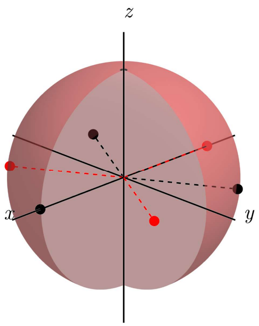
Robust dynamical decoupling using the Majorana constellation



Colin Read, Eduardo Serrano-Ensástiga and John Martin

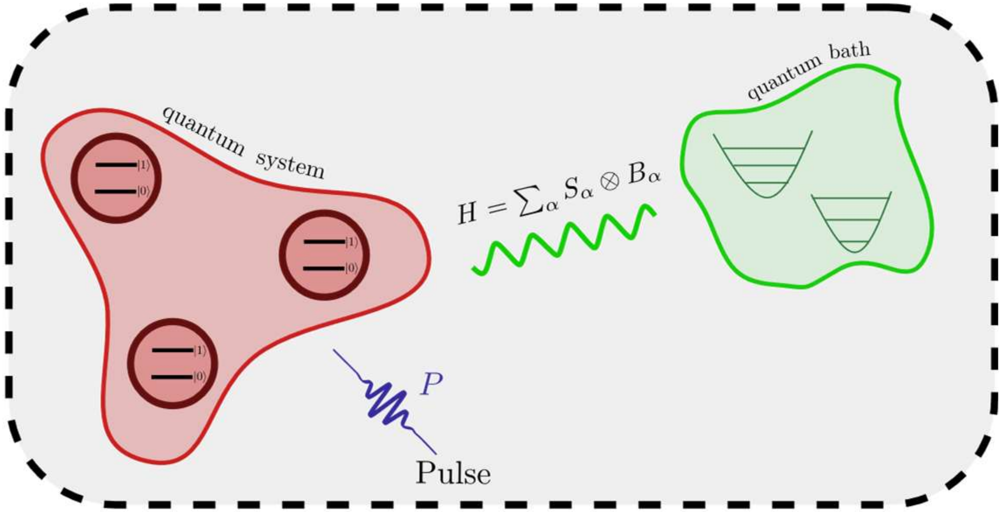
Institut de Physique Nucléaire, Atomique et de Spectroscopie, CESAM, University of Liège, Belgium

Quantum 9, 1661 (2025)

Outline

- Introduction to dynamical decoupling
- Symmetries in Majorana constellations
- Application for spin systems

System-bath interaction

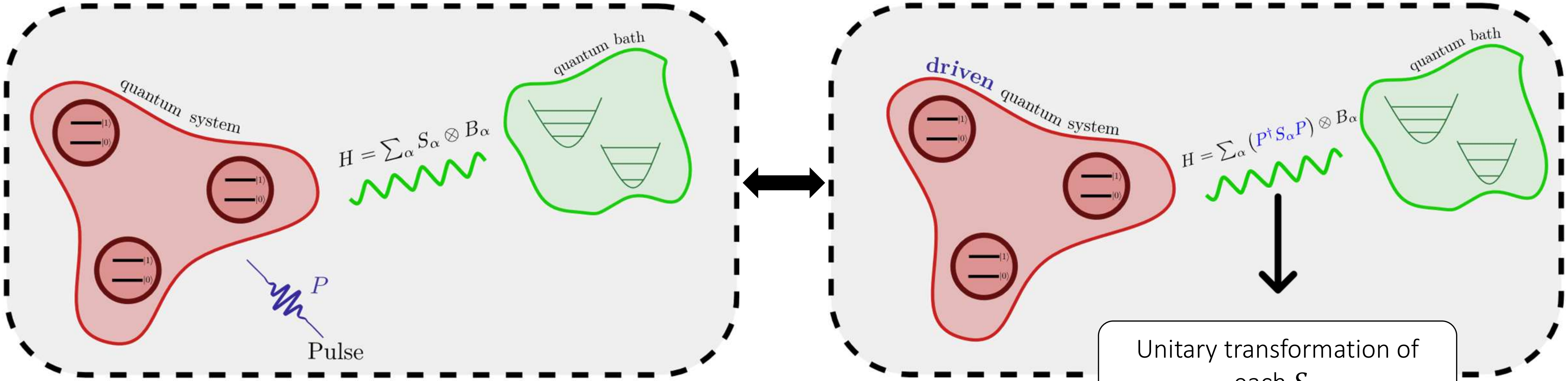


Unwanted dynamics

Engineering of
the interaction

- Longer coherence time
- Hamiltonian engineering

System-bath interaction



Unitary transformation of each S_α

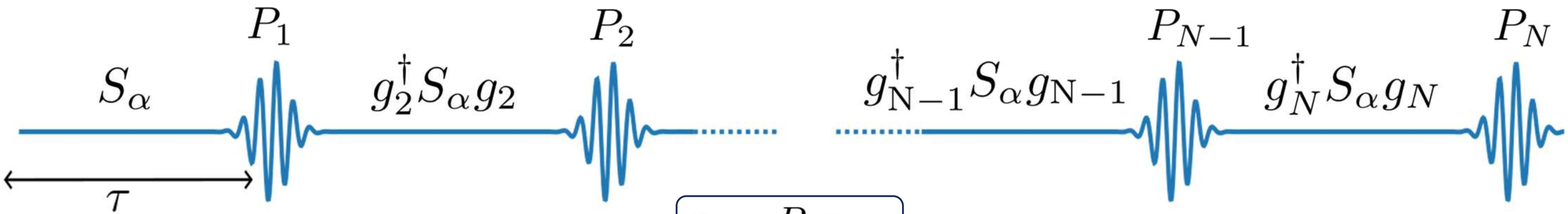
Symmetrization

$$\boxed{g_j = P_j g_{j-1}}$$

Sequence of unitary transformation → Average Hamiltonian Theory (AHT)

L. Viola *et al.*, Phys. Rev. Lett. **82**, 2417 (1999)

Symmetrization



$$\boxed{g_j = P_j g_{j-1}}$$

Sequence of unitary transformation → Average Hamiltonian Theory (AHT)

$$S_\alpha \quad \longmapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N} \sum_k g_k^\dagger S_\alpha g_k$$

