Reduced Order Modeling Research Challenge 2023: Nonlinear Dynamic Response Predictions for an Exhaust Cover Plate

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Abstract A variety of reduced order modeling (ROM) methods for geometrically nonlinear structures have been developed over recent decades, each of which takes a distinct approach, and may have different advantages and disadvantages for a given application. This research challenge is motivated by the need for a consistent, reliable, and ongoing process for ROM comparison. In this chapter, seven state-of-the-art ROM methods are evaluated and compared in terms of accuracy and efficiency in capturing the nonlinear characteristics of a benchmark structure: a curved, perforated plate that is part of the exhaust system of a large diesel engine. Preliminary results comparing the full-order and ROM simulations are discussed. The predictions obtained by the various methods are compared to provide an understanding of the performance differences between the ROM methods participating in the challenge. Where possible, comments are provided on insight gained into how geometric nonlinearity contributes to the nonlinear behavior of the benchmark system.

Keywords Nonlinear dynamics · Geometric nonlinearity · Reduced order modeling

Introduction

Recently developed reduced order models (ROMs) have been beneficial for analyzing the dynamics of geometrically nonlinear structures, significantly alleviating the computational burden [1]. This ROM Research Challenge is the community's first attempt in recent decades to apply a wide range of state-of-the-art ROM methods to the same problem and to compare them in terms of accuracy and efficiency. The Research Challenge covers various ROM methods, mainly categorized into

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implicit condensation (IC) [2–6], modal derivatives (MDs) [7–9], invariant manifold [10–14], and machine-learning-based data-driven approaches [15, 16].

The ROM methods are applied to a benchmark structure (described below) to capture its nonlinear characteristics as the system energy increases. This chapter presents a list of the methods and participants and a small sampling of the results of the ROM predictions characterized in terms of nonlinear normal modes (NNMs). The next section describes the benchmark structure. Section "Preliminary Results: Nonlinear Normal Modes" presents the NNM backbone curves of the structure predicted by the different ROM methods. This chapter concludes with a summary and future works in section "Conclusion."

Benchmark Problem

The benchmark structure is a perforated cover plate, 317.5 mm in diameter, that is part of the exhaust system of a large diesel engine. The plate is of engineering interest because it experienced fatigue failures in service. During durability testing, the plate was found to behave nonlinearly, and it was later used to validate a nonlinear model updating approach in [17]. Figure 1 illustrates the structure and the finite element model (FEM) approximating the perforated cover as a thin, curved stainless steel (unperforated) plate whose density and modulus were adjusted to account for the holes. The curvature and geometry were measured with 3D digital image correlation and mapped onto the plate model. The plates were annealed prior to testing to minimize residual stresses. The FEM mesh is comprised of 1440 shell elements of 1.5-mm thickness (8,566 free degrees of freedom (DOF)), an elastic modulus of 96 GPa, a Poisson ratio of 0.3, and a density of 5,120 kg m^{-3}. The model is assumed to have weak structural damping with a constant modal damping ratio of 0.000425, and the welded boundary is approximated by a series of 80 linear springs in the radial direction each having a stiffness of 650 kN m^{-1}.

In [17], a low-order ROM (i.e., 2-DOF IC ROM) of the structure was able to capture some interesting nonlinear behaviors with increasing response amplitude, including a softening–hardening behavior and a nonlinear modal interaction that resulted in increased stresses at the center of the plate. These observations explained experimental failures at that location. Based on these findings, the structure is expected to exhibit complex nonlinearities at large deflection. The Research Challenge described here aims to evaluate and compare the ability of recent ROM methods to accurately reproduce the plate's nonlinear behavior as compared with the high-fidelity FE analysis.

Preliminary Results: Nonlinear Normal Modes

The ROM methods were applied to the benchmark structure to predict the nonlinear normal modes (NNMs), which are an efficient metric for describing the characteristics of geometrically nonlinear structures [18]. NNMs are preferred to simply comparing time histories, as they provide for a more rigorous comparison [19].

