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EFFICIENT CSM/CFD METHOD FOR AEROELASTICITY STABILITY ANALYS OF TURBOMACHNE BLADES

Y.M. Temis[1]*, L. Salles[1,2]

[1] Bauman Moscow State Technical University, Applied Mathematics
2nd Baumanskaia Str.5, Moscow, 105005, Russia
jmt@newmail.ru

[2] Laboratoire LTDS UMR 5513 Ecole Centrale de Lyon
36, Guy de Collongue, Ecully, 69130, France
loic.salles@ec-lyon.fr

Abstract

The problem of blade flutter prediction is relevant on the engine design stage. Modern technology of multidisciplinary simulation that combines the vibrating blade model and the response model of gas flow in gas dynamic tract is used in present paper to determine the influence of flow parameters on natural modes and frequencies. Geometrically nonlinear aeroelastic blade models (pre-twisted beam, shell and 3D solid body**)** that consider aerodynamic stiffness in the blade assembly are implemented. Aerodynamic stiffness coefficients are calculated on the base of CFD simulation results. Stability analysis for typical compressor and turbine blades is carried out. It is demonstrated, that centrifugal loads and flow speed change blade natural frequencies and modes, what has an influence on blade sensitivity to flutter. It is shown that connection between blade second bending and first torsion natural modes defines the condition of blade bending-torsion flutter initiation.

Introduction

Vibration of blades of compressors and turbines is one of the most complex and acute problems which arise during design of transport or stationary steam turbine. Vibration treatment is still very lengthy and requires large expenditure of money and time when turbomachines are produced. If up to a certain time this problem was characteristic for light and highly stressed aviation and transport power installations, then at the present time problems of vibrational strength are just as acute in stationary energy installations: the traditional course of ensuring vibrational strength of stationary machines - design of the blades with a low level of static flexural stresses – cannot be used when producing installations of great power together with high peripheral speeds and relatively thin blades. The complexity of this problem to a large extent is determined by its involved character: problems of unsteady aerodynamics and problems of vibration of thin pre-twisted blade in elastic interaction with other elements of the rotor.

Compressor and turbine bladed-disk vibrations are among the most topical issues that can be encountered during the design of steam turbine and aeroengines. The most common types of vibration problem that concern bladed-disk include resonant vibration occurring at a multiple of rotation speed and flutter, an aeroelastic instability, whose frequency does not depend on rotation speed. In this paper second type of vibration is studied, particularly coupled bending-torsion blade flutter.

Blade flutter in axial turbomachines is defined in [13] as an aerodynamic instability caused by an interaction between the vibratory motions of an assembly of blades and fluid dynamic forces resulting from these motions. Different compressor blade flutter phenomena aspects are reviewed in [4,13]. The majority of investigations have been until recently devoted to the aerodynamic aspects of this problem, and only some have studied the compressor as an interdependent mechanical aerodynamic system. Most of studies consider that compressor blade flutter differs substantially from classic wing flutter [4], and it is related with a single structural mode of vibrations due to high stiffness of blade. However, such neglect may not be entirely justified in cases of frequencies close to the second and third forms of natural blade vibrations [5]. Present work gives additional investigation of this question and propose method of multidisciplinary simulation to numerically observe this kind of flutter [16,17].

Analysis is based on coupling between a geometrically nonlinear finite element blade model for structural dynamics and resolution of unsteady Navier-Stoke equation for fluid problem. Stability analysis of blade under non-stationary gas loading is conducted and its dynamic bifurcation mechanism is investigated. Blade as structural system has intern damping characteristics, which depend on material properties. Another form of damping can occur in bladed-disk, which is due to friction at joint interfaces. Friction damping occurs in bladed-disk dovetail attachments [2, 18]. If structural damping is not sufficient to offset the aerodynamic effects, external friction dampers are used. Different kind of damper can be used including underplatform dampers [10] or friction ring damper [8]. There are few studies in literature [1] on coupled aeroelastic analysis of bladed-disk taking into account non-linear effects of contact with friction. In this paper we will present two approaches that allows to include structural nonlinearity in flutter analysis.

Governing equation of aeroelastic problem

Each engine operational regime is characterized by rotating frequency ω_r and corresponding flow gas-dynamic parameters set $\overline{P}_G$ (pressure ratio, mass flow…).

