TECHNOLOGICAL ATTRACTION POLES

FINAL REPORT

STATIC AND DYNAMIC DESIGN ANALYSIS PROCEDURES FOR STRUCTURES WITH UNCERTAIN PARAMETERS

PA-31

LAMSENS Paul

Centre de Rech. S&T de l'Industrie des Fabrications Métal.

DE VOS MONIKA

Centre Scientifique et Technique de la Construction

DE ROECK Guido

Katholieke Universiteit Leuven, Afdeling Bouwmechanica

VANDEPITTE Dirk

Katholieke Universiteit Leuven, Afdeling Productietechnieken, machinebouw en automatisering (PMA)

ROCHUS Pierre

Université de Liège, Centre Spatial de Liège

BELGIAN SCIENCE POLICY

CRIF-WTCM

KATHOLIEKE UNIVERSITEIT LEUVEN

CSL

BELGIAN SCIENCE POLICY

Rue de la Science 8
Wetenschapsstraat 8
B-1000 Brussels
Belgium
Tel: +32 (0)2 238 34 11 – Fax: +32 (0)2 230 59 12
http://www.belspo.be

Contact person:
Dimitri Harmegnies
Secretariat: +32 (0)2 238 37 61

TAP 31
Report part 2.3
Development of a consistent fuzzy finite element method for static analysis using interval arithmetic

David Moens
K.U.Leuven, Dept. Mechanical Engineering, PMA
Celestijnenlaan 300B, B-3001 Heverlee, Belgium
david.moens@mech.kuleuven.ac.be

1 Introduction

This annex of the project report describes the study on the applicability of interval arithmetic procedures for the implementation of the fuzzy finite element method. As described in annex 2.1 of this report, the core of such a methodology is an efficient implementation of the interval procedure that has to be performed on all considered sublevels in the membership range. Therefore, this annex focusses on the development of an IFE methodology for the calculation of intervals on the output of a typical structural FE analysis.

In order to apply the interval arithmetic procedure, the starting point, i.e., the deterministic finite element procedure, has to be clearly defined. Therefore, this report starts in section 2 with a short general description of the numerical scheme behind a typical structural FE solution procedure. Section 3 then gives a conceptual description of the required input and output of the IFE analysis. Based on the deterministic scheme, the corresponding interval arithmetic approach is studied by translating each step in the deterministic algorithm to its interval algebraic equivalent. This interval translation requires a methodology for the calculation of the range of the outcome of numerical algorithms, which is described in section 4. The effective interval translation of the general FE solution scheme follows in section 5. The conceptual study focusses principally on the system matrix assembly phase. Because of the high degree of similarity between the assembly phase of the static and dynamic analysis, the study that has been performed has been generalized immediately to structural dynamic finite element analysis. Section 6 finally describes a hybrid methodology, which was proven to be of high practical value in the dynamic response analysis (see annex 2.4 of this project).

2 Deterministic FE algorithm

This section discusses briefly the main aspects of the numerical algorithm for the deterministic dynamic FE analysis based on system matrices. This algorithm is

model definition

$$\textbf{element matrices} \rightarrow \quad \begin{array}{cccc} [K^{e_1}] & [K^{e_2}] & \cdots & [K^{e_n}] \\ [M^{e_1}] & [M^{e_2}] & & [M^{e_n}] \\ [C^{e_1}] & [C^{e_2}] & & [C^{e_n}] \\ \{f^{e_1}\} & \{f^{e_2}\} & \cdots & \{f^{e_n}\} \end{array}$$

$$\textbf{system matrices} \rightarrow \quad [K], [M], [C], \{f\}$$

$$\downarrow$$

$$\textbf{analysis} \rightarrow \{y\} = f\left([K], [M], [C], \{f\}\right)$$

Figure 1: Elementary overview of the algorithm for deterministic dynamic FE analysis

the basis for the definition of the allowed input and required output of the IFE analysis and provides the methodology for the interval matrices assembly described in section 5. Figure 1 gives an elementary description of the four basic steps of the algorithm: the model definition, the element matrix computation, the system matrix assembly and the analysis performed on the system matrices.

The definition of a deterministic FE model consists of the definition and discretisation of the geometry, the definition of the material properties, constraints and loads and the specification of the result request. Each element's contribution to the global system matrices follows from the model definition using the general equations [1]:

$$[K^e] = \int_{V^e} [B]^T [D] \; [B] \; dV^e \tag{1}$$

$$[M^e] = \int_{V^e} [N]^T \rho \; [N] \; dV^e \tag{2}$$

$$[C^e] = \int_{V^e} [N]^T \mu \; [N] \; dV^e \tag{3}$$

with $[B] = \partial[N]$, ∂ a linear operator, $[N]$ the element shape functions, $[D]$ the material stiffness matrix, V^e the volume of an element, ρ the material mass density and μ the material damping factor. For simple elements, analytical expressions of the element matrices generally are available. For more complex elements, a numerical integration scheme gives a good approximation of the element matrices.

When the analysis aims at the behaviour of the model under external loading conditions, the external force vector equals:

$$\{f^e\} = \int_{V^e} [N]^T \{b\} \; dV^e + \int_{A^e} [N]^T \{t\} \; dA^e \tag{4}$$

with $\{b\}$ and $\{t\}$ representing the external volume and surface forces.