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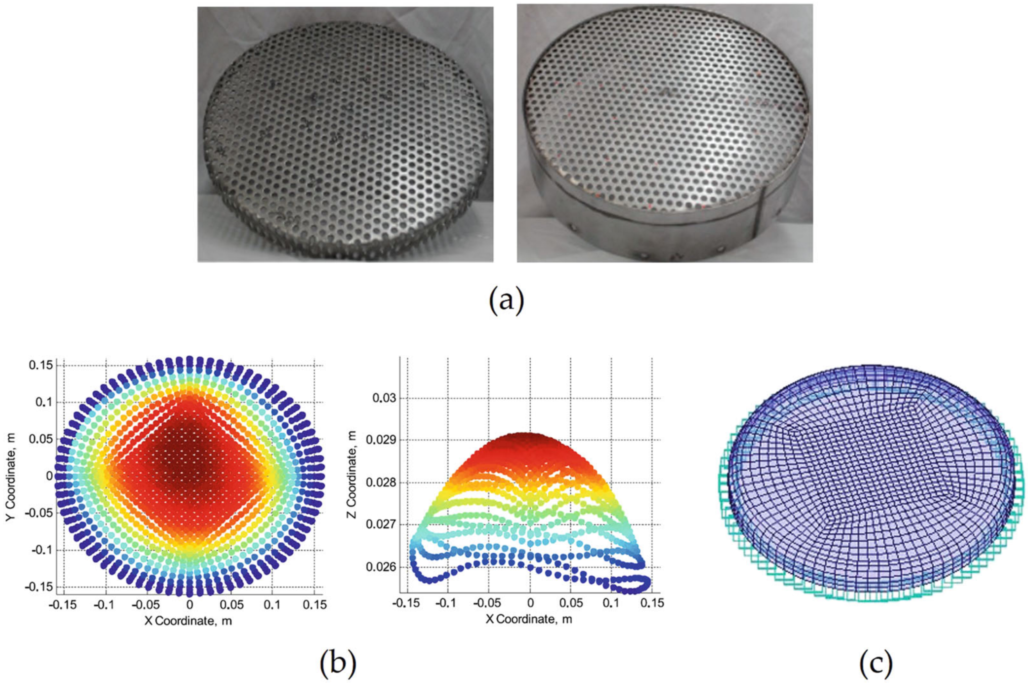

Fig. 1 The benchmark exhaust cover plate. (**a**) A perforated plate before being welded to the test configuration (left) and after welding to the supporting cylinder (right), (**b**) measured surface geometry, and (**c**) meshed plate FEM model based on the measured geometry. (figures adapted from [17] with permission)

Figure 2 presents the first NNM backbone curves for a subset of the ROM methods in the frequency-energy and frequency-peak center deflection plane. The total energy can be either conservative or nonconservative, depending on whether damping is considered in a ROM method. The full-order FEM was used to compute the backbone curves that served as ground truth to evaluate the ROMs. The FEM-based backbone curves were computed using the multi-harmonic balance (MHB) method [20] with five harmonics. While this is taken to be the "ground truth" result, the accuracy is, in fact, limited by the assumption that five harmonics are sufficient to describe all behaviors of interest. Additionally, the MHB algorithm used was not able to obtain a converged solution above an energy level of 0.3 J. This is presumed to occur due to an internal resonance in this vicinity or due to coupling between the underlying linear modes, both of which increase the demands on the algorithm.

The ROM methods considered so far captured the softening–hardening transition of the benchmark structure quite well. The implicit condensation-based ROM (i.e., ICE-GPR and ICE-IC ROM) required four bending modes in order to accurately predict the softening–hardening behavior of the backbone curve. The GPR ROM was trained with static solutions applied by random forces in the forcing range corresponding to [0.25, 3.00] times the plate thickness. The confidence interval of the GPR ROM prediction gauges the sensitivity of the IC ROMs with respect to the level of applied forces in the static sets, which gradually increased after the snap-through. The ICE with inertial compensation (ICE-IC) method accounted for the kinetic energy and nonconservative forces on the quasi-statically coupled modes of the benchmark system [5, 7]. The backbone curves were computed using the computational continuation core (COCO) [21] and had a good agreement with the GPR ROM curves. The curves after a sudden change of the center deflection in a reverse direction (at a peak center deflection of 6.5 mm) indicate a severe multi-mode coupling in the system.

