Transducers and Cantor bases
Notebook companion permits to compute the finite transducers discussed in the paper
E. Charlier, P. Popoli, M. Rigo (2025)

Part of the abstract: We develop a new approach where a single transducer associated with a fixed real number r, computes the B-expansion of r but for an infinite family of Cantor bases B given as input. This point of view contrasts with traditional computational models for which the numeration system is fixed.

Main functions to be executed first

```
(* auxiliary function called to build graphs *)
```

```
Given a real number r and a list l ={β1,β2,...}the function outputs a list of digits a1,
```

```
In[1]:= f[{r_, l_}] := Module[
  {digit = 0,
   (* consider the sequence of partial products in l *)
   seq = Table[Apply[Times, Take[l, i]], {i, 1, Length[l]}],
   remainder = r, output = {}},
  For[i = 1, i ≤ Length[l],
   digit = 0;
   (* find the maximal digit to remove *)
   While[FullSimplify[remainder - digit / seq[[i]]] > 0,
    digit++]; digit--;
   remainder = FullSimplify[remainder - digit / seq[[i]]];
   (* we add a new digit to the output sequence *)
   AppendTo[output, digit];
   i++;
  ];
  (* when done, a normalization occurs in the β-transformation,
  Last[seq] contains the product of those elements *)
  {output, FullSimplify[Last[seq] * remainder]}
 ]
```

Here is an example :

```
In[•]:= f[{Pi / 4, {Sqrt[2], 2, 2, 2, 2, 2}}]
Out[•]= {{1, 0, 0, 0, 1, 1}, -35 + 8 √2 π}
```

This means that Pi/4 can be written as 1/Sqrt[2] + 1/(Sqrt[2] 2^4) + 1/(Sqrt[2] 2^5) and here is the normalized remainder :

In[∘]:=
```
(Pi / 4 - 1 / Sqrt[2] - 1 / (Sqrt[2] 2 ^ 4) - 1 / (Sqrt[2] 2 ^ 5))
  Apply[Times, {Sqrt[2], 2, 2, 2, 2, 2}] // Simplify
```

Out[∘]= $-35 + 8\sqrt{2}\,\pi$

Algorithm 2 Algorithm producing $\mathcal{T}^*_{E,r}$

```
procedure QUASI-GREEDY TRANSDUCER(E,r)
    Q ← ∅                                        ▷ Set of states of the transducer
    L ← {r}                                      ▷ List of states to be processed
    while L ≠ ∅ do
        Let q be the first element in L
        Remove q from L and add it to Q
        for β in E do
            if T*_β(q) ∉ Q ∪ L then add q to L   ▷ Detection of a new state
            end if
            add a transition (q, β | ⌈βq − 1⌉, T*_β(q))   ▷ Defining transitions
        end for
    end while
end procedure
```

In[5]:=
```
(* the arguments are the starting vertex and wlist is the
 alphabet containing the real bases, the last argument can be omitted,
it permits to track the progress of computations if set to True *)

buildTransducer[start_, wlist_, verbose_ : False] :=
 (* q contains the lists of states of the transducers *)
 Module[{q = {start}, toprocess = {start}, g = {}, detect, new},
  (* toprocess is the list of states to be processed ;
  g will contain the list of transitions *)
  While[Length[toprocess] > 0,
   (* for the first state to be processed,
   we apply all the bases from wlist and record the "detected states" *)
   detect = Table[
     FullSimplify[Last[f[{First[toprocess], {x}}]]], {x, wlist}];
   (* we update the list g of transitions accordingly *)
   AppendTo[g,
    Table[
     {First[toprocess] ⟷ FullSimplify[Last[f[{First[toprocess], {x}}]]],
      (* our choice of labeling transitions *)
      Row[{Subscript["e", Position[wlist, x][[1, 1]]],
        "|", StringPart[ToString[First[f[{First[toprocess], {x}}]]], 2]}]},
     {x, wlist}]];
   (* newly detected states are those in detect but not
    obtained in previous stages so, already stored in q or toprocess *)
   new = Complement[detect, q, toprocess];
   q = Union[q, detect];
   (* this print option permits to follow computations *)
   If[verbose, Print[Length[q]]];
   (* the fist element to be processed may be
    removed from the list and the newly detected states added *)
   toprocess = Drop[toprocess, 1];
   toprocess = Union[toprocess, new];
  ];
  (* output the transitions of the graph *)
  DeleteDuplicates[Flatten[g, 1]]
 ]
```

An example

The result is a list whose elements are pairs of the form {a⟷b, e_i|d} where a, b are vertices (real numbers), the e_i's represent the element of the set E of bases and finally d is the output digit

In[81]:=
```
t = buildTransducer[1/2, {GoldenRatio, GoldenRatio^3}]
```

Out[81]=

$$\left\{\left\{\frac{1}{2} \mapsto \frac{\text{GoldenRatio}}{2}, e_1|0\right\}, \left\{\frac{1}{2} \mapsto \frac{1}{2}\left(-4+\text{GoldenRatio}^3\right), e_2|2\right\},\right.$$

$$\left\{\frac{\text{GoldenRatio}}{2} \mapsto \frac{1}{2}\left(-2+\text{GoldenRatio}^2\right), e_1|1\right\}, \left\{\frac{\text{GoldenRatio}}{2} \mapsto \frac{1}{2}\left(-6+\text{GoldenRatio}^4\right), e_2|3\right\},$$

$$\left\{\frac{1}{2}\left(-2+\text{GoldenRatio}^2\right) \mapsto \frac{1}{2}, e_1|0\right\}, \left\{\frac{1}{2}\left(-2+\text{GoldenRatio}^2\right) \mapsto \frac{1}{4}\left(-1+\sqrt{5}\right), e_2|1\right\},$$

$$\left\{\frac{1}{4}\left(-1+\sqrt{5}\right) \mapsto \frac{1}{2}, e_1|0\right\}, \left\{\frac{1}{4}\left(-1+\sqrt{5}\right) \mapsto \frac{1}{4}\left(-1+\sqrt{5}\right), e_2|1\right\},$$

$$\left\{\frac{1}{2}\left(-4+\text{GoldenRatio}^3\right) \mapsto \frac{1}{2}\,\text{GoldenRatio}\left(-4+\text{GoldenRatio}^3\right), e_1|0\right\},$$

$$\left\{\frac{1}{2}\left(-4+\text{GoldenRatio}^3\right) \mapsto \frac{1}{2}, e_2|0\right\}, \left\{\frac{1}{2}\,\text{GoldenRatio}\left(-4+\text{GoldenRatio}^3\right) \mapsto \frac{1}{4}\left(-1+\sqrt{5}\right), e_1|0\right\},$$

$$\left\{\frac{1}{2}\,\text{GoldenRatio}\left(-4+\text{GoldenRatio}^3\right) \mapsto \frac{1}{4}\left(1+\sqrt{5}\right), e_2|0\right\}, \left\{\frac{1}{4}\left(1+\sqrt{5}\right) \mapsto \frac{1}{4}\left(-1+\sqrt{5}\right), e_1|1\right\},$$

$$\left\{\frac{1}{4}\left(1+\sqrt{5}\right) \mapsto \frac{1}{4}\left(-5+3\sqrt{5}\right), e_2|3\right\}, \left\{\frac{1}{4}\left(-5+3\sqrt{5}\right) \mapsto \frac{1}{4}\left(5-\sqrt{5}\right), e_1|0\right\},$$

$$\left\{\frac{1}{4}\left(-5+3\sqrt{5}\right) \mapsto \frac{1}{4}\left(1+\sqrt{5}\right), e_2|1\right\}, \left\{\frac{1}{4}\left(5-\sqrt{5}\right) \mapsto \frac{1}{2}\left(-2+\sqrt{5}\right), e_1|1\right\},$$

$$\left\{\frac{1}{4}\left(5-\sqrt{5}\right) \mapsto \frac{3}{4}\left(-1+\sqrt{5}\right), e_2|2\right\}, \left\{\frac{1}{2}\left(-2+\sqrt{5}\right) \mapsto \frac{1}{2}\left(-2+\sqrt{5}\right)\text{GoldenRatio}, e_1|0\right\},$$

$$\left\{\frac{1}{2}\left(-2+\sqrt{5}\right) \mapsto \frac{1}{2}, e_2|0\right\}, \left\{\frac{3}{4}\left(-1+\sqrt{5}\right) \mapsto \frac{1}{2}, e_1|1\right\}, \left\{\frac{3}{4}\left(-1+\sqrt{5}\right) \mapsto \frac{3}{4}\left(-1+\sqrt{5}\right), e_2|3\right\},$$

$$\left\{\frac{1}{2}\left(-2+\sqrt{5}\right)\text{GoldenRatio} \mapsto \frac{1}{4}\left(-1+\sqrt{5}\right), e_1|0\right\}, \left\{\frac{1}{2}\left(-2+\sqrt{5}\right)\text{GoldenRatio} \mapsto \frac{1}{4}\left(1+\sqrt{5}\right), e_2|0\right\},$$

$$\left\{\frac{1}{2}\left(-6+\text{GoldenRatio}^4\right) \mapsto \frac{1}{2}\,\text{GoldenRatio}\left(-6+\text{GoldenRatio}^4\right), e_1|0\right\},$$

$$\left\{\frac{1}{2}\left(-6+\text{GoldenRatio}^4\right) \mapsto \frac{1}{4}\left(1+\sqrt{5}\right), e_2|1\right\},$$

