# Bidding in day-ahead electricity markets: a dynamic programming framework

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**Abstract**

Strategic bidding problems have gained a lot of attention with the introduction of deregulated electricity markets where producers and retailers trade electricity in a day-ahead market run by a Market Operator (MO). All actors propose bids composed of a unit production price and a quantity of electricity to the MO. Based on these bids, the MO selects the most interesting ones and defines the spot price of electricity at which all actors are paid. As the bids of all actors determine the price of electricity, a bidding Generation Company (GC) faces a high risk regarding its profit when placing bids as the bids of competitors are not known in advance. This paper proposes a novel dynamic programming framework for a GC's Stochastic Bidding Problem (SBP) in the day-ahead market considering uncertainty over the competitor bids. We prove this problem is NP-hard and study two variants of this problem solved with the dynamic programming framework. Firstly, a relaxation provides an upper bound solved in polynomial time (SBP-R). Secondly, we consider a bidding problem using fixed bidding quantities (SBP-Q) that has previously been solved through heuristic methods. We prove that SBP-Q is NP-hard and solve it to optimality in pseudo-polynomial time. SBP-Q is solved on much larger instances than in previous studies. We show on realistic instances that its optimal value is typically under 1% of the optimal value of SBP by using the upper bound provided by SBP-R.

*Keywords:* Deregulated electricity markets, Strategic bidding, Dynamic Programming, Stochastic optimization

## 1. Introduction

Electricity markets have significantly evolved over the past years following the introduction of deregulated systems where Generation Companies (GCs) and retailers trade electricity according to a standardized procedure, to maximize the social welfare (Hobbs et al., 2000), through increased competition between the

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agents. In this context, we consider a wholesale electricity market run by a market operator (MO) within a day-ahead scheme. In this context, producers and retailers propose to the MO, on the day before delivery, hourly bids corresponding to a unit production cost, together with associated quantities. Based on this information, the MO selects the optimal bids and sets the resulting hourly spot prices on delivery day. For a GC, the bids are associated with generators and represent the minimum unit price accepted by the GC for a generator and the quantity that can be produced at that price. For a retailer, the bid price corresponds to the maximum price it is willing to pay for electricity. Based on all bids, the MO selects the cheapest production bids and the most expensive retailer bids for each hour until demand is met. Finally, the maximum price of the production bids accepted defines the hourly spot price (Hobbs & Helman, 2004; Ramos et al., 1999).

Wholesale electricity markets lead to challenging problems for electricity producers (Conejo & Prieto, 2001; Kahn, 1995). As spot prices depend on the bids of all agents, a GC is exposed to a high risk regarding its pricing strategy. Indeed, low bids may yield a decreasing spot price, thus negatively impacting revenues. Alternatively, high bids could be rejected and result in lost sales as analyzed in Brotcorne et al. (2022). Striking the right balance requires a thorough understanding of the market mechanism that determines the spot price of electricity (Esmaeili Aliabadi et al., 2017; Panda & Kumar Tiwari, 2018).

This paper addresses the issue of determining an optimal bidding strategy for a GC in a day-ahead market. This problem is complex for several reasons. First, its mathematical formulation is highly nonconvex (Kwon & Frances, 2012). Next, uncertainty concerning the competing bids induces uncertainty concerning the spot price (Madani & Van Vyve, 2015). Thirdly, the electricity production planning problem for the GC called Unit Commitment (UC), which determines the production cost of electricity, is also a challenging problem in itself (Tahanan et al., 2015).

Most of the literature concerning the bidding problem for a GC uses a deterministic approach where competitors' bids are considered as known. This removes the uncertainty over the spot prices defined by the bids of the GC (de la Torre et al., 2002). The focus is then on integrating a UC model to have realistic production costs (Steeger et al., 2014) in a uniform pricing market. Further market regulations features have also been considered, such as bidding curves associated to generators (Bakirtzis et al., 2007), or coupled day-ahead markets linked by a transmission network, such as the European network (Kardakos et al., 2014; Brotcorne et al., 2022). Mathematically, the problem is most commonly formulated as a bilevel optimization problem involving the bidding GC at the upper level and the MO at the lower level, where hourly spot prices based on the bids of the GC are determined (Bakirtzis et al., 2007; Fampa et al., 2008; Ruiz & Conejo, 2009; Zhang et al., 2011; Kardakos et al., 2014; Dalby, 2017; Brotcorne et al., 2022). Based on the characterization of lower-level optimality via the Karush-Kuhn-Tucker conditions, the problem can be reformulated and solved (for small instances) as an equivalent mixed integer linear program (MILP).

Other approaches explicitly embed uncertainty within the formulation while simultaneously relaxing some production or regulation constraints. For instance, Baillo et al. (2004) consider uncertain residual demand functions to estimate the possible spot prices depending on the bidding quantities of the GC under the assumption of linear production costs. Computationally, the model is formulated as a MILP and solved using a Benders decomposition algorithm. Beraldi et al. (2008) consider a UC formulation with transmission constraints, where uncertainty over competitor bids is modeled through scenarios. The drawback of this approach is that it uses predetermined spot prices. The reformulation of the problem as a MILP is then solved on a set of small instances. In Fampa et al. (2008), production costs are linear, and competing bids are represented by a set of scenarios. The resulting model SBP-Q is then solved by a primal-dual heuristic algorithm whose performance is assessed on instances that can be solved to optimality. For large instances, Fampa & Pimentel (2015) propose a genetic algorithm, while, in Fampa & Pimentel (2017), the upper bound obtained by solving a linear program is validated on instances of reduced size. Ostadi et al. (2020) use risk-based portfolio optimization to determine bidding strategies for a GC. Fixed bidding prices are used as chromosomes in a genetic algorithm. The associated bidding quantities are determined through a Markowitz model with an associated risk in terms of acceptance. Deterministic bids of competitors are estimated through historical data and used in the Markowitz model. The bidding problem from the point view of a retailer is studied by Song & Amelin (2018). From this side, no production cost must be considered; the new difficulty lies in the real-time demand of the consumers. The problem is modeled through a MILP formulation, optimizing the Condition Value-at-Risk of the profit. The uncertainty is modeled through a set of scenarios consisting of the aggregated production of GCs, and an elasticity matrix is used to represent the deviation in real-time demand. This leads to a large formulation considering only a limited number of scenarios.

In the present paper, we address the *Stochastic Bidding Problem* (SBP) to determine a bidding strategy that maximizes the expected profit of a GC. Solving SBP to optimality is very challenging (and an open question), we tackle this problem by studying two variants of this problem: the first one provides a heuristic solution of high quality, and the second one provides a tight upper bound that allows to assess the quality of the heuristic solution. We assume that production costs are linear, and uncertainty enters the model via a set of scenarios representing competitors' bids. The MO uses uniform pricing, and network constraints are not considered as in Fampa et al. (2008); Fampa & Pimentel (2015, 2017). The resulting model is a generalization of SBP-Q that embeds the bidding quantities as decision variables. Problem SBP-Q is used as a proxy model for SBP. The contribution of this work is twofold. First, we prove that SBP and SBP-Q are both NP-hard. Next, based on properties of the bidding prices, we develop an exact dynamic programming (DP) framework for two variants of SBP. The first one (SBP-R) consists of a relaxation of SBP where the GC may place as many bids as desired, independently from the generators. It is solved by DP in polynomial time, providing an upper bound on SBP. The second one is the problem studied by Fampa et al. (2008) using predetermined bidding quantities (SBP-Q), which is solved to optimality by DP in pseudo-polynomial time with respect to the number of generators. On both variants, DP performs better than previously proposed algorithms. In numerical experiments, we observe that the proxy model SBP-Q finds near-optimal solutions for SBP, the solution quality being assessed by the upper bound provided by SBP-R.

The rest of the paper is organized as follows. Section 2 provides a description of SBP, its two variants, as well as some general properties of the spot price and bid prices. We also show that SBP is NP-hard. Section 3 proposes a DP framework for SBP, which is adapted to compute an upper bound on the optimal value of the problem in Section 3.2 and to solve SBP-Q to optimality in Section 3.3. Numerical results are presented in Section 4. Section 5 concludes with suggestions for algorithmic improvements.

## 2. The stochastic bidding problem

### *2.1. Problem Description*

The *Stochastic Bidding Problem* faced by a GC consists in placing revenue-maximizing bids towards the market operator, together with productions for its generators, within the day-ahead market. The day-ahead market consists of a trading platform for producers and retailers, which is run by a *Market Operator* (MO) using uniform pricing and ignoring transmission constraints. Hourly bids composed of a unit price and a quantity are proposed by all actors to the MO the day before delivery. Once all bids are received, the MO selects the bids maximizing the social welfare, settles quantities traded between actors, and fixes the resulting spot price for electricity for each time period (O'Neill et al., 2005). Without loss of generality, we consider only bidding producers and a fixed demand; this will be detailed in Section 2.5. In the day-ahead market, the spot price occurs at the intersection of the aggregate production and demand curves, as illustrated in Figure 1. In this example, the demand is equal to 10, and there are four bids: (2,4), (5,3),(8,1) and (10,3). Bids below the spot price are traded, bids above the spot price are not traded, while bids that match the spot price are partially traded to meet the demand. All bids are fully traded in this example, except the last one for which only a quantity of 2 is traded.

Throughout this paper, we assume that production costs are linear and that the bids of the competing GCs and the fixed total demand are estimated through a set of scenarios. It follows from the linearity assumption that the problem can be decomposed by time period. From now on, GC will refer to the optimizing GC.

Let the GC own a set of generators $J$, $|J| = m$ and place bids $\{(\pi_j, q_j)\}_{j\in J}$ on a day-ahead market, $\pi_j$ representing the unit bid price of electricity and $q_j$ the bidden quantity. Each generator $j \in J$ has an associated unit production cost $c_j$ and a maximum production capacity $\overline{q}_j$. The MO sets a maximum bid price of $\overline{\pi}$, and we assume that $c_j \leq \pi_j \leq \overline{\pi}$ for all $j \in J$. This assumption is made without loss of generality

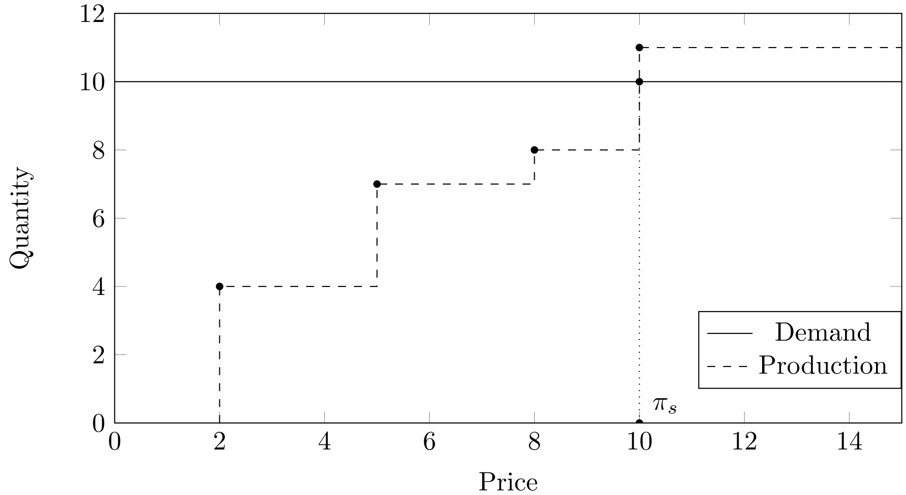



Figure 1: Aggregated production curve and spot price

as bids not respecting this condition are either refused by the MO or cannot produce a positive income. Generators in $J$ are indexed in non-decreasing order of cost, that is, $c_j \leq c_{j'}$ if $j, j' \in J, j < j'$.

The competitors own a set of generators $\tilde{J}$, $|\tilde{J}| = \tilde{m}$. The uncertainty over the total demand and competing bids can be represented through a set of scenarios $S$ (Birge & Louveaux, 2011) each consisting of:

- $p_s$: the probability of scenario $s$,
- $d_s$: the total demand on the market,
- $\{(\tilde{\pi}^s_j, \tilde{q}^s_j)\}_{j \in \tilde{J}}$: the bids of the competitors, with $0 < \tilde{\pi}^s_j \leq \overline{\pi}$ and $0 < \tilde{q}^s_j$. We assume that $\sum_{j \in \tilde{J}} \tilde{q}^s_j > d_s$ to ensure the demand can be met in each scenario. We consider that the bidding prices of competitors are distinct as competitor bids at the same price can be aggregated into a single bid from the perspective of the GC.