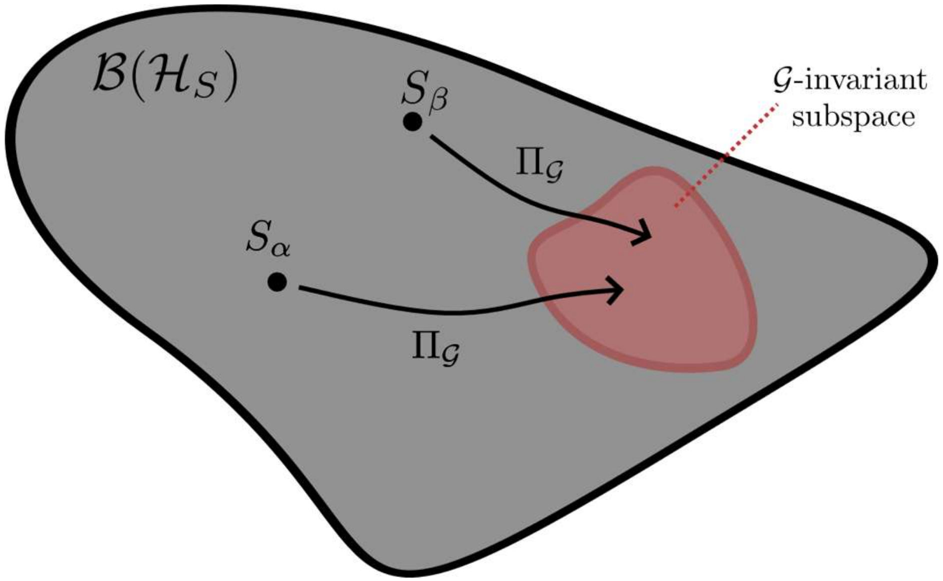
« symmetrization operation »

L. Viola *et al.*, Phys. Rev. Lett. 82, 2417 (1999)

Symmetrization

$$S_\alpha \quad \longmapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

If $\mathcal{G} = \{g_k\}$ is a group…



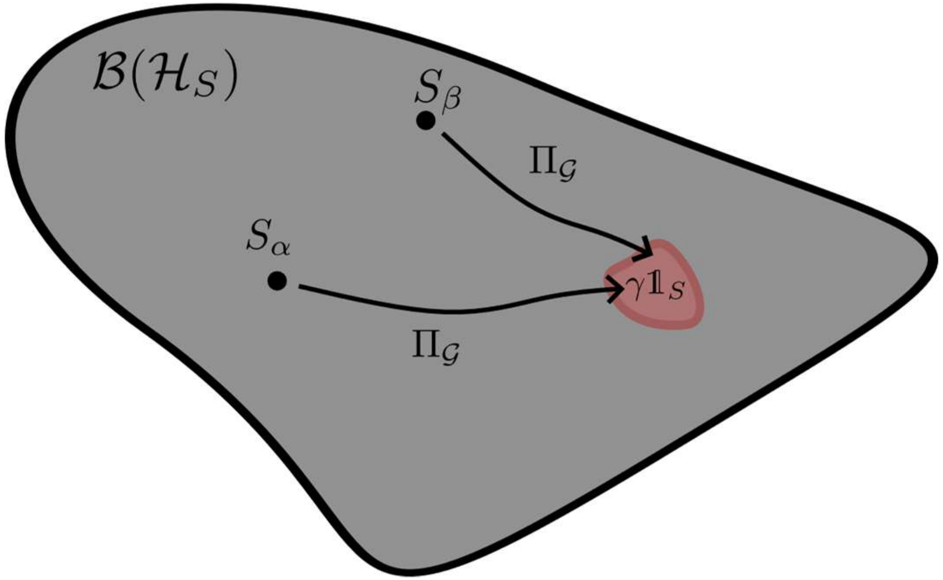
L. Viola *et al.*, Phys. Rev. Lett. 82, 2417 (1999)
P. Zanardi, Phys. Lett. A 258, 77 (1999)

Symmetrization

$$S_\alpha \quad \longmapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

If $\mathcal{G} = \{g_k\}$ is a group...

If the symmetry is inaccessible, the system is decoupled from its environment



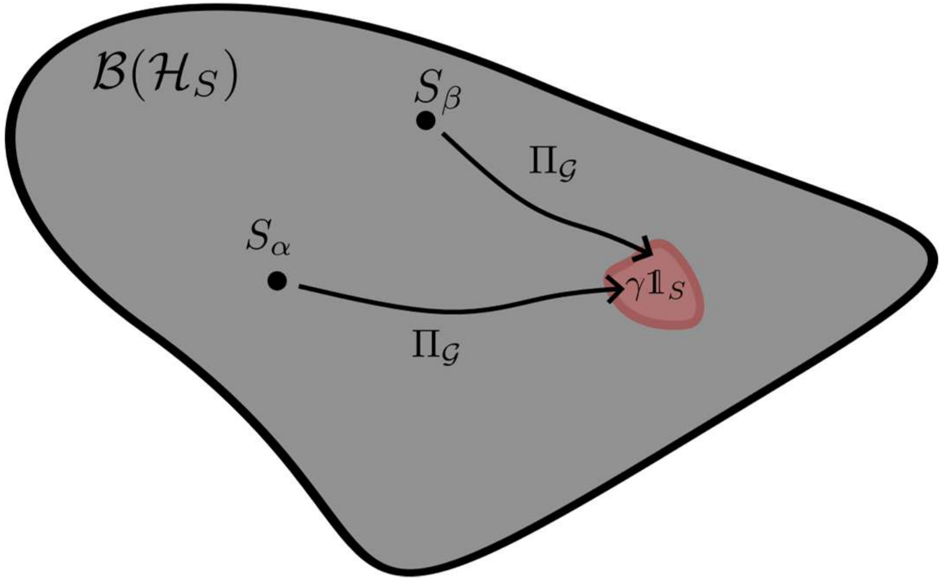
Symmetrization

$$S_\alpha \quad \longmapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

If $\mathcal{G} = \{g_k\}$ is a group…

If the symmetry is **inaccessible**, the system is decoupled from its environment

« **decoupling group** »



SU(2) symmetries

$$S_\alpha \quad \mapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

$$R(\boldsymbol{n}, \theta) = e^{-i\theta \boldsymbol{n}\cdot\boldsymbol{J}}$$

We focus on global SU(2) pulses

SU(2) symmetries

$$S_\alpha \longmapsto \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

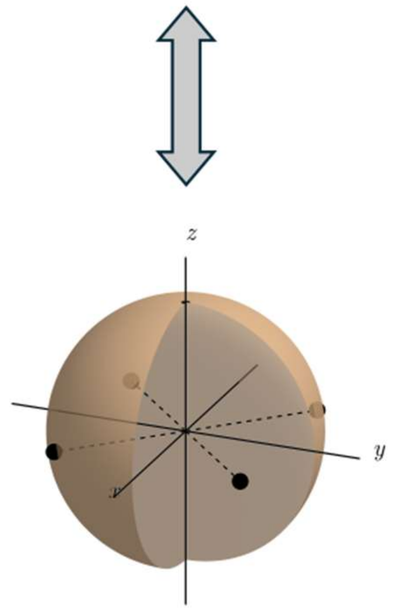
$$R(\boldsymbol{n}, \theta) = e^{-i\theta \boldsymbol{n}\cdot\boldsymbol{J}}$$

