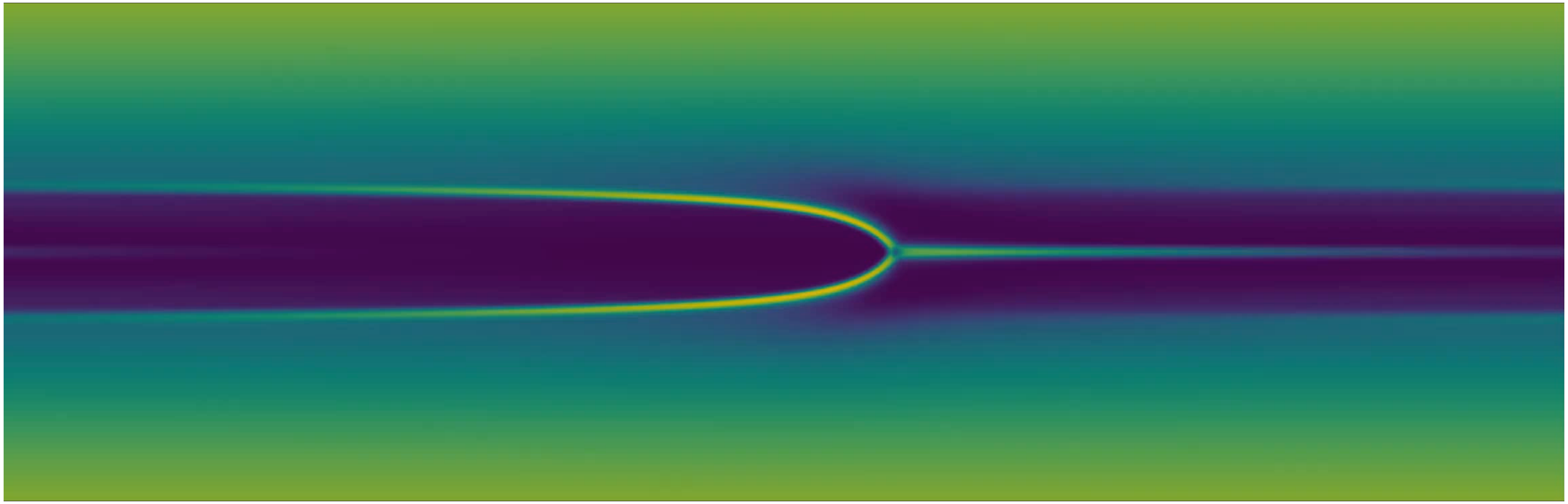
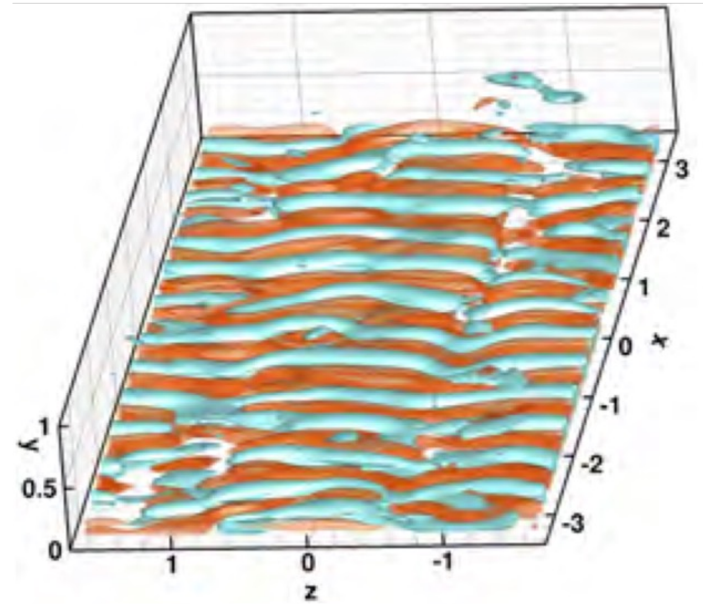
Characterization of steady arrowhead in periodic channel flow

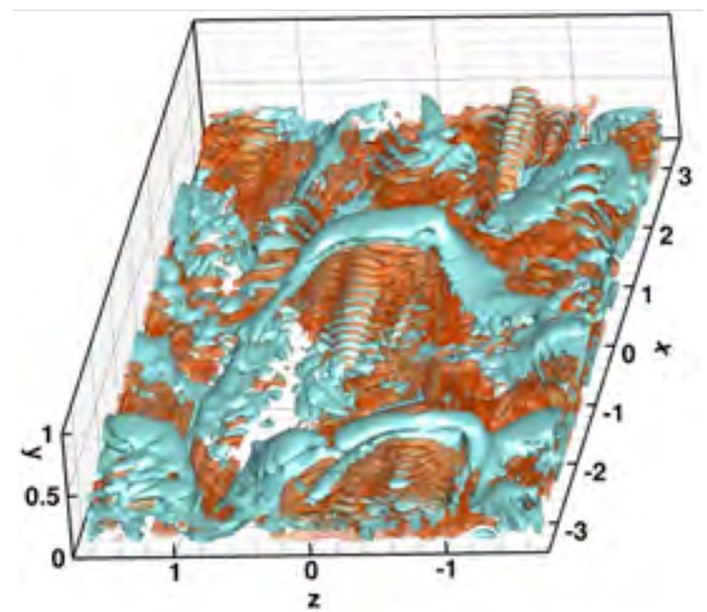


P.-Y. Goffin, **V.E. Terrapon**, Y. Dubief

Elasto-inertial turbulence

- **Chaotic** flow state sustained by polymer dynamics
- Observed both at **subcritical** and **supercritical** Re
- Coherent structures (e.g., Q) different from inertial ones
- Dynamics driven by **backward energy transfer** from **small polymer scales**





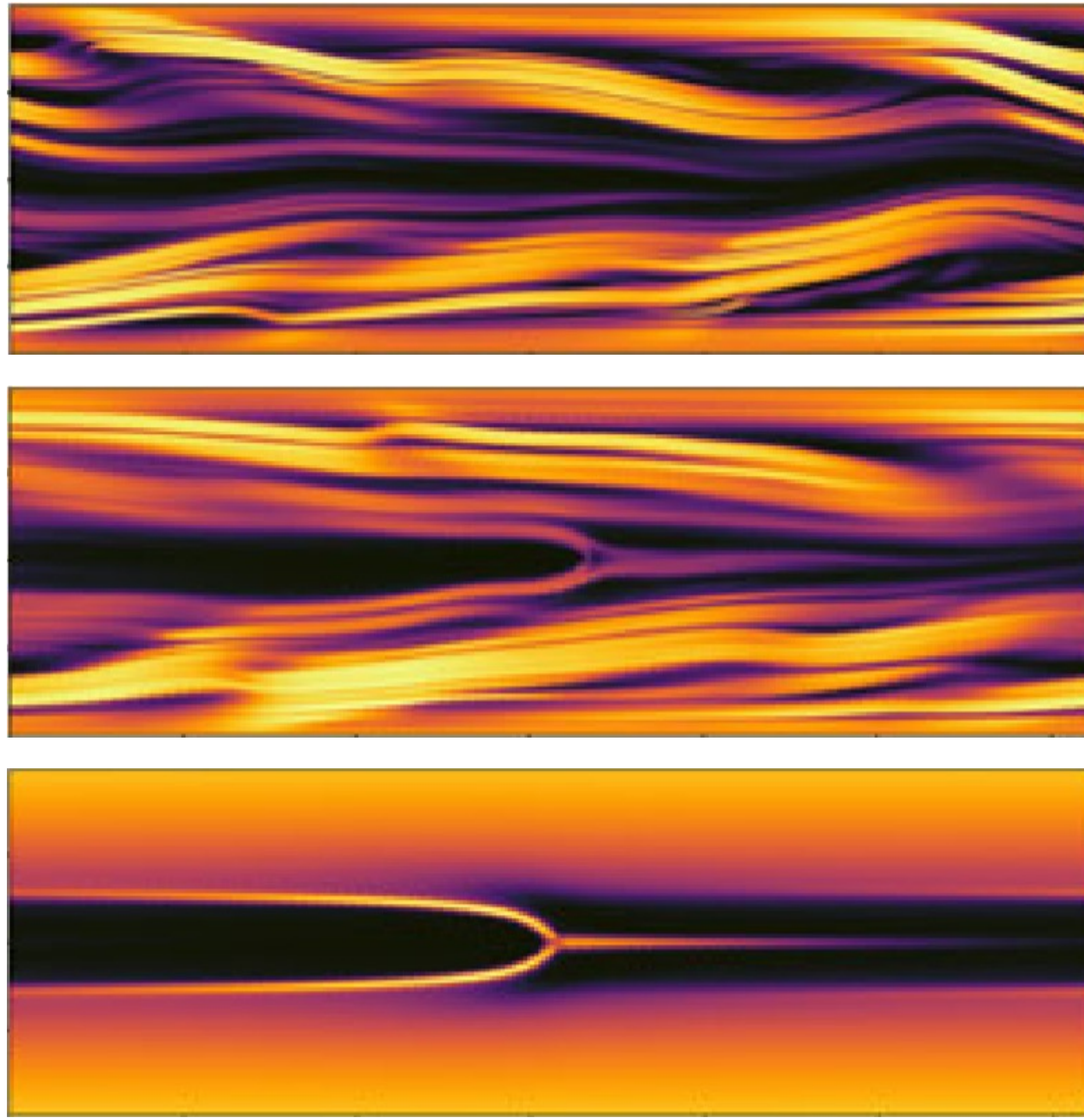
Three attractors in addition to laminar (L) state

EIT Elasto-inertial turbulence

CAR Chaotic arrowhead

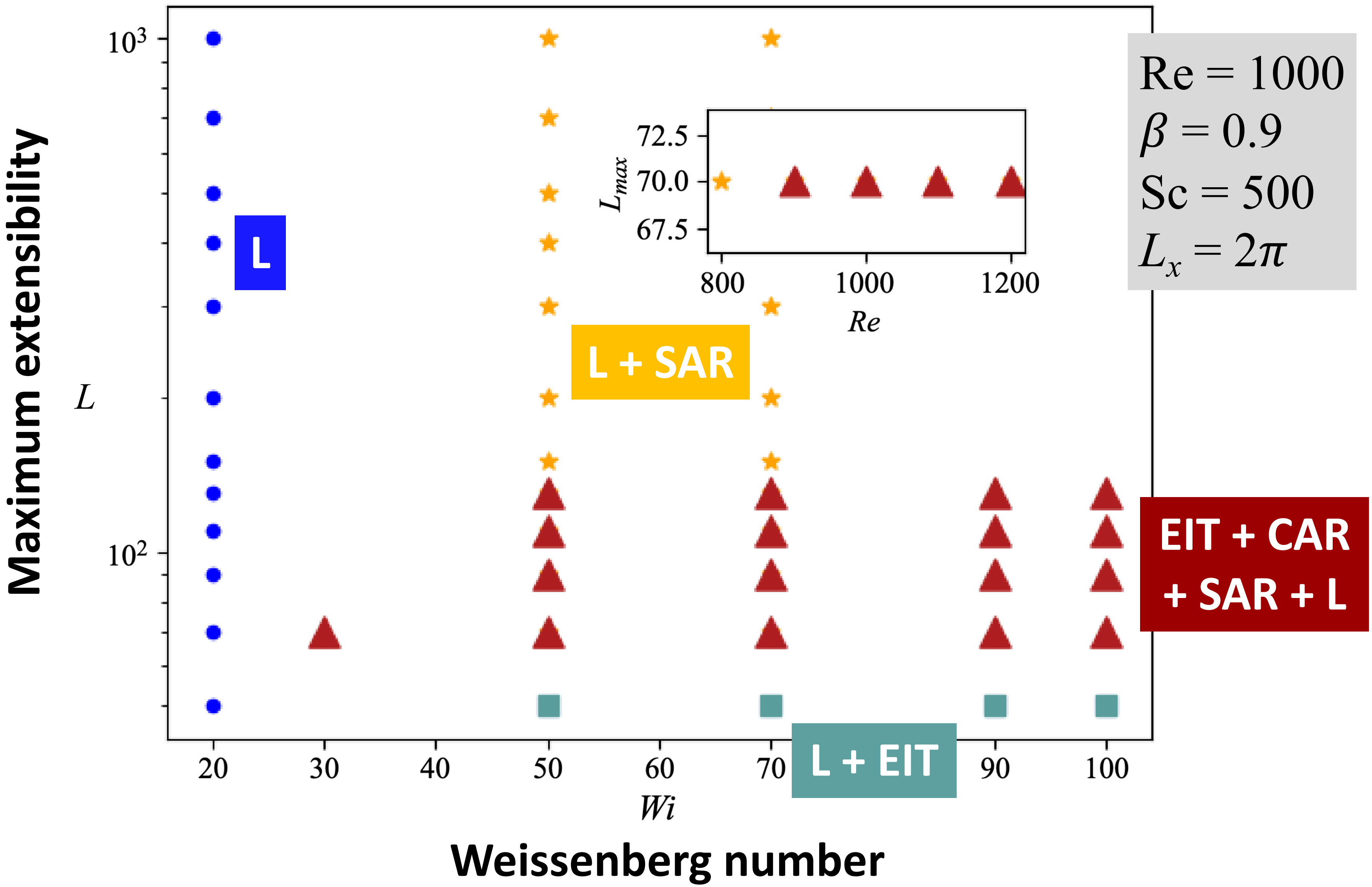
SAR Steady arrowhead

Polymer extension



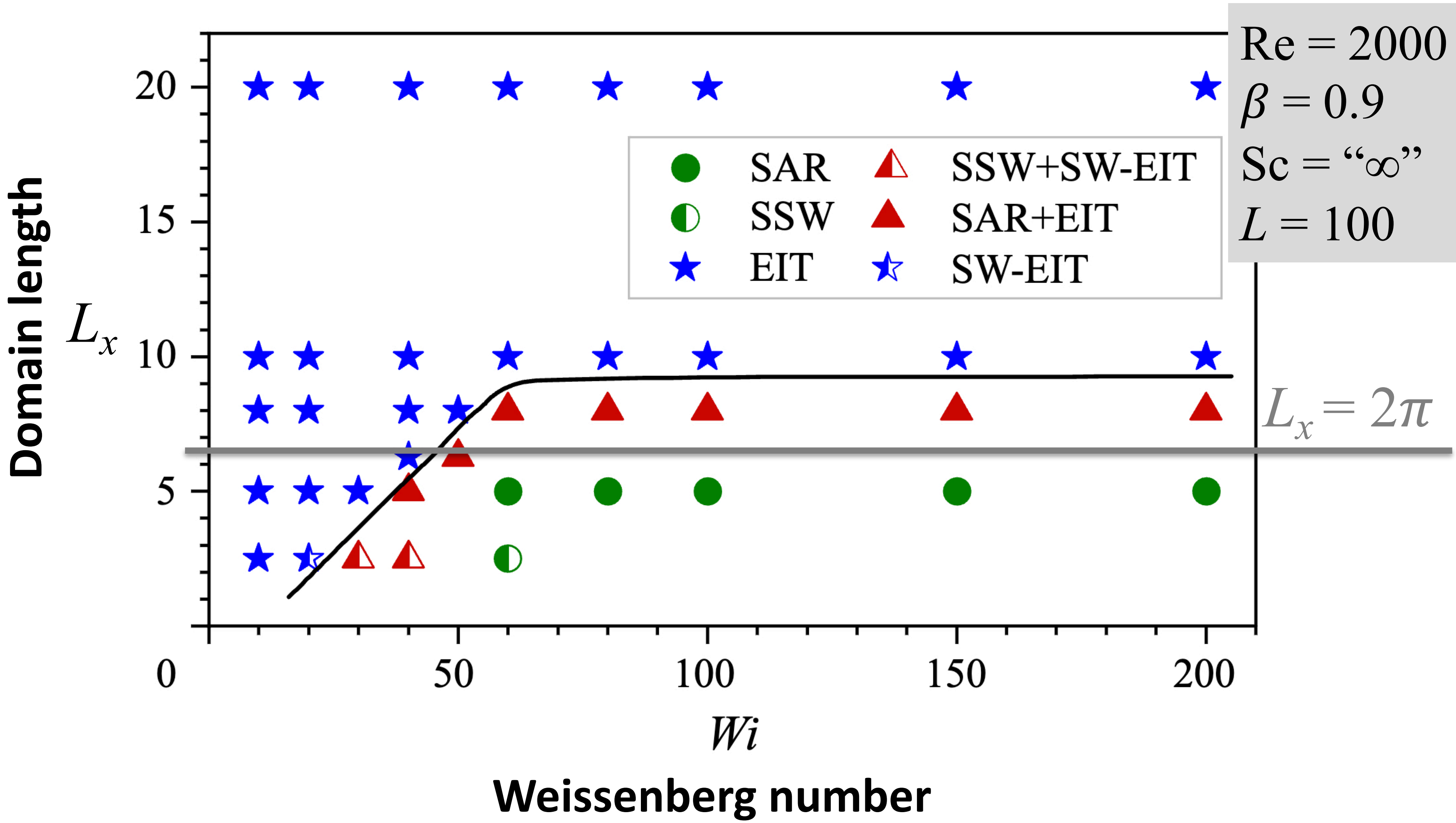
[Dubief *et al.* 2022; Beneitez *et al.* 2023]

All attractors can coexist in parameter space



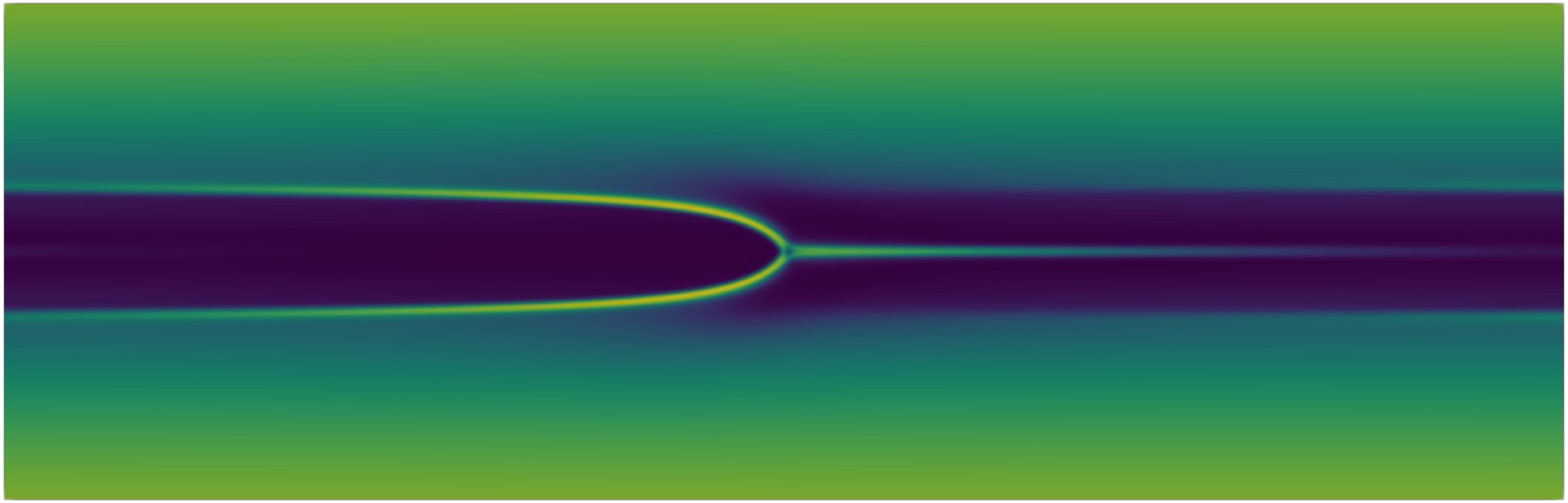
[Beneitez *et al.* 2023]