In this work dynamical behavior around equilibrium position will be studied. This position characterized equilibrium between centrifugal forces and static aerodynamic pressure. Coupled Aeroelastic problem is characterized by following system:

$$\begin{aligned}&\left[K_T(\mathbf{U}_s)\right]\mathbf{U} = \mathbf{F}_{\mathbf{aero}}(\mathbf{U},\dot{\mathbf{u}}) + \\ &\mathbf{F}_{\Omega} + \mathbf{F}_{\mathbf{j}}(\mathbf{U}+\mathbf{u}) + \mathbf{F}_{\mathbf{damp}}(\dot{\mathbf{u}}) + \mathbf{F}_{\mathbf{inertia}}(\ddot{\mathbf{u}}),\end{aligned} \tag{1}$$

where $\left[K_T(X)\right]$ is tangent stiffness matrix that includes prestress effects from centrifugal and aerodynamic loadings. $\mathbf{U},\mathbf{u}$ are respectively vector of static displacements and dynamical perturbation around static equilibrium position, whose dimension depends on chosen finite element method. $\mathbf{F}_{\mathbf{aero}},\mathbf{F}_{\Omega},\mathbf{F}_{\mathbf{j}},\mathbf{F}_{\mathbf{damp}},\mathbf{F}_{\mathbf{inertia}}$ are vector of stationary aerodynamic loads, vector of centrifugal loading, support forces, friction damping and inertia and as (1) will be linearized around equilibrium position these forces are defined by:

$$\begin{aligned}&\mathbf{F}_{\mathbf{aero}}(\mathbf{U},\dot{\mathbf{u}}) = \mathbf{F}_{\mathbf{aero}}(\mathbf{U}) - \left(\left[K_A\right]\mathbf{u} + \left[C_A\right]\dot{\mathbf{u}}\right),\\ &\mathbf{F}_{\mathbf{j}}(\mathbf{U}+\mathbf{u}) = \mathbf{F}_{\mathbf{j}}(\mathbf{U}) - \left[K_J\right]\mathbf{u},\\ &\mathbf{F}_{\mathbf{damp}}(\dot{\mathbf{u}}) = -\left[C_J\right]\dot{\mathbf{u}},\\ &\mathbf{F}_{\mathbf{inertia}}(\mathbf{u}) = \left[M\right]\ddot{\mathbf{u}}.\end{aligned} \tag{2}$$

Static equilibrium position is calculated considering $\mathbf{u} = \mathbf{0}$. Structural and stationary aerodynamic are performed separately. Linearized equation around equilibrium position of motion takes following form:

$$\begin{aligned}&\left[M\right]\ddot{\mathbf{u}} + \left(\left[C\right] + \left[C_A\right] + \left[C_J\right]\right)\dot{\mathbf{u}}\\ &+\left(\left[K\right] + \left[K_A\right] + \left[K_J\right]\right)\mathbf{u} = \mathbf{0},\end{aligned} \tag{3}$$

where $\left[C_A\right]$, $\left[C_J\right]$ are matrices of aerodynamic damping and friction damping and $\left[K_A\right]$, $\left[K_J\right]$ are matrices of aerodynamic and joint stiffness. As system (3) is linear and we are interested in vibratory motions, displacement will be harmonic with $\mathbf{u} = \mathbf{X}e^{\lambda t}$ with λ vibration frequency and $\mathbf{X}$ vector of displacement amplitude. Substituting this expression into equation (3) , generalized eigenvalue problem is obtained.

$$\{\lambda^2[M]+\lambda([C]+[C_A]+[C_J]) \\ +([K]+[K_A]+[K_J])\}\mathbf{X}=\mathbf{0}. \quad (4)$$

If $\lambda = a+ib$ a, b *are* real numbers. If $a>0$, then the vibrations are unstable, what initiates flutter. While generalized eigenvalue system can be solved directly, the authors of [16, 17] make the further simplification, neglecting the dependence of the unsteady aerodynamic gas loads with respect to the velocity of the blade. This assumption can be done because - speed of the airfoil motion in direction, transverse to the flow speed is very small compare to flow speed and for typical turbomachinery blades rotation angles of section significantly surpass ratio of blade linear speed and flow velocity, and from this point of view the second component of following expression, representing the aerodynamic damping, can be omitted.

$$L=\pi\rho V^2 c\left(\theta-\frac{\dot{x}}{V}\right), \quad (5)$$

where L is aerodynamic lift force. In works [17] authors have also assumed absence of mechanical damping component and they obtained standard eigenvalue problem, the solution of which gives natural frequencies and modes of blade vibrations in presence of gas flow:

$$\left\{\omega^2[M]+([K_T]+[K_A])\right\}\mathbf{X}=\mathbf{0}. \quad (6)$$

In order to form system (6) it is necessary to determine geometrically nonlinear system tangent matrix for blades and aerodynamic stiffness matrix parameters. Thus, system (6) combines calculation results of geometrically nonlinear structure and gas flow in compressor or turbine stage.

