DYNAMIC ANALYSIS OF FRETTING-WEAR IN JOINT INTERFACE BY A MULTISCALE HARMONIC BALANCE METHOD COUPLED WITH EXPLICIT OR IMPLICIT INTEGRATION SCHEMES

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ABSTRACT

Assembled bladed disks have many contact interfaces (blade-disk joint, blade shrouds, friction damper, etc). Because of relative displacements at these interfaces, fretting-wear can occur, which shortens the life expectancy of the structure. Moreover, vibrations that occur in bladed-disks can increase this fretting-wear phenomenon.

Two previous papers in Turboexpo have introduced a numerical method based on the Dynamical Lagrangian Frequency Time algorithm (DLFT) to calculate worn geometry, especially wear of bladed-disks' dovetail roots. Numerical investigations have illustrated the performances of this method and shown the coupling between dynamical and tribological phenomena.

The basic idea of the DLFT-with-wear method is to separate time in two scales, slow scale for tribological phenomena and fast scale for dynamics. In the present paper, implicit and explicit integration schemes on the slow time scale are compared. An ad hoc prediction-correction method is used in both methods to accelerate the convergence of the non-linear solver. Numerical experiments on bladed-disk show that the implicit scheme is more appropriate to deal with fretting-wear under dynamical loading.

NOMENCLATURE

$\tilde{\_}$ Multiharmonic vectors.
$\_^T, \_^N$ Tangential and normal direction.
p_N, p_n Normal pressures.
p_T, p_T Tangential shear.
u, v, g Displacement,displacement,initial gap.
$w, \boldsymbol{W}$ Wear depth, nodal wear depth vector.
$\_r$ Relative value.
$\boldsymbol{U}$ Time-domain nodal displacement vector.
$\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ Mass, viscous damping and stiffness matrices.
$\mathbf{Z}_r$ Reduced dynamic stiffness on relative displacements.
F_c Nodal force of contact.
λ, $\tilde{\lambda}$ Lagrangian multiplier in time and frequency domains.

INTRODUCTION

Turbo-machinery bladed disks operate in severe environments in terms of aerodynamic loads and may experience high vibratory stresses due to resonance or flutter. Their ability to withstand these stress levels, which may cause high cycle fatigue, mainly depends on the external inputs of damping. In bladed disk assemblies, friction in interfaces is the most widely used source of external damping; this includes shroud contact in blades or platform dampers. But relative displacements at the interface have a damaging aspect, since fretting-wear can appear and induce cracking. Moreover fretting-wear can change damping property of structure and high cycle fatigue can increase. Methods [1, 2] exist to study the vibrational response of bladed disks in the presence of friction in blade attachments. Fretting-wear studies in blade roots are usually performed in quasi-static situations. Thus, in the case of planes, they only calculate [3] or test [4] the take-off or landing situations. Nevertheless, dynamic wear can also occur when vibrations appear at cruising rate. In this latter case, in spite of the fact that only micro-slidings are observed, they are repeated during a much longer time. Recently [5, 6] was proposed a new method to couple the calculations of the vibrational response and of the wear kinetics based on the DLFT (Dynamical Lagrangian Frequency Time) method [2] and on a multi-scale approach. The present paper presents improvements of this method. This approach was used [7] to calculate modal wear using method of calculation of Complex Nonlinear Normal Modes proposed by Laxalde [8].

In this paper homogenization in time similar to space homogenization of periodic structure is performed to treat fretting-wear under dynamical loading. This strategy is very efficient and less time consuming. In this work the method proposed is applied to real bladed-disk with dovetail joint. Vibration response and wear kinetics are calculated. This study allows to understand coupling between fretting-wear and dynamics. Results show that fretting-wear experiment protocols are wide of ”real” (simulated) wearing process in bladed-disk attachments.

THEORY

Important theory assumption are explained in this section. A rapid review of different existing methods for contact problem and wear modelling is proposed. Thus reader can understand choice of modelling and resolution of the problem of fretting-wear under dynamical loading made in this study.

Contact problems with friction: models and simulations Two steps are very important in creation of contact model. The first step is to define how to take into account contact constrains. The second step is to find an optimal algorithm to solve the problem with contact constrains. Chabrand *et al.* [9] made a good review of different methods with comparaison between them. Explanation of almost methods can be founded in the book of Wriggers [10].

Contact formulation can be primal, dual or mixed. For each formulation different method can be used to take into account boundary conditions. The most common method is penalty method where penetration is control by penalty term on normal problem. This method is instrinsically not exact and results depend on choice of penalty term, even if residue of nonlinear function is exactly zero. This method allows penetration and microsliding during stick period. When penalty terms tend to infinity contact force solution tend to exact value. In frequency method because of microsliding during stick period solution displacements can be far from exact solution especially when more than two stop occur during one period. This method can be easily used in dynamical problem with HBM method and AFT procedure [11–13].

Another approach uses Lagrangian multiplier, which are new unknowns. The size of the system is increased. In static analysis different algorithm can be used to solve this problem. For example projected conjugated gradient, Lemke’s method,modified Gauss-Seidel method, etc These approaches can’t be used in frequency method with AFT procedure because contact conditions change during cycle. For this reason the condition of contact is not boolean, which is necessary for the previously presented methods to work.

