Phase Reconfiguration for Power Distribution Networks with High DERs Penetration

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***Abstract*—Power distribution networks are designed to operate as balanced three-phase systems, but achieving and maintaining network balance is challenging, particularly with the increasing adoption of distributed energy resources (DERs) such as photovoltaic panels (PVs), electric vehicles (EVs), and heat pumps (HPs). Phase unbalance introduces inefficiencies into the network, including increased energy losses and difficulties in maintaining voltage levels within acceptable limits. This paper proposes a computationally efficient methodology to optimize customer phase configurations in networks with high DER penetration. The approach focuses only on load unbalance, eliminating the need for power flow outputs and extensive smart meter (SM) installations across the network, typically required for voltage and current values. The methodology is validated using a real-world Belgian distribution network featuring high DER penetration. Results show improvements with a 46% reduction in load unbalance, improved voltage profiles, and a 12% decrease in line losses.**

***Index Terms*—Power Distribution Systems, Phase Balancing, Distributed Energy Resources, Network Optimization**

I. Introduction

The massive integration of distributed energy resources (DERs) into power grids, including photovoltaic panels (PVs), electric vehicles (EVs), and heat pumps (HPs), poses significant challenges for distribution system operators (DSOs). Among these challenges, managing network phase unbalance has become increasingly important for ensuring stable and efficient grid operation. Phase unbalance arises when the loads across the phases of a network are not evenly distributed, leading to unequal voltage and current between the phases. Both issues can accelerate equipment aging, increase energy losses, and raise operational costs for the DSOs.

Phase unbalance in power distribution networks is influenced by their structural characteristics. Factors such as asymmetrical feeder layouts, differences in line lengths, and impedance often create unbalances. The unbalanced issue is further intensified by uneven customer assignments to phases, variability in load consumption patterns, and the unpredictable behavior of distributed energy resources (DERs).

There are three main types of solutions for network unbalance: *i)* re-phasing, *ii)* phase balancers, and *iii)* active network management [1]. In general, the last two solutions require the deployment of new devices such as phase balancers, storage systems, or other controllable equipment. While effective, the additional equipment represents a significant capital investment and increases operational complexity for DSOs. In contrast, re-phasing focuses on changing customer phase configuration to balance the networks without additional physical equipment.

Re-phasing has been widely studied in the literature, with various strategies addressing the phase unbalance problem. The authors in [2] focus on re-phasing single-phase customers to improve the voltage and current unbalance while keeping the number of phase adjustments below a given number established by the DSO. Authors in [3] propose a statistical method to identify the networks that may benefit from re-phasing, considering two consumption scenarios: yearly average and day-by-day consumption. In [4] an optimization problem is built to identify the minimum number of phase connection changes in a network to meet a specific phase unbalance tolerance. In [5] authors apply a Vortex search algorithm to minimize the total power loss of several IEEE networks. The authors in [6] build a strategy to minimize the current unbalance, feeder energy-loss cost, customer-interruption cost, and labor cost. In [7] authors propose the application of devices such as soft open points (SOPs) and phase switch devices (PSDs) to re-phase the network in real-time and to improve network stability and reduce costs, including the energy curtailment costs of PVs and wind turbines (WTs).

Most studies on network re-phasing focus on short-term scenarios, typically considering only single-day balancing and limited integration of DERs. Table I provides a summary of the works in the field, categorizing them based on their approaches. Moreover, the table compares these studies with the methodology proposed in this paper, highlighting the contributions of our approach.

This work contributes to phase unbalance mitigation in power distribution networks by proposing an alternative optimization problem without relying on non-linear power flow calculation outputs. The proposed approach formulates a linear optimization problem aimed at minimizing the unbalance between customer load curves across phases while simultaneously reducing the number of required customer reconfigurations. Our methodology focuses on customer connections in single and multi-phase configurations, reflecting the structure of the DSO's network. The optimization is conducted over a year and accommodates different DER technologies such as PVs, EVs, and HPs, providing a robust a solution to manage phase unbalance.

The code and data of this work are available at *github.com/PhaseReconfiguration/PhaseReconfiguration*.