We focus on global SU(2) pulses

→ **Majorana constellation**

Majorana constellation for pure states

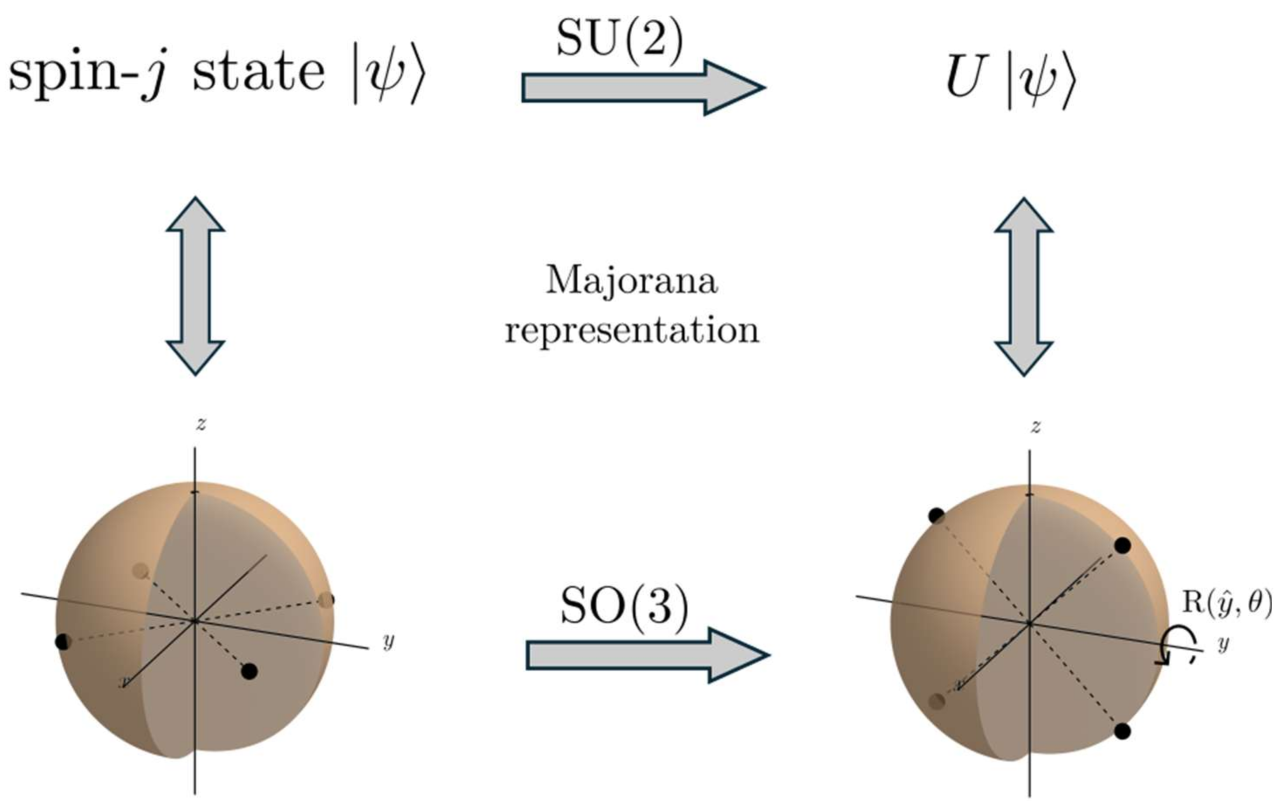
spin-j state $|\psi\rangle$



Information stored in the position of the **stars**

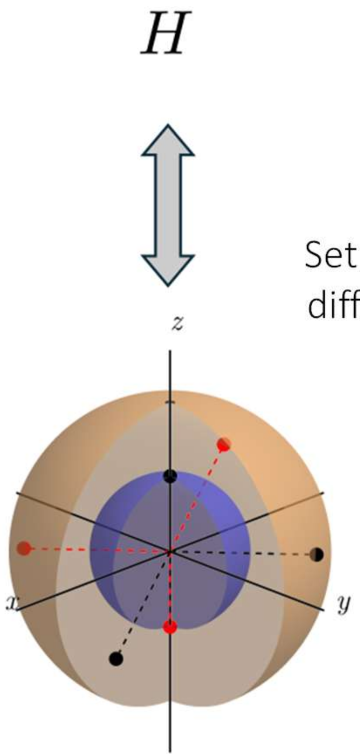
E. Majorana, *Nuovo Cim* 9, 43–50 (1932)

Majorana constellation for pure states



E. Majorana, *Nuovo Cim* 9, 43–50 (1932)

Majorana constellation of operators



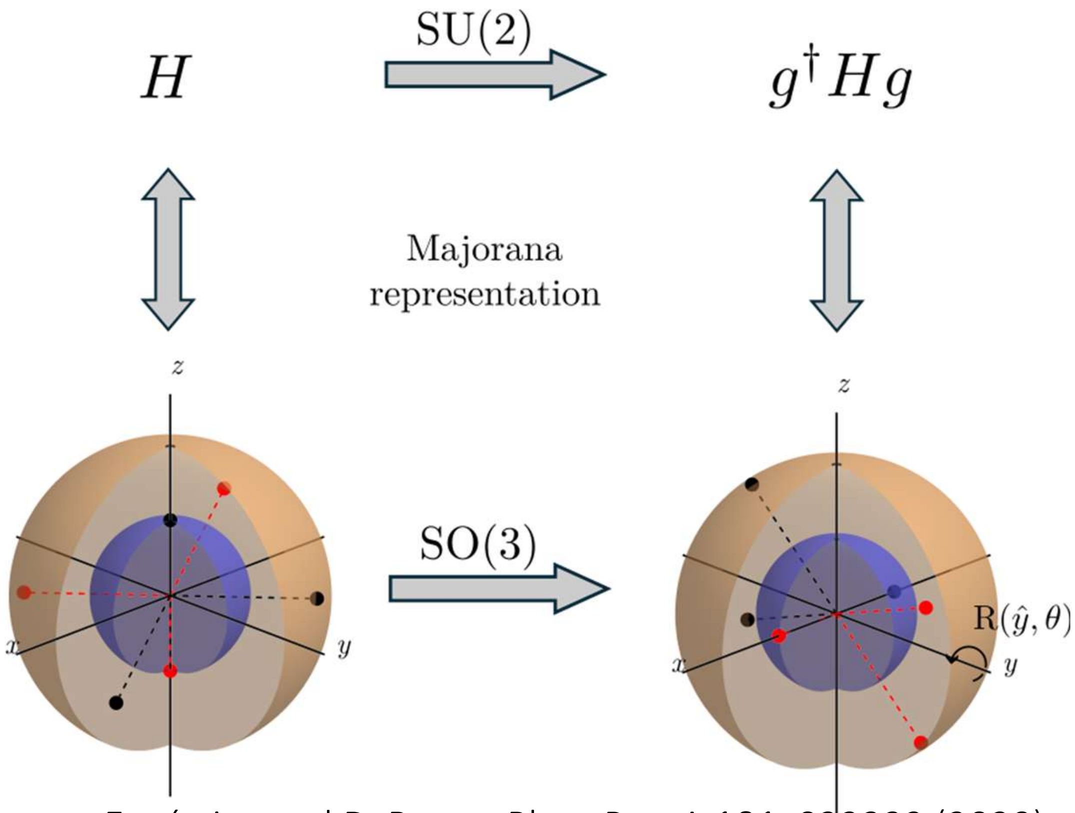
Set of constellations with different sizes and radius