The assembly of the deterministic dynamic system matrices from each element's stiffness, mass and damping matrices $[K^{e_i}]$, $[M^{e_i}]$ and $[C^{e_i}]$ yields:

$$[K] = \bigcup_{i=1...n} [K^{e_i}] \tag{5}$$

$$[M] = \bigcup_{i=1...n} [M^{e_i}] \tag{6}$$

$$[C] = \bigcup_{i=1...n} [C^{e_i}] \tag{7}$$

with $\bigcup$ representing the assembly over the n elements e_i of the FE model. Similarly, the assembly of the load vector yields:

$$\{f\} = \bigcup_{i=1...n} \{f^{e_i}\} \tag{8}$$

The final step consists of the dynamic analysis using the system matrices and load vector. The most frequent types of dynamic analyses are based on the dynamic equilibrium equation:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \tag{9}$$

The most popular numerical solution procedures are the following:

- **time domain analysis**: the analysis aims at the transient time analysis of the structure under well defined loading conditions. This involves the solution of equation (9) in the time domain.
- **frequency domain analysis**: the analysis aims at the description of the structure's dynamic behaviour through frequency response functions. For this purpose, the equilibrium expressed by equation (9) is transformed to its frequency domain counterpart:

$$\left([K] + \jmath\omega[C] - \omega^2[M]\right)\{X\} = \{F\} \tag{10}$$

  with ω varying over the considered frequency domain and $\{F\}$ and $\{X\}$ representing the amplitude of the harmonic excitation force and resulting displacement.
- **modal superposition**: the analysis first calculates the eigenfrequencies and eigenmodes of the structure. This involves the solution of the general eigenvalue equation:

$$[K]\{\phi\} = \lambda[M]\{\phi\} \tag{11}$$

  with λ the eigenvalue and $\{\phi\}$ the corresponding eigenvector. Based on this information, the frequency response function can be assembled using the modal superposition principle:

$$FRF_{ij} = \sum_{i=1}^{n} \frac{\phi_{ij}\,\phi_{ik}}{\{\phi_i\}^T[K]\{\phi_i\} - \omega^2\{\phi_i\}^T[M]\{\phi_i\}} \tag{12}$$

In the remainder of this work, damping is considered to be proportional. This means that the damping matrix follows directly from the global stiffness and mass matrix using:

$$[C] = \alpha_K\,[K] \;+\; \alpha_M\,[M] \tag{13}$$

with α_K and α_M proportional constants to be defined for each analysis. This is one of the most common models of damping (also referred to as RAYLEIGH damping). The assumption of proportional damping is purely made for mathematical convenience as it simplifies the solution process. Although the assumption has no real physical basis, in practice the material damping coefficients are rarely known in sufficient detail to justify a more complicated damping model. Furthermore, the proportional damping model has proved to be reliable for structures with damping below 10% of critical [2].

3 Overview of the IFE procedure

3.1 IFEM input: interval model description

Only the physical properties of the model quantified by the analyst are allowed to be subject to uncertainty or variability. All of these adopt an interval model. For uncertainties, the analyst defines the feasible domain of the uncertain property for the analysis. He is completely free to choose the interval which he judges interesting or necessary to analyse. Referring to the deterministic dynamic FE procedure described in section 2 the following properties are allowed to be non-deterministic:

- model geometry: prescribed tolerances on the design dimensions, thickness of plates, area of beams, ...
- material parameters: material Young's modulus, Poisson constant, damping properties, material density, ...
- constraints: refers to the unknown stiffness of the connection of the constrained DOFs to the fixed environment

In the framework of dynamic design validation and optimisation, this project aims at the eigenfrequency analysis and frequency response function calculation. The eigenfrequency analysis does not take external loads into consideration. The frequency response analysis considers a known deterministic harmonic excitation force or moment in a single node. Therefore, this study further does not take non-deterministic loading conditions into account.

3.2 IFEM output: required result

Requirements for the dynamic properties of a mechanical structure are commonly expressed in terms of eigenfrequencies. In order to study reliability, the range of the critical eigenfrequencies is pursued through an interval eigenvalue analysis.

However, rather than eigenfrequencies, the specification of maximal allowable responses at frequencies that are critical in the operating conditions of the structure would be a more realistic concept for dynamic design requirements. The IFE analysis of frequency response functions enables the use of such design requirements. For this analysis, the required output is an envelope function for the response, defining an upper and lower bound on the response in the considered frequency range.

Since generally design requirements are expressed in terms of the lower, easily identifiable modes, this work focusses on this range of the frequency domain. In the middle and higher frequency range, the high modal density generally renders a meaningful deterministic FE analysis impossible. Likewise, the effect of uncertainties on the dynamic behaviour in this frequency region is more complicated. It is the aim of this research to define a working methodology for IFE analysis in the lower frequency range. An extension to the higher frequency domain is left open for future research.

From the viewpoint of non-probabilistic design validation, the worst case scenario is pursued. Therefore, conservative results are preferable above underestimations. This is extremely important because of the crisp definition of reliability based on non-probabilistic analysis. An underestimated upper bound on a design validation property could wrongly declare the analysed input range allowable with respect to the design specifications. On the other hand, precaution is necessary in order to prevent exorbitant conservatism.

