Experimental study of the discharge of granular quasi-2D silo in a uniform magnetic field

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Abstract. We present an experimental study of the discharge of a monolayer of a ferromagnetic granular medium in a quasi-2D silo surrounded by vertical uniform magnetic field. We observe that the cohesion induced by the vertical magnetic field tends to canalize the flow above the outlet, modifying its morphology with little incidence on the mass flow rate. Using a direct measurement method of velocities based on particle tracking, we are able to extract an altitude-dependent diffusion length of the granular medium, previously introduced in the kinetic model, and propose a qualitative interpretation of its decay with cohesion.

1 Introduction

Granular media frequently shows complex cohesive behavior in various geophysical phenomena and industrial applications as in additive manufacturing [1] or in food processes [2]. The difficulty in apprehending such cohesive behavior lies in the hard, and often impossible, ways of controlling cohesion in granular materials. Although granular materials are used in many processes in the industry, its behavior remains quite obscure. Therefore, various standard experiments have been used in the past to extract rheological properties of granular materials, such as slump test [3], pile angles [4] or inclined planes flow [5]. In particular, silo discharge is a widely used system, some properties of which remain challenging to fully understand to this day. It has been extensively studied for cohesionless granular media [6–9] and more recent works focus on the cohesive case [10]. In particular the influence of aperture size on the flow rate was widely studied, firstly by Hagen in 1852 (translated in [6]) followed by more recent works [7–9]. Considering grains of size d falling freely through an opening of a given size $D \times W$, one can show that the mass flow rate Q may be written:

$$Q(D) \propto \sqrt{g} W D^{3/2} \tag{1}$$

Later, Beverloo [7] introduced a new parameter k to take account of grains obstructing part of the aperture, leading to the following expression :

$$Q(D) = C\rho\phi_b \sqrt{g} W (D - kd)^{3/2} \tag{2}$$

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Where ρ represents the density of the grains, ϕ_b denotes the volume fraction of the bulk, W is the width of the silo, D corresponds to the aperture size length, d is the mean grain diameter. C is a dimensionless fitting parameter ranging between 0.55 and 0.65 [7]. Since k accounts for the fraction of grains obstructing the aperture, it should be close to 1, which was shown by Mankoc et al 2007 [8] for large outlet (i.e. large D/d ratio) implying that dilatancy effects that cannot be neglected for small aperture are not captured by the Beverloo law.
The morphology of the velocity profile in the bulk of the silo was also investigated [11–15]. In that case, the flow is controlled by the cooperative dynamics of particles at the outlet and the kinetic model attempts to account for the resulting diffusion processes. This model is based on the hypothesis of the existence of a correlation length, denoted b, between the horizontal velocity v_x and the horizontal velocity gradient $\partial_x v_y$ such as:

$$v_x = b\frac{\partial v_y}{\partial x} \tag{3}$$

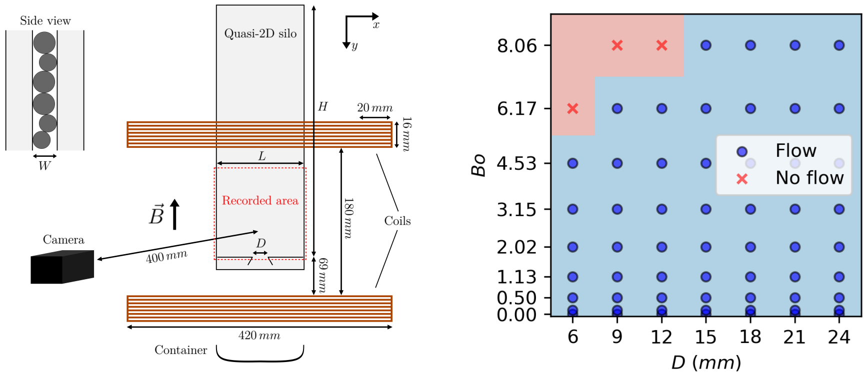
Assuming that the mass flow rate Q is constant, which is observed experimentally and the flow is mostly incompressible, the vertical velocity v_y can be calculated analytically such as:

$$v_y(x, y) = \frac{Q}{\sqrt{4\pi b y}}\, exp\left(\frac{-x^2}{4by}\right) \tag{4}$$

which correspond to a velocity profile close to a gaussian distribution. Since a strong hypothesis of the model is the incompressibility of the flow, this analytical formula do not capture the observed velocity profile close to the oultlet.
In this work, we investigate the flow of a remotely cohesion-controlled granular media in a quasi-2D silo, which allows us to observe the effect of the cohesion on the flow. We evidence the influence of cohesion on previous existing models.