But only EIT seems to exist in large domains



[Zhang *et al.* 2024]

Focus here on steady arrowhead



- Nonlinear saturation of linear **center mode** instability
- Strongly subcritical
- Perturbations at wall necessary to trigger chaotic state
- Does not seem to play a role in self-sustaining mechanisms of EIT

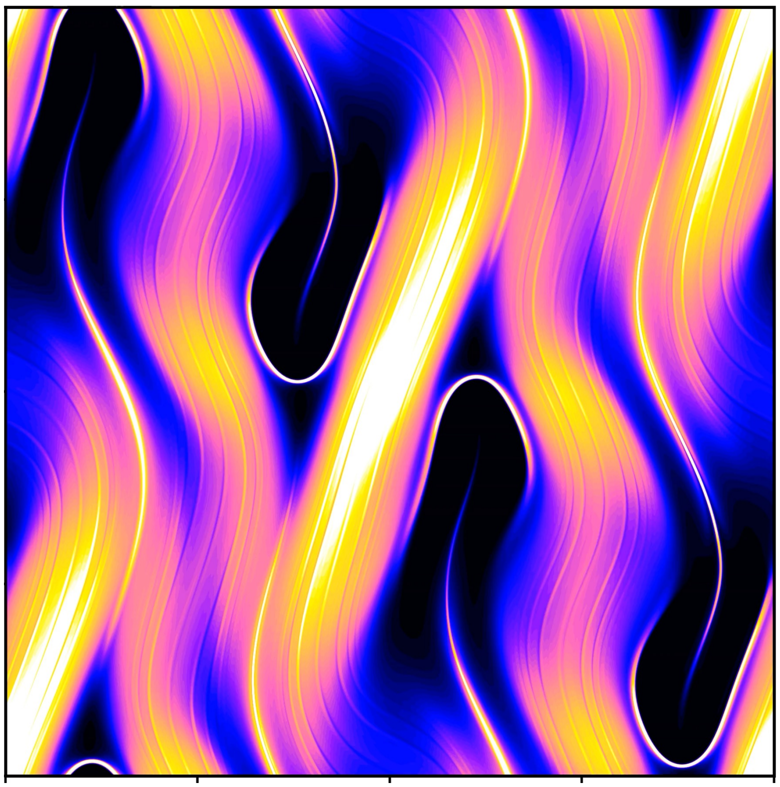
But

- coherent structure
- 2D and steady
- simpler laboratory to understand EIT
- found in other flows

[Beneitez *et al.* 2023]

Other flow configurations

Cellular flow



+ Kolmogorov flow
+ viscoelastic flow through porous materials
+ …

Objectives

WORK IN PROGRESS

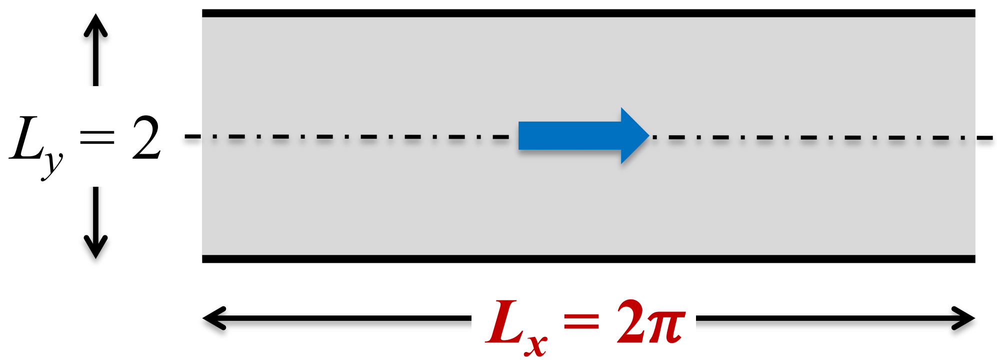
Characterize steady arrowhead in 2D periodic channel flow

- impact of flow topology on polymer field
- back-coupling of polymer field onto flow

$\Rightarrow$ Elucidate **self-sustaining** mechanism

Approach – Configuration

2D periodic viscoelastic channel flow



Constant mass flow rate

$$\partial_j u_j = 0$$

$\beta = 0.9$

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \frac{\beta}{\mathrm{Re}} \partial_j \partial_j u_i + \underbrace{\frac{1-\beta}{\mathrm{Re}} \partial_j \tau_{ij}^{P}}_{\text{Polymer}}$$

$\mathrm{Re_b} = 1000$

Approach – FENE-P

Polymer stress

$$\tau_{ij}^{P} = \frac{1}{\mathrm{Wi}}\left(\frac{C_{ij}}{1-\frac{C_{kk}}{L^2}} - \delta_{ij}\right)$$

Wi = 50

***L* = 90**

Conformation tensor

$$\partial_t C_{ij} + u_k \partial_k C_{ij} = C_{ik}\partial_k u_j + C_{kj}\partial_k u_i - \frac{1}{\mathrm{Wi}}\frac{C_{ij}}{1-\frac{C_{kk}}{L^2}} + \frac{1}{\mathrm{ReSc}}\partial_k \partial_k C_{ij}$$

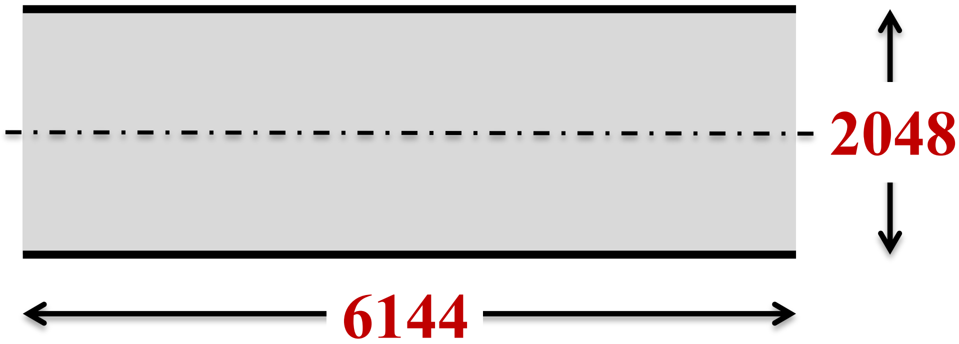
Sc = 500

Pe = 5×10^5

Approach – Discretization

Space

Pseudo-spectral (Dedalus)



$\Delta y = 2.9\times10^{-7}$

$\Delta y = 1.5\times10^{-3}$

$\Delta x = 10^{-3}$

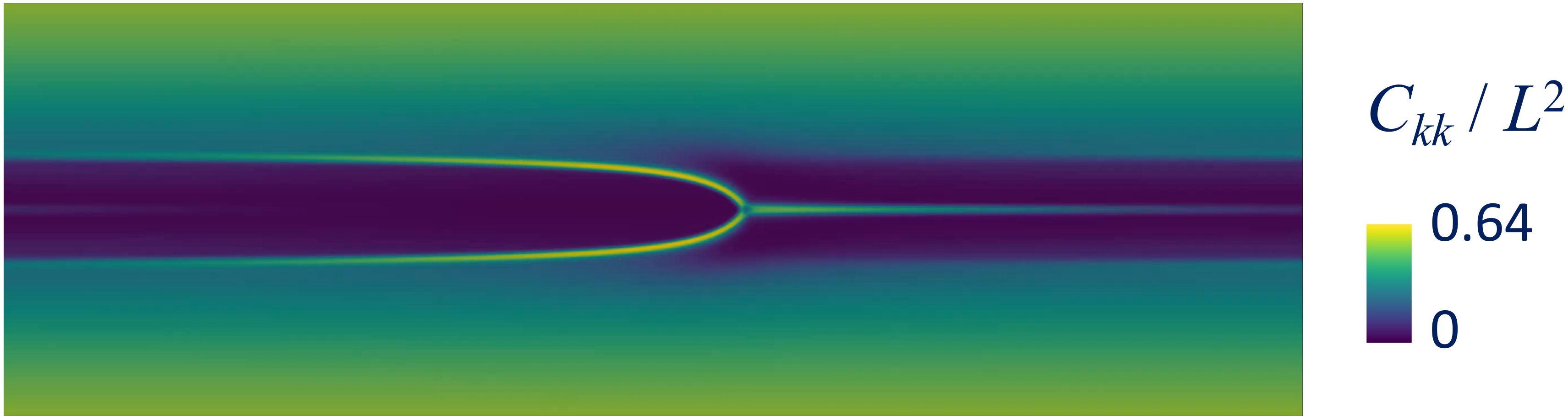
Time

4 steps 3rd order implicit/explicit Runge-Kutta (RK443)

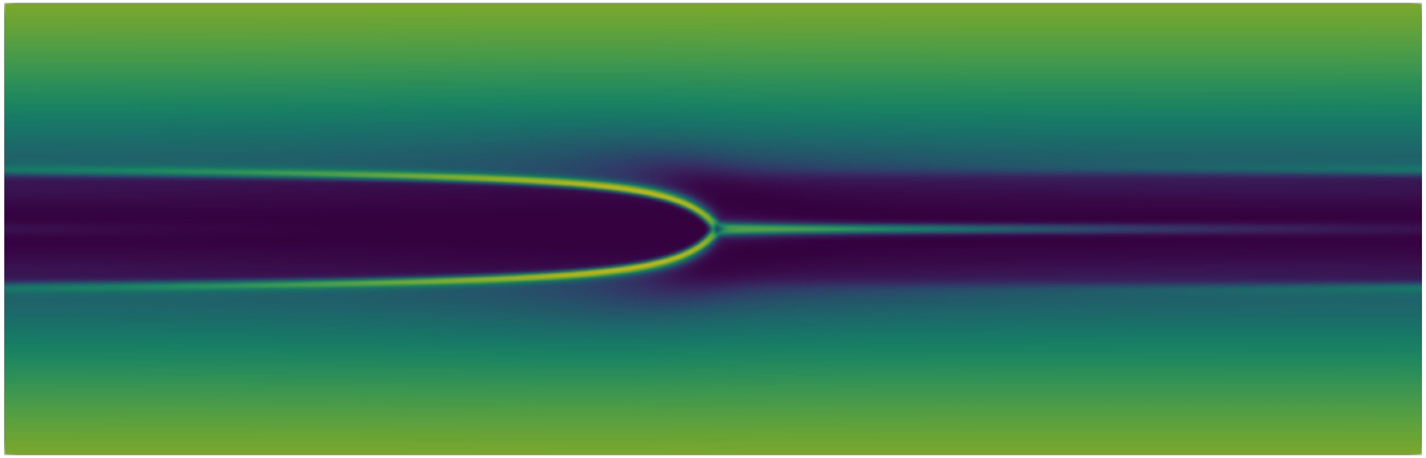
$\Delta t = 2.5\times10^{-4}$

Traveling wave

Fixed frame of reference



Moving frame of reference ($u_{\mathrm{SAR}} \simeq 1.4579$)

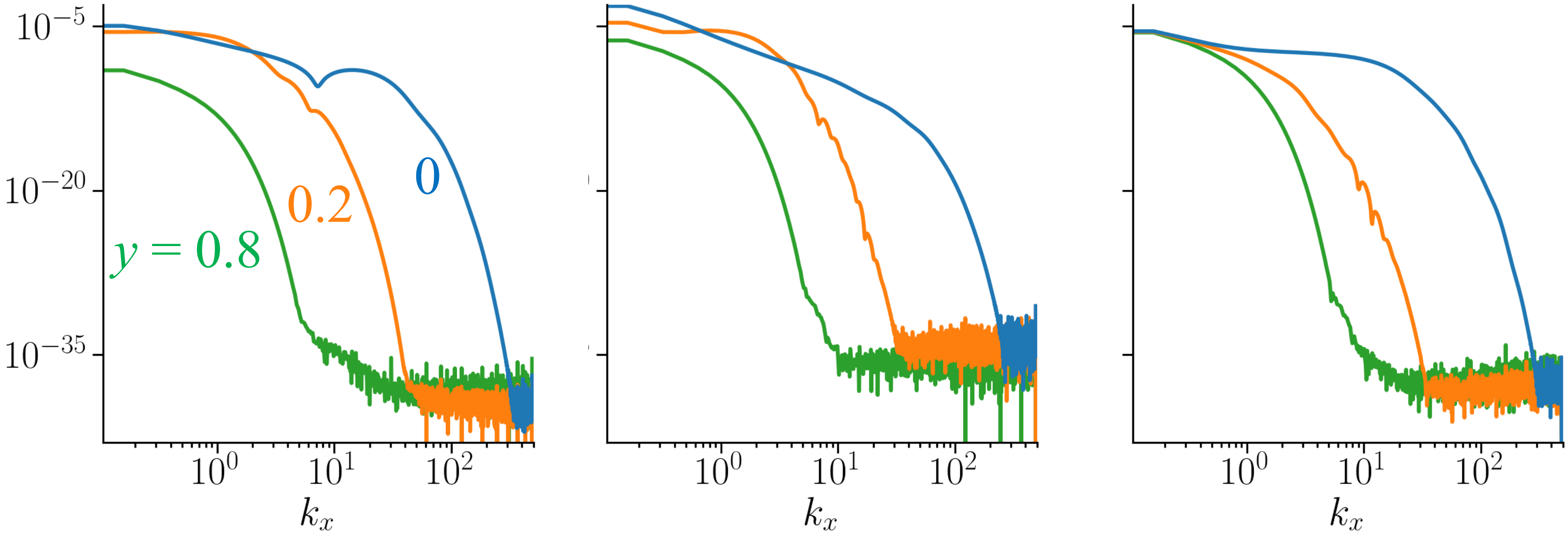


Steady!

Solution check – Spatial accuracy

Streamwise spectra

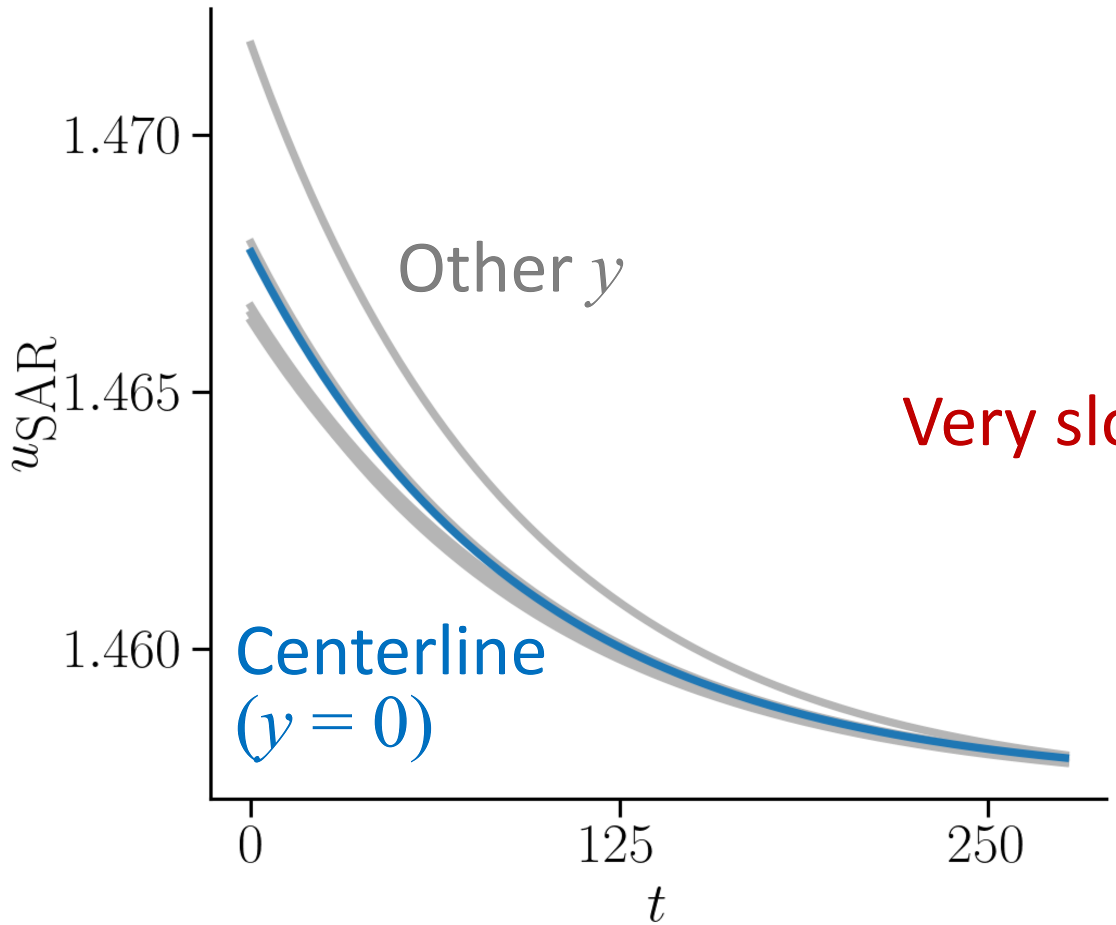
Elastic energy e_p | **Kinetic energy e_k** | **Pressure p**



Small scales fully resolved at $\mathrm{Sc} = 500$

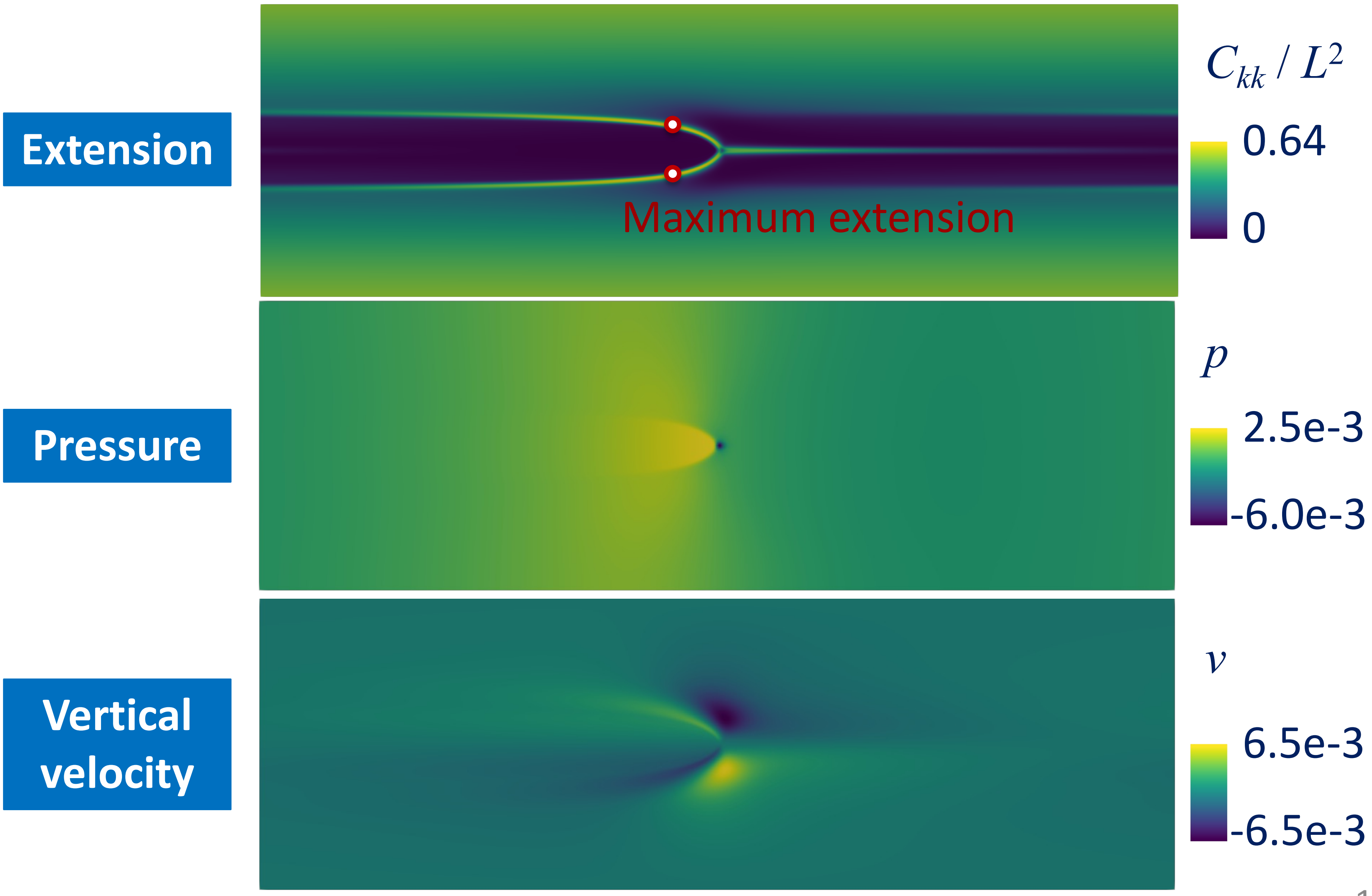
Solution check – Convergence to steady state

