Supplementary material

Model equations and parameter values

Differential Equations

Mean voltages, V_v and V_m, of the VLPO and MA

$$\tau_v \frac{dV_v}{dt} = v_{vm} Q_m - V_v + D_v \quad (1)$$

$$\tau_m \frac{dV_m}{dt} = v_{mv} Q_v - V_m + D_m + W \quad (2)$$

Homeostatic drive

$$\tau_H \frac{dH}{dt} = v_{Hm} Q_m - H \quad (3)$$

Circadian variables

$$\tau_X \frac{dX}{dt} = Y + \gamma\left(\frac{1}{3}X + \frac{4}{3}X^3 - \frac{256}{105}X^7\right) + v_{Xp} D_p + v_{Xn} D_n \quad (4)$$

$$\tau_Y \frac{dY}{dt} = D_p\left(v_{YY} Y - v_{YX} X\right) - \left(\frac{\delta}{\tau_c}\right)^2 X \quad (5)$$

Photoreceptor activity

$$\frac{dP}{dt} = \alpha(1 - P) - \beta P \quad (6)$$

Complementary Functions

Mean population firing rate

$$Q_i = \frac{Q_{max}}{1 + \exp\left(\left|\theta - V_i\right| / \sigma'\right)}, i = v, m \quad (7)$$

Wake effort

$$W = F_W \, max\left[0, V_{WE} - v_{mv} Q_v - D_m\right] \quad (8)$$

Total sleep drive

$$D_v = v_{vh} H + v_{vC} C + A_v \quad (9)$$

Circadian drive, sleep propensity model (Postnova et al., 2016)

$$C = 0.1\left(\frac{1 + X}{2}\right) + \left(\frac{3.1X - 2.5Y + 4.2}{3.7(X + 2)}\right)^2 \quad (10)$$

Nonphotic drive to the circadian

$$D_n = \left(S - \frac{2}{3}\right)\left[1 - \tanh(rX)\right] \quad (11)$$

Photic drive to the circadian

$$D_p = \alpha(1 - P)(1 - \varepsilon X)(1 - \varepsilon Y) \quad (12)$$

$$\alpha = \alpha_0 \frac{I}{I + I_1}\sqrt{\frac{I}{I_0}} \quad (13)$$

$$I = IS \quad (14)$$

State function

$$S = \begin{cases} 1, V_m > V_{th} \text{(wake)} \\ 0, V_m \le V_{th} \text{(sleep)} \end{cases} \quad (15)$$

Circadian Phase Calculation

$$t_{crit} = t_{\varphi crit} + t_0 \quad (16)$$

$$\varphi_{crit} = -2.98 = atan\left(\frac{Y}{X}\right) \quad (17)$$

Time constants

$\tau_v = \tau_m = 50\,\text{s}$

$\tau_H = 59\,x\,3600\,\text{s}$

$\tau_X = \tau_Y = 24\,x\,3600/2\pi\,\text{s}$

$\tau_C = 24.2\,x\,3600\,\text{s}$

Coupling strengths

$v_{vm} = -2.1\,\text{mV}$

$v_{mv} = -1.8\,\text{mV}$

$v_{Hm} = 4.57\,\text{s}$

$v_{Xp} = 37\,x\,60\,\text{s}$

$v_{Xn} = 0.032$

$v_{YY} = 1/3\,v_{Xp}\,\text{s}$

$v_{YX} = 0.55\,v_{Xp}\,\text{s}$

$v_{vH} = 1$

$v_{vC} = -0.5\,\text{mV}$

External neuronal drives

$A_v = -10.3\,\text{mV}$

$D_m = A_m = 1.3\,\text{mV}$

Circadian

$\gamma = 0.13$

$\delta = 24\,x\,3600/0.99729\,\text{s}$

$\beta = 0.007/60\,\text{s}^{-1}$

Nonphotic drive

$r = 10$

Photic drive

$\varepsilon = 0.4$

$I_0 = 100\,\text{lx}$

$I_1 = 9500\,\text{lx}$

$\alpha_0 = 0.1/60\,\text{s}^{-1}$

Firing rate

$Q_{max} = 100\,\text{s}^{-1}$

$\theta = 10\,\text{mV}$

$\sigma' = 3\,\text{mV}$

Wake effort

$V_{WE} = -0.07\,\text{mV}$

$V_{th} = -2\,\text{mV}$

$F_W = 1$ if forced wake and 0 otherwise

Circadian phase

MELpeak

$t_0 = 0.7\,x\,3600\,\text{s}$

CBTmin

$t_0 = 2.7\,x\,3600\,\text{s}$

For further details, see: Postnova, S.; Lockley, S.W.; Robinson, P.A. Sleep Propensity under Forced Desynchrony in a Model of Arousal State Dynamics. *J. Biol. Rhythms* 2016, *31*, 498–508, doi:10.1177/0748730416658806

