Practice report

Journal of Virtual Exchange

A proposal in STEM for virtual exchange held by Computer Science and Applied and Computational Mathematics programs

Gilcilene Sanchez de Paulo[1], Quentin Louveaux[2], Ana Cristina Biondo Salomão[3] and Gustavo Primo[4]

Abstract

In this practice report, we present a practical experience of a virtual exchange carried out by a Belgian and a Brazilian professor from two research intensive universities in Northwestern Europe and in Latin America, respectively. Particularly, this paper is focused on how we designed a common syllabus in a specific topic of STEM, aiming to explore math and computer science skills of the students, and how we implemented this virtual exchange step by step, despite the students being from different courses and different levels of degree. In addition, we discuss the pros and cons of the process from the instructors' perspective as well as under students' views, and provide some suggestions to be applied and tested for future virtual exchanges in this field. Overall, this practice report reinforces our deep reflection about the results and desire to share our STEM syllabus to encourage other colleagues to apply it (or modify it) for different courses.

Keywords: computer science; applied and computational mathematics; STEM education; cross-cultural collaboration; interdisciplinary teaching

1. Universidade Estadual Paulista Júlio de Mesquita Filho; gilcilene.sanchez@unesp.br; https://orcid.org/0000-0002-0146-9367
2. Université de Liège; q.louveaux@uliege.be; https://orcid.org/0000-0003-3458-2435
3. Universidade Estadual Paulista Júlio de Mesquita Filho; ana.salomao@unesp.br; https://orcid.org/0000-0002-1531-8551
4. Universidade Estadual Paulista Júlio de Mesquita Filho; gustavo.primo@unesp.br; https://orcid.org/0000-0002-8431-9144

How to cite this article: Paulo, G. S., Louveaux, Q., Salomão, A. C. B., & Primo, G. (2024). A proposal in STEM for virtual exchange held by Computer Science and Applied and Computational Mathematics programs. *Journal of Virtual Exchange*, *7*(SI-IVEC2023), 28–41, https://doi.org/10.21827/jve.7.41323

1. Introduction

In recent years, many higher education institutions have recognized the importance of Internationalization at Home (IaH) (Beelen & Jones, 2015), in particular those focusing on global learning and internationalization of the curriculum. In this direction, virtual exchange (VE) has stood out. VE is a pedagogically structured online collaborative learning modality between groups of students in different cultural contexts and/or geographic locations (O'Dowd, 2018). By combining intercultural dialogue with the broad reach of digital technology, VE can act as a catalyst in bringing new tools to support teaching and learning with an international perspective, fostering the development of competencies such as teamwork, problem solving, decision making, and communication in foreign languages.

While VE is intended for all areas of knowledge, there has been a predominance of use in humanities and language learning (Hilliker, 2022; Potolia & Derivry-Plard, 2023). This is probably due to the fact that VEs grew out of telecollaboration projects originating in the language teaching field (O'Dowd, 2018). Therefore, it is important to report and expand the knowledge base on VE experiences that may broaden educators' views of the possibilities of this type of modality.

This practice report focuses on a VE in STEM, involving math and computer science students. It was designed with an interdisciplinary approach for students from different careers and degree levels. We present and discuss the pros and cons from our perspective as well as from the students' viewpoints.

The authors state that this research was conducted ethically and responsibly. Each author contributed according to their role in the VE, bringing their time and expertise to the collaborative elaboration of the text.

2. Context

The collaboration described in this practice report was implemented between two research intensive universities, one in Brazil and the other in Belgium. On the Brazilian side, it was part of a nationwide virtual exchange program aimed at promoting the internationalization of Brazilian universities and seeking to promote cross-cultural understanding and cooperation towards the development of a more diverse and globally connected higher education community in Brazil (Salomão & Freire Junior, 2020). The main methodology used in this program is Collaborative Online International Learning

(COIL). On the Belgian side, no formal international virtual exchange program existed in the beginning of the project. The exchange program was a first attempt by the university to include such an international dimension into its curriculum.

The two courses involved were not exactly the same, but addressed STEM contents that could be considered complementary. The Belgian professor taught "*Introduction à l'algorithmique numérique*" (Introduction to numerical algorithms) to 45 students of a Computer Science undergraduate program. Most students were around 20 years of age and were in the second year of the program with a basic training in mathematics and in coding. Among the 45 students, there were 35 Belgians, three Dutch, two Italians, one Greek, one French, one Algerian and one from Cameroon. Among them, 40 students had finished their high school in Belgium while five had done it abroad. The Brazilian professor taught "*Análise Numérica*" (Numerical Analysis) to 16 students of an Applied and Computational Mathematics post-graduation program. The Brazilian post-graduation program is an inter-campus initiative, involving precisely two campuses of the university, and admits students from Latin America, with a particular emphasis on Peru. Among the 16 students enrolled, two were Peruvian and part of the cohort of eight students from Campus 1, while the remaining eight students were Brazilian and from Campus 2. Most of the students were around 23 years old, though the age range spanned from 22 to 41 years old.

In order to develop the collaborative project, the first task was to determine a topic that would fulfill the following properties:

- it should deal with a topic that was being covered in sufficient depth in both classes;
- it should be a topic in which the skills of students in both classes would be complementary.

Although both groups dealt with numerical analysis, their focus was a little different. In Belgium, the focus was solving linear systems, a topic that was not covered in that Brazilian course. On the other hand, the topic of regression and interpolation was covered only on the Brazilian side. We finally could identify an intersection in both courses: solving nonlinear equations and systems of nonlinear equations.

In a subsequent step, it was then useful to identify a project statement where the teams from Brazil and Belgium could collaborate and where their skills would bring something to the group. The background of the Belgian students in particular was very much computer oriented. This means that they were trained at writing accurate and robust codes whereas their mathematical background was relatively less comprehensive. On the other hand, Brazilian students were trained in applied mathematics but with a lesser focus on coding skills.