Traveling wave velocity
(from spatio-temporal correlations)

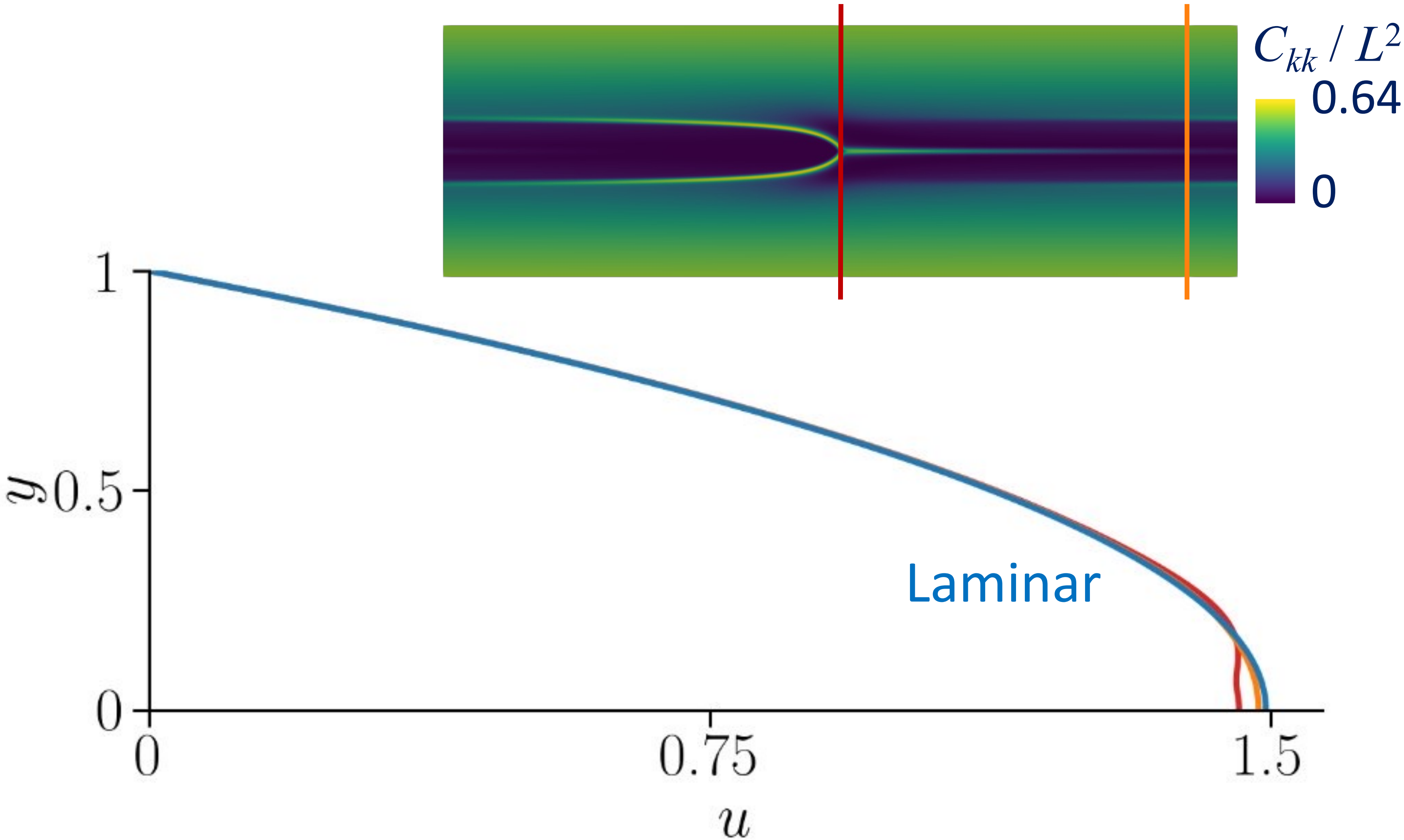


Very slow convergence!

Solution overview

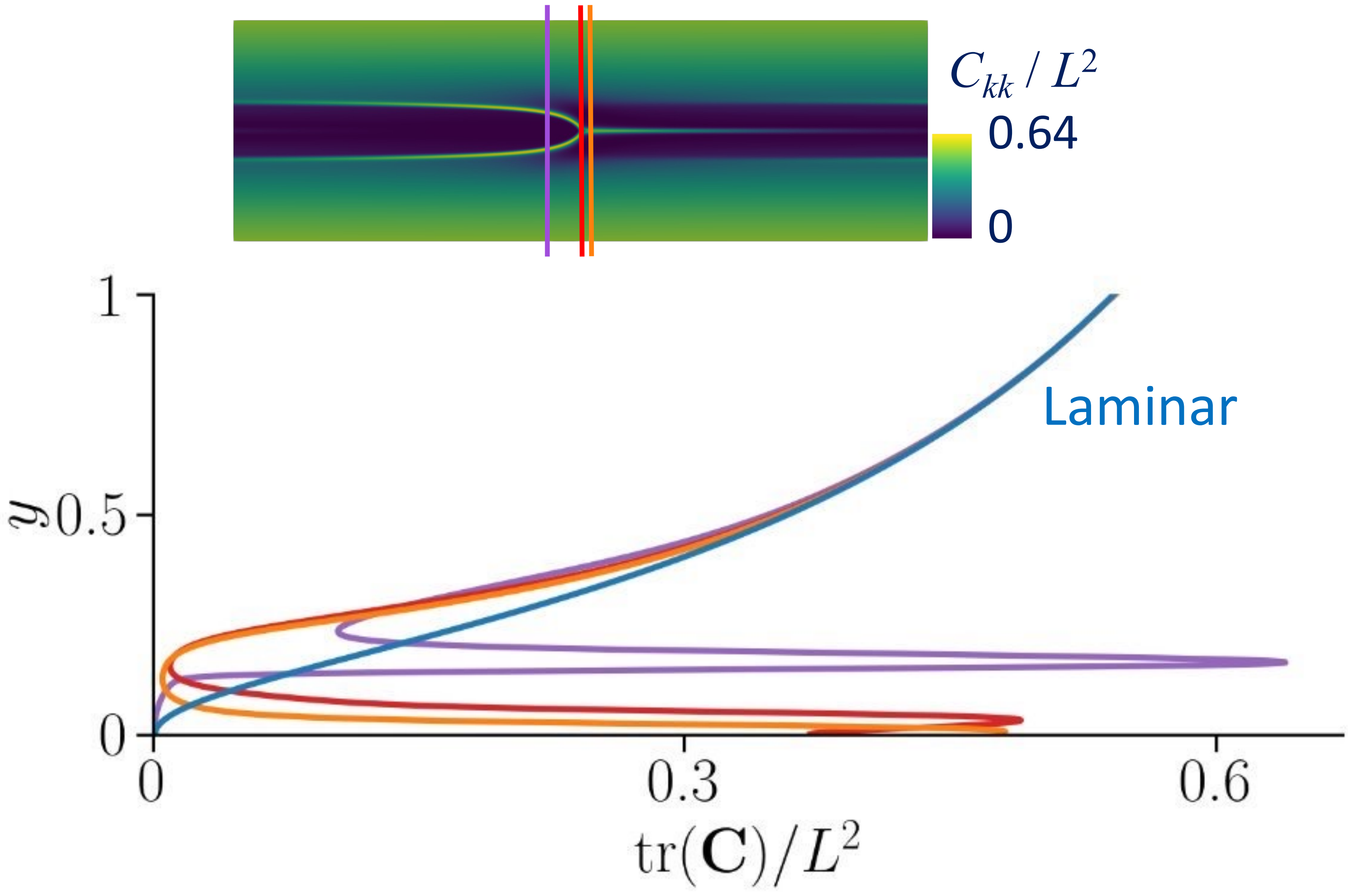


Profiles of streamwise velocity



Streamwise velocity very similar to laminar case…

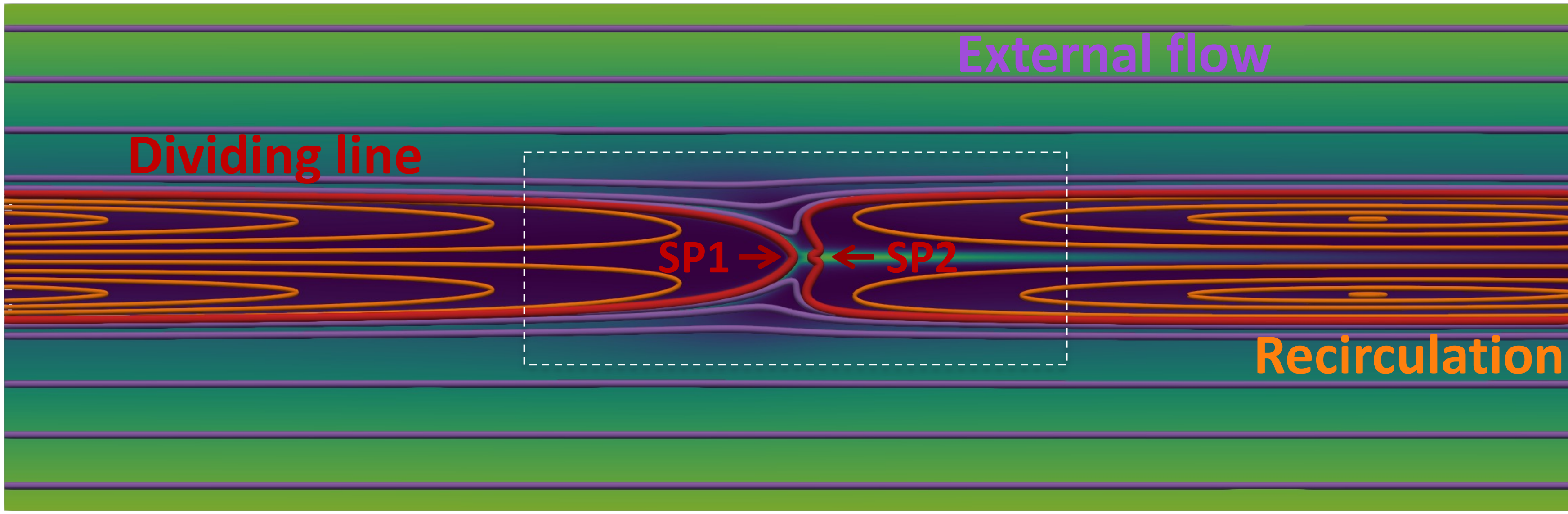
Profiles of polymer extension

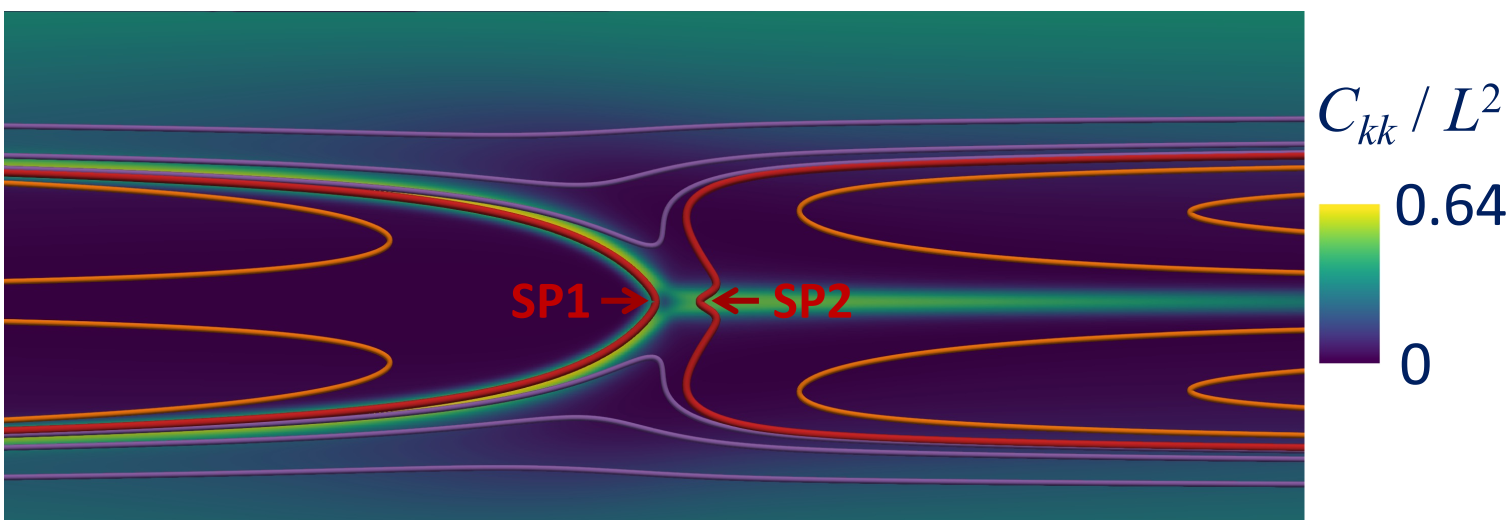


… but marked differences in polymer extension

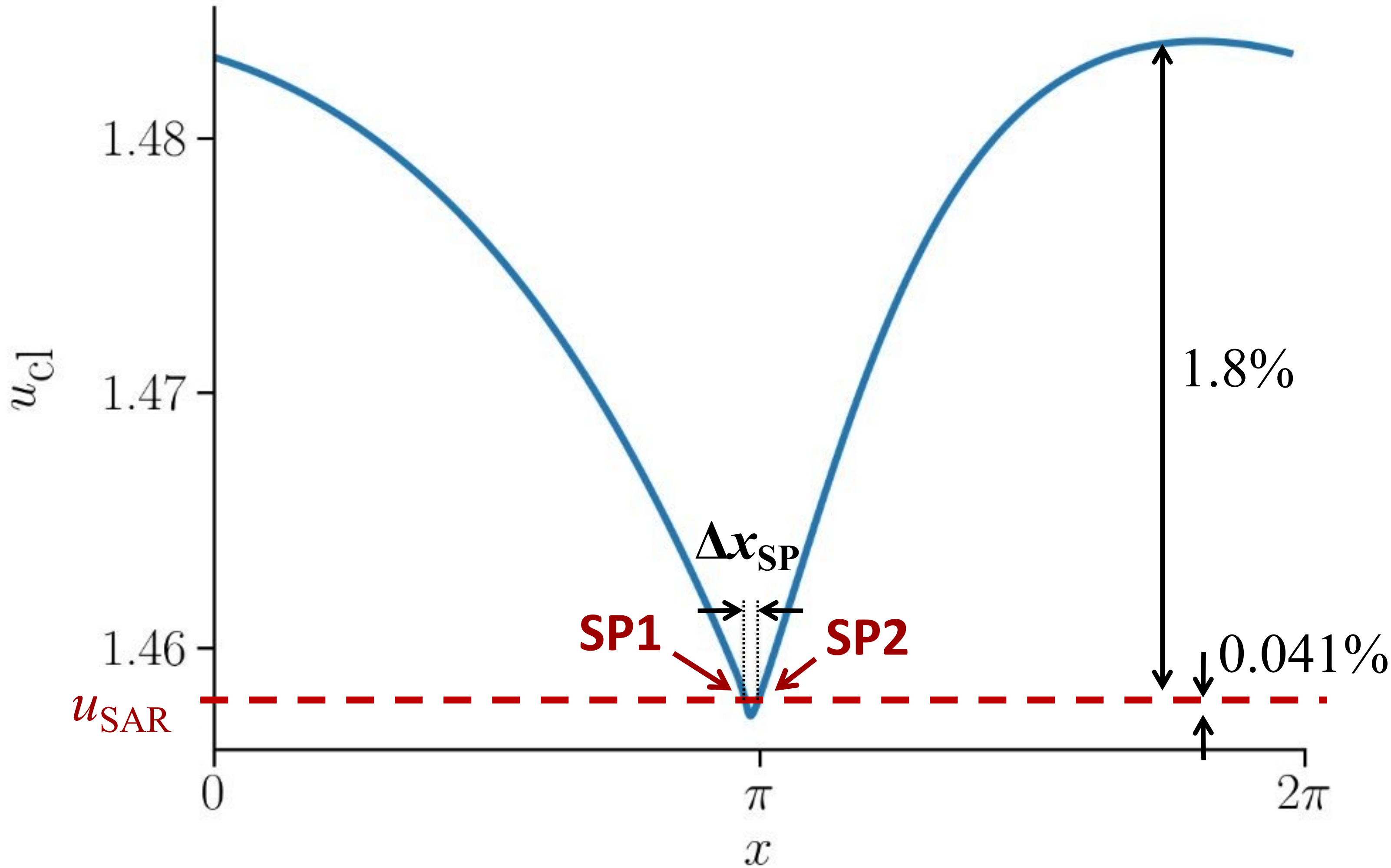
Streamlines in moving frame of reference

