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Research papers

Optimal design and performance investigation of latent heat thermal energy storage system integrated with nonuniformed topology fins

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ABSTRACT

The intrinsic low thermal conductivity of PCM restricts performance of phase change systems. Incorporation of metal fins represents an efficacious strategy to remediate the inferior thermal performance. In order to establish more efficient heat transfer, a nonuniform topology-based fin structure fitting for a rounded rectangle shell-tube phase change unit is designed in this paper. This paper investigated the effect of nonuniform tree-like fins on the melting characteristics of phase change units. The topology domain is achieved based on a density-based optimization methodology, with a heat source varying along the vertical direction innovatively. Numerical results reveal that the tree-like fin structure is distributed within the design domain. The varying heat source induces more metal fins placed in the bottom part of the phase change unit. Topology fins are capable of reducing the unmelted "dead zone" appearing in the bottom corners of the no-fin or even-fin phase change unit. The balance between thermal transient conduction and convection reduces the total melting time of PCMs in the topology phase change unit approximately 52.78 % and 23.62 % when compared to the no-fin and even-fin cases. The phase change unit with nonuniformly topologized fins ($n = 2$) is optimal with fastest melting rate and lowest average temperature of PCM. Fin volume, tube-shell ratio and heat flux are found to have positive effect on phase transition of the phase change unit. The melting rate of phase change unit is significantly accelerated by an increase in fin volume, tube-shell ratio and heat flux. In conclusion, this paper illustrates the heat transfer enhancement of a phase change unit integrated with the nonuniform topology fins, with noticeable benefit to the optimal design of phase change system.

1. Introduction

The global energy system is undergoing a significant transition, with renewable energy emerging as a crucial component in the face of mounting concerns about fossil fuel scarcity and environmental degradation [1]. High-quality development of renewable energy enables to facilitate green and low-carbon transformation and sustainable development, steadily paving the way for carbon neutrality [2,3]. Whereas, renewable energy is generally limited by the discontinuity and instability, which needs to be solved urgently [4,5].

Latent heat thermal energy storage system employs phase change materials (PCMs, which are usually solid-liquid PCMs) as the medium, through which thermal energy can be stored or released in the form of latent heat [6,7]. It has unparalleled advantages in terms of high energy storage density and nearly isothermal energy storage process, presenting numerous potential applications in the field of renewable energy [8,9]. However, the inherent low thermal conductivity of PCM greatly restricts its flow and heat transfer characteristics, exerting a negative effect on the corresponding charging/discharging performance of the latent heat thermal energy storage system [10–12]. Therefore, available methods are sought with large unmet engineering needs to address this limitation.

Insertion of the metal fins is reckoned as an effective strategy for enhancing heat transfer of the latent heat thermal energy storage system [13]. It is established that the high-temperature heat source transfers heat to the metal fins, which then quickly transfers to the low-

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temperature solid PCMs, establishing efficient heat transfer between the heat source and PCMs [14,15]. Previous research has assessed a variety of fin shapes (rectangular, annular, triangular, helical and branch), fin materials (Cu, Fe, Al, Ti and Ni) and fin arrangement (cylindrical shell-and-tube, rectangular, triple-tube and multitube) [16–21]. Numerous fin shapes (Y-shaped fins, twisted fins, eccentric V-shaped fins, honeycomb fins, arc-shaped fins, Petal-shaped fins, hook-shaped fins, tree shaped fins) are also developed [22–29]. Effects of fin parameters on thermal performance enhancement of latent heat thermal energy storage system are also explored. It is proposed that metal fins have the potential to enhance the thermal performance of a latent heat thermal energy storage system. The passive fins play a crucial role in remarkably augmenting the heat transfer rate in PCM-based systems [30]. Incorporating tree shaped fins is highly effective approach to promoting the solidification and melting of PCM systems in thermal management applications [31]. Tree-fin structures have drawn considerable attention due to their outstanding performance in LHTES storage systems [32]. BarikIt et al. [33] studied the melting of PCM in three-tube heat exchangers with different configuration of tree fins, and observed that the melting time was significantly shortened with the increase of fin structure. Shukla et al. [34] assessed the effect of tree fin parameters on the melting of PCM. Results showed that heat transfer was remarkably improved with implement of the fin structure. Ziaei et al. [35] studied the influence of complexity and freedom degree for the tree-like structure on melting of PCM. Results indicated that the heat transfer rate density increased with the growth of the complexity and the freedom degree. Whereas, these accessible results primarily concentrate on the performance of a prefabricated fin configuration, rather than on the development of an innovative design with regard to the variable demands.

Topology optimization is a mathematical method for optimizing distribution of materials within a specific design area based on the given load, constraints and performance indicators [36,37]. It can be conceptualized as a structural optimization to guide the shape design of fins in a phase change unit. Topology optimization is usually performed by taking the average temperature or maximum temperature of PCM as the objective function. The variable density-based topology optimization is sensitive to changes in structural topology, and is therefore being widely utilized in the optimization design of practical engineering structures. Topology optimization research mainly includes two distinct areas: continuum topology optimization and discrete structure topology optimization [38]. Both of them are strongly dependent on the finite element method (FEM). Numerous studies have applied topology optimization to design more efficient metal fins available to the latent heat thermal energy system [39,40].

Peremans et al. [41] used topology optimization to obtain optimal designs for rectangular PCM modules based on a conductive heat transfer model for the PCM to maximum mean charging power. The topology designs were particularly suitable to increase the performance for long compartments. Zhao et al. [42] numerical compared thermal performance of a phase change energy storage tank with topology fins and no-topology fins. The topology structure could reduce the thermal storage and release time by 70 % and 81 % compared to the 4-fin configuration. Iradukunda et al. [43] aimed to generate topology optimized fin structures that could be integrated into the phase change units to enhance the heat transfer rate. Experimental results indicated that the topology structure improve PCM performance significantly relative to a benchmark plate fin, reducing the peak temperature by up to 18.9 °C under a pulsed load of 50 W. Guibert et al. [44] employed the level-set method to topologically optimize the distribution of both PCM and high thermal conductivity materials within a heat sink. The phase change heat sink displayed superior thermal performance with a maximum temperature reduction of up to 42 % and a mean temperature reduction of up to 42 %, contributing to address crucial challenges in thermal management. Pizzolato et al. [45] designed a topology optimization framework of a thermal energy storage system involving phase change. The layout of a highly conductive material embedded in the phase change unit was optimized to maximize the heat exchange performance. The 3D optimized design yielded a discharge time reduction of roughly 20 % with respect to the 2D design. It can be concluded that the aforementioned investigations were focused on topology optimization based on thermal conduction with the objective to develop an excellent latent heat thermal energy system. An even fin layout is formed in order to achieve improved heat dissipation from the heat source. Compared to the single random arrangement of metal fins, the topology structures are proposed to exert beneficial influence on the phase change heat transfer of PCM.

There is substantial evidence that natural convection plays a significant role in influencing the phase transition process of PCM. The natural convection facilitates the flow of the melted liquid PCM. The effective heat transfer coefficient of the phase change unit incorporating natural convection is proven to be greater than that only considers thermal conduction. Given that natural convection of liquid PCM gradually emerges and intensifies over time during the melting process, numerous studies have conducted topology structure optimization through the integration of convection heat transfer.