In a day-ahead market, the GC places its bids before knowing what scenario occurs. It must thus consider the information of all scenarios in its bidding strategy. A single GC is likely to represent only a small proportion of the total number of generators bidding in this market. Without loss of generality, we consider in the following that $m < \tilde{m}$ to simplify the complexity analysis of the algorithms proposed.

In a given scenario $s \in S$, the profit of the GC depends on the quantity $q^s_j$ traded by the MO for each generator $j \in J$ and the spot price $\pi^s$ of electricity. The MO determines these values by maximizing the social welfare based on the bids received. The MO can trade any proportion of a bid from the GC or a competitor. If the MO receives a set of bids $\{(\tilde{\pi}_j, \tilde{q}_j)\}_{j \in J^{MO}}$ in scenario $s \in S$, including the bids of the GC, and trades a quantity $q_j^s$ for each $j \in J^{MO}$, $J^{MO} = J \cup \tilde{J}$, its problem can be formulated as the mathematical program

$$
\begin{aligned}
(\mathrm{SPOT}^s)\ \min \quad & \sum_{j \in J^{MO}} \pi_j q_j^s \\
\text{s.t.} \quad & \sum_{j \in J^{MO}} q_j^s = d_s \qquad (\pi^s) \qquad\qquad (1)\\
& 0 \leq q_j^s \leq \tilde{q}_j, \quad \forall j \in J^{MO}
\end{aligned}
$$

It has been shown by Baker & Taylor (1979); Balachandran & Ramakrishnan (1996) that the spot prices $\pi^s$ are the optimal values of the dual variables associated with the demand constraints (1) of ($\mathrm{SPOT}^s$).

As the MO solves the maximization welfare problem only once all bids are received, we face a two-stage Stackelberg game, where the leader GC places revenue-maximizing bids, anticipating the spot price and quantities traded determined by the MO. This problem has been presented by Fampa et al. (2008) as the following bilevel problem:

$$
\begin{aligned}
(\mathrm{SBP})\ \max \quad & \sum_{s \in S} p_s \sum_{j \in J} (\pi^s - c_j) q_j^s \\
\text{s.t.} \quad & 0 \leq \pi_j \leq \overline{\pi}, \quad \forall j \in J \\
& 0 \leq q_j \leq \overline{q}_j, \quad \forall j \in J \\
& (q_j^s, \pi^s) \in \arg\min \Big(\sum_{j \in J} \pi_j q_j^{s'} + \sum_{j \in \tilde{J}} \tilde{\pi}_j^s q_j^{s'}\Big) \qquad s \in S \\
& \qquad \text{s.t.} \quad \sum_{j \in J \cup \tilde{J}} q_j^{s'} = d_s \qquad (\pi^s) \\
& \qquad\qquad 0 \leq q_j^{s'} \leq q_j \qquad j \in J \\
& \qquad\qquad 0 \leq q_j^{s'} \leq \tilde{q}_j^s \qquad j \in \tilde{J}
\end{aligned}
$$

Note that this formulation is invalid as the spot prices $\pi^s$, which are not available in closed form, must enter the objective function. This problem is generally formulated as a Mathematical Program with Equilibrium Constraints (MPEC) in the literature to incorporate the quantities traded and the spot prices of the lower-level problem (Bakirtzis et al., 2007; Ruiz & Conejo, 2009; Kardakos et al., 2014; Dalby, 2017; Brotcorne et al., 2022). We rather chose an approach based on the characterization of the spot price. We assume that given equivalent lower-level solutions, the one maximizing the GC's revenue is selected. This hypothesis corresponds to the optimistic assumption (Loridan & Morgan, 1996) and ensures an equilibrium always exists, which is a common assumption for bilevel formulations or MPECs. Namely,

- If several spot prices are available, the highest one is selected, which is consistent with classical economic theory for the spot price of a resource (Littlechild, 1988).

- If several bid prices are equivalent for the MO, the one preferred by the GC will be selected. This can occur if several bids are made at the spot price. Practically, the GC can decrease the bidding price by a small value $\epsilon$ to ensure this priority with a negligible impact on profit if $\epsilon \to 0$.

### 2.2. Variants of SBP

Two variants of SBP are considered in this paper:

1. The *Stochastic Bidding Problem Relaxation* (SBP-R), in which the GC can place bids independently from generators. A solution of SBP-R consists in a set of bids $B = \{(\pi_k, q_k)\}_{k\in\{1,\dots,K\}}$ where $K$ is an arbitrary number of bids and such that $\sum_{k=1}^{K} q_k \leq \sum_{j\in J} \overline{q}_j$. Once the MO has selected the bids, the GC dispatches the production to its cheapest generators.
   A solution of SBP is feasible for SBP-R and can have a higher value in SBP-R, as the production of each generator can be split across multiple bids, increasing bidding flexibility. The optimal value of SBP-R thus provides an upper bound on SBP.
2. The *Stochastic Bidding Problem with fixed Quantities* (SBP-Q), a constrained version of SBP in which the bidden quantity of each generator is set to its maximum production capacity, $q_j = \overline{q}_j$ for all $j \in J$. A solution of SBP-Q consists in a set of bids $B = \{(\pi_j, \overline{q}_j)\}_{j\in J}$ where only bid prices $\pi_j$ need to be determined. In economic terms, this corresponds to a Bertrand model where a company tries to optimize its profit by adjusting the price of a resource.
   This problem is used as a proxy model for SBP and has previously been studied by Fampa et al. (2008), who presented a primal-dual heuristic and a genetic algorithm (Fampa & Pimentel, 2015) to find a feasible solution. The quality of this solution was first validated by using a strong MILP formulation to obtain the optimal value of SBP-Q. An upper bound on the optimal solution was found later by studying a relaxation of the problem and addressing it by a cutting plane method (Fampa & Pimentel, 2017).

Let $z^x$ be the optimal value of problem $x$. We have

$$z^{SBP-Q} \leq z^{SBP} \leq z^{SBP-R}.$$

The quality of a feasible solution for SBP provided by the proxy model SBP-Q can be evaluated through the optimal value of SBP-R.

### 2.3. Spot and bid prices

The spot prices defined in a day-ahead market must respect some basic rules to prevent trading at loss or ending up with a spot price strictly higher than bids not traded (Madani & Van Vyve, 2015). More formally,

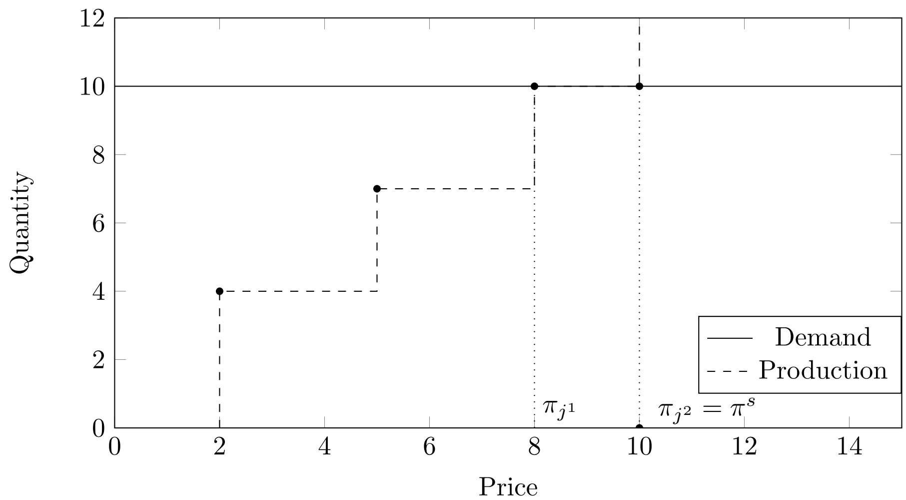



Figure 2: Multiple spot price scenario

for a set of bids $\{(\tilde{\pi}_j, \tilde{q}_j)\}_{j \in J^{MO}}$ ordered in non-decreasing order of price and an optimal solution of SPOT$^s$ with values $q_j^{s*}$ we must have that

$$\tilde{\pi}_{j^1} \leq \pi^s \leq \tilde{\pi}_{j^2}, \quad j^1 = \max\{j \in J^{MO} | q_j^{s*} > 0\}, \quad j^2 = \min\{j \in J^{MO} | q_j^{s*} = 0\}$$

Generator $j_1$ corresponds to the first generator meeting the demand while generator $j_2$ corresponds to the first generator exceeding the demand. Note that formulation SPOT$^s$, akin to a continuous knapsack problem, is solved by selecting the cheapest bids that meet the demand. As a result,

$$j^1 = \min\{j \in J^{MO} | \sum_{j' \in J^{MO}, j' \leq j} q_{j'}^s \geq d_s\},$$

$$j^2 = \min\{j \in J^{MO} | \sum_{j' \in J^{MO}, j' \leq j} q_{j'}^s > d_s\}$$

and the spot price lies in the interval $[\pi_{j^1}, \pi_{j^2}]$. In the example of Figure 1, $j^1 = j^2$ and $\pi_{j^1} = \pi_{j^2}$ as the demand is strictly met only at a price equal to 10. Alternatively, the demand can be exactly met for an interval of price values, in which case $j^1 \neq j^2$, as illustrated in Figure 2. In this example, the bids are (2,4), (5,3), (8,3) and (10,3). The quantity bidden at a price of 8 in Figure 1 has been increased to meet the demand at that price. It follows that the demand is met for a price between 8 and 10. As the highest spot price is always selected when there are several possibilities (Littlechild, 1988), the MO sets the spot price to $\pi_{j^2}$. These observations are summarized in the following lemma.

**Lemma 1.** *Given a demand $d_s$ and a set of bids $\{(\pi_j, q_j)\}_{j \in J^{MO}}$, the highest spot price $\pi^s$ is equal to $\pi_{j^*}$, where $j^* = \min\{j \in J^{MO} : \sum_{j' \in J^{MO}, \pi_{j'} \leq \pi_j} q_{j'} > d_s\}$.*

As a consequence of Lemma 1, the continuous bidding prices $\pi_j$ to consider in an optimal solution of SBP can be restricted to a finite set that depends on the competitors' bids throughout all possible scenarios, as shown by Fampa et al. (2008). Let $\Lambda$ denote this set of increasing and distinct bid prices of the competitors in scenarios of $S$, where the values 0 and $\overline{\pi}$ are added if not present in the scenarios, that is, $\Lambda = \{\tilde{\pi}_j^s | s \in S, j \in \tilde{J}\} \cup \{0, \overline{\pi}\}$. Let $I$ denote the set of price indices in $\Lambda$ with $n = |\Lambda| - 1$. The $i^{th}$ price in $\Lambda$ is denoted $\lambda_i$ with $\lambda_0 = 0, \lambda_n = \overline{\pi}$ and $\lambda_i < \lambda_{i'}$, for $i, i' \in I$ and $i < i'$. Without loss of generality, we consider that if the GC does not place a bid for some given generator $j \in J$, then $\pi_j = \overline{\pi}$. As we consider there are $\tilde{m}$ bidding prices in each scenario, $\tilde{m} \leq n$. We also consider $m < \tilde{m}$, so $m < n$. This observation is made to simplify the notation in the complexity analysis of the algorithms presented in Section 3.

For each price $i \in I$ and scenario $s \in S$, the residual demand $r_i^s$ is defined as the difference between the demand $d_s$ and the total bidden quantities of competitors at prices strictly smaller than $\lambda_i$:

$$r_i^s = d_s - \sum_{\substack{j \in \tilde{J} \\ \tilde{\pi}_j^s < \lambda_i}} \tilde{q}_j^s, s \in S, i \in I$$

Table 1 presents the residual demands for the example of Figure 2.

| $\lambda_i$ | 2 | 5 | 8 | 10 | $> 10$ |
|---|---|---|---|---|---|
| $r_i^s$ | 10 | 6 | 3 | 0 | $< 0$ |

Table 1: Residual demands for the example of Figure 2

.