2 separate regions with 2 stagnation points (SP) on centerline



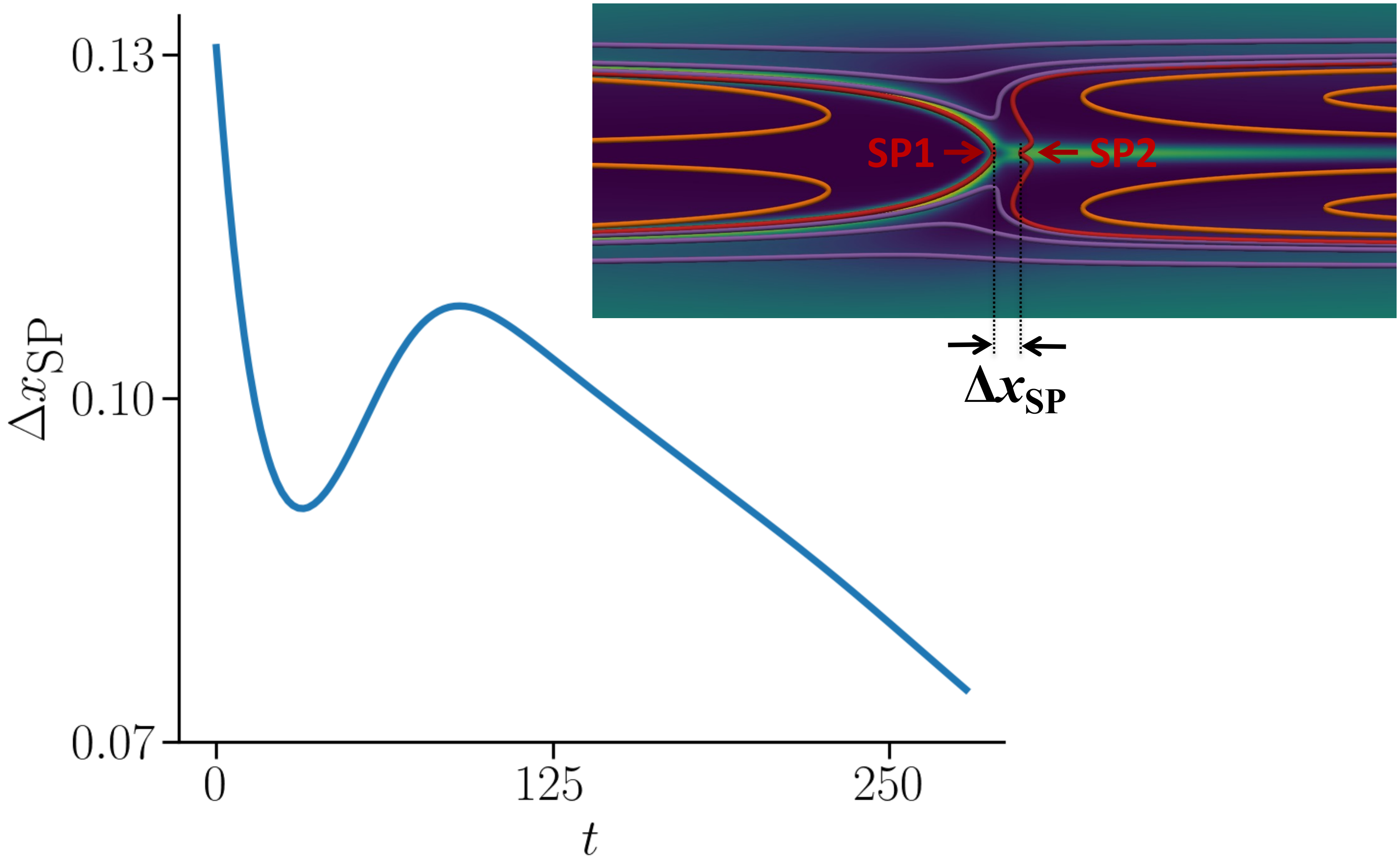


Streamwise velocity along centerline



Small error on u_{SAR} can have an important impact on topology!

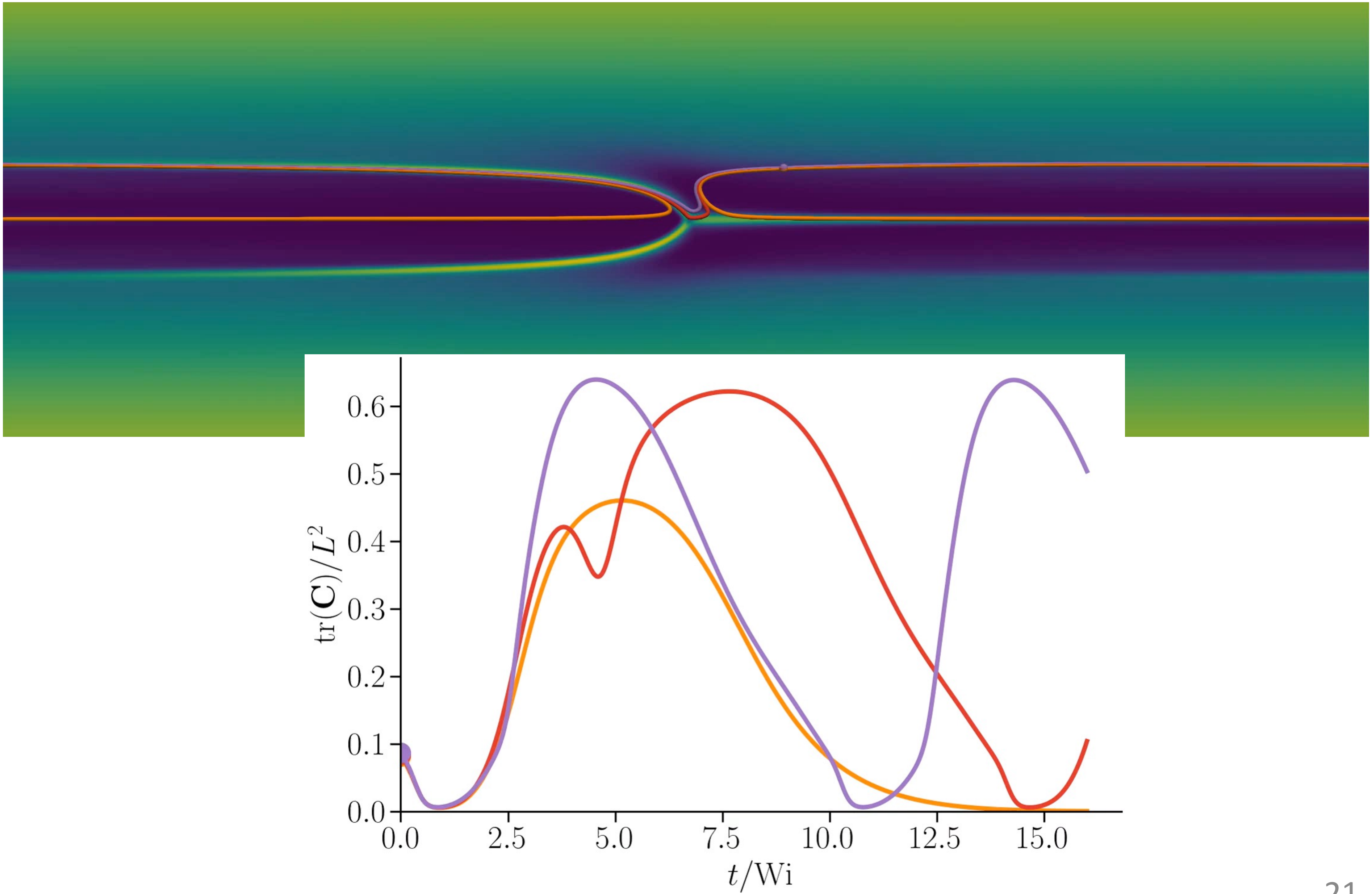
Distance between 2 stagnation points



No sign of convergence!

Could the two stagnation points merge?

Lagrangian trajectories



Key question 1

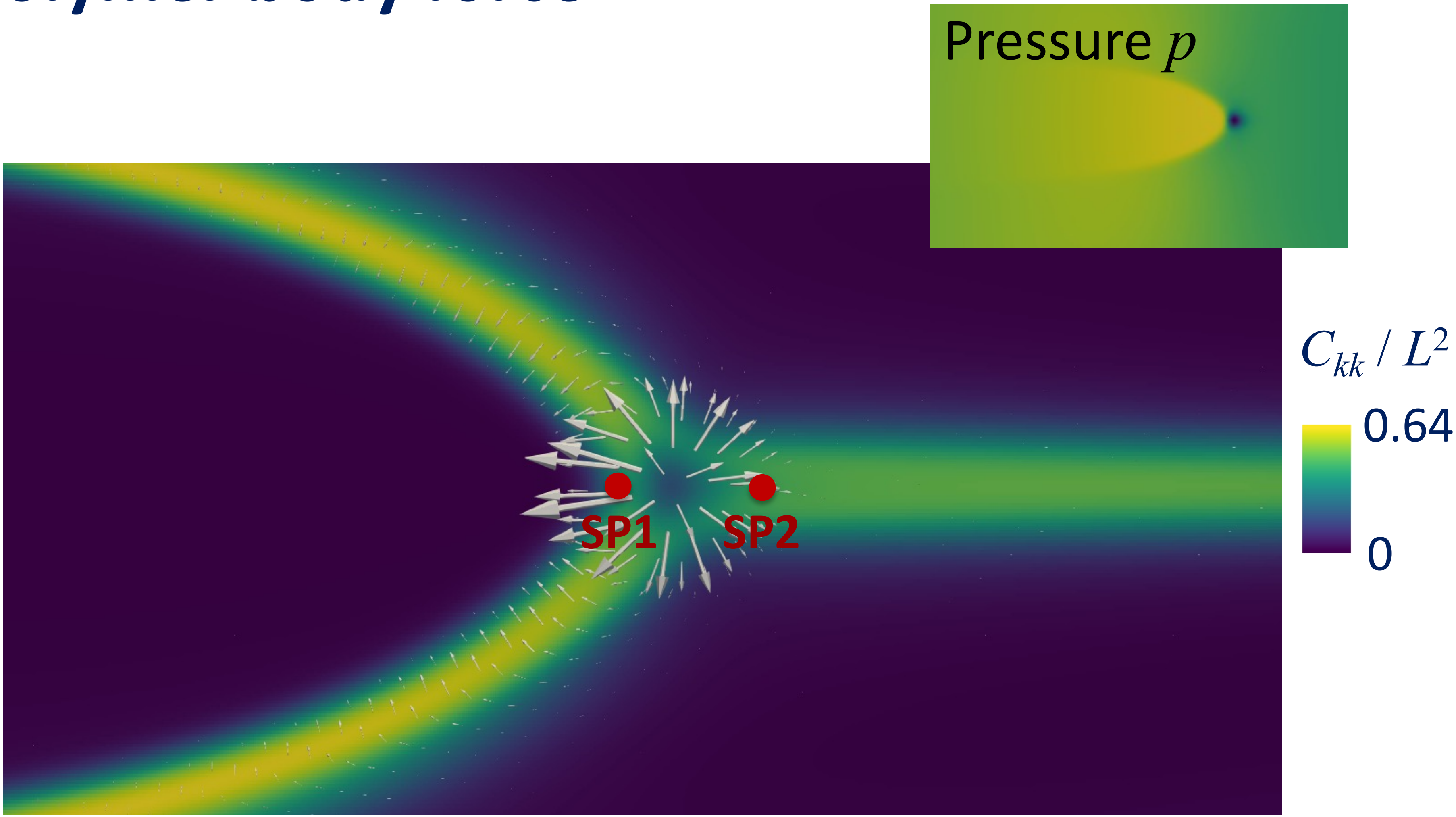
How do the polymers induce such a flow topology?

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \frac{\beta}{\mathrm{Re}}\Delta\mathbf{u} + \underbrace{\frac{1-\beta}{\mathrm{Re}}\nabla\cdot\boldsymbol{\tau}^p}_{\text{Polymer body force } \mathbf{f}^p}$$

Polymer body force $\mathbf{f}^p$

$$\mathbf{f}^p = \frac{1-\beta}{\mathrm{Re}}\left[(\partial_x\tau^p_{xx} + \partial_y\tau^p_{xy})\mathbf{e}_x + (\partial_x\tau^p_{xy} + \partial_y\tau^p_{yy})\mathbf{e}_y\right]$$

Polymer body force



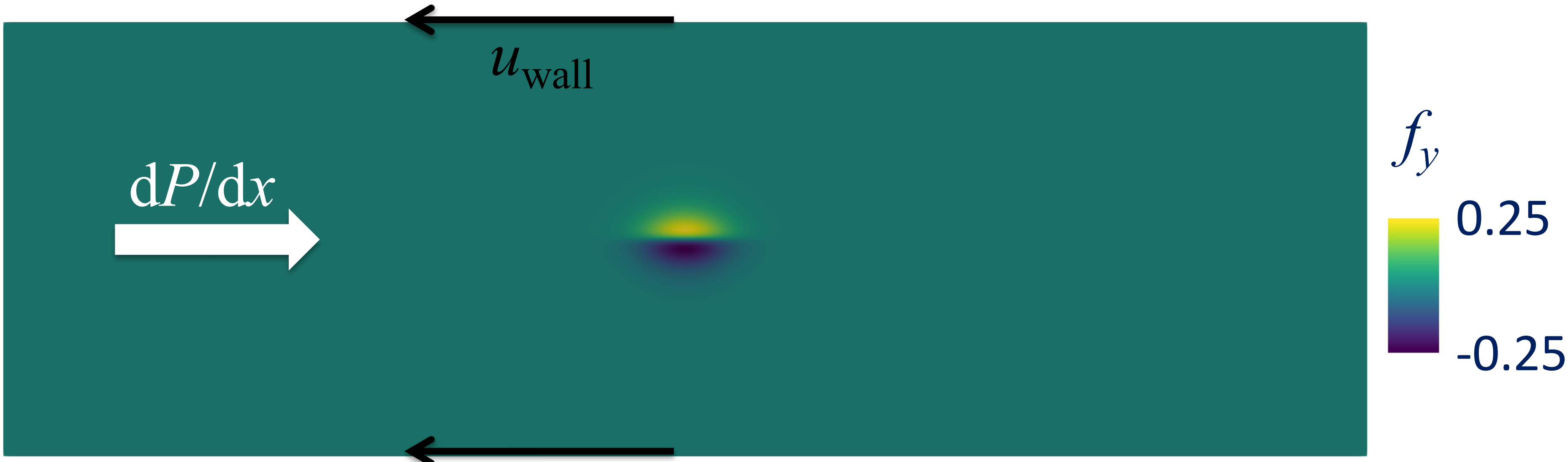
Large body force concentrated between stagnation points (ring around minimum pressure zone)

Newtonian flow with localized body force

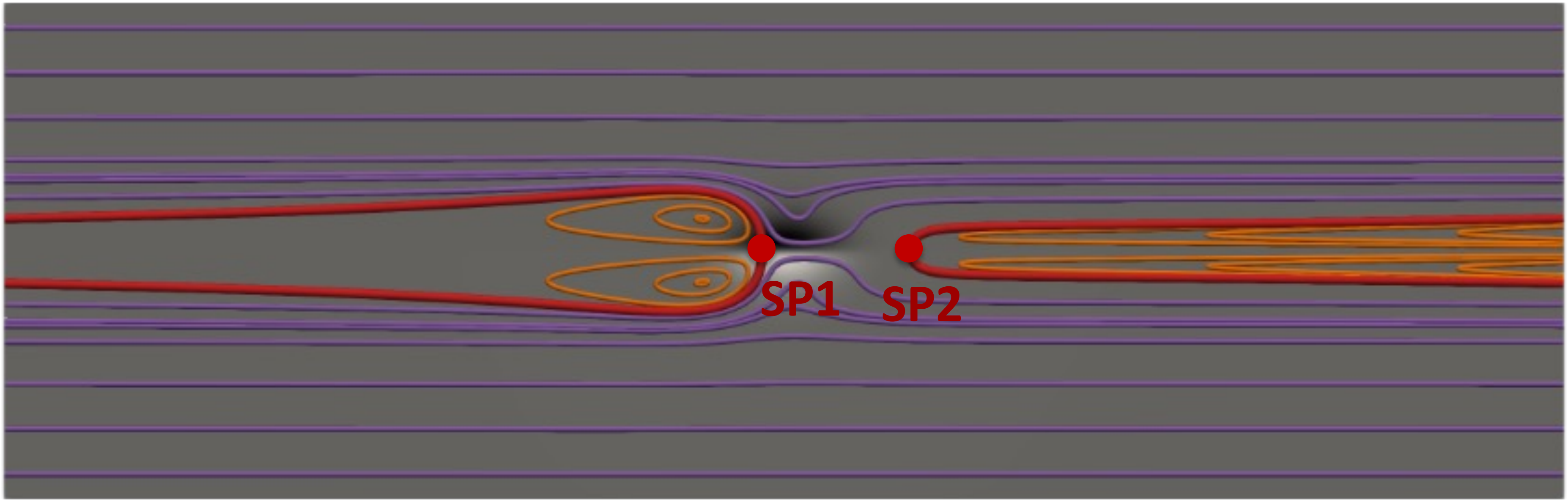
Can we reproduce this flow topology in Newtonian case with an external localized body force?

Vertical body force in **moving** frame of reference

$$f_y = c\mathrm{e}^{-\left(\frac{\sin(\pi x/L_x)}{\sigma_x}\right)^2} y(1-y^2)\mathrm{e}^{-\frac{|y|}{\sigma_y}}$$



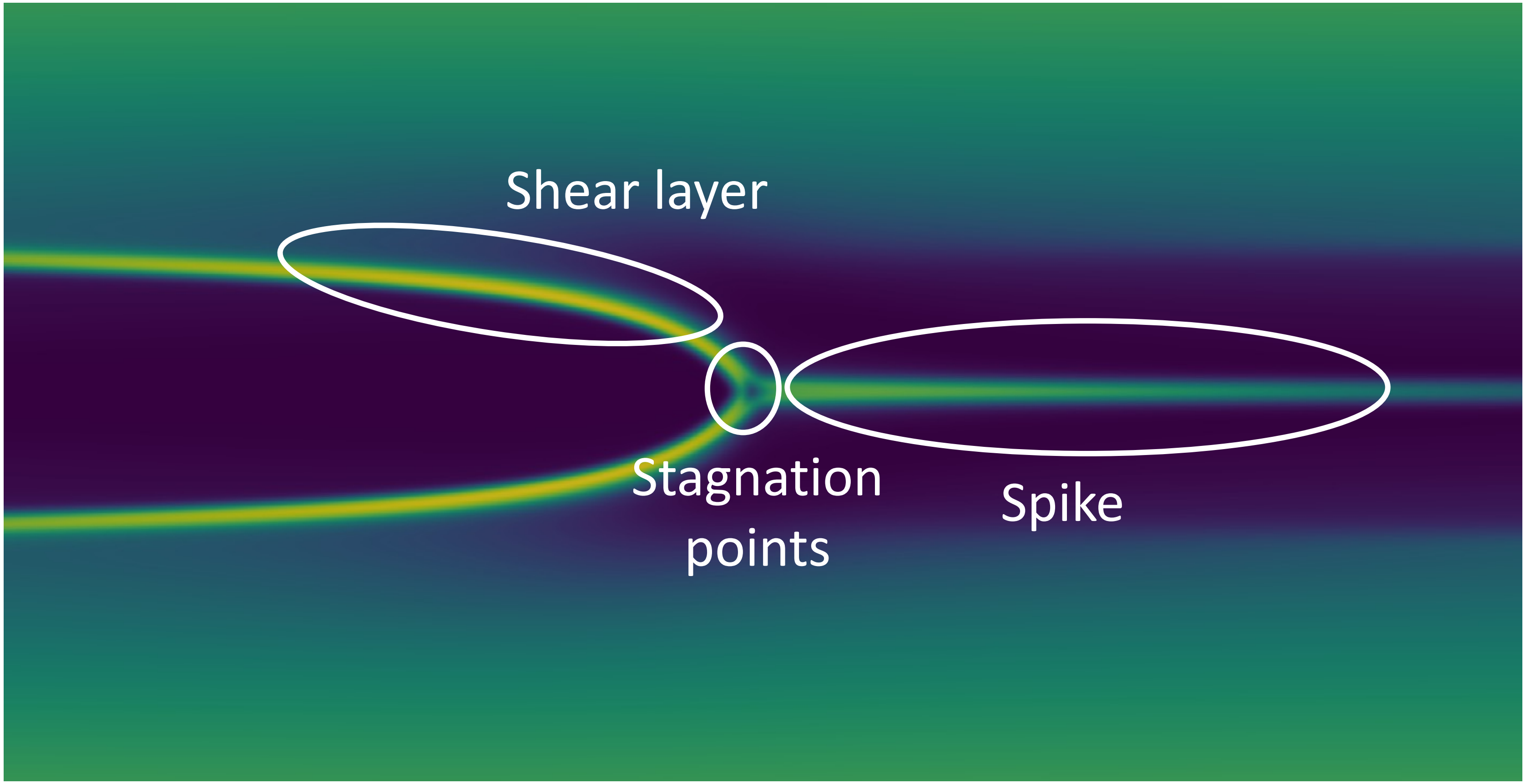
Newtonian flow with localized body force



A localized vertical body force on the centerline seems to be sufficient to create this flow topology

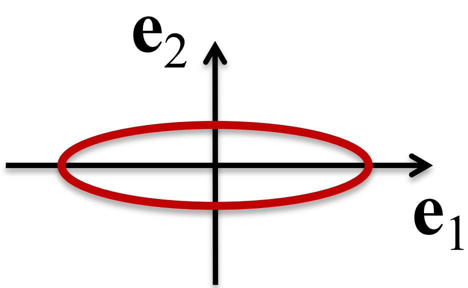
Key question 2

How does this velocity field stretch the polymers?