The last formulation explained in this section is augmented Lagrangian formulation of contact forces. Augmented Lagrangian approach is based on use of Lagrangian multiplier with addition of penalty term. To solve obtained problem different methods exist. Based on mixed formulation with displacement and contact forces as unknowns Alart and Curnier [14] have proposed a generalized newton method to solve problem. Only one augmentation of Lagrangian multiplier is performed. Other popular method to solve contact problem with augmented Lagrangian is Uzawa procedure. Equations of motion are solved by newton algorithm and a fixed point method is used to update contact forces (Lagrangian multipliers). Both methods can be used with frequency method and AFT procedure. Nacivet *et al.* [2] proposed method based on augmented Lagrangian with Newton-like solver. The problem is formulated as a primal problem. The method is similar to Alart’s method but the approach was adapted to frequency domain framework. In order to avoid prohibitive computation time, the augmented Lagrangians are given in frequency need to be updated using the non-linear contact forces at the previous frequency, hence requiring a fairly large penalty coefficient. This means that range of penalty coefficient is very limited. So primal formulation was preferred to obtain better stability of algorithm. Lagrangian multiplier are obtained in time domain and introduce in frequency domain by an prediction/correction approach. The final equation which has to be solved only depends on displacements. Laxalde [8] proposed a new method based on augmented Lagrangian with adapted Uzawa procedure to solve unilateral contact problem in frequency domain formulation.

Fretting-wear models Fretting wear is a surface degradation process in which removal of material is induced by small-amplitude oscillatory movement between contacting components. The main parameters affecting fretting wear are reported to be normal load, slip amplitude, frequency, contact geometry, surface roughness and material properties. The fretting map approach, established by Vingsbo and Soderberg has shown that fretting damage evolution depends strongly on the fretting regime. Debris is also a critical factor influencing fretting wear. It was reported that, once debris accumulates on the contact surfaces and forms a compact oxide layer, the wear rate is significantly reduced . In recent years, Godet and co-workers developed the theories of third-body tribology and velocity accommodation mechanisms to explain the role of wear debris in specific fretting conditions. Compared with the development of qualitative understanding, quantitative assessment of fretting wear is less well advanced. One of the difficulties is the absence of a universal and well-formulated wear model. According to [15] around 180 laws of wear have been proposed. In addition, it is not clear how to incorporate the effect of wear debris into such a quantitative model. For some situations, where wear debris is more easily eliminated from the contact area and more metal-to-metal contact is maintained, the influence of the debris can be reasonably neglected. In such cases, fretting wear can therefore be regarded as a purely contact-based wear problem. At the microscopic level, models of differential elements are used to study wear mechanisms. Macroscopic modelling uses wear rates or other models of wear as inputs to finite element analysis. To quantify wear, the Archard's model [16] is the most commonly used. It considers the wear volume is linked to the product of the normal force and of the sliding displacement. A local derivated form of this model will be used in this paper. Recently, a new model based on dissipated energy has been proposed to quantify wear directly [17]. This approach can be used with proposed method in this paper without many modifications. Research on simulating wear, from the available literature, can be divided broadly into two approaches. The first method is to implement directly a wear model into the material law of an FE analysis. However this should be implemented at the element level, since the information about the surface normal is only available at this level. Legrand [18] has proposed a macroscopic wear behaviour of abradable in contact rotor/stator problem numerically approximated through a piecewise linear plastic constitutive law. In our case a faster and efficient approach is to process (post-processing) the FE results obtained from the solution of a general contact problem with a suitable wear model to compute the progress of wear for a given time interval/sliding distance.

In proposed method the microscopic effects like asperities deformations and material tearing are not directly considered. The influence of these microscopic phenomena are taken into account through a macroscopic wear coefficient in Archard's law. The temperature effects are neglected. The plastic deformations and the influence of friction on the contact pressure distribution are considered to be negligible.

Finite Element formulation of the problem Two elastic solids in contact with friction are considered and are

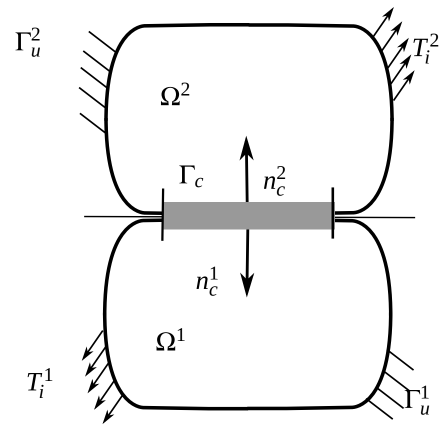


Figure 1: Description of the problem of contact between two solids

shown on scheme Fig. 1 . Both solids occupy two distinct domains Ω^l with smooth boundaries $\partial\Omega^l$.The boundary $\partial\Omega^l$ is divided into three disjoint parts Γ_t^l, Γ_u^l and Γ_c. Traction forces are imposed on the first part of the boundary, displacements on the second and contact conditions on the third part of the surface. The wear phenomena occur on this part of the body. Wear is characterised by lost of matter in this zone. Strömberg [19] has proposed a method to effectively simulate fretting-wear. Proposed approach in this work is based on his method. More details and profound explanations can be founded in previous papers [5, 6]. When two structures $(l = 1,2)$ are in contact the equation of motion is in FE approach:

$$\mathbf{M}\ddot{\boldsymbol{U}}^{(l)} + \mathbf{C}\dot{\boldsymbol{U}}^{(l)} + \mathbf{K}\boldsymbol{U}^{(l)} + \boldsymbol{F_c}^{(l)}(\boldsymbol{U}^{(l)}, \dot{\boldsymbol{U}}^{(l)}, \boldsymbol{W}^{(l)}) = \boldsymbol{F}_{\boldsymbol{ex}}^{(\boldsymbol{l})}, \quad (1)$$

$\boldsymbol{F_{ex}}$ is the vector of external forces (periodic excitation at frequency ω) and $\boldsymbol{F_c}$ represents the non-linear contact forces due to friction and unilateral condition. They also depend on the wear and on the materials in contact. Wear is calculated at the nodes (marked M) of the interface:

$$\dot{W}^M = \frac{k_w}{\alpha^M} \left|P_N^M\right| \left\|\dot{\boldsymbol{U}}_T^M\right\|, \quad (2)$$