We decided to ask the students to implement the Newton method. The iteration of the Newton method is composed of two steps: computing the derivative (or the Jacobian in the case of a system) and solving a linear system in order to find the next iterate. Questions of convergence and of the starting point of the method are also interesting. Clearly, implementing the method in one dimension is easier than implementing it (and having it run properly) in large dimensions. This could provide a possible progression in the project. Finally, in order to suggest some kind of complementarity between the students, it was also needed to introduce a part that would be more mathematically oriented and a part that would be more related with coding skills. The mathematically oriented part had to be related to the computation of the derivatives. In order for that part to be challenging enough, we broke it into three parts.

In the first part, we asked the students to deal with trigonometric functions. In fact, this is a standard question but not very much adapted to systems of equations. The second part was related to polynomials. It is well known that computing the derivative of one-dimensional polynomials is rather straightforward, but in order to introduce some challenge, we asked the students to go to multivariate polynomials. This is also interesting in its own, since the Belgian students never dealt with multivariate polynomials before and, in Brazil, it is a specific research area, which is not usually taught in general courses (undergraduate students may study multivariate polynomials in Scientific Initiation, under the supervision of a Professor in this field, or later, in themes of a master's thesis or doctoral dissertation). In a third step, we asked the students to think about the computer representation of those functions, which is more relevant for coding. Questions like the generality or the sparsity of the representation are pertinent for students willing to do more coding in the future. Finally, the implementation of the Newton method for these specific functions was the closing step of the project. We believe that, in this way, the project statement has a balanced integration of mathematics and coding skills. Indeed, there is a progression in the project, starting from simple problems to more complex ones, where numerical analysis becomes increasingly pertinent.

3. Objectives

The main objective of this VE was to promote international collaborative online learning among students from different courses and levels of education in the STEM area. Both students and professors had never participated in this modality of teaching-learning before. In this way, it would be a whole new experience and an immersion of discovery for both sides involved.

4. Project design

During approximately one month (August-September 2022), we met and planned a collaborative international virtual exchange for supporting our students to solve a common problem computationally. The VE took place from September 26th to October 28th of the same year.

The collaborative course was organized on Padlet, an online tool that works as a hub of interactive boards where users (the students and the professors) can publish texts, images, videos, links and documents. In this case, Padlet was used to share basic information about the participants, instructions about the activities and a calendar with important dates and deadlines. It was also used as a starting point for the students to get to know the other members of their teams, introduce themselves with a short bio and periodically publish results of their work. Students were allowed to use other platforms and applications to make the interaction easier, such as Google tools, social media, WhatsApp, etc. In order to develop and share the code in Python programming language, Google Colaboratory, Visual Code or Github were the main options, and to produce and also share the reports, the Overleaf editor of LaTeX was strongly suggested.

One pedagogical strategy used to keep the students motivated and productive was to divide the problem into smaller parts. The initial task focused on studying the differentiation of polynomial and trigonometric functions in the theoretical and numerical views. The proposal of studying it theoretically aimed to leverage the Brazilian students' expertise in applied mathematics, whereas the numerical perspective utilized the skills of the Belgian students in computer science. Therefore, the theoretical elements were thought to use the Brazilian students' skills on applied mathematics, whereas the numerical exercise would benefit the computer science students on the Belgian side.

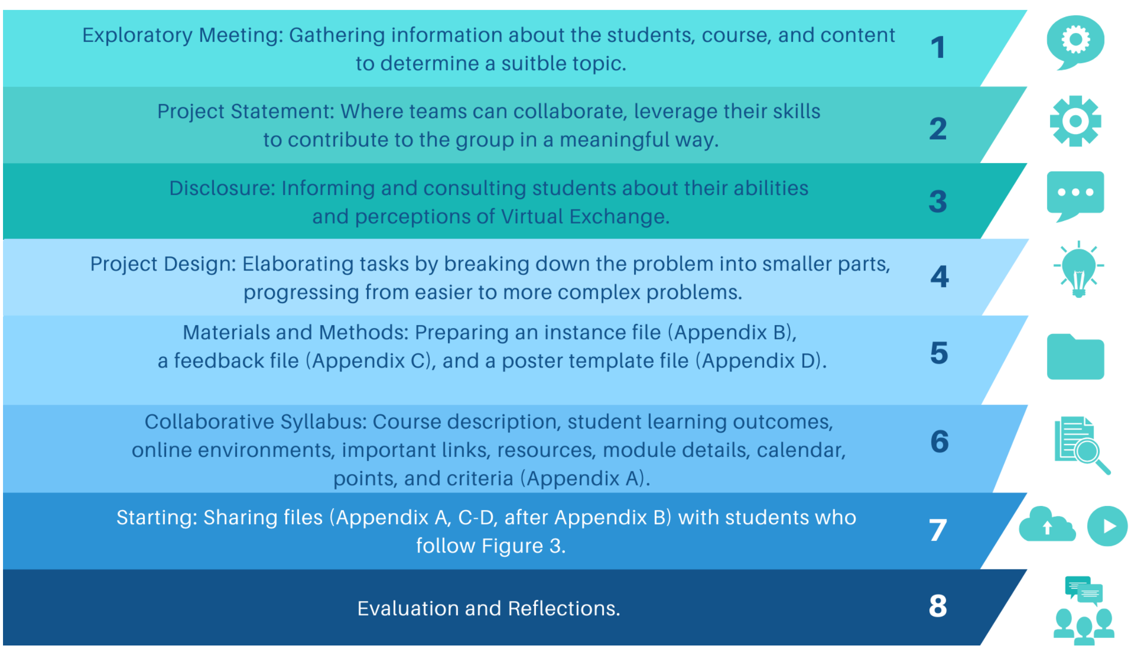
In order to contribute to intercultural and collaborative interaction, the tasks were done in groups formed by Belgian and Brazilian students and their deadlines were mostly weekly. The formation of the groups was challenging because of the different size of the class on each side, as there were almost four times more Belgian students than Brazilians.