TABLE I. Key features of various phase reconfiguration methods proposed in the literature, compared to our approach.
V = Voltage, I = Current, RC = Reconfiguration count, L = Energy losses, FL = Feeder load, DI=Distance Impact

Feature | paper	[2]	[3]	[4]	[5]	[6]	[7]	ours
1/2/3-phase customer connections	1	3	1, 2	3	1, 2, 3	3	1, 2, 3
Objective function	V, I, RC	I	FL, RC	V	I, L	V, I, L	FL, RC, DI
One year optimization	○	◐	○	○	○	○	●
Real-time reconfiguration	○	○	○	○	○	●	○
DERs	∅	∅	∅	∅	∅	PVs, WTs	PVs, EVs, HPs

The rest of the paper is organized as follows. Section II introduces the mathematical formulation of power distribution networks, which serves as the foundation for our method. Section III presents the definition of the problem. Section IV shows the methodology applied to an example network. The results of the real Belgian network are reported in Section V. Section VI concludes the work and discusses possible future works. Appendix A describes the process of assigning time series data to each customer.

II. Network mathematical formalization

A. Network graph

A power distribution network can be modeled as a directed graph, denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ represents the set of nodes and $\mathcal{E}$ represents the set of directed edges. Each node corresponds to a network component, such as a transformer or a customer. Each edge $e \in \mathcal{E}$, commonly referred to as lines, establishes a connection between two nodes of the set $\mathcal{N}$.

B. Network phases

Generally, power distribution networks operate as three-phase systems, where power is distributed across three phases to ensure balanced operation and maximize efficiency. Both nodes and lines are associated with one or more phases. The set of phases in the network is denoted as Φ, with $\Phi = \{A, B, C\}$, representing the three distinct electrical phases.

We define the set of all possible phase configurations, Ψ, as the collection of all customer phase assignments.

For networks with 1-phase, 2-phase, and 3-phase connections, the set Ψ contains 15 distinct possible configurations, representing permutations of the phase without repetition. Here are included some of the possible configurations:

$$\begin{aligned}\Psi = \{&(A), (B), (C), \\ &(A,B), (A,C), (B,A), (B,C), (C,A), (C,B), \\ &(A,B,C), (A,C,B), (B,A,C), ..., (C,B,A)\}.\end{aligned} \tag{1}$$

For a given node $n \in \mathcal{N}$ or line $e \in \mathcal{E}$, their respective phase configurations are denoted as n_ψ and e_ψ, with $\psi \in \Psi$. A single phase of a configuration $\psi \in \Psi$ is denoted as $\phi \in \psi$.

C. Network feeders

In typical power distribution networks, particularly the ones designed in a radial configuration, each feeder radiates outward from a single source, such as a transformer, toward consumption points, such as customers. The set of all feeders is defined by $\mathcal{F}$, where each feeder $f \in \mathcal{F}$ is a subgraph of $\mathcal{G}$, $f \subset \mathcal{G}$.

Every node and line in the network belongs to a specific feeder. A node of a given feeder is denoted as $n \in \mathcal{N}_f$ with $f \in \mathcal{F}$ and $\mathcal{N}_f \subset \mathcal{N}$. Given a feeder $f \in \mathcal{F}$ and a customer in that feeder $c \in \mathcal{C}_f$, the phase configuration of the customer is indicated as c_ψ with $\psi \in \Psi$.

D. Phase matrices

Some matrices are used to represent the initial, feasible, and track the phase configurations of the customers in the network.

1) Initial configuration matrix: the initial phase configuration of each customer is represented by the matrix $\mathbf{B}^{init} \in \{0,1\}^{|\mathcal{C}|\times|\Psi|}$, where a value of 1 indicates a connection to a specific phase configuration, and 0 indicates no connection. An example of the matrix $\mathbf{B}^{init}$ is given hereafter:

$$\mathbf{B}^{init}_{example} = \begin{array}{c} \\ c_1 \\ \vdots \\ c_i \\ \vdots \\ c_{|\mathcal{C}|} \end{array} \begin{array}{c} \begin{array}{cccccc} \psi_1 & \psi_2 & \cdots & \psi_{10} & \cdots & \psi_{|\Psi|} \end{array} \\ \begin{pmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & \cdots & 0 \end{pmatrix} \end{array}. \tag{2}$$

Equation 2 shows how each customer $c \in \mathcal{C}$ is connected to a particular phase configuration $\psi \in \Psi$. For example, customer c_1 is connected to a single phase A (configuration $\psi_1 \in \Psi$), while the last customer, $c_{|\mathcal{C}|}$, is connected to the three-phase configuration (A, B, C) (configuration $\psi_{10} \in \Psi$).