A more accurate method based on nonlinear complex normal mode [7] can be used to study coupled non-linear structural and aeroelastic system. Displacements **u** are decomposed in generalized Fourier series,

$$u(t)=X_0+\sum_{n=1}^{Nh} e^{-nat}\left(X_n^c\cos(nbt)+X_n^s\sin(nbt)\right) \quad (7)$$

assuming that damping term e^{-nat} is independent of frequency. N_h is harmonic order of Fourier series. Expression (7) is introduced in (1) and Galerkine procedure is performed, which gives non-linear algebraic system:

$$[Z(a,b)]\tilde{\mathbf{X}}+\tilde{F}_c(\tilde{\mathbf{X}})+\tilde{F}_{aero}(\tilde{\mathbf{X}})=\mathbf{0}, \quad (8)$$

where $\tilde{\mathbf{X}}$ is multiharmonic vector of displacement, $\tilde{F}_c$ and $\tilde{F}_{aero}$ are multiharmonic vector of contact force and aerodynamic forces. Both structural and aerodynamic problem are solved using harmonic balance method (HBM). If problem is solved in frequency domain an alternating frequency time procedure must be performed to calculate non-linear forces [2]. Another approach consists in using $2N_h+1$ equal separated time variable as unknowns. This method is known as Time Spectral Method [11]. This approach is used to find steady state of unsteady RANS system [12]. Solving system (8) gives $\tilde{\mathbf{X}}$ and (a,b), which allows to know behavior of system. If $a<0$ system is unstable and if $a=0$ there is a LCO situation (Limit Cycle Oscillation). Some authors have used HBM to find LCO solution of aeroelastic blade system with structural nonlinearity [10]. HBM method is less time consuming than accurate time integration, indeed URANS system can be converge to steady state in small number of cycle but speed of convergence for solving system of structural dynamics with contact and friction will be at least a factor ten times more important. Moreover numerical scheme can often product numerical damping which will make impossible flutter analysis. Frequency methods avoid this difficulty.

General blade model

Geometric nonlinear solid or shell finite element models are most widely used for blade dynamics and allow today to get result rapidly for one parameter set. For a given rotating speed Ω static equilibrium position is found by incremental approach.:

-First ,solution $\mathbf{U}_0$ is found solving (1) for static problem without nonlinear behavior:

-Then nonlinear behavior is taking into account and Newton-Raphson method is performed to find $\mathbf{U}_s$:

$$\mathbf{U}_s = \mathbf{U}_0 + \sum \Delta\mathbf{U}_k \text{ with } [K_T]\Delta\mathbf{U}_k = \beta_k \mathbf{\Psi}_k, \qquad (9)$$

where tangent matrix $[K_T]$ for these models considering, initial stress stiffness $[K_\sigma]$, geometrical nonlinearity stiffness $[K_{geom}]$ and centrifugal stiffness $[K_\Omega]$ and eventually contact stiffness $[K_c]$re represented as follow:

$$[K_T] = [K_{elastic}] + [K_{geom}] + [K_\sigma] + [K_\Omega] + [K_c],$$

$\mathbf{\Psi}_k$ is sum of surface and interior forces for step *k*.

However, if large series of calculation are conducted for geometry involving a lot of parameters in a process of optimization, accurate computational methods are required. Efficiency of geometrically nonlinear pre-twisted beam finite element [15] with 2 nodes, shown in Fig. 1, that allows to get quite accurate results faster than with solid or shell FE is investigated.

Pre-twisted beam model

Non-linear pre-twisted beam model is characterized by following expression of deformation:

$$\varepsilon_\varsigma = u_{,\varsigma} - \xi v_{,\varsigma\varsigma} - \eta w_{,\varsigma\varsigma} + \varphi_0 \rho^2 \Phi_{,\varsigma} + \frac{1}{2}\left(u_{,\varsigma}^2 + v_{,\varsigma}^2 + w_{,\varsigma}^2 + \rho^2 \Phi_{,\varsigma}^2\right), \qquad (10)$$

where ρ vector of shear center position relative to center of coordinate system, (u,v,w) are displacement in (x,y,z) -direction Φ is torsion angle of pre-twisted beam. Main characteristics of pre-twisted beam FE [15] are length l, cross-section area S, pre-twist per unit length $\alpha = A/l$, central axial moments of inertia J_ξ, J_η, centrifugal moment of inertia J_ξ, polar moment of inertia, etc.