Chen et al. [46] designed the inner fins of spherical phase change capsules by applying topology optimization techniques. The fin model with minimized dissipation of thermal conductivity as the objective function was found to have the best performance of reducing the melting time by 34.22 %. Zhang et al. [47] applied topology optimization to design of the fin structure of the latent heat thermal energy storage system. Effects of parameters including penalty factor, filter radius, steepness factor and threshold value on the topology were analyzed. Results showed that the topology fins provided a larger heat transfer area and a more reasonable heat transfer path. Chen et al. [48] obtained novel fins in a shell-tube phase change accumulator considering natural convection through the topology optimization method. Results indicated that natural convection was favorable to obtain a better fin structure to enhance thermal performance. When the weight ratio of the maximum average temperature and the minimum temperature difference was 1.5, performance of the shell-tube phase change accumulator was further improved. Abate et al. [49] obtained an optimized fin structure for a passive PCM-based heat sink under a heat flux of 1 W/cm^2. It was found that topology optimization could be an effective tool to optimize the structures of heat sinks coupled with PCMs. The peak temperature reached by the source plate in the thermal transient considered was approximately 20 °C lower than that reached without any enhancers. See et al. [50] developed a topology-optimized phase change heat sink suitable to the electronics cooling. Results showed that the natural convection topology-optimized heat sink had a lower base temperature compared to the conventional no-fin heat sink, but a higher base temperature than the topology-optimized heat sink based on heat conduction. Ho et al. [51] obtained that the better thermal performance of the topologically tree-like structure heat sink was due to its optimized heat conduction paths that allowed heat from heat source to be efficiently dissipated to the PCM. The topology optimized structures generated by considering both heat conduction and phase transition differed limited from the solo-thermal conduction case. Pizzolato et al. [52] presented a heat transfer intensification to the shell-and-tube latent heat thermal energy storage unit by means of high conducting topology fins. Compared results indicated that accounting for fluid flow in design optimization studies was crucial to thermal performance. Topologically designed fins with specific features remarkably benefited phase transition of PCMs.

Based on the above literatures, it is arrived that topology optimization of the phase change unit considering convection produces an uneven fin layout within the phase change unit. This special structure is beneficial to the development of natural convection for liquid PCMs, which subsequently promotes thermal performance of the phase change unit. While, effect of natural convection on phase transition of PCM is a transient process that gradually increases with the augment of the charging time. PCMs undergo solid-liquid transition during the melting process and liquid flow at low velocity (magnitude of 10^{-3}) occurs within the phase change unit owing to the driving force of thermal buoyancy. The metal fins obtained from topology optimization have tree-like structure, resulting in serious obstruction to flow of liquid PCMs. Topology optimization methodology regarding the weak convection of PCM allows for significant design freedom, which leads to deficiencies of long calculation period, low computational efficiency and numerical instability, etc.

It is evident that natural convection of liquid PCM is more intensive in the upper part of the phase change unit, accelerating the melting of PCM and creating a temperature decline trend along the vertical upward direction. Inspired by the research phenomenon, this paper configures a varying heat source in the design domain of conventional topology optimization in order to correspond the temperature distribution of PCMs. This paper aims to design nonuniform topologized fins within the phase change system and the formed fins could effectively utilize thermal conduction and convection to improve the phase transition process. A phase change unit with rounded rectangle configuration is built in this investigation. It has the potential to function as an independent, small-scale energy storage system or to be stacked together to form a large-scale energy storage system. It is anticipated that this special topology structure will facilitate the collaborative utilization of convection and conduction. Parametric analysis is also conducted to assess effects of fin volume, tube-shell ratio and heat flux on thermal performance of the phase change units. Obtained results are beneficial to develop topological fin phase change units with excellent heat transfer performance.

2. Methodology

Fig. 1 illustrates that the latent heat thermal energy unit is designed as a concentric structure which comprises an interior surface with a high-temperature heat flux. PCM is filled in the space between the interior and exterior surfaces, acting as the medium for storing thermal energy. Various copper fins are fixed on the interior surface of the latent heat thermal energy unit in order to promote the melting of PCM. Thermophysical properties of PCM and copper are listed in Table 1. The LHTES structure can be rationally simplified into a two-dimensional physical model without considering the depth. This paper aims to arrange the copper fins according to the topology optimization.

2.1. Heat transfer analysis

2.1.1. Physical model

A two-dimensional physical model of the LHTES unit for heat transfer analysis is built in Fig. 1(b). The concentric structure has an inner radius (R_1) of 5 mm and an outer radius (L) of 25 mm, with the middle space filled with PCM. The incorporation of copper fins serves to accelerate the heat transfer from the heat source to the PCM, thereby enhancing the phase change heat transfer of PCM and consequently achieving a latent heat based thermal energy storage with improved performance.

Table 1
Thermophysical properties of utilized PCM and copper.

Properties	PCM		Copper
	Solid	Liquid	
ρ(kg/m^3)	900	850	8960
c_p(J/(kg·°C))	1950	2100	385
κ(W/(m·°C))	0.25	0.20	400
μ(Pa·s)	–	0.0035	–
L(kJ/kg)	190		–
T_m(K)	315.15	321.15	–
β(1/K)		0.0008	–

2.1.2. Governing equations

The transfer of thermal energy from the heat source to the PCM or copper fins occurs via two mechanisms: thermal conduction and convection. Phase transition from a solid to a liquid state appears in the PCM during the thermal energy storage process. This paper terms PCM as an incompressible Newtonian liquid that follows the Boussinesq approximation. Local thermal equilibrium is accomplished through the coupling between the solid and liquid phases [14].

(1) For copper fins

Only energy conservation equation of copper fins is considered in this investigation.

$$\frac{\partial T}{\partial \tau} = \alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\phi}{\rho_c c_{p,c}} \tag{1}$$

where τ is the time; α, ρ_c and $c_{p,c}$ are the thermal diffusivity, density and specific heat of copper; ϕ is the source term.

(2) For PCM

An enthalpy-porosity method is utilized to evaluate related phase change heat transfer of PCM. The whole computational domain is treated as a porous zone with the porosity of each cell characterized by liquid fraction (f). The obtained governing equations, which pertain to continuity, momentum and energy conservation are separately shown in Eqs. (2)–(4).

$$\frac{\partial(\rho_f)}{\partial \tau}+\nabla(\rho_f \vec{u}) = 0 \tag{2}$$

$$\frac{\partial(\rho_f \vec{u})}{\partial \tau}+\nabla(\rho_f \vec{u}\,\vec{u}) = -\nabla p+\mu_f \nabla^2 u+\rho_f g\beta_f(T_f-T_s)+S_{u,f} \tag{3}$$

$$\frac{\partial(\rho_f H)}{\partial \tau}+\nabla(\rho_f \vec{u} H) = \nabla(\lambda_f \nabla T)+S_{r,f} \tag{4}$$

where $\vec{u}$ is the velocity of liquid PCM; ρ_f, μ_f, λ_f, p and β_f denote the density, dynamic viscosity, thermal conductivity, pressure and thermal expansion of PCM; $S_{u,f}$ and $S_{r,f}$ are the source terms in momentum and

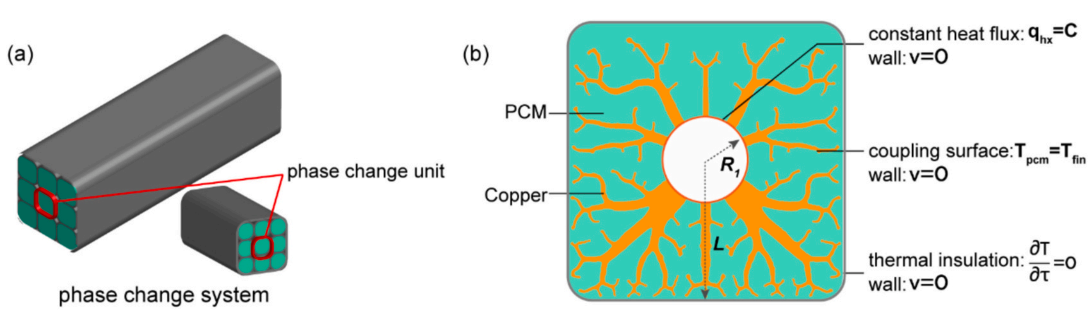

Fig. 1. Schematic of the phase change units, (a) Structure diagram and (b) Two-dimensional physical model.

energy conservation equations, $S_{u,f} = -uA_{mushy}\frac{(1-f)^2}{f^3+\psi}$, $A_{mushy} = 10^5$, $\psi = 0.001$; H represents the PCM enthalpy that equals to the sum of sensible heat (H_s) and latent heat (L).

$$H = H_s + fL \tag{5}$$

$$H_s = H_{ref} + \int_{T_{ref}}^{T} c_p dT \tag{6}$$

where H_{ref} and T_{ref} are the reference enthalpy and temperature; The f is used to indicate the volume proportion of melted PCM during the research. f will linearly change within the scope of 0 and 1 for PCM in the mushy region. $f = 0$ or 1 represents PCM is in solid or liquid state.

$$f = \begin{Bmatrix} 0, T_f < T_s \\ \frac{T_f - T_s}{T_l - T_s}, T_s < T_f < T_l \\ 1, T_f > T_l \end{Bmatrix} \tag{7}$$

where T_s and T_l are the initial and terminal temperatures of the PCM mushy zone.