Taking into account a better pedagogical strategy to facilitate collaborative interaction due to time zone and other activities of the students, we decided to avoid big groups. Hence, the groups were formed with only one Brazilian student and about four Belgian students each. This arrangement caused a lack of Brazilian students in some groups due to dropouts from the course.

The complete work plan that we elaborated together and provided to the students is in Appendix A. In order to have a quick reference guide to understanding the whole collaborative project, Figure 1 displays the step-by-step process of the project from the instructor's perspective.

In the next section we will focus on the implementation of the activities of our VE as well as reflections and ideas for VEs in STEM.

Figure 1. Main steps of the collaborative process from the instructor's perspective.

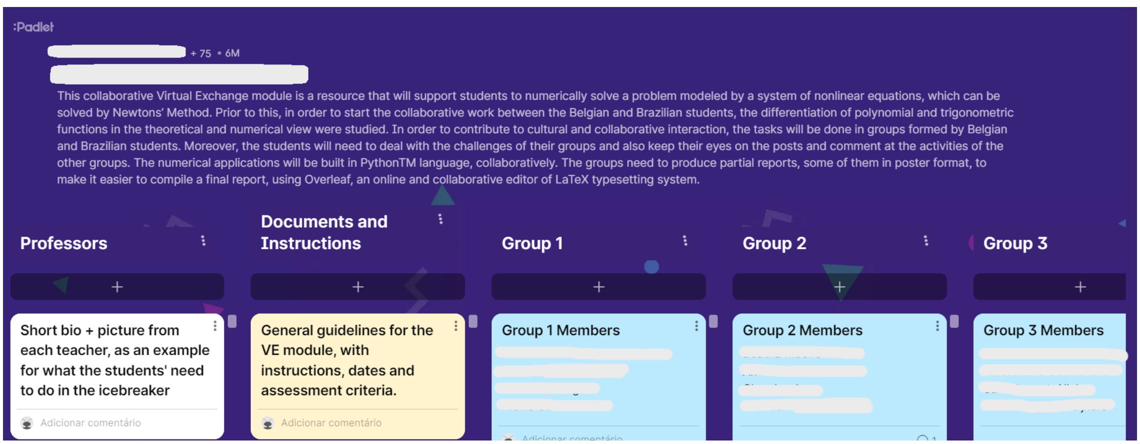


5. Implementation and discussion of outcomes

While the Belgian and Brazilian instructors were planning the syllabus of this VE, they were also preparing and informing the students about the new component of the international collaboration that would take place in their class. In this context, the words "new component" had an important meaning for both the Belgian and Brazilian side, because it was their first experience taking part in a totally virtual international exchange. Therefore, both instructors and students were available to learn and practice this methodology even if their lack of knowledge brought doubts and fears.

The specialized staff of the university started the management of a Google Sheet to organize the groups of the students and their respective emails. Then, they used the Padlet application to arrange the groups in column format, as shown in Figure 2.

Figure 2. A screenshot of the Padlet application used as a virtual learning environment. The groups (from group 1 to group 16) were arranged in column format.



The first column of the Padlet board was reserved for an introduction of the instructors, and the second column was reserved for general guidelines for the VE component, instructions, deadlines and assessment criteria. Then, from the 3rd to the 18th columns, the groups were distributed in ascending order, each one with the names of its members (who were anonymized in Figure 2). The option of comments from other participants on their posts was enabled, in order to motivate students to check the solutions proposed by other groups and comment on the points that interested them.

Module 1 (Meeting people) was an icebreaker activity. The students participated by posting their favorite picture and a short description of their lifestyle, and, in addition, they participated by commenting on the posts of other students.

Module 2 (Differentiation: theoretical part) was designed to explore the skills of the Brazilian students in mathematics. This module provided a moment for Brazilian students to act as leaders of their teams in this subject, collaborating with Belgian classmates to enhance their analytical knowledge. The instances[5] for guiding the students are presented in Appendix B.

Students compiled a rather lengthy partial report on this topic, and not all of them were complete, as we expected. Therefore, a suggestion of the solutions to the tasks from Module 2 was provided to the students. This aimed at providing feedback[6] from the instructors to the students in order to complete as much as possible the process of teaching-learning, a technical part in VE. The file was sent to the students after they concluded the module in question, and it can be found in the Appendix C.

In this module, the level of interaction with different groups decreased compared to the interaction with different groups in module 1. It was observed that the Brazilian students provided comments on approximately 66% of their peers' reports, while the Belgian students commented on about 55% of their peers' reports.

Module 3 (Differentiation: numerical part) made more use of Belgian students' skills in programming, using Python language. To complete this module successfully, it was necessary to seek support from the Brazilian students about the previous module (Module 2), in order to understand the correct meaning of each variable into the analytical expressions. It was expected that Belgian students would lead this module and the Brazilian students would improve their skills in programming.

Instructors offered a poster template[7] for students, who then organized the development of this module following a presentation pattern. This template is available in the Appendix D.

A more complete explanation of the elements and tasks that were designed and implemented is provided in the Appendices: Appendix A presents the Collaborative Syllabus, Appendix B includes several instances, Appendix C shares feedback for Module 2, and finally, Appendix D shows a poster template.[8]

5. Available in LaTeX at Overleaf: https://pt.overleaf.com/read/rqxpscdcktgr
6. Available in LaTeX at Overleaf: https://pt.overleaf.com/read/kynznbgbftyy
7. Available in LaTeX at Overleaf: https://pt.overleaf.com/read/wkfmbhkwnfrp
8. The materials in the Appendices are also available for download through the link https://drive.google.com/drive/folders/1EsietgL293v-tLndXj7fZrLkUC54dofG?usp=drive_link.