Diagonalization of polymer stress tensor

$$\begin{bmatrix} \tau_{xx}^{p} & \tau_{xy}^{p} \\ \tau_{xy}^{p} & \tau_{yy}^{p} \end{bmatrix} \blacktriangleright \begin{bmatrix} \tau_{1}^{p} & 0 \\ 0 & \tau_{2}^{p} \end{bmatrix}$$

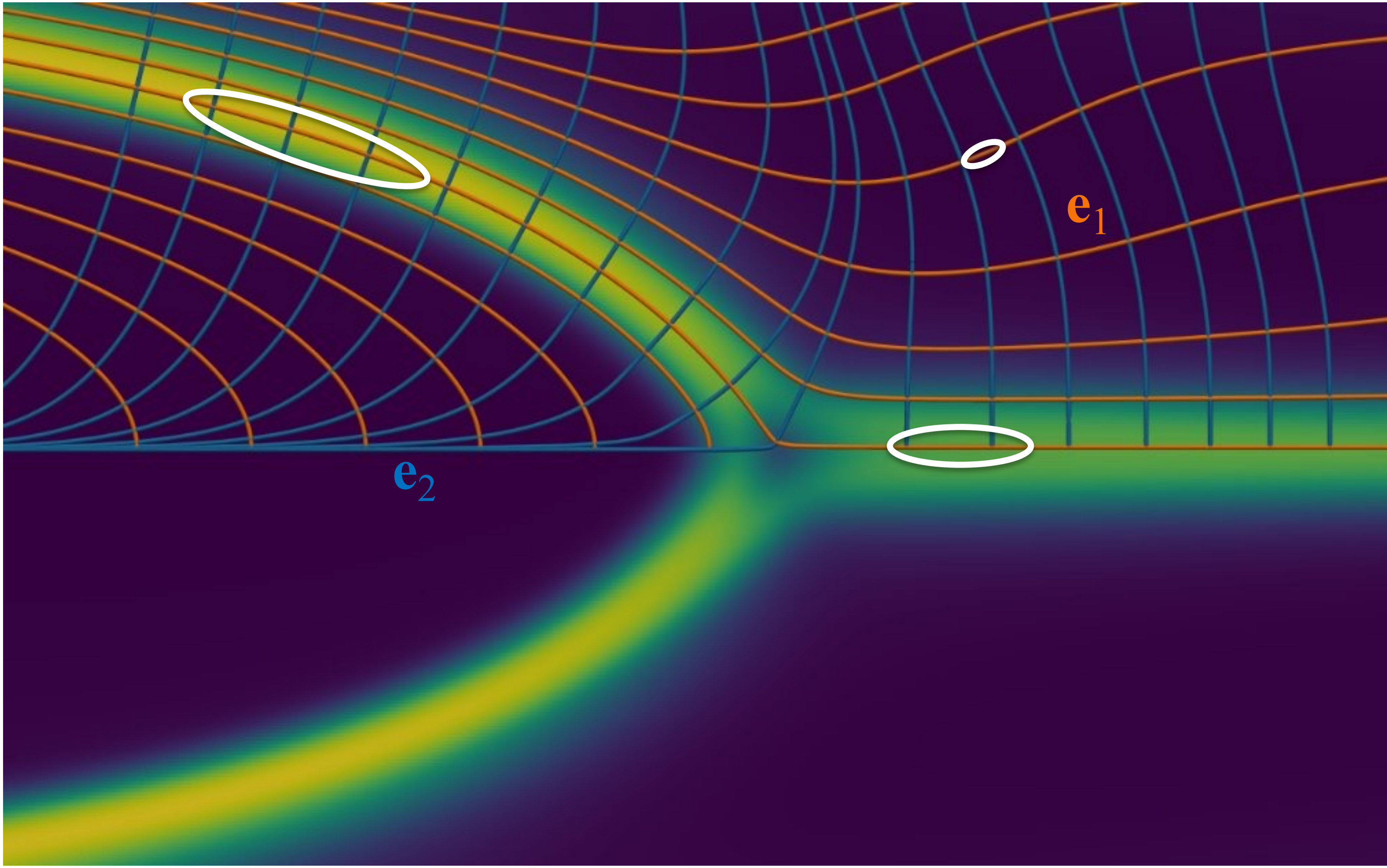


Principal stresses with $\tau_1^p > \tau_2^p$

along principal axes $\mathbf{e}_1$ and $\mathbf{e}_2$

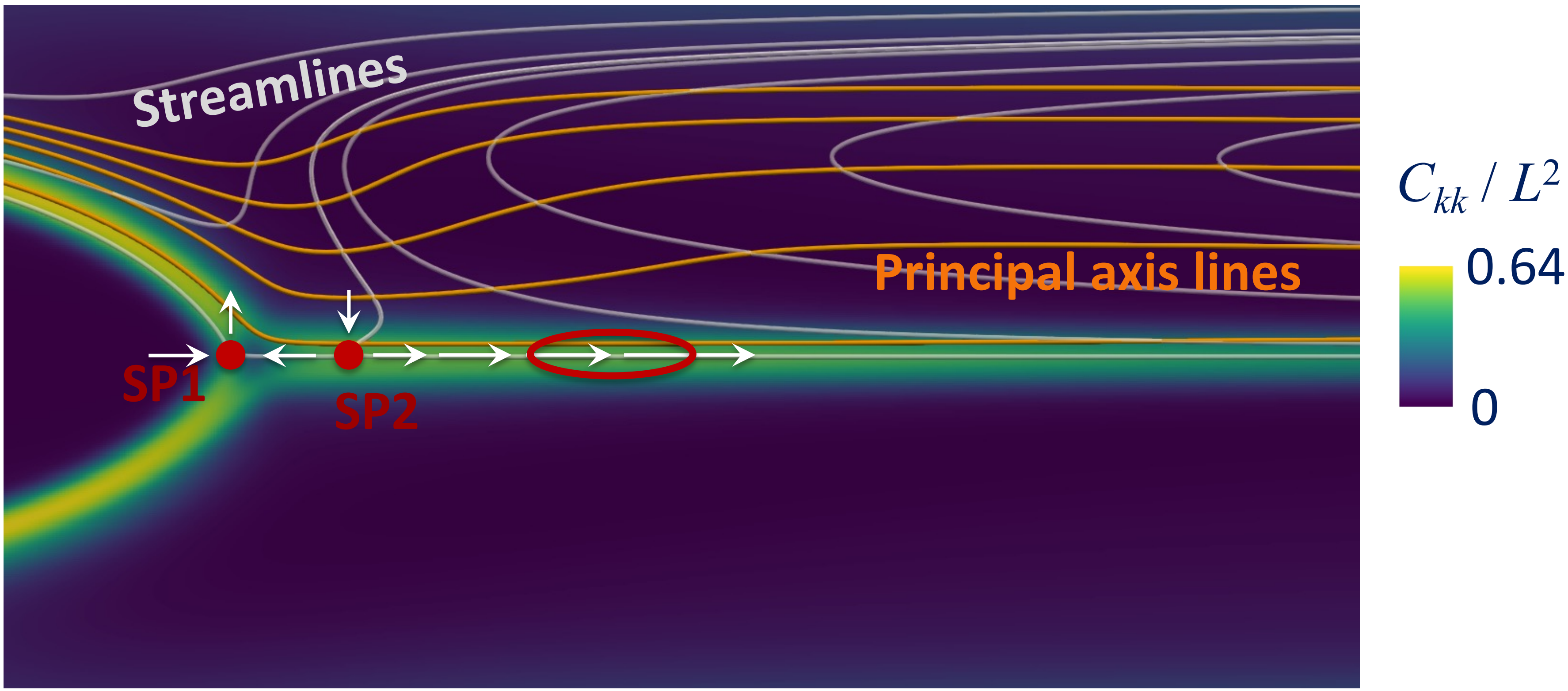
- Visualize **principal axis lines** (similar to streamlines)

Principal axis lines



Polymer extension in the spike region

Pure extensional flow on centerline (symmetry) $\nabla \boldsymbol{u}|_{\mathrm{cl}} = \begin{bmatrix} \dot{\varepsilon} & 0 \\ 0 & -\dot{\varepsilon} \end{bmatrix}$

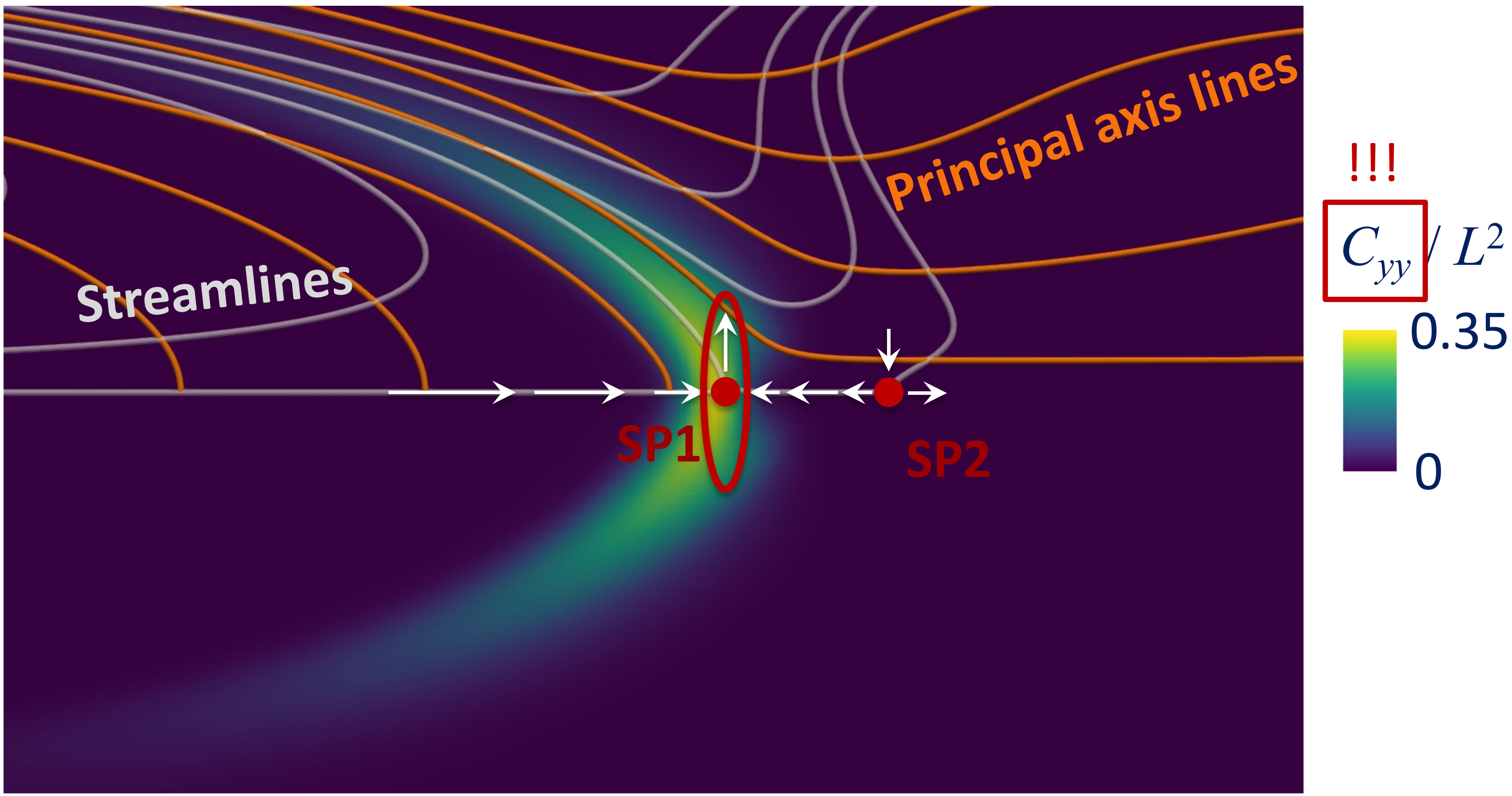


Polymers aligned with the flow along centerline

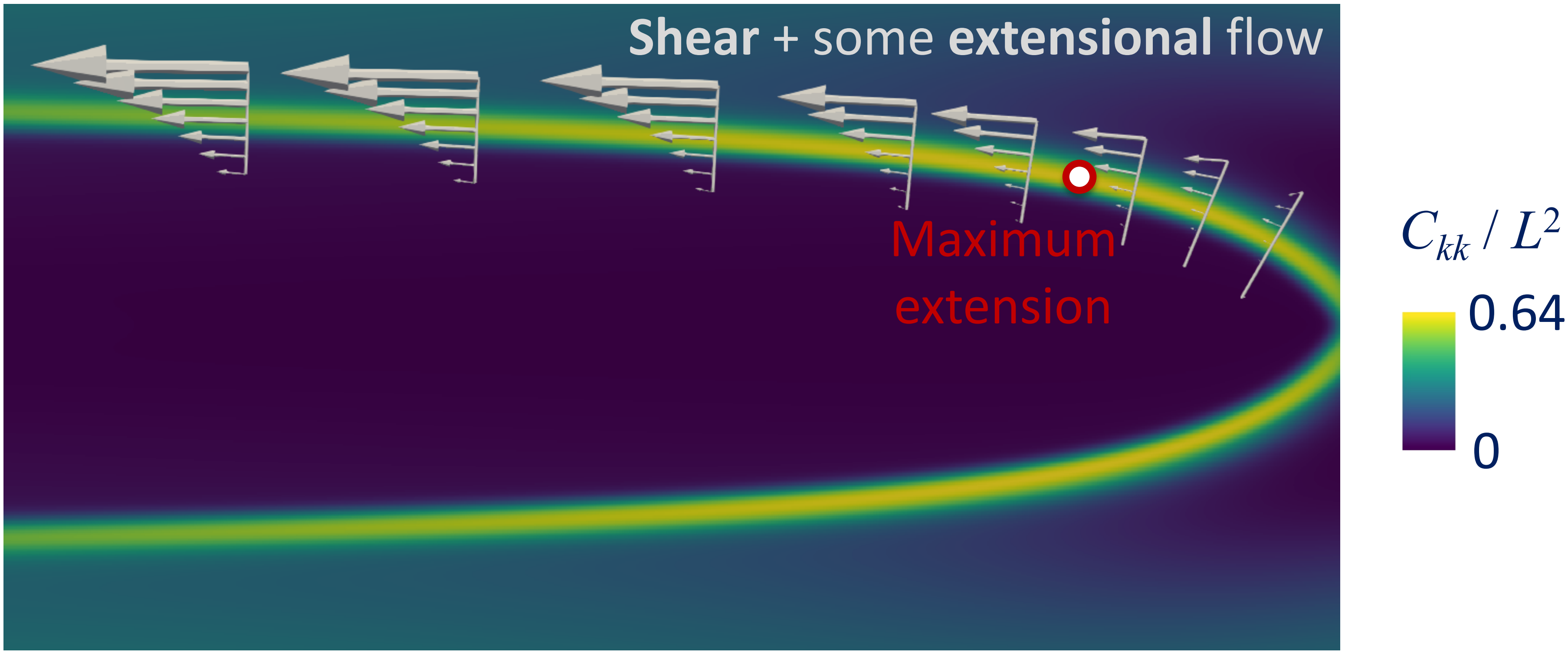
Tip fades away as $\dot{\varepsilon}$ decreases in x

Polymer extension at SP1

Pure extensional flow on centerline (symmetry) $\nabla \boldsymbol{u}|_{\mathrm{cl}} = \begin{bmatrix} \dot{\varepsilon} & 0 \\ 0 & -\dot{\varepsilon} \end{bmatrix}$, $\dot{\varepsilon} < 0$

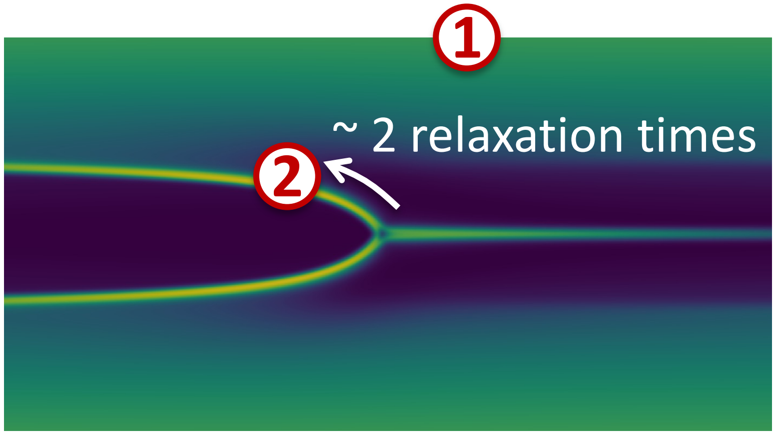


Polymer extension in shear layer



- Strong steady shear?
- Advection of initial extension at SP1?
- Extensional flow?
- Transient overshoot due to initial misalignment?

