Numerical treatment of nonsmooth frictional beam-to-beam contact

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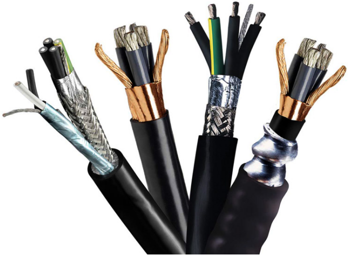
Fraunhofer ITWM

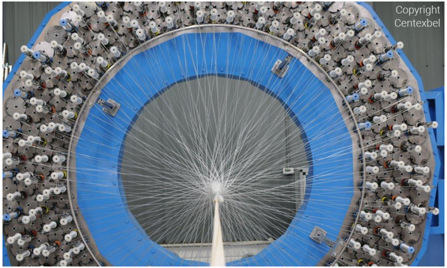
Context and applications[1,2]



- Material behaviour as Simulation input: Need for substantial experimental data!
- Mesoscopic physical processes poorly understood

[1] http://blog.latrivenetacavi.com/fr/conseils-sur-lisolation-des-cables-electrique-a-linterieur-et-a-lexterieur/
[2] https://media03.comptoir-du-cable.com/

Context and applications [3][4][5]



[3] https://thread-etn.eu/
[4] https://learnodo-newtonic.com/
[5] https://media.beckman.com/

Modeling approach and literature

Slender structures

- FE beam models with rigid cross-sections → finite rotations: Simo(1985), Jelenic(1997), Sonneville(2014)

Distributed contact interactions → penalty approaches

- Gauß point-to-segment (*no friction*): Meier(2016, 2017, 2017b)
- Collocation method (*quasi-static friction*): Durville(2011)

Our contribution

- Nonsmooth frictional contact: mortar Bosten(2021) & collocation
- Local frame formulation on $SE(3)$
- Quasi-static and dynamic

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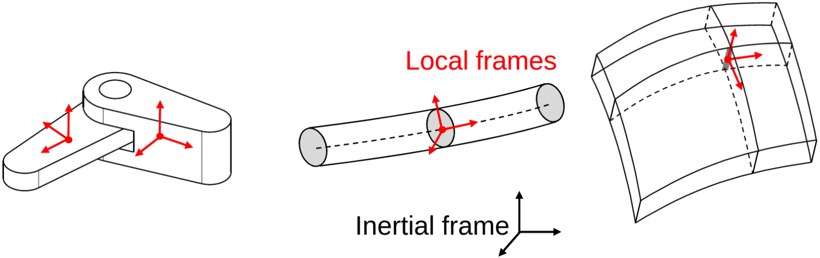
Geometric FEM Strategy

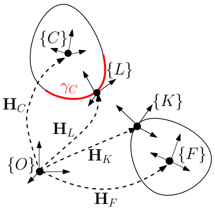
- Represent finite motions as frame transformations
- Consider these frame transformations as elements of $SE(3)$: $\mathbf{H}_{\bullet}$
- Work with left invariant derivatives and solve the dynamics in the local frame
- Exploit modern numerical methods on Lie groups



Quasi-static contact conditions and notations

Association of two frames $\mathbf{H}_L, \mathbf{H}_K$ over the potential contact region γ_C. The variations are

$$\delta\mathbf{H}_L = \mathbf{H}_L S(\delta\boldsymbol{\pi}_L), \quad \delta\mathbf{H}_K = \mathbf{H}_K S(\delta\boldsymbol{\pi}_K)$$
$$S(\delta\boldsymbol{\pi}_L), S(\delta\boldsymbol{\pi}_K) \in \mathfrak{se}(3)$$



Unilateral restriction of relative motion ensured via a reaction force

$$g_N(\mathbf{H}_{LK}) \geq 0, \quad \lambda_N \geq 0, \quad g_N\lambda_N = 0$$

It derives from non-smooth potentials

$$g_N \in \partial\psi_{\mathbb{R}^-}(\lambda_N) \quad \text{or} \quad \lambda_N \in \partial\psi_{\mathbb{R}^+}(g_N)$$

