2)} \end{bmatrix} \mathbf{f}_T + \begin{bmatrix} \mathbf{B}_N^{(1)} \\ \mathbf{B}_N^{(2)} \end{bmatrix} \mathbf{f}_N = \mathbf{f}_{ext}, \quad (1)$$

where the contact forces are defined by

$$\mathbf{f}_T = \mathbf{f}(\mathbf{f}_T, \mathbf{x}_{rel}, \mathbf{f}_N, \boldsymbol{\mu}, \mathbf{k}_T, \mathbf{x}_{rel} = \mathbf{B}_T^{(1)T} \mathbf{x}^{(1)} - \mathbf{B}_T^{(2)T} \mathbf{x}^{(2)}) \quad (2)$$

$$\mathbf{f}_N = \mathbf{f}(\underbrace{\mathbf{B}_N^{(1)T} \mathbf{x}^{(1)} - \mathbf{B}_N^{(2)T} \mathbf{x}^{(2)}}_g, \mathbf{k}_{N,0}, \mathbf{k}_{N,1}). \quad (3)$$

Equation 1 is solved by the single harmonic balance method. The contact forces are linearized assuming that the normal forces are constant

$$F_T(\mathbf{x}_{rel}) \approx k_{HBM} \mathbf{x}_{rel} + d_{HBM} \dot{\mathbf{x}}_{rel}. \quad (4)$$

with

$$k_{HBM} = \frac{\omega}{\pi \hat{\mathbf{x}}_{rel}} \int_0^{2\pi/\omega} F_T(\mathbf{x}_{rel}) \cos(\omega t) dt \quad \text{and} \quad d_{HBM} = -\frac{1}{\pi \hat{\mathbf{x}}_{rel}} \int_0^{2\pi/\omega} F_T(\mathbf{x}_{rel}) \sin(\omega t) dt. \quad (5)$$

The nonlinear system that defines the frequency response is obtained by introducing (4) in (1) and considering the response of the system to be $\mathbf{x} = \tilde{\mathbf{x}} e^{\omega t}$:

$$\hat{\mathbf{x}} = \mathbf{H}_{HBM}^{-1}(\hat{\mathbf{x}}) \hat{\mathbf{f}}, \quad (6)$$

The transfer function for the HBM method

$$\mathbf{H}_{HBM}(\hat{\mathbf{x}}) = ((\mathbf{K} + \mathbf{K}_{HBM}(\hat{\mathbf{x}})) + i\omega(\mathbf{D} + \mathbf{D}_{HBM}(\hat{\mathbf{x}})) - \omega^2 \mathbf{M}). \quad (7)$$

The system (6) is then solved using a modified version of a Newton-Raphson solver to obtain the nonlinear dynamic response.

2.3. Imperial approach

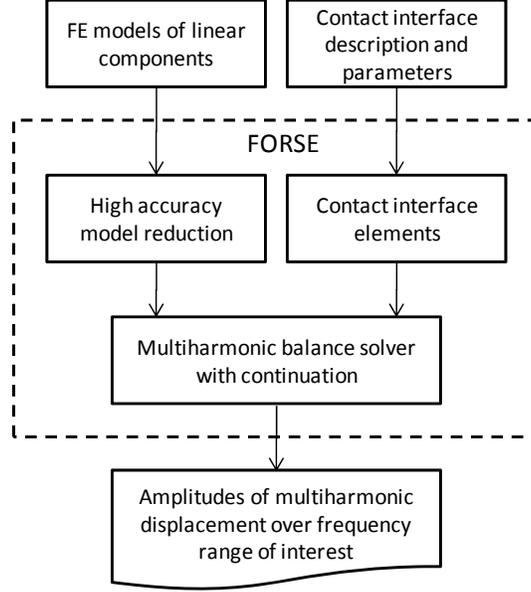


Figure 2 Scheme of the forced response analysis

The Imperial-developed code, FORSE, used for the analysis of the nonlinear response of flange joints is based on the multiharmonic representation of the steady-state response and allows large scale realistic friction interface modelling. Major features of the methodology were described in [8] and only an overview of the analysis is presented in this paper. The equation of motion consists of a linear part, which is independent of the vibration amplitudes, and the nonlinear part due to the friction interfaces at the interface joint. The nonlinear equation of motion can be written as

$$\mathbf{K}q(t) + \mathbf{C}\dot{q}(t) + \mathbf{M}\ddot{q}(t) + \mathbf{f}(q(t)) - \mathbf{p}(t) = \mathbf{0}, \quad (8)$$

where q is a vector of displacements; \mathbf{K} , \mathbf{C} , and \mathbf{M} are stiffness, damping and mass matrices, respectively, of the linear model; \mathbf{f} is a vector of nonlinear friction interface forces, which is dependent on displacements and velocities of the interacting nodes, and \mathbf{p} is a vector of periodic exciting forces. The variation of the displacements in time is represented by a restricted Fourier series, which can contain as many harmonic components as it is necessary to approximate the solution, i.e.