Polymer extension in shear layer

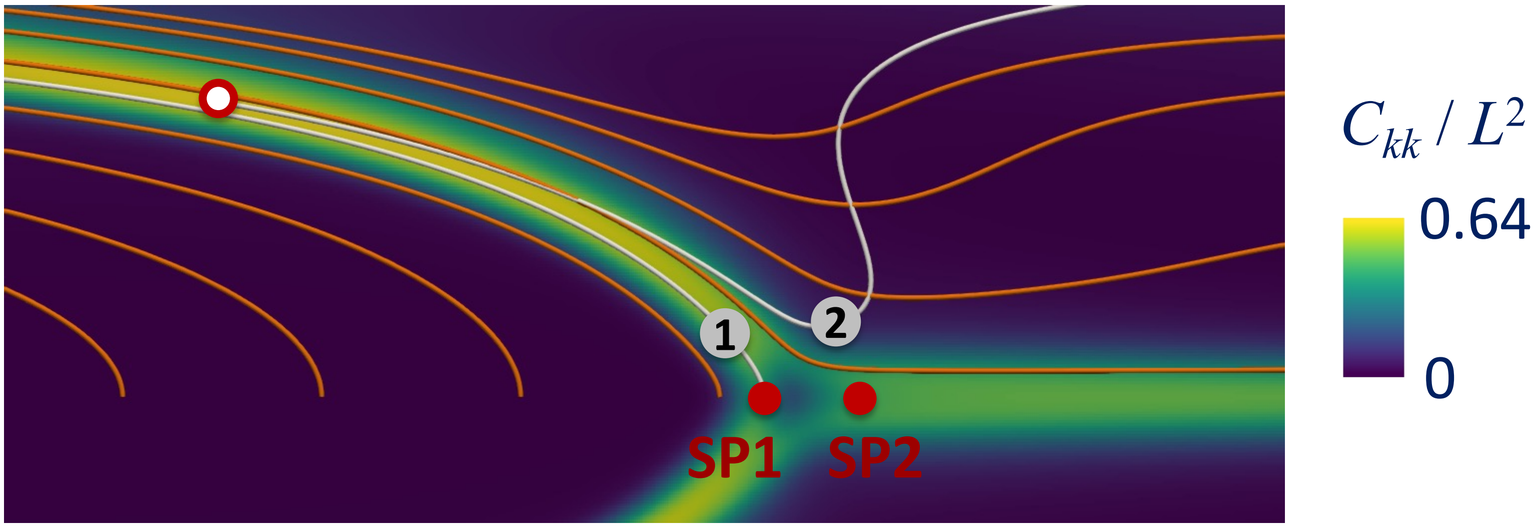


Strong steady shear? ✗

		①		②
	$\dot{\gamma}$	3.080	>	0.370
	C_{kk} / L^2	0.543	<	0.639
If steady ▶	C_{kk} / L^2	0.543	>>	0.004

Polymer extension in shear layer

Advection from initial extension at SP1?	✗
Extensional flow?	?

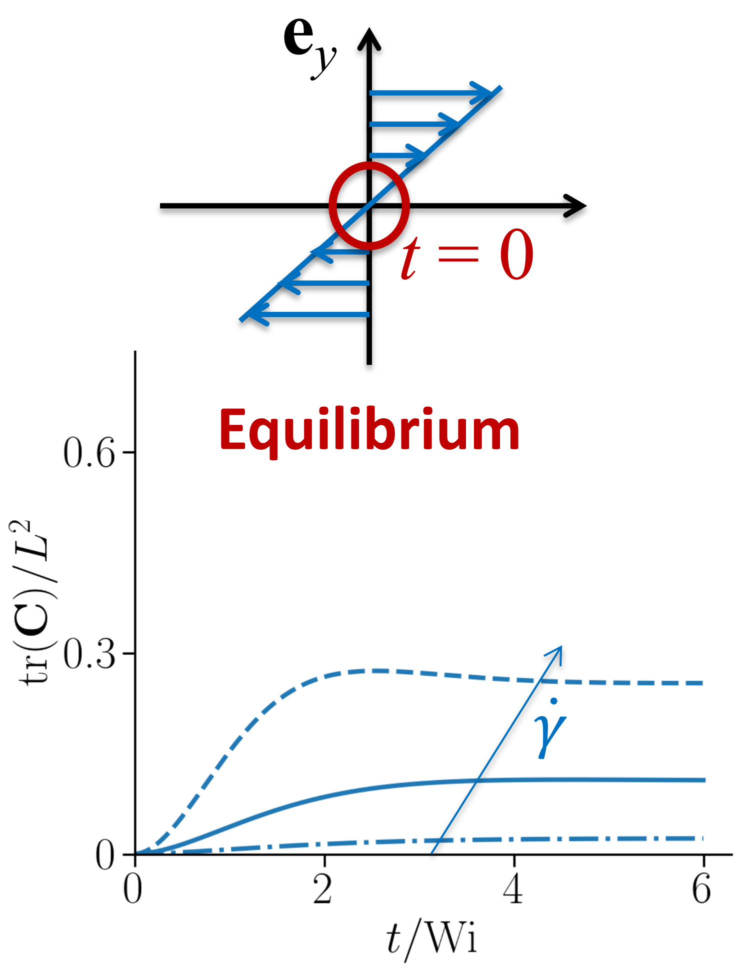


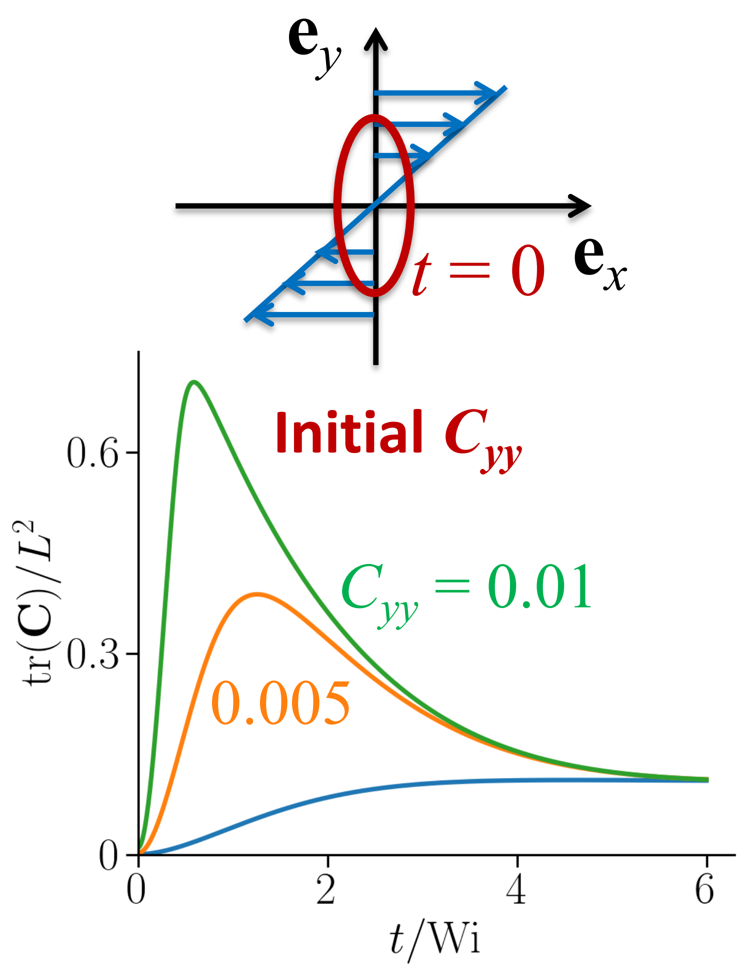
- Streamline leading to maximum extension is not the stagnation streamline (1) but streamline (2)
- No significant initial pre-stretching for streamline (2)
- But some extensional flow along streamline

Polymer extension in shear layer

Transient overshoot due to initial misalignment?

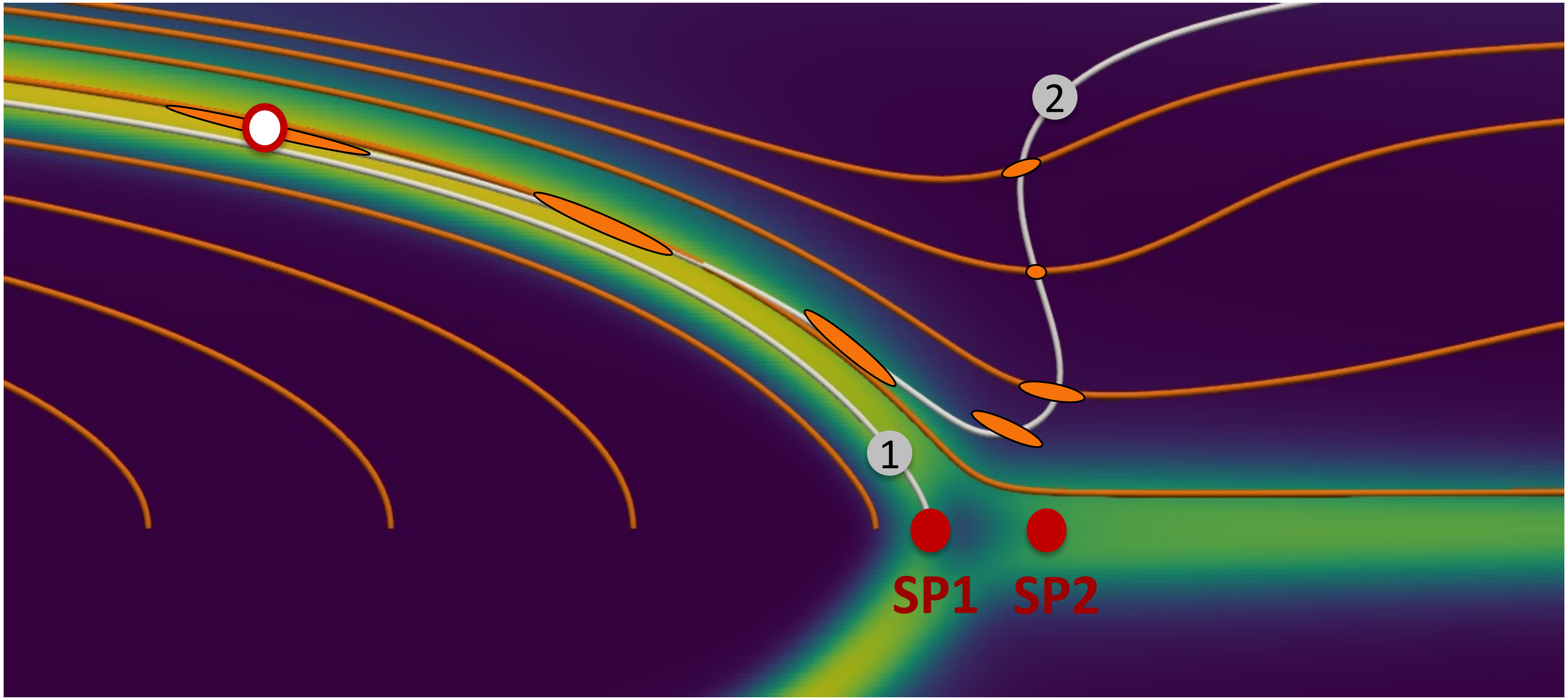
Inception of steady shear flow from different initial states





Polymer extension in shear layer

Transient overshoot due to initial misalignment? ?



Key question 3

How does the polymer field lead to the body force?

Polymer body force $\mathbf{f}^p$ in principal stress axes

Curvature of principal stress lines

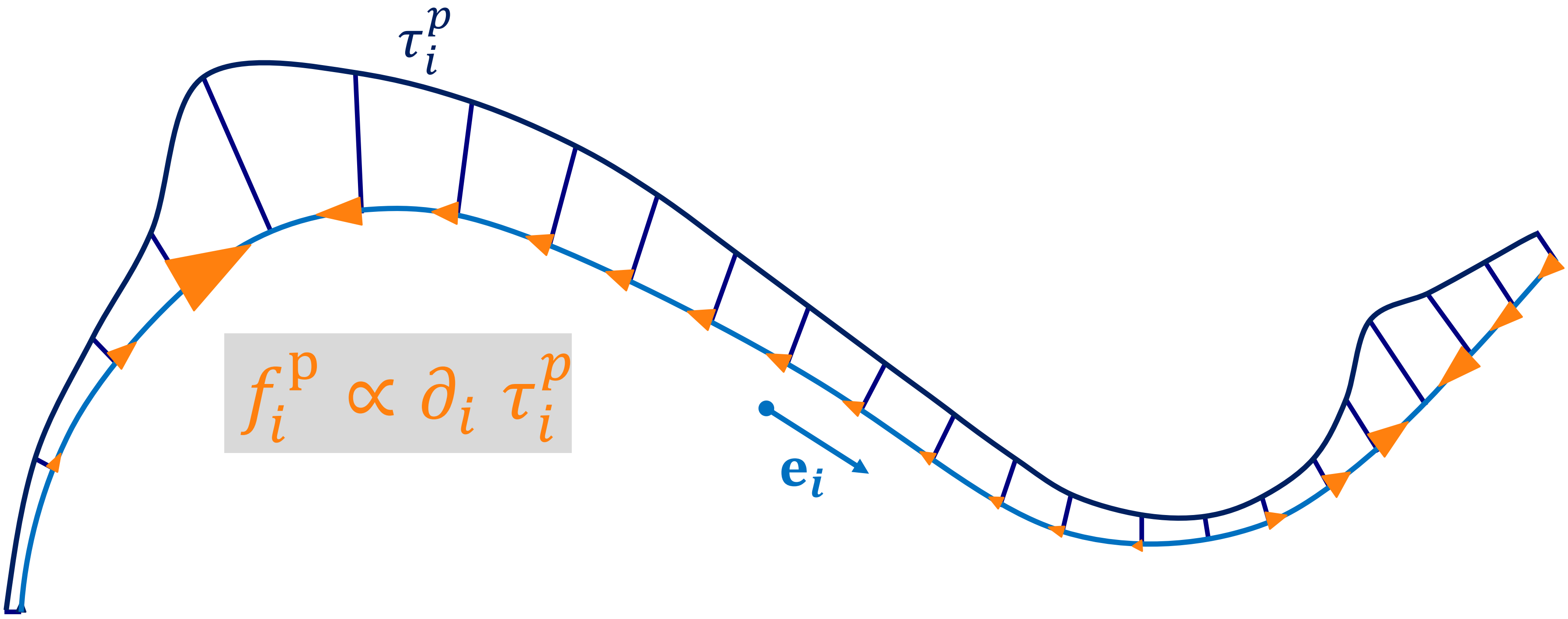
$$f_1^p = \frac{1-\beta}{\mathrm{Re}}\left[\partial_1 \tau_1^p - \kappa_2 N_1\right]$$

$$f_2^p = \frac{1-\beta}{\mathrm{Re}}\left[\partial_2 \tau_2^p + \kappa_1 N_1\right]$$

Directional derivative along principal stress

1st normal stress difference: τ_1^p - $\tau_2^p > 0$

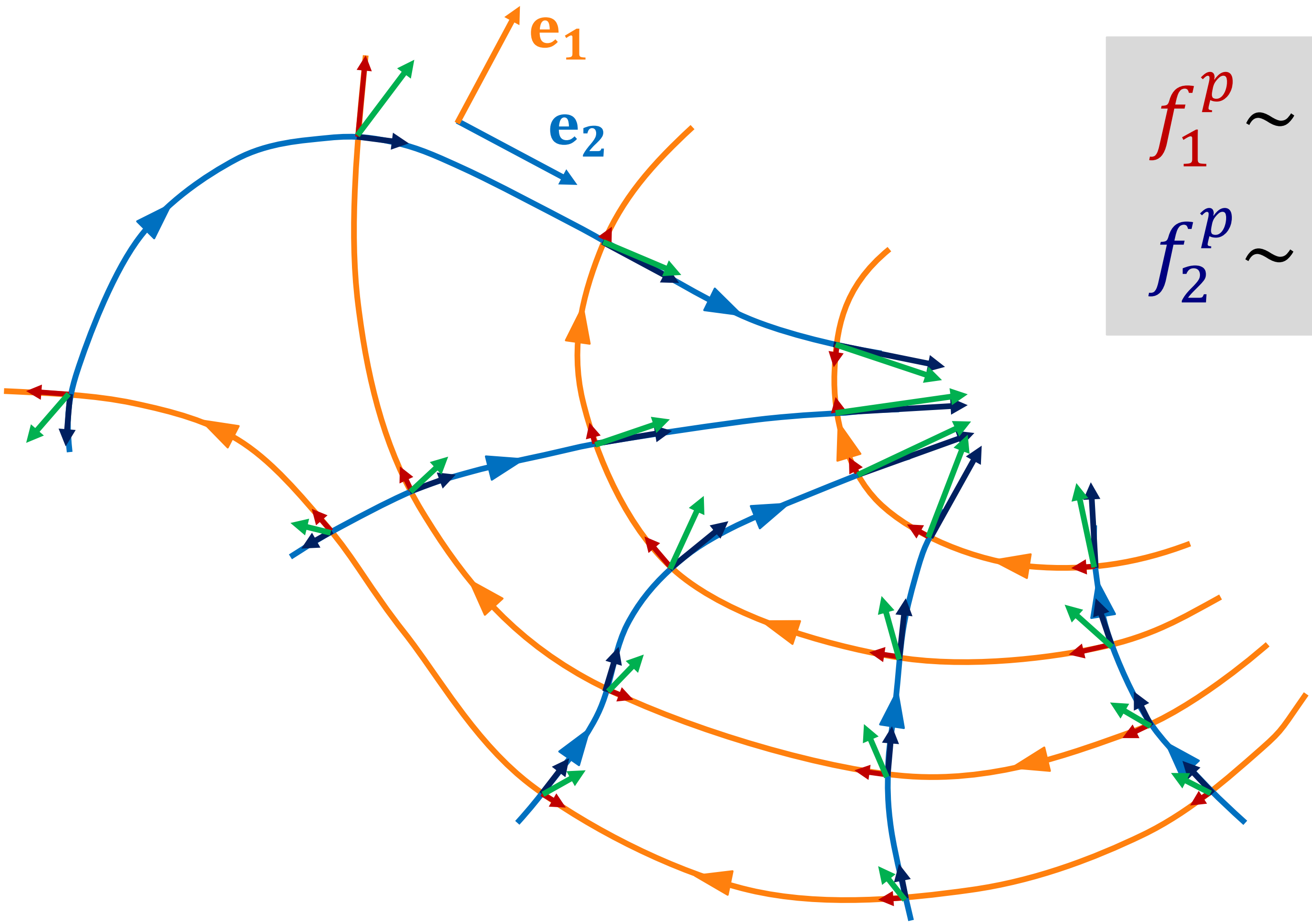
Polymer body force – Directional derivative



- Analogy with an elastic material
- Spring force differential induce forcing on the flow
- Force towards regions of high stress