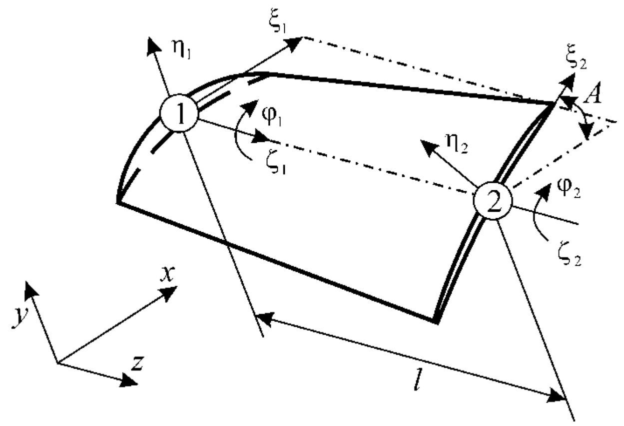


Fig. 1 Pre-twisted beam finite element

Stiffness matrices for pre-twisted beam finite element are calculated using following expressions:

$$[K_{elastic}] = \int_0^l [A_0]^T [D^*] [A_0] d\zeta, \qquad (11)$$

where

$$[D^*] = \int_F \begin{bmatrix} E & -\xi E & -\eta E & \varphi_0 E \rho^2 \\ -\xi E & \xi^2 E & \xi\eta E & -\varphi_0 E \xi \rho^2 \\ -\eta E & \xi\eta E & \eta^2 E & -\varphi_0 E \eta \rho^2 \\ \varphi_0 E \rho^2 & -\varphi_0 E \xi \rho^2 & -\varphi_0 E \eta \rho^2 & G\left(\Theta_{,\eta} + \Theta_{,\xi}\right) + \varphi_0^2 \rho^4 E \end{bmatrix} dF$$

and

$$[A_0] = \left[\frac{\partial N_u}{\partial \zeta}, \frac{\partial^2 N_v}{\partial \zeta^2}, \frac{\partial^2 N_\Phi}{\partial \zeta^2}\right],$$

with (N_u, N_v, N_θ) shape function of beam finite element.

$$[K_{geom}] = \int_0^l [A_0]^T [B^*] d\zeta + E\int_0^l \int_F [B_1]^T [B_1] dF d\zeta + \int_0^l [B^*]^T [A_0] d\zeta, \qquad (12)$$

with

$$[B^*] = \int_F \begin{bmatrix} B_1 \\ -\xi B_1 \\ -\eta B_1 \\ \varphi_0 \rho^2 B_1 \end{bmatrix} dF,$$

where

$$[B_1] = u_{,\zeta} \frac{\partial N_u}{\partial \zeta} + v_{,\zeta} \frac{\partial N_v}{\partial \zeta} + w_{,\zeta} \frac{\partial N_w}{\partial \zeta} + \rho^2 \Phi_{,\zeta} \frac{\partial N_\Phi}{\partial \zeta}.$$