2) Complement configuration matrix: we introduce the matrix $\overline{\mathbf{B}}^{init}$ as the complement of the matrix $\mathbf{B}^{init}$ where each value is inverted. This is expressed as:

$$\overline{\mathbf{B}}^{init} = \mathbb{1} - \mathbf{B}^{init}, \quad (3)$$

where $\mathbb{1}$ is a matrix of ones with the same dimensions as $\mathbf{B}^{init}$. The operation results in swapping the values of 0 and 1 in $\mathbf{B}^{init}$. The matrix $\overline{\mathbf{B}}^{init}$ is used later in Eq. 10 to count the number of reconfigurations of customer phases.

3) Solution configuration matrix: DSOs can change the phase configuration of each customer to achieve a more balanced network. The matrix $\mathbf{B}$ represents the final phase configuration matrix obtained after solving the optimization problem, as described in Section III. Each element of $\mathbf{B}$ indicates the assigned phase configuration for a customer.

4) Feasible configuration matrix: not all phase configuration changes are allowed, as there are constraints on customer connections. Specifically, a customer must maintain a configuration with the same number of phases as their initial setup, meaning a customer assigned to a single-phase connection cannot be switched to a multi-phase configuration, and vice versa. For instance, a customer connected to the single phase A (e.g., $\mathbf{B}^{init}_{c_1,\psi_1} = 1$) can therefore only be switched to another single phase, but cannot be connected to a multi-phase configuration, such as (A, B, C).
The matrix $\mathbf{B}^{feas}$ represents the set of feasible phase configuration changes. Considering the example matrix $\mathbf{B}^{init}_{example}$ in Eq. 2, $\mathbf{B}^{feas}_{example}$ is given as follows:

$$\mathbf{B}^{feas}_{example} = \begin{array}{c} \\ c_1 \\ \vdots \\ c_{|\mathcal{C}|} \end{array} \begin{array}{c} \begin{array}{cccccc} \psi_1 & \psi_2 & \psi_3 & \psi_4 & \cdots & \psi_{|\Psi|} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \end{array}. \quad (4)$$

Equation 4 shows the matrix $\mathbf{B}^{feas}$, with a focus on the customer c_1. Since this customer has a one-phase connection, the feasible configurations are only A, B or C ($\mathbf{B}^{feas}_{c_1,\psi_1} = \mathbf{B}^{feas}_{c_1,\psi_2} = \mathbf{B}^{feas}_{c_1,\psi_3} = 1$). For all other phase configurations, the corresponding values are 0.

E. Customer power dynamics

The DSO evaluates the behavior of the network over a finite number of time steps. The set of all time steps is denoted as $\mathcal{T}$, and a single time step is denoted as t, with $t \in \mathcal{T}$.
At any time step t, customers can consume or produce a given amount of power. The term $P^{+}_{\phi,c,t}$ represents the power consumed on the phase $\phi \in c_\psi$ by the customer $c \in \mathcal{C}_f$ of the feeder $f \in \mathcal{F}$ during the time step $t \in \mathcal{T}$. This power consumption includes miscellaneous loads, EVs, and HPs consumption (with $P^{+}_{\phi,c,t} \geq 0$). Similarly, $P^{-}_{\phi,c,t}$ represents the power produced by PVs (with $P^{-}_{\phi,c,t} \leq 0$). The total power for each phase of each customer at any given time step is calculated as the sum of the power consumed and the power produced, expressed as:

$$P_{\phi,c,t} = P^{+}_{\phi,c,t} + P^{-}_{\phi,c,t}. \quad (5)$$

The matrix $P_{\phi,c,t}$ captures the net power flow for each customer, phase, and time step, providing a representation of the network dynamic behavior. The values in the matrix can be positive or negative, depending on whether consumption exceeds production or vice versa.