Polymer body force – Curvature

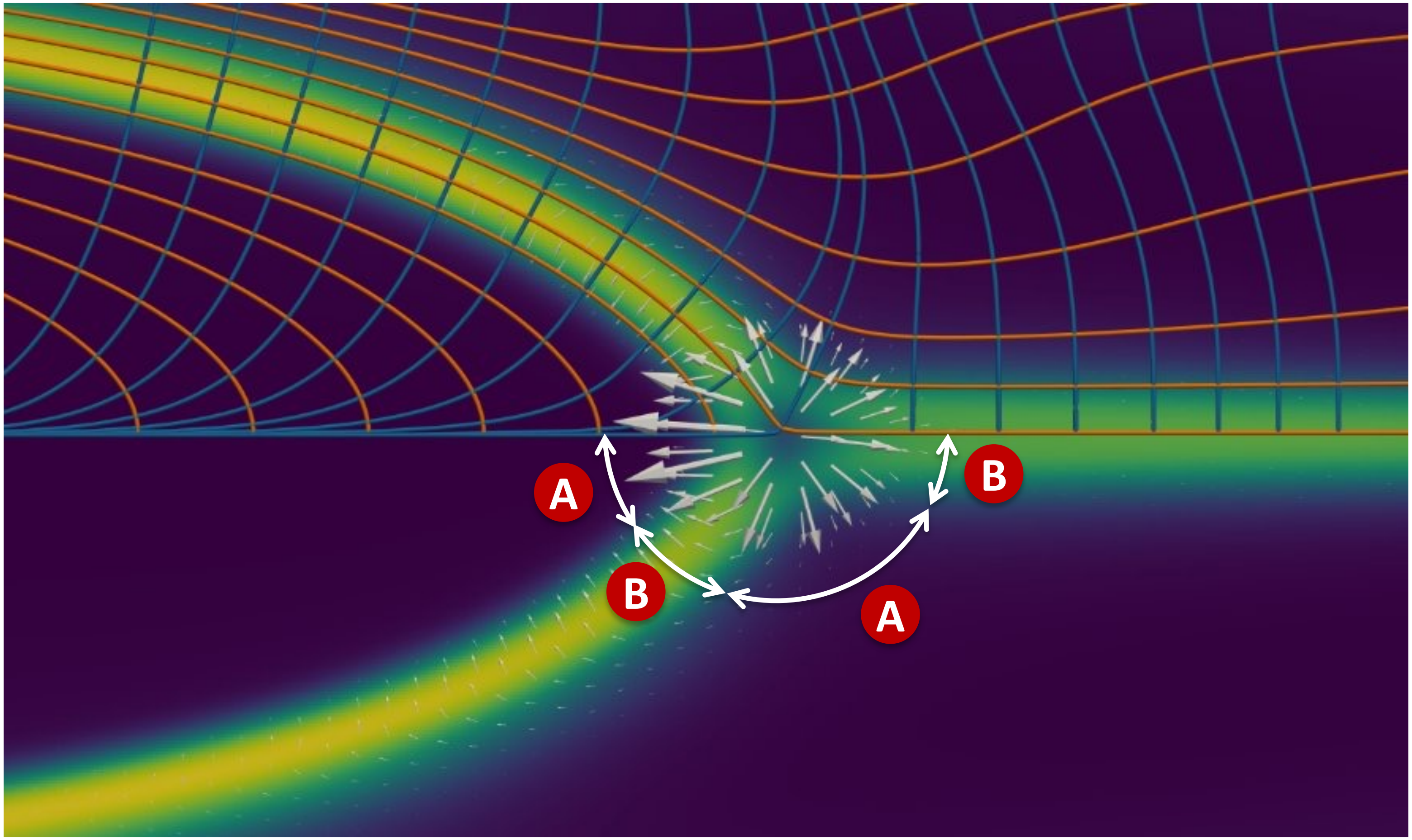
Focus on curvature ⇒ constant $\tau_1^p > \tau_2^p$



$$f_1^p \sim -\kappa_2 N_1$$

$$f_2^p \sim +\kappa_1 N_1$$

Polymer body force



A Stress line curvature

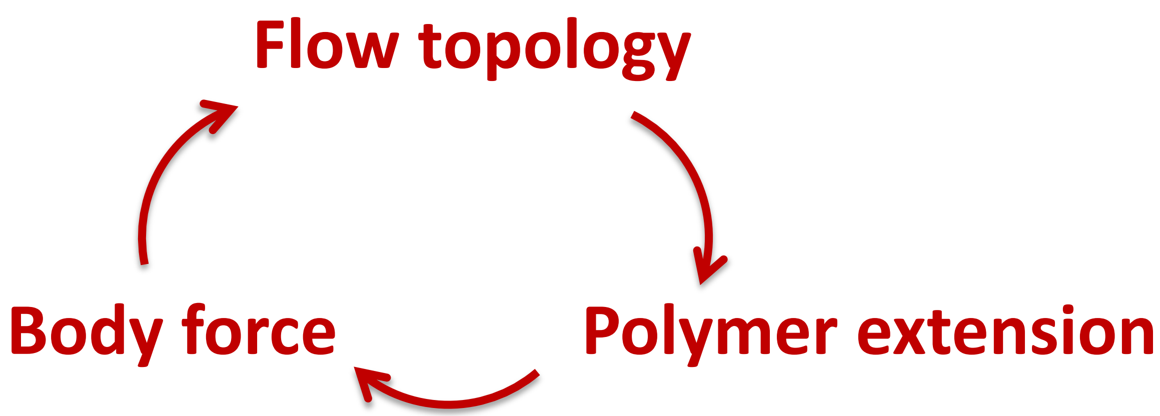
+

B Stress directional derivative

Forcing around SP

Conclusion

- **Slow convergence** to steady state
- Characteristic **flow topology** created by localized **body force** at the head of the arrowhead
- **Head** of arrowhead is key, ”legs” are a by-product
- Physically intuitive **formulation** of body force in polymer **principal axes**
- A first attempt at explaining **self-sustained cycle**



Open questions

- Does this apply to **other cases** (parameters, configurations)?
- What is the **path** to this self-sustained cycle?
- What are the **necessary ingredients** for arrowhead ("parabolic" velocity profile, localized body force…)?

  $\Rightarrow$ **Chicken and egg problem!**
- Does the steady state correspond to the **merging** of the two stagnation points?
- Effect of **periodicity**?

Acknowledgment

fnrs
FREEDOM TO RESEARCH

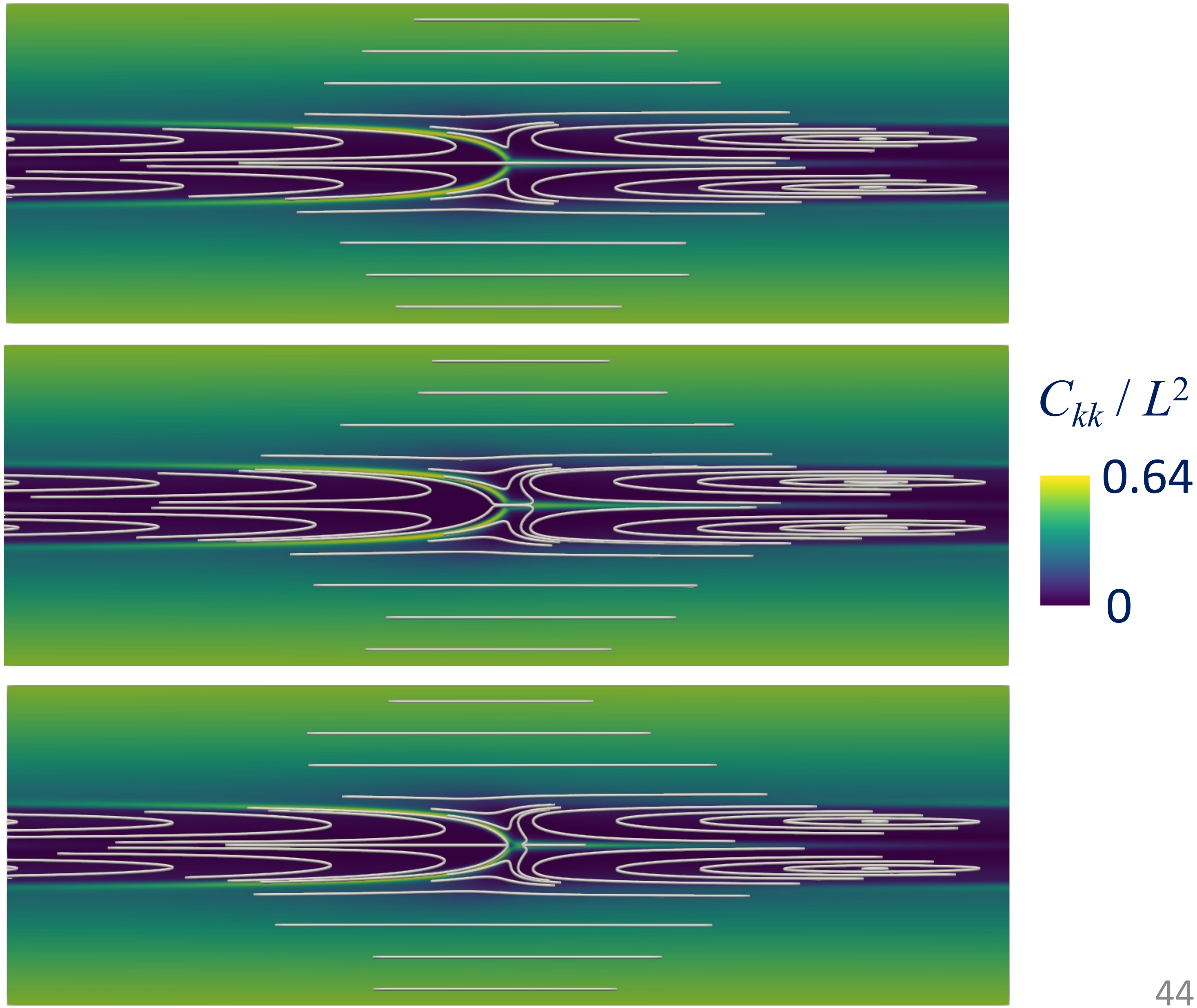
FÉDÉRATION WALLONIE-BRUXELLES

C.E.C.I

Wallonie recherche SPW

BACKUP

Convergence of flow topology

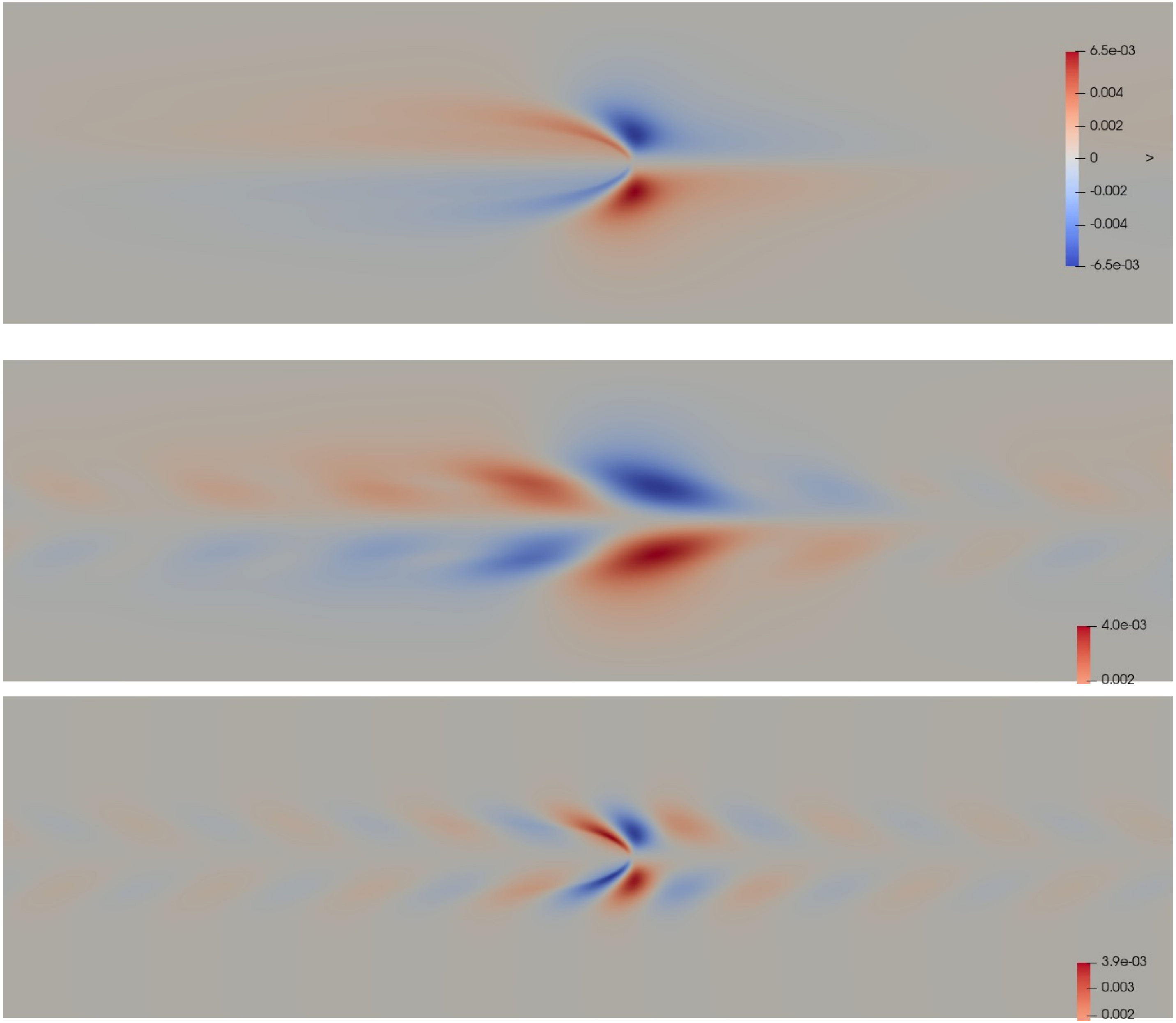


$u_{\mathrm{SAR}} = 1.4598$

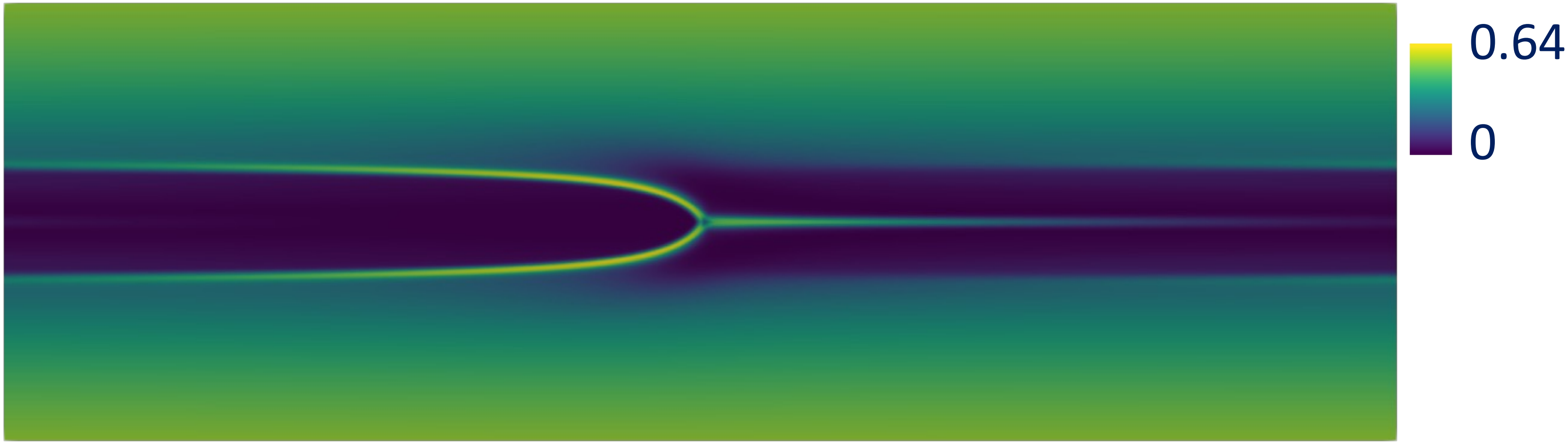
$u_{\mathrm{SAR}} = 1.4583$

$u_{\mathrm{SAR}} = 1.4568$

Spectral filtering



Approach – Configuration

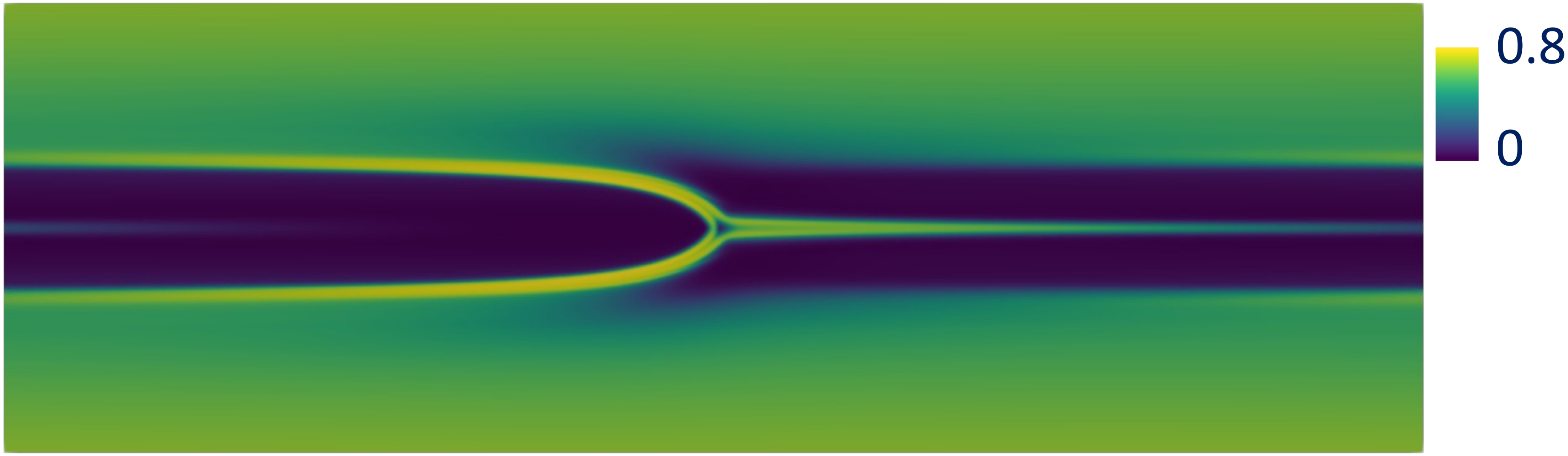


$\mathrm{tr}(\boldsymbol{C})/L^2$

$\mathrm{Re} = 1000$
$\mathrm{Wi} = 50$
$L_x = 2\pi$
$\beta = 0.9$
$L = 90$
$\mathrm{Sc} = 500$

- Viscoelastic 2D periodic channel flow
- FENE-P
- Solution symmetry not used (stabilizing agent)