α^M are the weighting factors for each node M, P_N^M is the nodal normal pressure and $\dot{\boldsymbol{U}}_T^M$ is the tangential velocity of the node M. The weighting factors depend on used quadrature rule to calculate integrale on the contact area. The constraints introduced by Signorini's contact conditions and Coulomb's friction are, in the FEM formalism:

$$^t(\boldsymbol{U}_N - \boldsymbol{W} - \boldsymbol{G})\left(\boldsymbol{P}'_N - \boldsymbol{P}_N\right) \leq 0 \quad \forall \boldsymbol{P}'_N \in \mathcal{K}_N^h, \quad (3)$$

$$^t\dot{\boldsymbol{U}}_T\left(\boldsymbol{P}'_T - \boldsymbol{P}_T\right) \leq 0 \quad \forall \boldsymbol{P}'_T \in \mathcal{F}^h(\boldsymbol{P}_N), \quad (4)$$

where $\mathcal{K}_N^h = \left\{P_N^M : P_N^M \geq 0\right\}$ is the approximation of $\mathcal{K}_N$ and $\mathcal{F}^h(\boldsymbol{P}_N) = \left\{\boldsymbol{P}_T^M : \left\|\boldsymbol{P}_T^M\right\| - \mu|\boldsymbol{P}_N^M| \leq 0\right\}$ is the approximation of $\mathcal{F}(p_N)$ with $\mathcal{W}^M = \boldsymbol{P}_N^M$. Definition of $\mathcal{K}_N$ and $\mathcal{F}(p_N)$ can be founded in [6].

Multiple time scale approach Wear is a very slow phenomenon and the depth of wear is very small compared with the dimension of the structures in contact. We can neglect the modifications of the mass and stiffness matrices due to wear. In this paper, we present a new paradigm for multi-scale modelling of fretting-wear under dynamical loading based on multiple temporal scale asymptotic analysis. The proposed methodology hinges on the hypothesis of local periodicity in the time domain (dynamic phenomena). This approach can be equated with homogenisation theory of space periodic structure. In the temporal homogenization proposed here, both the applied loads and the response fields are assumed to depend on a slow time co-ordinate η (slow modification of contact interfaces due to fretting-wear), as well as on the fast time co-ordinate, τ (due to locally periodic dynamical loading). Wear appears here as a permanent normal displacement of the interface at the scale of one period in the response of the system. On the fast time scale response of the structure is still periodic and consequently on one period Fourier series can describe displacements and contact forces. On a long duration Fourier coefficients will evolve as functions of time variable η linked with the fretting-wear phenomenon.

In this introduction necessary assumptions and proposed theoretical approaches were presented. In the next section DLFT-wear procedure is explained.

DLFT-WEAR PROCEDURE

Frequency domain formulation Using homogenization theory in time and average method Fourier series of $\boldsymbol{U}$ can written as follow:

$$\boldsymbol{U}(\tau,\eta) = \tilde{\boldsymbol{U}}_0(\eta) + \sum_{n=1}^{Nh} \tilde{\boldsymbol{U}}_{n,c}(\eta)\cos(n\tau) + \tilde{\boldsymbol{U}}_{n,s}(\eta)\sin(n\tau) \quad (5)$$

where τ is the fast time scale linked with the periodic response and η the slow time scale linked with fretting-wear phenomena. Wear depth doesn't depend on τ because it doesn't evolve on a single period and so wear depends only on η.

$$W^M = W^M(\tau,\eta) = W^M(\eta), \quad (6)$$

The evolution of wear during one period is for each node in contact:

$$\Delta W^M(\eta) = \frac{K_w}{\alpha^M} \int_{Period} |P^{N,M}(\eta,\tau)|.\|\dot{\boldsymbol{U}}_T^M(\eta,\tau)\| d\tau \quad (7)$$

The equation of motion in the frequency domain is:

$$\mathbf{Z_r}\tilde{\boldsymbol{U}}_{\boldsymbol{r}}(\eta) + \tilde{\lambda}(\eta) = \tilde{\boldsymbol{F}}_{\boldsymbol{r}} \quad (8)$$

The term concerning wear will appear in the augmented Lagrangian used to solve this equation with the constrains of contact with friction. This procedure is described in the next part.

Dynamical Lagrangian formulation of contact In the next equations, the term η is omitted in the expression of Fourier coefficients. In the frequency domain, the Lagrange multiplier λ is formulated as a penalization of the equations of motion on the tangential and normal directions:

$$\tilde{\lambda}^T = \tilde{\boldsymbol{F}}_{\boldsymbol{r}}^T - \mathbf{Z_r}\tilde{\boldsymbol{U}}_{\boldsymbol{r}} + \varepsilon_T\left(\tilde{\boldsymbol{U}}_{\boldsymbol{r}}^T - \tilde{\boldsymbol{X}}_{\boldsymbol{r}}^T\right), \quad (9a)$$

$$\tilde{\lambda}^N = \tilde{\boldsymbol{F}}_{\boldsymbol{r}}^N - \mathbf{Z_r}\tilde{\boldsymbol{U}}_{\boldsymbol{r}} + \varepsilon_N\left(\tilde{\boldsymbol{U}}_{\boldsymbol{r}}^N - \boldsymbol{W} - \tilde{\boldsymbol{X}}_{\boldsymbol{r}}^N\right), \quad (9b)$$

ε_T and ε_N are penalty coefficients, $\tilde{\boldsymbol{X}}_{\boldsymbol{r}}$ is a new vector of relative displacements, which is computed in the time domain. It will be seen that it corresponds to the relative displacements which fulfil the contact and friction law in the time domain.

Solving (8) requires to know the expression of the contact forces in the frequency domain. Unfortunately it's not possible to calculate a priori the multi-harmonic vector $\tilde{\lambda}$ of Lagrange multipliers in the Galerkin procedure, indeed the function is not a priori defined. Transition steps are not a priori known. Displacements and velocities are calculated in the frequency domain and transformed into the time domain using an inverse DFT procedure (iDFT). In the time domain, contact forces may be evaluated. In DLFT method the calculation of contact forces in frequency domain requires prediction/correction procedure. A time-marching procedure in the time domain is also required.

