Platonic dynamical decoupling for multi-spin systems

C. Read, E. Serrano-Ensástiga and J. Martin

Institut de Physique Nucléaire, Atomique et de Spectroscopie, CESAM, Université de Liège, 4000 Liège, Belgique

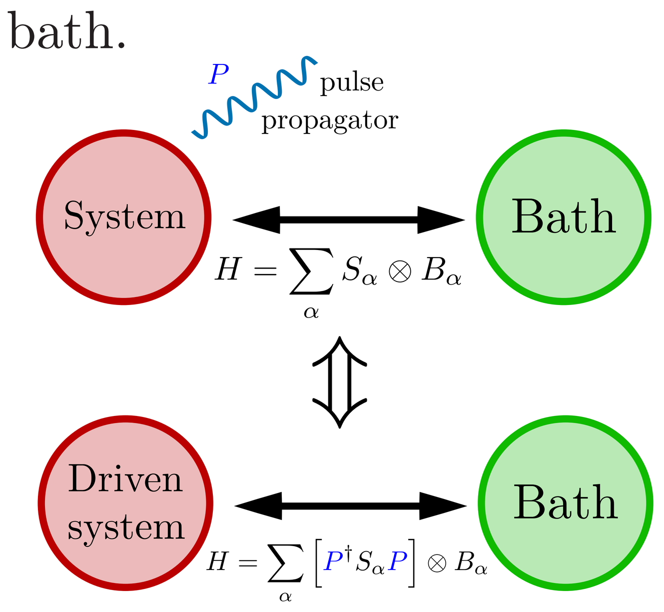
Motivation

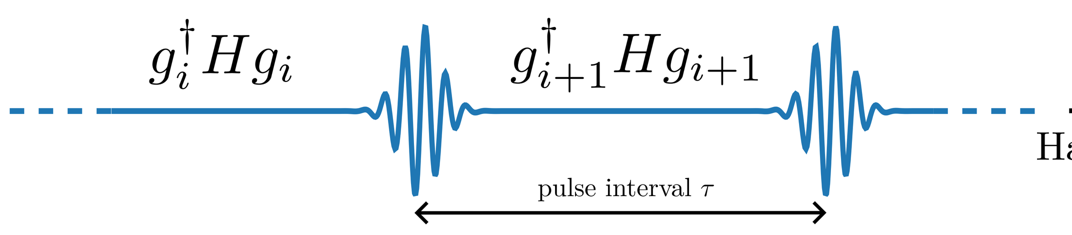
- A dynamical decoupling (DD) protocol is a pulse sequence which decouples a quantum system from its environment, keeping it from decohering [1].
- In this work, we present novel DD sequences constructed from the three Platonic point groups and composed of global SU(2) rotations. We explain which one should be used based on the system of interest.

Dynamical decoupling

Consider a quantum system coupled to some bath.

- The system operators S_α span an *interaction subspace* $\mathcal{I}_S = \text{span}(\{S_\alpha\})$.
- With a periodic sequence of N equidistant pulses, we can design the interaction Hamiltonian on each pulse interval.
- An average Hamiltonian H_{av} governs the effective dynamics of the open quantum system.





$$H_{\text{av}} \approx \sum_\alpha \Pi_{\mathcal{G}}(S_\alpha) \otimes B_\alpha$$

$$\Pi_{\mathcal{G}}(S) = \frac{1}{N}\sum_k g_k^\dagger S g_k$$

If $\mathcal{G} = \{g_k\}$ forms a group, $[\Pi_{\mathcal{G}}(S), g_k] = 0 \ \forall k, S$.

$\Rightarrow$ Each S_α is projected onto a $\mathcal{G}$-invariant subspace $\Pi_{\mathcal{G}}(\mathcal{I}_S)$.