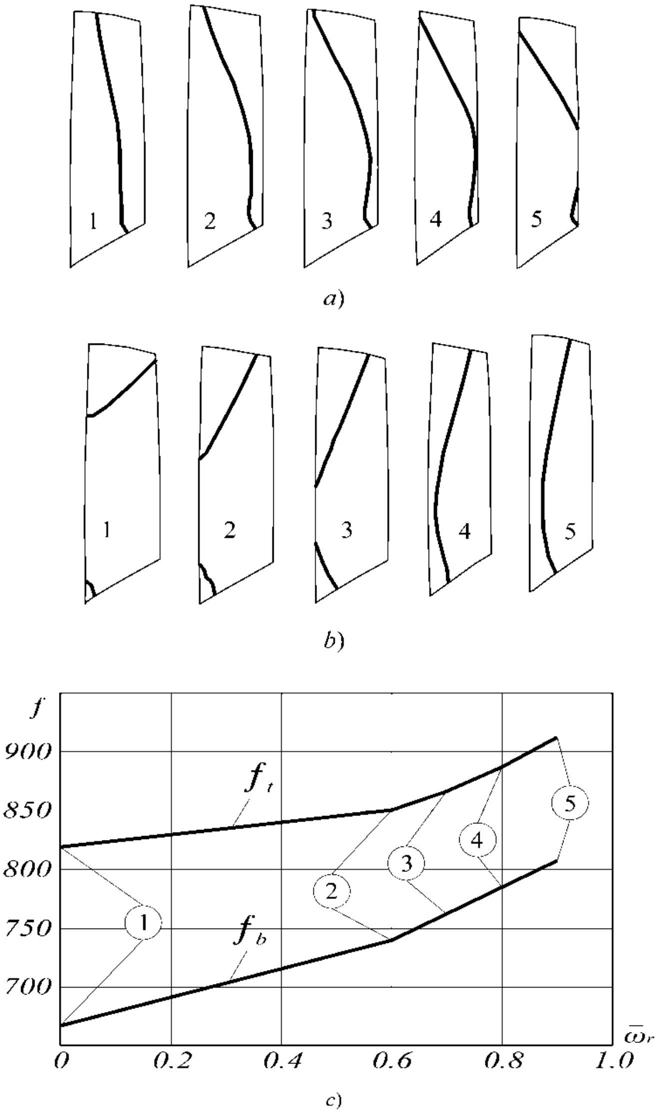


Fig. 2 Inversion of blade natural forms versus *rotating velocity*

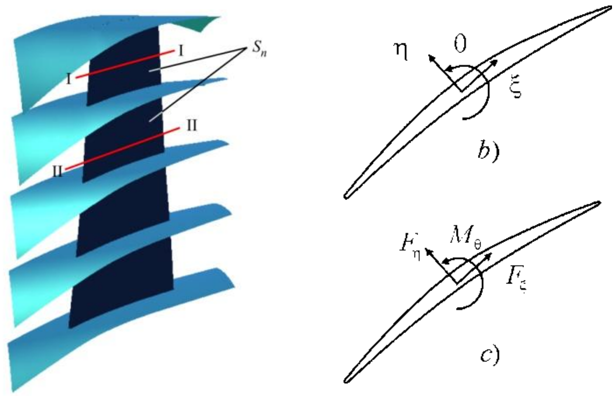
a - first torsion form; *b* - the second bending form; *c* - blade natural values

Cross section characteristics of beam and integrals in stiffness matrices can be calculated using finite element method [15] or boundary element method [14]. Other approaches were proposed but they are less accurate. Boundary element method give best results and is less Cpu time consuming.

Blade Vibration

Calculation of blade natural frequency taking into account geometrical nonlinearities and centrifugal loads recovering effect allows to better determine zone where veering occurs. Veering phenomena occurs when bending mode switches to torsion mode and torsion mode switches to bending mode. Such recombination, which is shown in Fig. 2 , is one of the diagnostic factors of possible bending torsion flutter appearance.

Aerodynamic model



(a)

Fig. 3 Blade model (a) with different retained sections, definition of profile displacements (b) and aerodynamics forces (c)

Calculation of $[K_A]$ is performed around static equilibrium position. One way to simplify calculation of this matrix is to transform 3D unsteady aerodynamic problem in a 2D aerodynamic problem. In this case a set of cylindrical cross-sections based on streamlines should be selected. This approach is less time consuming than whole 3D calculation.

Displacement of airfoil in each section is characterized by following vector (u_n, v_n, θ) displacement in ξ direction along the blade chord, displacement in η direction perpendicular to blade chord and in plane section rotation angle. Generalized aerodynamic forces and moment associated with these displacements are defined by following expression:

$$F_\xi = \int_{S_n} p l_\xi dS, \qquad F_\eta = \int_{S_n} p l_\eta dS,$$
$$M_\theta = \int_{S_n} p\left(\eta l_\xi + \xi l_\eta\right) dS, \qquad (13)$$

where l_ξ , l_η are direction cosines of surface S_n . $[K_A]$ is calculated using finite difference of each force and moment components in local coordinate system, attached to each blade section:

$$[K_A] = \begin{bmatrix} k_{\xi\xi} & k_{\xi\eta} & k_{\xi\theta} \\ k_{\eta\xi} & k_{\eta\eta} & k_{\eta\theta} \\ k_{\theta\xi} & k_{\theta\eta} & k_{\theta\theta} \end{bmatrix}, \tag{14}$$

where $k_{\xi\xi} = \dfrac{F_\xi(\Delta\xi,0,0) - F_\xi(0,0,0)}{\Delta\xi}$ for example.