Prediction/correction in the time domain Now let us present the prediction/correction procedure in the time domain. This procedure have already been explained in other papers [1, 2, 6]. In order to make more comprehensible this study prediction/correction procedure is presented again. The contact forces are calculated in time domain where the transition criteria between the three possible states are easily formulated. Equation (9) is reformulated as:

$$\tilde{\lambda} = \begin{pmatrix} \tilde{\lambda}^T \\ \tilde{\lambda}^N \end{pmatrix} = \underbrace{\tilde{\boldsymbol{F}}_{\boldsymbol{r}} - \mathbf{Z}_r\tilde{\boldsymbol{U}}_{\boldsymbol{r}} + \varepsilon\left(\tilde{\boldsymbol{U}}_{\boldsymbol{r}} - \boldsymbol{W}\right)}_{\tilde{\lambda}_u} - \underbrace{\varepsilon\tilde{\boldsymbol{X}}_{\boldsymbol{r}}}_{\tilde{\lambda}_x}, \quad (10)$$

The period is split into N time steps. In this time domain each multi-harmonic vector has a counterpart. $\tilde{\lambda}$,$\tilde{\lambda}_u$ and $\tilde{\lambda}_x$ have respectively $\{\lambda^n\}_{n=1..N}$, $\{\lambda_u^n\}_{n=1..N}$ and $\{\lambda_x^n\}_{n=1..N}$ as time domain equivalents. These vectors are obtained from frequency domain variables through an iDFT procedure. A prediction/correction is used to compute the contact forces. At each time increment it assumes that the contact node is in stick situation, thus the node doesn't move and $\lambda_x^{n,T} = \lambda_x^{n-1,T}$ and $\lambda_x^{n,N} = 0$. The predicted contact forces are:

$$\lambda_{pre}^{n,T} = \lambda_u^{n,T} - \lambda_x^{n-1,T}, \quad \lambda_{pre}^{n,N} = \lambda_u^{n,N}, \tag{11}$$

The corrected contact forces will be:

$$\lambda^n = \lambda_u^n - \lambda_x^n \tag{12}$$

λ_x^n will we be calculated to satisfy the contact and friction laws.

1. Separation: $\lambda_{pre}^{n,N} \geq 0$, the contact is lost and the forces should be zero

$$\lambda_x^n = \lambda_u^n, \tag{13}$$

2. Stick: $\lambda_{pre}^{n,N} < 0$ and $\left\|\lambda_{pre}^{n,T}\right\| < \mu \left|\lambda_{pre}^{n,N}\right|$
In this case, the prediction verifies the contact conditions:

$$\lambda_x^{n,N} = 0, \quad \lambda_x^{n,T} = \lambda_x^{n-1,T}, \tag{14}$$

3. Slip: $\lambda_{pre}^{n,N} < 0$ and $\left\|\lambda_{pre}^{n,T}\right\| \geq \mu \left|\lambda_{pre}^{n,N}\right|$
Again,there is no normal relative displacement. The correction is made assuming that the tangential contact force has the same direction as the tangential predicted force. The definition of relative velocity and the respect of the Coulomb's law give:

$$\lambda_x^{n,N} = 0, \quad \lambda_x^{n,T} = \lambda_x^{n-1,T} + \lambda_{pre}^{n,T} \left(1 - \mu \frac{|\lambda_{pre}^{n,N}|}{\|\lambda_{pre}^{n,T}\|}\right), \tag{15}$$

The final step consists of transforming back the time domain updated Lagrangian in the frequency domain using DFT algorithm.

Jacobian matrix calculation In a Newton's like algorithm Jacobian matrix is necessary. It's possible to numerically evaluate it by finite difference but it's time consumming. We have proposed in precedent study [6] analytical evaluation of Jacobian matrix in the case of Dynamic Lagrangian formulation. In this paper method is summarized. More details can be founded in [6].

Jacobian is define as follow :

$$J = \frac{\partial f(\tilde{\boldsymbol{U}}_r)}{\partial \tilde{\boldsymbol{U}}_r} \tag{16}$$

f consist of linear part and nonlinear part.

$$J = -\mathbf{Z}_r - \frac{\partial \tilde{\lambda}}{\partial \tilde{\boldsymbol{U}}_r} = -\mathbf{Z}_r - \left(\frac{\partial \tilde{\lambda}_u}{\partial \tilde{\boldsymbol{U}}_r} - \frac{\partial \tilde{\lambda}_x}{\partial \tilde{\boldsymbol{U}}_r}\right) \tag{17}$$

Using equations(13-15) gradient of λ_x in $\tilde{\boldsymbol{U}}_r$ can be found.

The expression $\frac{\partial \lambda_x}{\partial W}$ can also be computed and it is used to express gradient of f in W,denoted J_w:

$$\mathbf{J_w} = -\varepsilon \mathbf{I}_{nodes} + \frac{\partial \tilde{\lambda}_x}{\partial W} \tag{18}$$

In [6] multiharmonic vector displacement for updated worn geometry was predicted by $U_r^{l+1} = U_r^l + \delta U_r$ with:

$$\delta U_r = -J^{-1} J_w \delta W \tag{19}$$

To correct prediction DLFT algorithm was used. U_r^{l+1} was initial vector of Hybrid Powell's algorihm for new system with updated worn geometry. In next part we will see that gradient expression are very useful in implicit scheme on slow time scale.

Explicit and implicit scheme on slow time scale
On the slow time scale integration scheme is used to calculate evolution of wear with homogenization of wear evolution during one cycle. Two types of scheme can be used explicit and implicit. In this part will be presented properties of both approaches. The study is limited to Euler Scheme: explicit Euler scheme and implicit backward scheme. In dynamical contact problem it was shown [20] that higher order scheme are less stable so Euler schemes are good candidate to solve our problem.