The ROM based on modal derivatives (MDs) used the first six vibration modes and the corresponding modal derivatives, which made the ROM feature 27 DOF. The ROM could accurately capture the backbone curve at small amplitudes (total energy up to 0.4 J). Note that similar to other methods, MD ROMs also had a convergence issue related to the continuation scheme at large amplitudes.

The two ROM methods based on the direct invariant manifold parameterization were also applied to the benchmark model (i.e., DNF and DPIM ROM). The ROMs contained the first two axisymmetric modes (Mode 1 and 6) that had a strong modal coupling with a ratio of 3:1. The DNF ROM used a third-order truncation [12]. Since the DNF method relies on asymptotic expansions around the fixed point, the accuracy is limited to the basin of attraction of the fixed point. This might be an explanation for the poor behavior of the method at large amplitudes. The DPIM ROM used a seventh-order truncation in graph style [14]. The higher order improved the prediction, as expected. However, as the software for DPIM is only available

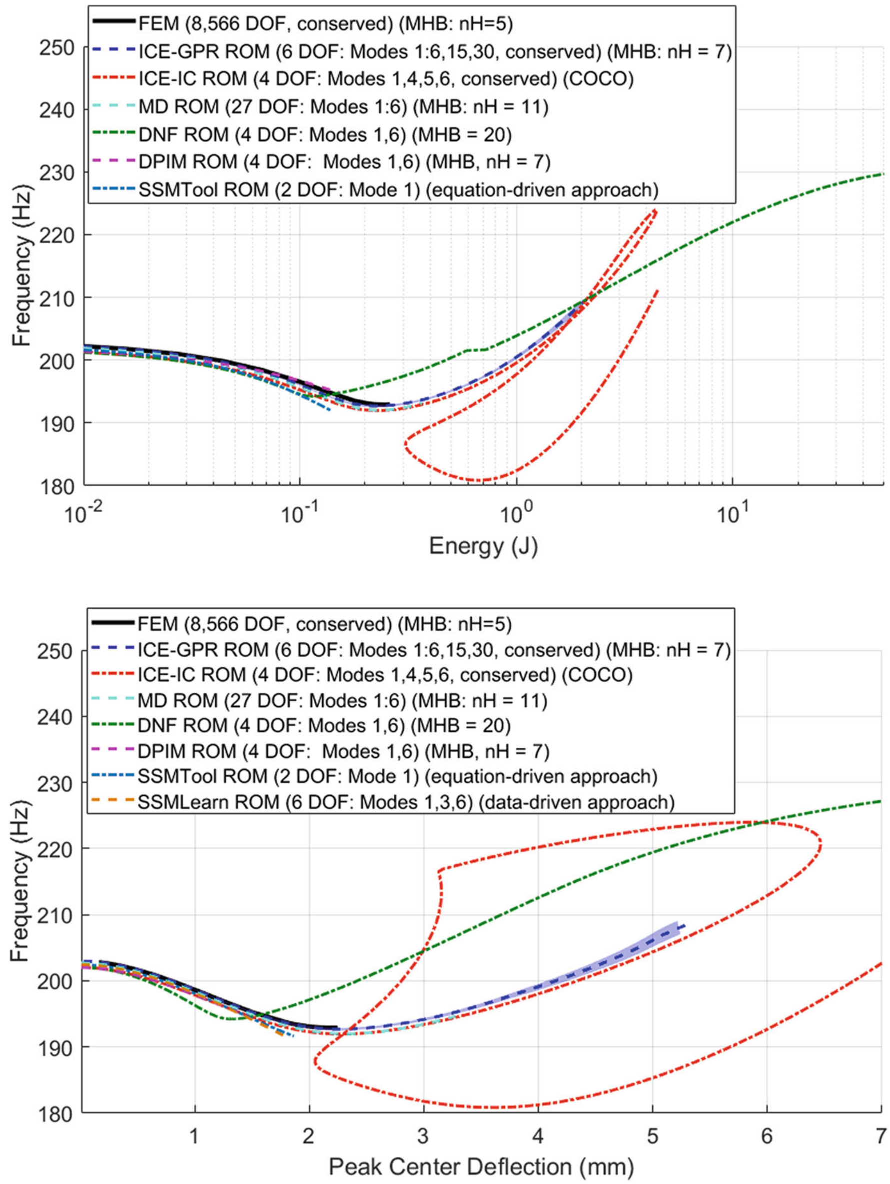

Fig. 2 The first NNM backbone curves of the plate model computed using the full-order FEM and some of the considered ROM methods. The curves are represented on the two different planes: the frequency-energy *(top)* and frequency-peak center displacement plane *(bottom)*. *"Conserved"* indicates the model neglected the dissipative term of the benchmark system. *"nH"* indicates the number of harmonics used in the MHB algorithm. The gray shading corresponds to the 95% confidence interval of the GPR ROM prediction. The predictions using other ROM methods are in progress and will be discussed during the conference presentation

for 3D FEM, the model used for such prediction was a 3D adaptation of the 2D model provided. This explains the shift in the linear frequency as compared to the other methods.

The SSM ROM method was also investigated on the benchmark, which takes advantage of the smoothest nonlinear continuations of spectral subspaces [10, 11]. Two approaches were studied: a data-driven approach that identifies invariant manifolds based on unforced trajectory data obtained from FE simulations and an equation-driven method that computes nonautonomous SSMs in a completely nonintrusive manner. Both methods captured softening behavior and then encountered some issues when the dynamics became sophisticated at snap-through. The equation-driven method suffered from the convergence issue, while the data-driven method could not fit a good dynamics at peak center deflection around 2 mm in the ring-down simulation. These require further investigation.