3.3 IFE analysis procedures

As described in annex 2.1 of this project, there are two basic strategies for the implementation of the IFE analysis: the global optimisation strategy and the interval arithmetic approach. The applicability of either of these for the IFE analysis strongly depends on the intended analysis type.

When design requirements are stated in the form of eigenfrequency regions that should be avoided, the eigenfrequency analysis focusses on a limited number of modes. The eigenvalue and eigenvector derivatives with respect to an input parameter can be expressed using the corresponding analytical system matrices derivatives, which are generally available. This makes the optimisation approach a valuable procedure for IFE eigenfrequency analysis. On the other hand, the interval arithmetic approach based on interval system matrices requires the calculation of the set:

$$\langle\lambda\rangle = \left\{ \lambda \mid \Big([K] \in [\mathbf{K}]\Big)\Big([M] \in [\mathbf{M}]\Big)\Big([K]\{\phi\} = \lambda\,[M]\{\phi\}\Big) \right\} \tag{14}$$

There are general analytical solution schemes available for the solution of this interval problem.

The interval frequency response analysis pursues a continuous description of the response interval over the considered frequency domain. This requires the optimisation to be performed successively on a large number of discrete frequencies. Furthermore, the analytical derivative of a response to an input parameter is not generally available. This renders the optimisation approach very unattractive for the IFE FRF analysis. The interval arithmetic approach requires the calculation of the following set:

$$\begin{aligned}\langle\{X\}\rangle \quad = \quad & \Big\{\{X\} \mid \Big([K] \in [\mathbf{K}]\Big)\Big([M] \in [\mathbf{M}]\Big)\ldots \\ & \qquad \Big(\left([K] + \jmath\,\omega\,[C] - \omega^2\,[M]\right)\{X\} = \{F\}\Big)\Big\}\end{aligned} \tag{15}$$

This set needs to be calculated for a large number of discrete frequencies in order to obtain an envelope function. The basic numerical problem at each frequency is equivalent to that of the equilibrium IFE analysis. Interval arithmetic solution procedures for this problem were reviewed in annex 2.1 of this project.

4 The set algorithm translation concept

The IFE analysis based on interval arithmetic requires a tool to calculate the range of the result of the FE algorithm based on interval input properties. For this purpose, the algorithm translation methodology is developed. This forms the basis of the interval arithmetic IFE procedure. This section starts from the definition of the interval and set functions to facilitate the mathematical description of the developed methods.

4.1 Interval and set functions

The range of a general function $f\left(x_1, \ldots x_n\right)$ with reference to the set vector $\big\langle\{x\}\big\rangle$ is denoted by:

$$\langle f\left(x_1, \ldots x_n\right)\rangle_{\big\langle\{x\}\big\rangle} \tag{16}$$

The range of the function is defined as the set of all results of the function considering all possible combinations of the function's argument inside the defined vector $\big\langle\{x\}\big\rangle$:

$$\langle f\left(x_1, \ldots x_n\right)\rangle_{\big\langle\{x\}\big\rangle} = \Big\{f\left(x_1, \ldots x_n\right) \mid \left(x_i \in \langle x_i\rangle, i = 1 \ldots n\right)\Big\} \tag{17}$$

This definition of the range of a function considers the argument sets mutually independent. This is of great importance when considering the conservatism of the resulting range of the function for specific applications, as discussed in section 4.4. In order to enhance the readability the notation of equation (16) is simplified using the *set function* $\breve{f}$:

$$\breve{f}\big(\langle x_1\rangle, \ldots \langle x_n\rangle\big) = \langle f\left(x_1, \ldots x_n\right)\rangle_{\big\langle\{x\}\big\rangle} \tag{18}$$

Both the arguments and the result of a set function are sets. Equation (18) in conjunction with equation (17) defines the required result of $\breve{f}$. The advantage of this notation is that the set function $\breve{f}$ defines an action directly on the set arguments of the function. Therefore, it can be analytically defined, for instance as an action on the bounds of the argument sets. However, an analytical expression of the result of a set function exists only for very simple functions and depends strongly on the complexity of the argument sets. This means that an implementation of a set

$$
\begin{array}{ccc}
\text{inputs} & \longrightarrow & \{x\} = \{x_1, x_2, \dots x_n\} \\
1^{st}\ \text{step} & \longrightarrow & \{z_1\} = f_1\Big(\{x\}\Big) \\
2^{nd}\ \text{step} & \longrightarrow & \{z_2\} = f_2\Big(\{z_1\}\Big) \\
\dots & \longrightarrow & \dots \\
m^{th}\ \text{step} & \longrightarrow & \{y\} = \{z_m\} = f_m\Big(\{z_{m-1}\}\Big)
\end{array}
$$

Figure 2: General algebraic description of an algorithm

function based on a specific argument set type is not necessarily extendable to all other types of argument sets.