According to Lemma 1, the spot price is the lowest price $\lambda_i$ such that the total quantity bidden by the GC up to this price is strictly greater than $r_{i+1}^s$.

### *2.4. Theoretical complexity*

In this section, we consider the *decision version of SBP* (DBP) that consists in determining whether SBP has an optimal solution with a given value $V$ and show that it is strongly NP-complete through a reduction from the 3-Partition problem (see Garey & Johnson (1979)). Given a positive integer $B$ and a set $A$ of $3n$ positive integers $\{a_1, \ldots, a_{3n}\}$ such that $B/4 < a_j < B/2$ for all $j \in \{1, \ldots, 3n\}$ and $\sum_{j=1}^{3n} a_j = nB$, the 3-Partition problem consists in determining whether the set $A$ can be partitioned into $n$ sets $A_1, \ldots, A_n$, each containing exactly three elements, such that $\sum_{a \in A_i} a = B$.

**Theorem 2.** *Problem SBP is strongly NP-hard.*

*Proof.* The result follows from the reduction of 3-Partition to SBP. Given an instance of 3-Partition, a corresponding instance of DBP can be constructed as follows:

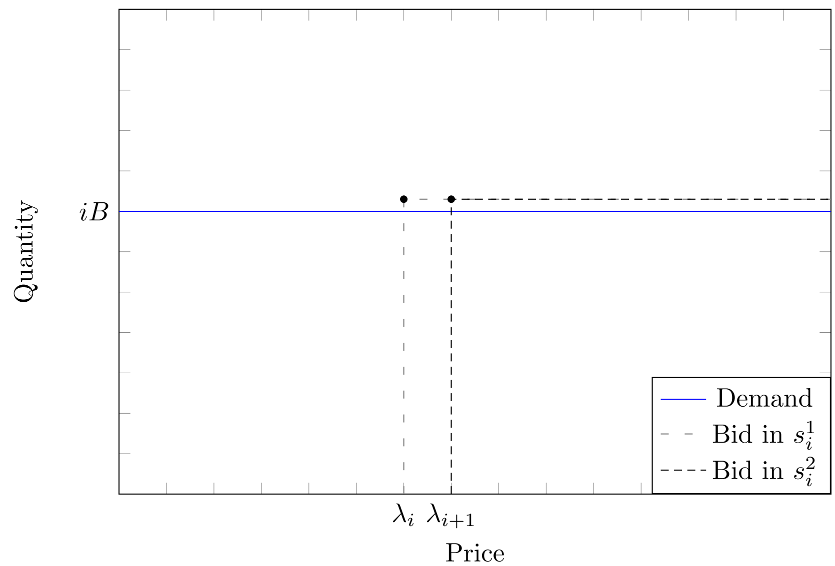



Figure 3: Scenarios $s_i^1$ and $s_i^2$ generated from 3-Partition

- A generator $j$ is created for each element in the set $A$ of the 3-Partition problem. The maximum production capacity $\overline{q}_j$ equal to $a_j$, the $j^{th}$ smallest value $a_j$ in $A$, and the production cost is set to 0. The total production capacity of the GC is then $\sum_{j=1}^{3n} \overline{q}_j = nB$.
- $2n$ scenarios $S = \{s_i^1, s_i^2 | i \in \{1, \ldots, n\}\}$ are created. For each couple $s_i = (s_i^1, s_i^2)$, we set:
  - the probabilities $p_{s_i^1} = p_{s_i^2} = \frac{1}{2n}$ to have equiprobable scenarios,
  - the demands $d_{s_i^1}$ and $d_{s_i^2}$ are equal to value $iB$,
  - scenario $s_i^1$ contains a unique bid $(i, iB + 1)$ from the competition,
  - and scenario $s_i^2$ contains a unique bid $(i + 1, iB + 1)$ from the competition.

  As a consequence, there exists a competitor bid at each price between 1 and $n + 1$. We thus have $\Lambda = \{0, 1, ..., n + 1 = \overline{\pi}\}$ and $\lambda_i = i$.
- The value $V$ used as potential optimal value for DBP is defined by $V = \frac{B}{2n} \sum_{i=1}^{n} i(2i + 1)$.

Figure 3 illustrates a pair of scenario $s_i^1$and $s_i^2$.

We can observe that if the GC does not place any bid, the spot prices in scenarios $s_i^1$and $s_i^2$ are respectively $\lambda_i$ and $\lambda_{i+1}$. These values are the highest possible spot prices that can be achieved in these scenario as an additional bid of the GC could only potentially decrease them.

Consider a set of bids of the GC, let $Q_i$ be the total bidden up to price $\lambda_i$ included, $\mathcal{Q} = \{Q_0, Q_1, \ldots, Q_n\}$ the set of cumulative bidding quantities at each price $\lambda_i$, and $\overline{R}_s(\mathcal{Q})$ an upper bound on the maximum profit in scenario $s$ based on $\mathcal{Q}$. The upper bounds $\overline{R}_s(\mathcal{Q})$ can be computed as follow for each pair of scenarios $s_i^1$ and $s_i^2$:

$$\overline{R}_{s_i^1}(\mathcal{Q}) = \begin{cases} \lambda_i Q_i & \text{if } Q_i < iB, \\ \lambda_i iB & \text{if } Q_i = iB, \\ \lambda_i iB & \text{if } Q_i > iB. \end{cases} \qquad \overline{R}_{s_i^2}(\mathcal{Q}) = \begin{cases} \lambda_{i+1} Q & \text{if } Q_i < iB, \\ \lambda_{i+1} iB & \text{if } Q_i = iB, \\ \lambda_i iB & \text{if } Q_i > iB. \end{cases}$$

For $\overline{R}_{s_i^1}(\mathcal{Q})$, if $Q_i < iB$, the demand is not met by the bid of the GC who trades the total quantity $Q_i$ due to priority over its competitors. If $Q_i = iB$ or $Q_i > iB$, then the quantity meeting the demand $iB$ is traded. In all cases, the spot price is at most $\lambda_i$. The same observations can be made for $\overline{R}_{s_i^1}(\mathcal{Q})$ with a difference between the two last case as if $Q_i > iB$, the demand will be exceeded at price $\lambda_i$, making the spot price fall to this value by Lemma 1. For both scenarios $s_i^1$ and $s_i^2$, if $Q_i = iB$, then the profit is the highest. This is true for all $i \in \{1, ..., n\}$.

An upper bound on the optimal value of SBP is thus obtained if the upper bound is reached in all scenarios if $Q_i = iB$ for $i \in \{1, ..., n\}$. Since $\lambda_i = i$, this upper bound is equal to

$$V = \frac{\sum_{i=1}^{n} (\lambda_i + \lambda_{i+1}) iB}{2n} = \frac{\sum_{i=1}^{n} (2i+1) iB}{2n}.$$

An instance of DBP built from a 3-Partition instance has a solution if $Q_i = iB$ for all $i \in \{1, \ldots, n\}$, i.e., the total production bidden at each price $\lambda_i$ is equal to $B$ for all $i \in \{1, \ldots, n\}$. If such a solution exists, all generators are thus bidding their maximum capacity since $\sum_{j=1}^{3n} \overline{q}_j = nB$. With such a set of bids, the upper bound $\overline{R}_s(\mathcal{Q})$ is reached in every scenario, and the average profit is equal to $V$. The quantities of generators $j \in J$ bidden at price $\lambda_i$ for $i \in \{1, \ldots, n\}$ correspond to the integers values $a_j$ composing set $A_i$ in the solution of the original 3-Partition problem. □

**Theorem 3.** *Problem SBP-Q is strongly NP-hard.*

*Proof.* The proof is the same as for Theorem 2 as all generators are bidding their maximum capacity in the reduction. □

### *2.5. Considering a fixed demand*

We mentioned in the problem description that considering a fixed demand is not restrictive. Indeed, if there exists a scenario where retailer bids are considered, it can be transformed into a scenario with a constant demand and only production bids as follows:

- add the bidden quantity of all retailer bids to obtain the fixed demand,
- consider each retailer bid as a competitor production bid.

The residual demands obtained at each bidding price are identical in both scenarios, as well as the resulting spot price. An illustration of this procedure is provided in Figure 4.

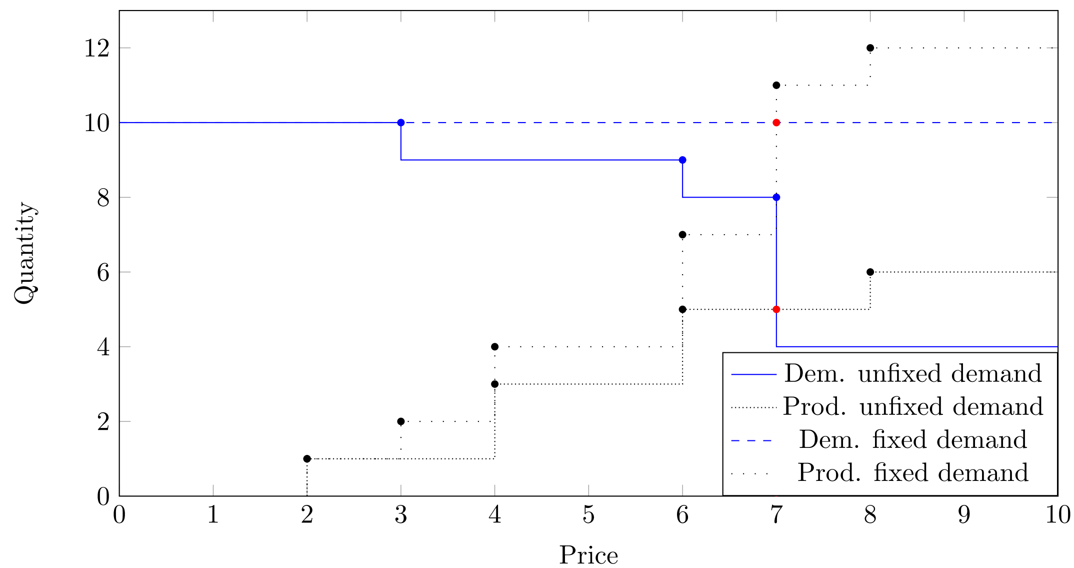



Figure 4: Identical scenario examples with unfixed and fixed demand

## 3. A dynamic programming framework

In this section, we introduce a dynamic programming framework for determining optimal bid policies among a finite number of such policies provided by Lemma 1. This framework is based on a recursive equation for computing the profit associated with a given set of bids and is used to solve to optimality SBP-R, providing an upper bound on SBP, as well as the proxy model SBP-Q. The bidding prices in $\Lambda$ are used as stages of the dynamic programming framework. The states used for SBP-R and SBP-Q are described in the corresponding sections below. We first summarize the notation used throughout this paper in Table 2.

### *3.1. Recursive profit computation*

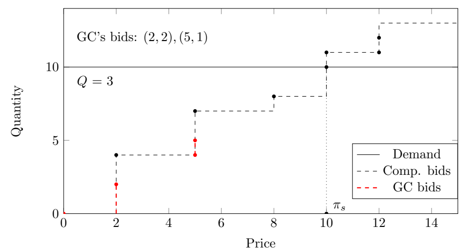
Our dynamic programming framework builds a solution by non-decreasing bidding prices. When adding a bid $(\pi_j, q_j)$ with a price higher or equal to existing bids of the GC, the MO trades a quantity $q_j^{s*}$ of this new bid, minimizing its objective in each scenario $s \in S$ and defining new spot prices $\pi^s$ as illustrated in Figure 5. Initially, the competitors place bids $(2,2), (5,2), (8,1), (10,3)$ and $(12,2)$ in this scenario, while the GC places bids $(2,2)$ and $(5,1)$. With these bids, the spot price is set at 10, the price at which the demand is met. This situation is illustrated in the top left figure.