Having the results of the previous modules on how to describe the derivatives and the Jacobian matrix analytically and computationally, in Module 4 the groups of students were asked to focus on the structure of Newton's Method for solving nonlinear systems, being mindful of a very important fact for this method: the choice of the initial guess. Furthermore, checking the convergence rate was also a task to make the studies more complete. This module required collaborative work between the two sides, Belgian and Brazilian. We had planned that the teams shared the code working properly with us before the online meeting; however, to give extra time to the students, we postponed the delivery of this activity to the final report.

In Module 5, we carried out the closure of the project. An online meeting was held with all participants, in which the students presented the solutions of the tasks and a final report compiling all the tasks.

The online meeting was held synchronously at 13:30 from Brazil and 18:30 from Belgium. Finding a common timeframe was a hard task due to the time difference and the various schedules of the students involved with other courses. However, 13:30 was the regular lecturing time in Brazil and 18:30 was just at the end of the normal course day in Belgium. In this respect, the time difference facilitated the identification of a window during which all students were available, although it was a little late for the Belgian students.

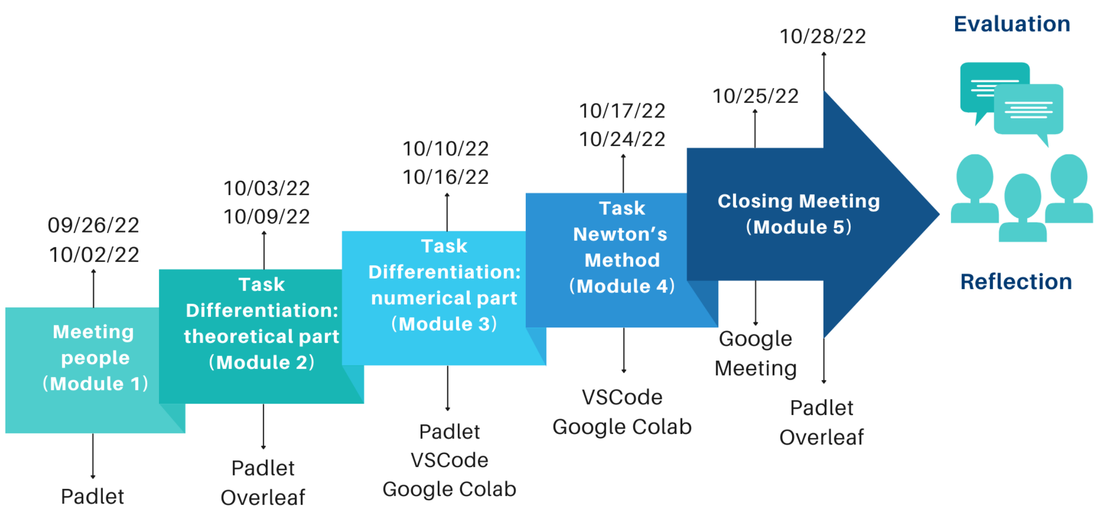
There were 16 groups tasked with presenting their technical results. We estimated seven minutes per group, translating to just over two minutes per student. Beforehand, we had recommended that all students had a moment to speak, even if it was just a few minutes. Time keeping proved to be a challenge for several groups. Some groups were well-prepared and demonstrated their coordination even with the geographical distance, while in some other groups it was difficult to come up with a cohesive presentation, leading to an uneven distribution between the different subtasks within their project.

On the Belgian side, everyone participated from their preferred locations, such as their homes, due to the time difference – and because those were autumn evenings. The Brazilian participants decided to be together in a videoconference office from the University, which works supported by a specific system, named Cisco Jabber. However, we had to transition to Google Meet and use only wireless connections. This way, the signal oscillated less during the presentations. It was possible for all students to present, but the strategy employed by the Belgium side was more linear and efficient. Such technological issues must be taken into account in VEs.

After the meeting, the students were asked to compile a final report with all results, including those obtained in the previous modules. We recommended a limit of 10 pages and the use of Overleaf to make typing equations and math formulas easier.

A summary of the whole process that was indeed achieved is illustrated by Figure 3, which displays a timeline of each module and the minimum of its respective tools required.

Figure 3. Step-by-step of the international VE between STEM courses from Belgian and Brazilian research universities in Northwestern Europe and Latin America, respectively.



5. Evaluation

Each instructor used an independent form of evaluation, normalizing the score according to their institutions' grading systems. In both courses, the component of the VE counted as one-third of the total grade of the course.

Before the classes started, the Brazilian instructor had sent the students a survey to ask them about their experience in some programming language, English level and perceptions of VE. Most students had some knowledge of MATLAB and/or Python, and a basic level of English, with some of them having an intermediate level of English. Many students received the new idea of "virtual exchange" with positive enthusiasm, but the majority were surprised and concerned about the challenge.

After the conclusion of this VE, the Brazilian students answered another survey as a reflection of their experience and also to provide an instrument of evaluation about our planning, implementation and evaluation.

In qualitative terms:

a. All students agreed that:
- the VE presented to them a new perspective and new ways of learning the content;
- the VE enlarged their perception about other culture and country and made them more flexible considering new perspectives;
- the tools suggested were very useful and enough to develop the tasks collaboratively;
- they would recommend a course with a VE component to other students.

b. Most students agreed that:
- the VE provided skills and knowledge that they will use in the future;
- they met international students with whom they intend to keep in touch after the end of this course;
- the VE could affect their career and/or professional choices.

c. Few students:
- did not feel prepared for the cultural aspects of collaborating with participants from the partner institution;
- did not feel prepared for the technological demands;
- were not able to communicate satisfactorily in a foreign language with their partners (they used Google Translate and communicated in Portuguese, French and English);
- had had the opportunity to travel to another country.