The ROM methods and research groups contributed so far to the ROM Research Challenge are presented in Table 1. The ROM methods all dramatically reduced the cost of computing the backbone curves, orders of magnitude faster than

Table 1 Research groups and ROM methods that contributed so far to the ROM research challenge with the preliminary results

Research institute	ROM method	References
ETH Zurich, Switzerland	SSM	[10, 11]
ETH Zurich, Switzerland	MD	[7–9]
Imperial College London, UK/University of Exter, UK/University of Liege, Belgium	DNF	[12]
Politecnico di Milano, Italy/Institut Polytechnique de Paris, France/University of Exeter, UK	DPIM	[13, 14]
University of Bristol, UK	ICE-IC	[5, 6]
University of Minnesota, USA/Brigham Young University, USA	ICE-GPR	[4]

computing the full-order FEM solutions. Note that differences in FEM configuration (e.g., Rayleigh damping), FEA solver, and NNM continuation method used for each ROM method could contribute to the gaps between the curves (e.g., the gap between the curves at the linear frequency). The results from different methods so far capture the nonlinear behavior of the same benchmark model using each of their distinct characteristics. Further analysis is needed regarding the accuracy of the truth model as compared to each ROM and the tradeoff between efficiency and accuracy made in each method before one could rank their performance. The presentation will seek to present a more in-depth analysis and hence additional insights on the strengths and limitations of each method in capturing the complicated nonlinear behaviors exhibited by this structure.

Conclusion

This chapter presented the preliminary results of the 2023 ROM Research Challenge. Various ROM methods were used to predict the nonlinear dynamic responses of an exhaust cover plate structure in an effort to understand the performance of state-of-the-art ROM methods. Our future work will focus on more detailed, in-depth analysis and comparison of the participating ROM methods by predicting additional types of nonlinear response of the benchmark problem. These may include investigating the effect of different ROM formulations and parameter selections on the performance of each of the ROM methods.

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