When placing a new bid $(\pi_j, q_j)$ at a price $\lambda_i \in \Lambda$ higher or equal to 5, there are three possibilities regarding the impact on the GC's profit:

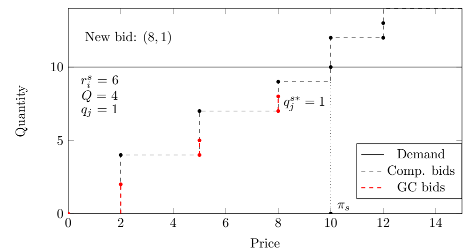
- If the total bidden quantity $Q$ is lower or equal to the residual demand $r_i^s$, then the new bid is entirely traded, as illustrated in the top right figure. The spot price does not change in this example, but this

| Notation | Definition |
|---|---|
| $J$ | Generators of the bidding GC. |
| $m = \|J\|$ | Number of generators of the bidding GC. |
| $c_j$ | Unit production cost of generator $j \in J$. |
| $\overline{q}_j$ | Maximum production capacity of generator $j \in J$. |
| $\overline{q}^J$ | Total generation capacity of the GC, $\sum_{j \in J} \overline{q}_j$. |
| $(\pi_j, g_j)$ | Unit price and quantity of the bid associated to generator $j \in J$. |
| $\tilde{J}$ | Generators of the competitors. |
| $\tilde{m} = \|\tilde{J}\|$ | Number of generators of the competitors. |
| $J^{MO} = J \cup \tilde{J}$ | Set of generators bidding to the MO. |
| $\overline{\pi}$ | Maximum bidding price allowed by the MO. |
| $S$ | Set of scenarios. |
| $p_s$ | Probability of scenario $s \in S$. |
| $d_s$ | Demand in scenario $s \in S$. |
| $\{(\tilde{\pi}^s_j, \tilde{q}^s_j)\}_{j \in \tilde{J}}$ | The set of competitor bids in scenario $s \in S$. |
| $q^s_j$ | Quantity of bid $j \in J^{MO}$ and $s \in S$ traded by the MO. |
| $\pi^s$ | Spot price of electricity in scenario $s \in S$. |
| $\Lambda$ | Set of bidding prices of competitors in all scenarios $\cup\{0, \overline{\pi}\}$. |
| $n$ | Number of prices in $\Lambda$. |
| $I = \{0, 1, ..., n\}$ | Indices of increasing price in $\Lambda$. |
| $\lambda_i$ | $i \in I, i^{th}$ price of $\Lambda$ ordered by value. |
| $r^s_i$ | Residual at price $i \in I$ and scenario $s \in S$. Maximum quantity the GC can trade with priority over competitors up to price $\lambda_i$ in $s$. |
| $\overline{q}^J_i$ | Total generation capacity up to price $\lambda_i$, $\sum_{j \in J, c_j \leq \lambda_i} \overline{q}_j$. |
| $B = \{(\pi_j, q_j)\}_{j \in J'}$ | Set of bids placed by the GC. |
| $P(B)$ | Profit of a set of bids $B$. |
| $R_i(\overline{J}, Q)$ | Maximum profit by bidding a total quantity $Q$ with generators $\overline{J}$ up to price $\lambda_i$. |
| $\Delta^s(i, j, Q, q_j)$ | Impact on profit in scenario $s$ of bidding a quantity $q_j$ with generator $j \in J$ at price $\lambda_i$, assuming existing bids have a lower price and a total quantity $Q$. |

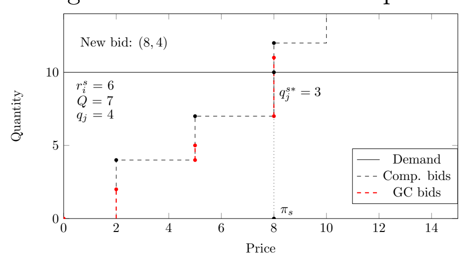
Table 2: Notation for SBP



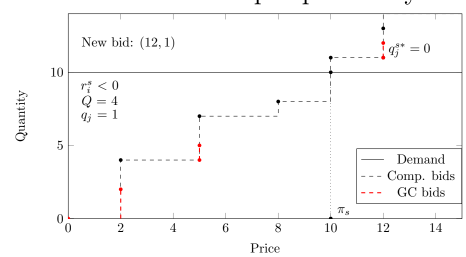

Existing bids of a GC and its competitors.




New bid at most at the spot price fully traded.




New bid at most at the spot price partially traded.




New bid strictly above the spot price.

Figure 5: Impact on profit when adding a new bid at a higher price than others.

is not always true. If the bid (5,4) had been added instead, the demand would have been exceeded at a price of 5, making the spot price fall to this value.

- If the residual demand $r_i^s$ is nonnegative, the total bidden quantity prior to the new bid is strictly smaller than $r_i^s$, and the total bidden quantity $Q$ exceeds it with the new bid, then the new bid is partially traded, as illustrated in the bottom left figure. In this case, the spot price falls to the bidden price as the new bid exceeds the demand.

- Otherwise, the residual demand $r_i^s$ is already met by the total bidden quantity $Q$ prior to the new bid, the new bid is not traded, and the spot price remains unchanged.

In the first two cases, a higher quantity is traded but at a potentially lower price. In the third case, the profit remains unchanged. What can be observed is that in a given scenario $s \in S$, the impact on profit of bid $(\pi_j, q_j)$ only depends on the total bidden quantity $Q$ and the residual demand $r_i^s$. These values allow us to compute the spot prices with and without the new bid, the total quantity traded, and the production cost of the new bid that is traded. The impact on profit of bid $(\pi_j, q_j)$ in each scenario can thus be computed, as well as the impact on the total expected profit. We formalize this to compute the profit of a set of bids by recursively removing the bids with the maximum price.

Consider a set of bids $B = \{(\pi_j, q_j)\}_{j \in J(B)}$ placed by the GC, where $J(B)$ denotes the set of generators associated with $B$, and $P(B)$ is the profit of the solution to SBP obtained by setting $(\pi_j, q_j) = (0, 0)$, for all $j \in J \setminus J(B)$. Let $Q = \sum_{j \in J(B)} q_j$ be the total bidden quantity, $B^{-j} = B \setminus \{(\pi_j, q_j)\}$ be the set of bids excluding generator $j$ for $j \in J(B)$, and $J^{\max} = \{j \in J(B) : \pi_j = \lambda_i \geq \pi_k, k \in J(B)\}$ the maximum bidden price in $B$.

**Lemma 4.** *The profit of a set of bids $B$ can be computed recursively as follows:*

$$P(B) = \max_{j \in J^{\max}} \{P(B^{-j}) + \sum_{s \in S} p^s \Delta^s(i, j, Q, q_j)\},$$

*where $\Delta^s(i, j, Q, q_j)$ is the impact on profit in scenario $s \in S$ adding a bid $(\lambda_i, q_j)$ with generator $j$ to $B^{-j}$ and is computed by*

$$\Delta^s(i, j, Q, q_j) = \pi^s(Q)(q_j^{s*} + Q - q_j) - \pi^s(Q - q_j)(Q - q_j) - c_j q_j^{s*} \tag{2}$$

*if $Q - q_j \leq r_i^s$ (0 otherwise),*
*the quantity of $(\pi_j, q_j)$ traded is*

$$q_j^{s*} = \min\{q_j, r_i^s - Q + q_j\},$$

*and the spot price in scenario $s$ for a total bidden quantity $x$ in (2) is*

$$\pi^s(x) = \min\{\lambda_{i'} \in \Lambda : x > r_{i'+1}^s\}. \tag{3}$$

*Proof.* For each scenario $s \in S$, the incorporation of the bid $(\pi_j, q_j)$ to the set $B^{-j}$ results in one of the following two cases:

- $Q - q_j > r_i^s$: the bids $(\pi_k, q_k) \in B^{-j}$ are sufficient to satisfy the whole demand. The new bid is not traded, and the profit remains constant.
- $Q - q_j \leq r_i^s$: all bids $(\pi_k, q_k) \in B^{-j}$ are traded and the new bid is partially or fully traded. The difference in profit $P(B) - P(B^{-j})$ is the sum of the differences in income and production costs. The difference in production cost depends on the quantity $q_j^{s*} = \min\{q_j, r_i^s - Q + q_j\}$ produced with generator $j$ that is traded. Concerning the difference in income, the total quantity traded will increase by $q_j^{s*}$, and the spot price, which either stays constant or decreases, is determined according to Equation (3).

□

From Lemma 4, we can conclude that the impact on the expected profit of adding a bid with a strictly higher price to a set of existing bids (i.e., $|J^{max}| = 1$) depends only on the new bid, the residual demand, and the total bidden quantity. This change is computed by averaging quantities $\Delta^s(i, j, Q, q_j)$ representing the difference in profit when adding a bid in each scenario following their probability distribution. In the case we add a bid with a price equal to the highest price of a set of existing bids (i.e., $|J^{max}| > 1$), the same computation can be performed for all generators in $J^{max}$. Furthermore, quantities $\Delta^s(i, j, Q, q_j)$ can be determined in logarithmic time based on residual demands. Now, let $R_i(\overline{J}, Q)$ denote the maximum expected profit for the GC if it places bids $(\pi_j, q_j)$ only for generators $j \in \overline{J}$ up to price $\lambda_i$ with a total quantity $Q$, that is, $\pi_j \leq \lambda_i$ for all $j \in \overline{J}$ and $\sum_{j \in \overline{J}} q_j = Q$. We consider $R_i(\emptyset, 0) = 0$ for all $i \in I$ and $R_0(\overline{J}, Q) = 0$ for all $\overline{J} \subseteq J$ and $0 \leq Q \leq \overline{q}^{\overline{J}} = \sum_{j \in \overline{J}} \overline{q}_j$.

Consider the set of bids $B$ bidden up to price $\lambda_i$ leading to a profit $R_i(\overline{J}, Q)$. Either no bid is placed at price $\lambda_i$, and $R_i(\overline{J}, Q) = R_{i-1}(\overline{J}, Q)$, or there exists a bid $(\pi_j, q_j)$ placed at price $\lambda_i$. In the second case, by Lemma 4, $R_i(\overline{J}, Q)$ can be computed recursively if we know the bid $(\pi_j, q_j)$. The bidden generator $j$ is a generator in $\overline{J}$, and the quantity $q_j$ must respect the production capacity of $j$ as well as the total quantity $Q$. The following proposition shows how to determine the expected profits $R_i(\overline{J}, Q)$ recursively.

**Proposition 5.** *Let $\overline{J}$ be a subset of generators of $J$ and $Q$ be such that $0 \leq Q \leq \overline{q}^{\overline{J}}$. Then,*

$$R_i(\overline{J}, Q) = \max\{R_{i-1}(\overline{J}, Q), \tag{4}$$

$$\max_{j \in \overline{J}} \max_{0 \leq q \leq \min\{\overline{q}_j, Q\}} R_i(\overline{J} \backslash \{j\}, Q - q) + \sum_{s \in S} p^s \Delta^s(i, j, Q, q)\}. \tag{5}$$

*Proof.* We distinguish two cases.

CASE 1:

In the optimal set of bids for the GC corresponding to $R_i(\overline{J}, Q)$, all bid prices are strictly lower than $\lambda_i$. Then $R_i(\overline{J}, Q) = R_{i-1}(\overline{J}, Q)$.

CASE 2:

There exists a bid $(\pi_j, q_j)$ such that $\pi_j = \lambda_i$. It follows that $0 < q_j \leq \min\{\overline{q}_j, Q\}$, and Lemma 4 applies. The maximum profit is then evaluated by taking the maximum over all possible indices $j \in \overline{J}$ and all possible values of $q$.

□

To compute $R_i(\overline{J}, Q)$ with Equation 4, we would need to discretize the values to consider for $q$ for a recursive computation of $R_i(\overline{J}, Q)$. The optimal value of SBP would be $R_n(J, Q)$, but the total bidden quantity $Q$ in an optimal solution of SBP remains unknown, preventing us from solving SBP. In the following sections, we answer these questions on the two variants of SBP and derive an upper bound on the optimal value of the problem through SBP-R, together with a feasible solution from the proxy model SBP-Q.