$$q(t) = \mathbf{Q}_0 + \sum_{j=1}^n \mathbf{Q}_j^c \cos m_j \omega t + \mathbf{Q}_j^s \sin m_j \omega t. \quad (9)$$

In Eq. 9, \mathbf{Q} are vectors of harmonic coefficients for the system degrees of freedom (DOFs), n are numbers of harmonics that are used in the multiharmonic displacement representation, and ω is the principal vibration frequency. The flowchart of the calculations performed with the code is presented in Fig. 2. The contact interface elements developed in [17] (see Fig. 1.c) are used for modelling of nonlinear interactions at contact interfaces and analytical expressions for the multiharmonic representation of the nonlinear contact forces and stiffnesses. The nonlinear algebraic system of the reduced model is obtained using a hybrid method of reduction developed by Petrov [17, 18]. The nonlinear system in the frequency domain is

$$\tilde{\mathbf{Q}} = A(\omega) \left(\tilde{\mathbf{F}} - \tilde{\mathbf{F}}_{nl}(\tilde{\mathbf{Q}}) \right), \quad (10)$$

with $\tilde{\mathbf{Q}}$ defined as the vector of the Fourier coefficients of the displacements at the interface, $A(\omega)$ the frequency response, $\tilde{\mathbf{F}}$ is the vector of the Fourier coefficients of the excitation force and $\tilde{\mathbf{F}}_{nl}$ is the vector of the Fourier coefficients of the nonlinear contact forces.

3. THE LAP JOINT TEST CASE

The test case chosen for the Round Robin investigation is the Brake-Reuss beam [19], a simple straight beam consisting of two pieces connected with a lap joint (see Fig. 3). The beam is made of 304 stainless steel, and has an assembled dimension of 720x25x25mm. The lap joint is located in the centre of the beam and has a length of 120mm. Three bolt holes, spaced 30.2mm apart, with a diameter of 8.4mm allow the application of the required contact pressure in the joint.

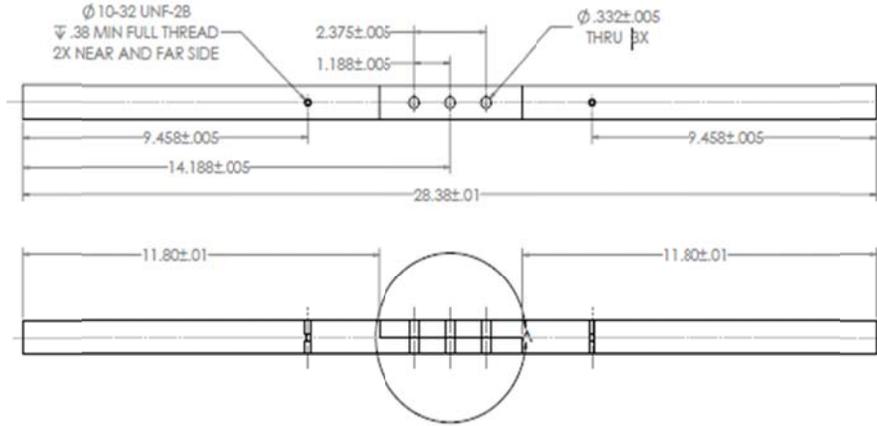


Figure 3: Dimensions of the round robin beam (dimensions given in inches).

3.1. The finite element mesh

Representative linear finite element models for each side of the beam were created in Hyper Mesh and used for all four approaches (Fig. 4). Each side of the beam consists of 10184 8-node hexahedral elements, with particular care taken to ensure that the mesh is congruent on either side of the contact interface. This allows for an accurate application of the nonlinear meshes for each of the methods in the round robin exercise. The contact interface has 931 nodes, as shown in Fig. 5. Both ends of the beam are clamped, to reduce complications in the nonlinear analysis of the joint. The nodes around the through-hole on one side of the beam are all given the same dynamic forcing load, and a single node on the hole on the other side of the interface becomes the output node of interest.

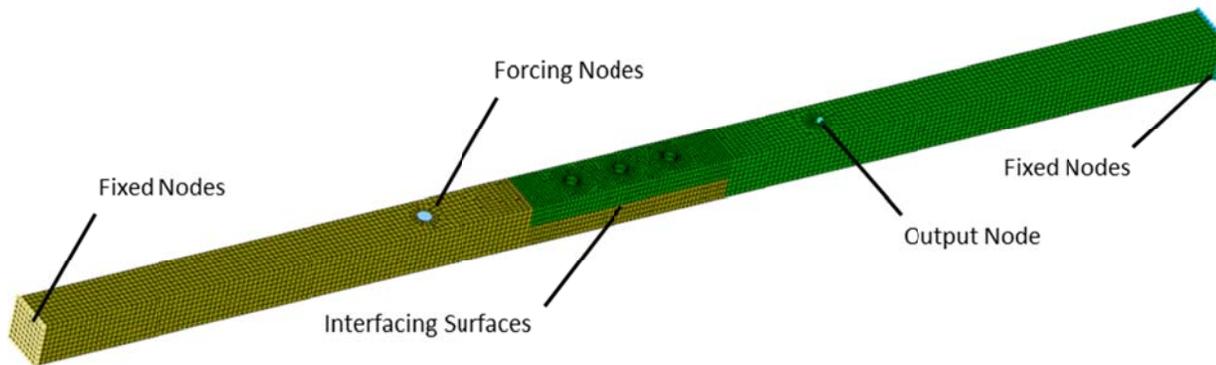


Figure 4: Round Robin beam finite element model setup.

