A Nonsmooth Approach to Frictionless Beam-to-Beam Contact

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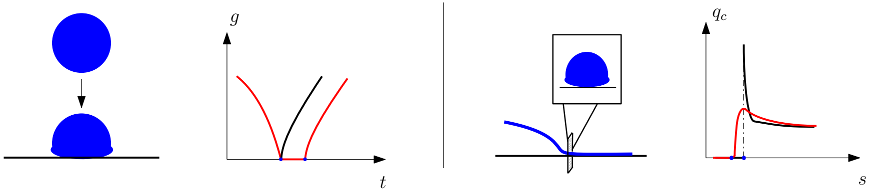
Outline

- **Introduction**
  - Kinematic hypotheses and discontinuity
  - Need of nonsmooth methods for beam contact
- **Time integration scheme and contact formulation**
  - NSGA
  - Mortar for line-to-line contact
- **Test cases**
  - 3 fiber twisting
  - Bouncing parallel beams
  - Swinging beam with point contact

Kinematic hypotheses and discontinuity

Continuum Mechanics

- Impacts of finite duration, Continuous surface tractions
- Ensured via "local scale" deformability



Multibody components

- Reduced kinematics introduces nonsmoothness [1, 2]
- Analyst chooses time and spatial scales of interest

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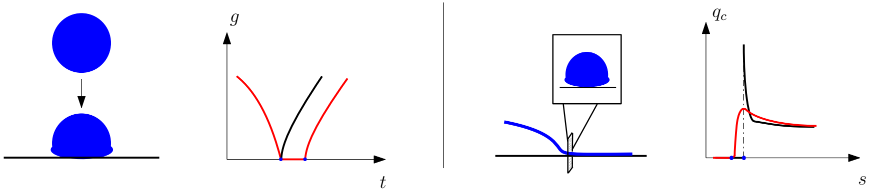

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Need of nonsmooth methods for beam contact

Nonsmooth behaviour in time

- Coupling with rigid bodies (discontinuous velocities & impulses)
- Rigidity assumption of cross-sections
- FE-discretization: Numerical impacts between nodes
- Stick-slip transitions in frictional contact

Nonsmooth behaviour in space

- FE-discretization leads to non-matching grids
- Discontinuous distributed contact forces and steep gradients

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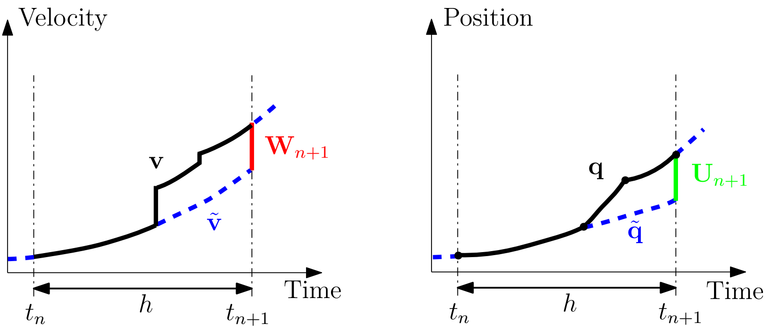
(Decoupled) nonsmooth generalized-α [3]



$$\mathbf{v}(t) = \widetilde{\mathbf{v}}(t_n; t) + \mathbf{W}(t_n; t) \rightarrow \boldsymbol{\mu}$$
$$\mathbf{q}(t) = \widetilde{\mathbf{q}}(t_n; t) + \mathbf{U}(t_n; t) \rightarrow \boldsymbol{\nu}$$

- Corrections $\mathbf{W}_{n+1}$ and $\mathbf{U}_{n+1}$ integrated with $\Theta(h)$
- Smooth predictions $\tilde{\mathbf{q}}$ and $\widetilde{\mathbf{v}}$ integrated with $\Theta(h^2)$

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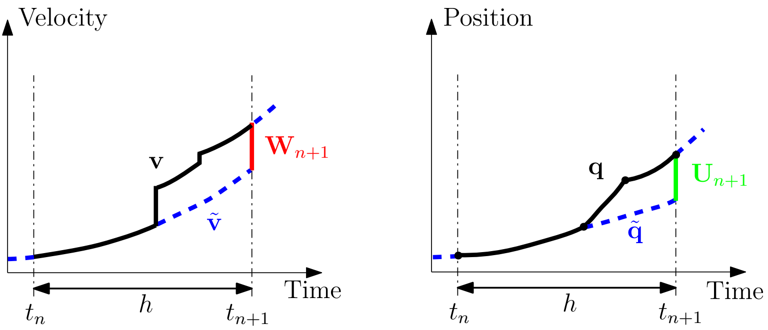