Information stored in :

- Position of the stars
- Radius of the sphere
- Coloring of the stars

E. Serrano-Ensástiga and D. Braun, Phys. Rev. A **101**, 022332 (2020)

Majorana constellation of operators



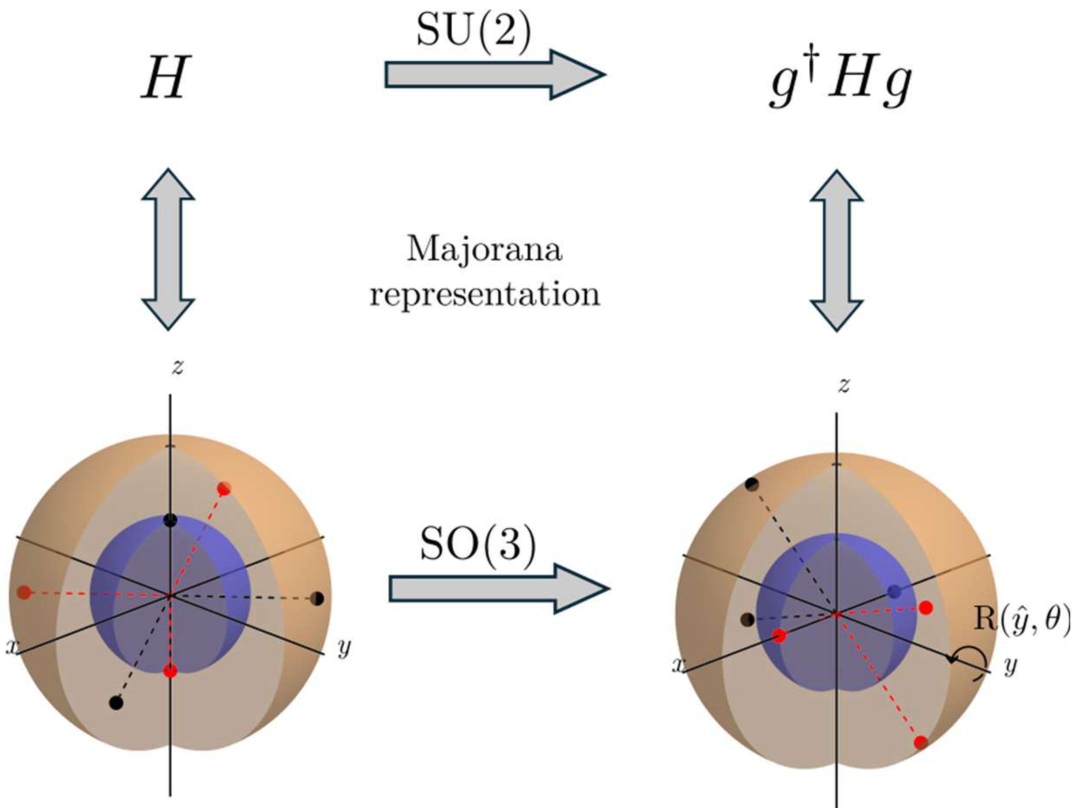
SU(2) symmetries

↕

Rotational symmetries in the constellation

E. Serrano-Ensástiga and D. Braun, Phys. Rev. A **101**, 022332 (2020)

Majorana constellation of operators



SU(2) symmetries

↕

Rotational symmetries in the constellation

→ The highest accessible symmetry depends on the number of points in the constellation