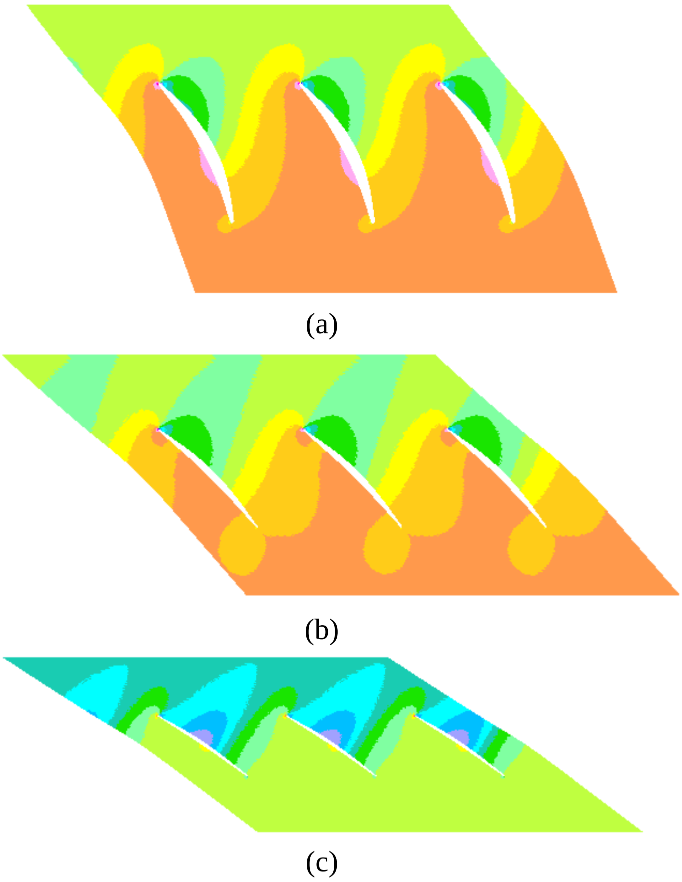


Fig. 4 Static pressure distribution in the hub (*a*), middle (*b*) and tip (*c*) blade cross-sections

In the present study values of aerodynamic loads were obtained on the basis of proposed approach in this paragraph for 5 sections of the blade, a 2D fluid dynamics analysis was performed using a commercial CFD tool: Navier-Stokes, standard k-w turbulence model.

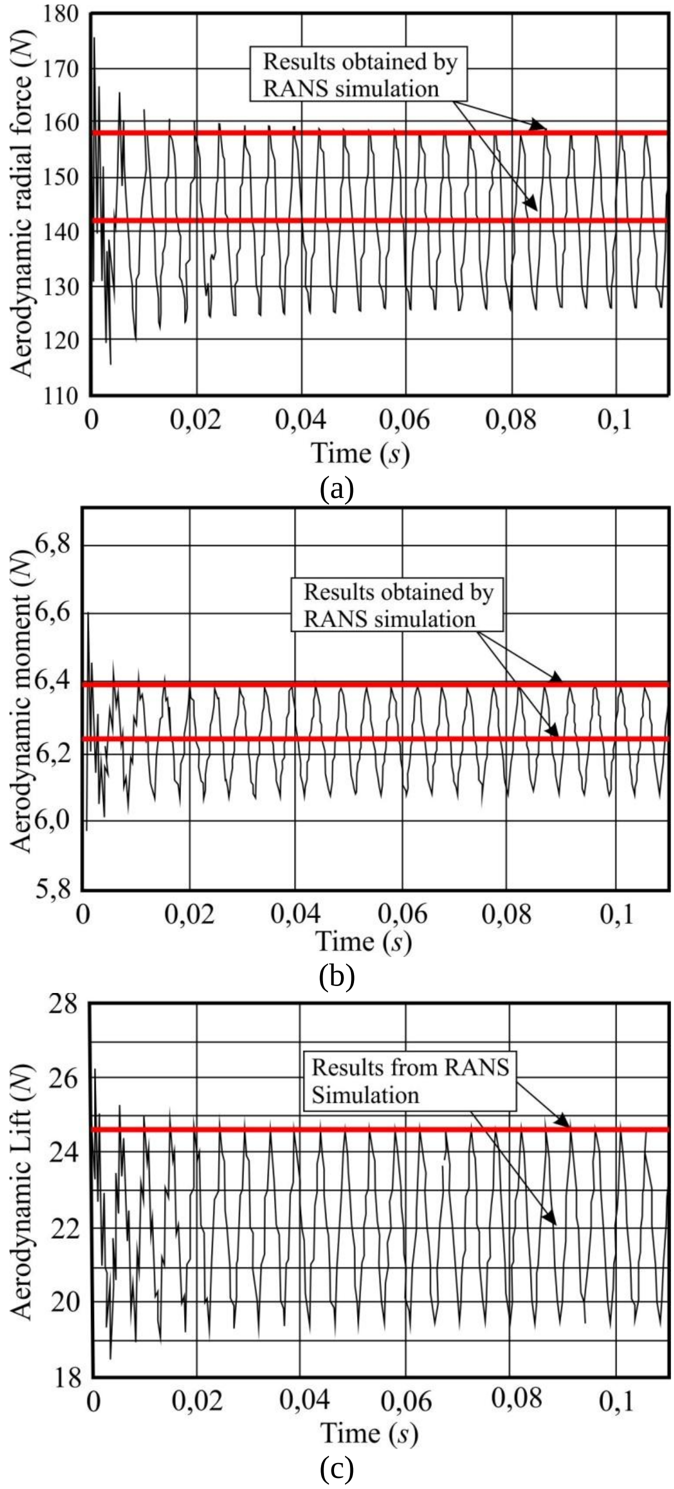


Fig. 5 Generalized aerodynamic forces: (a) radial forces, (b) lift and (c) moment for vibrated profile with RANS and U-RANS simulation

A steady-state response was obtained for both nominal blade position and small blade deflection $(\Delta\xi, \Delta\eta, \Delta\theta)$ in each principal direction This approach is valid if blade vibration reduced frequency ($\gamma = b\omega/V$) is small and quasi-static approach can be assumed; b is half blade chord, ω

is vibration frequency and V is flow relative velocity. Fig. 5 shows that this approach is valid, indeed those figures show good equivalency between RANS simulations and unsteady RANS simulations for calculation of aerodynamic forces.