Supplemental Figures

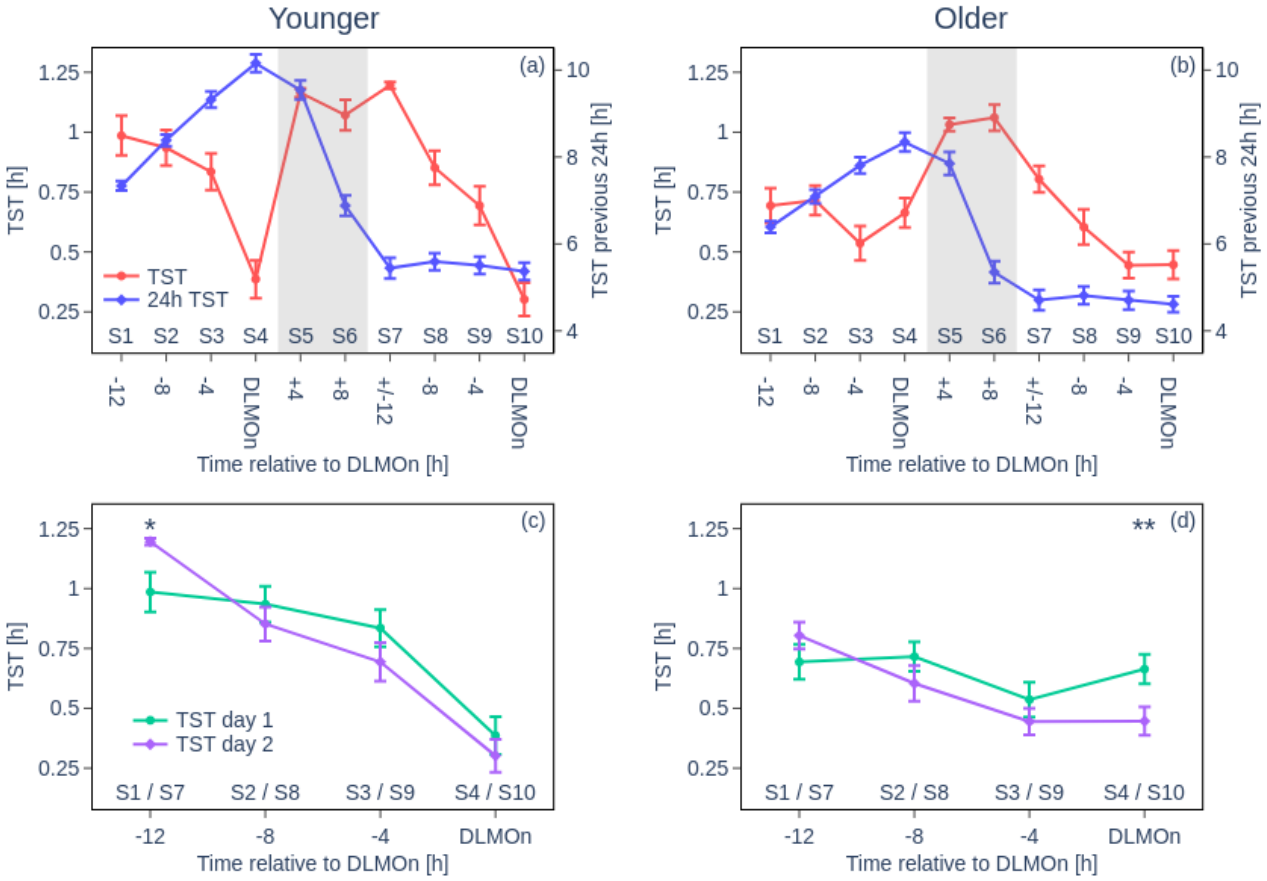

Figure S1: Sleep duration and sleep history over the multiple nap protocol relative to melatonin onset. Left hand panels show the group mean total sleep time (TST) per sleep opportunity and mean cumulative sleep time over the prior 24 hours (±SEM) over the protocol. The right hand panels compare the TSTs during the sleep opportunities appearing at the same circadian phase on the first (S1-S4) and the second (S7-S10) day (with significant differences indicated with * for $p<0.05$, ** for $p<0.01$). Panels (a) and (b) are for the younger group, and (c),(d) for the older group.