III. Problem Statement

The DSO aims to determine optimal customer phase configurations in the distribution network, balancing consumption across phases while minimizing the associated reconfiguration costs. The optimization problem is defined as follows:

$$\min_{\mathbf{B}} \; \lambda^{P} \cdot P^{unb} + \lambda^{\Psi} \cdot \Psi^{chg} + \lambda^{D} \cdot D^{wgt} \quad (6a)$$

$s.t.$

$$\sum_{\psi}^{\Psi} \mathbf{B}_{c,\psi} = 1, \quad \forall c \in \mathcal{C} \quad (6b)$$

$$\mathbf{B}_{c,\psi} \leq \mathbf{B}^{feas}_{c,\psi}, \quad \forall c \in \mathcal{C}, \forall \psi \in \Psi \quad (6c)$$

$$\mathbf{B}_{c,\psi} \in \{0,1\}, \quad \forall c \in \mathcal{C}, \forall \psi \in \Psi \quad (6d)$$

A. Objective terms

- **Phase unbalance impact**: represents the sum over time of the deviations of the load in each phase from the average load across the three phases. Minimizing the term in Eq. 7 aims to achieve a more balanced load distribution across the phases of each feeder.

$$P^{unb} = \sum_{t}^{\mathcal{T}} \sum_{f}^{\mathcal{F}} \sum_{\phi}^{\Phi} |\mathcal{A}_{\phi,f,t} - \mu^{\mathcal{A}}_{f,t}| \quad (7)$$

  where:
  - $\mathcal{A}_{\phi,f,t}$ represents the aggregate load of the customers connected to phase ϕ of feeder f at time t:

$$\mathcal{A}_{\phi,f,t} = \sum_{c}^{\mathcal{C}_f} P_{\phi,c,t} \quad (8)$$

  - and $\mu^{\mathcal{A}}_{f,t}$ is the average load across phases of feeder f at time t:

$$\mu^{\mathcal{A}}_{f,t} = \frac{\sum_{\phi}^{\Phi} \mathcal{A}_{\phi,f,t}}{|\Phi|}. \quad (9)$$

- **Reconfiguration impact**: counts the number of customer phase reconfigurations:

$$\Psi^{chg} = \sum_{c}^{\mathcal{C}} \sum_{\psi}^{\Psi} \mathbf{B}_{c,\psi} \cdot \overline{\mathbf{B}}^{init}_{c,\psi}. \quad (10)$$

- **Customer distance impact**: considers the influence of customer distance from the transformer on the network performance:

$$D^{wgt} = \sum_{c}^{\mathcal{C}} \frac{1}{c_{dist}} \cdot \sum_{\psi}^{\Psi} \mathbf{B}_{c,\psi} \cdot \overline{\mathbf{B}}_{c,\psi}^{init} \tag{11}$$

where c_{dist} represents the distance of the customer c from the transformer, with $c_{dist} > 0$.
Customers farther from the transformer have a greater impact on network performance due to higher voltage drops and line losses. Maximizing Eq. 11 prioritizes configuration changes that have the most significant impact on the network, typically those located farther away from the transformer, often at the end of the feeder.

B. Scaling factors

The factors λ^P, λ^Ψ, and λ^D are scaling parameters:

- λ^P reflects the cost of phase unbalance, including technical losses and operational inefficiencies caused by uneven load distribution between the phases.
- λ^Ψ represents operational costs associated with changing customer phase configurations, including labor and potential downtime.
- λ^D represents the importance of reconfigurations for customers farther from the transformer, where changes have a greater impact on the network due to voltage drops and line losses.

C. Constraints

The objective function in Eq. 6 is constrained by Eqs. 6b and 6c, which guarantee the feasibility of the solution.

- **Unique configuration assignment (Eq. 6b)**: ensures each customer is assigned to one and only one phase configuration.
- **Feasibility (Eq. 6c)**: restricts phase changes to feasible configurations, maintaining the original number of phases for each customer.

IV. Conceptual Example

This section illustrates the methodology for customer phase reconfiguration using a simplified conceptual example of a power distribution network. The example demonstrates the impact of phase configuration optimization on network unbalance.

A. Network setup

The example network consists of a 3-phase system with main lines configured for 3-phase connections, while customer connections may be single-phase or multi-phase.
Figure 1 provides a visualization of the network and the phase configurations. The primary components include the transformer, feeders, lines, and customer nodes.
Each customer is initially assigned a phase configuration, shown as blocks of different colors. The height of each block corresponds to the phase load relative to its total capacity.