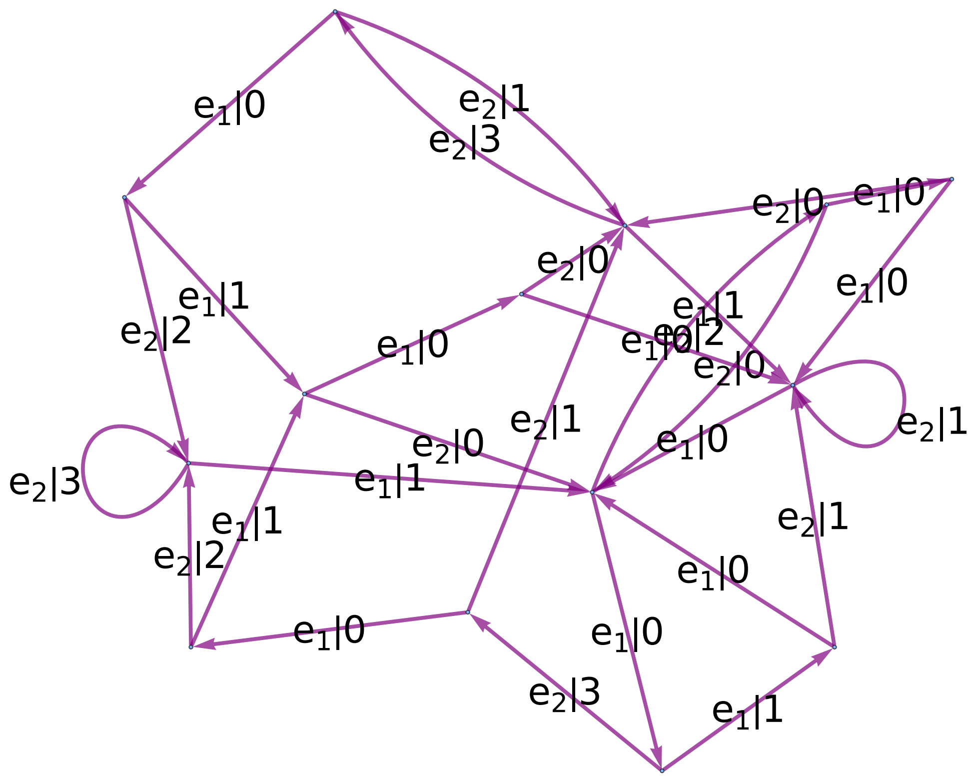
$$\left\{\frac{1}{2}\,\text{GoldenRatio}\left(-6+\text{GoldenRatio}^4\right) \mapsto \frac{1}{2}\left(-2+\sqrt{5}\right), e_1|1\right\},$$

$$\left.\left\{\frac{1}{2}\,\text{GoldenRatio}\left(-6+\text{GoldenRatio}^4\right) \mapsto \frac{3}{4}\left(-1+\sqrt{5}\right), e_2|2\right\}\right\}$$

It is then easy to represent the graph, one can use the VertexLabels -> “Name” to add the value of the vertices

In[86]:=
```
GraphPlot[t,
 VertexSize → 0.03, EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[86]=



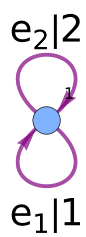
Examples from the paper

Figure 3

In[6]:=
```
t = buildTransducer[1, {2, 3}];
```

```
In[7]:= GraphPlot[t,
         VertexLabels → "Name", VertexSize → 0.03,
         EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

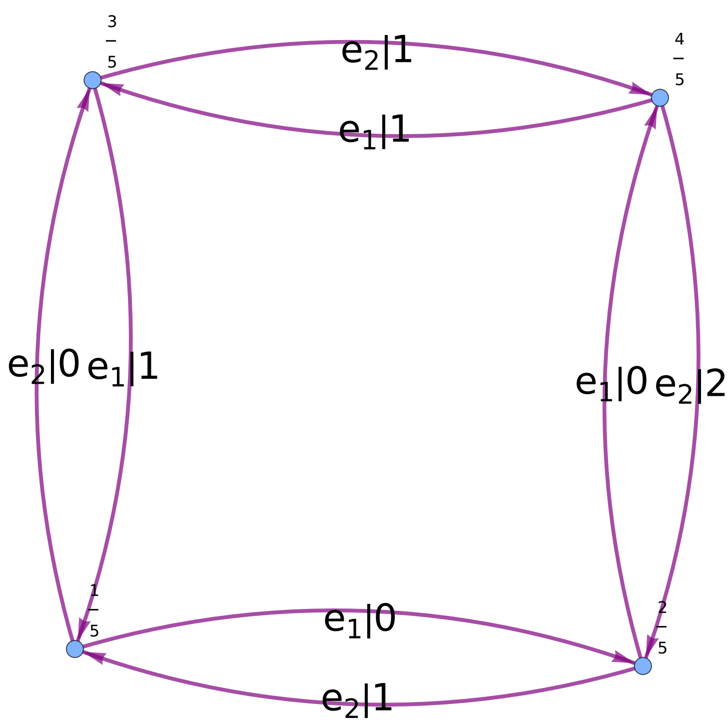
Out[7]=




```
In[•]:= t = buildGraph[1/5, {2, 3}, False];
```

```
In[•]:= GraphPlot[t,
         VertexLabels → "Name", VertexSize → 0.03,
         EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[•]=

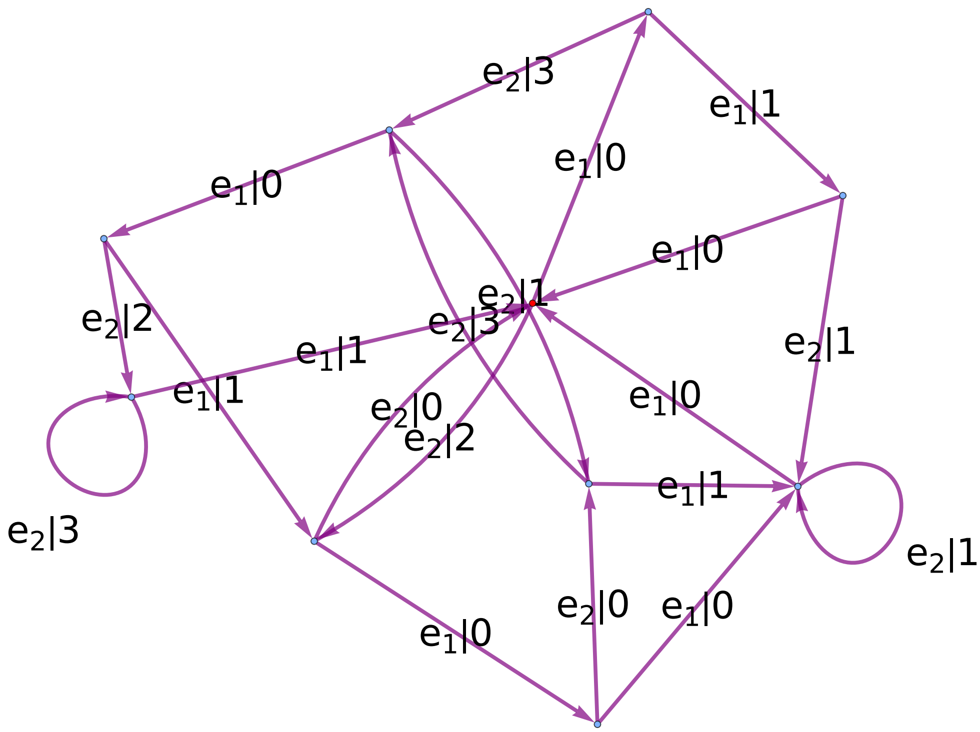


Example 3.15

```
In[8]:= t = buildTransducer[1/2, {GoldenRatio, 2 GoldenRatio + 1}, False];
```

In[9]:=
```
GraphPlot[t,
 VertexLabels → None, VertexStyle → {1/2 → Red}, VertexSize → 0.04,
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[9]=



Example 3.13 with 180 states

```
t = buildTransducer[932/3885, {2, 3}];
```

A few elements of the list :

In[14]:=
```
Take[t, 5]
```

Out[14]=

$$\left\{\left\{\frac{932}{3885} \leftrightarrow \frac{1864}{3885}, e_1|0\right\}, \left\{\frac{932}{3885} \leftrightarrow \frac{932}{1295}, e_2|0\right\}, \left\{\frac{1864}{3885} \leftrightarrow \frac{3728}{3885}, e_1|0\right\}, \left\{\frac{1864}{3885} \leftrightarrow \frac{569}{1295}, e_2|1\right\}, \left\{\frac{569}{1295} \leftrightarrow \frac{1138}{1295}, e_1|0\right\}\right\}$$

Here is the list of states of this transducer :