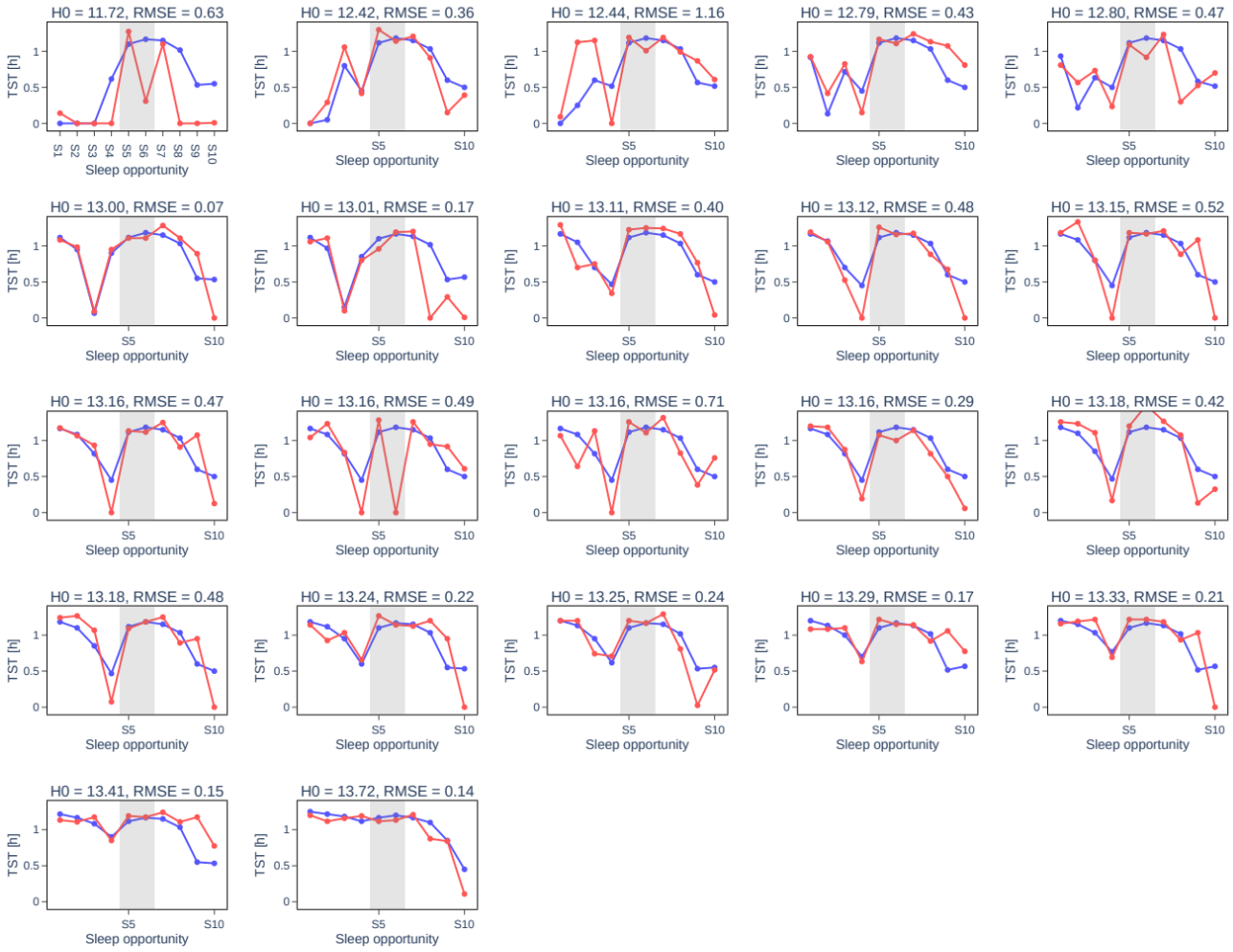

Figure S2: Data (red) and best fits (blue) for younger participants. Participants are ordered by the best fit initial value of the homeostatic drive, H0. The root-mean squared error over the first four sleep opportunities is provided.