Moreover, customers can install DER technologies, including PVs, EVs, and HPs. The penetration rates for these DERs are as follows:

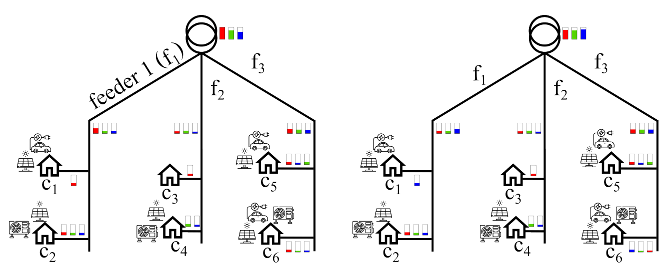
- 83% for PVs (5 out of 6 customers in Fig. 1),
- 50% for EVs, and
- 50% for HPs.

Time series for PVs, EVs, and HPs are assigned following the methodology explained in Appendix A.

B. Optimization process

Figure 1 visualizes the network's phase configuration both before (Fig. 1a) and after (1b) applying the proposed optimization. The initial configuration, $\mathbf{B}_{example}^{init}$, is presented in Eq. 2, with the feasible configurations specified in Eq. 4.

In this scenario, the DSO identified voltage issues on the A phase of the transformer, primarily due to overloading. To address these issues, an optimization process is used to mitigate phase unbalance, following the methodology described in Section III.
The optimization algorithm adjusts customer phase configurations to reduce stress on the overloaded A phase, leading to a more balanced load distribution among the three phases. For instance, Fig. 1b shows that the configurations for customers c_1 and c_6 were updated from A and (A, B, C) to C and (C, B, A), respectively.



(a) Before optimization (b) After optimization

Fig. 1. Phase configurations before and after rebalancing. Each color represents a phase: A, B and C.

The resulting solution matrix, $\mathbf{B}_{example}$, is given in Eq. 12. This matrix represents the final configuration, reducing voltage stress on the A phase and achieving a more balanced load across all phases.

$$\mathbf{B}_{example} = \begin{array}{c} \\ c_1 \\ \vdots \\ c_3 \\ \vdots \\ c_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} \psi_1 & \psi_2 & \psi_3 & \cdots & \psi_{10} & \cdots & \psi_{|\Psi|} \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix} \end{array} \quad (12)$$

V. Case study results

The test network used for this study is a digital representation of a Belgian distribution network [8]. It is a 3-phase network, featuring 3-phase connections in main lines, with customer connections being either single-phase or multi-phase.

A specific test scenario is designed to evaluate the performance of the proposed method and to demonstrate the impact of the methodology. In this scenario, the penetration rates for different DERs are set as follows: 85% for PVs, 75% for EVs, and 70% for HPs. Time series for PVs, EVs, and HPs are assigned following the methodology explained in Appendix A.

Table II shows some details about the considered network and scenario.

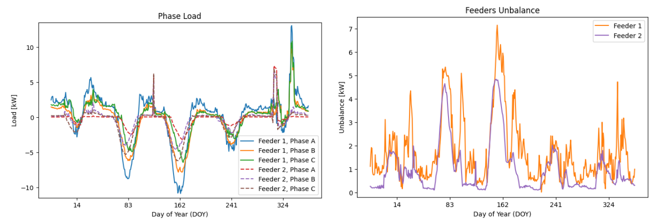
TABLE II. Description of the considered low-voltage distribution network.

Category	Number of elements
transformer	1
feeders	2
nodes	44
lines	42
customers	23
customers with PV	20
customers with EV	17
customers with HP	16

The optimization is performed over one year with a 15-minute resolution time series, therefore $|\mathcal{T}| = 35040$. For visualization purposes, data from only five days are selected for the figures. These days correspond to the 15th, 40th, 162nd, 241st, and 324th days of the year (DOY).

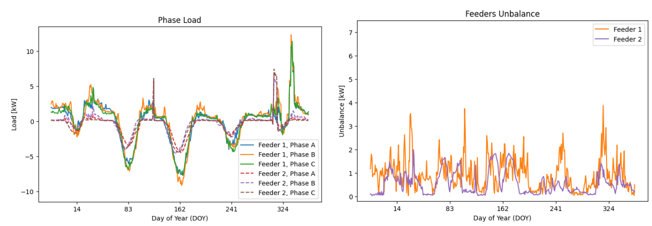
Figure 2 shows the load unbalances for each feeder and phase before the optimization. Figure 2a presents the load distribution across feeders and phases, while Fig. 2b provides a view of the unbalances for each feeder. The unbalances are calculated using Eq. 7. Load peaks can be observed, including high peaks primarily caused by EV charging and HP usage during periods of low PV production, and low peaks resulting during periods of low consumption combined with high PV production.