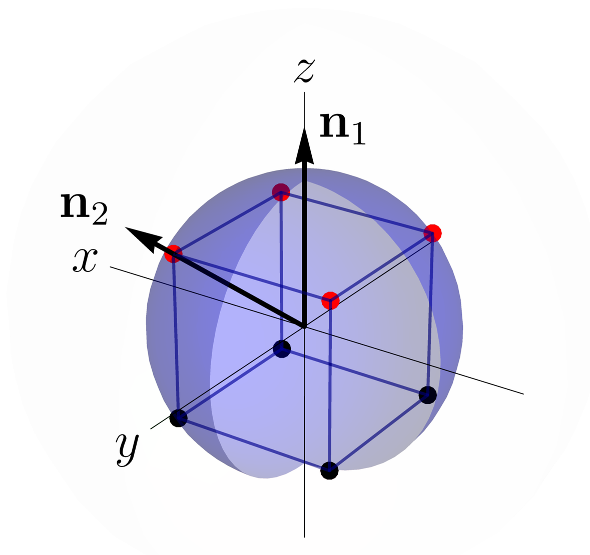
$\Rightarrow$ If $\Pi_{\mathcal{G}}(\mathcal{I}_S)$ only contains the identity, $\mathcal{G}$ is a decoupling group for $\mathcal{I}_S$.

> We find decoupling groups by searching for inaccessible symmetries in spaces of operators.

The DD sequence forms an Eulerian path on the Cayley graph of the group [2].

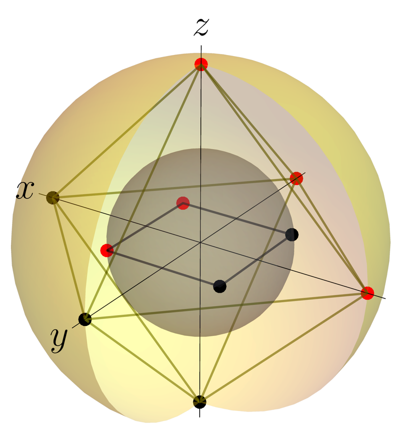
Set of maximal point groups

The space of operators of any quantum system can be written as a direct sum of SU(2)-irrep $\mathcal{B}(\mathcal{H}_S) = \bigoplus_L \mathscr{B}^{(L)}$.



- Hermitian operators of $\mathscr{B}^{(L)}$ can be represented by a (bi-colored) constellation of $2L$ points on a sphere [3].
- Certain constellations have high rotational symmetries (*i.e.*, point groups).
- Due to finite number of points, some point groups are not accessible.

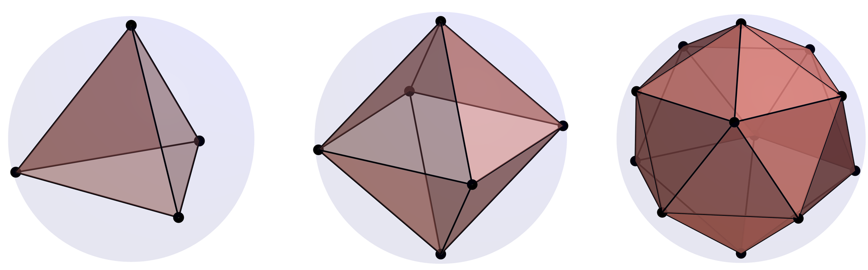
- Any operator of $\mathcal{B}(\mathcal{H}_S)$ can be represented by a set of constellations.
- The list of accessible point groups consists of all point groups of the individual constellations.



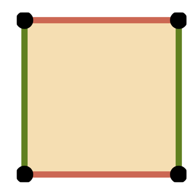
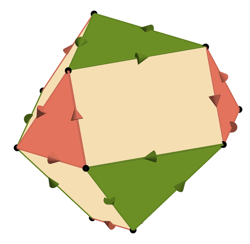
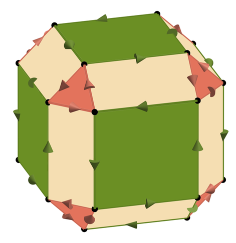
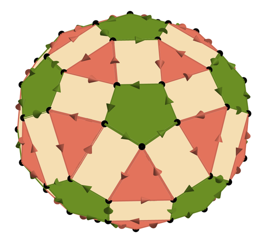
> By finding all possible point groups of the set of constellations, we can find the inaccessible rotational symmetries of any space of operators.