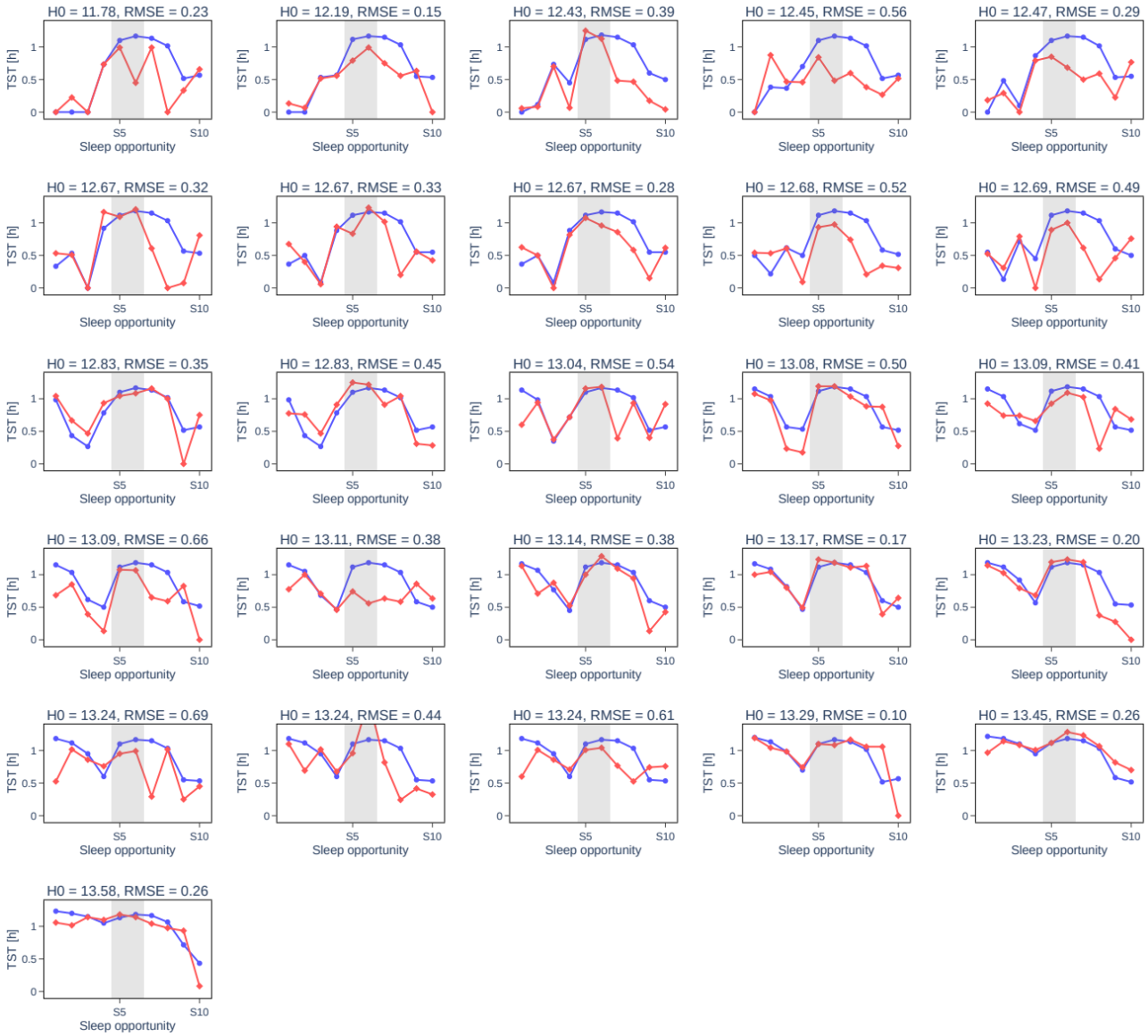

Figure S3: Data (red) and best fits (blue) for older participants. Participants are ordered by the best fit initial value of the homeostatic drive, H0. The root-mean squared error over the first four sleep opportunities is provided.