### *3.2. Dynamic programming for SBP-R*

In the *Stochastic Bidding Problem Relaxation* (SBP-R), we consider bids must not be assigned to generators. The GC can place as many bids as desired without exceeding its total production capacity. By Lemma 1, the bidding prices can be restricted to $\Lambda$, and are used as stages in our dynamic programming algorithm. A solution $B = \{(\lambda_i, q_i)\}_{i \in I}$ of SBP-R consists in determining what quantity $q_i$ to bid at each price $\lambda_i \in \Lambda$. Table 3 lists the main notation used for SBP-R.

| Notation | Definition |
|---|---|
| $B_\Lambda = \{(\lambda_i, q_i)\}_{i \in I}$ | Set of bids associated to prices in $\Lambda$. Each bid has a price $\lambda_{i'} \in \Lambda$ and quantity $q_{i'}$ representing the bidden quantity at that price. |
| $c(Q)$ | Minimum production cost for quantity $Q$. |
| $Q_i$ | Total quantity bidden up to price $\lambda_i$, $Q_i = \sum_{i' \in I, i' \leq i} q_{i'}$. |
| $P(B_\Lambda)$ | Profit of a set of bids $B_\Lambda$ |
| $R_i^*(Q)$ | Maximum profit by bidding a total quantity $Q$ up to price $\lambda_i$. This is similar to $R_i(\overline{J}, Q)$, $\overline{J}$ is omitted as bids are not assigned to generators. |
| $\Delta^s(i, Q, q_i)$ | Impact on profit in scenario $s$ of bidding a quantity $q_i$ at price $\lambda_i$. This is similar to $\Delta^s(i, j, Q, q_j)$ for SBP except $j$ can be omitted as bids are not assigned to generators. |
| $P^s(\lambda_i, q_i)$ | Single bid profit in scenario $s \in S$ when placing a unique bid $(\lambda_i, q_i)$. |

Table 3: Notation for SBP-R

Besides the unlimited number of bids, another main difference with SBP is the way the production costs are settled, as the bids are not associated with generators. When the MO clears the market, it communicates the total quantity traded to the GC. The GC then dispatches the production to its cheapest generators. Let $Q^s$ be the total quantity the MO trades from the GC in scenario $s \in S$. Let us order the generators of the GC $j \in J = \{1, \dots, m\}$ in non decreasing order of unit cost $c_j$ and define $j(Q^s) = \min\{j : \sum_{j'=1}^{j} \overline{q}_{j'} \geq Q^s\}$ the set of generators of minimum cost required to produce $Q^s$. All generators in $j(Q^s)$ produce at their maximum capacity except the last one, which completes the production to $Q^s$. The production cost of the GC is then given by:

$$c(Q^s) = \sum_{j'=1}^{j(Q^s)-1} c_{j'}(\overline{q}_{j'}) + c_{j(Q^s)}(Q^s - \sum_{j'=1}^{j(Q^s)-1} \overline{q}_{j'}). \tag{6}$$

The complexity of computing $c(Q^s)$ is in $O(\log m)$ if the generators are sorted by non-decreasing bidding price.

Note that, as $c_j < \overline{\pi}$ for all $j \in J$, the unused capacity can be bidden at price $\overline{\pi}$ in any optimal solution of SBP-R without any risk of production at loss.

In SBP-R, we focus on the quantities $q_i$ to bid at each price $\lambda_i \in \Lambda$. Let $Q_i$ be the cumulative quantity offered by the GC up to price $\lambda_i$, i.e., $Q_i = \sum_{i' \in I, i' \leq i} q_{i'}$. Problem SBP-R is equivalent to determining cumulative quantities $Q_i$, $i \in I$, such that $Q_i \leq Q_{i+1}$ and $Q_n = \overline{q}^J$ that maximize the expected profit. The values that will be considered for $Q_i$ will represent the states at stage $\lambda_i$ in our dynamic programming algorithm. The spot price in scenario $s$ is equal to the price $\lambda_{i(s)}$, where $i(s) = \min\{i \in I : Q_i > r_{i+1}^s\}$, which is a reformulation of Lemma 1. Moreover, the total quantity traded by the MO, $Q^s = \min\{Q_{i(s)}, r_{i(s)}^s\}$ is the total bidden quantity up to the spot price, or the residual demand if it exceeds it.

The properties and equations of the previous section can be simplified for SBP-R. As we do not associate bids to generators and the production costs are determined only once the MO decides $Q^s$, the prices at which bids are placed up to the spot price lose their importance. In Figure 6, we take back the bottom left example of Figure 5 in which a bid of (8,4) was added to two existing bids. In SBP, the quantity the MO trades for the generators bidden at prices 2, 5, and 8 are 2, 1, and 3. The production costs would be computed for these quantities. In SBP-R, what only matters is that a total quantity of 6 is traded, a quantity that would be dispatched to the cheapest generators. All bids up to the spot price can be aggregated, as illustrated in the right figure, without changing the profit. The profit of such aggregated bids representing the profit

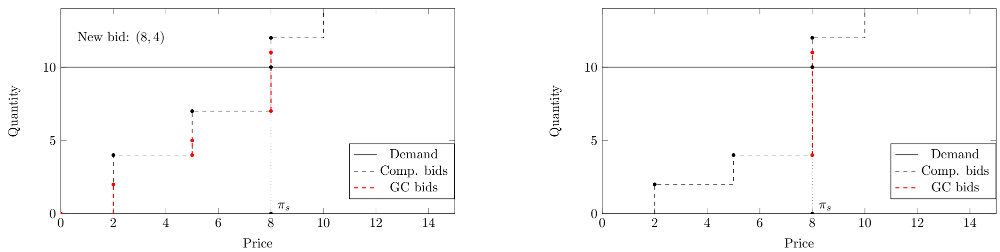



Figure 6: Aggregation of bids up to the spot price.

obtained in scenario $s$ by placing a single bid $(\lambda_i, q_i)$ at price $\lambda_i$ with a quantity $q_i$ are called *single bid profits* and denoted $P^s(\lambda_i, q_i)$.

Single-bid profits can easily be computed as follows:

$$P^s(\lambda_i, q_i) = \begin{cases} 0 & \text{if } r_i^s < 0, \\ \lambda_i r_i^s - c(r_i^s) & \text{if } 0 \leq r_i^s < q_i, \\ \pi^s(q_i) q_i - c(q_i) & \text{if } q_i \leq r_i^s. \end{cases} \tag{7}$$

In the first case of (7), the demand is satisfied by competitor bids before $\lambda_i$ as the residual demand is negative and the bid is not traded. In the second case, bid $(\lambda_i, q_i)$ will exceed the demand and is partially traded as the residual demand is nonnegative. The spot price is set at $\lambda_i$, and the residual demand $r_i^s$ meeting the demand is traded. In the last case, the bidden quantity $q_i$ does not meet the demand and is fully traded and the spot price is computed based on $q_i$ with Equation (3).

Lemma 4 can be streamlined based on single bid profits with a similar proof.

**Lemma 6.** *Let $B_\Lambda = \{(\lambda_k, q_k)\}_{k \in I}$ be a set of bids associated to prices in $\Lambda$. Let $\lambda_i$ be the highest bidden price in $B_\Lambda$, $Q$ the cumulative bidden quantity up to price $\lambda_i$ included, and $B_\Lambda^{-i}$ denote the subset of bids in $B_\Lambda$ without bid $(\lambda_i, q_i)$. We have that*

$$P(B_\Lambda) = P(B_\Lambda^{-i}) + \sum_{s \in S} p^s \Delta^s(i, Q, q_i). \tag{8}$$

*with*

$$\Delta^s(i,Q,q_i) = \begin{cases} 0 & \text{if } Q - q_i > r_i^s, \\ P^s(\lambda_i, Q) - P^s(\lambda_i, Q - q_i) & \text{if } Q - q_i \leq r_i^s, \end{cases} \tag{9}$$

*Proof.* For each scenario $s \in S$, the incorporation of the bid $(\lambda_i, q_i)$ to the set $B_\Lambda^{-i}$ results in one of the following two cases:

- $Q - q_i > r_i^s$: the demand is met at price $\lambda_i$, the new bid is not traded, and the profit remains constant.
- $Q - q_i \leq r_i^s$: the new bid is partially or fully traded, and the spot price is at least $\lambda_i$. In $B_\Lambda$ and $B_\Lambda^{-i}$, all bids can be aggregated into single bids $(\lambda_i, Q)$ and $(\lambda_i, Q - q_i)$ without changing their profit. The difference in profit is then the difference between these two single-bid profits.

□

Note that values $\Delta^s(i, Q, q_i)$ are computed in constant time based on single bid profits, and a single bid profit is computed in $O(\log n)$ based on residual demands $r_i^s$.

Next, let $R_i^*(Q)$ represent the maximum expected profit for the GC if it places bids up to price $\lambda_i$ for a total cumulative quantity $Q_i = Q$. This value is similar to $R_i(\overline{J}, Q)$ in SBP. As we assume $c_j \leq \overline{\pi}, j \in J$ the optimal value of SBP-R is:

$$z^* = R_n^*(\overline{q}^J) \tag{10}$$

Moreover, we set $R_i^*(0) = 0$ for all $i \in I$.

Proposition 5 can also be easily adapted to SBP-R.

**Proposition 7.** *Let $Q$ be such that $0 \leq Q \leq \overline{q}^J$. Then,*

$$R_i^*(Q) = \max_{q \in \Theta_i(Q)} \{R_{i-1}^*(Q - q) + \sum_s \Delta^s(i, Q, q)\}, \tag{11}$$

*where $\Theta_i(Q) \subset [0, Q]$.*

We will prove further that $\Theta_i(Q)$ can be discretized, allowing to compute $R_n^*(\overline{q}^J)$ recursively.

As we do not make a distinction between generators, we do not need to consider the case where several bids are placed at the same price. The maximum revenue by placing bids up to price $\lambda_i$ included is computed only based on the maximum revenue at price $\lambda_{i-1}$ and the possible bids $(\lambda_i, q)$ at price $\lambda_i$.

When placing a bid, we want to trade as much as possible without decreasing too much the spot price. We have already seen in Figure 2 the limit at which a spot price decreases: when the demand is strictly exceeded. A strategy for the GC would then be to bid quantities that match residual demands. It would then bid the maximum possible quantity before decreasing the spot price.

The following proposition shows that we can restrict the set $\Theta_i(Q)$ of values to consider for $q$ in the recurrence relations (11) to a set of polynomial size based on residual demands and maximum production capacities.

**Proposition 8.** *There exists an optimal solution $Q_i^*, i \in I$ to (SBP-R) such that*

$$Q_i^* \in \{r_{i+1}^s : s \in S, r_{i+1}^s \leq \overline{q}_i^J\} \cup \{\overline{q}_i^J, Q_{i+1}^*\}, \tag{12}$$

*with $Q_{n+1}^* = \overline{q}_n^J$ and $\overline{q}_i^J = \sum_{j \in J, c_j \leq \lambda_i} \overline{q}_j$.*

*Proof.* Consider an optimal solution in which some cumulative quantity $Q_i^*$ does not satisfy (12). First, let $i' \in I$ be the smallest index such that $Q_{i'}^* \geq \overline{q}_{i'}^J$. Then, profit will increase or stay unchanged if $Q_{i'}^*$ is decreased to $\overline{q}_{i'}^J$. Applying this transformation iteratively, one obtains an optimal solution in which all cumulative quantities $Q_i^*$ are lower than or equal to the maximum capacity $\overline{q}_i^J$ without producing at loss.

Second, let $i'$ be the largest index for which the cumulative quantity $Q_{i'}^*$ is not equal to one of the candidate values given by (12), and denote by $a$ the smallest candidate value greater than $Q_{i'}^*$. We are going to increase $Q_{i'}^*$ up to $a$ while preserving optimality. Since $Q_{i'}^* < \overline{q}_i^J$, increasing $Q_{i'}^*$ cannot induce production at loss. Furthermore, given that $Q_{i'+1}^*$ satisfies (12), if follows that $a \leq Q_{i'+1}^*$. The profit $P^s$ of each scenario $s \in S$ obtained by increasing $Q_{i'}^*$ will depend on its spot price $\pi^s$:

- $\pi^s < \lambda_{i'}$: demand $d_s$ is satisfied using bids with price smaller than $\lambda_{i'}$, $P^s$ stays unchanged after increasing $Q_{i'}$ up to $a$.

- $\pi^s > \lambda_{i'+1}$: all bids up to price $\lambda_{i'+1}$ are traded. Increasing $Q_{i'}$ up to $a$ does not change the spot price. As a consequence, $P^s$ stays unchanged.

- $\pi^s = \lambda_i$: all bids lower than price $\lambda_{i-1}$, in addition to some production at price $\lambda_i$, are required to meet the demand $d_s$. Increasing $Q_{i'}$ up to $a$ increases or leaves $P^s$ unchanged since the spot price stays at $\lambda_{i'}$ .