Equivalent weak variational inequality

$$\lambda_N \in \mathcal{M}_N, \quad \int_{\gamma_C} (\delta\lambda_N - \lambda_N) g_N \, \mathrm{d}\gamma \geq 0, \quad \forall \delta\lambda_N \in \mathcal{M}_N$$

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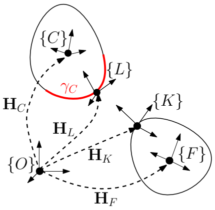

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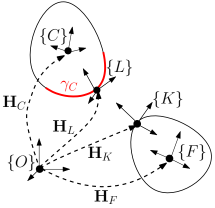

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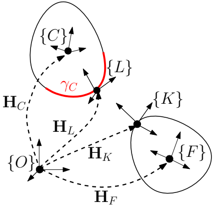

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Quasi-static contact conditions and notations

Coulomb model that incorporates tangential stick and slip: in terms of slip increments!

$$\mu\lambda_N - \|\boldsymbol{\lambda}_T\| \geq 0, \quad \|\mathrm{d}\mathbf{g}_T\| \left(\mu\lambda_N - \|\boldsymbol{\lambda}_T\|\right) = 0$$

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$$\boldsymbol{\lambda}_T \in \mathcal{M}_T, \quad \int_{\gamma_C} \left[\left(\delta\boldsymbol{\lambda}_T - \boldsymbol{\lambda}_T\right)^T \mathrm{d}\mathbf{g}_T \right] \mathrm{d}\gamma \geq 0, \quad \delta\boldsymbol{\lambda}_T \in \mathcal{M}_T$$

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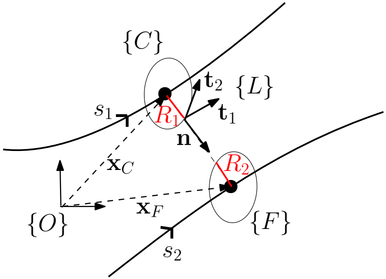
Kinematics for thin circular beams



$$\mathbf{n} \approx \frac{\mathbf{x}_F - \mathbf{x}_C}{\|\mathbf{x}_F - \mathbf{x}_C\|}, \quad \mathbf{t}_1 \approx \mathbf{m} - \left(\frac{\mathbf{m}^T\mathbf{n}}{\|\mathbf{n}\|^2}\right)\mathbf{n},$$

$$\mathbf{t}_2 \approx -\frac{S(\mathbf{n})\mathbf{t}_1}{\|S(\mathbf{n})\mathbf{t}_1\|}$$

$$\mathbf{R}_L = \{\mathbf{n}, \mathbf{t}_1, \mathbf{t}_2\},$$

$$\mathbf{H}_L = \begin{bmatrix} \mathbf{R}_L & \mathbf{x}_L \\ \mathbf{0} & 1 \end{bmatrix}, \quad \mathbf{H}_K = \begin{bmatrix} \mathbf{R}_L & \mathbf{x}_K \\ \mathbf{0} & 1 \end{bmatrix}$$

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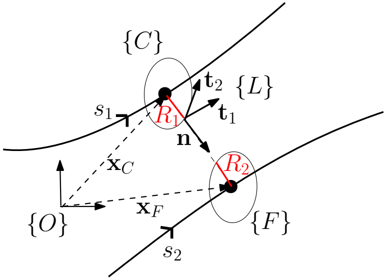

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Kinematics for thin circular beams

Relative configuration of the contact points:

$$\mathbf{H}_{LK} = \mathbf{H}_L^{-1}\mathbf{H}_K$$

Construct the normal gap and the tangential slip increment:

$$g_N = \mathbf{n}^T(\mathbf{x}_F - \mathbf{x}_C) - R_1 - R_2$$
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- The procedure can be applied for other flexible components

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Contact force

$$\mathbf{f}^{\text{con}}(\mathbf{H}_{LK}, \boldsymbol{\lambda}) = \int_{\gamma_c} (\delta\boldsymbol{\pi})^T \, \mathbf{G}^T \boldsymbol{\lambda} \, \mathrm{d}\gamma$$
$$= \mathbf{B}^T \boldsymbol{\Lambda}$$