In terms of positive and negative points under the students' views, we can summarize in qualitative data as:

a. The students most liked:
- the exchange of views on different technologies;

- the ways the same subject was taught in other countries;
- communication with colleagues in another language and being able to discuss the tasks.

b. The students most disliked:

- the time zone difference of five hours, which made the communication more difficult;
- the short period of the calendar reserved to the component of the VE;
- the way the groups were formed with only one Brazilian per group;
- the description of the activities, which could have contained more details.

In a similar fashion, the Belgian students answered a survey to collect their views about the way the project went. The main points raised by the students were very similar to those pointed out by the Brazilian students.

In particular, the students liked:

- The concept of VE and in particular the cultural aspect of it. They felt thrilled to be able to do a joint project with students from that far away. Actually, some students were exempt from the project and regretted not being able to take part in it.
- The students liked the fact that the due dates were close to each other and that the project was carried out in a relatively short time frame, not to interact too much with the other duties from other courses they had, especially since the project took much more time than they had expected.

The students disliked:

- The way the groups were created. Since the groups were drawn randomly in the Belgian side, they were not able to work with their own friends. Furthermore, there was only one Brazilian in each group, which was an issue for some groups. Indeed in some cases, the Brazilian student dropped out, leaving the group without any cultural and exchange aspect. In some other cases, the students did not get along very well which hindered the smooth running of the group.
- Many students found that the amount of work was badly evaluated by the instructors, leading to a project taking too much of their time compared to the amount of time normally devoted to the project in the course (40 hours of personal work).
- Many students complained that the statement of the project was not precise enough. They would have liked to have more instructions about what was expected from the tests to be made and the contents of the various reports.

Although we planned rubrics and criteria, as shown in the "Calendar Tasks and Activities" table in Appendix A, the criteria and expectations of both countries were sometimes different and not completely transparent for the students, which resulted in specific complaints about the evaluation process. We believe that it is a point that should be taken care of with great detail when considering a VE. For future iterations of VE, a weekly notification of rubrics with clear criteria and tasks should be emphasized.

Finally, regarding the cultural aspect of the VE, the timeframe of the project was relatively brief and within this timeframe, the students had little time to finish their duties, which resulted in missing opportunities to get to know each other better. When designing such a project, there is a tradeoff to make between the academic expectations and the intercultural communication aspect attached to it. In a future version of this project, enough time should be devoted to potential exchanges between the students.

6. Conclusions and implication

Using COIL/VE in a Numerical Analysis course offers several advantages that can enhance students' learning experience and prepare them for the global and interdisciplinary nature of numerical analysis. COIL/VE provides opportunities for students to collaborate with peers from different countries and cultural backgrounds, bringing diverse perspectives to problem solving. This collaboration encourages innovation and expands students' understanding of numerical methods.

Taking into account the first experience of the instructors, incorporating a VE project in their curriculum was very beneficial and brought a lot of learning throughout the process. It was necessary to consider an interdisciplinary approach and elaborate strategies to involve different students, encompassing the details of the planning and its implementation, until the closure of the project.

Some Brazilian students evaded the course because they were not enrolled full-time in the program. Hence, it produced a lack of Brazilian students in some groups. We can corroborate that a good balance between the number of students from each cohort will lead to a more successful collaboration.

To minimize the chances of misunderstanding, it is necessary to explain all the phases of the project in detail to the students and as frequently as possible.

Another pedagogical strategy that could be interesting for promoting more cultural learning is to deliver the tasks through project-based learning (PBL), taking into account cultural information and a mathematical toy-model. However, the timeframe for planning this kind of collaborative syllabus would need to be extended accordingly.

References

Beelen, J., & Jones, E. (2015). Redefining internationalization at home. In A. Curai, L. Matei, R. Pricopie, J. Salmi & P. Scott (Eds.), *The European higher education area: Between critical reflections and future policies* (pp. 59-72). Springer. https://link.springer.com/chapter/10.1007/978-3-319-20877-0_5

Hilliker, S. (Ed.) (2022). *Second language teaching and learning through virtual exchange*. De Gruyter Mouton.

O'Dowd, R. (2018). From telecollaboration to virtual exchange: State-of-the-art and the role of UNICollaboration in moving forward. *Journal of Virtual Exchange, 1*, 1-23. https://doi.org/10.14705/rpnet.2018.jve.1

Potolia, A., & Derivry-Plard, M. (2023). *Virtual exhange for intercultural language learning and teaching*. Routledge.

Salomão, A. C. B., & Freire Junior, J. C. (Eds.). (2020). *Perspectivas de internacionalização em casa: Intercâmbio virtual por meio do Programa BRaVE/Unesp*. Cultura Acadêmica. http://hdl.handle.net/11449/202860

Journal of
Virtual Exchange

University
of Groningen
Press

Published by University of Groningen Press | UGP, a not-for-profit press
Groningen, The Netherlands | UGP@rug.nl

Journal of Virtual Exchange 2024
Special issue IVEC 2023
Edited by Stephanie Swartz and Ana Cristina Biondo Salomão

Publication date: 2024/11/25

Journal of Virtual Exchange (JVE) is an online, open-access, peer-reviewed journal aimed at practitioners and researchers in the field known variously as virtual exchange, telecollaboration, or online intercultural exchange. It is the official journal of UNICollaboration, the international academic organisation dedicated to supporting and promoting telecollaboration and virtual exchange in higher-level education.

Typeset by LINE UP Boek en Media

Noto fonts are open source. All Noto fonts are published under the SIL Open Font License, Version 1.1. Noto is a trademark of Google Inc.