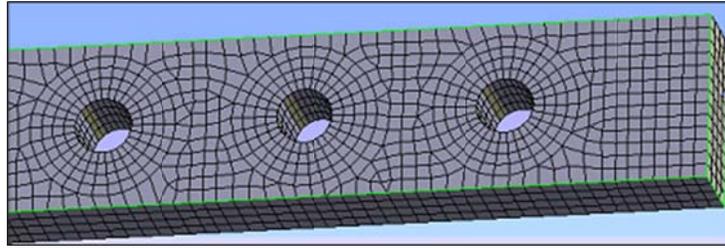


Figure 5: Finite element mesh at the contact interface.

3.2. Nonlinear Static Analysis

A static stress analysis is used to determine the contact stress distribution in the contact interface using Code Aster and the continuous contact augmented Lagrangian formulation ($\mu=0.6$) to obtain the most accurate stress distribution. The bolt torque is subsequently applied as a constant pressure load on the outer surfaces of the beam, and several loads (between 1000 and 12500 N) are chosen for each bolt. The resulting pressure distribution for a load of 1000 N can be seen in Fig. 6a). The high stress levels close to the bolt holes (pressure cone) reduces concentrically leading to an area of lower compression between the holes, and no stress towards the two ends of the lap joint. Figure 6b) shows the out-of-plane displacement in the contact surface, indicating a very small opening of a gap at the two ends of the lap joint, which explains the absence of pressure in the area and leads to some interesting initial conditions for the analysis.

The linear finite element model and the calculated nonlinear static stress distribution in the lap joint are used as a starting point for the generation of the nonlinear dynamic models.

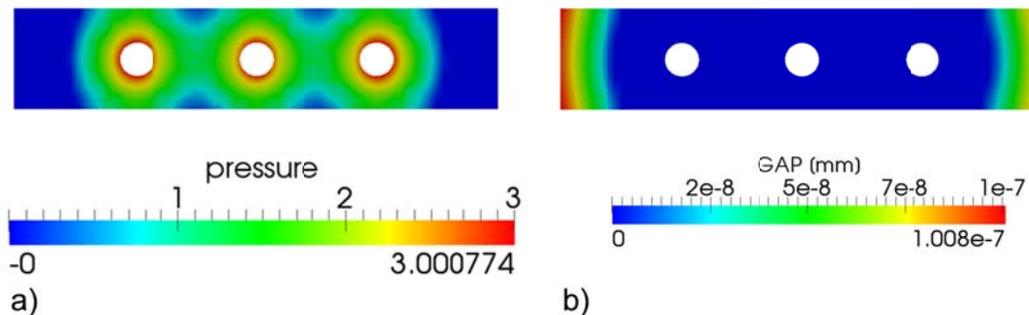


Figure 6 The a) calculated contact pressure and b) deformation in the contact zone.

3.3. Nonlinear Dynamic Model

Two different modelling approaches are investigated by Sandia based on the high fidelity finite element model and a reduced order model. For both models, the joint is modelled with a four parameter Iwan model, the parameters of which (F_S , K_T , χ , and β) must be tuned to match the damping and frequency behaviour of the other methods for a meaningful comparison. The slip force F_S and tangent stiffness K_T have equivalent parameters in each of the other methods; however, the energy dissipation power law slope χ and exponent β have no equivalent representation in either of the HBMs studied in this round robin. The slip force can be interpreted as the amount of shear force necessary to instigate slip between the two interfacing surfaces and is derived from Coulomb's law. With a friction coefficient of $\mu = 0.3$ and a total normal load from the bolts of $F_N = 30000$ N the friction force $F_S = \mu F_N$ becomes 9000 N.

The tangent stiffness value, K_T , is linked to the tangential stiffness parameters, $k_t = 50\text{kN/mm}^3$, from the Stuttgart and Imperial approaches, but it must be adopted to represent the contact conditions in Iwan models correctly. The last two Iwan parameters, χ and β , have no known direct correspondence with the parameters of the other methods, so previous experimental data from a similar system [6] is used to derive their values.