In the particular case where all non-deterministic argument sets of the considered function are interval objects, the set function is referred to as the *interval function.* Its definition is:

$$\breve{f}\left(\mathbf{x_1}, \mathbf{x_2}, \dots \mathbf{x_n}\right) = \langle f\left(x_1, x_2, \dots x_n\right)\rangle_{\{\mathbf{x}\}} \tag{19}$$

Compared to a general set function, an interval function is generally easier to implement using the lower and upper bounds of the argument intervals. This is explicitly the case for the interval functions for the four elementary operations: addition, subtraction, multiplication and division:

$$
\begin{aligned}
\mathbf{a}+\mathbf{b} &= \left[\underline{a}+\underline{b}, \overline{a}+\overline{b}\right] && (20)\\
\mathbf{a}-\mathbf{b} &= \left[\underline{a}-\overline{b}, \overline{a}-\underline{b}\right] && (21)\\
\mathbf{a}\times\mathbf{b} &= [\min(\underline{a}\times\underline{b}, \underline{a}\times\overline{b}, \overline{a}\times\underline{b}, \overline{a}\times\overline{b})\dots \\
&\qquad \max(\underline{a}\times\underline{b}, \underline{a}\times\overline{b}, \overline{a}\times\underline{b}, \overline{a}\times\overline{b})] && (22)\\
\mathbf{a}/\mathbf{b} &= \mathbf{a}\times\left[\frac{1}{\overline{b}}, \frac{1}{\underline{b}}\right], \ \text{if } 0 \notin \mathbf{b} && (23)
\end{aligned}
$$

A conservative approximation of the range of a function $f\left(\{x\}\right)$ referring to the set object $\left\langle\{x\}\right\rangle$ is denoted by $\langle\!\langle f\left(\{x\}\right)\rangle\!\rangle_{\left\langle\{x\}\right\rangle}$ similar to the conservative set and interval object approximations.

4.2 Conceptual overview of the set algorithm translation

Any numerical analysis is based on some deterministic algorithm applied on a number of inputs. The output of the algorithm is the required analysis result. In order for the algorithm to be of any practical use, it should be possible to implement it as a sequence of analytical functions. Figure 2 describes a general algorithm with the conventions applied in this work. There are n inputs denoted by a vector $\{x\} = \{x_1, x_2, \dots x_n\}$. The total algorithm is represented by a function $f\left(x_1, x_2, \dots x_n\right)$. The final requested result is denoted in the vector $\{y\}$. The total algorithm is split into substeps. Each step has its own *subfunction* f_i which calculates the required variables for the next step of the algorithm $\{z_i\}$ based on the result of the previous step's results $\{z_{i-1}\}$. The vector $\{z_i\}$ is referred to as the i^{th} step's *intermediate variables.* Applying all subfunction in right order on the inputs

$$
\begin{array}{ccc}
\text{basic parameters} & \longrightarrow & \{\mathbf{x}\} = \{\mathbf{x_1}, \mathbf{x_2}, \dots \mathbf{x_n}\} \\
1^{st} \text{ step} & \longrightarrow & \langle\{z_1\}\rangle = \breve{f}_1\Big(\{\mathbf{x}\}\Big) \\
2^{nd} \text{ step} & \longrightarrow & \langle\{z_2\}\rangle = \breve{f}_2\Big(\langle\{z_1\}\rangle\Big) \\
\dots & \longrightarrow & \dots \\
m^{th} \text{ step} & \longrightarrow & \langle\{y\}\rangle = \breve{f}_m\Big(\langle\{z_{m-1}\}\rangle\Big)
\end{array}
$$

Figure 3: General algebraic description of a set algorithm resulting from the set algorithm translation concept

yields the total algorithm expressed as a series of nested functions:

$$
f\left(x_1, x_2, \dots x_n\right) = f_m\left(f_{m-1}\left(\dots f_2\left(f_1\left(x_1, x_2, \dots x_n\right)\right)\dots\right)\right) \tag{24}
$$

The goal of the set algorithm translation concept is to obtain a general applicable computation methodology to determine the range of the result of a deterministic algorithm when a number of inputs are defined as interval scalars. Consider n uncertain independent inputs referred to as the *basic parameters*, the uncertainty of which is described by an interval. The set algorithm translation concept translates all subfunctions of the deterministic algorithm to the domain of set arithmetic. This means that for every subfunction in the deterministic algorithm, the corresponding set subfunction is implemented. The arguments of this set subfunction are sets resulting from the previous set subfunctions. Initially, these are the intervals defined for the basic parameters. The result of a set subfunction is again a set vector containing the sets used as argument for the next set subfunction. By applying this methodology consecutively on all algorithm subfunctions, the range of the final result of the algorithm is calculated. Figure 3 describes the set algorithm corresponding to the deterministic algorithm described in figure 2.

4.3 The inclusion property as source of conservatism

The methodology described in the previous section generally does not result in the exact description of the range of the algorithm. A simple example illustrates this. Consider the algorithm with two inputs x_1 and x_2 defined by $f_2\left(f_{11}\left(x_1\right), f_{12}\left(x_1, x_2\right)\right)$. The uncertainty on the inputs is defined by the basic parameters $\mathbf{x_1}$ and $\mathbf{x_2}$. Figure 4 describes the deterministic and corresponding set algorithm.