In Euler explicit scheme non-linear algorithm based on HBM and AFT procedure allows to calculate wear rate δW on one fretting cycle. Using this wear rate is possible to update geometry.

$$W_{k+1}^M = W_k^M + hg(U_k^M, W_k^M) \tag{20}$$

where,

$$g(U_k^M, W_k^M) = k_w \int_0^T P_N^M(\tau, W_k^M).\|U_T^M(\tau, W_k^M)\| d\tau \tag{21}$$

where T is the period of force excitation. System responds at the same frequency as the the frequency of excitation.

On the slow time scale time integration step h is arbitrarily chosen. Another approach is to arbitrarily define maximum authorized wear depth on the whole contact area during these cycles. Wear depth is incremented instead of time. This wear depth is noted δW^*. A relation (22) links δW^* and h.

$$h = \min_M \left(\frac{\delta W^*}{\boldsymbol{g}(U_k^M, W_k^M)} \right) \tag{22}$$

Using h worn geometry can be updated to next slow time moment η^{l+1}.

$$\boldsymbol{W}(\eta^{l+1}) = \boldsymbol{W}(\eta^l) + h\boldsymbol{g}(U_k^M, W_k^M) \tag{23}$$

Implicit scheme is another approach. One of the primary advantages of this implicit wear model is that it is not necessary to compute as many cycles. Backward Euler discretization of wear evolution is:

$$W_{k+1}^M = W_k^M + hg(U_{k+1}^M, W_{k+1}^M) \tag{24}$$

where,

$$g(U_{k+1}^M, W_{k+1}^M) = k_w \int_0^T P_N^M(\tau, W_{k+1}^M).\|U_T^M(\tau, W_{k+1}^M)\| d\tau \tag{25}$$

With this sheme non-linear system is solved at each step (slow time scale). Unknown vector is $X = \left\{ \begin{array}{c} \boldsymbol{U}_r^{k+1} \\ \boldsymbol{W}^{k+1} \end{array} \right\}$. System to solve becomes:

$$\mathbf{Z_r}\tilde{\boldsymbol{U}}_{\boldsymbol{r}}^{k+1} + \tilde{\lambda}^{k+1} = \tilde{\boldsymbol{F}}_{\boldsymbol{r}} \tag{26a}$$
$$W^{k+1} - g(\tilde{\lambda}^{k+1}, \tilde{\boldsymbol{U}}_{\boldsymbol{r}}^{k+1}, W^{k+1}) = W^k \tag{26b}$$

To solve (26) Jacobian matrix J is necessary:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{U}_r} & \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{W}} \\ \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{U}_r} & \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{W}} \end{bmatrix}$$

Explicit and implicit scheme have respectively their advantages and disadvantages. For explicit scheme advantages are:

1. the number of equations doesn't change,
2. the Choice of wear law is not important, because jacobian needn't to be computed.
3. there is no change in non-linear solver.

ans disadvantages are:

1. the scheme can be unstable,
2. results are very dependent of h

For implicit backward Euler scheme non-linear systems and wear equation are solved together and the advantages are:

1. the choice of h is easier,
2. the scheme is more stable,
3. results are more reliable.

and the disadvantages are :

1. the method is less flexible because gradient of wear rate function must be calculated,
2. there are more unknowns in non-linear system at each slow time scale step.

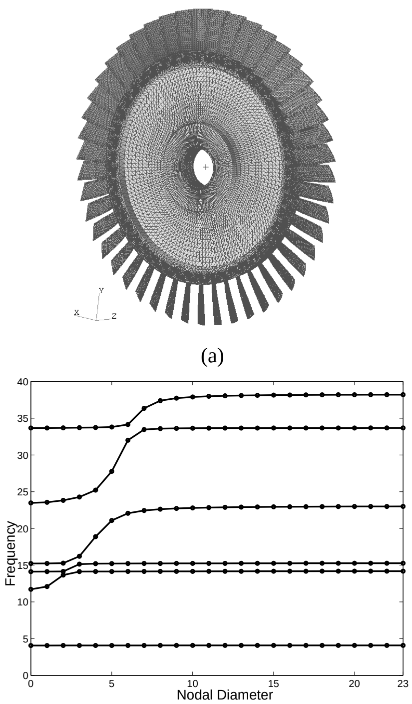
The accuracy of results with HBM/AFT methods depends of number of harmonics. The method to estimate order of harmonics proposed in [6] can be used with explicit and implicit scheme.

NUMERICAL EXAMPLE

The method developed for the coupled calculation of both the wear kinetics and the vibrational response has been applied to a compressor's bladed disk. This structure have already been studied in [6]. Both schemes explicit and implicit have been implemented.

The fretting-wear zones are the interfaces between disk and blade in the dovetail attachment. The disk (Fig. 2a) is made of 47 sectors Fig. 3a. Its natural frequencies (normalized) are shown in Fig. 2b. In Fig. 3b the top grid shows numbering for contact area of the same side as the intrados of the blade and the bottom grid shows the numbering for the extrados side of the blade. Left side of the grid is leading edge of the blade and right side is trailing edge of the blade. The friction coefficient is $\mu = 0.5$, corresponding to a contact between parts made of titanium without coating. The wear rate Kw is chosen equal to $1.110^{-11} Pa^{-1}$. The first mode for 0 diameter (a flexural mode) is excited through an harmonic force applied perpendicularly at the top of the blade. Its amplitude is $0.5N$. The time step on slow time scale is $h = 1\,10^3$ cycles.