2.2. Topology optimization

The cooper fins are situated within the PCM region via a topology optimization methodology [47]. Heat transfer between solid (copper fin) and liquid (PCM) are coupled together. Thermal energy is expected to store in the form of latent heat in the LHTES unit, resulting in an approximate isothermal rapid charging process. Accordingly, an objective function (Eq. (8)) of minimizing the average temperature PCM is developed during the optimization (Fig. 2).

$$\begin{cases} minimize : T_{\Omega,ave} \\ s.t. 0 < \gamma < 1 \\ \frac{1}{V_\Omega}\int \gamma d\Omega \le V_f \end{cases} \tag{8}$$

where $T_{\Omega,ave}$ is the average temperature of PCM in the design domain; V_f is the designed volume fraction of the copper fin.

2.2.1. Density-based topology optimization

A density-based topology optimization method is adopted with the design domain exhibiting characteristics similar to a porous medium material [53]. A design parameter (γ) which shifts between 0 and 1 is used to delineate the material of the cells. $\gamma = 0$ or 1 denotes that the cell corresponds to the PCM or copper fin. The material properties within the design domain can continuously vary from 0 to 1 through an interpolation function. The sensitivity of the parameter makes it convenient to use a gradient algorithm for topology optimization.

Considering the natural convection of liquid PCM enabling to play a critical role during its phase transition, the Solid Isotropic Microstructures with Penalization (SIMP) interpolation model is utilized in this investigation, with a modified nonlinear thermal boundary to reflect the effect of natural convection on thermal performance [41,54]. The topological density and thermal conductivity of materials in the design domain are determined by the related formula in Eqs. (9)–(10).

$$\rho = \rho_f + \gamma_p(\rho_s - \rho_f) = \rho_f + (\gamma_{min} + (1-\gamma_{min})^{p_{SIMP}})(\rho_s - \rho_f) \tag{9}$$

$$\kappa = \kappa_f + \gamma_p(\kappa_s - \kappa_f) = \kappa_f + (\gamma_{min} + (1-\gamma_{min})^{p_{SIMP}})(\kappa_s - \kappa_f) \tag{10}$$

where ρ_s and κ_s are the density and thermal conductivity of the PCM; ρ_f and κ_f are the density and thermal conductivity of the copper fin; P_{SIMP} is the SIMP exponent; γ_{min} is the minimum penalized volume fraction.

2.2.2. Helmholtz filtration

The design domain is discretized into numerous cells through the FEM [42]. In order to circumvent the numerical instability caused by the mesh dependency and checkerboard problem, the optimization process necessitates the Helmholtz partial differential equations (Eq. (11)) to accurately filter out any inherent fuzzy patterns that may be observed in the obtained optimization results.

$$-r^2\nabla^2\tilde{\gamma} + \tilde{\gamma} = \gamma \tag{11}$$

where r is the filter radius, which generally equals to the mesh size; $\tilde{\gamma}$ is the filtered design variable.

2.2.3. Hyperbolic tangent projection

In consideration of the fact that the Helmholtz filtering method produces noticeable grayscales to the topology process, a smooth step function of projection is introduced with the objective of reducing the grayscales. This paper selects a hyperbolic tangent projection to eliminate the intermediate grayscales in the design domain [55,56]. The projection quantity can be well regulated by means of the slope (β) and projection point (γ_β).

$$\tilde{\gamma} = \frac{tanh(\beta(\tilde{\gamma} - \gamma_\beta)) + tanh(\beta\gamma_\beta)}{tanh(\beta(1-\gamma_\beta)) + tanh(\beta\gamma_\beta)} \tag{12}$$

2.3. Numerical solving

The numerical calculation is divided into two steps: topological optimization and thermal performance analysis. The steady topological optimization is initially performed through the FEM embedded in COMSOL Multiphysics software. Subsequently, the obtained topologized structure of copper fins is implemented into the PCM domain. The transient thermal performance analysis is then mathematically solved based on the FEM.

2.3.1. Boundary condition and initial condition

(1) For steady topological optimization

The interior surface is termed as a constant temperature boundary condition. Correspondingly, the exterior surface is assumed as an adiabatic boundary condition. In order to accurately reflect the uneven melting of PCM in the vertical direction, heat flux of the heat source is defined as a function of the y-coordinate in design domain (Eq. (13)).

$$\begin{cases} internal\ surface : T = T_{int} \\ exterior\ surface : \frac{\partial T_{ext}}{\partial \tau} = 0 \\ desgin\ domain : q_{PCM} = \frac{q_0}{2R_i}|y - 2R_i|^n \end{cases} \tag{13}$$

where T_{int} is the temperature of internal surface, 273.15 K; q_0 is the

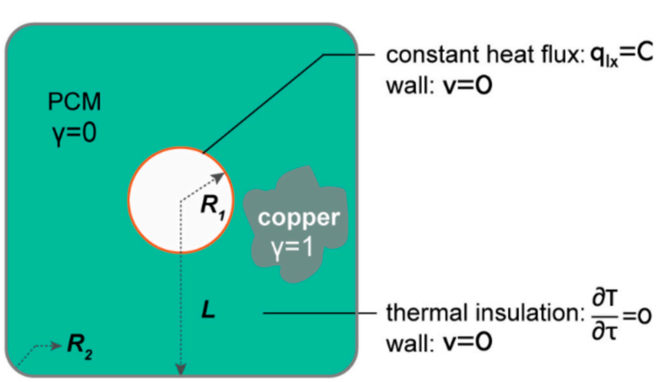

Fig. 2. The schematic of the topology optimization.

average heat flux of the design domain, 10^6 W/m^2; n is the characteristic exponent to adjust the distribution of heat sources, $n = 0$ or $n \neq 0$ means that the heat source exists uniformly or nonuniformly.

(2) For transient thermal analysis

The movement of liquid PCM is driven by thermal buoyancy. The fin surface, interior surface devoid of metal fins and exterior surface are treated as wall boundary condition in the velocity field. The whole interior surface is designated as a constant heat flow boundary in heat transfer field. The exterior surface is well insulated and functions as an adiabatic boundary condition.

$$\begin{cases} internal\ surface: q = q_{int}, \vec{u} = 0 \\ exterior\ surface: \dfrac{\partial T_{ext}}{\partial \tau} = 0, \vec{u} = 0 \\ middle\ space: coupling\ of\ solid\ and\ liquid\ heat\ transfer \end{cases} \tag{14}$$

where q_{int} is the heat flux of heat source, 2500–10,000 W/m^2.

2.3.2. Solving method

The computational region is discretized through the staggered grid technology. The governing equations and topological optimization are numerically solved through the FEM. The topology field of design variable is optimized by using the Method of Moving Asymptotes (MMA), with an adjoint approach to calculate the sensitivities of the objective function and constraints. The relative tolerance of the topology optimization is set to 10^{-5}.

The governing equations of heat transfer adopt variable time steps, which are regulated by a free step implicit backward differentiation formula (BDF). A parallel direct solver with relative residual of 10^{-5} is about to solve the continuity, momentum and energy conservation equations. The relative residual is checked at each time step of the numerical calculation in order to maintain a highly accurate resolution.

2.4. Independence test and model verification

2.4.1. Mesh independence

Liquid fraction and temperature of PCM at the upper wall surface are compared through three mesh numbers (125,640, 190,232 and 251,731) in Fig. 3. It is identified that only slight changes are produced with increase of the mesh number. This paper adopts the mesh number of 190,232 in the numerical study based on the balance of both calculation accuracy and calculation period.