From the definition in equation (18) it is known that a set function considers all its argument sets independently. In this case, this does not comply with reality since the argument sets $\langle z_{11}\rangle$ and $\langle z_{12}\rangle$ are coupled through the common basic parameter $\mathbf{x_1}$. Consequently, the set resulting from application of $\breve{f}_2$ on the sets $\langle z_{11}\rangle$ and $\langle z_{12}\rangle$ is possibly an overestimation of the exact range of the total algorithm. The inclusion property of the range of nested set functions generalises this observation:

$$
\Big\langle f\Big(g_1\left(\{x\}\right), \dots g_m\left(\{x\}\right)\Big)\Big\rangle_{\langle\{x\}\rangle} \subseteq \breve{f}\Big(\breve{g}_1(\langle\{x\}\rangle), \dots \breve{g}_m(\langle\{x\}\rangle)\Big) \tag{25}
$$

The inclusion property can be explained intuitively. The set on the left-hand side is assembled from all results of the nested function for which the arguments $\{x\}$ are

deterministic algorithm	**set algorithm**
$$\begin{tikzcd}[column sep=tiny] & x_1 \arrow[dl] \arrow[dr] & & x_2 \arrow[dl] \\ z_{11} = f_{11}(x_1) \arrow[dr] & & z_{12} = f_{12}(x_1, x_2) \arrow[dl] & \\ & f_2(z_{11}, z_{12}) & & \end{tikzcd}$$	$$\begin{tikzcd}[column sep=tiny] & \mathbf{x_1} \arrow[dl] \arrow[dr] & & \mathbf{x_2} \arrow[dl] \\ \langle z_{11}\rangle = \breve{f}_{11}(\mathbf{x_1}) \arrow[dr] & & \langle z_{12}\rangle = \breve{f}_{12}(\mathbf{x_1}, \mathbf{x_2}) \arrow[dl] & \\ & \breve{f}_2(\langle z_{11}\rangle, \langle z_{12}\rangle) & & \end{tikzcd}$$

Figure 4: Comparison of a simple deterministic algorithm with the corresponding set translated algorithm

$$\begin{array}{ccc}
\text{basic parameters} & \longrightarrow & \{\mathbf{x}\} = \{\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}\} \\
1^{st} \text{ step} & \longrightarrow & \left\langle\!\!\left\langle \{z_1\} \right\rangle\!\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}_1\Big(\{\mathbf{x}\}\Big) \\
2^{nd} \text{ step} & \longrightarrow & \left\langle\!\!\left\langle \{z_2\} \right\rangle\!\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}_2\left(\left\langle\!\!\left\langle \{z_1\} \right\rangle\!\!\right\rangle_{\{\mathbf{x}\}}\right) \\
\ldots & \longrightarrow & \ldots \\
m^{th} \text{ step} & \longrightarrow & \left\langle\!\!\left\langle \{y\} \right\rangle\!\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}_m\left(\left\langle\!\!\left\langle \{z_{m-1}\} \right\rangle\!\!\right\rangle_{\{\mathbf{x}\}}\right)
\end{array}$$

Figure 5: Conservative approximations inside the general algebraic description of a set algorithm

inside $\big\langle\{x\}\big\rangle$. These arguments are equal for all inner functions g_i. The right-hand expression of this inequality considers all argument sets of the outer set function $\breve{f}$ independently. This is equivalent to allowing different values for the arguments for each inner function g_i. Consequently, the right-hand expression adds to the exact range the result of combinations of the inner set values which are artificial.

Referring to the algorithm translation concept, the inclusion property states that whenever a set subfunction is applied on argument sets which are related through common basic parameters, possibly an overestimation of the set subfunction's range is made by considering the argument sets as independent. By applying the inclusion property on every subfunction of a general algorithm represented as in equation (24), we can state that the final set resulting from a set algorithm contains the exact range of the corresponding deterministic algorithm, but we have no information on the accuracy of the predicted bounds. Only in the specific case where the arguments of each and every subfunction are not correlated through a common basic parameter does the set algorithm translation certainly yield the exact range of the corresponding deterministic algorithm. Figure 5 describes the set algorithm using the appropriate notation for conservative approximations.

4.4 Predicting the degree of conservatism of a set algorithm

At this point, an important question arises concerning the size of the overestimation incorporated in the result of the set algorithm. This overestimation should not be too large in order for the result to be of any practical use. Furthermore, the degree of conservatism could be an important benchmark to compare different set algorithm implementations based on different deterministic algorithms of the same numerical analysis. When different set algorithms are available, the one with the least conservative result is preferable, although other computational aspects could play an important role in the selection of the algorithm. It is, however, very difficult to predict the degree of conservatism of a set algorithm. A simple example illustrates this. Consider the function $f(x) = x$ with x ranging in $\mathbf{x} = [0, 1]$, which is obviously also the exact range of the function. Now consider an algorithm $f(x) = f_{11}(x) + f_{12}(x)$ with:

$$\begin{aligned} f_{11}(x) &= \frac{2x}{3} \\ f_{12}(x) &= \frac{x}{3} \end{aligned}$$

The set algorithm on f_{11} and f_{12} yields:

$$\begin{aligned} \breve{f}_{11}(\mathbf{x}) &= \left[0, \tfrac{2}{3}\right] \\ \breve{f}_{12}(\mathbf{x}) &= \left[0, \tfrac{1}{3}\right] \end{aligned}$$

and the final result yields $\left(\left[0, \frac{2}{3}\right] + \left[0, \frac{1}{3}\right]\right) = [0, 1]$, which is the exact solution. Now consider an alternative algorithm $f(x) = f_{11}(x) + f_{12}(x)$ which yields the same deterministic result:

$$\begin{aligned} f_{11}(x) &= \frac{3x}{2} \\ f_{12}(x) &= -\frac{x}{2} \end{aligned}$$