ISSN: 2647-4832 (online only)

UNICollaboration

Appendices

Appendix A: Collaborative Syllabus

Collaborative Syllabus

ANÁLISE NUMÉRICA
Gilcilene Sanchez de Paulo
São Paulo State University, UNESP, Presidente Prudente - SP, Brazil

INTRODUCTION À L'ALGORITHMIQUE NUMÉRIQUE
Quentin Louveaux
Université de Liège, Liège, Belgium

COURSE DESCRIPTION

This collaborative course is a resource that will support students to numerically solve a problem modeled by a system of nonlinear equations, which can be solved by Newtons' Method. Prior to this, in order to start the collaborative work between the Belgian and Brazilian students, the differentiation of polynomial and trigonometric functions in the theoretical and numerical view will be studied. In order, to contribute to cultural and collaborative interaction, the tasks will be done in groups formed by Belgian and Brazilian students. Moreover, the students will need to deal with the challenges of their groups and also keep their eyes looking and commenting at the activities of the other groups. The numerical application will be built in Python™ language, collaboratively. The groups need to produce partial reports, some of them in poster format, to make easy to compile a final report, using Overleaf, an online and collaborative editor of LaTeX typesetting system.

STUDENT LEARNING OUTCOMES

According to the program, it is expected to improve student's skills in

1. the application of numerical methods to obtain approximate solutions for problems.
2. the programming language Python™.
3. the tools to perform, store, and share computer codes, such as Google Collaboratory, Github, etc.
4. the editor of the LaTeX typesetting system, Overleaf.

5. a foreign language and cultural knowledge.

ONLINE ENVIRONMENTS

The collaborative course will take place with a Padlet application. Other platforms and applications might be used in order to make the interaction easier, such as Google tools, social media, WhatsApp, etc. In order to develop and share the code in Python™ programming language, Google Collaboratory or Github might be used, and to produce and also share the reports, the Overleaf editor of LaTeX is strongly suggested.

IMPORTANT LINKS[1]	
List of students (Google Sheets)	https://www.google.com/sheets/about/
Padlet	https://padlet.com/
Tasks (Overleaf)	https://www.overleaf.com/
Poster Template (Overleaf)	https://www.overleaf.com/
Online Presentation (Google Meet)	https://apps.google.com/meet/

RESOURCES

1. Ascher, U. M. and Greif, C., A First Course in Numerical Methods; Computational Science and Engineering, Society for Industrial and Applied Mathematics, Philadelphia, 2011.
2. Burden, R. L. and Faires, J. D., Numerical Analysis, Brooks/Cole, CENGAGE Learning, Boston, 2010.
3. Chapra, S. C. and Canale, R. P., Numerical Methods for Engineers, , McGraw Hill Education, New York, 2015.
4. Johansson, R., Numerical Python: Scientific Computing and Data Science Applications with Numpy, Scipy and Matplotlib, Apress, California, 2018.

DETAILS OF THE STATEMENT OF THE PROJECT

1 Here are the general links from the platforms. From students we provided our specific link.

MODULE 1

In this module, the students are just asked to submit a picture or a video of themselves with a short presentation on the Padlet. Comments on the other submissions are also welcome.

TECHNICAL PART – INTRODUCTION

In this project, the students are asked to make a program that allows them to solve a series of nonlinear equations and systems of nonlinear equations with a specific format. The program will run in python, possibly with the use of the capabilities of numpy. There are a few steps needed in order to achieve the expected outcome: study the different forms of functions that we ask to tackle, being able to represent these forms in a clean computer format, then being able to compute the derivative (matrix of derivatives for multivariate functions), and finally implement the Newton method in order to find one solution of the equation or the system.

There are four general forms to tackle: equations with trigonometric functions, equations with a polynomial, systems of n equations with n multivariate quadratic functions, systems of n equations with n multivariate polynomials. There are a few examples of instances for such equations and systems but the final goal is to be able to handle the general form.

MODULE 2

In Module 2, you must study the four different general forms and be able to compute analytically their derivatives. Be aware that for the systems of equations, you are expected to provide the Jacobian matrix, i.e. the matrix of first partial derivatives. Towards the end of Module 2, you should be able to think about the way to represent the four different forms in the computer.

MODULE 3

In Module 3, you must come up with a way to represent the four general forms in the computer. You must also implement a function that receives one of the four general forms as input as well as a point (number or vector) and outputs the derivative (number or matrix) of the given function. Be aware that the derivative must be computed analytically. In this module, you are not allowed to use a python library that computes the derivative automatically.

MODULE 4

In Module 4, you must implement the Newton method in order to solve one of the four general forms of equations and systems. You are allowed to use the functions from the np.linalg library from python in order to tackle the linear algebra (checking that the matrix is nonsingular, solving the linear systems). Pay attention to use the right initial point and to the right convergence criterion. Moreover, include the checking of the convergence rate.

CALENDAR AND ACTIVITIES TASKS

Modules	Tools	Weeks	Evidence on performance	Points	Criteria
Module 1 Meeting people	Padlet	September 26th to October 02th	• a photo plus personal information or a short video • comments on other participants' posts.	**10** • own profile = 3 points. • each comment to other student = 1 point	• Clarity of ideas • Quality of interaction
Module 2 Differentiation: theoretical part	Padlet	October 03th to October 09th	• analytical derivative – polynomials • present a report (partial report of the COIL course) • comments on other participants' posts.	**20** • partial report = 15 points • each comment to other team = 1 point	• Clarity of ideas • Coherence • Cohesion • Format
Module 3 Differentiation: numerical part	Padlet	October 10th to October 16th	• be able to represent the derivatives in the computer • be able to implement in Python language • present a poster • comments on other participants' posts.	**20** • application run in Python = 8 points • partial report = 8 points • each comment to other team= 1 point	• Clarity of ideas • Coherence • Cohesion • Accuracy • Convergence • Format
Module 4 Solving numerically a non-linear system	Private	October 17th to October 23th	• implement the Newton´s Method according to the problems at the list prepared by the instructors.	**20** • code is working well = 20 points.	• Accuracy • Convergence
Module 5 Gathering the knowledgments from COIL course	Overleaf Google Meet	October 25th (presentation) October 28th (final report)	• online presentation / synchronously 13:30BR/18:30 Belgium • final report with all results, including the previous (use graphics, etc). • comments on other participants' posts.	**30** • participation on the presentarion = 15 points • final report = 10 points. • each comment to other team = 1 point	• Clarity of ideas • Coherence • Cohesion • Sufficiency • Format