For an interaction subspace $\mathcal{I}_S \subseteq \bigoplus_{l=1}^{L} \mathscr{B}^{(l)}$, we find

L	1	2	3	4	5	≥ 6
Smallest inaccessible point group	D_2	T	O	I	I	none



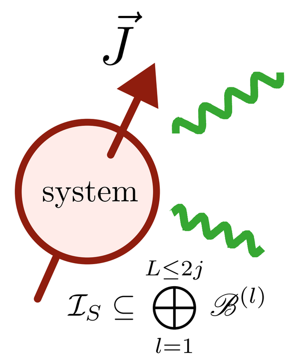
Platonic sequences

Group	D_2	T	O	I
Generators in axis-angle notation	$R_1 = ((1,0,0), \pi)$ $R_2 = ((0,1,0), \pi)$	$R_1 = \left((0,0,1), \frac{2\pi}{3}\right)$ $R_2 = \left(\left(\frac{\sqrt{2}}{3}, \sqrt{\frac{2}{3}}, \frac{1}{3}\right), \frac{2\pi}{3}\right)$	$R_1 = \left((0,0,1), \frac{\pi}{2}\right)$ $R_2 = \left(\frac{1}{\sqrt{3}}(1,1,1), \frac{2\pi}{3}\right)$	$R_1 = \left(\frac{1}{\sqrt{\Phi+2}}(0,-1,\Phi), \frac{2\pi}{5}\right)$ $R_2 = \left(\frac{1}{\sqrt{3}}(1-\Phi,0,\Phi), \frac{2\pi}{3}\right)$ $\Phi = \frac{\sqrt{5}+1}{2}$
Cayley graph				
Number of pulses	8	24	48	120
Sequence name	EDD [2]	TEDD	OEDD	IEDD

Some properties (see Ref. [4]) $\longrightarrow$

- Robust to finite-duration pulses.
- High robustness to systematic control errors.
- Suitable for concatenation and dynamically corrected gates.

Application for a single spin-j



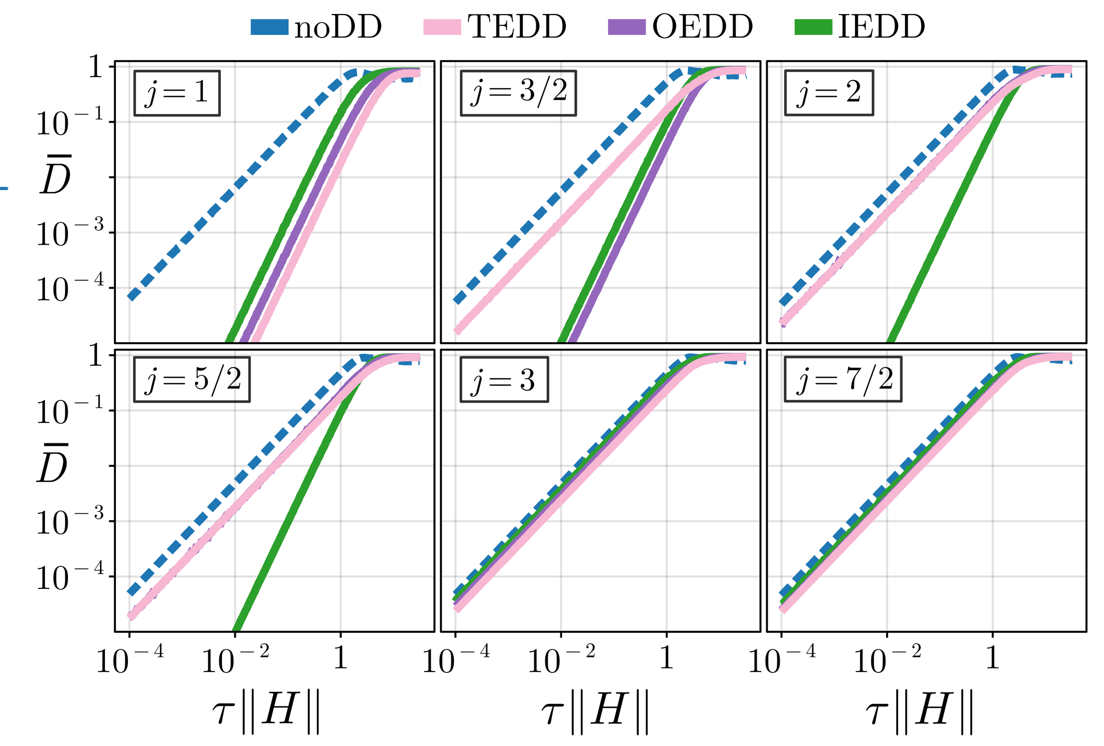
- We find universal$^{(*)}$ decoupling sequences for $j \leq 5/2$.
- Smaller decoupling groups can be used when $L < 2j$.