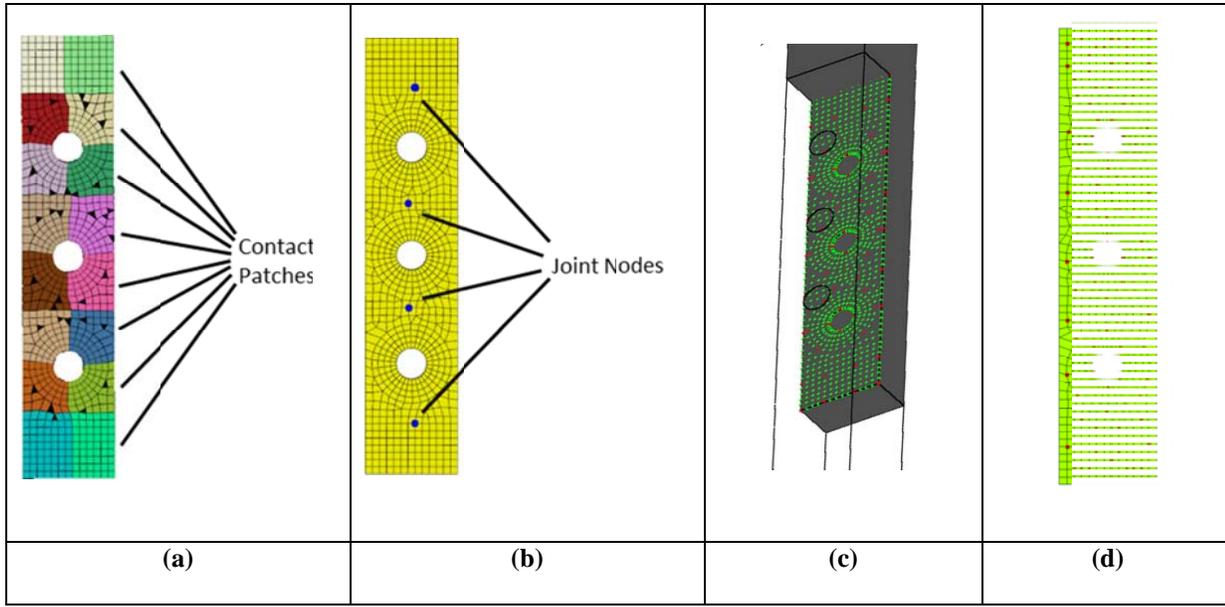


Figure 7 The nonlinear models (a) Iwan-full , (b) Iwan-reduced, (c) HBM-Stuttgart, and (d) HBM-Imperial College model.

For the simulation in Sierra [12], the two interfacing surfaces are divided into 16 contact patches each as shown in Fig. 7(a) (Iwan-full model). The two surface-tangent translational DOF are assigned 16 discrete Iwan models tuned to the parameters discussed before. The two surface-tangent rotational DOF are automatically constrained due to a zero-gap constraint in the surface-normal direction, and the surface-normal rotational DOF is given a spring with stiffness 10^9 N/radian to approximate a fixed rotation constraint due to bolt pinning. The joint model in the reduced model simulation uses four nonlinear nodes on each contact surface as indicated in Fig. 7 (b) (Iwan-reduced model) with the same input parameters as above. In addition, a penalty spring model with a stiffness of 10^6 N/mm is assigned to the surface-normal DOF for each node interface. The rotational joint models do not apply for the reduced model because the rotational DOF from every node are discarded during the Craig-Bampton reduction.

The Friction Coefficient and the contact stiffness for the Stuttgart and Imperial simulations are the same as used for the Iwan models with a friction coefficient, $\mu = 0.3$, and a tangential contact stiffness, $k_t = 50\text{kN/mm}$. The model created by Stuttgart (HBM-Stuttgart, Fig. 7(c)) has 702 nonlinear nodes at the contact interface and uses a single harmonic for the solution. The models created by Imperial College (HBM-Imperial, Fig. 7(d)) consisted of 86, 118 and 903 nonlinear elements respectively, allowing for a faster nonlinear computation with the first model, and a much more detailed analysis with the latter one. Each element had its own normal preload attributed, based on the static stress distribution from Fig. 6(a) and both a single- and a multi-harmonic analysis are used to predict the nonlinear dynamic response.

4. NONLINEAR DYNAMIC RESULTS

For both the Iwan-full and Iwan-reduced models, the displacement frequency response functions (FRF) at the output node of interest are obtained by numerically integrating the system equations of motion in the time domain, and then using the Fourier transform on the response solution. To observe the nonlinear characteristics of the Iwan joint in the frequency domain, the transient response to several impulse load excitation cases are simulated. For the Iwan-full model, the response to a Heaviside force impulse with a period of 0.01 seconds is simulated for peak loads of 1 kN, 10 kN, 30 kN, 100 kN, 300 kN, and 1 MN. The Iwan-reduced used a forcing period of 0.001 seconds for peak loads of 10 N, 100 N, and 1000 N. The responses for both models are integrated over a simulation time of 0.35 seconds to ensure adequate frequency step size resolution after using the Fourier transform.

Figure 8 shows a plot of the displacement FRF for each excitation amplitude up to 100 kN, normalized to their forcing magnitude, from the Iwan-full model. As the forcing magnitude increases, the natural frequencies of the first three modes

decrease and the damping increases for the first and second modes. This suggests that the joint becomes less stiff and energy dissipation increases as more energy is added to the system, which is expected. The exception to this trend appears to occur around a 10,000 N excitation since both the first and third modes show a marked decrease in damping from the 1000 N case; however, this anomaly could be the result of a simulation error or further evidence of the nonlinearity of this system, and warrants further attention.