The set algorithm on f_{11} and f_{12} yields:

$$\begin{aligned} \breve{f}_{11}(\mathbf{x}) &= \left[0, \tfrac{3}{2}\right] \\ \breve{f}_{12}(\mathbf{x}) &= \left[-\tfrac{1}{2}, 0\right] \end{aligned}$$

and the final result yields $\left(\left[0, \frac{3}{2}\right] + \left[-\frac{1}{2}, 0\right]\right) = \left[-\frac{1}{2}, \frac{3}{2}\right]$. This alternative set algorithm's solution is conservative.

While both algorithm implementations above are extremely simple and only differ in the definition of the inner functions, there is an important difference in their approximation of the exact result range. It seems that the choice of the second algorithm is unfortunate, since the unlinking of f_{11} and f_{12} in this case has an important influence on the outer function, while in the first algorithm it had no influence whatsoever. This is due to the unlinking of the single argument x from itself over the two subfunctions. This is actually a special case of the inclusion property. Generally, when the range of a function with multiple occurrences of one set argument is calculated, the argument is unlinked from itself, resulting in an overestimation. Simplifying the function to an alternative form which cancels multiple occurrences of set arguments generally solves this problem. However, this is only possible for very basic functions.

This simple example shows that predicting a set algorithm's degree of conservatism requires an extensive analytical study of each set subfunction regarding the effect on the unlinked set arguments, where properties as monotonicity, linearity and convexity play an important role. In the case of the numerical procedure of the FE analysis, this would be an extremely cumbersome and problem dependent task. While this analytical study might be feasible for some academic examples, it is definitely not suited for implementation in an efficient IFEM code. The most important conclusion here is that it is difficult to make general statements about the degree of conservatism of set algorithms based only on their analytical description. It is, however, important to detect the sources of conservatism in a set algorithm. This only results in a qualitative statement regarding the conservatism, but it might already be a clear indication regarding the applicability of the developed set algorithm.

Still, it is possible to study the degree of conservatism for a set algorithm by comparing the set algorithm range with other approximations. A Monte Carlo simulation could be used for this purpose. Introducing uniform probability distributions over the intervals describing the basic parameters results in a probability distribution on the result of the analysis. However, while the Monte Carlo analysis aims at a good description of the first statistical moments and, therefore, mainly concentrates on the centre of the resulting range, the interval analysis is aimed at the bounds of the range. This implies that a study of the range of an algorithm through Monte Carlo simulation can only be meaningful if enough samples are taken. Furthermore, if the result of the deterministic algorithm is strong non-linearly coupled to the basic parameters, the uniform distribution could yield misleading results. In this case, it is mandatory to repeat the Monte Carlo analysis for a variety of probability distributions on the basic parameters in order to incorporate as much of the result range as possible. The fact remains, however, that one can never predict how much of the exact range really is covered by a Monte Carlo analysis.

5 Dynamic IFEM based on interval system matrices

The IFE analysis based on interval system matrices results from the application of the set algorithm translation on the deterministic FE procedure for dynamic analysis as described in section 2. This section describes the effect of the set algorithm translation on the first three principal steps of the FE numerical procedure: the model definition, the element matrix calculation and the system matrix assembly phase. It only briefly discusses the analysis phase. It focusses in particular on all possible sources of conservatism during these steps. The following chapters treat the analysis phase in detail for specific applications.

5.1 IFE model definition

The translation of the modelling phase to interval analysis requires the identification and quantification of the properties of the deterministic algorithm which are subject to uncertainty. These basic parameters should be deduced to a set of mutually independent closed intervals. The vector $\{x\}$ denotes the vector of basic parameters which represent the uncertainties in the model. The interval vector $\{\mathbf{x}\}$ expresses the interval uncertainty defined for each basic parameter. The certain model properties are constants in the set algorithm and, therefore, not mentioned explicitly. The required analysis result depends on the nature of the problem. Representing the total deterministic FE procedure by $\{y\} = \mathcal{F}(\{x\})$, the IFE analysis aims at the solution of:

$$\langle\{y\}\rangle_{\{\mathbf{x}\}} = \left\{\mathcal{F}(\{x\}) \mid \{x\} \in \{\mathbf{x}\}\right\} \tag{26}$$

5.2 IFEM element matrices

The calculation of an element stiffness or mass matrix as expressed in equations (1) and (2) consists of an integration of an integrand over the volume of the element. This volume could be uncertain due to geometrical uncertainties in the input. The dependency upon these uncertainties is expressed explicitly as $V_e(\{x\})$. The integrand consists of factors based on the element shape functions and on material properties. These material properties could also be uncertain, expressed explicitly as $D(\{x\})$ and $\rho(\{x\})$. Substituting these into equations (1) and (2) yields:

$$[K^e(\{x\})] = \int_{V^e(\{x\})} [B]^T \; [D(\{x\})] \; [B] \; dV^e \tag{27}$$

$$[M^e(\{x\})] = \int_{V^e(\{x\})} [N]^T \; \rho(\{x\}) \; [N] \; dV^e \tag{28}$$

These equations express explicitly the direct dependency of the element matrices on the basic parameters. The treatment of these expressions distinguishes thoroughly between 1D and more complex element types. Therefore, they are treated separately here.