In[23]:=
```
VertexList[Graph[Map[#[[1]] &, t]]]
```

Out[23]=

$$\Big\{\frac{932}{3885}, \frac{1864}{3885}, \frac{932}{1295}, \frac{3728}{3885}, \frac{569}{1295}, \frac{1138}{1295}, \frac{412}{1295}, \frac{824}{1295}, \frac{1236}{1295}, \frac{353}{1295}, \frac{1177}{1295}, \frac{706}{1295},$$
$$\frac{1059}{1295}, \frac{117}{1295}, \frac{823}{1295}, \frac{234}{1295}, \frac{351}{1295}, \frac{468}{1295}, \frac{702}{1295}, \frac{1053}{1295}, \frac{936}{1295}, \frac{109}{1295}, \frac{218}{1295}, \frac{327}{1295},$$
$$\frac{436}{1295}, \frac{654}{1295}, \frac{981}{1295}, \frac{872}{1295}, \frac{13}{1295}, \frac{26}{1295}, \frac{39}{1295}, \frac{52}{1295}, \frac{78}{1295}, \frac{104}{1295}, \frac{156}{1295}, \frac{208}{1295},$$
$$\frac{312}{1295}, \frac{416}{1295}, \frac{624}{1295}, \frac{832}{1295}, \frac{1248}{1295}, \frac{577}{1295}, \frac{1154}{1295}, \frac{667}{1295}, \frac{811}{1295}, \frac{1174}{1295}, \frac{369}{1295}, \frac{1201}{1295},$$
$$\frac{738}{1295}, \frac{1107}{1295}, \frac{181}{1295}, \frac{919}{1295}, \frac{362}{1295}, \frac{543}{1295}, \frac{724}{1295}, \frac{1086}{1295}, \frac{334}{1295}, \frac{668}{1295}, \frac{1002}{1295}, \frac{41}{1295},$$
$$\frac{709}{1295}, \frac{82}{1295}, \frac{123}{1295}, \frac{164}{1295}, \frac{246}{1295}, \frac{328}{1295}, \frac{492}{1295}, \frac{656}{1295}, \frac{984}{1295}, \frac{17}{1295}, \frac{673}{1295}, \frac{34}{1295},$$
$$\frac{51}{1295}, \frac{68}{1295}, \frac{102}{1295}, \frac{153}{1295}, \frac{136}{1295}, \frac{204}{1295}, \frac{306}{1295}, \frac{272}{1295}, \frac{408}{1295}, \frac{459}{1295}, \frac{612}{1295}, \frac{544}{1295},$$
$$\frac{816}{1295}, \frac{918}{1295}, \frac{1224}{1295}, \frac{1088}{1295}, \frac{337}{1295}, \frac{674}{1295}, \frac{1011}{1295}, \frac{541}{1295}, \frac{1082}{1295}, \frac{53}{1295}, \frac{727}{1295}, \frac{106}{1295},$$
$$\frac{159}{1295}, \frac{212}{1295}, \frac{318}{1295}, \frac{477}{1295}, \frac{424}{1295}, \frac{636}{1295}, \frac{954}{1295}, \frac{848}{1295}, \frac{1272}{1295}, \frac{613}{1295}, \frac{1226}{1295}, \frac{877}{1295},$$
$$\frac{886}{1295}, \frac{1153}{1295}, \frac{401}{1295}, \frac{1249}{1295}, \frac{802}{1295}, \frac{1203}{1295}, \frac{309}{1295}, \frac{1111}{1295}, \frac{618}{1295}, \frac{927}{1295}, \frac{559}{1295}, \frac{1118}{1295},$$
$$\frac{382}{1295}, \frac{764}{1295}, \frac{1146}{1295}, \frac{233}{1295}, \frac{997}{1295}, \frac{466}{1295}, \frac{699}{1295}, \frac{103}{1295}, \frac{206}{1295}, \frac{449}{1295}, \frac{898}{1295}, \frac{501}{1295},$$
$$\frac{167}{1295}, \frac{191}{1295}, \frac{573}{1295}, \frac{443}{1295}, \frac{587}{1295}, \frac{869}{1295}, \frac{881}{1295}, \frac{467}{1295}, \frac{934}{1295}, \frac{731}{1295}, \frac{743}{1295}, \frac{941}{1295},$$
$$\frac{1013}{1295}, \frac{1019}{1295}, \frac{1157}{1295}, \frac{3571}{3885}, \frac{3257}{3885}, \frac{2629}{3885}, \frac{1373}{3885}, \frac{2746}{3885}, \frac{1607}{3885}, \frac{3214}{3885}, \frac{2543}{3885}, \frac{1201}{3885},$$
$$\frac{2402}{3885}, \frac{919}{3885}, \frac{1838}{3885}, \frac{3676}{3885}, \frac{3467}{3885}, \frac{3049}{3885}, \frac{2213}{3885}, \frac{541}{3885}, \frac{1082}{3885}, \frac{2164}{3885}, \frac{443}{3885}, \frac{886}{3885},$$
$$\frac{1772}{3885}, \frac{3544}{3885}, \frac{3203}{3885}, \frac{2521}{3885}, \frac{1157}{3885}, \frac{2314}{3885}, \frac{743}{3885}, \frac{1486}{3885}, \frac{2972}{3885}, \frac{2059}{3885}, \frac{233}{3885}, \frac{466}{3885}\Big\}$$

Let us check that we have indeed 180 states :

In[24]:=
```
Length[%]
```

Out[24]=

180

Building by hand the subgraph obtained from 932/3885 when only reading e1 e2 or e2 e1

In[28]:=
```
temp = Select[t, MatchQ[#[[1]], 932/3885 ↔ _] &]
```

Out[28]=

$$\left\{\left\{\frac{932}{3885} \leftrightarrow \frac{1864}{3885}, e_1|0\right\}, \left\{\frac{932}{3885} \leftrightarrow \frac{932}{1295}, e_2|0\right\}\right\}$$

In[29]:=
```
Select[t, MatchQ[#[[1]], temp[[1, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 2], "|", _]]] &]
```
Out[29]=
$$\left\{\left\{\frac{1864}{3885} \leftrightarrow \frac{569}{1295}, e_2|1\right\}\right\}$$

In[30]:=
```
Select[t, MatchQ[#[[1]], temp[[2, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 1], "|", _]]] &]
```
Out[30]=
$$\left\{\left\{\frac{932}{1295} \leftrightarrow \frac{569}{1295}, e_1|1\right\}\right\}$$

In[31]:=
```
temp = Select[t, MatchQ[#[[1]], %[[1, 1, 2]] ⟷ _] &]
```
Out[31]=
$$\left\{\left\{\frac{569}{1295} \leftrightarrow \frac{1138}{1295}, e_1|0\right\}, \left\{\frac{569}{1295} \leftrightarrow \frac{412}{1295}, e_2|1\right\}\right\}$$

In[32]:=
```
Select[t, MatchQ[#[[1]], temp[[1, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 2], "|", _]]] &]
```
Out[32]=
$$\left\{\left\{\frac{1138}{1295} \leftrightarrow \frac{824}{1295}, e_2|2\right\}\right\}$$

In[33]:=
```
Select[t, MatchQ[#[[1]], temp[[2, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 1], "|", _]]] &]
```
Out[33]=
$$\left\{\left\{\frac{412}{1295} \leftrightarrow \frac{824}{1295}, e_1|0\right\}\right\}$$

In[34]:=
```
temp = Select[t, MatchQ[#[[1]], %[[1, 1, 2]] ⟷ _] &]
```
Out[34]=
$$\left\{\left\{\frac{824}{1295} \leftrightarrow \frac{353}{1295}, e_1|1\right\}, \left\{\frac{824}{1295} \leftrightarrow \frac{1177}{1295}, e_2|1\right\}\right\}$$

In[35]:=
```
Select[t, MatchQ[#[[1]], temp[[1, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 2], "|", _]]] &]
```
Out[35]=
$$\left\{\left\{\frac{353}{1295} \leftrightarrow \frac{1059}{1295}, e_2|0\right\}\right\}$$

In[36]:=
```
Select[t, MatchQ[#[[1]], temp[[2, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 1], "|", _]]] &]
```
Out[36]=
$$\left\{\left\{\frac{1177}{1295} \leftrightarrow \frac{1059}{1295}, e_1|1\right\}\right\}$$

In[37]:=
```
temp = Select[t, MatchQ[#[[1]], %[[1, 1, 2]] ⟷ _] &]
```
Out[37]=
$$\left\{\left\{\frac{1059}{1295} \leftrightarrow \frac{823}{1295}, e_1|1\right\}, \left\{\frac{1059}{1295} \leftrightarrow \frac{587}{1295}, e_2|2\right\}\right\}$$

In[38]:=
```
Select[t, MatchQ[#[[1]], temp[[1, 1, 2]] ⟷ _] && MatchQ[#[[2]], Row[List[Subscript["e", 2], "|", _]]] &]
```
Out[38]=
$$\left\{\left\{\frac{823}{1295} \leftrightarrow \frac{1174}{1295}, e_2|1\right\}\right\}$$

In[39]:=
```
Select[t, MatchQ[#[[1]], temp[[2, 1, 2]] ↔ _] && MatchQ[#[[2]], Row[List[Subscript["e", 1], "|", _]]] &]
```

Out[39]=

$$\left\{\left\{\frac{587}{1295} \leftrightarrow \frac{1174}{1295}, e_1|0\right\}\right\}$$

In[40]:=
```
temp = Select[t, MatchQ[#[[1]], %[[1, 1, 2]] ↔ _] &]
```

Out[40]=

$$\left\{\left\{\frac{1174}{1295} \leftrightarrow \frac{1053}{1295}, e_1|1\right\}, \left\{\frac{1174}{1295} \leftrightarrow \frac{932}{1295}, e_2|2\right\}\right\}$$

In[41]:=
```
Select[t, MatchQ[#[[1]], temp[[1, 1, 2]] ↔ _] && MatchQ[#[[2]], Row[List[Subscript["e", 2], "|", _]]] &]
```

Out[41]=

$$\left\{\left\{\frac{1053}{1295} \leftrightarrow \frac{569}{1295}, e_2|2\right\}\right\}$$

In[42]:=
```
Select[t, MatchQ[#[[1]], temp[[2, 1, 2]] ↔ _] && MatchQ[#[[2]], Row[List[Subscript["e", 1], "|", _]]] &]
```

Out[42]=

$$\left\{\left\{\frac{932}{1295} \leftrightarrow \frac{569}{1295}, e_1|1\right\}\right\}$$

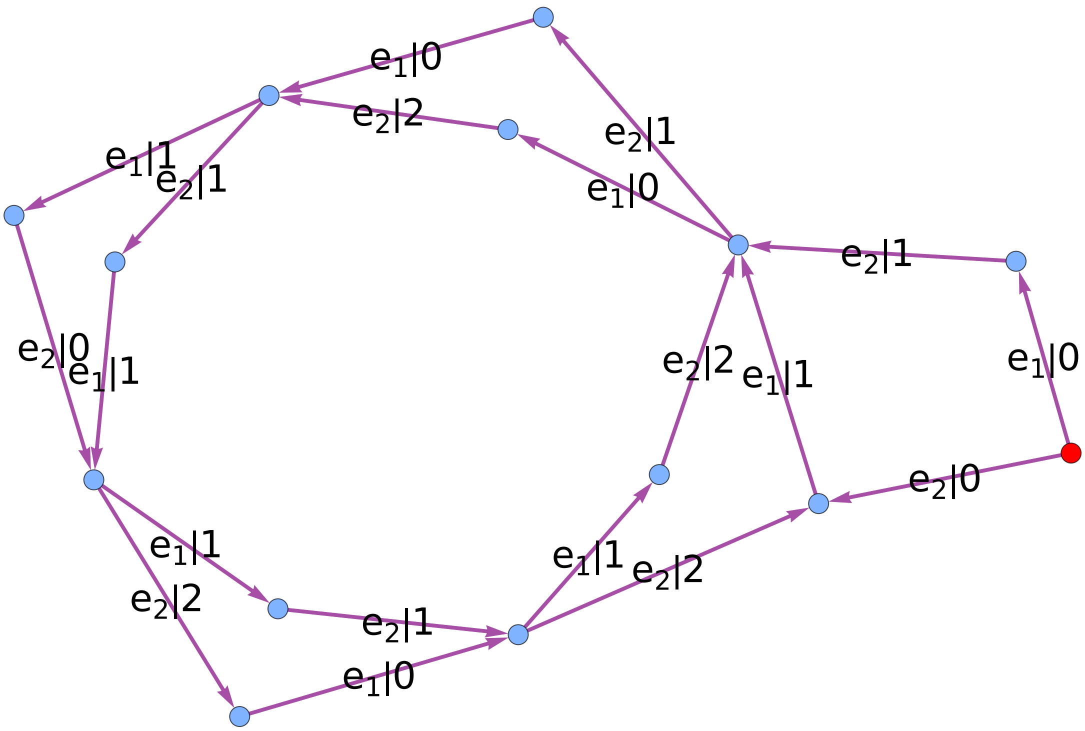
In[45]:=
```
t = Join[%28, %29, %30, %31, %32, %33, %34,
    %35, %36, %37, %38, %39, %40, %41, %42] // DeleteDuplicates
```