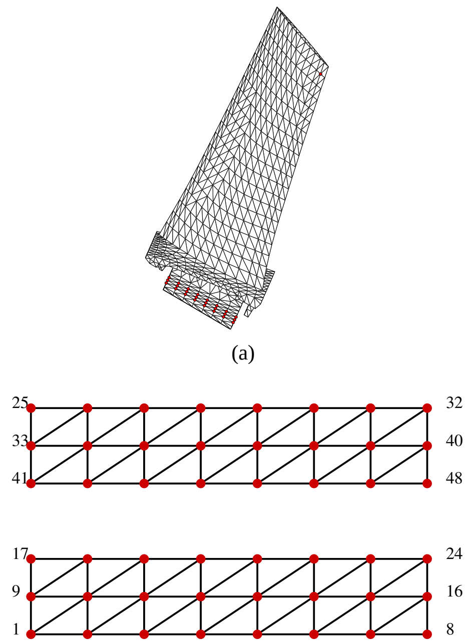
Comparison between both schemes Fig. 4 and Fig. 5 were obtained at the resonance with both scheme. Conclusion is evident for the same h explicit and implicit scheme give different results. Explicit scheme gives more important worn volume. Some nodes which are really in stick situation can begin



(a)

(b)

Figure 2: Analyzed bladed disk: finite element geometry (a) and natural frequencies (b)



(a)

(b)

Figure 3: Calculated bladed sector: finite element geometry (a) and contact nodes (b) on the intrados (top) and on the extrados (bottom)

slip and wear. Implicit scheme permit to verify contact equation and wearing process at the same time. Obviously a error estimator can be proposed to adapt time step but this method can't give warranty of stability of scheme. Difficulty of fretting-wear problem under dynamical loading is caused by unilateral constrain and its verification during fretting cycle.

Wear evolution on frequency range All the results presented in this section have been obtained using multi-scale HBM with implicit scheme. Fig. 6 and Fig. 7 show evolution of worn volume and wear rate on frequency range and during fretting cycles. Roots wear near the resonance. Near the resonance of non linear system final worn volume is nearly the same. Worn volume function in frequency is not a continuous function. It is interesting to see on Fig. 7 that before the resonance evolution along fretting cycle is quite complex. Wearing process starts very slowly and accelerate during a short period.

The final obtained worn volume is shown on Fig. 8. It is shown that wear evolution can be separated by range of frequencies. It is due to phenomenon stick-slip. During some frequencies situation in the contact interface is quite the same and consequently wear evolution is nearly the same. To conclude this section FRF were drawn on Fig. 9. Fretting-wear modifies clearly dynamical response of structure. In this case resonance disappears but a new resonance appears in lower frequencies. FRF for worn roots is not very smooth. It is caused by stick-slip phenomenon in interface and contact loss. Non-linear aspect increases with fretting-wear.

Wear evolution at the resonance Now we focus on evolution at the resonance. On Fig. 10 and Fig. 11 a comparison between mean pressure during first cycle and at 10^5 is made. Pressure profile is modified by fretting-wear. In the wearing zone pressures relax to tend to zero. In this example it's not very clear because model has few contact nodes. Pressure increases at the boundary between slip zone and stick zone. In these points cracking can start. Waves in pressure profile are caused by spline interpolation used to post-treat pressure obtained on the model with 8 by 3 nodes on each contact interfaces. Worn profile at 10^5 cycles are drawn on Fig. 12. These profiles are very asymmetrical. Last point addressed in this part is study of modification of fretting loop at node 41. It allows to understand modification of contact mechanics during fretting-wear process. In following graphics all drawn displacements are relative displacements of node 41. Fig. 13 shows fretting loop of node 41 in x-direction and y-direction; Problem is 3D and (xy) are the plans tangential to root surfaces. Fretting loop is clearly very complex and is not

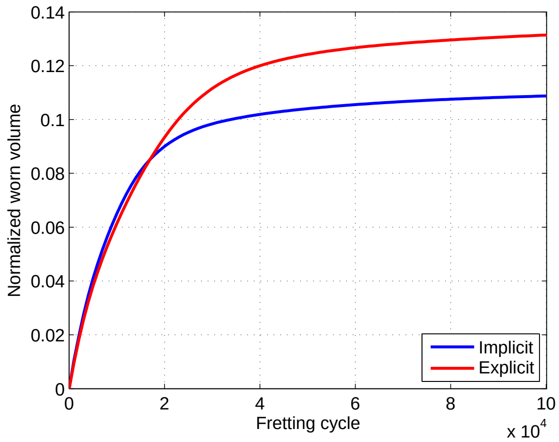


Figure 4: Comparison of wear evolution between explicit and implicit scheme

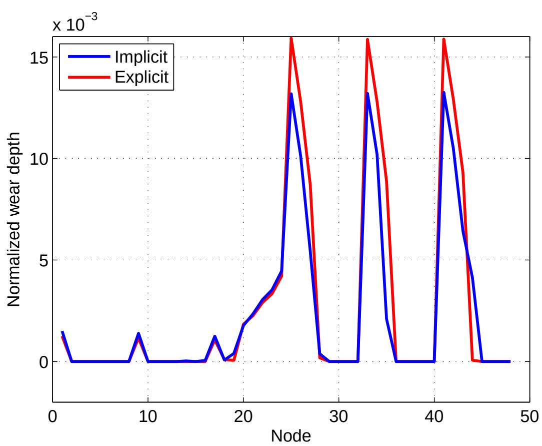


Figure 5: Comparison of wear depth at 10^5 cycles between explicit and implicit scheme

similar to fretting loop using during experiments [4]. It is caused by coupled aspect of variable normal loading and stick-slip situation during cycle. After 10^5 cycles fretting loop is closed because normal pressure has severely decreases. It's a little bit surprising but the amplitude of displacements at node 41 has decreased. It must be due to contact loss during fretting cycle. Fig. 14 shows that after fretting process contact opens during cycle. In this study we have used Archard's model of wear which does not take into account contact loss. A new model taking into account micro impact is necessary to model fretting wear during vibration of bladed-disk. Before carry out experiment it would be better to create a mathematical model. Loading is too complex to understand wearing process during experiment without math-