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Smooth prediction

$$\mathbf{M}(\widetilde{\mathbf{q}})\dot{\widetilde{\mathbf{v}}} - \mathbf{f}(\widetilde{\mathbf{q}}, \widetilde{\mathbf{v}}, t) = \mathbf{0}$$

Position correction

$$\begin{aligned} \mathbf{M}(\widetilde{\mathbf{q}})\mathbf{U} - h^2\mathbf{f}^p(\mathbf{q}, \widetilde{\mathbf{q}}, \mathbf{v}, \widetilde{\mathbf{v}}, t) - \mathbf{g}_{\mathbf{q}}^T\nu &= \mathbf{0} \\ \mathbf{0} \leq \mathbf{g} \perp \nu &\geq \mathbf{0} \end{aligned}$$

Velocity jump

$$\begin{aligned} \mathbf{M}(\mathbf{q})\mathbf{W} - h\mathbf{f}^*(\mathbf{q}, \widetilde{\mathbf{q}}, \mathbf{v}, \widetilde{\mathbf{v}}, \dot{\widetilde{\mathbf{v}}}, t) - \mathbf{g}_{\mathbf{q}}^T\mu &= \mathbf{0} \\ \text{if } g^j(\mathbf{q}) \leq 0 \text{ then} \qquad \mathbf{0} \leq g_{\mathbf{q}}^j\mathbf{v} \perp \mu^j &\geq 0 \end{aligned}$$

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Characteristics

- Sequence of three decoupled problems
- Second order approximation of smooth terms and control of dissipation
- Exact enforcement of constraints at position and velocity level (*GGL* type formulation)
- Acceleration constraints may be included [4, 5]

Solution scheme for each subproblem

- Monolithic augmented *Lagrangian* approach with a semismooth *Newton* algorithm (*Robustness for many contacts*? [6])
- Other option: *Gauß-Seidel* type + *Fisher-Burmeister* functional [7]
- Implemented in *Odin* [8]

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Mortar for line-to-line contact [9]

Distributed contact force:

$$\lambda(s,t) = \phi(s)\boldsymbol{\lambda}(t),$$

$$\boldsymbol{\lambda} = \left[\lambda^1 \dots \lambda^j \dots \lambda^m\right]^T, \boldsymbol{\phi} = \left[\phi^1 \dots \phi^j \dots \phi^m\right]^T, m = \deg(\phi) + 1$$

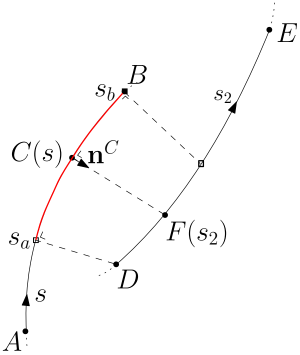
We assume $\boldsymbol{\lambda}(t) \in SLBV(\mathbb{R};\mathbb{R}^m)$

Position constraint enforced in a weak sense:

$$g^j(t) = \int_{s_a}^{s_b} g(s,t)\phi^j(s)\,\mathrm{d}s$$

Associated velocity constraint:

$$g^j_{\mathbf{q}}(t)\mathbf{v} = \int_{s_a}^{s_b} g_{\mathbf{q}}(s,t)\mathbf{v}\phi^j(s)\,\mathrm{d}s$$



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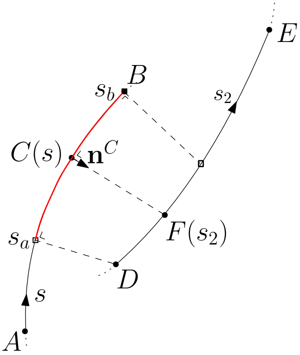