Out[45]=

$$\left\{\left\{\frac{932}{3885} \leftrightarrow \frac{1864}{3885}, e_1|0\right\}, \left\{\frac{932}{3885} \leftrightarrow \frac{932}{1295}, e_2|0\right\}, \left\{\frac{1864}{3885} \leftrightarrow \frac{569}{1295}, e_2|1\right\},\right.$$
$$\left\{\frac{932}{1295} \leftrightarrow \frac{569}{1295}, e_1|1\right\}, \left\{\frac{569}{1295} \leftrightarrow \frac{1138}{1295}, e_1|0\right\}, \left\{\frac{569}{1295} \leftrightarrow \frac{412}{1295}, e_2|1\right\}, \left\{\frac{1138}{1295} \leftrightarrow \frac{824}{1295}, e_2|2\right\},$$
$$\left\{\frac{412}{1295} \leftrightarrow \frac{824}{1295}, e_1|0\right\}, \left\{\frac{824}{1295} \leftrightarrow \frac{353}{1295}, e_1|1\right\}, \left\{\frac{824}{1295} \leftrightarrow \frac{1177}{1295}, e_2|1\right\}, \left\{\frac{353}{1295} \leftrightarrow \frac{1059}{1295}, e_2|0\right\},$$
$$\left\{\frac{1177}{1295} \leftrightarrow \frac{1059}{1295}, e_1|1\right\}, \left\{\frac{1059}{1295} \leftrightarrow \frac{823}{1295}, e_1|1\right\}, \left\{\frac{1059}{1295} \leftrightarrow \frac{587}{1295}, e_2|2\right\}, \left\{\frac{823}{1295} \leftrightarrow \frac{1174}{1295}, e_2|1\right\},$$
$$\left.\left\{\frac{587}{1295} \leftrightarrow \frac{1174}{1295}, e_1|0\right\}, \left\{\frac{1174}{1295} \leftrightarrow \frac{1053}{1295}, e_1|1\right\}, \left\{\frac{1174}{1295} \leftrightarrow \frac{932}{1295}, e_2|2\right\}, \left\{\frac{1053}{1295} \leftrightarrow \frac{569}{1295}, e_2|2\right\}\right\}$$

In[46]:=
```
GraphPlot[t,
 VertexLabels → None, VertexStyle → {932/3885 → Red}, VertexSize → 0.18,
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[46]=



Example 4.9 with 2-walk property

In[47]:=
```
β = (X /. Solve[X ^ 3 - X - 1 == 0, X])[[1]]
```

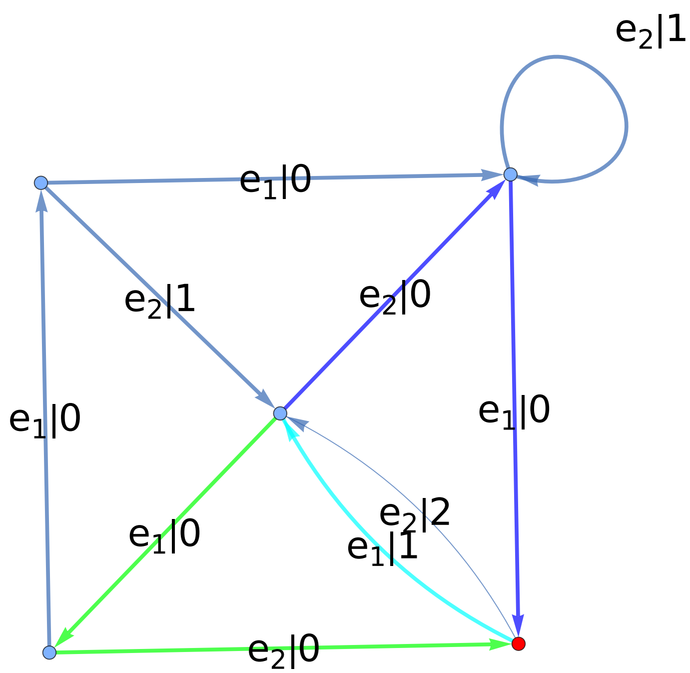
Out[47]=

```
∛ 1.32...
```

In[48]:=
```
t = buildTransducer[1, {β, β ^ 3}];
```

In[50]:=
```
GraphPlot[t,
 VertexLabels → None, VertexStyle → {1 → Red},
 VertexSize → 0.04, EdgeLabelStyle → Directive[20],
 EdgeStyle → {1 ↔ 0.325... → {Thickness[Large], Cyan}, 0.325... ↔ 0.430... →
    {Thickness[Large], Green}, 0.325... ↔ 0.755... → {Thickness[Large], Blue},
   0.430... ↔ 0.570... → {Thickness[Large]}, 0.430... ↔ 1 → {Thickness[Large], Green},
   0.570... ↔ 0.755... → {Thickness[Large]},
   0.570... ↔ 0.325... → {Thickness[Large]}, 0.755... ↔ 1 → {Thickness[Large], Blue},
   0.755... ↔ 0.755... → {Thickness[Large]}}]
```

Out[50]=



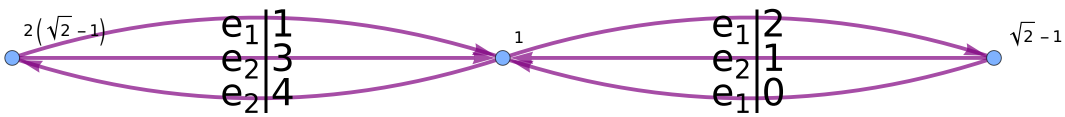
Examples of Section 5 about connectedness

In[51]:=
```
γ1 = 1 + Sqrt[2];
γ2 = 2 + 2 Sqrt[2];
γ3 = 4 + 3 Sqrt[2];
γ4 = 5 + 4 Sqrt[2];
γ5 = 1 + 2 ^ (1 / 3) + 4 ^ (1 / 3);
```

In[58]:=
```
t = buildTransducer[1, {γ1, γ2}];
```

In[59]:=
```
GraphPlot[t,
 VertexLabels → "Name", VertexSize → 0.03,
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[59]=



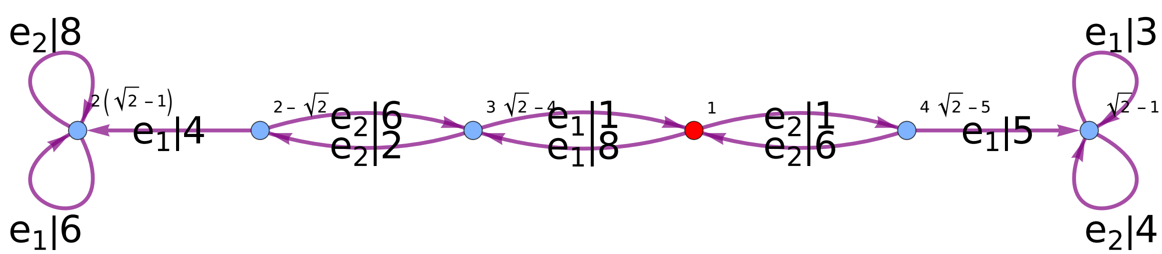
In[62]:=
```
ConnectedGraphQ[Graph[Map[#[[1]] &, t]]]
```

Out[62]=
```
True
```

In[63]:=
```
t = buildTransducer[1, {γ3, γ4}];
```

In[66]:=
```
GraphPlot[t,
 VertexLabels → "Name", VertexSize → 0.1, VertexStyle → {1 → Red},
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[66]=



In[65]:=
```
ConnectedGraphQ[Graph[Map[#[[1]] &, t]]]
```

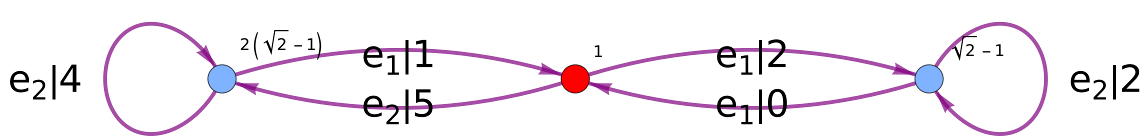
Out[65]=
```
False
```

```
(* also depicted in Fig. 8*)
```

In[68]:=
```
t = buildTransducer[1, {γ1, γ1^2}];
```

In[69]:=
```
GraphPlot[t,
 VertexLabels → "Name", VertexSize → 0.08, VertexStyle → {1 → Red},
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