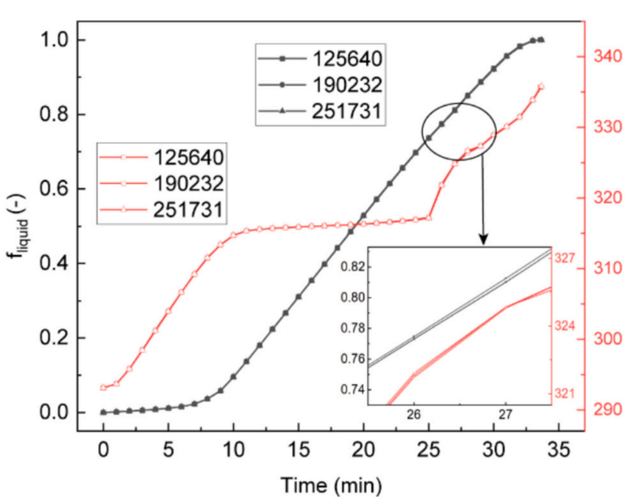

Fig. 3. Mesh independence test of the built model.

2.4.2. Model verification

The numerical model is verified through the comparison between the experimental and simulated results. A circular shell and tube phase change unit is fabricated in this paper with the purpose of evaluating the melting characteristics (Fig. 4). The radius of the inner tube and outer tube are respectively 40 mm and 80 mm, with the outside wrapped in an insulation layer.

The melting process is conducted through the heating from the interior surface of the phase change unit. A flexible electric heater is attached in the interior surface to maintained a constant heat source of 353.15 K, while the exterior surface is maintained as an adiabatic condition. The intermediate space between inner and outer tubes is fully filled with PCM ($T_m = 317.15$ K, L = 173.8 kJ/kg, $\lambda = 0.147$ W/(m·K), Cp = 2300 J/(kg·K), $\beta = 0.000615$). The comparison indicates that the simulated liquid fraction and temperature of PCM are almost consistent with those of the experimental determination. Slight discrepancy is mainly caused by incomplete adiabatic conditions and delayed capture of the phase interface. It is thus rationally inferred that the built numerical model is accurate to be applied in the following investigation towards performance of phase change units.

3. Results and discussion

3.1. Topology structure of PCM units

The copper fins layout is designed through a topology optimization. n is a parameter to regulate the distribution of heat source. $n = 0$ means that the heat source exists uniformly. Obtained results (case T1) in Fig. 5 illustrate that fins are distributed evenly within the designed domain, extending from the interior to the exterior in a tree-branched configuration. This structure is conducive to promoting the heat transfer along the specified direction. PCMs stored in the phase change unit then melt rapidly, thereby achieving superior performance of latent heat energy storage system. Considering the high-temperature liquid PCM will flow upwards under the influence of thermal buoyancy, it is necessary to enhance heat transfer of the low-temperature solid PCM at the bottom part. This paper proposes a novel nonuniform heat source in order to generate uneven copper fin arrangement in the vertical direction. Case T2-T5 in Fig. 5 are separately corresponding to the topology fin structures when n changes from 0.5 to 2.0. Larger n indicates more heat is concentrated in the negative direction of y-axis. The copper fins tend to be placed in the bottom part of the phase change units to dissipate more thermal energy from the heat source, facilitating the phase transition of PCMs.

3.2. Addition of topology fins

Melting performance of the topology phase change unit is compared with two conventional phase change units (no-fin and even-fin cases), with the detailed structures presented in Fig. 6(a). The volume ratios of copper fins are maintained at a constant of 0.2 in both the even-fin and top-fin cases. Thermal energy is rapidly accumulated in the interior surface owing to the low thermal conductivity of PCMs. PCMs in the no-fin phase change unit then start to melt, leading to the fastest rising rate of the liquid fraction in the early stage. Even or topologized distributed copper fins provide effective heat transfer paths and more contacting areas between the heat source and PCMs. Thermal energy is able to transfer to the PCM domain and store as latent heat in the PCMs, resulting in the lower liquid fraction in the early stage. More thermal energy is stored over the whole PCM domain with elapse of time. The liquid fractions of PCMs exhibit a pronounced increase in phase change units with metal fins. Conversely, the thermal buoyancy, which is a function of density and temperature of the PCMs, causes a greater flow of high-temperature liquid PCMs to the upper part of the domain. Which is adverse to the melting of solid PCMs in the bottom part. The rate of increase in the liquid fraction of PCMs gradually declines over time. The

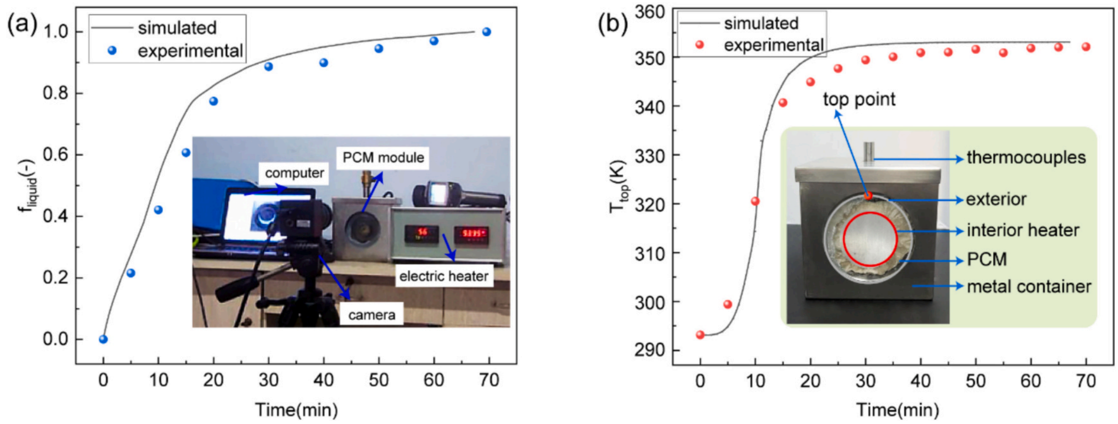

Fig. 4. Model verification between simulation and experiments.

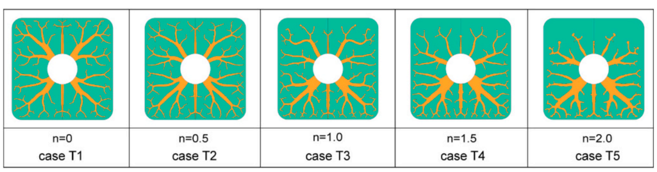

Fig. 5. Configuration of phase change units with topologized copper fins.

bottom part evolves into a “dead zone”, wherein thermal conduction dominates heat transfer. This zone has a markedly detrimental impact on the phase change heat transfer of PCMs. Although evenly distributed fins could enable to mitigate this issue, there still appears a noticeable “dead zone” in the even-fin case as time continues to rise.

Correspondingly, nearly no “dead zone” can be found in the top-fin case, indicating that topologized fins are more efficient to improve the heat transfer during the later stage of melting for phase change units in contrast to no-fin or even-fin cases. It is calculated in Fig. 6(c) that the no-fin phase change unit requires 71.258 min to complete the melting process. Whereas, the total melting time is reduced to 44.057 and 33.648 min for the even-fin and top-fin cases, with corresponding time saving ratios of 0.382 and 0.528, respectively.

It is found in Fig. 6(d) that the average temperatures of PCM in phase change units with fins (even-fin and top-fin cases) are lower than that in the no-fin phase change unit. This is due to the fact that the metal fins functioning as effective heat transfer paths are capable of evenly dissipating thermal energy to PCMs. In initial stage, PCMs primarily store thermal energy in the form of sensible heat, resulting in a reduction in liquid fractions of PCMs within the fin phase change units in comparison to those observed in the no-fin phase change unit, as illustrated in Fig. 6 (b). It is further discovered that liquid fraction of PCM in the no-fin phase change unit shows a remarkable increase over time. This is due to the limited heat transfer between the heat source and PCMs under the no fin condition. The rise in average temperature of the liquid PCM can be attributed to the phenomenon of overheating. The mean temperature of the PCM in the top-fin phase change unit is lower than that of the PCM in the even-fin phase change unit, which can be attributed to the more rational and effective layout of metal fins.