5.2.1 1D elements

For 1D elements, the analytical expressions of the element matrices are generally available in reference to the element coordinate system. This is illustrated here using a bar element in a two-dimensional FE analysis. For this case, the analytical descriptions of the stiffness and consistent mass matrix in the local coordinate system yield:

$$[K^e_{local}] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{29}$$

and

$$[M^e_{local}] = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{30}$$

with E the Young's modulus, I the area moment of inertia, L the length, ρ the mass density and A the cross section area of the element. When either of the physical properties involved in a matrix entry calculation is a basic parameter, the interval element matrices $[\mathbf{K^e_{local}}]$ and $[\mathbf{M^e_{local}}]$ result from substituting the corresponding intervals directly into these analytical expressions.

Generally, a rotation on the local matrices is necessary to compensate for the angle between the global and the local coordinate system. Referring to the example of the bar element this rotation numerically yields:

$$\begin{aligned} \left[K^e_{global}\right] &= [A]^T \; [K^e_{local}] \; [A] \quad (31) \\ \left[M^e_{global}\right] &= [A]^T \; [M^e_{local}] \; [A] \quad (32) \end{aligned}$$

with

$$[A] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix} \quad (33)$$

and θ the angle between the local and the global coordinate system. Whenever this angle is subject to uncertainty, the rotation matrix is an interval matrix $[\mathbf{A}]$. Substituting the interval matrices $[\mathbf{K^e_{local}}]$, $[\mathbf{M^e_{local}}]$ and $[\mathbf{A}]$ in equations (31) and (32) yields the general expression for the global interval element matrices:

$$\begin{aligned} \left[\mathbf{K^e_{global}}\right] &= [\mathbf{A}]^T \, [\mathbf{K^e_{local}}] \, [\mathbf{A}] \quad (34) \\ \left[\mathbf{M^e_{global}}\right] &= [\mathbf{A}]^T \, [\mathbf{M^e_{local}}] \, [\mathbf{A}] \quad (35) \end{aligned}$$

The resulting matrices are interval matrices since the calculation of the global interval matrix entries involves only multiplications and additions of interval scalars. Whether the interval entries describe the exact range or are conservative approximations depends on the basic parameters.

A basic parameter which simultaneously affects multiple of the element's geometry and material properties appearing in one matrix entry causes an overestimation in the corresponding matrix entry range calculation. It is clear from equation (29) that for the stiffness matrix, this would imply that some sort of correlation exists between the length, the area moment of inertia and the Young's modulus through a common basic parameter. It is plausible to assume that these types of uncertainties do not occur in the model. In that case, the interval matrices obtained for 1D elements based on the analytical expression of the local element matrices describe the exact range for each entry in the matrix. However, equations (34), (35) and the definition of $[A]$ in equation (33) indicate that there are multiple occurrences of the angle θ and the length L in the calculation of each global interval matrix entry through the matrix multiplication. This unlinks these properties during this substep of the algorithm. Therefore, applying the rotation introduces conservatism if the orientation or length of the bar is a basic parameter. Furthermore, when the global geometry of the model is uncertain, defined by interval scalars for the uncertain element's nodal coordinates, there exists a correlation between the orientation angle and the length of an element. The matrix rotation neglects this correlation, which gives rise to yet another source of conservatism.

To summarise, it is plausible to state that an uncertain nodal geometry of 1D elements is the only possible source of conservatism in the interval element matrices calculation. The above argumentation can be easily generalised to 1D elements in three-dimensional FE analysis.

5.2.2 2D and 3D elements

For 2D and 3D elements, equations (1) and (2) are generally solved by applying a numerical integration scheme on the integrands. The deterministic Gauss-Legendre numerical integration is based on the evaluation of the integrand in a predefined number of Gauss points in the element's natural coordinate system. For 3D elements, this is stated as:

$$\begin{aligned} [K^e] &\simeq \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} H_i H_j H_k \left[f_K\left(\xi_i, \eta_j, \zeta_k\right)\right] && (36) \\ [M^e] &\simeq \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} H_i H_j H_k \left[f_M\left(\xi_i, \eta_j, \zeta_k\right)\right] && (37) \end{aligned}$$

with

$$\begin{aligned} \left[f_K\left(\xi,\eta,\zeta\right)\right] &= \left[B\left(\xi,\eta,\zeta\right)\right]^T [D] \left[B\left(\xi,\eta,\zeta\right)\right] \det\left[J\left(\xi,\eta,\zeta\right)\right] && (38) \\ \left[f_M\left(\xi,\eta,\zeta\right)\right] &= \left[N\left(\xi,\eta,\zeta\right)\right]^T \rho \left[N\left(\xi,\eta,\zeta\right)\right] \det\left[J\left(\xi,\eta,\zeta\right)\right] && (39) \end{aligned}$$

with (ξ_i, η_j, ζ_k) the evaluation coordinates in the natural coordinate system, H_i,H_j and H_k the corresponding weights from the Gauss-Legendre integration scheme and $[J]$ the Jacobian describing the transformation between the element's natural coordinate system and the global coordinate system. For isoparametric elements, the Jacobian following the classic deterministic calculation scheme yields:

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \cdots & \frac{\partial N_n}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_n}{\partial \eta} \\ \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \cdots & \frac{\partial N_n}{\partial \zeta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \cdots & \cdots & \cdots \\ x_n & y_n & z_n \end{bmatrix} \tag{40}$$

with (x_i, y_i, z_i) describing the nodal coordinates of the element.