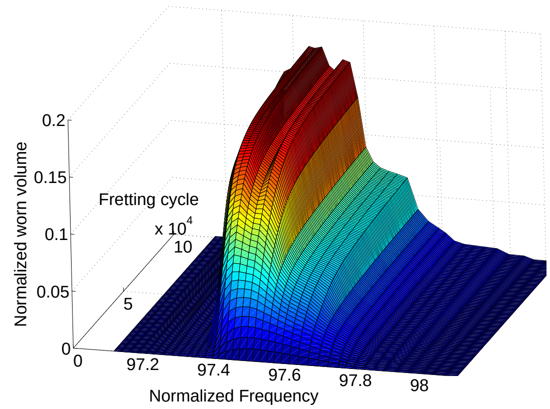


Figure 6: Worn volume evolution on frequency range

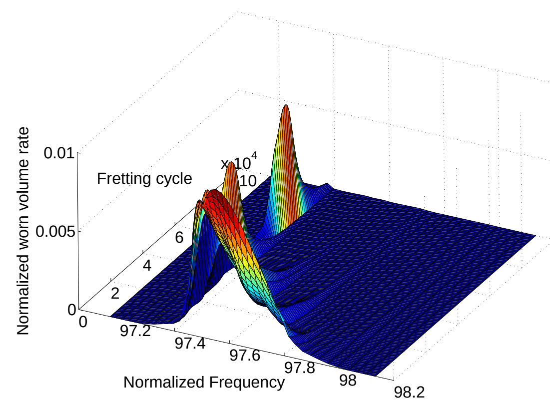


Figure 7: Worn volume rate evolution on frequency range

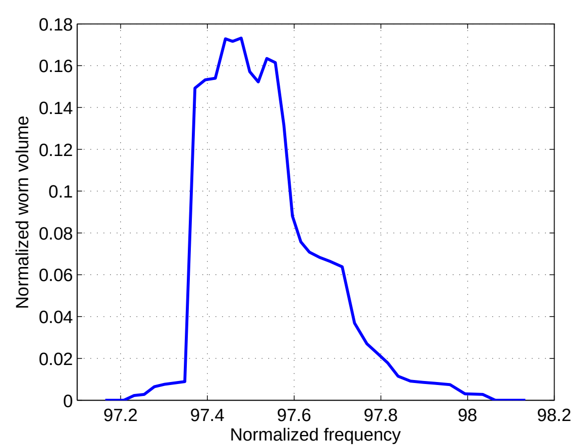


Figure 8: Worn volume at 10^5 cycles on frequency range

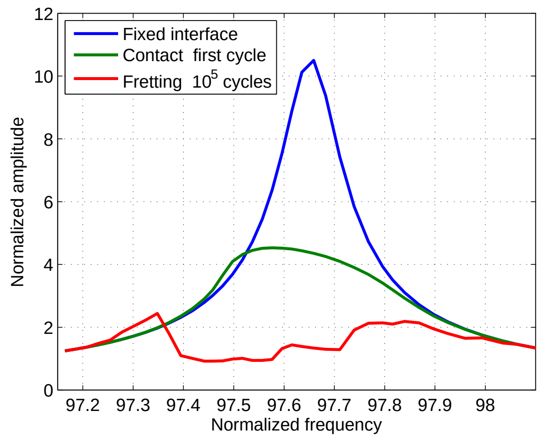
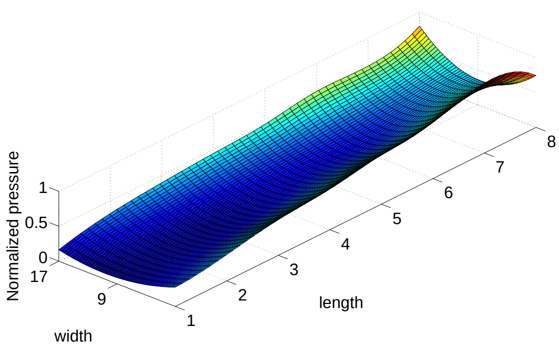
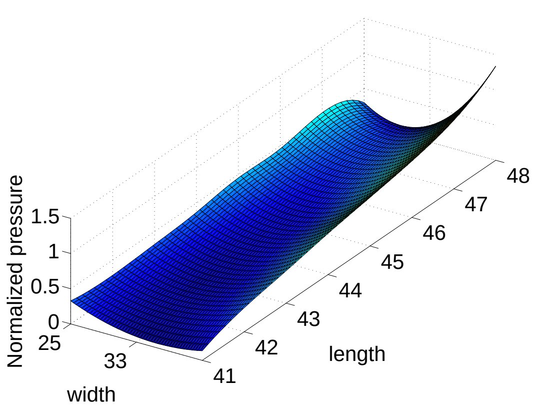


Figure 9: FRF during first cycle and at 10^5 cycles

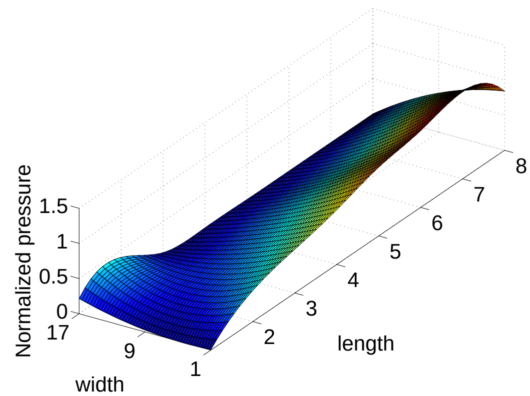


(a)

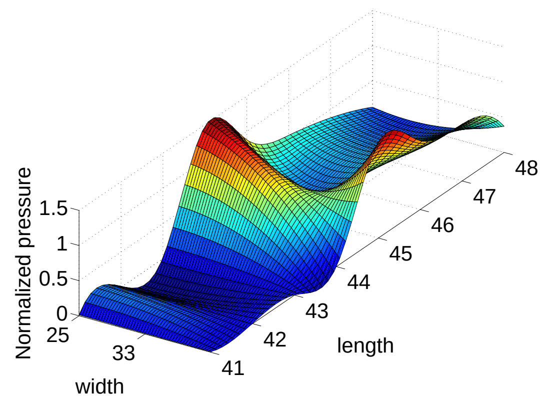


(b)