2 Experiments

A monolayer of steel beads (mean diameter d = 1.1 mm ± 0.1 mm) is confined in a quasi-2D silo made of conductor polycarbonate plates. The dimensions of the cavity are H = 300 mm (height), L = 100 mm (width) and W = 1.25 mm (depth) (Fig. 1 (a)).



(a) (b)

Figure 1. (a) Sketch of the experimental setup. (b) Phase diagram for the outlet flow in the parameter space (D, Bo).

We characterize the flow for aperture size D from 6 mm to 24 mm. Images of the discharge recorded by a high-speed camera are analysed by using PTV to extract the velocity v, the volume fraction ϕ and reconstruct the flow rate Q at each timestep. Coils in Helmholtz configuration subjected to a current I allow one to apply a uniform magnetic field $\vec{B}$ across the opening of the silo. Cohesion is controlled by adjusting the current passing through them (up to 4.0 A) which corresponds to a range of applied magnetic field from 0 G to 27.2 G. Steel particles placed in a magnetic field acquire magnetic dipolar moments, and behave like small magnets. They attract each other in the vertical direction and repel each other in the horizontal direction as shown in previous work [16]. To quantify cohesion, we define a magnetic Bond number, denoted as $Bo = F_m/mg$ where F_m is the normal magnetic force and m is the mass of a particle. As discussed in [17], the expression of the Bond number may be reduced to:

$$Bo = \frac{I^2}{I_{ref}^2} \tag{5}$$

where I_{ref} is the reference current required for the magnetic force to equal the weight of a single grain ($Bo = 1$). Measurements lead to a reference current of $I_{ref} = 1.409 \pm 0.144$ A and Bond numbers from 0 to 8.06.

3 Results

3.1 Outlet flow

We observe a flow threshold emerging for a critical cohesion at a given aperture D, as shown in Fig. 1 (b). The flow does not occur beyond this threshold. In the cohesion-less case, it is known that velocity and volume fraction profiles are self-similar [9]. Below this threshold, our results show that cohesion does not change the shape of velocity profiles (Fig.2 (a)) and have only a little incidence on volumic fraction profiles (Fig.2 (b)).

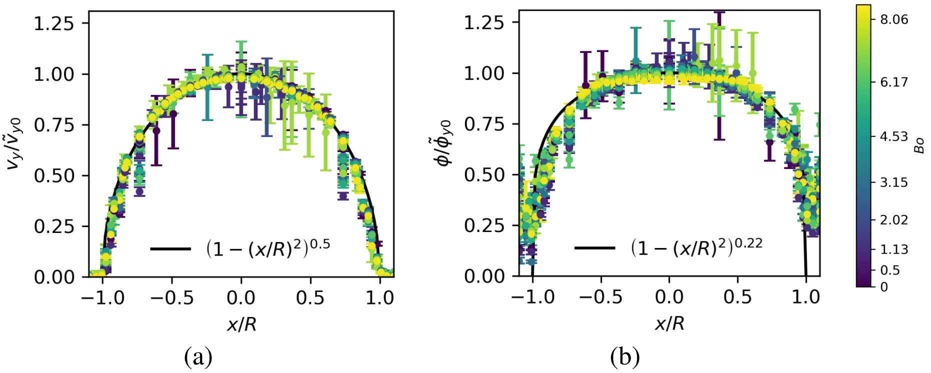
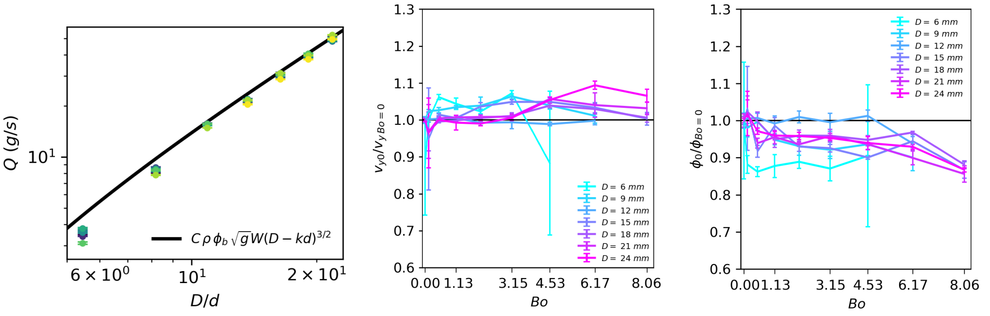