If the value of reduced frequency is closed or greater than 1, stiffness matrix coefficients may be obtained by unsteady gas dynamic simulation applying modal motion to blade or group of blades. After getting cyclic steady state $[K_A]$ and $[C_A]$ can be identified using least square method [9] or POD method [3].

Flutter Analysis.

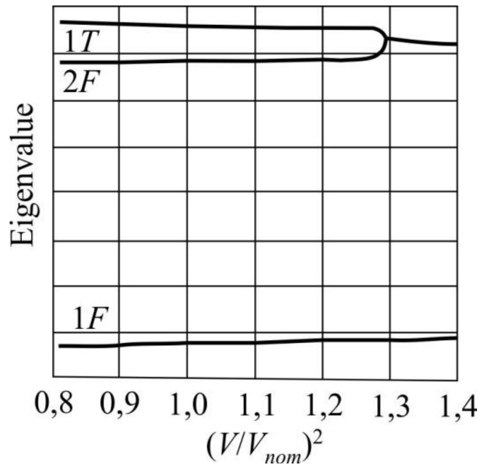


Fig. 6 Bending-torsion flutter initiation

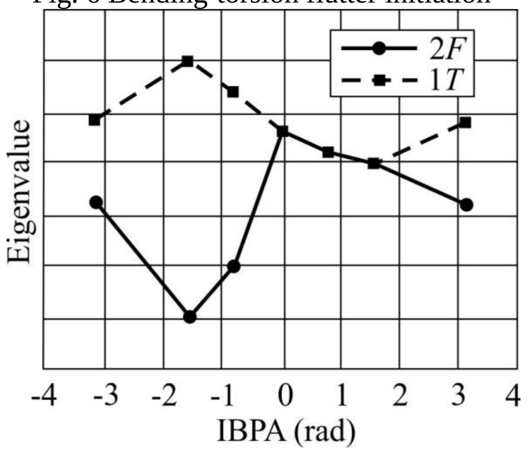


Fig. 7 Eigenvalue versus interblade phase angle (IBPA)

Proposed method was used to perform flutter analysis of long ratio blade. Neglecting damping effect map of frequency versus flow speed can be draw. Fig. 6 shows that unstable mode of vibration occurs after $V^2 = 1.3V_{nom}{}^2$

Another study was performed using analogy with wing flutter. To estimate the flutter boundaries and use it for design process, a certain characteristic, which is continuous with respect to blade geometrics, is necessary. As for wing flutter problem [14], where flow aerodynamic stiffness is proportional to the square of flow velocity, the natural frequencies behavior can be investigated:

$$[M]\ddot{\mathbf{q}} + \left([K_T] + \beta^2 [K_A]\right)[\mathbf{q}] = \mathbf{0}, \qquad (15)$$

where β is independent parameter. It's obvious that if $\beta = 0$, all natural frequencies of system (15) are pure imaginary. The value of parameter β that gives the frequency with positive real part has a meaning of flutter boundary. In Fig. 8 the aerodynamic stiffness influence on blade natural modes and frequencies is shown for non-rotating system. Five sections have been used (0, 0.25, 0.5, 0.75 and 1 of the blade height) for tangent matrix calculation, aerodynamics stiffness matrices for 3 sections (0, 0.5 and 1) have been calculated, for other sections these matrices have been obtained using interpolation considering section rotation. Hub section has been fixed. Blade stiffness matrix was obtained without considering rotation. In case of two imaginary frequencies combination a couple of roots $\pm\lambda + i\omega$ appears, one mode is stable, another is unstable. These eigenvalue have imaginary natural modes, what corresponds to motion $q(t) = e^{at}\left(q^c \cos(bt) + q^s \sin(bt)\right)$, ie. vibration in two bending-torsion modes with phase shift $\pm\pi/2$. Small instability zone doesn't necessary occur for second and third natural modes combination. The inversion of these modes may happen without frequencies combining. Increasing parameter β over the working area leads to frequencies combining and flutter.

In Fig. 9 natural frequencies and modes modification is shown. The blade tangent stiffness matrix was calculated considering rotation of rotor. Second bending and torsion modes inversion occurred, because, unlike Fig. 8, second and third frequencies are not combined.