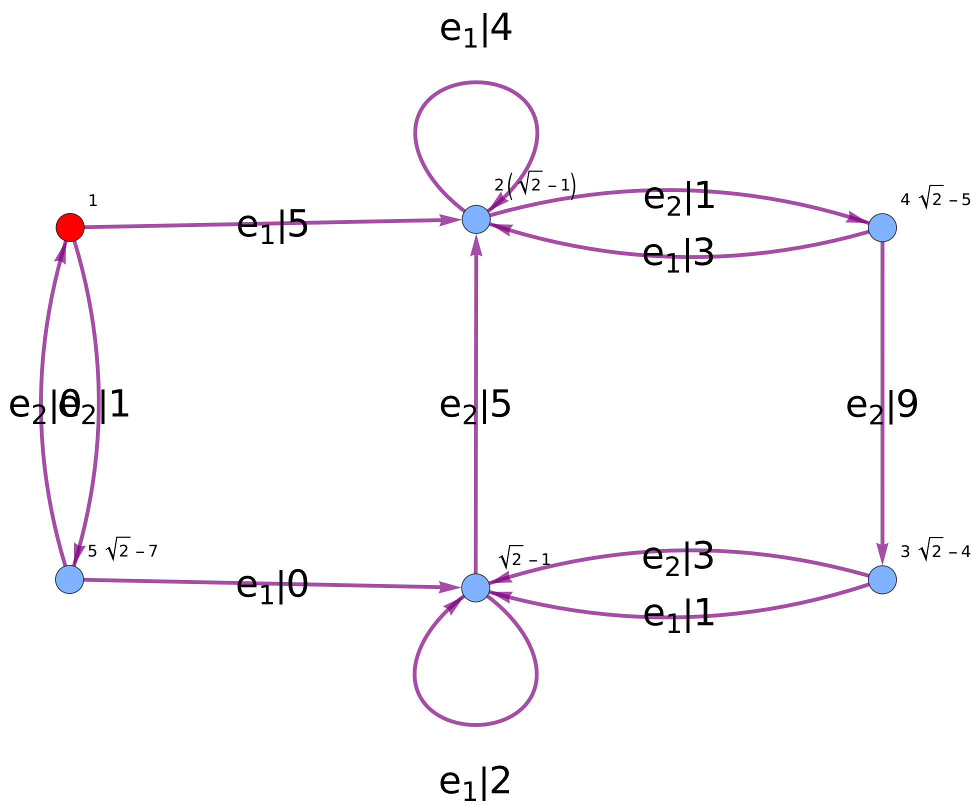
Out[69]=



In[70]:=
```
t = buildTransducer[1, {γ1^2, γ1^3}];
```

In[71]:=
```
GraphPlot[t,
 VertexLabels → "Name", VertexSize → 0.08, VertexStyle → {1 → Red},
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

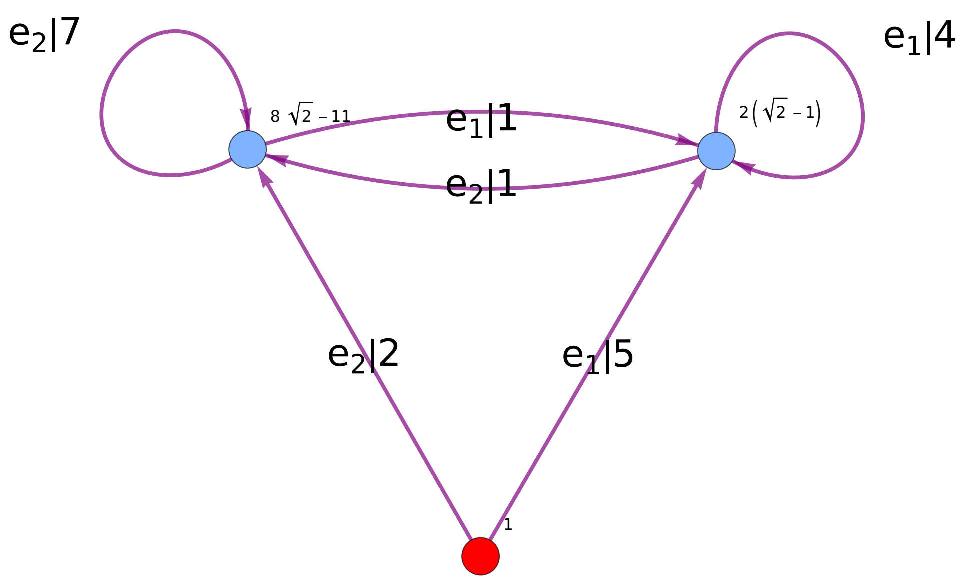
Out[71]=



In[72]:=
```
t = buildTransducer[1, {γ1^2, γ2^2}];
```

In[73]:=
```
GraphPlot[t,
 VertexLabels → "Name", VertexSize → 0.08, VertexStyle → {1 → Red},
 EdgeStyle → {{Thick, Purple}}, EdgeLabelStyle → Directive[20]]
```

Out[73]=



The following transducer is discussed in the paper but not represented because of its size

In[88]:=
```
Timing[t = buildTransducer[1, {γ5^2, γ5^3}]]
```

Out[88]=

$\{120.207, \{\{1 \leftrightarrow -9 + 4 \times 2^{1/3} + 3 \times 2^{2/3}, e_1|1\},$

$\{1 \leftrightarrow -37 + 15 \times 2^{1/3} + 12 \times 2^{2/3}, e_2|5\}, \{-9 + 4 \times 2^{1/3} + 3 \times 2^{2/3} \leftrightarrow 2 (-4 + 2^{1/3} + 2 \times 2^{2/3}), e_1|1\},$

$\{-9 + 4 \times 2^{1/3} + 3 \times 2^{2/3} \leftrightarrow -30 + 13 \times 2^{1/3} + 9 \times 2^{2/3}, e_2|4\},$

$\{2 (-4 + 2^{1/3} + 2 \times 2^{2/3}) \leftrightarrow 2 (-4 + 2^{1/3} + 2 \times 2^{2/3}), e_1|1\},$

$\{2 (-4 + 2^{1/3} + 2 \times 2^{2/3}) \leftrightarrow -33 + 14 \times 2^{1/3} + 10 \times 2^{2/3}, e_2|4\},$

$\{-30 + 13 \times 2^{1/3} + 9 \times 2^{2/3} \leftrightarrow -9 - 2^{1/3} + 7 \times 2^{2/3}, e_1|9\},$

$\{-30 + 13 \times 2^{1/3} + 9 \times 2^{2/3} \leftrightarrow -25 + 13 \times 2^{1/3} + 6 \times 2^{2/3}, e_2|3\},$

$\{-25 + 13 \times 2^{1/3} + 6 \times 2^{2/3} \leftrightarrow -12 + 2^{1/3} + 7 \times 2^{2/3}, e_1|1\},$

$\{-25 + 13 \times 2^{1/3} + 6 \times 2^{2/3} \leftrightarrow -34 + 16 \times 2^{1/3} + 9 \times 2^{2/3}, e_2|5\}, \{-9 - 2^{1/3} + 7 \times 2^{2/3} \leftrightarrow -7 + 2^{1/3} + 4 \times 2^{2/3}, e_1|1\},$

$\{-9 - 2^{1/3} + 7 \times 2^{2/3} \leftrightarrow -33 + 14 \times 2^{1/3} + 10 \times 2^{2/3}, e_2|4\}, \{-7 + 2^{1/3} + 4 \times 2^{2/3} \leftrightarrow -6 + 2^{1/3} + 3 \times 2^{2/3}, e_1|9\},$

$\{-7 + 2^{1/3} + 4 \times 2^{2/3} \leftrightarrow -23 + 10 \times 2^{1/3} + 7 \times 2^{2/3}, e_2|3\}, \{-6 + 2^{1/3} + 3 \times 2^{2/3} \leftrightarrow 2^{1/3} (-1 + 2^{1/3}), e_1|0\},$

$\{-6 + 2^{1/3} + 3 \times 2^{2/3} \leftrightarrow -1 + 2^{1/3}, e_2|1\}, \{-1 + 2^{1/3} \leftrightarrow -2 + 2^{1/3} + 2^{2/3}, e_1|3\},$

$\{-1 + 2^{1/3} \leftrightarrow -9 + 4 \times 2^{1/3} + 3 \times 2^{2/3}, e_2|1\}, \{2^{1/3} (-1 + 2^{1/3}) \leftrightarrow -2 + 2^{1/3} + 2^{2/3}, e_1|4\},$

$\{2^{1/3} (-1 + 2^{1/3}) \leftrightarrow -12 + 5 \times 2^{1/3} + 4 \times 2^{2/3}, e_2|1\}, \{-2 + 2^{1/3} + 2^{2/3} \leftrightarrow -8 + 3 \times 2^{1/3} + 3 \times 2^{2/3}, e_1|1\},$

$\{-2 + 2^{1/3} + 2^{2/3} \leftrightarrow -32 + 13 \times 2^{1/3} + 10 \times 2^{2/3}, e_2|4\}, \{-8 + 3 \times 2^{1/3} + 3 \times 2^{2/3} \leftrightarrow -6 + 2^{1/3} + 3 \times 2^{2/3}, e_1|8\},$

$\{-8 + 3 \times 2^{1/3} + 3 \times 2^{2/3} \leftrightarrow -20 + 9 \times 2^{1/3} + 6 \times 2^{2/3}, e_2|3\}, \{-12 + 5 \times 2^{1/3} + 4 \times 2^{2/3} \leftrightarrow -7 + 2^{1/3} + 4 \times 2^{2/3}, e_1|9\},$

$\{-12 + 5 \times 2^{1/3} + 4 \times 2^{2/3} \leftrightarrow -24 + 11 \times 2^{1/3} + 7 \times 2^{2/3}, e_2|3\},$

$\{-20+9\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -10+2^{1/3}+6\times 2^{2/3},\ e_1|1\},$
$\{-20+9\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -33+15\times 2^{1/3}+9\times 2^{2/3},\ e_2|4\},\ \{-10+2^{1/3}+6\times 2^{2/3}\leftrightarrow -7+2^{1/3}+4\times 2^{2/3},\ e_1|1\},$
$\{-10+2^{1/3}+6\times 2^{2/3}\leftrightarrow -30+13\times 2^{1/3}+9\times 2^{2/3},\ e_2|4\},\ \{-12+2^{1/3}+7\times 2^{2/3}\leftrightarrow -3-2^{1/3}+3\times 2^{2/3},\ e_1|5\},$
$\{-12+2^{1/3}+7\times 2^{2/3}\leftrightarrow (3+2^{1/3})(-5+4\times 2^{1/3}),\ e_2|2\},\ \{(3+2^{1/3})(-5+4\times 2^{1/3})\leftrightarrow -3-2^{1/3}+3\times 2^{2/3},\ e_1|2\},$
$\{(3+2^{1/3})(-5+4\times 2^{1/3})\leftrightarrow -6+4\times 2^{1/3}+2^{2/3},\ e_2|9\},\ \{-6+4\times 2^{1/3}+2^{2/3}\leftrightarrow -7+2\times 2^{1/3}+3\times 2^{2/3},\ e_1|9\},$
$\{-6+4\times 2^{1/3}+2^{2/3}\leftrightarrow -23+10\times 2^{1/3}+7\times 2^{2/3},\ e_2|3\},\ \{-3-2^{1/3}+3\times 2^{2/3}\leftrightarrow -4+2^{1/3}+2\times 2^{2/3},\ e_1|7\},$
$\{-3-2^{1/3}+3\times 2^{2/3}\leftrightarrow -19+8\times 2^{1/3}+6\times 2^{2/3},\ e_2|2\},\ \{-4+2^{1/3}+2\times 2^{2/3}\leftrightarrow -4+2^{1/3}+2\times 2^{2/3},\ e_1|6\},$
$\{-4+2^{1/3}+2\times 2^{2/3}\leftrightarrow -16+7\times 2^{1/3}+5\times 2^{2/3},\ e_2|2\},\ \{-7+2\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -3+2\times 2^{2/3},\ e_1|4\},$
$\{-7+2\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -11+5\times 2^{1/3}+3\times 2^{2/3},\ e_2|1\},\ \{-3+2\times 2^{2/3}\leftrightarrow -1+2^{2/3},\ e_1|2\},$
$\{-3+2\times 2^{2/3}\leftrightarrow -6+3\times 2^{1/3}+2\times 2^{2/3},\ e_2|9\},\ \{-1+2^{2/3}\leftrightarrow -5+2\times 2^{1/3}+2\times 2^{2/3},\ e_1|8\},$
$\{-1+2^{2/3}\leftrightarrow -22+9\times 2^{1/3}+7\times 2^{2/3},\ e_2|3\},\ \{-5+2\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -7+2\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},$
$\{-5+2\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -26+11\times 2^{1/3}+8\times 2^{2/3},\ e_2|3\},$
$\{-6+3\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow (2+2^{1/3})(-5+4\times 2^{1/3}),\ e_1|1\},$
$\{-6+3\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -36+15\times 2^{1/3}+11\times 2^{2/3},\ e_2|5\},$
$\{(2+2^{1/3})(-5+4\times 2^{1/3})\leftrightarrow (-1+2^{1/3})(1+2\times 2^{1/3}),\ e_1|1\},$
$\{(2+2^{1/3})(-5+4\times 2^{1/3})\leftrightarrow -5+3\times 2^{1/3}+2^{2/3},\ e_2|7\},\ \{(-1+2^{1/3})(1+2\times 2^{1/3})\leftrightarrow -8+3\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},$
$\{(-1+2^{1/3})(1+2\times 2^{1/3})\leftrightarrow -35+14\times 2^{1/3}+11\times 2^{2/3},\ e_2|5\},\ \{-5+3\times 2^{1/3}+2^{2/3}\leftrightarrow -4+2^{1/3}+2\times 2^{2/3},\ e_1|5\},$
$\{-5+3\times 2^{1/3}+2^{2/3}\leftrightarrow -13+6\times 2^{1/3}+4\times 2^{2/3},\ e_2|2\},\ \{-11+5\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow (-1+2^{1/3})(1+2\times 2^{1/3}),\ e_1|0\},$
$\{-11+5\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow 2(-1+2^{1/3}),\ e_2|3\},\ \{2(-1+2^{1/3})\leftrightarrow -5+2\times 2^{1/3}+2\times 2^{2/3},\ e_1|7\},$
$\{2(-1+2^{1/3})\leftrightarrow -19+8\times 2^{1/3}+6\times 2^{2/3},\ e_2|2\},\ \{-13+6\times 2^{1/3}+4\times 2^{2/3}\leftrightarrow -10+2\times 2^{1/3}+5\times 2^{2/3},\ e_1|1\},$