Figure 3 displays the load unbalances in the network after optimization. Specifically, Fig. 3a shows that the load curves across the phases of each feeder are now much closer to the mean, significantly reducing the deviations observed before optimization.



(a) (b)

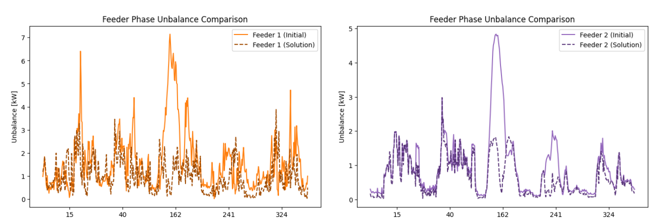
Fig. 2. Feeder load distribution and unbalances before optimization.



(a) (b)

Fig. 3. Feeder load distribution and unbalances after optimization.

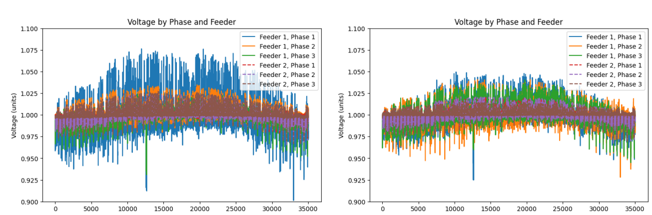
Figure 4 compares the unbalance in the network before and after the optimization for the two feeders. It is possible to see that in both feeders the unbalance was reduced. In particular, the load unbalances were decreased by 46% for feeder 1 and 44% for feeder 2, achieved with 7 and 1 configuration changes, respectively.



(a) (b)

Fig. 4. Load distribution and unbalances across feeders.

A power flow is executed before and after the optimization to test whether reducing the load unbalance affects the voltage and losses in the network.



(a) (b)

Fig. 5. Voltage curves of the phases across feeders.

Table III presents the results of the power flow analysis before and after the optimization. The results show that

TABLE III. Voltage and line losses comparison before and after the optimization.

Feeder	Phase	Voltage max [p.u.]		Voltage min [p.u.]		Line Losses [kW]	
		before	after	before	after	before	after
1	A	1.0763	1.0495	0.9011	0.9247	3926	2170
	B	1.0347	1.0408	0.9611	0.9279	951	2170
	C	1.0241	1.0404	0.9310	0.9448	1068	1501
2	A	1.0104	1.0200	0.9732	0.9698	136	362
	B	1.0238	1.0208	0.9653	0.9667	519	404
	C	1.0273	1.0161	0.9657	0.9695	516	309

minimizing the load curve unbalance has a positive effect on the network's voltage profile and line losses. For feeder 1, the found customer configurations improve voltage conditions in phase A, which was overloaded before optimization. The maximum voltage decreases from 1.0763 p.u. to 1.0495 p.u. bringing them inside the acceptable limits of 1.05, while the minimum voltage increases from 0.9011 p.u. to 0.9247 p.u. However, a side effect is observed in phase B of the same feeder, where the minimum voltage increases from 0.9611 p.u. to 0.9279 p.u. In terms of line losses, the optimization reduces the total losses across the network by approximately 12%. However, some phases experience slight increases in losses (for example, phase C in feeder 1) as a trade-off for achieving better overall balance and operational efficiency.

VI. Conclusion

Phase imbalance in electrical distribution networks can lead to inefficiencies, increased losses, and operational challenges for distribution system operators (DSOs), especially in networks with high penetration of distributed energy resources (DERs). Addressing these issues by reconfiguring customer phase connections offers a possible solution to improve network performance.

In this work, we presented an optimization problem for addressing phase unbalance in power distribution networks by reconfiguring customer phase connections without relying on power flow outputs. While detailed power flow-based optimization could provide highly accurate solutions, the computational burden of solving such an optimization for a year-long horizon with 15 minutes resolution makes it mostly impractical. This paper proposes a computationally efficient alternative that delivers robust solutions to improve network conditions. The optimization reduced phase unbalance by 46% in feeder 1 and 44% in feeder 2, with a total of 8 customer reconfigurations. Voltage profiles improved particularly in overloaded phases, bringing maximum voltages within acceptable limits. However, a slight worsening in voltage and losses was observed in some phases, highlighting the trade-offs inherent in phase balancing.