we can list all inaccessible symmetries for each constellation

E. Serrano-Ensástiga and D. Braun, Phys. Rev. A **101**, 022332 (2020)

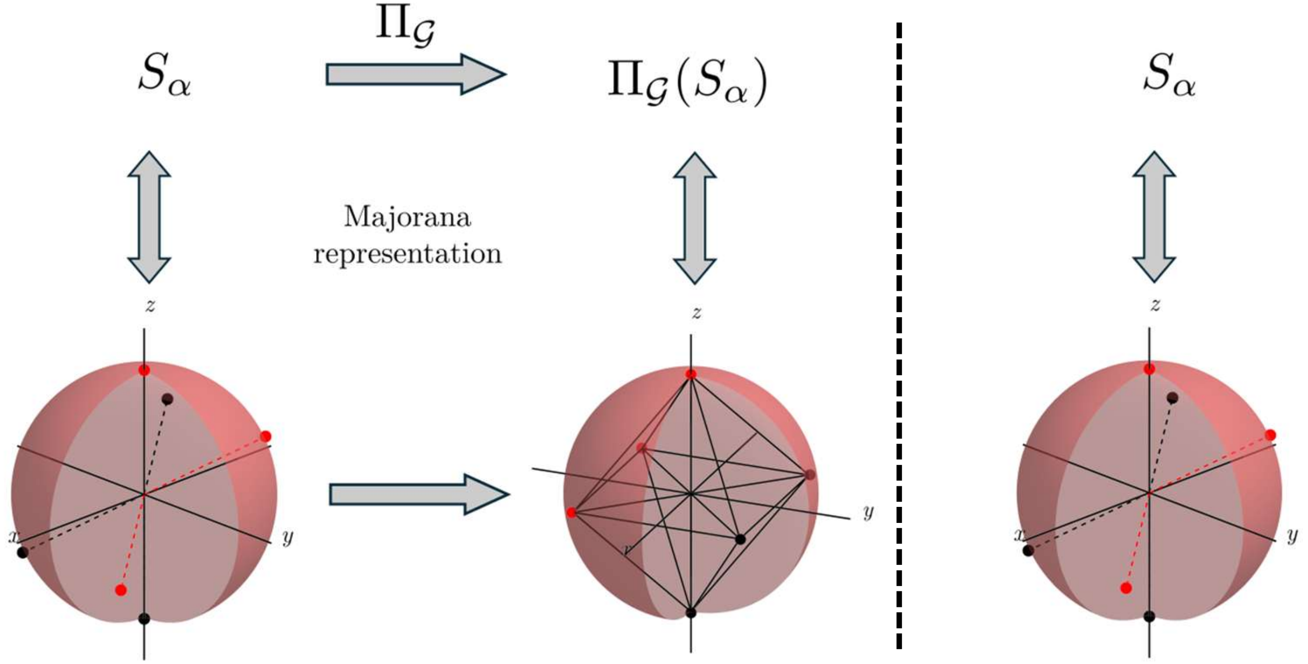
Symmetrizing a Majorana constellation

$$S_\alpha \quad \mapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

$$R(\boldsymbol{n},\theta) = e^{-i\theta\boldsymbol{n}\cdot\boldsymbol{J}}$$

Accessible symmetry

Inaccessible symmetry



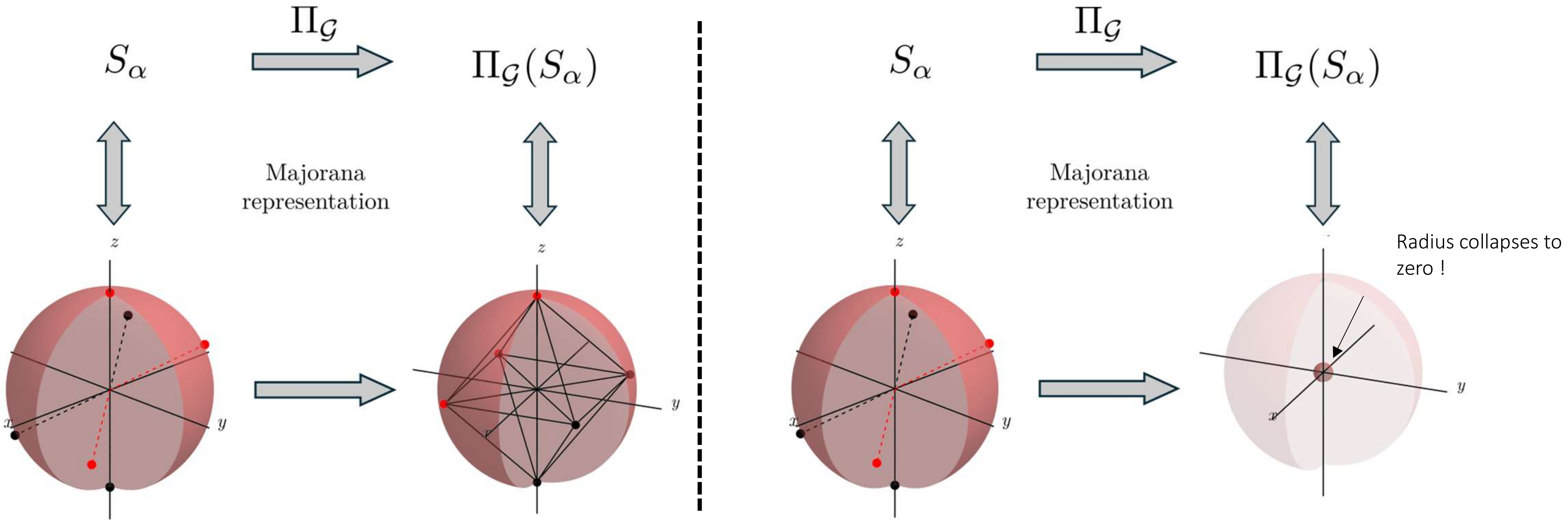
Symmetrizing a Majorana constellation

$$S_\alpha \quad \mapsto \quad \Pi_{\mathcal{G}}(S_\alpha) = \frac{1}{N}\sum_k g_k^\dagger S_\alpha g_k$$

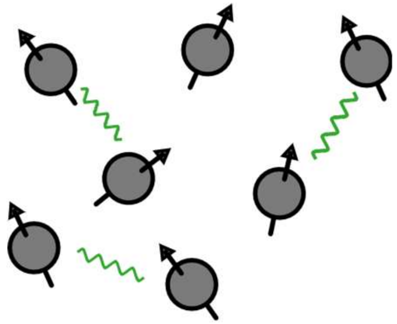
$$R(\boldsymbol{n}, \theta) = e^{-i\theta \boldsymbol{n}\cdot\boldsymbol{J}}$$

Accessible symmetry

Inaccessible symmetry



Example : interacting spins

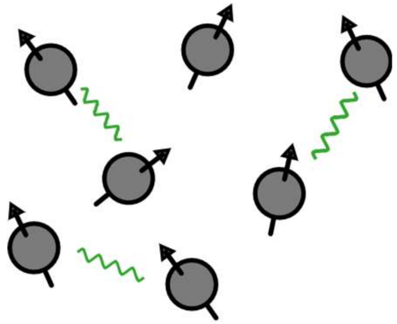


$$H = H_{\mathrm{dis}} + H_{\mathrm{dip}}$$

$$H_{\mathrm{dis}} = \sum_i \delta_i \mathbf{m}_i \cdot \mathbf{J}^i$$

$$H_{\mathrm{dip}} = \sum_{i<j} \Delta_{ij} \left(3 \left[\mathbf{e}_{ij} \cdot \mathbf{J}^i\right] \left[\mathbf{e}_{ij} \cdot \mathbf{J}^j\right] - \mathbf{J}^i \cdot \mathbf{J}^j\right)$$

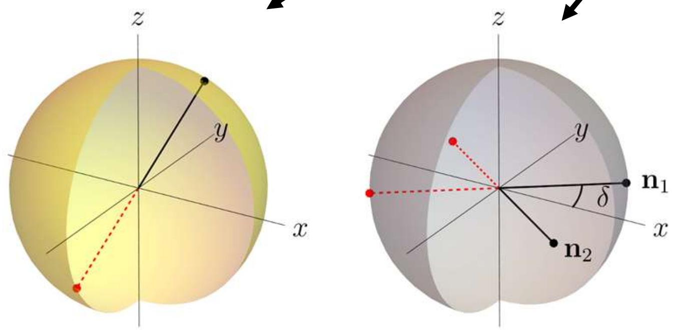
Example : interacting spins



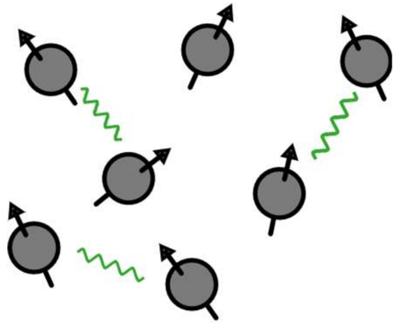
$$H = H_{\mathrm{dis}} + H_{\mathrm{dip}}$$

$$H_{\mathrm{dis}} = \sum_i \delta_i \mathbf{m}_i \cdot \mathbf{J}^i$$

$$H_{\mathrm{dip}} = \sum_{i<j} \Delta_{ij} \left(3 \left[\mathbf{e}_{ij} \cdot \mathbf{J}^i\right] \left[\mathbf{e}_{ij} \cdot \mathbf{J}^j\right] - \mathbf{J}^i \cdot \mathbf{J}^j\right)$$