Constraint gradient: $\mathbf{G}(\mathbf{H}_{LK})$

- Gives the direction of the contact force expressed in the respective local frames attached to the beam centerlines.
- Relative transformation from frames $\{L\}$,$\{K\}$ to $\{C\}$,$\{F\}$.
- Adjoint operator on $SE(3)$

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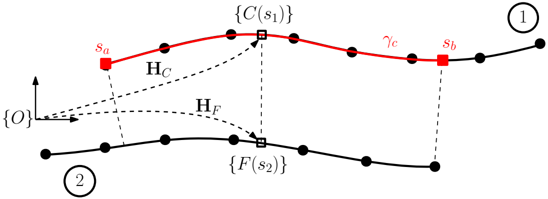
Discrete constraints: Collocation vs Mortar



Simple evaluation of all functions at the collocation points:

$$\begin{bmatrix} g_N^j \\ \mathbf{g}_T^j \end{bmatrix} = \begin{bmatrix} \mathbf{n}^T(\mathbf{x}_F(s_{2,j}^c) - \mathbf{x}_C(s_{1,j}^c)) - R_1 - R_2 \\ \mathbf{t}_1^T(\Delta\mathbf{x}_F(s_{2,j}^c) - \Delta\mathbf{x}_C(s_{1,j}^c)) \\ \mathbf{t}_2^T(\Delta\mathbf{x}_F(s_{2,j}^c) - \Delta\mathbf{x}_C(s_{1,j}^c)) \end{bmatrix}$$

For mortar the constraints are weighted integrals!

$$\begin{bmatrix} g_N^j \\ \mathbf{g}_T^j \end{bmatrix} = \int_{\gamma_c} \phi^j \begin{bmatrix} \mathbf{n}^T(\mathbf{x}_F - \mathbf{x}_C) - R_1 - R_2 \\ \mathbf{t}_1^T(\Delta\mathbf{x}_F - \Delta\mathbf{x}_C) \\ \mathbf{t}_2^T(\Delta\mathbf{x}_F - \Delta\mathbf{x}_C) \end{bmatrix} \mathrm{d}\gamma.$$

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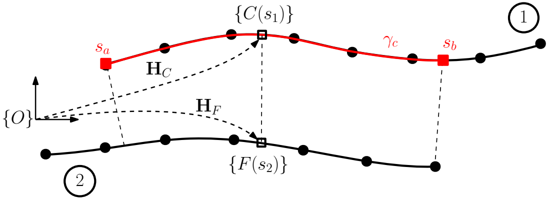

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Discrete frictional contact problem

- Discrete system configuration variable $q = \{\cdots, \mathbf{H}_{\bullet}, \cdots\}$
- Discrete Lagrange multipliers: $\boldsymbol{\Lambda}$

At each load step solve:

$$
\begin{aligned}
\mathbf{f}(q) + \mathbf{f}^{\text{con}}(q, \boldsymbol{\Lambda}) &= \mathbf{0}, \\
g_N^j &\in \partial\psi_{\mathbb{R}^-}(\lambda_N^j), \\
\text{if } g_N^j \leq 0: \quad \mathbf{g}_T^j &\in \partial\psi_{C(\lambda_N^j)}(\boldsymbol{\lambda}_T^j), \quad \forall j = 1, 2, ..., N_C.
\end{aligned}
$$

Augmented Lagrangian

Augmented multipliers:

$$\xi_N^j = k\lambda_N^j - p g_N^j, \quad \boldsymbol{\xi}_T^j = k\boldsymbol{\lambda}_T^j - p\mathbf{g}_T^j.$$

Lagrangian functional:

$$\mathcal{L}(q, \boldsymbol{\Lambda}) = \sum_{j=1}^{N_C} - k g_N^j \lambda_N^j + \frac{p}{2}(g_N^j)^2 - \frac{1}{2p}\mathrm{dist}^2\left[\xi_N^j, \mathbb{R}^+\right]$$

$$- k\mathbf{g}_T^j \cdot \boldsymbol{\lambda}_T^j + \frac{p}{2}\|\mathbf{g}_T^j\|^2 - \frac{1}{2p}\mathrm{dist}^2\left[\boldsymbol{\xi}_T^j, C_{\xi_N^j}\right],$$