Figure 2. (a) Vertical velocity profiles and (b) volumic fraction profiles against cohesion Bo normalized by their fitted center value $\tilde{v}_{y0}$ and $\tilde{\phi}_0$.

From these data, we investigate the evolution of the flow rate Q for different aperture size D and different cohesion by computing the integral of these profiles. Our experiments shows that the evolution of the flow rate is well captured by the Beverloo model (Fig.3 (a)) even with cohesion, with a little deviation for small values of D/d which is consistent with the literature. A more detailed analysis shows that cohesion slightly increases the velocity v_{y0} at the center of the aperture (Fig.3 (b)) but reduces the volume fraction ϕ_{y0} (Fig.3 (c)), which implies that magnetic cohesion has little influence on the flow rate.

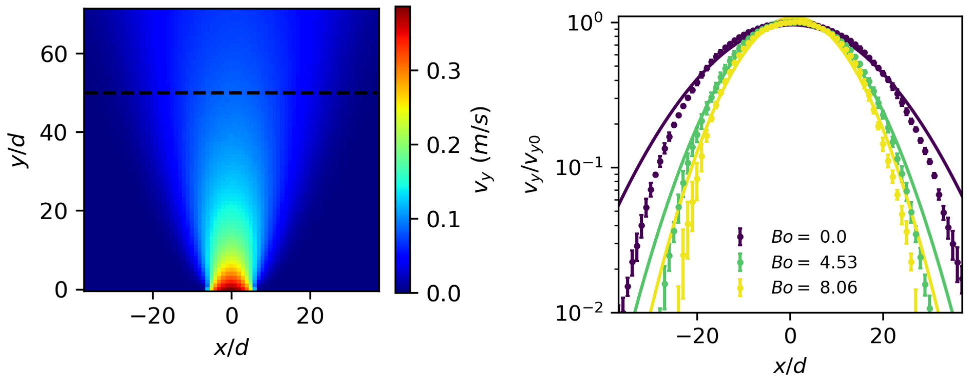


(a) (b) (c)

Figure 3. (a) Flow rate versus the aperture D and cohesion Bo and Beverloo law for $C = 0.65$ and $k = 1.5$. Evolution of (b) the velocity and (c) the volume fraction at the center of the outlet versus cohesion.

3.2 Bulk flow

Our results show that, in the bulk of the silo, the vertical velocity profiles resemble Gaussian distributions for cohesionless granular materials, which is consistent with the literature [13], and also for cohesive granular material (Fig. 4 (a) and (b)). Moreover, we observe that the vertical magnetic field does not alter the Gaussian-like shape of the velocity profiles (Fig. 4 (b)), although it channels the flow (Fig. 5). This effect of cohesion extends the dead zone where the velocity is close to zero, thereby making the velocity field more vertical. This channeling effect has already been observed by Gans *et al.* [10] for the flow of cohesive polymer-coated granular materials, and it is interesting to note that we observe quite the same behavior, although the origin of cohesion is different.