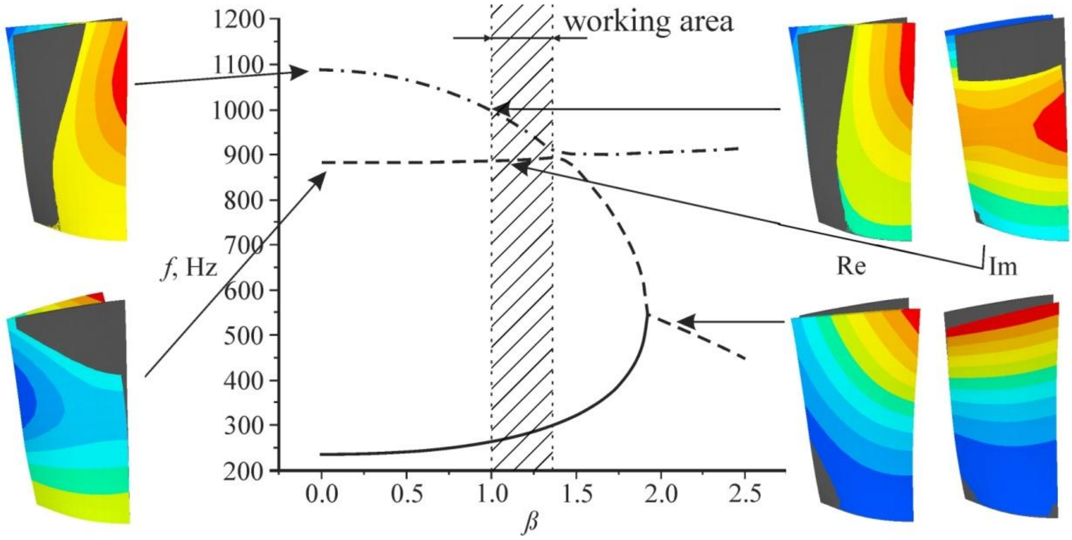


Fig. 8 Aeroelastic influence for non rotating blade

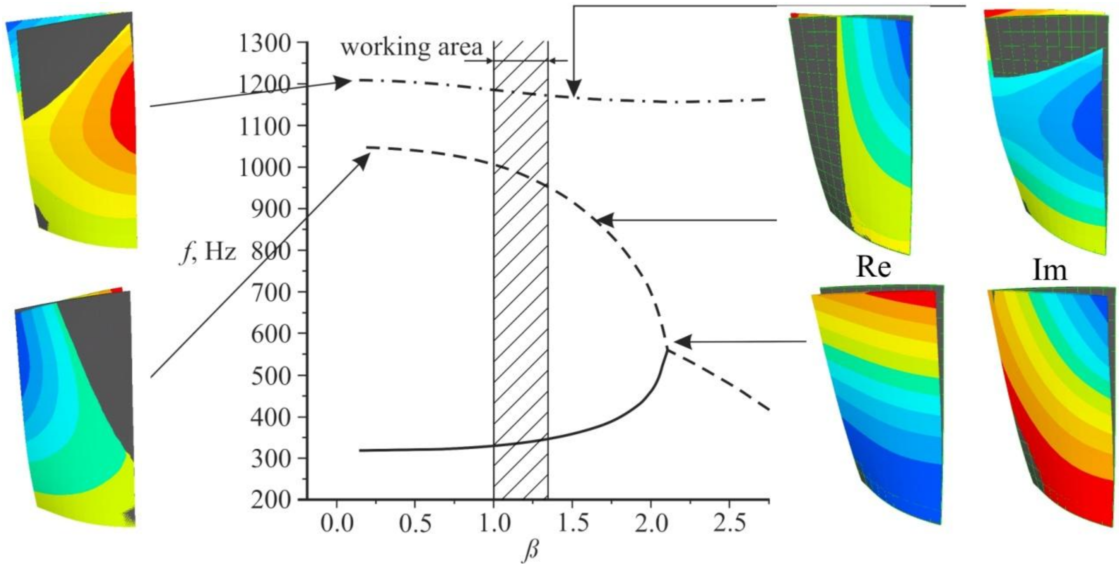


Fig. 9 Aeroelastic influence for rotating blade

Conclusions

In this paper was proposed coupling CSM/CFD method to perform bending-torsion flutter analysis of blade around static equilibrium position. Structural model is based on non-linear finite element analysis with non-linear pre-twisted beam model to accelerate calculation and aerodynamic model is based on linearized equivalent around equilibrium position of U-RANS equation. Flutter analysis have shown that speed velocity via veering play a role in prediction of stability boundaries. Flutter stability versus flow velocity can also be predicted by wing flutter like approach. Analysis show that centrifugal forces produce veering without decrease of frequency.

Proposed non-linear pre-twisted beam model can be used in analysis of mistuned bladed-disk.

Future works will concern analysis of flutter of bladed-disk taking into account damping, linear and nonlinear, which needs methods based on complex modal analysis or nonlinear harmonic analysis.

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