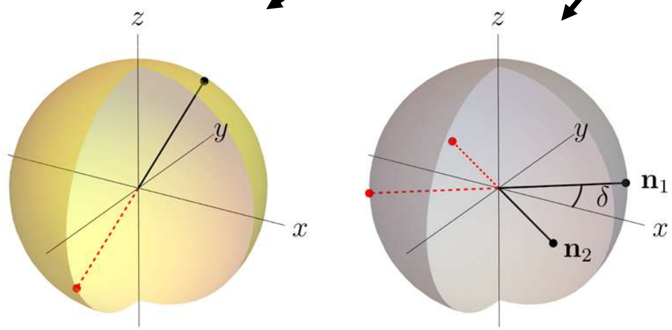
Example : interacting spins

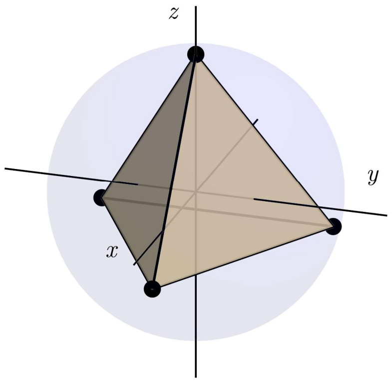


$$H = H_{\text{dis}} + H_{\text{dip}}$$

$$H_{\text{dis}} = \sum_i \delta_i \mathbf{m}_i \cdot \mathbf{J}^i$$

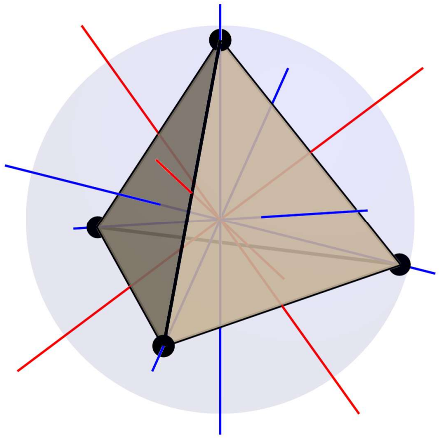
$$H_{\text{dip}} = \sum_{i<j} \Delta_{ij} \left(3 \left[\mathbf{e}_{ij} \cdot \mathbf{J}^i\right] \left[\mathbf{e}_{ij} \cdot \mathbf{J}^j\right] - \mathbf{J}^i \cdot \mathbf{J}^j\right)$$





Tetrahedral symmetry inaccessible !

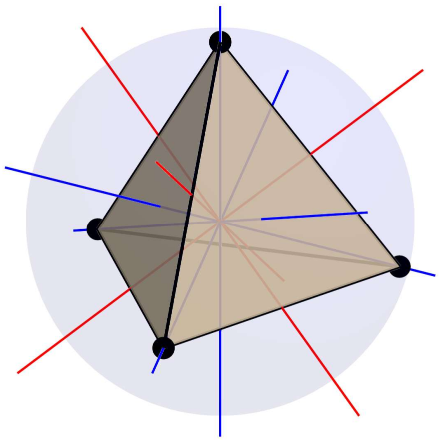
Example : interacting spins

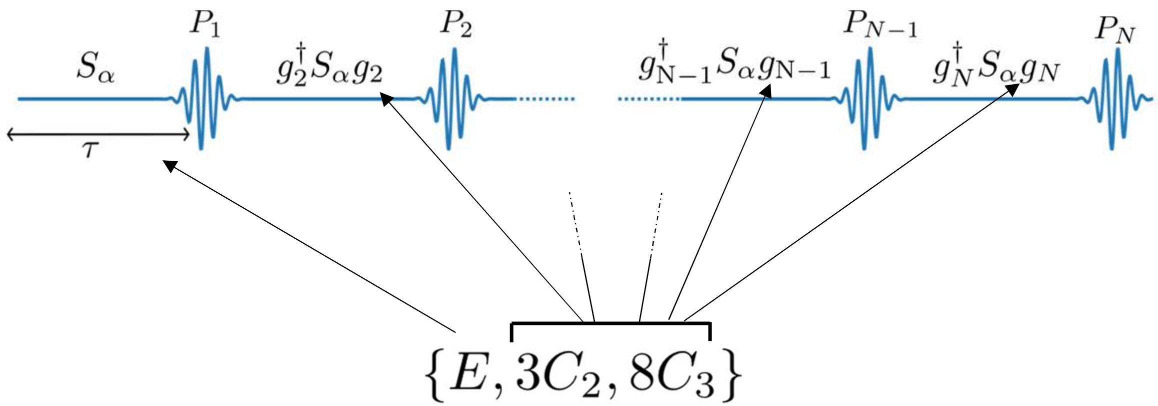


$$\{E, \underline{3C_2}, \underline{8C_3}\}$$

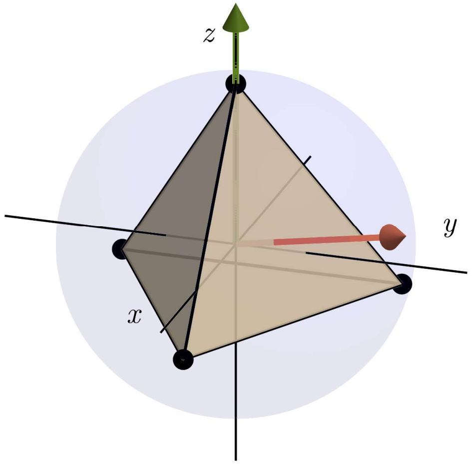
$\longrightarrow$ Decoupling group !