- $\pi^s = \lambda_{i'+1}$: Then, $Q_{i'} < r_{i'+1}^s$. Since $\min\{r_{i'+1}^s, \overline{q}_i^J\}$ is one of the candidate values, $a \leq \min\{r_{i'+1}^s, \overline{q}_i^J\}$. By increasing $Q_{i'}$ to $a$, the spot price remains constant, and the profit $P^s$ increases.

In conclusion, for all scenarios $s \in S$, increasing $Q_{i'}$ to its next candidate value $a$ either increases the profit $P^s$ or leaves it unchanged. This procedure can be iterated until all cumulative quantities $Q_i$ satisfy (12). □

Proposition 8 provides a polynomial number of states $Q_i^*$ to consider when searching for an optimal solution of SBP-R using Equation (11). The first set in (12) contains the residual demands at price $\lambda_{i+1}$

respecting the production capacity. In the second set lies the maximum production capacity and the quantities at the next price $Q^*_{i+1}$. This means the potential values for $Q^*_i$ are all residual demand $r^s_{i'}$ with $i' > i$ respecting production capacities. We are thus searching for bids with cumulative quantities reaching residual demands throughout the scenarios.

Set $\Theta_i(Q)$ contains $O(S)$ values that will lead to cumulative quantities $Q^*_i$ lying in (12) and can be defined by:

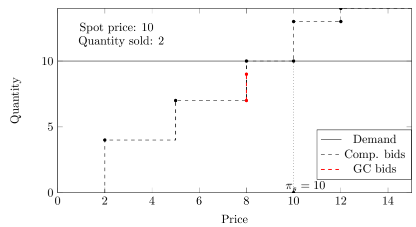
$$\Theta_i(Q) = \{Q - r^s_i : s \in S, r^s_i \leq \min\{\overline{q}^J_{i-1}, Q\}\} \cup \{\min\{Q - \overline{q}^J_{i-1}, 0\}\} \tag{13}$$

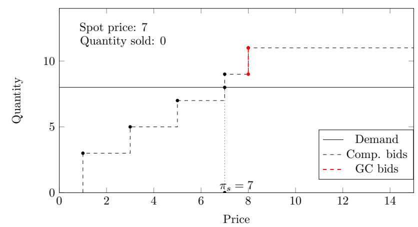
The values in the first set of $\Theta_i(Q)$ are the values leading to states equal to a residual demand $r^s_i$ at stage $\lambda_{i-1}$. The second set of $\Theta_i(Q)$ is the value leading to a state equal to the maximum production capacity at stage $\lambda_{i-1}$ or to the same state as at stage $\lambda_i$, in which case no bid is placed at stage $\lambda_{i-1}$.

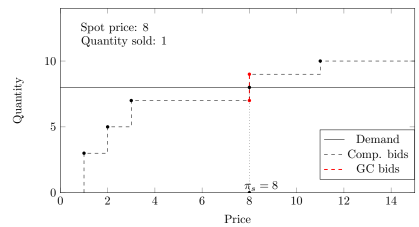
Figure 7 illustrates a situation in which it is interesting to place bids reaching a residual demand in some scenarios and the impact in other scenarios. Each column corresponds to a scenario, and each line corresponds to the bidding strategy of the GC. We consider that the GC places a single bid with a generator that has a production cost equal to 4. In scenario 1, the residual demand at a price of 10 is equal to 2, meaning the GC can bid 2 units up to a price of 10 and sell the full bidden quantity. By Proposition 8, we know this quantity of 2 units should be bidden at most up to the previous bidding price, which is equal to 8. On the first line, the GC bids a quantity of 2 at a price of 8, which is fully sold in scenario 1 at a price of 10. Consequently, this bid is not sold in scenario 2 and partially in scenario 3. The average profit is $\frac{12+0+4}{3} = \frac{16}{3}$. On the second line, the GC bids at the lowest possible price of 4 to try to sell the highest possible quantity. In scenario 1, the spot price and quantity sold are unchanged; in scenario 2, the bid is fully sold; in scenario 3, the bid is partially sold at a lower spot price than on the first line. The average profit is of $\frac{12+2+0}{3} = \frac{14}{3}$. The total quantity sold increases, but the spot prices are decreased in scenarios 2 and 3, leading to a lower expected profit. On the last line, the GC bids at an intermediate price of 7. The spot price and quantity sold are unchanged in scenario 1, and in scenarios 2 and 3, the quantity sold is equal to 1, but the spot price is increased to 7. The average profit is of $\frac{12+3+3}{3} = 6$. The optimal bidding prices are thus not necessarily the smallest or biggest ones when aiming to meet the residual demand at a given price. At each price, we aim to bid a cumulative quantity that will meet the residual demand at a higher price. This is the case on the third line of Figure 7 in which the GC aims at meeting the residual demand at a price of 8 but places its bids at a lower price.

| | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|

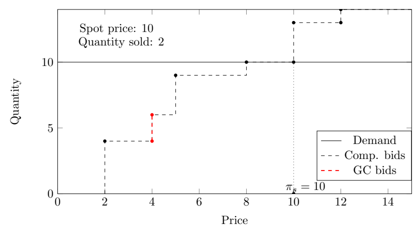
GC's bid : (8,2)
Bid at a price meeting
the demand in sc. 1.

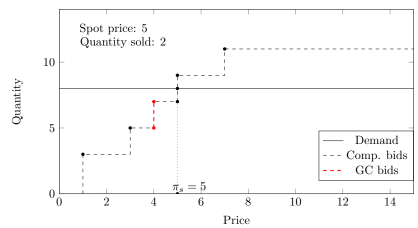


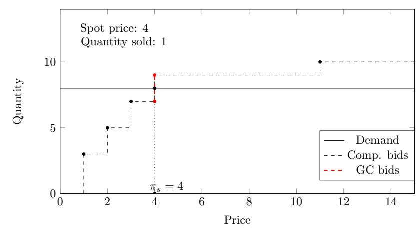



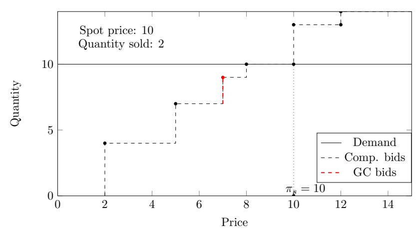

GC's bid : (4,2)
Bid at the production cost
of the generator

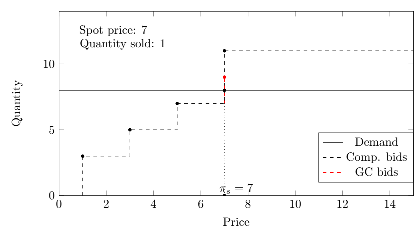



GC's bid : (7,2)
Bid at an
intermediate price



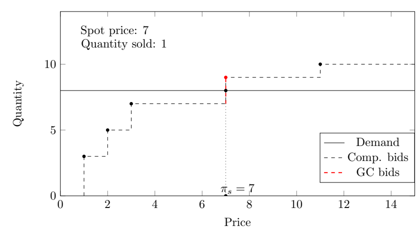




Figure 7: Impact of a GC's bid in several scenarios

Computing the optimal value of SBP-R using (11) with quantities given by (13) can be interpreted as searching for the longest path in a directed graph. The nodes of the graph are the states at the different stages given by (12). Arcs are defined by the quantities provided by $\Theta_i(Q_i)$, linking states $Q_{i-1}$ and $Q_i$. The weight of each arc corresponds to $\sum_{s\in S} p_s\Delta^s(i, Q_i, q_i)$ representing the average impact on profit when adding a bid $(\lambda_i, q_i)$ to state $Q_{i-1}$ at stage $\lambda_{i-1}$. The searched path goes from state $Q_0 = 0$ at stage $\lambda_0 = 0$ to state $\overline{q}^J$ at stage $\lambda_n$. An illustration of the graph of an instance with three scenarios is provided in Figure 8 in which stage $\lambda_i$ is equal to a price of $i$. The residual demands in each scenario are given under each price, the square nodes in the graph represent quantities $Q_i$ equal to a residual demand at the next price $r^s_{i+1}$.

Some arcs can easily be discarded by excluding some values from $\Theta_i(Q)$:

- If $Q \leq \min\{r^s_i | s \in S\}$ for $i \in I$, the bids preceding price $\lambda_i$ are all traded and can be made at price $\lambda_i$ without changing the profit, thus $\Theta_i(Q) = \{0\}$.

  For example, in Figure 8, at stage $\lambda_3$, all states are smaller than the minimum residual demand at this price which is equal to 6. Thus, all incoming arcs at stage $\lambda_3$ come from state 0 at stage $\lambda_2$.

- As soon as a state reaches the maximum residual demand $r^s_i$ throughout all scenarios at a stage $\lambda_{i-1}$, any quantity bidden at the next stages cannot be traded. In this situation, the only quantities that need to be considered for bidding at the following stages are quantities leading to state $\overline{q}^J$. Quantities in $\Theta_i(Q)$ leading to a state $\overline{q}^J_{i-1}$ exceeding this maximum residual demand but smaller than the maximum production capacity can be eliminated.

The path in red in Figure 8 corresponds to the three following bids from the GC: ($\lambda_5$,2), ($\lambda_6$,1), and ($\lambda_7$,4). The GC has three generators with capacities and production costs given in the description of the figure.

All nodes representing cumulative quantities $Q_i$ are either the maximum capacity at the corresponding price or a residual demand $r^s_{i'}$ for $i' > i$. For instance, 3 is the residual demand at price $\lambda_7$ in the third scenario. A node representing this residual demand appears at price $\lambda_6$ and all smaller prices until the production capacity is violated. The incoming arcs of a node are all residual demands at the current price and either the node at the previous price with the same quantity or the maximum capacity at this price. For instance, the residual demands at price $\lambda_7$ are 0, 2, and 3. The incoming arcs of node ($\lambda_7$,7) are thus the nodes at price $\lambda_6$ with quantities equal to 0, 2, 3, or 7. When placing a bid reaching a square node, such as bid $(\lambda_2, 2)$, the demand is exactly met at this price in the corresponding scenario. With this bid, the spot price will be equal to $\lambda_6$ in the second scenario. The next bid, $(\lambda_6, 1)$, will meet exactly the demand at this price in scenario 3, fixing the spot price at $\lambda_7$. This is as in the first column of Figure 7 in which we attempt to meet exactly the demand in some scenarios to push the spot price at a higher value.

Regarding the computational complexity, the computation of $R^*_n(\overline{q}^J)$ can be decomposed into two steps.

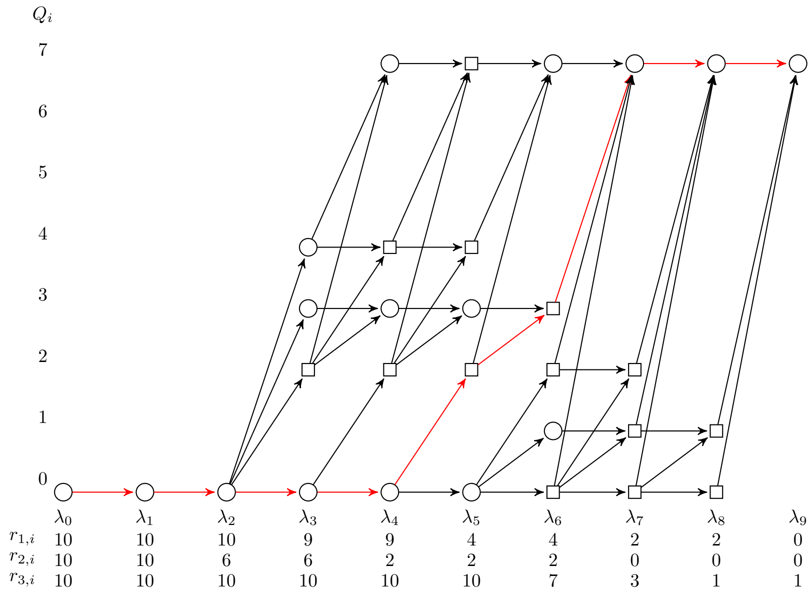



Figure 8: Graph of SBP-R instance with $\overline{q}_1 = 2$, $\overline{q}_2 = 2$, $\overline{q}_3 = 3$, $c_1 = 1$, $c_2 = 3$, $c_3 = 5$.