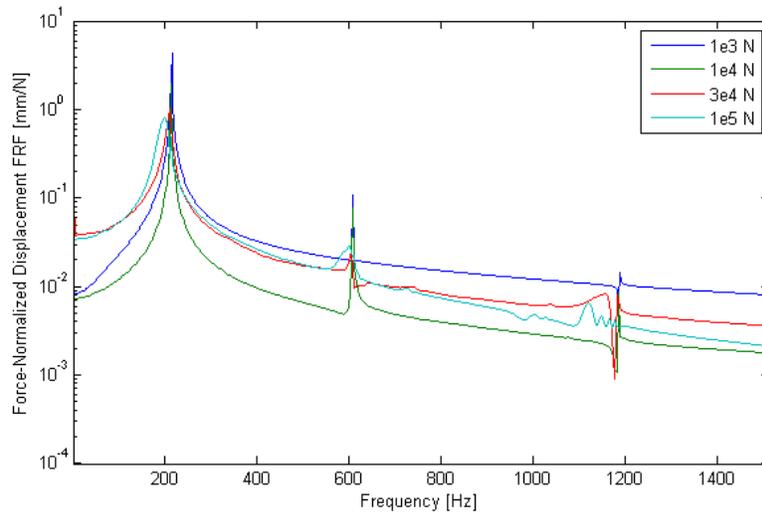


Figure 8: Iwan-full model displacement frequency response functions for different forcing amplitudes, showing the evolution of the first three natural frequencies as the excitation magnitude is increased.

Similar to Fig. 8, Fig. 9 shows the displacement FRF for the Iwan-reduced simulations. The nearly identical responses suggest that none of the load cases (with a maximum of 1 kN) added enough energy to the system to excite the nonlinearity in the joint. Further simulations at higher excitation amplitudes are necessary to study the effect of the joint dynamics on the response of the system.

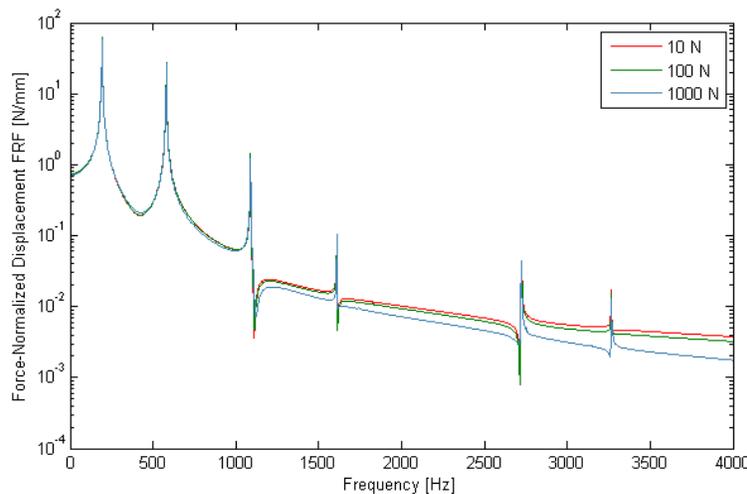


Figure 9: Reduced model displacement frequency response functions for different forcing amplitudes. Figure suggests too little energy is put into the system for the given force amplitudes to excite nonlinearity in the joint.

Figure 10 shows the FRF near the first natural frequency obtained by the HBM-Stuttgart model with 712 contact nodes. Results for four different bolt preloads (1, 5, 10 and 15kN) and a harmonic excitation of 100N are shown in Fig. 10(a). These results show that the behavior is nonlinear for the considered loading, in contrast to the Iwan-reduced predictions. It is interesting to observe, that the damping seems to increase with an increase in bolt load, which could potentially indicate more

micro-slip (and asperity interactions) in the contact zone due to a stronger static contact deformation. Figure 11b shows the FRF near the first natural frequency for a bolt loading of 1kN and three excitation levels (1, 10 and 100 N). In this case the damping increases with an increasing excitation, following a more traditional understanding of the joint behavior.