Figure 10: Mean pressure distribution during first cycle: on the extrados (a) and on the intrados (b)



(a)



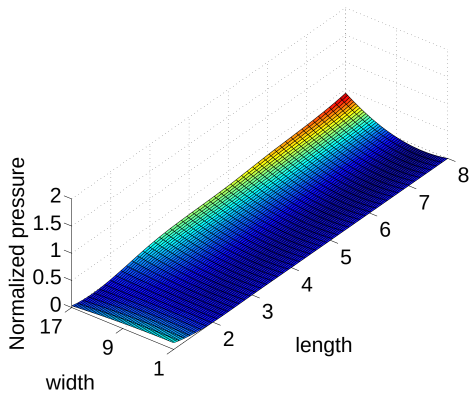
(b)

Figure 11: Mean pressure distribution during 10^5 th cycle: on the extrados (a) and on the intrados (b)

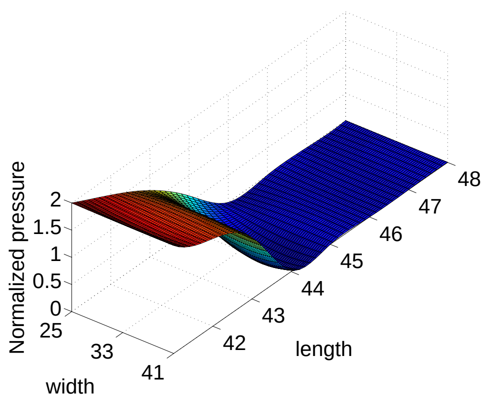
ematical model. Static relative position of node 41 is modified with fretting-wear. Fig. 15 also shows that the cinematic loop of node 41 is closing during fretting-wear process. As was said previously dynamic of contact zone during fretting-wear under dynamic loading is more complex than experiment carried out in laboratory on contact punch/surface which is a 1D system.

CONCLUSIONS AND PROSPECTS

A method has been proposed to calculate simultaneously wear and vibrations of bladed disks. The proposed methodology is based on the hypothesis of local periodicity in the time domain (dynamic phenomena). On slow time scale two integration schemes have been proposed. Numerical results have shown that implicit scheme is more reliable. Jump cycle strategy is not necessary. Choice of integration step depends on convergence rate of non linear solver. The new approach has been applied to



(a)



(b)

Figure 12: Worn geometry profile: (a) in extrados (b) in intrados

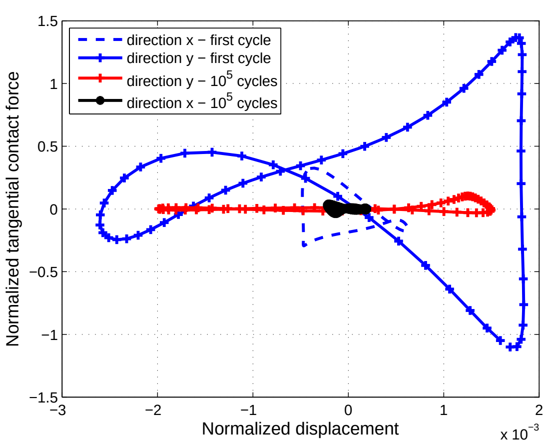


Figure 13: Fretting cycle of node 41

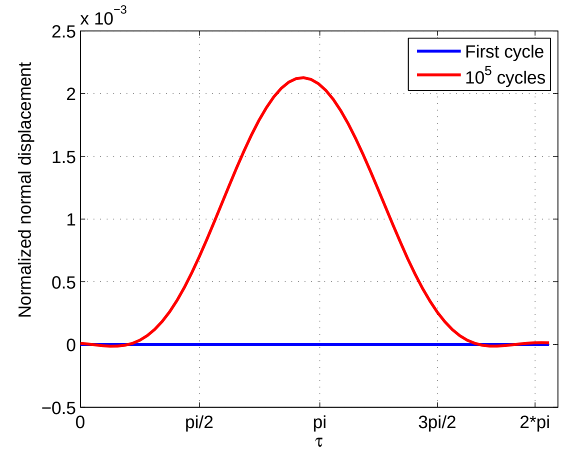


Figure 14: Contact loss during fretting-wear for node 41

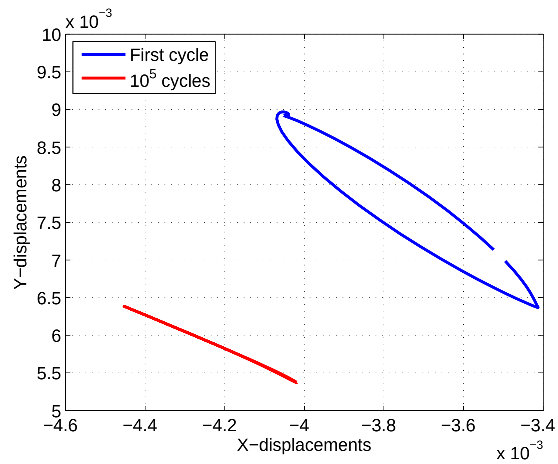


Figure 15: Movement of contact node 41

the finite element model of an assembled bladed disk with dovetail joints. This example has revealed a important modification of contact mechanics during fretting-wear. Wear model used in this simulation don't seem adapted to give quantitative assessment of fretting wear. Only a qualitative understanding is obtained by the proposed method.

Future works will focus on development of a new method to find directly worn profile without integration on slow time scale. The final worn profile depends on loading history, so asymptotic method are not adapted. This method could be based on optimal control theory. The key-point of this method would be to write functional to minimize to find the same worn profile obtained by time integration.

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