First, all single bid profits $P^s(\lambda_i, q_i)$ (7) are used to compute differences on profit $\Delta^s(i, Q, q_i)$ (9) faster, before computing $R_n^*(\overline{q}^J)$ recursively using (11) and candidate sets $\Theta_i(Q)$.

Since each scenario includes $\tilde{m}$ bids from the competitors, the total number of states for each of the $n$ stages is $O(\tilde{m}|S|)$ according to Proposition 8. As a single bid profit in a given scenario is computed in $O(\log n)$, all single bid profits are computed for all scenarios in $O(n\tilde{m}|S|^2 \log n)$. Based on residual demands, each of the $O(n\tilde{m}|S|)$ values $R_i^*(Q)$ being computed in $O(S^2)$ time (there are $O(|S|)$ states in $\Theta_i(Q)$ and the difference in profit is computed for the $|S|$ scenarios). This yields a complexity of $O(n\tilde{m}|S|^3)$ for the computation of $R_n^*(\overline{q}^J)$. The overall complexity is in $O(n\tilde{m}|S|^2(\log n + |S|))$.

**Lemma 9.** *SBP-R can be solved in polynomial time.*

### *3.3. Dynamic programming for SBP-Q*

The *Stochastic Bidding Problem with fixed Quantities* (SBP-Q) is a constrained version of SBP where quantities bidden by generators are fixed to their maximum capacity. It can be used as a proxy model to find feasible solutions for SBP. A solution $B = \{(\pi_j, \overline{q}_j)\}_{j \in J}$ consists in determining the bidden prices $\pi_j$ for each generator as the associated bidden quantity is trivial. As for SBP-R, the bidding prices $\pi_j$ can be restricted to $\Lambda$ by Lemma 1 and are used as stages. The states of SBP-Q are the generators bidden up to a the corresponding stage included. Table 4 lists the main notation used for SBP-Q.

Lemma 4 holds for SBP-Q. We simplify the notation $\Delta_s(i, j, Q, q_j)$ to $\Delta_s(i, j, Q)$ where the fixed quantity $\overline{q}_j$ has been dropped as they are predetermined. Let $R_i^Q(\overline{J})$ denote the maximum expected profit for the

| Notation | Definition |
|---|---|
| $R_i^Q(\overline{J})$ | Maximum profit by bidding generators $\overline{J}$ up to price $\lambda_i$. |
| | This is similar to $R_i(\overline{J}, Q)$ with $Q$ omitted as bidden quantities are fixed. |
| $\Delta^s(i, j, Q)$ | Impact on profit in scenario $s$ of bidding generator $j \in J$ at price $\lambda_i$. |
| | This is similar to $\Delta^s(i, j, Q, q_j)$ for SBP except $q_j$ can be omitted |
| | as bidden quantities are fixed. |

Table 4: Notation for SBP-Q

GC if it places bids $(\pi_j, \overline{q}_j)$ only for generators $j \in \overline{J}$, up to price $\lambda_i$ included. As we assume $c_j < \overline{\pi}, j \in J$ the optimal value of SBP-Q is:

$$z^* = R_n^Q(J) \tag{14}$$

Moreover, we set $R_n^Q(\emptyset) = 0$ for all $i \in I$ and $R_0^Q(\overline{J}) = 0$.

The notation of Proposition 5 can trivial be adapted to compute $R_i^Q(\overline{J})$, the proof is identical by setting $q_j = \overline{q}_j$.

**Proposition 10.** *For any subset $\overline{J}$ of $J$, there holds that*

$$R_i^Q(\overline{J}) = \max\{R_{i-1}^Q(\overline{J}), \max_{j \in \overline{J}}(R_i^Q(\overline{J}^{-j}) + \sum_{s \in S} p^s \Delta^s(i, j, Q))\} \tag{15}$$

By using (14) and Proposition 10, we can compute the optimal value of SBP-Q by adding generators one by one by non-decreasing price.

As for SBP-R, solving SBP-Q can be interpreted as searching for the longest path in a directed graph. Figure 9 illustrates the graph explored when computing $R_Q^*(J)$ on the same instance used in Figure 8. The solution plotted in red represents bids $(\lambda_3, 2)$, $(\lambda_6, 2)$ and $(\lambda_4, 3)$ for generators 1, 2 and 3 respectively. Some states obtained by bidding generators at price $\lambda_i$ in Equation (10) when computing $R_Q^*(J)$ can easily be eliminated based on the maximum and minimum residual demands:

- If $\overline{q}^{\overline{J}} > \max\{r_i^s | s \in S\}$, then a part of the total production of generators $\overline{J}$ would not be sold if some are bidden at stage $\lambda_i$.
  - If $\overline{J}$ contains the cheapest generator in $J$, then if a bid is placed at stage $\lambda_i$, they are placed by non-decreasing price $c_j$ to trade in priority the cheapest generators if their quantity is to be traded. For example, in Figure 9, there is no arc from state {2,3} to state {1,2,3} at stages $\lambda_8$ and $\lambda_9$.
  - Otherwise, $\overline{J}$ does not contain the cheapest generator in $J$ (because it has already been bidden), say $j_{min}$, its production is not traded. Thus if a bid is made with a generator $j \in \overline{J}$ at stage $\lambda_i$,

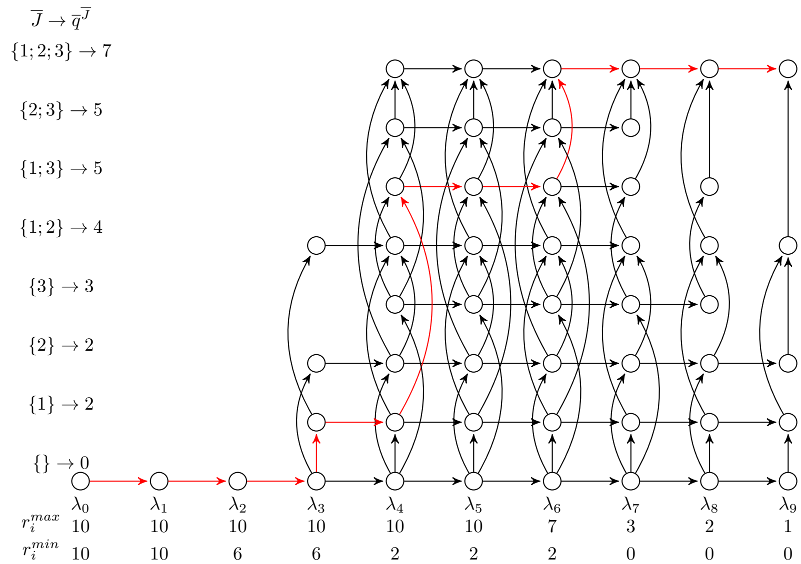



Figure 9: Graph of SBP-Q instance with $\overline{q}_1 = 2$, $\overline{q}_2 = 2$, $\overline{q}_3 = 3$, $c_1 = 1$, $c_2 = 3$ and $c_3 = 5$.

this cannot lead to a higher profit than having bidden $j_{min}$ at stage $\lambda_i$ (and swapped the two bids of $j$ and $j_{min}$). Thus, in this situation, no bid is placed at price $\lambda_i$ with generators in $\overline{J}$ as we are computing the maximum revenue. This is the case in Figure 9 for state {2,3} at stage $\lambda_7$ that has no incoming arc from the same stage.

- If $\overline{q}^{\overline{J}} < \min\{r_i^s | s \in S\}$, $\overline{J} \neq \emptyset$ and all generators in $\overline{J}$ have a production cost at most equal to $\lambda_i$, then bidding a generator at price $\lambda_i$ or a lower price will lead to the same profit. As any bid yields a positive profit, there is at least one bid between stages $\lambda_0$ and $\lambda_i$. Thus, we force a bid at price $\lambda_i$ in this situation to eliminate states at a lower price. Hence there is no arc between two nodes at the same stage before stage $\lambda_3$ in Figure 9.

The computational complexity of finding the optimal value $R_n^Q(J)$ of SBP-Q is established similarly for SBP-R. The spot prices used to compute the difference on profit $\Delta^s(i, j, Q)$ in (2) are precomputed for the $O(2^m)$ possible subsets $\overline{J} \subseteq J$ in $O(n|S|2^m \log n)$. Based on these spot prices, $\Delta^s(i, j, Q)$ is computed in $O(\log m)$. The value of $R_n^Q(J)$ is computed recursively using (15). A total of $O(n2^m)$ values $R_i^Q(\overline{J})$ are computed, each of them in $O(m|S| \log m)$, for a total computational complexity in $O(nm|S|2^m \log m)$ for the recurrence relation. The overall complexity is in $O(n|S|2^m(\log n + m \log m))$.

**Lemma 11.** *SBP-Q can be solved in polynomial time for a fixed number of generators.*

As previously mentioned, SBP-Q can be used as a proxy model for obtaining heuristic solutions for SBP. Figure 10 illustrates a case where the value of the heuristic is at most $\frac{2}{3}$ of the optimal value of SBP. However, our numerical experiments in Section 4 show that in practice, SBP-Q is much closer to SBP, leading to very reasonable gaps.

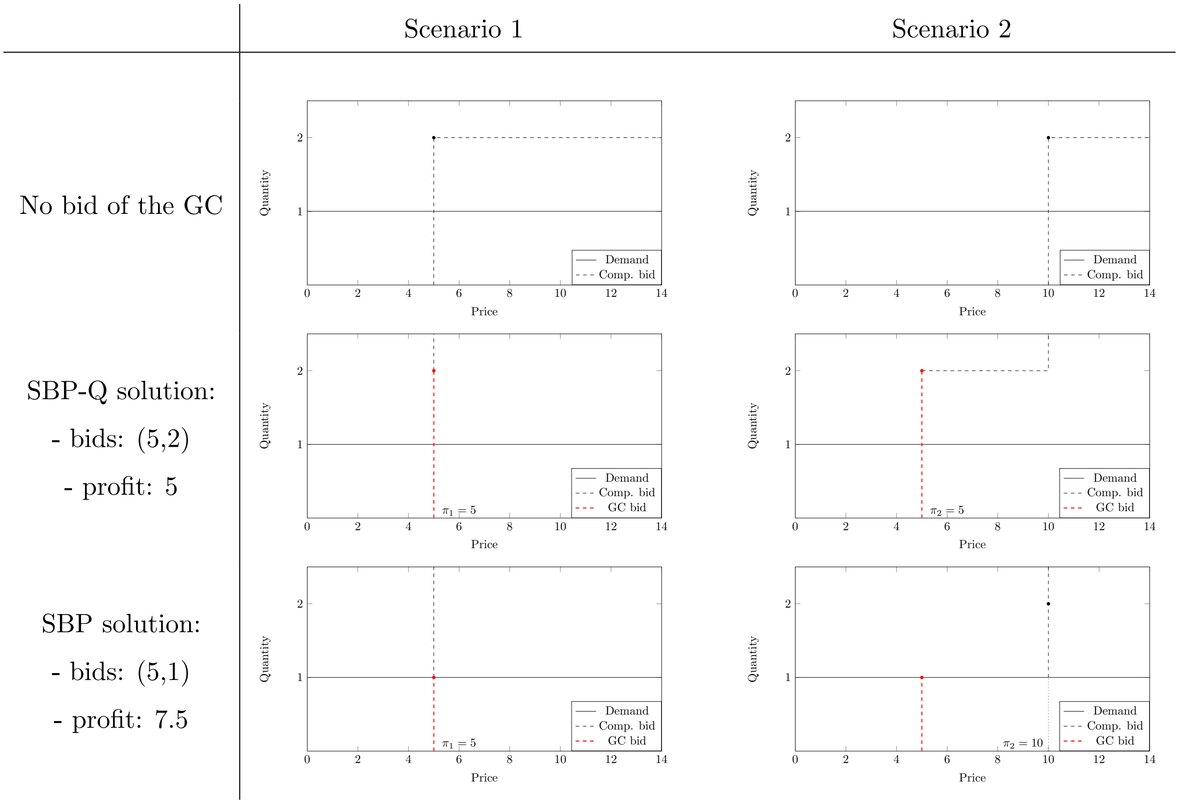



Figure 10: Illustration of worse-case performance of SBP-Q vs SBP

Consider an instance with two scenarios having equal probability and illustrated at the first line of the figure. Both have a demand of 1 and a single bid from the competitors, (5,2) and (10,2), respectively, exceeding the demand. The GC has a single generator with a maximum production capacity of 2 and no production cost. This is a generic example to represent what can occur if the demand is exceeded in two scenarios by competitor bids at prices with a large difference. In SBP-Q, the GC has to place a bid with a quantity of 2 units. A bid of (5,2) is an optimal bidding strategy in which the profit is equal to 5 in both scenarios and is illustrated in the second line. The drawback of bidding with fixed quantities is decreasing the spot price in scenario 2 as the production exceeds the demand with the bid of the GC. Note that (10,2) would also be an optimal bidding strategy, not selling the bid in scenario 1 and generating a profit of 10 in scenario 2. In the last line, an optimal solution of SBP is illustrated with a bid of (5,1) and an average profit of 7.5. Reducing the bidden quantity can preserve a higher spot price in scenario 2 with a profit of 10 while generating the same profit of 5 in scenario 1 as for SBP-Q. The optimal solution with fixed quantities has therefore a value of only $\frac{2}{3}$ of the optimal solution of SBP. Note that the low performance in this situation is due to the large gap between the bidding prices of the competitor, situation that is unlikely in practice.