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Monolithic nonsmooth Newton

Stationarity:

$$\delta\mathcal{L} = 0 \rightarrow \mathbf{F}^{\mathcal{L}}(q, \boldsymbol{\Lambda}) = \mathbf{0}$$

- $\mathbf{F}^{\mathcal{L}}$ is only piecewise differentiable

Newton update:

$$\Delta\mathbf{q} = -\mathbf{S}_t^{-1}\mathbf{F}^{\mathcal{L}}, \quad \text{with} \quad \mathbf{S}_t \in \partial\mathbf{F}^{\mathcal{L}}$$

- $\mathbf{S}_t$: generalized Jacobian $\rightarrow$ active set method

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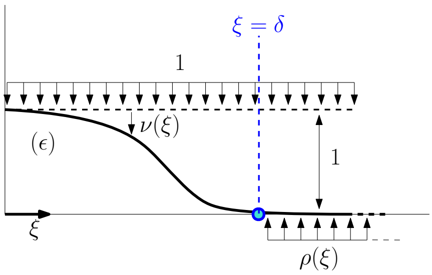

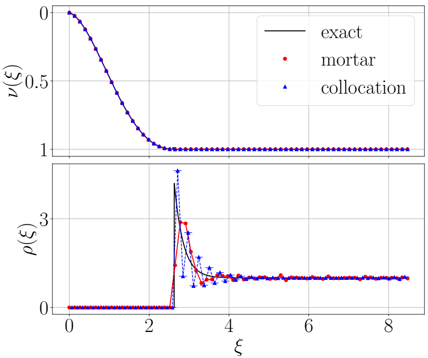

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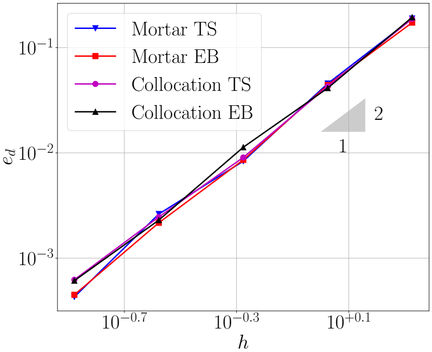
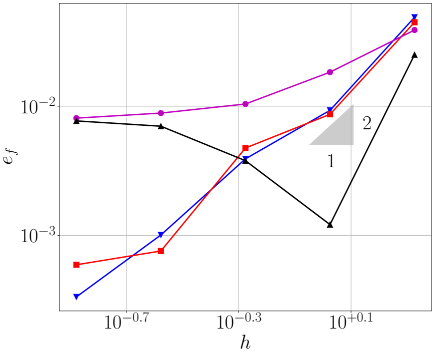
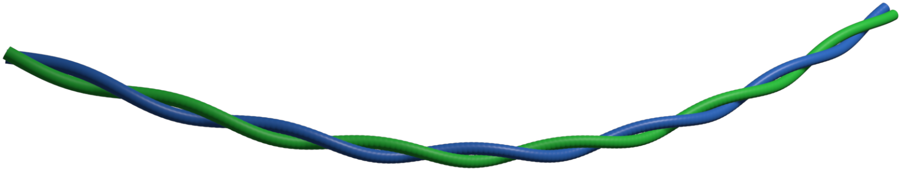




Bending of a pretwisted double helix: $\mu = 0.2$



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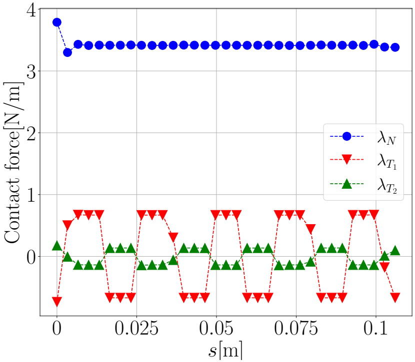