3.3. *Nonuniform topology fins*

Natural convection induces the flow of liquid PCMs upwards, thereby forming two distinct temperature zones: a high-temperature upper zone and a low-temperature bottom zone occurred in the no-fin and even-fin phase change units. The temperature difference increases with elapses of melting time, which has apparent negative effect on the melting of PCM. Five nonuniform topology fins are designed (Fig. 7) with the objective of achieving a balance between the dynamic natural convection and thermal conduction during the PCM melting process. Liquid fraction of various phase change units is plotted in Fig. 7. It is detected that nonuniform layouts of copper fins enable to exert diverse influence on liquid fraction of PCMs, contingent on the value of the parameter n. The larger n produces higher liquid fraction of PCMs in the initial stage of melting. This observation can be attributed to the fact that the placement of fewer copper fins in the upper part of the phase change units is beneficial to convection.

The synthetic effects of conduction and convection result in a nearly identical liquid fraction of PCMs in the middle stage of melting with the elapse of time. More tensive thermal conduction is formed in the bottom of phase change units as a consequence of more copper fins installed under the larger n condition. Consequently, the solid PCMs situated at

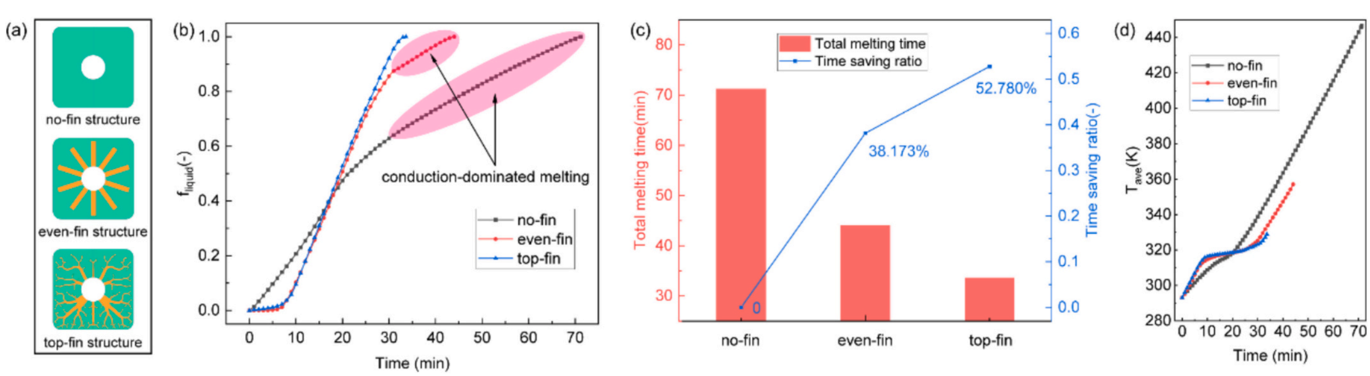

Fig. 6. Performance of phase change units with and without fins. (a) Phase change unit configuration; (b) liquid fraction, (c) Melting time saving and (d) Average temperature of PCM.

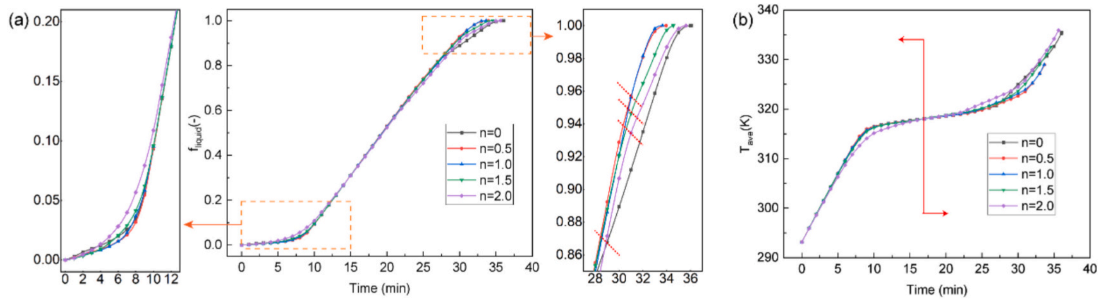

Fig. 7. Liquid fraction of phase change units with various nonuniform topologized fins.

the bottom part of the phase change units will melt at a faster rate in the larger n case, resulting in an increase in the liquid fraction of PCMs upon the end of the melting process.

Actually, there also appears varying degrees of "dead zone" in Fig. 7, which is primarily attributed to the discrepancy between enhancement to thermal conduction in bottom part and enhancement to convection in the upper part generated from topologized fins. It is determined that the "dead zone" firstly decreases and then increases with the augment of parameter n. The optimal melting performance is achieved when the parameter n is set to 1.0, saving approximately 6.715 % and 23.626 % melting time compared to the conventional uniformed top-fin case ($n = 0$) and even-fin case. The investigation will continue to proceed with the non-uniform topology structure ($n = 1.0$).

Fig. 7(b) indicates the average temperature of PCM in the phase change units. It is observed that the average temperature of PCM almost remains unchanged with the variation of n in the early stage ($t < 17$ min). Only the phase change unit with a nonuniform topologized structure ($n = 2.0$) exhibits slight lower average temperature of PCM than the other cases. This is attributed to the thermal convection in the upper part enhanced by the nonuniform topologized structure (n = 2.0), achieving a lower temperature rise of PCM compared to the slighter nonuniform topologized structures. As a consequence of weakened thermal conduction in the upper part of the phase change units with larger n, the melted PCMs will become overheated in the subsequent stage ($t > 17$ min) which then leads to increase of the average temperature of PCM. Fig. 7(b) also demonstrates that average temperatures of PCMs in the phase change units integrated with nonuniform topologized structures ($n = 0.5$ and 1.0) are nearly consistent as a result of the balance between thermal conduction and convection.

Phase change contours of phase change units integrated with various nonuniformly topologized fins are displayed in Fig. 8. It is revealed that nonuniform topologized fins have the capacity of accelerating the PCM melting in the vertical direction, with slowdown in PCM melting in the horizontal direction. This sight is driven by the nonuniform layout of copper fins under various parameter n ($n > 0$). Less copper fins in the upper part weaken the thermal conduction, while promoting the natural convection of liquid PCMs. Copper fins are inclined to place in the low position within the phase change units, according to the uneven heat source. The volume of copper fins rises as the depth of phase change units increases. The presence of more copper fins in the bottom part of the phase change units serves to reinforce the process of thermal conduction, while simultaneously limiting the natural convection of the liquid PCMs. It is particular that PCMs in the bottom part of the phase change unit melt fastest in the case of parameter $n = 1.0$, achieving near-simultaneous total melting with the PCMs in the upper part.