$\{-13+6\times 2^{1/3}+4\times 2^{2/3}\leftrightarrow -34+15\times 2^{1/3}+10\times 2^{2/3},\ e_2|5\},\ \{-10+2\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -4+3\times 2^{2/3},\ e_1|6\},$
$\{-10+2\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -18+8\times 2^{1/3}+5\times 2^{2/3},\ e_2|2\},\ \{-4+3\times 2^{2/3}\leftrightarrow -7+2\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},$
$\{-4+3\times 2^{2/3}\leftrightarrow -29+12\times 2^{1/3}+9\times 2^{2/3},\ e_2|4\},\ \{-16+7\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -9+2^{1/3}+5\times 2^{2/3},\ e_1|1\},$
$\{-16+7\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -29+13\times 2^{1/3}+8\times 2^{2/3},\ e_2|4\},$
$\{-9+2^{1/3}+5\times 2^{2/3}\leftrightarrow (-1+2^{1/3})(1+2\times 2^{1/3}),\ e_1|2\},\ \{-9+2^{1/3}+5\times 2^{2/3}\leftrightarrow 2(-4+2\times 2^{1/3}+2^{2/3}),\ e_2|1\},$
$\{2(-4+2\times 2^{1/3}+2^{2/3})\leftrightarrow -3+2\times 2^{2/3},\ e_1|3\},\ \{2(-4+2\times 2^{1/3}+2^{2/3})\leftrightarrow 2(-4+2\times 2^{1/3}+2^{2/3}),\ e_2|1\},$
$\{-18+8\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -2-2\times 2^{1/3}+3\times 2^{2/3},\ e_1|0\},\ \{-18+8\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -2^{1/3}(-2+2^{1/3}),\ e_2|0\},$
$\{-2^{1/3}(-2+2^{1/3})\leftrightarrow -9+4\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},\ \{-2^{1/3}(-2+2^{1/3})\leftrightarrow -35+14\times 2^{1/3}+11\times 2^{2/3},\ e_2|5\},$
$\{-2-2\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -1+2^{2/3},\ e_1|3\},\ \{-2-2\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -9+4\times 2^{1/3}+3\times 2^{2/3},\ e_2|1\},$
$\{-19+8\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -7+5\times 2^{2/3},\ e_1|8\},\ \{-19+8\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -23+11\times 2^{1/3}+6\times 2^{2/3},\ e_2|3\},$
$\{-7+5\times 2^{2/3}\leftrightarrow 2(-4+2^{1/3}+2\times 2^{2/3}),\ e_1|1\},\ \{-7+5\times 2^{2/3}\leftrightarrow -36+15\times 2^{1/3}+11\times 2^{2/3},\ e_2|5\},$
$\{-23+11\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow (1+2^{1/3})(-6+5\times 2^{1/3}),\ e_1|5\},$
$\{-23+11\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -14+8\times 2^{1/3}+3\times 2^{2/3},\ e_2|2\},$
$\{(1+2^{1/3})(-6+5\times 2^{1/3})\leftrightarrow -6+2^{1/3}+3\times 2^{2/3},\ e_1|1\},\ \{(1+2^{1/3})(-6+5\times 2^{1/3})\leftrightarrow -26+11\times 2^{1/3}+8\times 2^{2/3},\ e_2|3\},$
$\{-14+8\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -10+2\times 2^{1/3}+5\times 2^{2/3},\ e_1|1\},$
$\{-14+8\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -31+14\times 2^{1/3}+9\times 2^{2/3},\ e_2|4\},$
$\{-22+9\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow (1+2^{1/3})(-6+5\times 2^{1/3}),\ e_1|6\},$
$\{-22+9\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow -17+9\times 2^{1/3}+4\times 2^{2/3},\ e_2|2\},\ \{-17+9\times 2^{1/3}+4\times 2^{2/3}\leftrightarrow -9+2^{1/3}+5\times 2^{2/3},\ e_1|1\},$

$\{-17+9\times2^{1/3}+4\times2^{2/3}\leftrightarrow-26+12\times2^{1/3}+7\times2^{2/3},\ e_2|3\}$, $\{-23+10\times2^{1/3}+7\times2^{2/3}\leftrightarrow-9+6\times2^{2/3},\ e_1|1\}$,
$\{-23+10\times2^{1/3}+7\times2^{2/3}\leftrightarrow-27+13\times2^{1/3}+7\times2^{2/3},\ e_2|4\}$, $\{-9+6\times2^{2/3}\leftrightarrow-4+3\times2^{2/3},\ e_1|7\}$,
$\{-9+6\times2^{2/3}\leftrightarrow-20+9\times2^{1/3}+6\times2^{2/3},\ e_2|2\}$, $\{-24+11\times2^{1/3}+7\times2^{2/3}\leftrightarrow-12+2^{1/3}+7\times2^{2/3},\ e_1|1\}$,
$\{-24+11\times2^{1/3}+7\times2^{2/3}\leftrightarrow-37+17\times2^{1/3}+10\times2^{2/3},\ e_2|5\}$,
$\{-26+12\times2^{1/3}+7\times2^{2/3}\leftrightarrow-5-2\times2^{1/3}+5\times2^{2/3},\ e_1|3\}$,
$\{-26+12\times2^{1/3}+7\times2^{2/3}\leftrightarrow-9+6\times2^{1/3}+2^{2/3},\ e_2|1\}$, $\{-9+6\times2^{1/3}+2^{2/3}\leftrightarrow-3+2\times2^{2/3},\ e_1|2\}$,
$\{-9+6\times2^{1/3}+2^{2/3}\leftrightarrow-5+3\times2^{1/3}+2^{2/3},\ e_2|8\}$, $\{-5-2\times2^{1/3}+5\times2^{2/3}\leftrightarrow-3+2\times2^{2/3},\ e_1|6\}$,
$\{-5-2\times2^{1/3}+5\times2^{2/3}\leftrightarrow-16+7\times2^{1/3}+5\times2^{2/3},\ e_2|2\}$, $\{-27+13\times2^{1/3}+7\times2^{2/3}\leftrightarrow-8-2^{1/3}+6\times2^{2/3},\ e_1|7\}$,
$\{-27+13\times2^{1/3}+7\times2^{2/3}\leftrightarrow2\left(-9+5\times2^{1/3}+2\times2^{2/3}\right),\ e_2|2\}$,
$\{2\left(-9+5\times2^{1/3}+2\times2^{2/3}\right)\leftrightarrow2\left(-6+2^{1/3}+3\times2^{2/3}\right),\ e_1|1\}$,
$\{2\left(-9+5\times2^{1/3}+2\times2^{2/3}\right)\leftrightarrow2\left(-18+8\times2^{1/3}+5\times2^{2/3}\right),\ e_2|5\}$,
$\{2\left(-6+2^{1/3}+3\times2^{2/3}\right)\leftrightarrow2\times2^{1/3}\left(-1+2^{1/3}\right),\ e_1|0\}$, $\{2\left(-6+2^{1/3}+3\times2^{2/3}\right)\leftrightarrow2\left(-1+2^{1/3}\right),\ e_2|2\}$,
$\{2\times2^{1/3}\left(-1+2^{1/3}\right)\leftrightarrow-5+2\times2^{1/3}+2\times2^{2/3},\ e_1|9\}$, $\{2\times2^{1/3}\left(-1+2^{1/3}\right)\leftrightarrow-25+10\times2^{1/3}+8\times2^{2/3},\ e_2|3\}$,
$\{2\left(-18+8\times2^{1/3}+5\times2^{2/3}\right)\leftrightarrow-4-4\times2^{1/3}+6\times2^{2/3},\ e_1|0\}$,
$\{2\left(-18+8\times2^{1/3}+5\times2^{2/3}\right)\leftrightarrow-1+4\times2^{1/3}-2\times2^{2/3},\ e_2|1\}$,
$\{-1+4\times2^{1/3}-2\times2^{2/3}\leftrightarrow-9+4\times2^{1/3}+3\times2^{2/3},\ e_1|1\}$,
$\{-1+4\times2^{1/3}-2\times2^{2/3}\leftrightarrow-32+13\times2^{1/3}+10\times2^{2/3},\ e_2|4\}$, $\{-4-4\times2^{1/3}+6\times2^{2/3}\leftrightarrow-3+2\times2^{2/3},\ e_1|7\}$,
$\{-4-4\times2^{1/3}+6\times2^{2/3}\leftrightarrow-19+8\times2^{1/3}+6\times2^{2/3},\ e_2|2\}$, $\{-8-2^{1/3}+6\times2^{2/3}\leftrightarrow\left(-1+2^{1/3}\right)\left(1+2\times2^{1/3}\right),\ e_1|3\}$,
$\{-8-2^{1/3}+6\times2^{2/3}\leftrightarrow-11+5\times2^{1/3}+3\times2^{2/3},\ e_2|1\}$, $\{-25+10\times2^{1/3}+8\times2^{2/3}\leftrightarrow-5-2\times2^{1/3}+5\times2^{2/3},\ e_1|4\}$,
$\{-25+10\times2^{1/3}+8\times2^{2/3}\leftrightarrow-11+7\times2^{1/3}+2\times2^{2/3},\ e_2|1\}$,
$\{-11+7\times2^{1/3}+2\times2^{2/3}\leftrightarrow-11+3\times2^{1/3}+5\times2^{2/3},\ e_1|1\}$,
$\{-11+7\times2^{1/3}+2\times2^{2/3}\leftrightarrow-37+16\times2^{1/3}+11\times2^{2/3},\ e_2|5\}$,
$\{-11+3\times2^{1/3}+5\times2^{2/3}\leftrightarrow-7+2^{1/3}+4\times2^{2/3},\ e_1|1\}$,
$\{-11+3\times2^{1/3}+5\times2^{2/3}\leftrightarrow-27+12\times2^{1/3}+8\times2^{2/3},\ e_2|4\}$,
$\{-26+11\times2^{1/3}+8\times2^{2/3}\leftrightarrow-8-2^{1/3}+6\times2^{2/3},\ e_1|8\}$,