We leave the generalization of this example to find a lower bound on the approximation performance of SBP-Q for future research.

## 4. Numerical results

In this section, we demonstrate the efficiency of the dynamic programming algorithm. Test instances used by Fampa & Pimentel (2015) based on data from the Brazilian Electric System National Operator were kindly provided by W. Pimentel. The instances involve 6 generators, 108 competitor bids per scenario, and a number of scenarios ranging from 10 to 70. In our presentation, the focus is mainly on the number of generators and scenarios. Larger instances are built by clustering or splitting the production of generators of the GC, and we generate new scenarios by modifying 20% or less of the prices and quantities of competing bids. Five instances are generated for each pair of values $(|J|, |S|)$. All instances are available on the GitHub repository `https://github.com/jdeboeck/SBP`. Numerical results reported are averages over these five instances. For each type of instance, we provide the computing times and, in the case of the proxy model SBP-Q, the difference ('gap') between the optimum achieved and the upper bound corresponding to the optimum of the relaxed problem SBP-R. This gap is an upper bound on the optimality gap in SBP of an optimal solution of SBP-Q.

The algorithms have been implemented in Python 3.8.2 and run on a 4-core i7 2.30 GHz processor with 64Go RAM. In Section 4.1, previous numerical results of SBP-Q are outlined, while the numerical results for the dynamic programming framework are discussed in Section 4.2.

### *4.1. Comparison with previous studies*

Fampa & Pimentel (2015) have proposed a genetic algorithm to solve SBP-Q, based on the MILP formulation introduced in Fampa et al. (2008) to test its efficiency. The MILP formulation could solve to optimality instances involving up to 30 scenarios. On larger instances, the running time was limited to 16,000 seconds, with an optimality gap always less than 3%, while the genetic algorithm could find within a few seconds (7 to 35) a solution within 0.01% of the best feasible MILP solution.

A relaxation of SBP-Q has also been shown by Fampa & Pimentel (2017) to yield solutions within 10% of optimality in an average CPU time of 70 seconds on small instances involving 4 generators and up to 3 scenarios of 10 competitor bids.

The performance of the dynamic programming framework is illustrated in Table 5, where CPU times (in seconds). As we have not implemented these methods described in Fampa & Pimentel (2017), a direct comparison of the performance with the dynamic programming algorithm for SBP-Q is impossible. Still, as it solves these instances in under 2 seconds, it outperforms a MILP-based approach and seems significantly faster than the genetic algorithm while providing proof of optimality. The optimal value of SBP-R is an upper bound on SBP-Q. The gap reported in Table 5 is the relative gap between the optimal values of SBP-Q and SBP-R. The gap is, on average, similar to the results of Fampa & Pimentel (2017), but significantly larger instances are considered, which are solved in a reasonable time.

| instance | SBP-R | | SBP-Q | | | Fampa | | |
|---|---|---|---|---|---|---|---|---|
| $\lvert S\rvert$ | time | SD | time | SD | gap (%) | Solved | time | SD |
| 10 | 0.17 | 0.07 | 0.27 | 0.01 | 1.54 | 5 | 38.25 | 20.96 |
| 20 | 0.88 | 0.14 | 0.62 | 0.05 | 2.04 | 5 | 442.36 | 162.96 |
| 30 | 2.14 | 0.43 | 0.78 | 0.06 | 2.61 | 5 | 824.31 | 681.97 |
| 40 | 6.25 | 2.08 | 1.28 | 0.12 | 2.12 | 3 | 1233.43 | 584.95 |
| 50 | 8.34 | 0.93 | 1.36 | 0.05 | 2.09 | 4 | 2778.9 | 415.6 |
| 60 | 12.53 | 2.63 | 1.42 | 0.10 | 2.60 | 1 | 1142.33 | 0 |
| 70 | 19.34 | 2.65 | 1.71 | 0.08 | 2.11 | 0 | - | - |

Table 5: Impact of the number of scenarios for a fixed number of generators $m = 6$. Time units are seconds and 'SD' stands for the standard deviation of CPU time.

### *4.2. Results on larger instances*

The data displayed in Tables 6 and 7 illustrate the performance of the dynamic programming algorithms on larger instances. Besides CPU times, they report the average number of bids in the solution obtained at termination.

| instance | SBP-R | | | SBP-Q | | | |
|---|---|---|---|---|---|---|---|
| $m$ | time | SD | bids | time | SD | bids | gap (%) |
| 2 | 76.46 | 8.00 | 4.8 | 0.17 | 0.01 | 1.8 | 2.02 |
| 4 | 75.90 | 9.64 | 4.8 | 0.84 | 0.06 | 2.2 | 1.79 |
| 6 | 73.32 | 10.16 | 10.6 | 3.35 | 0.16 | 3.8 | 2.10 |
| 8 | 72.07 | 9.58 | 7.4 | 15.4 | 0.54 | 4.4 | 0.97 |
| 10 | 58.63 | 7.28 | 8.0 | 34.32 | 5.33 | 4.2 | 0.80 |
| 12 | 71.82 | 8.35 | 9.6 | 275.68 | 11.87 | 6.4 | 0.77 |
| 14 | 71.32 | 7.79 | 10.2 | 1128.4 | 44.93 | 6.6 | 0.81 |
| 16 | 70.88 | 6.80 | 9.6 | 4840.87 | 244.48 | 7.0 | 0.38 |

Table 6: Impact of the number of generators, for a fixed number of scenarios $\lvert S\rvert = 100$.

| instance | SBP-R | | | SBP-Q | | | |
|---|---|---|---|---|---|---|---|
| $\|S\|$ | time | SD | bids | time | SD | bids | gap (%) |
| 50 | 8.74 | 0.9 | 6.6 | 15.11 | 1.14 | 4.4 | 1.83 |
| 100 | 58.63 | 7.28 | 8.0 | 34.32 | 5.33 | 4.2 | 0.80 |
| 150 | 172.25 | 33.38 | 7.8 | 50.99 | 8.37 | 3.0 | 0.05 |
| 200 | 372.15 | 57.54 | 10.4 | 75.25 | 10.45 | 4.4 | 0.19 |
| 250 | 648.24 | 89.48 | 10.6 | 80.78 | 9.89 | 4.2 | 0.24 |
| 300 | 1128.79 | 147.97 | 11.2 | 111.57 | 18.04 | 3.2 | 0.05 |
| 350 | 1661.03 | 219.19 | 12.6 | 119.22 | 14.48 | 3.8 | 0.05 |
| 400 | 2367.06 | 250.76 | 10.6 | 122.26 | 13.04 | 3.8 | 0.19 |

Table 7: Impact of the number of scenarios for a fixed number of generators $m = 10$.

Table 6 illustrates the impact of the bidding GC's number of generators for a fixed number of 100 scenarios. Note that the solution times of SBP-R do not vary much with the number of generators, as the theoretical complexity ($O(n\tilde{m}|S|^2(\log n + |S|))$) does not depend on $m$. In contrast, the solution times for SBP-Q reflect the exponential factor in the complexity ($O(n|S|2^m(\log n + m \log m))$).

Table 7 shows that the computing times increase cubically (respectively linearly) with the number of scenarios involved in SBP-R (respectively SBP-Q). While the efficiency of the algorithm for solving SBP-Q decreases as the number of generators gets large, it outperforms the MILP-based algorithm of Fampa et al. (2008), which required over an hour of CPU on instances involving up to 6 generators, and at most 20 scenarios. For one, the linear growth of the complexity of SBP-Q in the number of scenarios allows us to solve instances involving much larger scenarios than in previous studies. As for SBP-R, the dynamic programming framework can address much larger instances than those considered in Fampa & Pimentel (2017), which were limited to 3 scenarios and 4 generators for the GC. Furthermore, our upper bound is, on average, much tighter than the one of Fampa & Pimentel (2017), with an optimality gap of less than 1% on large instances.

From an economic viewpoint, it is interesting to note that the number of bids placed is always larger in the solutions of SBP-R versus those of SBP-Q. This illustrates the financial interest for GC to place bids independently from generators and be more flexible.

An open question was to know the quality of an optimal solution of SBP-Q considering fixed bidden quantities compared to an optimal solution of SBP where variable bidden quantities are allowed as in classical day-ahead markets. As the algorithm for SBP-R provides an upper bound to problem SBP, comparing the optimal solutions of SBP-R and SBP-Q can answer this question. The gaps are generally under 1% on large instances, which illustrates that SBP-Q is an efficient proxy model for SBP.

## 5. Conclusion

The bidding problem SBP has been mentioned in the literature as a theoretical problem (Fampa et al., 2008) but lacks practical methods to solve it. Only the variant with fixed bidding quantities SBP-Q was solved through heuristic methods on realistic instances (Fampa et al., 2008; Fampa & Pimentel, 2015). As no method could provide an upper bound on SBP, the gap to optimality of a solution of SBP-Q could not be tracked.

The general dynamic programming framework proposed in this paper to find quality solutions for SBP allows us to solve two variants of this problem. First, problem SBP-R provides an upper bound on the optimal value of SBP, which is found in polynomial time. This upper bound is tighter than previous work on large instances (Fampa & Pimentel, 2017). Second, problem SBP-Q, previously studied in the literature (Fampa et al., 2008; Fampa & Pimentel, 2015), is solved to optimality and showed empirically to be an efficient proxy model for SBP. Our method to solve SBP-Q in a polynomial time for a fixed number of generators provides an optimal solution in significantly less time than previous studies and allows us to consider larger instances. The numerical results are the first to illustrate that an optimal solution found when bidding the maximum capacity of each generator is in a 1% gap of the optimal solution of SBP, a problem that has not yet been solved to optimality. We showed in this paper that the heuristic solution provided by the optimal solution of SBP-Q can have an objective value as bad as $\frac{2}{3}$ of the optimal value of SBP. Future research should explore lower bounds to this ratio to determine if SBP-Q is a constant-factor approximation of SBP. Identifying the structure of instances on which SBP-Q performs close to SBP (as in our numerical experiments) is also a possible avenue for research.

Our dynamic programming framework cannot practically be applied to SBP because an infinite number of states are to be considered at each step of constructing a solution. In future research, one could try to discretize the states as for SBP-R and use them to solve SBP to optimality by using the same dynamic programming framework.

Problem SBP-Q was presented by Fampa et al. (2008) as a Bertrand model of problem SBP, optimizing the bidding price for fixed quantities. A Cournot model could also be studied by optimizing the bidding quantities for fixed prices (SBP-P). Our dynamic programming framework could be adapted to this problem by finding an appropriate set of states. This could lead to an improvement in the optimal solution of SBP-Q by considering the bidding prices of this solution as fixed and optimizing the bidding quantities.

The linear production costs should also be generalized to convex production costs to make the problem more realistic. This could be done in SBP-Q, which could be solved using the same DP algorithm we proposed. It is easy to see that the properties used to solve SBP-Q can be generalized to convex production costs while preserving the solutions' optimality. Only the complexity would be impacted regarding the computation of production costs.

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