While the re-phasing solution does not require significant upfront investments, it is not entirely cost-free, as there may be operational expenses involved in reconfiguring customer connections, such as labor cost. Additionally, it is a temporary solution that may need to be repeated periodically as customer demand patterns evolve over the years. This makes it potentially less efficient than more permanent solutions like phase balancers or active network management. Therefore, future work could explore options combining re-phasing strategies and strategies requiring more significant investments.

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APPENDIX

A. Time series and phases assignation

In this section, we provide details on the implementation of the optimization problem introduced in Section III. In particular, this section explains how time series (TS) data for load, PV generation, EV charging, and HP usage are assigned to each customer.

Different information is available for each customer. In particular, each customer's total annual consumption, household size, and the number of phases they are connected to are known. For customers equipped with SMs, additional real-time data is available, including:

- TS information on the customer's load;
- the presence of PVs;
- TS generation profiles for customers with PV installations.

For customers without SM, detailed information on their connections and consumption/generation is generally unavailable to the DSOs. In such cases, assumptions are made regarding phase configurations and TS curves for load, PVs, EVs, and HPs when the information is missing. The assumption process is summarized in Fig. 6.

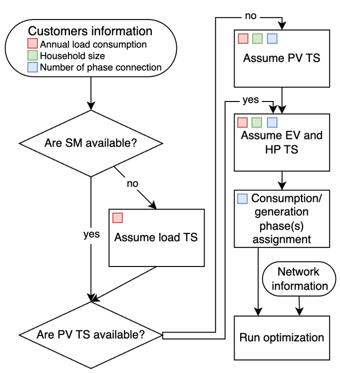

Fig. 6. Flowchart of data availability and assumption processes.

The assumptions are primarily driven by the annual customer consumption, household size, and the number of phases the customer is connected to, as represented by the red, green, and blue squares in the flowchart.

The installation size of the technologies can be different for the customers. In particular, for each technology, there are two main sizes, summarized in Table IV.

The score for a single customer $c \in \mathcal{C}$, considered in Table IV, is calculated as follows:

$$score_c = \frac{c_{\text{hh\_size}}}{\mu_{\text{hh\_sizes}}} + \frac{|c_\psi|}{\mu_\Phi} \tag{13}$$

where:

- $c_{\text{hh\_size}}$ is the customer household size;
- $\mu_{\text{hh\_sizes}}$ is the average of the household group sizes;
- $|c_\psi|$ represents the number of phases the customer is connected to;
- μ_Φ represents the average number of phases available.

The score evaluates how likely a customer is to install a technology of a given size.

TABLE IV. Installation sizes and requirements for technology installations

Technologies	Small installation		Large installation	
	Size	Requirements	Size	Requirements
PVs	2 kWp	$score_c \leq 3$ & $\lvert c_\psi \rvert \leq 2$	18 kWp	$score_c > 3$ & $\lvert c_\psi \rvert > 2$
EVs	3 kW		15 kW	
HPs	3.5 kW		13 kW	

After collecting (or assuming) the TS for each customer, a phase configuration is assigned to those without SM. Figure 7 illustrates the methodology used to assign consumption and production for each technology (load, PVs, EVs, and HPs) to the phase(s).

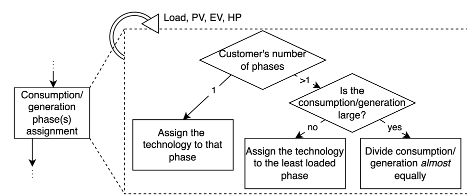

Fig. 7. Phase configuration assignment methodology.

The process for phase assignment follows these steps:

- **Single-phase configuration**: If the customer is connected to only one phase, all technology consumption and generation are assigned to that phase.
- **Multi-phase configuration**: If the customer is connected to more than one phase, the assignment process depends on the size of the installation:
  - If the consumption or generation is large, the load, or generation, is divided *almost equally* across the three phases where *almost equal* means that the phase distribution is based on a random variable drawn from a normal distribution.
    This randomized approach ensures a realistic division of consumption or generation across the phases, accounting for slight variations occurring in real-world power systems.
  - If the consumption or generation is small, it is assigned to the least loaded phase.
    This scenario reflects the practical reality that distributing small amounts of power across phases often requires specialized equipment (for example, 3-phase inverters for PVs), which may not be cost-effective.