$\{-26+11\times2^{1/3}+8\times2^{2/3}\leftrightarrow-21+11\times2^{1/3}+5\times2^{2/3},\ e_2|3\}$,
$\{-21+11\times2^{1/3}+5\times2^{2/3}\leftrightarrow-10+2^{1/3}+6\times2^{2/3},\ e_1|1\}$,
$\{-21+11\times2^{1/3}+5\times2^{2/3}\leftrightarrow2\left(3+2^{1/3}\right)\left(-5+4\times2^{1/3}\right),\ e_2|4\}$,
$\{2\left(3+2^{1/3}\right)\left(-5+4\times2^{1/3}\right)\leftrightarrow-7-2\times2^{1/3}+6\times2^{2/3},\ e_1|5\}$,
$\{2\left(3+2^{1/3}\right)\left(-5+4\times2^{1/3}\right)\leftrightarrow-13+8\times2^{1/3}+2\times2^{2/3},\ e_2|1\}$, $\{-13+8\times2^{1/3}+2\times2^{2/3}\leftrightarrow-4+3\times2^{2/3},\ e_1|3\}$,
$\{-13+8\times2^{1/3}+2\times2^{2/3}\leftrightarrow-9+5\times2^{1/3}+2\times2^{2/3},\ e_2|1\}$, $\{-9+5\times2^{1/3}+2\times2^{2/3}\leftrightarrow-6+2^{1/3}+3\times2^{2/3},\ e_1|7\}$,
$\{-9+5\times2^{1/3}+2\times2^{2/3}\leftrightarrow-18+8\times2^{1/3}+5\times2^{2/3},\ e_2|2\}$, $\{-7-2\times2^{1/3}+6\times2^{2/3}\leftrightarrow\left(-1+2^{1/3}\right)^2,\ e_1|0\}$,
$\{-7-2\times2^{1/3}+6\times2^{2/3}\leftrightarrow-1+2^{1/3},\ e_2|0\}$, $\{\left(-1+2^{1/3}\right)^2\leftrightarrow1,\ e_1|0\}$,
$\{\left(-1+2^{1/3}\right)^2\leftrightarrow-2+2^{1/3}+2^{2/3},\ e_2|3\}$, $\{-27+12\times2^{1/3}+8\times2^{2/3}\leftrightarrow-11+7\times2^{2/3},\ e_1|1\}$,
$\{-27+12\times2^{1/3}+8\times2^{2/3}\leftrightarrow-31+15\times2^{1/3}+8\times2^{2/3},\ e_2|4\}$, $\{-11+7\times2^{2/3}\leftrightarrow2\times2^{1/3}\left(-1+2^{1/3}\right),\ e_1|1\}$,
$\{-11+7\times2^{2/3}\leftrightarrow-5+3\times2^{1/3}+2^{2/3},\ e_2|6\}$, $\{-29+13\times2^{1/3}+8\times2^{2/3}\leftrightarrow-4-3\times2^{1/3}+5\times2^{2/3},\ e_1|1\}$,
$\{-29+13\times2^{1/3}+8\times2^{2/3}\leftrightarrow-3+4\times2^{1/3}-2^{2/3},\ e_2|4\}$, $\{-3+4\times2^{1/3}-2^{2/3}\leftrightarrow-5+2\times2^{1/3}+2\times2^{2/3},\ e_1|6\}$,
$\{-3+4\times2^{1/3}-2^{2/3}\leftrightarrow-16+7\times2^{1/3}+5\times2^{2/3},\ e_2|2\}$, $\{-4-3\times2^{1/3}+5\times2^{2/3}\leftrightarrow2^{1/3}\left(-1+2^{1/3}\right),\ e_1|2\}$,

$\{-4-3\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -6+3\times 2^{1/3}+2\times 2^{2/3},\ e_2|8\},\ \{-31+15\times 2^{1/3}+8\times 2^{2/3}\leftrightarrow -9-2^{1/3}+7\times 2^{2/3},\ e_1|8\},$
$\{-31+15\times 2^{1/3}+8\times 2^{2/3}\leftrightarrow -23+12\times 2^{1/3}+5\times 2^{2/3},\ e_2|3\},$
$\{-23+12\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -3-2\times 2^{1/3}+4\times 2^{2/3},\ e_1|0\},$
$\{-23+12\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -2+3\times 2^{1/3}-2^{2/3},\ e_2|3\},\ \{-2+3\times 2^{1/3}-2^{2/3}\leftrightarrow -2+2^{1/3}+2^{2/3},\ e_1|2\},$
$\{-2+3\times 2^{1/3}-2^{2/3}\leftrightarrow -6+3\times 2^{1/3}+2\times 2^{2/3},\ e_2|1\},\ \{-3-2\times 2^{1/3}+4\times 2^{2/3}\leftrightarrow -7+2\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},$
$\{-3-2\times 2^{1/3}+4\times 2^{2/3}\leftrightarrow -32+13\times 2^{1/3}+10\times 2^{2/3},\ e_2|4\},$
$\{-29+12\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -7-2\times 2^{1/3}+6\times 2^{2/3},\ e_1|6\},$
$\{-29+12\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -16+9\times 2^{1/3}+3\times 2^{2/3},\ e_2|2\},$
$\{-16+9\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -3-2^{1/3}+3\times 2^{2/3},\ e_1|1\},\ \{-16+9\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow 3\left(-1+2^{1/3}\right),\ e_2|5\},$
$\{3\left(-1+2^{1/3}\right)\leftrightarrow -8+3\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},\ \{3\left(-1+2^{1/3}\right)\leftrightarrow -29+12\times 2^{1/3}+9\times 2^{2/3},\ e_2|4\},$
$\{-31+14\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -12+8\times 2^{2/3},\ e_1|1\},\ \{-31+14\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -35+17\times 2^{1/3}+9\times 2^{2/3},\ e_2|5\},$
$\{-12+8\times 2^{2/3}\leftrightarrow -6+4\times 2^{2/3},\ e_1|1\},\ \{-12+8\times 2^{2/3}\leftrightarrow -27+12\times 2^{1/3}+8\times 2^{2/3},\ e_2|3\},$
$\{-6+4\times 2^{2/3}\leftrightarrow -3+2\times 2^{2/3},\ e_1|5\},\ \{-6+4\times 2^{2/3}\leftrightarrow -13+6\times 2^{1/3}+4\times 2^{2/3},\ e_2|1\},$
$\{-33+15\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -5-3\times 2^{1/3}+6\times 2^{2/3},\ e_1|2\},\ \{-33+15\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -7+6\times 2^{1/3},\ e_2|1\},$
$\{-7+6\times 2^{1/3}\leftrightarrow -7+2\times 2^{1/3}+3\times 2^{2/3},\ e_1|8\},\ \{-7+6\times 2^{1/3}\leftrightarrow -20+9\times 2^{1/3}+6\times 2^{2/3},\ e_2|3\},$
$\{-5-3\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -6+2^{1/3}+3\times 2^{2/3},\ e_1|1\},\ \{-5-3\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -29+12\times 2^{1/3}+9\times 2^{2/3},\ e_2|4\},$
$\{-34+16\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -8-2\times 2^{1/3}+7\times 2^{2/3},\ e_1|6\},$
$\{-34+16\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -17+10\times 2^{1/3}+3\times 2^{2/3},\ e_2|2\},$
$\{-17+10\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -6+4\times 2^{2/3},\ e_1|5\},\ \{-17+10\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -13+7\times 2^{1/3}+3\times 2^{2/3},\ e_2|2\},$
$\{-13+7\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -7+2^{1/3}+4\times 2^{2/3},\ e_1|8\},$
$\{-13+7\times 2^{1/3}+3\times 2^{2/3}\leftrightarrow -22+10\times 2^{1/3}+6\times 2^{2/3},\ e_2|3\},$
$\{-22+10\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -3-2\times 2^{1/3}+4\times 2^{2/3},\ e_1|1\},\ \{-22+10\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -5+4\times 2^{1/3},\ e_2|7\},$
$\{-5+4\times 2^{1/3}\leftrightarrow -1+2^{2/3},\ e_1|0\},\ \{-5+4\times 2^{1/3}\leftrightarrow -1+2^{1/3},\ e_2|2\},$
$\{-8-2\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow -4+3\times 2^{2/3},\ e_1|8\},\ \{-8-2\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow -23+10\times 2^{1/3}+7\times 2^{2/3},\ e_2|3\},$
$\{-35+17\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -11-2^{1/3}+8\times 2^{2/3},\ e_1|1\},$
$\{-35+17\times 2^{1/3}+9\times 2^{2/3}\leftrightarrow -27+14\times 2^{1/3}+6\times 2^{2/3},\ e_2|4\},$
$\{-27+14\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -5-2\times 2^{1/3}+5\times 2^{2/3},\ e_1|2\},\ \{-27+14\times 2^{1/3}+6\times 2^{2/3}\leftrightarrow -6+5\times 2^{1/3},\ e_2|9\},$
$\{-6+5\times 2^{1/3}\leftrightarrow -4+2^{1/3}+2\times 2^{2/3},\ e_1|4\},\ \{-6+5\times 2^{1/3}\leftrightarrow -11+5\times 2^{1/3}+3\times 2^{2/3},\ e_2|1\},$
$\{-11-2^{1/3}+8\times 2^{2/3}\leftrightarrow -3-2^{1/3}+3\times 2^{2/3},\ e_1|6\},\ \{-11-2^{1/3}+8\times 2^{2/3}\leftrightarrow -18+8\times 2^{1/3}+5\times 2^{2/3},\ e_2|2\},$