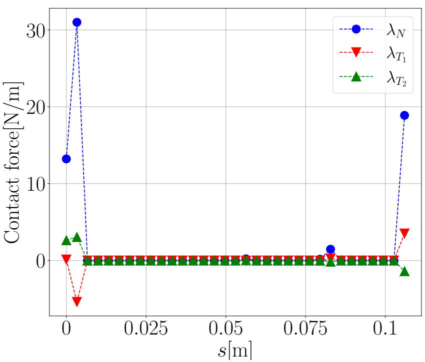

Dynamic nonsmooth effects

Origins

- Rigid bodies $\longrightarrow$ discontinuous velocities & impulsive forces
- Flexible bodies $\longrightarrow$ discontinuous velocities
- FE discretization $\longrightarrow$ numerical impacts between nodes

What to do?

- Regularize (penalty) & keep classical mathematical tools
- Mathematical framework for nonsmooth systems $\longrightarrow$ new class of time integration schemes

NSGA for systems with finite rotations and friction

A. Cosimo, F. J. Cavalieri, A. Cardona and O. Brüls, A general purpose formulation for nonsmooth dynamics with finite rotations, *J. Comput. Nonlinear Dyn.*, 16:031001-1 (2021)

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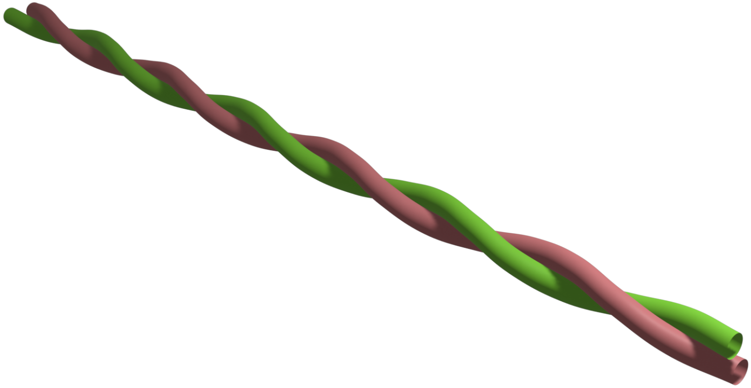
NSGA for systems with finite rotations and friction

A. Cosimo, F. J. Cavalieri, A. Cardona and O. Brüls, A general purpose formulation for nonsmooth dynamics with finite rotations, *J. Comput. Nonlinear Dyn.*, 16:031001-1 (2021)

Dynamic twisting: $\mu = 0.2$



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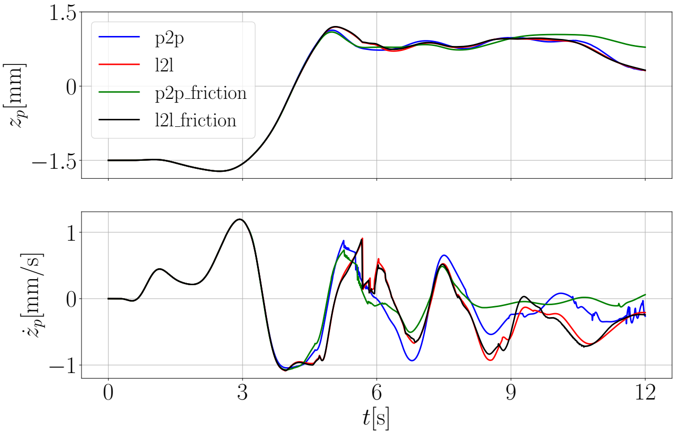

Conclusion and perspectives

- Application driven interest for distributed beam-to-beam contact
- The model contains spatial and temporal nonsmooth effects that impact numerical results
- We integrated a collocation and a mortar method into the local frame approach and the NSGA

- Collocation and mortar yield similar results for displacements
- Convergence rate of the contact forces is not optimal for collocation
- Presence of oscillations in the contact forces

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`https://doi.org/10.5281/zenodo.7468114`.