Appendix B: Instances

Available in LaTeX at Overleaf: https://pt.overleaf.com/read/rqxpscdcktgr

Collaborative Tasks

1 Equations - trigonometric functions

(a) $\cos(2x) + \cos(3x) + \cos(4x) = 1$

(b) $2\cos(x) - \sin(2x) + \cos(-x) = 0$

(c) $4\sin x + 4\cos x - 4\sin(2x) = 5$

(d) General form : $\sum_{i=1}^{k} a_i \cos(ix) + \sum_{j=1}^{l} b_j \sin(jx) = c,\ a_i, b_j, c \in \mathbb{R}.$

2 Equations - polynomials

(a) $x^4 - 3x^3 + x - 1 = 0$

(b) $x^5 + 3x^3 - x + 1 = 0$

(c) $x^3 - 4x^2 + 3x + 1 = 0$

(d) General form : $\sum_{i=0}^{n} a_i x^i = 0,\ a_i \in \mathbb{R}.$

3 Systems of equations - polynomials

(a) $\begin{cases} xy^2 - xy = 1 \\ x^2 + y^2 = 4 \end{cases}$

(b) $\begin{cases} xyz - x^3 + y^2x - 3xy = 2 \\ x^2 - y^2 + z^2 = 0 \\ xz^2 - z + 3xyz = 0 \end{cases}$

(c) General form : System of n quadratic equations with n unknowns.

(d) General form : System of n polynomial equations with n unknowns.

Appendix C: Feedback

Available in LaTeX at Overleaf: https://pt.overleaf.com/read/kynznbgbfty

Feedback for Module 2

1 Equations - trigonometric functions

(a) $f(x) = \cos(2x) + \cos(3x) + \cos(4x) - 1 = 0$
$f'(x) = -2\sin(2x) - 3\sin(3x) - 4\sin(4x)$

(b) $f(x) = 2\cos(x) - \sin(2x) + \cos(-x) = 0$
$f'(x) = -2\sin(x) - 2\cos(2x) + \sin(-x)$

(c) $f(x) = 4\sin(x) + 4\cos(x) - 4\sin(2x) - 5 = 0$
$f'(x) = 4\cos(x) - 4\sin(x) - 8\cos(2x)$

(d) General form : $f(x) = \sum_{i=1}^{k} a_i \cos(ix) + \sum_{j=1}^{l} b_j \sin(jx) + c = 0,\ a_i, b_j, c \in \mathbb{R}$.

$$f'(x) = \sum_{i=1}^{k} -i \cdot a_i \sin(ix) + \sum_{j=1}^{l} j \cdot b_j \cos(jx)$$

2 Equations - polynomials

(a) $f(x) = x^4 - 3x^3 + x - 1 = 0$
$f'(x) = 4x^3 - 9x^2 + 1$

(b) $f(x) = x^5 + 3x^3 - x + 1 = 0$
$f'(x) = 5x^4 + 9x^2 - 1$

(c) $f(x) = x^3 - 4x^2 + 3x + 1 = 0$
$f'(x) = 3x^2 - 8x + 3$

(d) General form : $f(x) = \sum_{i=0}^{n} a_i x^i = 0,\ a_i \in \mathbb{R}$.

$$f'(x) = \sum_{i=1}^{n} i \cdot a_i x^{i-1}$$

3 Systems of equations - polynomials

(a) $\begin{cases} f_1(x,y) = xy^2 - xy - 1 = 0 \\ f_2(x,y) = x^2 + y^2 - 4 = 0 \end{cases}$

$$\begin{pmatrix} \frac{\partial f_1(x,y)}{\partial x} & \frac{\partial f_1(x,y)}{\partial y} \\ \frac{\partial f_2(x,y)}{\partial x} & \frac{\partial f_2(x,y)}{\partial y} \end{pmatrix} = \begin{pmatrix} y^2 - y & 2xy - x \\ 2x & 2y \end{pmatrix}$$

(b) $\begin{cases} f_1(x,y,z) = xyz - x^3 + y^2x - 3xy - 2 = 0 \\ f_2(x,y,z) = x^2 - y^2 + z^2 = 0 \\ f_3(x,y,z) = xz^2 - z + 3xyz = 0 \end{cases}$

$$\begin{pmatrix} \frac{\partial f_1(x,y,z)}{\partial x} & \frac{\partial f_1(x,y,z)}{\partial y} & \frac{\partial f_1(x,y,z)}{\partial z} \\ \frac{\partial f_2(x,y,z)}{\partial x} & \frac{\partial f_2(x,y,z)}{\partial y} & \frac{\partial f_2(x,y,z)}{\partial z} \\ \frac{\partial f_3(x,y,z)}{\partial x} & \frac{\partial f_3(x,y,z)}{\partial y} & \frac{\partial f_3(x,y,z)}{\partial z} \end{pmatrix} = \begin{pmatrix} yz - 3x^2 + y^2 - 3y & xz + 2yx - 3x & xy \\ 2x & -2y & 2z \\ z^2 + 3yz & 3xz & 2xz - 1 + 3xy \end{pmatrix}$$

dratic equations with n unknowns.