$\{-32+13\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -5-3\times 2^{1/3}+6\times 2^{2/3},\ e_1|3\},$
$\{-32+13\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -10+7\times 2^{1/3}+2^{2/3},\ e_2|1\},$
$\{-10+7\times 2^{1/3}+2^{2/3}\leftrightarrow -6+2^{1/3}+3\times 2^{2/3},\ e_1|6\},\ \{-10+7\times 2^{1/3}+2^{2/3}\leftrightarrow \left(3+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right),\ e_2|2\},$
$\{-33+14\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -8-2\times 2^{1/3}+7\times 2^{2/3},\ e_1|7\},$
$\{-33+14\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow \left(4+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right),\ e_2|2\},$
$\{\left(4+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right)\leftrightarrow \left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right),\ e_1|3\},\ \{\left(4+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right)\leftrightarrow -7+5\times 2^{1/3}+2^{2/3},\ e_2|1\},$
$\{\left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right)\leftrightarrow 2^{1/3}\left(-1+2^{1/3}\right),\ e_1|1\},\ \{\left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right)\leftrightarrow -4+2\times 2^{1/3}+2^{2/3},\ e_2|5\},$
$\{-4+2\times 2^{1/3}+2^{2/3}\leftrightarrow -1+2^{2/3},\ e_1|1\},\ \{-4+2\times 2^{1/3}+2^{2/3}\leftrightarrow -4+2\times 2^{1/3}+2^{2/3},\ e_2|6\},$
$\{-7+5\times 2^{1/3}+2^{2/3}\leftrightarrow \left(2+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right),\ e_1|1\},\ \{-7+5\times 2^{1/3}+2^{2/3}\leftrightarrow -33+14\times 2^{1/3}+10\times 2^{2/3},\ e_2|5\},$
$\{-34+15\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -11-2^{1/3}+8\times 2^{2/3},\ e_1|1\},$

$\{-34+15\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -30+15\times 2^{1/3}+7\times 2^{2/3},\ e_2|4\},$
$\{-30+15\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow -4-3\times 2^{1/3}+5\times 2^{2/3},\ e_1|0\},\ \{-30+15\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow 3\times 2^{1/3}-2\times 2^{2/3},\ e_2|0\},$
$\{3\times 2^{1/3}-2\times 2^{2/3}\leftrightarrow -6+3\times 2^{1/3}+2\times 2^{2/3},\ e_1|8\},\ \{3\times 2^{1/3}-2\times 2^{2/3}\leftrightarrow -22+9\times 2^{1/3}+7\times 2^{2/3},\ e_2|3\},$
$\{-37+17\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -7-3\times 2^{1/3}+7\times 2^{2/3},\ e_1|4\},$
$\{-37+17\times 2^{1/3}+10\times 2^{2/3}\leftrightarrow -11+8\times 2^{1/3}+2^{2/3},\ e_2|1\},$
$\{-11+8\times 2^{1/3}+2^{2/3}\leftrightarrow 2\left(-4+2^{1/3}+2\times 2^{2/3}\right),\ e_1|9\},\ \{-11+8\times 2^{1/3}+2^{2/3}\leftrightarrow -24+11\times 2^{1/3}+7\times 2^{2/3},\ e_2|3\},$
$\{-7-3\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow \left(-1+2^{1/3}\right)\left(1+2\times 2^{1/3}\right),\ e_1|4\},$
$\{-7-3\times 2^{1/3}+7\times 2^{2/3}\leftrightarrow -13+6\times 2^{1/3}+4\times 2^{2/3},\ e_2|1\},$
$\{-35+14\times 2^{1/3}+11\times 2^{2/3}\leftrightarrow -4-4\times 2^{1/3}+6\times 2^{2/3},\ e_1|1\},$
$\{-35+14\times 2^{1/3}+11\times 2^{2/3}\leftrightarrow -4+5\times 2^{1/3}-2^{2/3},\ e_2|5\},$
$\{-4+5\times 2^{1/3}-2^{2/3}\leftrightarrow -8+3\times 2^{1/3}+3\times 2^{2/3},\ e_1|1\},\ \{-4+5\times 2^{1/3}-2^{2/3}\leftrightarrow -26+11\times 2^{1/3}+8\times 2^{2/3},\ e_2|4\},$
$\{-36+15\times 2^{1/3}+11\times 2^{2/3}\leftrightarrow -7-3\times 2^{1/3}+7\times 2^{2/3},\ e_1|5\},$
$\{-36+15\times 2^{1/3}+11\times 2^{2/3}\leftrightarrow -14+9\times 2^{1/3}+2\times 2^{2/3},\ e_2|2\},$
$\{-14+9\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -7+2^{1/3}+4\times 2^{2/3},\ e_1|7\},\ \{-14+9\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -19+9\times 2^{1/3}+5\times 2^{2/3},\ e_2|2\},$
$\{-19+9\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow \left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right),\ e_1|4\},$
$\{-19+9\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow 2\left(-5+3\times 2^{1/3}+2^{2/3}\right),\ e_2|1\},\ \{2\left(-5+3\times 2^{1/3}+2^{2/3}\right)\leftrightarrow 2\left(-4+2^{1/3}+2\times 2^{2/3}\right),\ e_1|1\},$
$\{2\left(-5+3\times 2^{1/3}+2^{2/3}\right)\leftrightarrow -27+12\times 2^{1/3}+8\times 2^{2/3},\ e_2|4\},$
$\{-37+16\times 2^{1/3}+11\times 2^{2/3}\leftrightarrow 2\left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right),\ e_1|9\},$
$\{-37+16\times 2^{1/3}+11\times 2^{2/3}\leftrightarrow -24+13\times 2^{1/3}+5\times 2^{2/3},\ e_2|3\},$
$\{2\left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right)\leftrightarrow 2\times 2^{1/3}\left(-1+2^{1/3}\right),\ e_1|2\},$
$\{2\left(1+2^{1/3}\right)\left(-5+4\times 2^{1/3}\right)\leftrightarrow 2\left(-4+2\times 2^{1/3}+2^{2/3}\right),\ e_2|1\},$
$\{-24+13\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow \left(1+2^{1/3}\right)\left(-6+5\times 2^{1/3}\right),\ e_1|4\},$
$\{-24+13\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -11+7\times 2^{1/3}+2\times 2^{2/3},\ e_2|1\},$
$\{-37+15\times 2^{1/3}+12\times 2^{2/3}\leftrightarrow -13-2^{1/3}+9\times 2^{2/3},\ e_1|1\},$
$\{-37+15\times 2^{1/3}+12\times 2^{2/3}\leftrightarrow 9\left(-4+2\times 2^{1/3}+2^{2/3}\right),\ e_2|5\},$
$\{9\left(-4+2\times 2^{1/3}+2^{2/3}\right)\leftrightarrow -14+9\times 2^{2/3},\ e_1|1\},\ \{9\left(-4+2\times 2^{1/3}+2^{2/3}\right)\leftrightarrow 9\left(-4+2\times 2^{1/3}+2^{2/3}\right),\ e_2|5\},$
$\{-14+9\times 2^{2/3}\leftrightarrow -2-2\times 2^{1/3}+3\times 2^{2/3},\ e_1|4\},\ \{-14+9\times 2^{2/3}\leftrightarrow 3\left(-4+2\times 2^{1/3}+2^{2/3}\right),\ e_2|1\},$
$\{3\left(-4+2\times 2^{1/3}+2^{2/3}\right)\leftrightarrow -4+3\times 2^{2/3},\ e_1|4\},\ \{3\left(-4+2\times 2^{1/3}+2^{2/3}\right)\leftrightarrow 3\left(-4+2\times 2^{1/3}+2^{2/3}\right),\ e_2|1\},$
$\{-13-2^{1/3}+9\times 2^{2/3}\leftrightarrow 1-3\times 2^{1/3}+2\times 2^{2/3},\ e_1|0\},\ \{-13-2^{1/3}+9\times 2^{2/3}\leftrightarrow 2\left(-1+2^{1/3}\right),\ e_2|1\},$
$\{1-3\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -2+2^{1/3}+2^{2/3},\ e_1|5\},\ \{1-3\times 2^{1/3}+2\times 2^{2/3}\leftrightarrow -15+6\times 2^{1/3}+5\times 2^{2/3},\ e_2|2\},$
$\{-15+6\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -6+4\times 2^{2/3},\ e_1|7\},\ \{-15+6\times 2^{1/3}+5\times 2^{2/3}\leftrightarrow -19+9\times 2^{1/3}+5\times 2^{2/3},\ e_2|2\}\}\}$

In[89]:=
```
VertexList[Graph[Map[#[[1]] &, t]]] // Length
```
Out[89]=

127

In[90]:=
```
ConnectedGraphQ[Graph[Map[#[[1]] &, t]]]
```
Out[90]=

True