(a) (b)

Figure 4. (a) Average vertical velocity field $v_y(x, y)$ for $Bo = 0$ and $D = 15$ mm and (b) normalized vertical mean velocity profiles for $D = 15$ mm for three different cohesion. The profiles are taken at $y = 50d$ (black doted line on (a)).

In the framework of the kinematic model [15], it is possible to account for the phenomenon of channeling through the characteristic diffusion length b using (4). First we observe that the value of b is not constant with the altitude in the bulk which is also consistent with the literature [14]. However we observe that the value of b decreases with cohesion for each aperture size (Fig. 6). This implies a more vertical motion of the grains leading to a higher velocity at the outlet (Fig.3 (c)). Our observation suggests that there is a critical Bond number *Bo* different for each outlet diameter where the value of b becomes constant with the altitude in the bulk for $y > 40d$. This result implies that for a critical cohesion value, the velocity profile in the bulk also becomes self-similar.

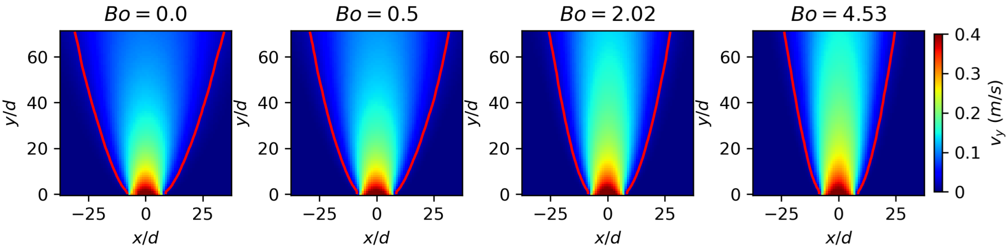


Figure 5. Velocity profiles $v_y(x, y)$ for different Bond number. The red line is the isovelocity line corresponding to 5% of the maximum velocity.

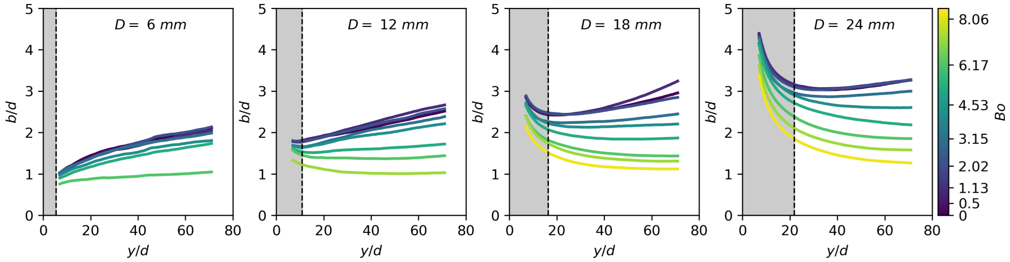


Figure 6. Fitting parameter of the kinetic model b for $D = 6$ *mm*, $D = 12$ *mm*, $D = 18$ *mm* and $D = 24$ *mm*. The gray zone corresponds to the region of the flow where the kinetic model cannot be considered valid due to the dilatancy close to the oultet that cannot be neglected.

4 Conclusion

Using magnetic field and ferromagnetic granular material, we investigated the effect of cohesion induced by a vertical magnetic field on the discharge of a quasi-2D silo. We reported a non-monotonic evolution of the flow rate with cohesion for a given aperture size, and an anti-correlated behavior of the velocity and the volume fraction at the outlet. Also our results show that, in the bulk, the velocity profile for a given altitude can be approximated by a gaussian-like profile determined by what may be seen as a diffusion length b for cohesionless granular media, which is consistent with the literature, and also for cohesive granular media, which is new. Moreover, our results show that cohesion tends to channel the flow which has already been observed in the literature, and suggest that there is a critical Bond number where the value of b becomes constant with the altitude in the bulk, a phenomenon that has never been observed before. More experiments are needed to fully apprehend the hidden mechanism behind this evolution of b with cohesion. We believe that this investigation on the effect of cohesion on the velocity profile, both at the outlet and in the bulk, constitute a promising perspective on the full comprehension of silo discharge.

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