Average distance between the identity and the propagator of a sample of 5000 spin-j Hamiltonians H under DD. $\leftarrow$

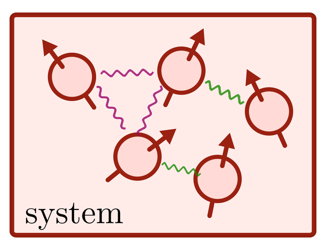
$$\overline{D} \propto (\tau\|H\|)^n, \quad \begin{cases} n=1 & \text{(noDD)} \\ n=2 & \text{(DD)} \end{cases}$$

(*) A decoupling group is universal if it decouples the entire space of operators.

Example : decoupling of arbitrary spin-j Hamiltonian



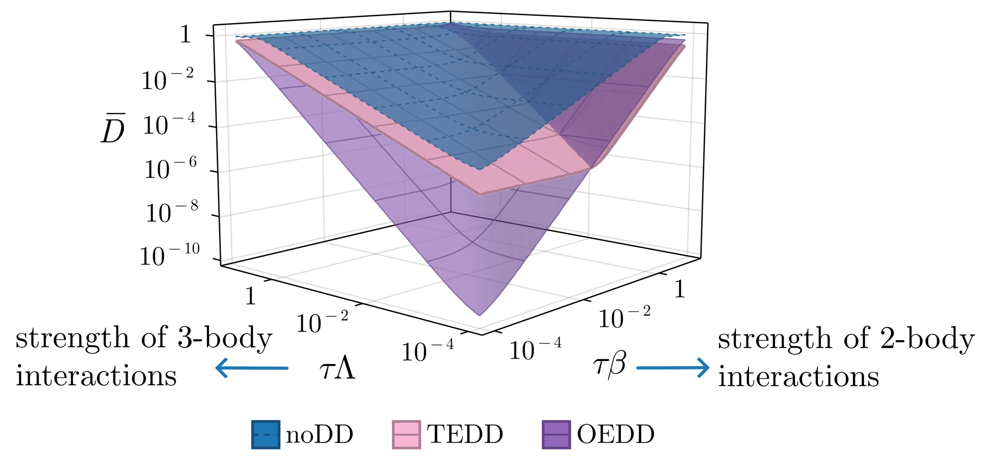
Application for an ensemble of spin-1/2



K	decoupling group
2	T
3	O
4,5	I
≥ 6	none

- We find decoupling sequences which suppress K-body interactions with $K \leq 5$.
- The number of global rotations is independent of the number of spins in the ensemble.
- Only isotropic interactions are not decoupled.

Example : decoupling of 2 and 3-body interactions



Conclusion

- We introduced new Eulerian sequences with great versatility and high robustness properties, constructed from the symmetry groups of the Platonic solids.
- They have direct applications for the decoupling of spin ensembles, large atomic spins and qudits for $d \leq 6$.

References

[1] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999)
[2] L. Viola, E. Knill, Phys. Rev. Lett. **90**, 037901 (2003)
[3] E. Serrano-Ensástiga and D. Braun, Phys. Rev. A. **101**, 022332 (2020)
[4] C. Read, E. Serrano-Ensástiga and J. Martin, arXiv:2409.04974 (2024)