Example : interacting spins

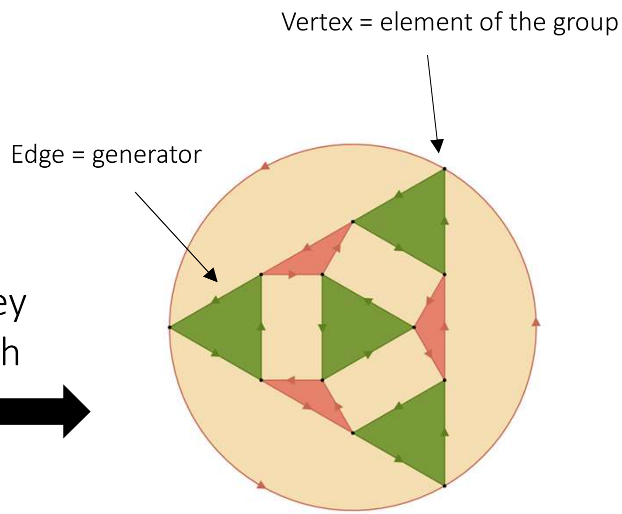




Example : interacting spins



Cayley
graph



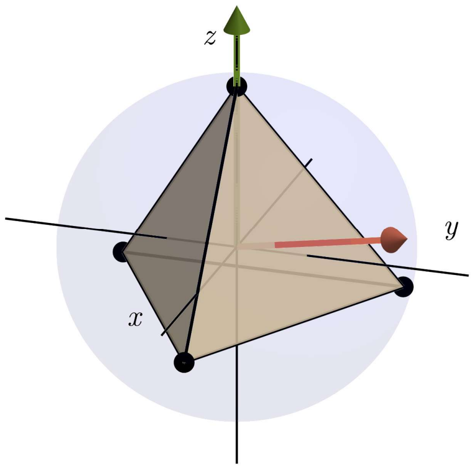
$$a = \left((0,0,1), \frac{2\pi}{3}\right)$$

$$b = \left(\left(\frac{\sqrt{2}}{3}, \sqrt{\frac{2}{3}}, \frac{1}{3}\right), \frac{2\pi}{3}\right)$$

$$\{E, 3C_2, 8C_3\}$$

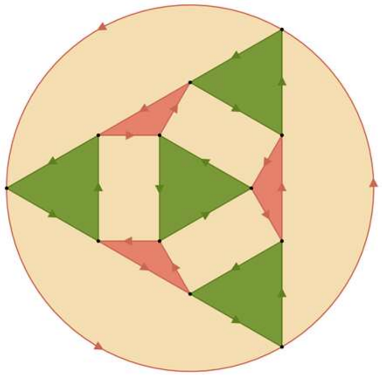
L. Viola *et al.*, Phys. Rev. Lett. **90**, 037901 (2003)

Example : interacting spins



$\{E, 3C_2, 8C_3\}$

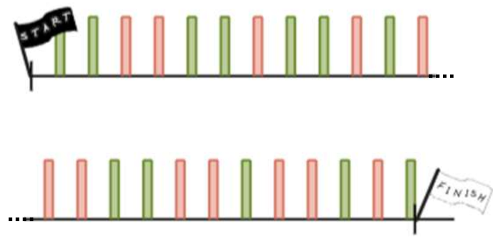
Cayley
graph



$a = \left((0,0,1), \frac{2\pi}{3}\right)$

$b = \left(\left(\frac{\sqrt{2}}{3}, \sqrt{\frac{2}{3}}, \frac{1}{3}\right), \frac{2\pi}{3}\right)$

Eulerian
path

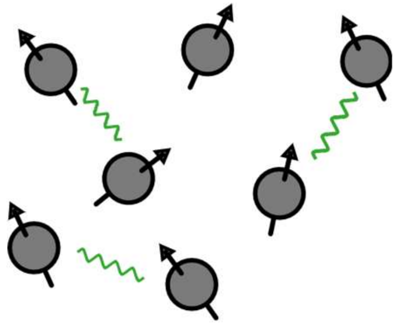


TEDD

L. Viola *et al.*, Phys. Rev. Lett. **90**, 037901 (2003)

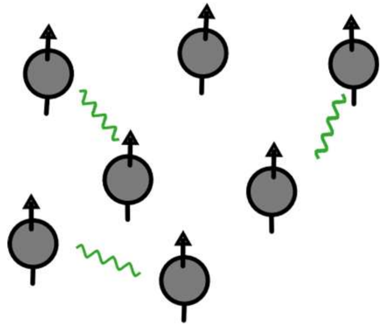
Leveraging existing symmetries

$$H = H_{\text{dis}} + H_{\text{dip}}$$



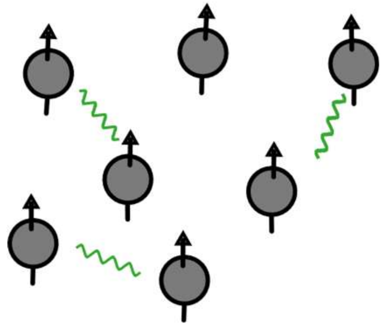
$$\begin{cases} H_{\text{dis}} = \sum_i \delta_i \mathbf{m}_i \cdot \mathbf{J}^i \\ H_{\text{dip}} = \sum_{i<j} \Delta_{ij} \left(3 \left[\mathbf{e}_{ij} \cdot \mathbf{J}^i\right] \left[\mathbf{e}_{ij} \cdot \mathbf{J}^j\right] - \mathbf{J}^i \cdot \mathbf{J}^j\right) \end{cases}$$

Rotating wave approximation



$$\begin{cases} H_{\text{dis}} = \sum_i \delta_i J_z^i \\ H_{\text{dip}} = \sum_{i<j} \Delta_{ij} \left(3 J_z^i J_z^j - \mathbf{J}^i \cdot \mathbf{J}^j\right) \end{cases}$$

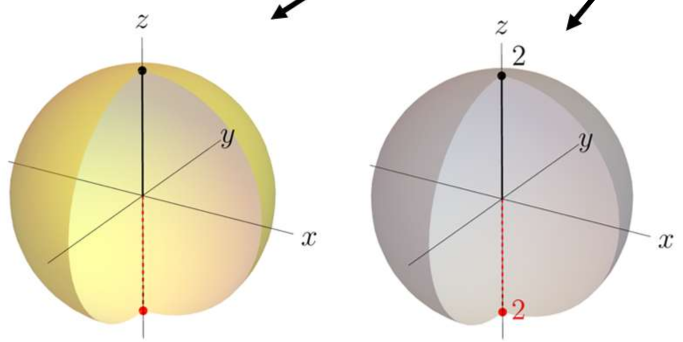
Leveraging existing symmetries



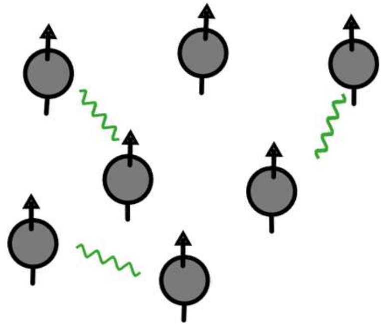
$$H = H_{\mathrm{dis}} + H_{\mathrm{dip}}$$

$$H_{\mathrm{dis}} = \sum_i \delta_i J_z^i$$

$$H_{\mathrm{dip}} = \sum_{i<j} \Delta_{ij} \left(3 J_z^i J_z^j - \mathbf{J}^i \cdot \mathbf{J}^j\right)$$