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Discontinuous velocities

Kinematics

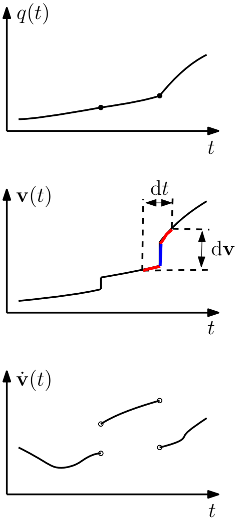
- q is continuous
- $\mathbf{v}$ has finite jumps
- $\dot{\mathbf{v}}$ defined for smooth motion

Substite time derivatives by differential measures

$$\mathrm{d}\mathbf{v} = \dot{\mathbf{v}}\,\mathrm{d}t + \sum_i \left[\mathbf{v}^+(t_i) - \mathbf{v}^-(t_i)\right]\delta_{t_i},$$

such that

$$\int_{(t_1,t_2]} \mathrm{d}\mathbf{v} = \mathbf{v}^+(t_2) - \mathbf{v}^+(t_1).$$



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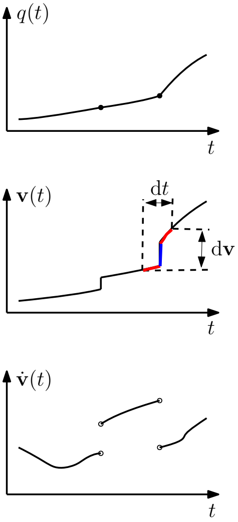

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such that

$$\int_{(t_1,t_2]} \mathrm{d}\mathbf{v} = \mathbf{v}^+(t_2) - \mathbf{v}^+(t_1).$$

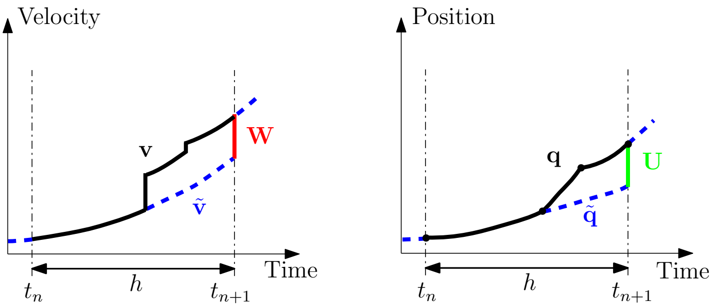
Contact forces

Similarly, we introduce the impulse measure of the contact reaction

$$\mathrm{d}\mathbf{i} = \boldsymbol{\Lambda}\,\mathrm{d}t + \sum_i \mathbf{p}_i \delta_{t_i},$$

- $\boldsymbol{\Lambda}$ is the vector of smooth Lagrange multipliers
- $\mathbf{p}_i$ is the impulse producing the jump at instant t_i

NSGA: One time step



$$\mathbf{v}(t) = \widetilde{\mathbf{v}}(t_n; t) + \mathbf{W}(t_n; t) \to \boldsymbol{\Lambda}$$
$$\mathbf{q}(t) = \widetilde{\mathbf{q}}(t_n; t) + \mathbf{U}(t_n; t) \to \boldsymbol{\nu}$$

- Smooth predictions $\tilde{\mathbf{q}}$ and $\widetilde{\mathbf{v}}$ integrated with generalized-α formulae

NSGA: Subproblems

$$\mathbf{M}(\widetilde{q}_{n+1})\dot{\widetilde{\mathbf{v}}}_{n+1} - \mathbf{f}(\widetilde{q}_{n+1}, \widetilde{\mathbf{v}}_{n+1}, t_{n+1}) = \mathbf{0}$$

$$\begin{aligned} \mathbf{M}(\widetilde{q}_{n+1})\mathbf{U}_{n+1} - h^2\mathbf{f}^p_{n+1} - \mathbf{B}^T_{n+1}\boldsymbol{\nu}_{n+1} &= \mathbf{0} \\ -g^j_{N,n+1} &\in \partial\psi_{\mathbb{R}^+}(\nu^j_{N,n+1}) \\ -\mathbf{g}^j_{T,n+1} \in \partial\psi_{C(\nu^j_{N,n+1})}(\boldsymbol{\nu}^j_{T,n+1}), \quad \text{if } g^j_{N,n+1} &\leq 0 \end{aligned}$$