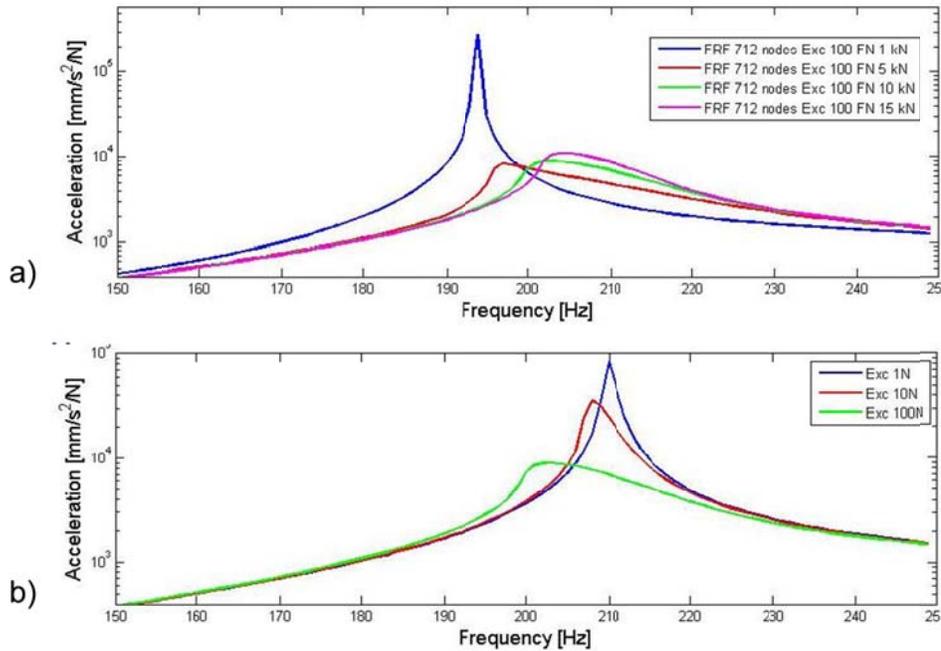


Figure 10 The results from the HBM-Stuttgart model: a) for varying normal load and b) for varying excitation forces with 10kN bolt loading

Figure 11 shows the FRF obtained by the HBM-Imperial model for different nonlinear models with 86 and 903 contact nodes and a linear FRF when the interface is modelled using spring elements. For varying excitation an increase in damping can be observed, which is particularly strong for the second mode. An increase in the number of the contact nodes leads to a slight stiffening of the response, and predicts more damping due to a more accurate representation of the contact surface. The contact condition during one period of vibration at the resonance for the first mode in Fig. 12 shows that most of the contact nodes have a nonlinear behavior during the vibration period by either slipping or separating.

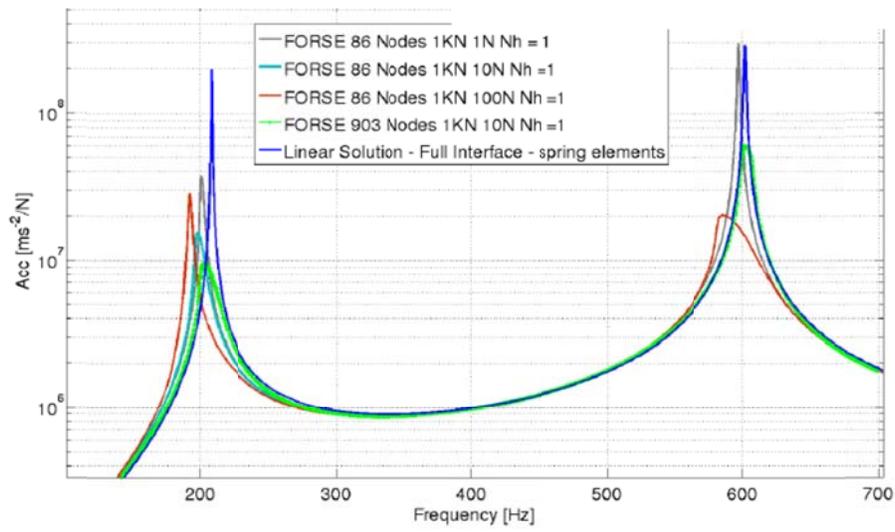


Figure 11: Nonlinear frequency response prediction from the HBM-Imperial model.

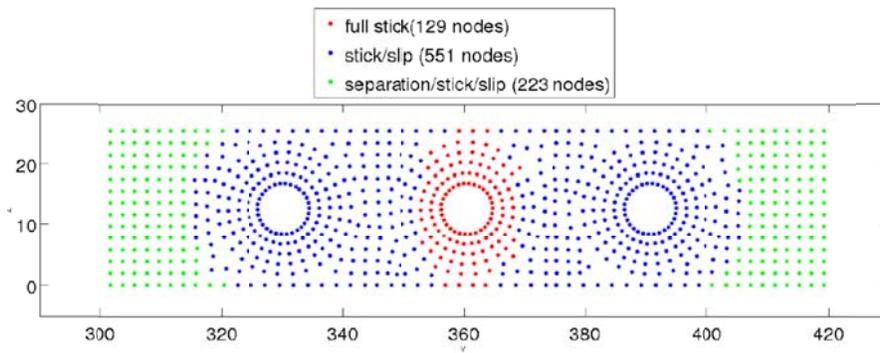


Figure 12 The contact conditions in the lap joint during vibration for the HBM-Imperial model.