Fig. 9 exhibits temperature contours and velocity contours of phase change units. It is determined that nonuniform fins generate an unequal distribution of temperature across the solid PCMs within the phase change units during the initial and intermediate stages of melting. This aforementioned unevenness of temperature increases with augment of

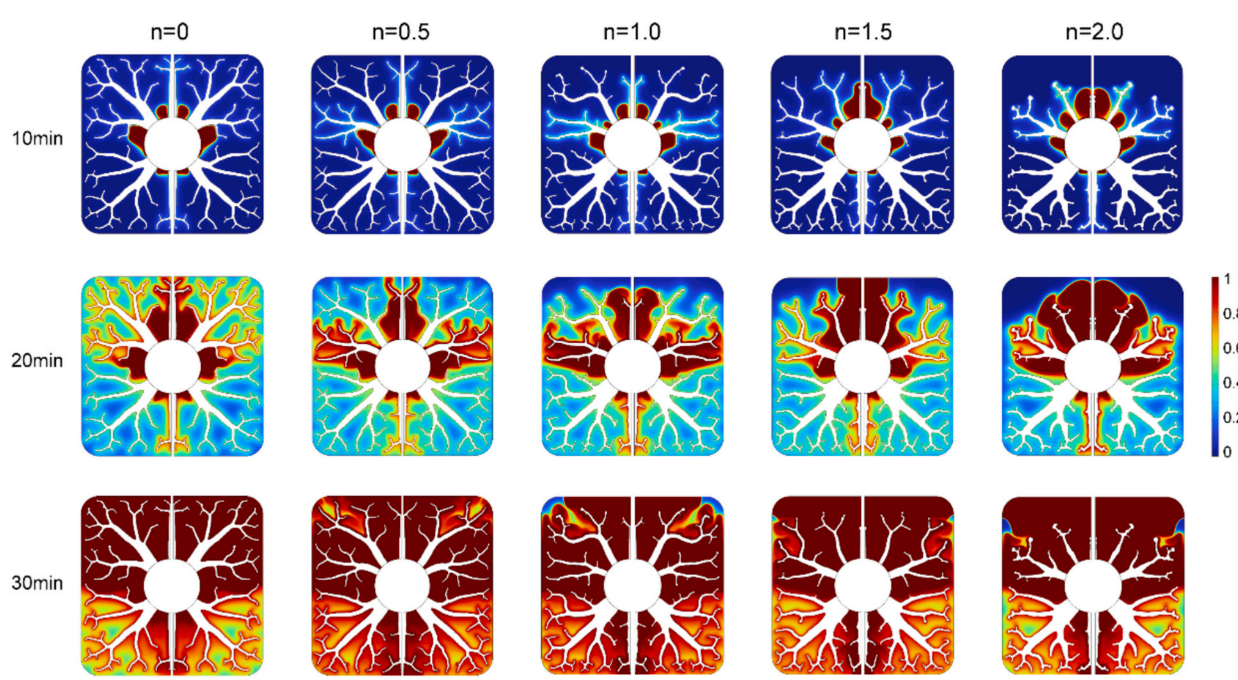

Fig. 8. Phase change contours of phase change units with nonuniform topologized fins.

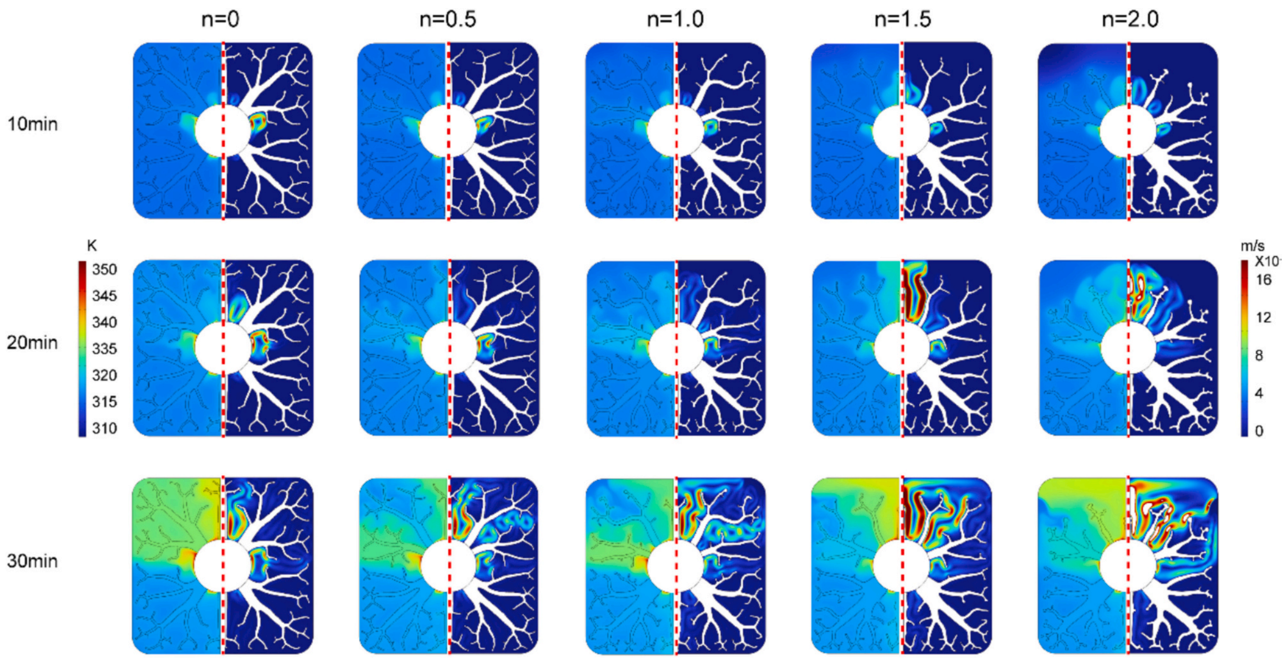

Fig. 9. Temperature and velocity contours of phase change units with nonuniform topology fins.

the parameter n. However, more liquid PCMs are formed with elapse of time, thereby producing a more intensive natural convection as evidenced by the velocity contours. The velocity of PCMs is observed to increase in phase change units with a higher parameter n. Enhancement in natural convection is favorable to the melting of PCMs, resulting in a lower temperature compared to the uniform topology fin case. It can therefore be concluded that non-uniform topology fins have an obvious impact on the thermal conduction and convection of PCMs.

3.4. Fin volume

Performance of the top-fin phase change units is found to vary in accordance with the fin volume (θ), as illustrated in Fig. 10. It is identified that more copper fins are introduced into the phase change units with augment of the fin volume, which effectively benefits heat transfer of incorporated PCMs. Fig. 10(b) shows that enhancement in liquid fraction of PCMs is predominantly concentrated in the middle and late stages of melting. This can be attributed to the enhancement of thermal conduction and the reduction of natural convection when more metal fins are integrated into the phase change units.

The total melting time is reduced from 41.693 min in the fin volume case of 0.1 to 33.648 min and 30.85 min in the fin volume case of 0.2 and 0.3, representing saving of approximately 19.296 % and 26.007 % in total melting time. The maximum temperatures of PCMs increase with augment of the fin volume, as illustrated in Fig. 10(d). It is specifical that the phase change unit with a fin volume of 0.1 exhibits a prolonged “dead zone”, necessitating a longer time for complete melting. The absorbed more thermal energy results in rapid increase in temperature of PCMs, which in turn demonstrates the highest maximum temperatures of the PCMs. Conversely, an increase in fin volume results in a more complex tree-branch structure of the topology fins, creating more contacting surface between fins and PCMs. A notable restriction is then imposed on the flow of liquid PCMs. The maximum velocity of PCM decreases with the augment of the fin volume accordingly. Fig. 10(f) exhibits phase change contours and velocity contours of phase change units. The phase change of PCM is observed to accelerate the melting in the vertical direction, with the increase of fin volume. The fins strengthen the heat conduction process, shorten the melting time, and gradually reduce the dead zone, so that the PCM in the region is almost completely melted at the same time. Velocity contours prove that the increase in fin volume limits natural convection and decreases the maximum velocity of PCM.

3.5. Tube-shell ratio

Performance of top-fin phase change units is assessed under various tube-shell ratios (ε). The volume of the copper fin is fixed at 0.2 in all cases, as indicated in Fig. 11(a). It is determined that there is a direct correlation between the tube-shell ratio and the PCM content loaded in the phase change units. A higher tube-shell ratio implies a less PCM content in the phase change unit, which leads to a faster PCM melting rate and a larger liquid fraction of PCM. The phase change unit requires 49.974 min to complete the melting of PCM at the tube-shell ratio of 0.2. The total melting time separately decreases to 33.648 and 24.373 min when the tube-shell ratios of phase change units rise to 0.3 and 0.4. The shortest melting time of merely 18.408 min is observed in the phase change unit with the largest tube-shell ratio of 0.5. It is further obtained that the time saving ratio substantially increases with augment of the tube-shell ratio, demonstrating approximately 32.699 %, 51.229 % and 63.165 % reduction in total melting time in contrast to the tube-shell ratio case of 0.2. Both the maximum temperature of PCM and maximum velocity of liquid PCM are found to generally increase with growth of the tube-shell ratio. These findings are attributable to the enhanced heat transfer and natural convection of PCMs under the larger tube-shell ratio. As shown in the phase change contours and velocity contours of phase change units in Fig. 11(f), with the increase of the tube-shell ratios, the liquid fraction of PCM increases significantly, and the melting rate is significantly faster. When $\varepsilon = 0.5$, most areas have been melted in 15 min.