Approach – Configuration

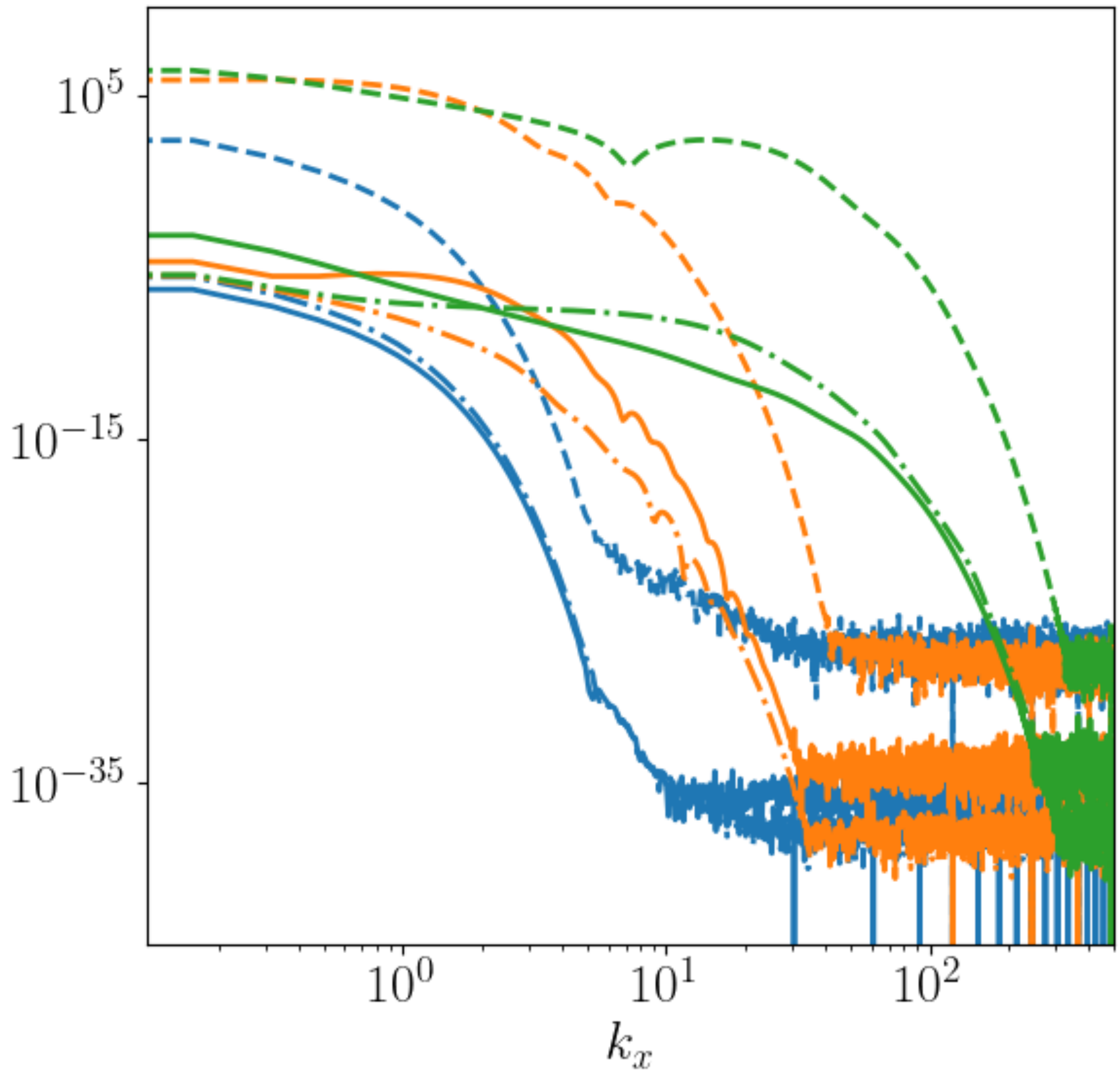


$\mathrm{tr}(\boldsymbol{C})/L^2$

$\mathrm{Re} = 1000$
$\mathrm{Wi} = 100$
$L_x = 2\pi$
$\beta = 0.9$
$L = 90$
$\mathrm{Sc} = 500$

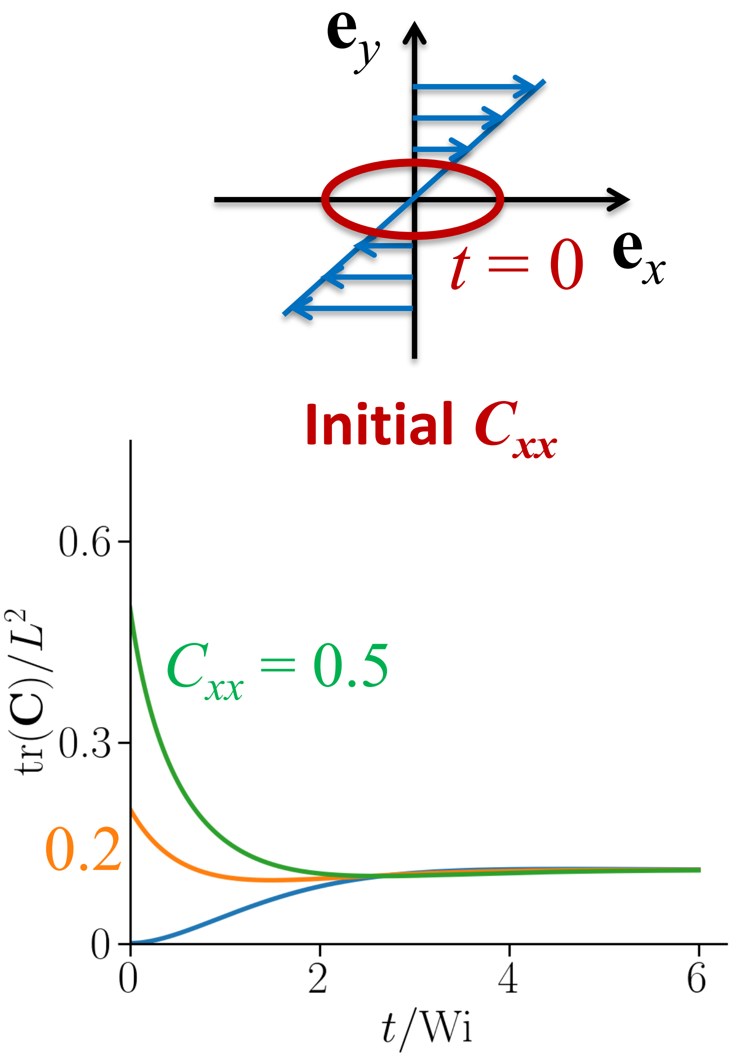
- Viscoelastic 2D periodic channel flow
- FENE-P
- Solution symmetry not used (stabilizing agent)

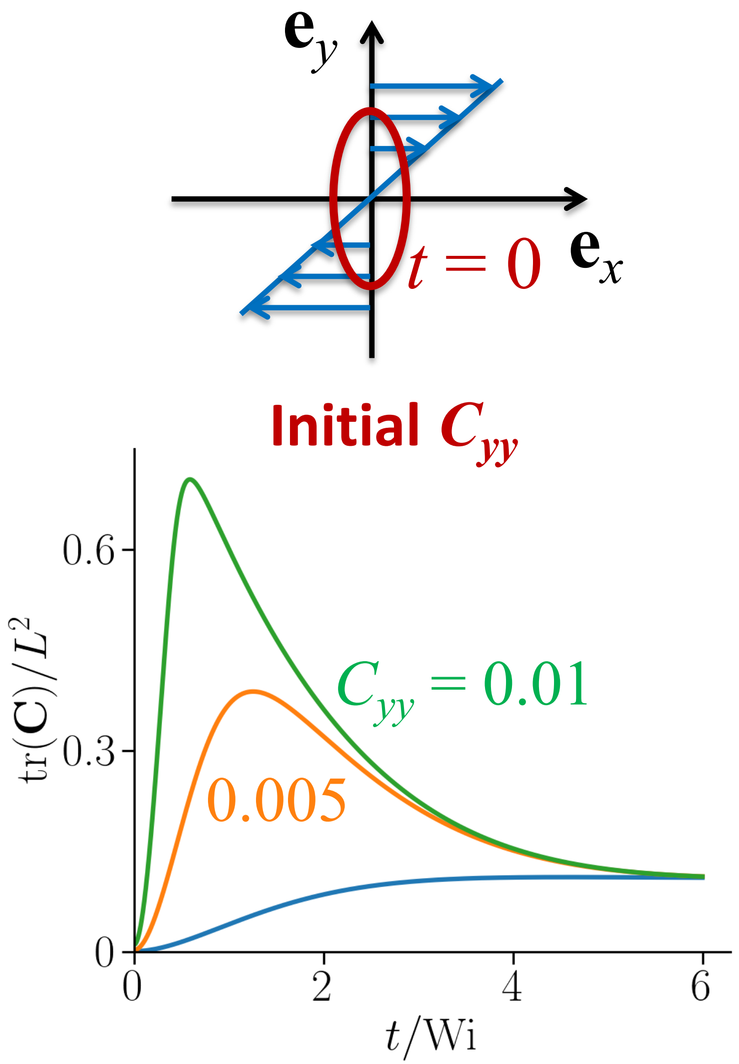
Solution description – Spatial accuracy



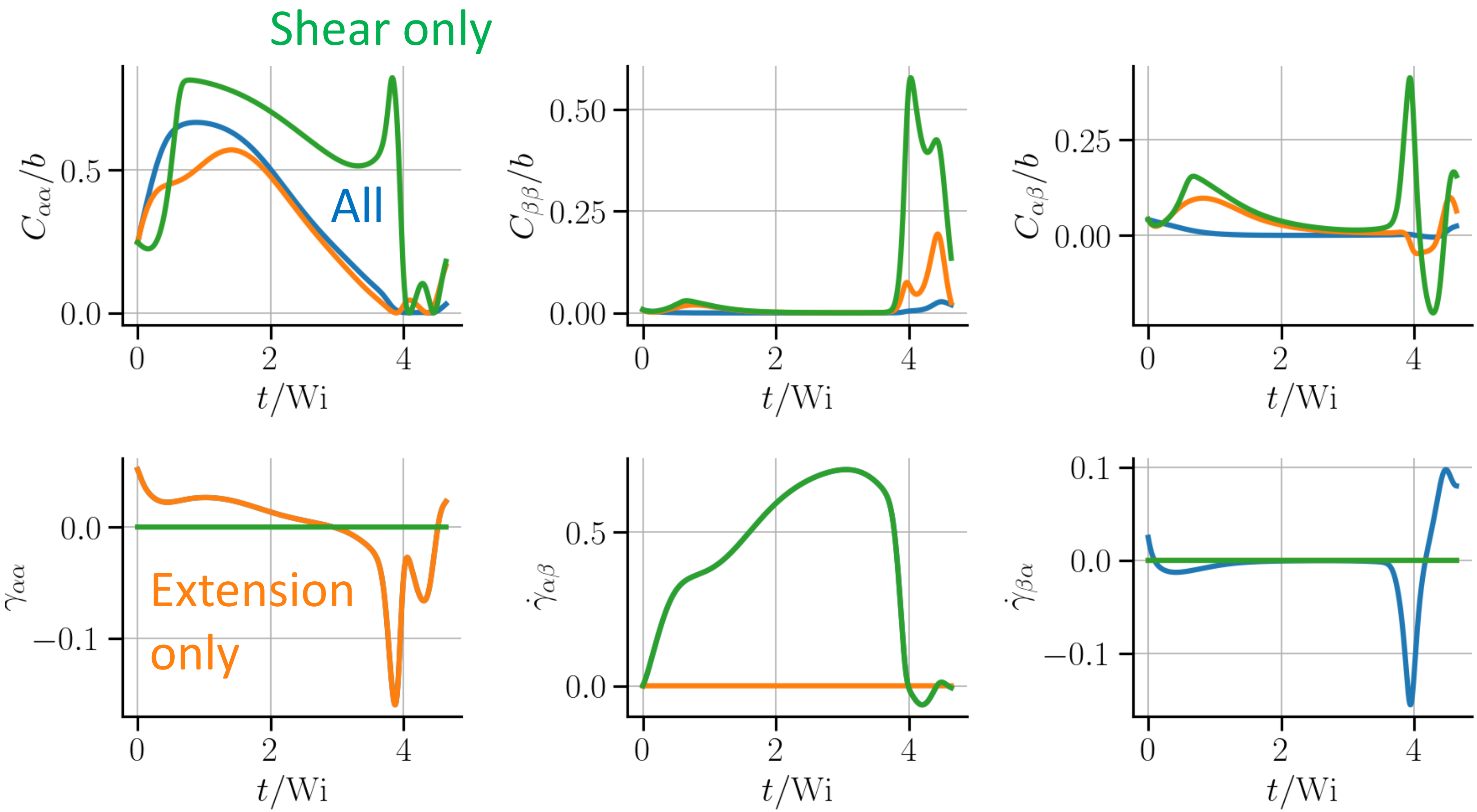
Small scales fully resolved at finite Sc = 500 (Pe = 5×10^{5})

Transient overshoot

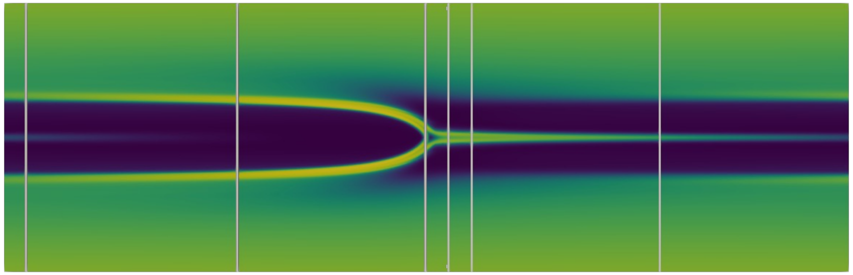


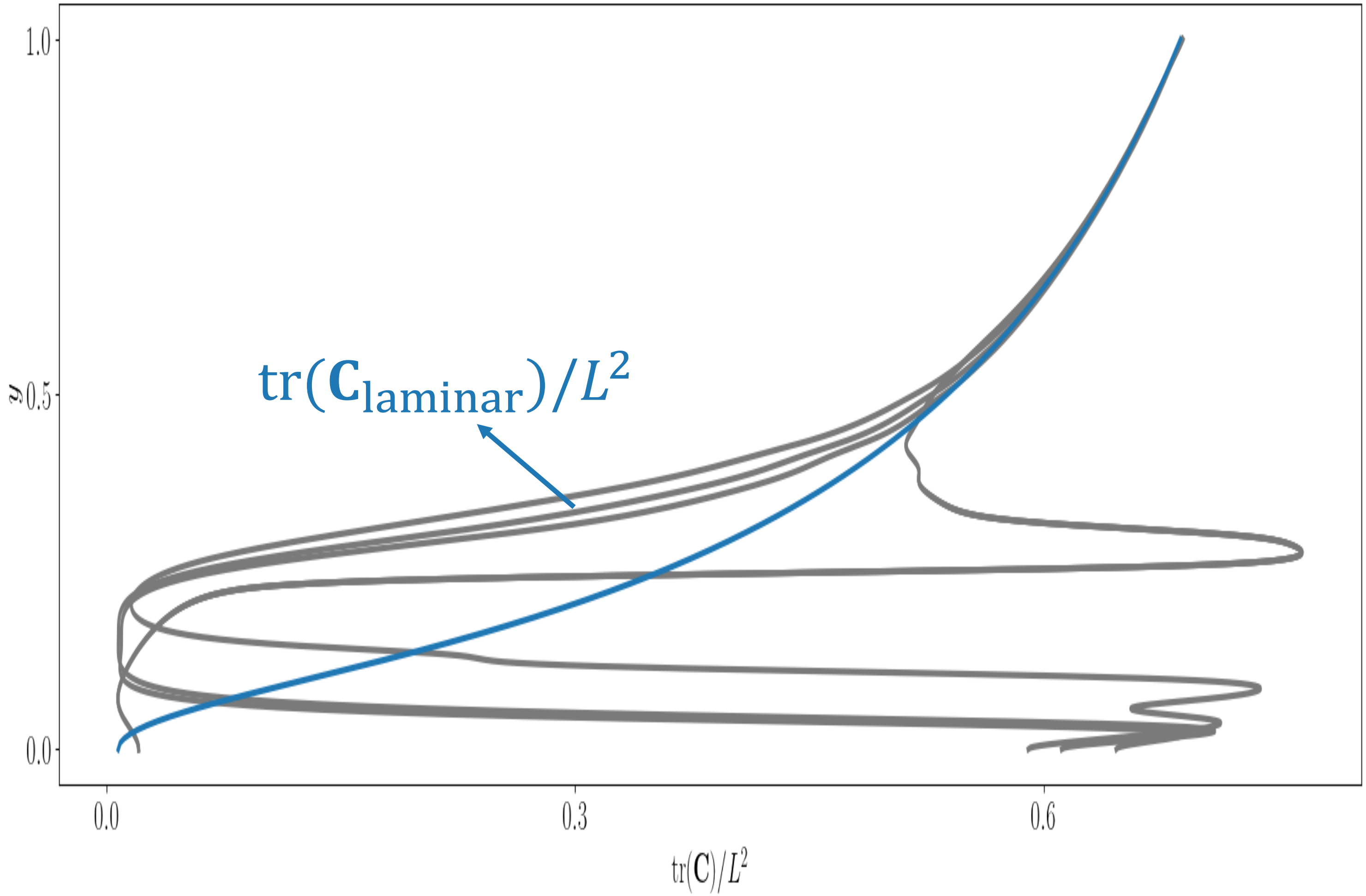


Transient overshoot



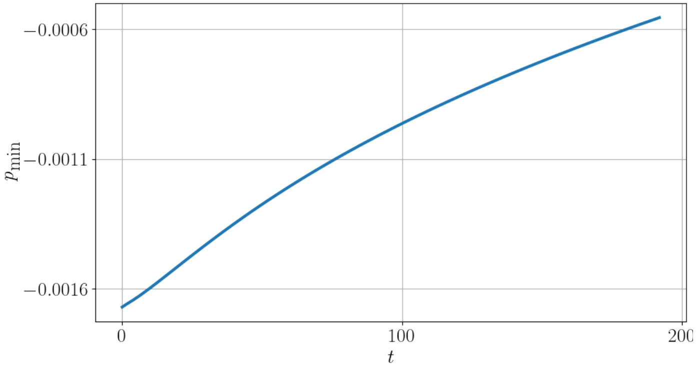
Wi=100



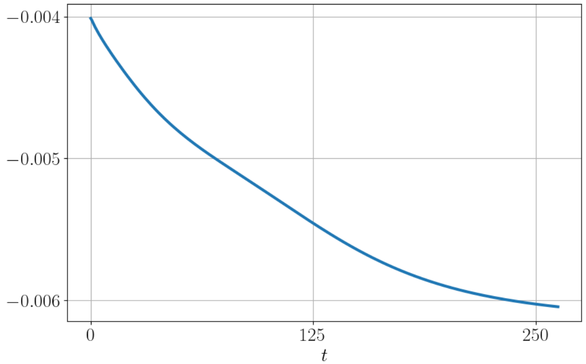


Convergence of minimum pressure

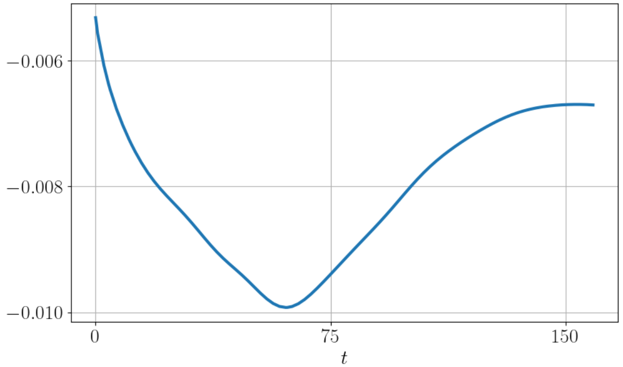
Wi = 30



Wi = 50



Wi = 100



Mean kinetic and mean elastic energy

Mean kinetic energy

$$\bar{e}_k = \frac{1}{TLxLy} \int_0^T \int_0^{Lx} \int_0^{Ly} \frac{u_i u_i}{2} \, dx \, dy$$

Mean elastic energy

$$\bar{e}_p = \frac{1}{TLxLy} \int_0^T \int_0^{Lx} \int_0^{Ly} -\frac{1-\beta}{2ReWi} L^2 \ln(1 - C_{kk}/L^2) \, dx \, dy$$

Reference scales

Taylor microscale

$$\lambda_T = \left(\frac{\overline{u_1'^2}}{\overline{\partial_1 u_1'^2}} \right)^{\frac{1}{2}}$$

initial curvature of velocity correlation function

Kolmogorov scale

$$\eta_K = \left(\frac{\nu^3}{\varepsilon} \right)^{\frac{1}{4}}$$

smallest structure of Newtonian turbulence

- Relevance of these scales in **non-Newtonian** turbulence unclear
- Here just a **reference** frame to our discussion