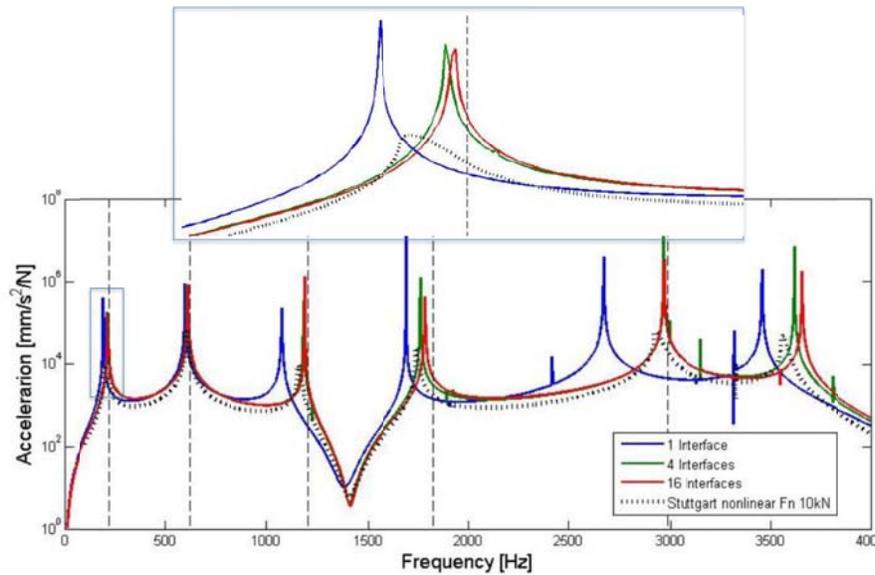


Figure 13: Predicted response for different Iwan-full models and the HBM-Stuttgart model.

Figure 13 shows FRFs obtained from the Iwan-full model for different number of joints (1, 4, and 16) in the interface and compares the results to the HBM-Stuttgart models with a bolt preload of 10 kN. The resonance frequencies agree reasonably well, also the Iwan-full model with a single element predicts the lowest interfacial stiffness. Further, the applied transient forces for the Iwan-full model do not seem to activate the nonlinear behaviour in a similar manner as in the HBM simulations.

Figure 14 shows the comparison between the FRFs obtained by the HBM-Stuttgart and HBM-Imperial models. A good agreement between both approaches is observed for the single harmonic case with a large number of nonlinear elements, which is not that surprising since both approaches use a similar method for the calculation of the nonlinear response. The FRF calculated with Imperial's multi-harmonic approach shows the effect of the higher harmonics since an additional peak in the response can be detected. As the interface has been reduced to 118 nodes for the multi-harmonic analysis a small frequency shift can be observed.

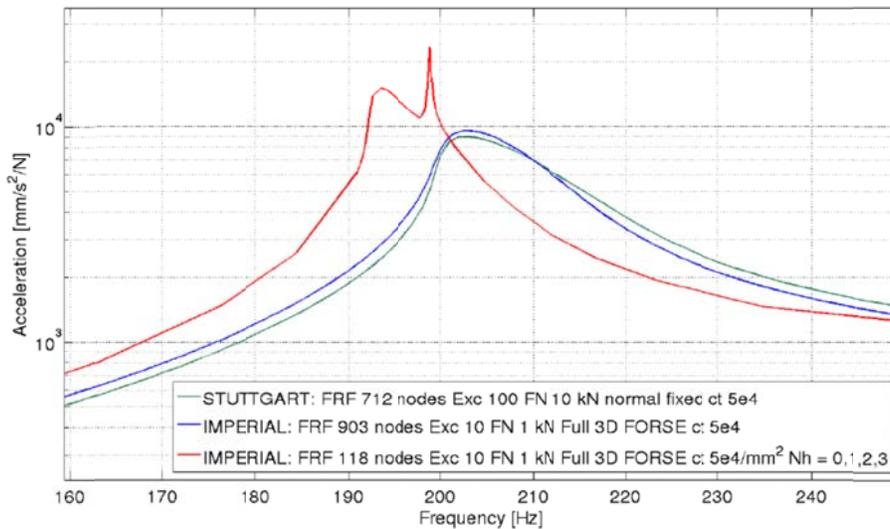


Figure 14 Comparison for the HBM-Stuttgart and HBM-Imperial predictions.

5. CONCLUSION

In this paper four approaches have been tested on a numerical benchmark to evaluate the current state of the art to model bolted lap joints. Two of them are frequency domain methods with contact elements based on Jenkins elements, while the other two are based on microslip Iwan elements, solved with a transient time domain analysis.

The selected lap joint beam had a strong nonlinear behaviour and it is consequently a good but quite challenging test case for the round robin exercise. Stick-slip and separation areas are present in the beam, providing a challenging task for the different approaches to capture the nonlinear response correctly. All investigated methods are able to predict a nonlinear response due to the contact surface, also some variations were observed especial between the time and the frequency domain approaches. The time domain approach seems to be unable to predict significant nonlinearity until dynamic force excitations up to three orders of magnitude higher than those of the frequency domain approach. The two methods based on frequency domain approaches give similar results for a single harmonic simulation but it could be observed that higher harmonics may be needed to capture the nonlinear dynamic behaviour of the joint correctly. ***The round robin exercise shows that the model reduction method of the contact interface can have a strong effect on the resulting nonlinear response and must be considered with care.***

Since different methods (frequency domain and time domain) with different excitation approaches are used for this preliminary round robin, a direct comparison of the results is rather difficult, and a new strategy is required to efficiently compare the different approaches. This study should be seen as a first step for a numerical round robin initiated by the ASME Joints Research Group to develop a proper benchmark for the validation of future nonlinear dynamic codes.