3.6. Heat flux

Fig. 12 elaborates the performance of top-fin phase change units as a function of the heat flux (q). An elevated heat flux from the heat source signifies a greater transfer of thermal energy to the phase change units, thereby markedly accelerating the phase transition of PCMs. Meanwhile, natural convection of liquid PCMs on the driven of thermal buoyancy will evolve into more intense under higher heat flux, which is positive to

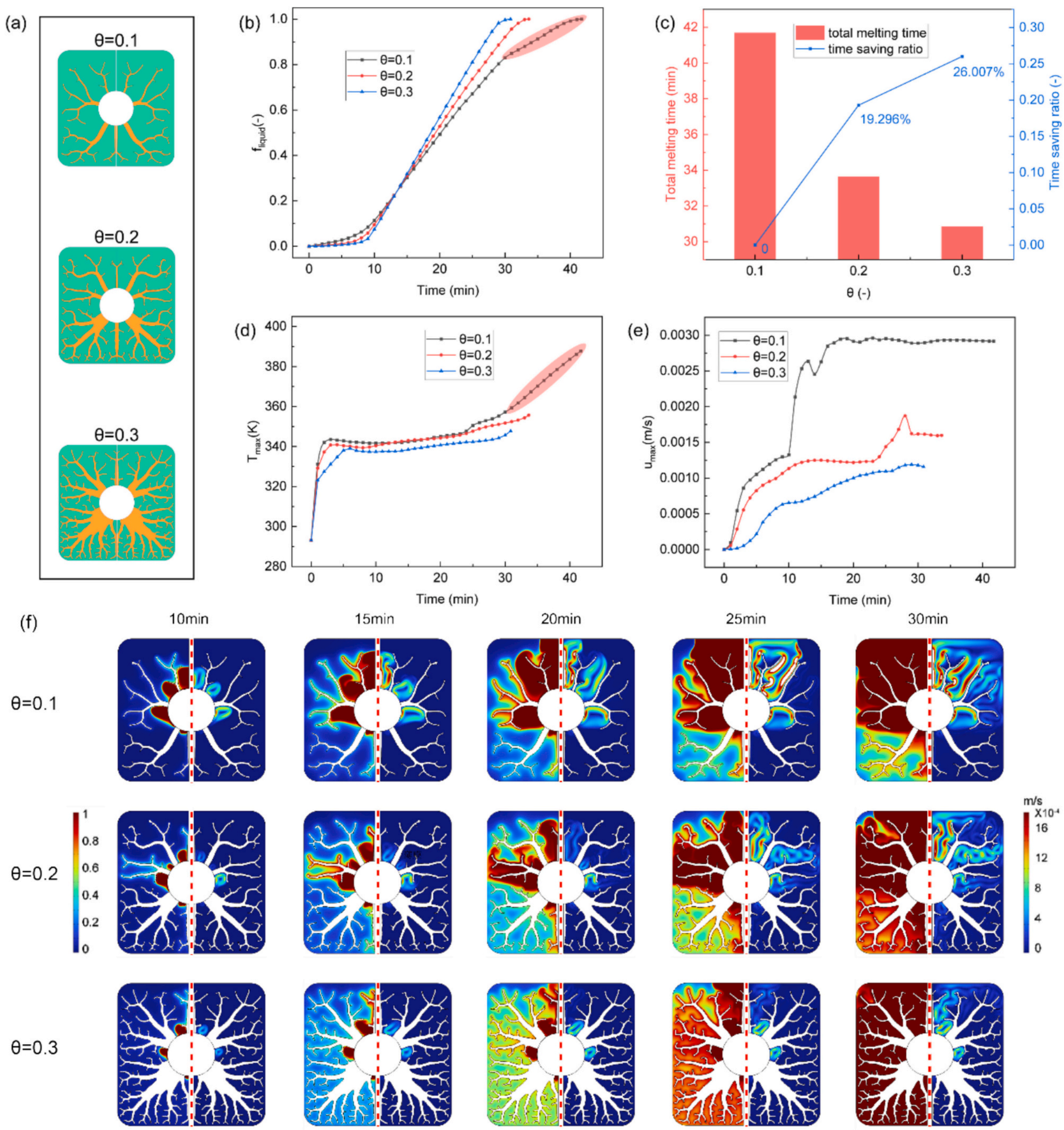

Fig. 10. Performance of top-fin phase change units with various fin volumes.

the melting of PCM. It is established that the total melting time is sharply reduced from 64.917 min in the top-fin phase change unit with the heat flux of 2500 W/m^2 to 33.648 min, 23.121 min and 17.791 min in top-fin phase change units with the heat fluxes of 5000, 7500 and 10,000 W/m^2, respectively. The corresponding time saving ratio demonstrates a notable increase as the heat flux continues to rise. Enhancement of 48.168 %, 64.384 % and 72.594 % is calculated in time saving ratio compared to the lowest heat flux case ($q = 2500$ W/m^2), as indicated in Fig. 12(c). Fig. 12(d) indicates that the highest maximum temperature of PCM is reached in the case of largest heat flux, due to the greatest absorption of thermal energy by the phase change unit. An increase in heat flux results in a greater flow of liquid PCMs within the melted zone of the phase change unit. This is reflected in the observation that the velocity of PCM rises with an increase in heat flux, as illustrated in Fig. 12(e). Fig. 12(f) exhibits phase change contours and velocity contours of phase change units. With the increase of heat flux, the liquid fraction of PCM increases, and more heat accumulates in the phase change unit. Moreover, natural convection obviously accelerates the melting rate of PCM and shortens the total melting time of PCM.

4. Conclusions

This paper presents a design of non-uniform topology fin structures for improving thermal performance of LHTES system. The topology domain is established based on a density-based topology optimization methodology in a rounded rectangle phase change unit, with a heat source varying in accordance with the temperature trend along the vertical direction. The numerical model is solved by the FEM and