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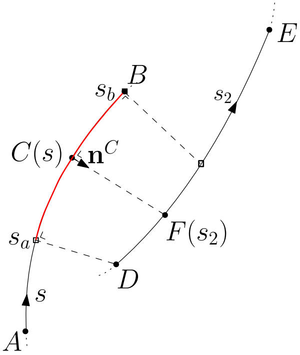
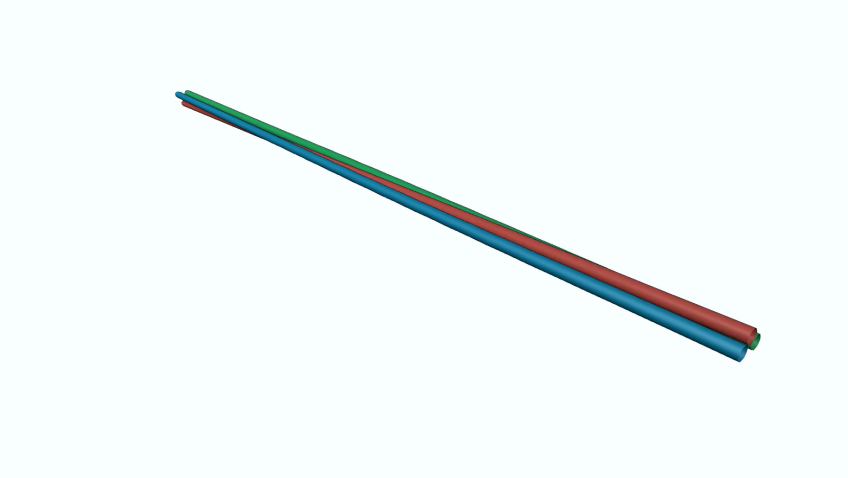


Mortar for line-to-line contact

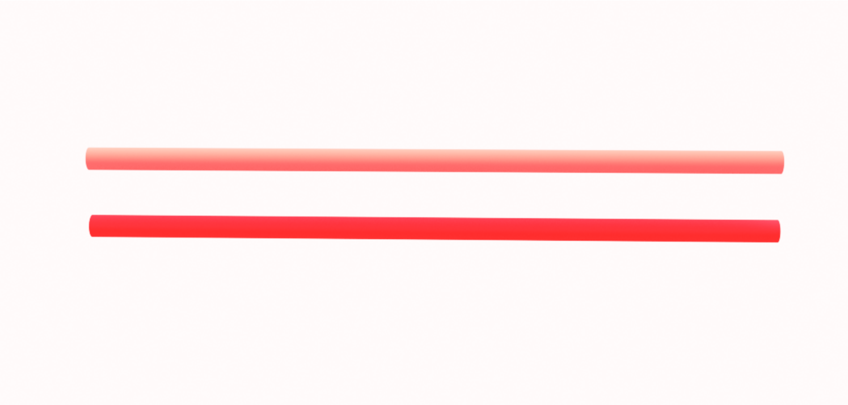
Characteristics

- Conservation of optimal spatial convergence rates
- The total contact force is well represented
- No locking or over constraining

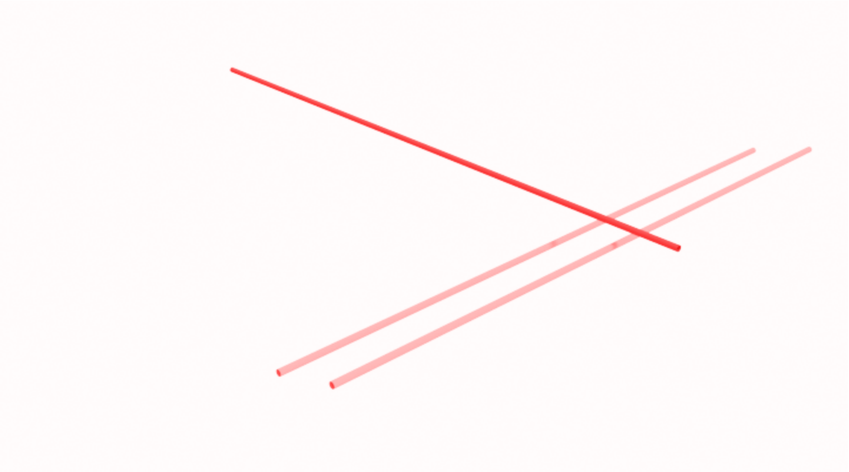
3 fiber twisting



Bouncing parallel beams



Swinging beam with point contact



Concluding remarks

Nonsmooth methods for beam-to-beam contact:

- Mortar for line-to-line contact extended to the dynamic case
- Some pathological situations can be dealt with using a point-to-point model

Perspectives:

- Upscaling towards more complex assemblies
- Gauß-Seidel
- Friction

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Thank you for your attention!

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