ACKNOWLEDGEMENTS

The authors would like to thank Sandia National Laboratories to support this research work during the 2014 Sandia Nonlinear Mechanics Summer Research Institute.

REFERENCES

- [1] Luan, Y., Guan, Z., Cheng, G., and Liu, S., 2011, "A Simplified Nonlinear Dynamic Model for the Analysis of Pipe Structures With Bolted Flange Joints," *Journal of Sound and Vibration*, **331**, pp. 325–344.
- [2] Boeswald, M., and Link, M., 2003, "Experimental and Analytical Investigations of Non-Linear Cylindrical Casing Joints Using Base Excitation Testing," In *IMAC XXI A Conference and Exposition on Structural Dynamics*, Kissimmee, FL, February 3–6.
- [3] Schwingshackl, C. W., Di Maio, D., Sever, I., and Green, J.S., 2013, "Modeling and Validation of the Nonlinear Dynamic Behavior of Bolted Flange Joints," *Transactions of the ASME: Journal of Engineering for Gas Turbines and Power*, **135**, pp. 122504-1-8.
- [4] Schwingshackl C.W., Petrov, E.P., and Ewins, D.J., 2012, "Measured and Estimated Friction Interface Parameters in a Nonlinear Dynamic Analysis," *Mechanical Systems and Signal Processing*, **28**, pp. 574-584.
- [5] Starr, M.J., Brake, M.R., Segalman, D.J., Bergman, L.A., Ewins, D.J., 2013, "Proceedings of the Third International Workshop on Jointed Structures," *SAND2013-6655*, Sandia National Laboratories, Albuquerque, NM.
- [6] Segalman, D.J., Gregory, D.L., Starr, M.J., Resor, B.R., Jew, M.D., Lauffer, J.P., and Ames, N.M., 2009 "Handbook on Dynamics of Jointed Structures," *SAND2009-4164*, Sandia National Laboratories, Albuquerque, NM.
- [7] Bograd, S., Reuss, P., Schmidt, A., Gaul, L., and Mayer, M., 2011, "Modeling the Dynamics of Mechanical Joints," *Mechanical Systems and Signal Processing*, **25**, pp. 2801-2826,
- [8] Petrov, E.P., and Ewins, D.J., 2002, "Analytical formulation of friction interface elements for analysis of nonlinear multiharmonic vibrations of bladed discs", *ASME Journal of Turbomachinery*, **125**, pp.364-371.
- [9] Segalman, D.J., 2005, "A Four-Parameter Iwan Model for Lap-Type Joints," *ASME Journal of Applied Mechanics*, **72**, pp. 752–760.
- [10] Soize, C., 2010, "Generalized Probabilistic Approach of Uncertainties in Computational Dynamics Using Random Matrices and Polynomial Chaos Decompositions," *International Journal for Numerical Methods in Engineering*, **81**, pp. 939-970.
- [11] Wang, X.Q., and Mignolet, M.P., 2014, "Stochastic Iwan-type Model of a Bolted Joint: Formulation and Identification". In *IMAC XXXII A Conference and Exposition on Structural Dynamics*. Orlando, FL.
- [12] Edwards, H. C., 2002. "Sierra Framework Version 3: Core Services Theory and Design." *SAND2002-3616*, Sandia National Laboratories, Albuquerque, NM.
- [13] Segalman, D.J., 2007, "Model Reduction of Systems with Localized Nonlinearities," *ASME Journal of Computational and Nonlinear Dynamics*, **2**, 249–266.

- [14] Brake, M.R., and Segalman, D.J. , 2013, “ Modeling Localized Nonlinear Constraints in Continuous Systems via the Method of Augmentation by Non-Smooth Basis Functions,” *Proceedings of the Royal Society A-Mathematical Physical and Engineering Sciences*, **469**, pp. 1–20.
- [15] Craig, R.R., and Bampton, M.C.C., 1968, “Coupling of Substructures for Dynamic Analyses”. *AIAA Journal*, **6**, pp. 1313–1319.
- [16] Milman, M.H., and Chu, C.-C., 1994, “Optimization Methods for Passive Damper Placement and Tuning,” *Journal of Guidance, Control, and Dynamics*, **17**, pp. 848-856.
- [17] Petrov, E.P., and Ewins, D.J., 2004, “Generic Friction Models for Time-Domain Vibration Analysis of Bladed Disks”, *Transactions of the ASME. Journal of Turbomachinery*, **126**, pp. 184-92.
- [18] Petrov, E.P., 2011, “A High-Accuracy Model Reduction for Analysis of Nonlinear Vibrations In Structures With Contact Interfaces”, *Transactions of the ASME: Journal of Engineering for Gas Turbines and Power*, **133**, pp. 102503-1 - 102503-10.
- [19] Brake, M. R., Reuss, P., Segalman, D.J., and Gaul, L., 2014, “Variability and Repeatability of Jointed Structures with Frictional Interfaces,” In *IMAC XXXII A Conference and Exposition on Structural Dynamics*. Orlando, FL.