$$\begin{aligned} \mathbf{M}(q_{n+1})\mathbf{W}_{n+1} - h\mathbf{f}^*_{n+1} - \mathbf{B}^T_{n+1}\boldsymbol{\Lambda}_{n+1} &= \mathbf{0} \\ -\mathbf{B}^j_{N,n+1}\mathbf{v}^j_{n+1} \in \partial\psi_{\mathbb{R}^+}(\Lambda^j_{N,n+1}), \quad \text{if } g^j_{N,n+1} &\leq 0 \\ -\mathbf{B}^j_{T,n+1}\mathbf{v}^j_{n+1} \in \partial\psi_{C(\Lambda^j_{N,n+1})}(\boldsymbol{\Lambda}^j_{T,n+1}), \quad \text{if } g^j_{N,n+1} &\leq 0 \end{aligned}$$

NSGA: Subproblems

$$\mathbf{M}(\widetilde{q}_{n+1})\dot{\widetilde{\mathbf{v}}}_{n+1} - \mathbf{f}(\widetilde{q}_{n+1}, \widetilde{\mathbf{v}}_{n+1}, t_{n+1}) = \mathbf{0}$$

$$\begin{aligned}
\mathbf{M}(\widetilde{q}_{n+1})\mathbf{U}_{n+1} - h^2\mathbf{f}^p_{n+1} - \mathbf{B}^T_{n+1}\boldsymbol{\nu}_{n+1} &= \mathbf{0}\\
-g^j_{N,n+1} &\in \partial\psi_{\mathbb{R}^+}(\nu^j_{N,n+1})\\
-\mathbf{g}^j_{T,n+1} \in \partial\psi_{C(\nu^j_{N,n+1})}(\boldsymbol{\nu}^j_{T,n+1}), &\quad \text{if } g^j_{N,n+1} \leq 0
\end{aligned}$$

$$\begin{aligned}
\mathbf{M}(q_{n+1})\mathbf{W}_{n+1} - h\mathbf{f}^*_{n+1} - \mathbf{B}^T_{n+1}\boldsymbol{\Lambda}_{n+1} &= \mathbf{0}\\
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\end{aligned}$$

NSGA: Subproblems

$$\mathbf{M}(\widetilde{q}_{n+1})\dot{\widetilde{\mathbf{v}}}_{n+1} - \mathbf{f}(\widetilde{q}_{n+1}, \widetilde{\mathbf{v}}_{n+1}, t_{n+1}) = \mathbf{0}$$

$$\begin{aligned} \mathbf{M}(\widetilde{q}_{n+1})\mathbf{U}_{n+1} - h^2\mathbf{f}^p_{n+1} - \mathbf{B}^T_{n+1}\boldsymbol{\nu}_{n+1} &= \mathbf{0} \\ -g^j_{N,n+1} &\in \partial\psi_{\mathbb{R}^+}(\nu^j_{N,n+1}) \\ -\mathbf{g}^j_{T,n+1} \in \partial\psi_{C(\nu^j_{N,n+1})}(\boldsymbol{\nu}^j_{T,n+1}), \quad \text{if } g^j_{N,n+1} &\leq 0 \end{aligned}$$

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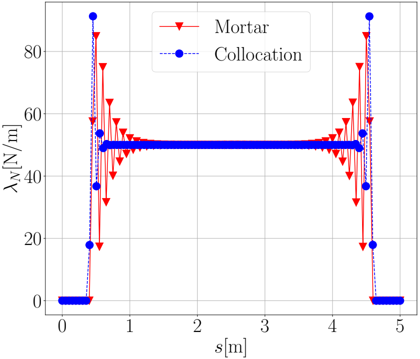
Twisting: $\mu = 0$



Twisting: $\mu = 0.2$