uadratic equation (k) can be written as:

$$\begin{aligned}(\mathbf{x}) = {} & a_1^{(1)}{x_1}^2 + a_2^{(1)}{x_2}^2 + a_3^{(1)}{x_3}^2 + \cdots a_n^{(1)}{x_n}^2 + \\ & + b_{12}^{(1)}x_1x_2 + b_{13}^{(1)}x_1x_3 + \cdots + b_{1n}^{(1)}x_1x_n + \\ & + b_{23}^{(1)}x_2x_3 + b_{24}^{(1)}x_2x_4 + \cdots + b_{2n}^{(1)}x_2x_n + \\ & + b_{34}^{(1)}x_3x_4 + b_{35}^{(1)}x_3x_5 + \cdots + b_{3n}^{(1)}x_3x_n + \\ & \vdots \\ & + b_{n-2,n-1}^{(1)}x_{n-2}x_{n-1} + b_{n-2,n}^{(1)}x_{n-2}x_n \\ & + b_{n-1,n}^{(1)}x_{n-1}x_n + \\ & + c_1^{(1)}x_1 + c_2^{(1)}x_2 + c_3^{(1)}x_3 + \cdots + c_n^{(1)}x_n + \\ & + d_1 = 0 \end{aligned} \qquad (1)$$

$$\begin{aligned}(\mathbf{x}) = {} & a_1^{(2)}{x_1}^2 + a_2^{(2)}{x_2}^2 + a_3^{(2)}{x_3}^2 + \cdots a_n^{(2)}{x_n}^2 + \\ & + b_{12}^{(2)}x_1x_2 + b_{13}^{(2)}x_1x_3 + \cdots + b_{1n}^{(2)}x_1x_n + \\ & + b_{23}^{(2)}x_2x_3 + b_{24}^{(2)}x_2x_4 + \cdots + b_{2n}^{(2)}x_2x_n + \\ & + b_{34}^{(2)}x_3x_4 + b_{35}^{(2)}x_3x_5 + \cdots + b_{3n}^{(2)}x_3x_n + \\ & \vdots \\ & + b_{n-2,n-1}^{(2)}x_{n-2}x_{n-1} + b_{n-2,n}^{(2)}x_{n-2}x_n \\ & + b_{n-1,n}^{(2)}x_{n-1}x_n + \\ & + c_1^{(2)}x_1 + c_2^{(2)}x_2 + c_3^{(2)}x_3 + \cdots + c_n^{(2)}x_n + \\ & + d_2 = 0 \end{aligned} \qquad (2)$$

$$\vdots$$

$$\begin{aligned}(\mathbf{x}) = {} & a_1^{(n)}{x_1}^2 + a_2^{(n)}{x_2}^2 + a_3^{(n)}{x_3}^2 + \cdots a_n^{(n)}{x_n}^2 + \\ & + b_{12}^{(n)}x_1x_2 + b_{13}^{(n)}x_1x_3 + \cdots + b_{1n}^{(n)}x_1x_n + \\ & + b_{23}^{(n)}x_2x_3 + b_{24}^{(n)}x_2x_4 + \cdots + b_{2n}^{(n)}x_2x_n + \\ & + b_{34}^{(n)}x_3x_4 + b_{35}^{(n)}x_3x_5 + \cdots + b_{3n}^{(n)}x_3x_n + \\ & \vdots \\ & + b_{n-2,n-1}^{(n)}x_{n-2}x_{n-1} + b_{n-2,n}^{(n)}x_{n-2}x_n \\ & + b_{n-1,n}^{(n)}x_{n-1}x_n + \\ & + c_1^{(n)}x_1 + c_2^{(n)}x_2 + c_3^{(n)}x_3 + \cdots + c_n^{(n)}x_n + \\ & + d_n = 0 \end{aligned} \qquad (3)$$

In a compact notation. Let be $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. For each $k = 1, 2, \ldots, n$:

$$f_k(\mathbf{x}) = \sum_{i=1}^{n} a_i{}^{(k)}{x_i}^2 + \sum_{i=1}^{n}\sum_{j>i}^{n} b_{ij}{}^{(k)}x_ix_j + \sum_{i=1}^{n} c_i{}^{(k)}x_i + d_k = 0$$

The terms of the Jacobian matrix are given by

$$J_{kj} = \frac{\partial f_k(\mathbf{x})}{\partial x_j} = 2 \cdot a_j{}^{(k)}x_j + \sum_{i=1}^{j-1} b_{ij}{}^{(k)}x_i + \sum_{i>j}^{n} b_{ji}{}^{(k)}x_i + c_j{}^{(k)}$$

(d) General form : System of n polynomial equations with n unknowns.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. For each $k = 1, 2, \ldots, n$:

$$f_k(\mathbf{x}) = \sum_{i=1}^{S} a_i^{(k)} x_1^{p_{1i}^{(k)}} x_2^{p_{2i}^{(k)}} \cdots x_n^{p_{ni}^{(k)}} = 0$$

$a_i^{(k)}, c_k \in \mathbb{R}$

$p_{1i}^{(k)}, p_{2i}^{(k)}, \ldots, p_{ni}^{(k)} \in \mathbb{N}$

$0 \le p_{1i}^{(k)} + p_{2i}^{(k)} + \cdots + p_{ni}^{(k)} \le q$, q is the dregree of polynomial

---

Each equation has a number maximum of terms, named S.

$S = C_{q,q} + C_{q,q-1} + C_{q,q-2} + \cdots + C_{q,2} + C_{q,1} + C_{q,0}$

where, $C_{m,r}$ represents combination with repetition

$$C_{m,r} = \frac{(m+r-1)!}{r!(m-1)!}.$$

The terms of the Jacobian matrix are given by

$$J_{kj} = \frac{\partial f_k(\mathbf{x})}{\partial x_j} = \sum_{\substack{i=1 \\ i \neq i_{ji}^{(k)}}}^{S} p_{ji}^{(k)} a_i^{(k)} x_1^{p_{1i}^{(k)}} x_2^{p_{2i}^{(k)}} \cdots x_{j-1}^{p_{j-1,i}^{(k)}} x_j^{p_{j,i}^{(k)}-1} x_{j+1}^{p_{j+1,i}^{(k)}} \cdots x_n^{p_{n,i}^{(k)}}$$

where $i_{ji}^{(k)}$ represents the i index that $p_{ji}^{(k)} = 0$.

Appendix D: Poster Template

Available in LaTeX at Overleaf: https://pt.overleaf.com/read/wkfmbhkwnfrp