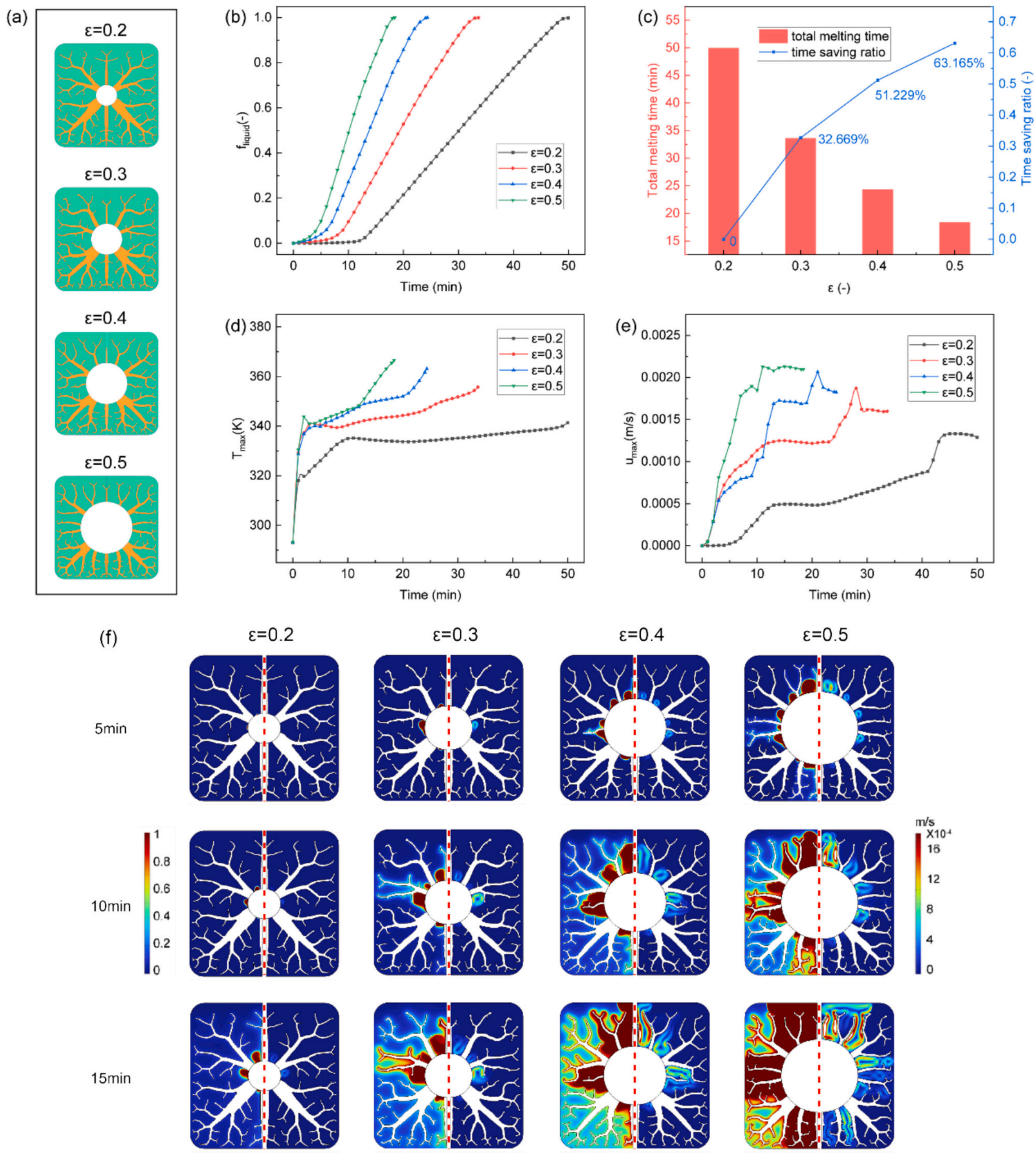

Fig. 11. Performance of top-fin phase change units with various tube-shell ratios.

verified with experimental determination. Several conclusions drawn from this investigation are listed as follows:

(1) The uneven heat source in the topology optimization induces production of tree-like fin structure. The nonuniform copper fins becomes more pronounced as the parameter n rises. More metal fins are placed in the bottom part, which enables to reduce the "dead zone" in the bottom corners of the phase change units with no fins or even fins.
(2) The non-uniform structure intensifies phase change heat transfer of PCMs, saving approximately 52.78 % and 23.62 % in total melting time compared to the no-fin and even-fin cases. Heat transfer also becomes homogenized, with the lowest average temperature observed in the topology phase-change unit.
(3) The phase change unit with nonuniformly topology fins ($n = 2$) is identified to express fastest melting rate and lowest average temperature of PCMs. The balance between thermal conduction and convection leads to remarkable variation in heat transfer at the initial and final stages of PCM melting.
(4) The PCM melting is significantly accelerated by an increase in fin volume, tube-shell ratio and heat flux. Rising the fin volume benefits the uniform melting process, leading to lower maximum

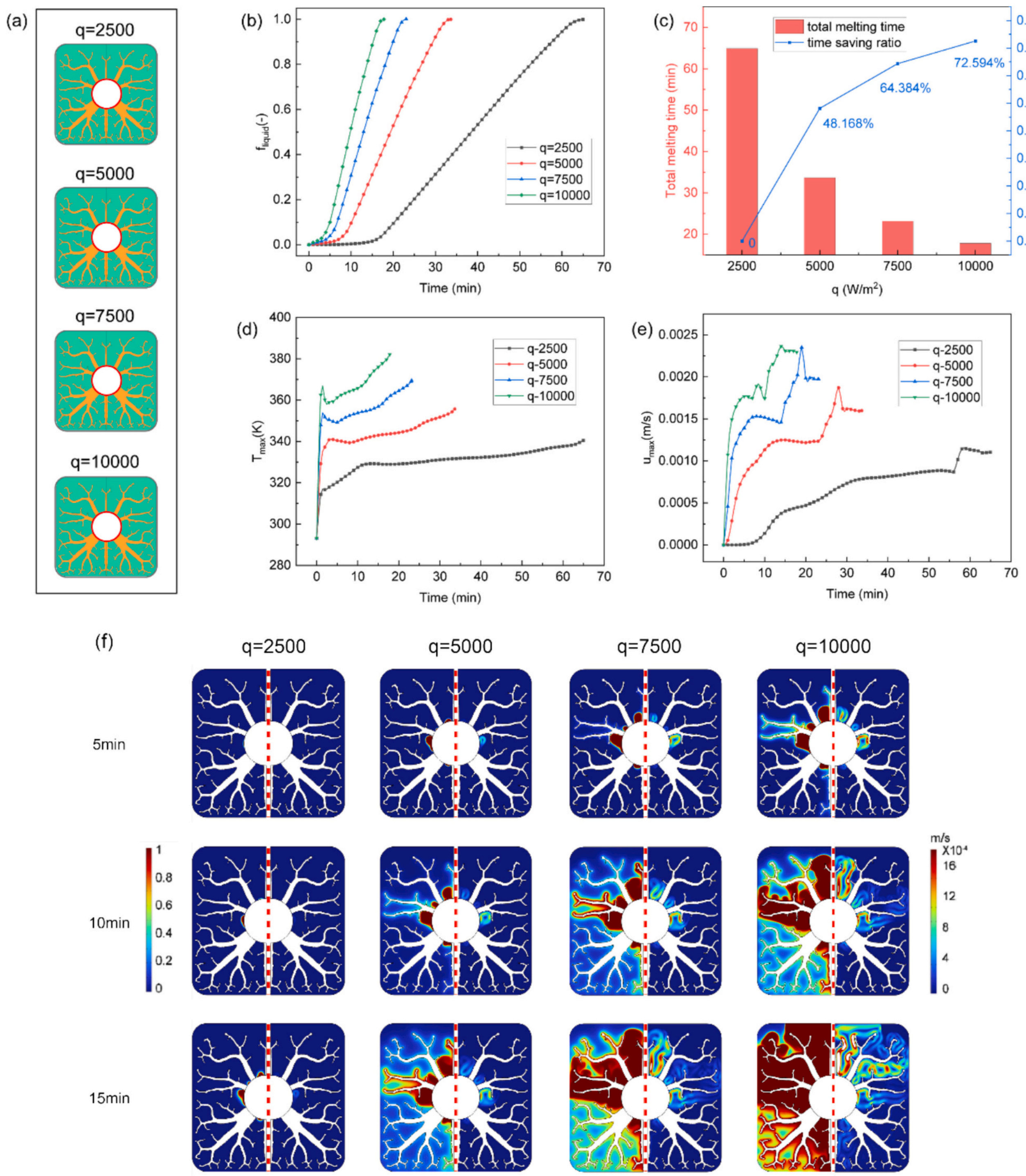

Fig. 12. Performance of top-fin phase change units with various heat fluxes.

temperature and average temperature of PCM. Increasing the tube-fin ratio and heat flux establishes higher maximum temperature and average temperature of PCM.

In conclusion, this paper reveals the heat transfer performance of the phase change units enhanced by the nonuniform topology optimization. Obtained findings could facilitate the advancement of high-efficient phase change system suitable for deployment in energy fields.

CRediT authorship contribution statement

Zhaoli Zhang: Writing – original draft, Visualization, Investigation. **Xinyu Chen:** Writing – review & editing, Validation, Methodology. **Nan Zhang:** Writing – review & editing, Visualization, Supervision. **Yanping Yuan:** Writing – review & editing, Visualization, Supervision. **Daniela Dzhonova-Atanasova:** Writing – review & editing, Visualization, Supervision. **Shady Attia:** Writing – review & editing, Visualization, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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