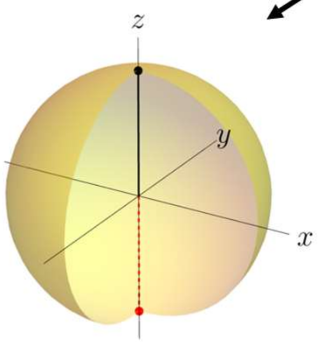
Leveraging existing symmetries

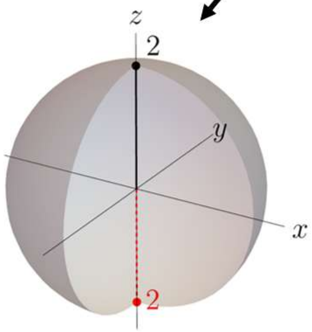


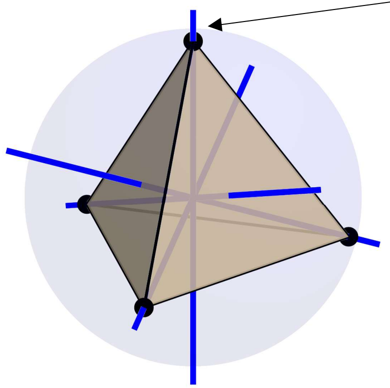
$$H = H_{\text{dis}} + H_{\text{dip}}$$

$$H_{\text{dis}} = \sum_i \delta_i J_z^i$$

$$H_{\text{dip}} = \sum_{i<j} \Delta_{ij} \left(3 J_z^i J_z^j - \mathbf{J}^i \cdot \mathbf{J}^j\right)$$



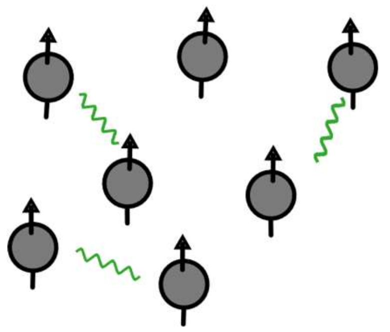




C_3 already there

T = C_3 D_2

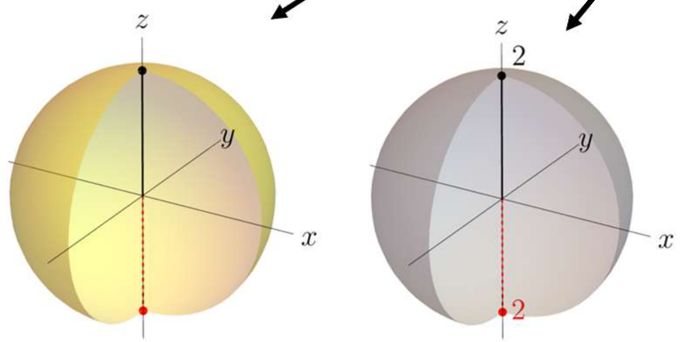
Leveraging existing symmetries

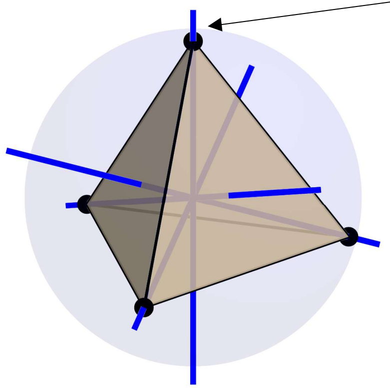


$$H = H_{\text{dis}} + H_{\text{dip}}$$

$$H_{\text{dis}} = \sum_i \delta_i J_z^i$$

$$H_{\text{dip}} = \sum_{i<j} \Delta_{ij} \left(3 J_z^i J_z^j - \mathbf{J}^i \cdot \mathbf{J}^j\right)$$





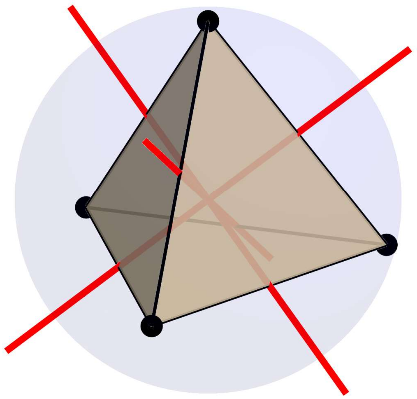
C_3 already there

$T = C_3\ D_2$

$\Downarrow$

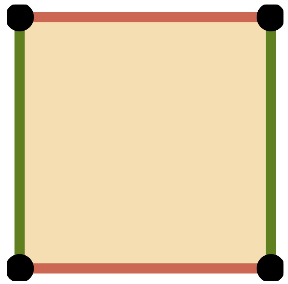
We need to symmetrize w.r.t. the remaining subgroup

Leveraging existing symmetries



$\{E, 3C_2\}$

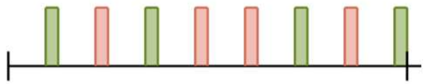
Cayley graph



$a = \left(\left(\sqrt{\tfrac{2}{3}}, 0, \tfrac{1}{\sqrt{3}}\right), \pi\right)$

$b = \left(\left(-\tfrac{1}{\sqrt{6}}, \tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{3}}\right), \pi\right)$

Eulerian path



Conclusion

- Decoupling groups $\leftrightarrow$ inaccessible symmetries

- SU(2) (operators) $\leftrightarrow$ SO(3) (Majorana constellation)

Perspectives

Conclusion

- Decoupling groups ↔ inaccessible symmetries

- SU(2) (operators) ↔ SO(3) (Majorana constellation)

→ **Novel sequences based on the symmetry groups of Platonic solids**

- Eulerian ↔ robust
- Versatile
  e.g. universal for spin-j<3, K-body interactions, qubit-qutrit,...
- Global SU(2) pulses

Perspectives

Conclusion

- Decoupling groups ↔ inaccessible symmetries
- SU(2) (operators) ↔ SO(3) (Majorana constellation)

➡ **Novel sequences based on the symmetry groups of Platonic solids**

Eulerian ↔ robust

Versatile

e.g. universal for spin-$j<3$, K-body interactions, qubit-qutrit,…

Global SU(2) pulses

Perspectives

- Optimization of these sequences (optimal Eulerian path, optimal generators, concatenation,…)

- Compatible with dynamically-corrected gates

- Any use for Hamiltonian engineering ?

Thank you !

« Platonic » dynamical decoupling

Quantum **9**, 1661 (2025)

Majorana constellation for operators

Phys. Rev. A **101**, 022332 (2020)