The calculation of the interval element matrix requires the set translation of this numerical integration scheme. The Gauss points in the natural coordinate system are analytically defined in reference to the natural element. Uncertainties in the model description do not affect this natural element since it is a model independent entity. Therefore, the uncertainty in the numerical integration is concentrated in the integrand defined in the natural coordinate system as described in equations (38) and (39).

Basic parameters for material properties in $[D]$ and ρ are implemented by substituting the corresponding interval objects directly in the integrand evaluation at the Gauss points. Basic parameters for geometry properties affect the Jacobian matrix. By substituting the uncertain geometry of the element's nodal coordinates in equation (40), the interval Jacobian matrix equals:

$$[\mathbf{J}] = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \cdots & \frac{\partial N_n}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_n}{\partial \eta} \\ \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \cdots & \frac{\partial N_n}{\partial \zeta} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} & \mathbf{y_1} & \mathbf{z_1} \\ \mathbf{x_2} & \mathbf{y_2} & \mathbf{z_2} \\ \cdots & \cdots & \cdots \\ \mathbf{x_n} & \mathbf{y_n} & \mathbf{z_n} \end{bmatrix} \tag{41}$$

The transformation of the $[B]$ matrix to the natural element's coordinate system requires the inverse and the determinant of the interval Jacobian. The only way to implement these is through a set algorithm translation of the deterministic matrix inversion and determinant calculation schemes. This requires every analytical operation on the matrix entries to be translated according to the interval arithmetic. This results in $[\mathbf{B}]$ and $\det[\mathbf{J}]$ in the GAUSS points.

Finally, after substituting all interval objects into equations (38) and (39) the interval element matrices yield:

$$
\begin{aligned}
[\mathbf{K^e}] &\simeq \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} H_i H_j H_k \left[\mathbf{f_K}\left(\xi_i, \eta_j, \zeta_k\right)\right] && (42)\\
[\mathbf{M^e}] &\simeq \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} H_i H_j H_k \left[\mathbf{f_M}\left(\xi_i, \eta_j, \zeta_k\right)\right] && (43)
\end{aligned}
$$

with

$$
\begin{aligned}
[\mathbf{f_K}(\xi,\eta,\zeta)] &= [\mathbf{B}(\xi,\eta,\zeta)]^T[\mathbf{D}][\mathbf{B}(\xi,\eta,\zeta)]\det[\mathbf{J}(\xi,\eta,\zeta)] && (44)\\
[\mathbf{f_M}(\xi,\eta,\zeta)] &= [N(\xi,\eta,\zeta)]^T\boldsymbol{\rho}\,[N(\xi,\eta,\zeta)]\det[\mathbf{J}(\xi,\eta,\zeta)] && (45)
\end{aligned}
$$

For these elements, there are numerous sources of conservatism resulting from geometrical uncertainties. The computation of the interval Jacobian as stated in equation (41) results only in the exact interval Jacobian if none of the nodal geometry uncertainties are physically coupled. This is for instance not the case when a basic parameter is used to introduce uncertainty on the orientation of one or a group of elements. This orientation represented by a single or two angles couples the coordinates of a possibly large number of nodes. Furthermore, the numerical inversion of the interval Jacobian used for the $[B]$ matrix calculation unlinks the Jacobian matrix entries. Since these are all based on the same uncertain geometry description, this introduces conservatism in the resulting $[\mathbf{B}]$ interval matrix. The same phenomenon occurs in the interval determinant calculation of the Jacobian. Finally, multiplying the interval matrices as in equation (44) again unlinks the $[\mathbf{B}]$ matrix entries from the determinant of the Jacobian, while all are based on the common uncertain geometry of the element.

Another source of conservatism results from the weighted averaging of the integrand intervals obtained at the GAUSS points as expressed in equations (42) and (43). The interval averaging considers the contribution of each GAUSS point independently, while in reality these are coupled through the uncertainties used to describe the integrand. Therefore, applying the GAUSS-LEGENDRE numerical integration introduces conservatism if it is performed with more than one GAUSS point.

5.2.3 Summary

Condensing the analytical procedure for 1D elements and the set algorithm translation of the numerical integration for more complex elements into the set functions $\breve{K}^e$ and $\breve{M}^e$ and applying the inclusion property of equation (25), we obtain:

$$
\begin{aligned}
\left\langle [K^e(\{x\})] \right\rangle_{\{\mathbf{x}\}} &\subseteq \breve{K}^e(\{\mathbf{x}\}) && (46)\\
\left\langle [M^e(\{x\})] \right\rangle_{\{\mathbf{x}\}} &\subseteq \breve{M}^e(\{\mathbf{x}\}) && (47)
\end{aligned}
$$

From equations (27) and (28) it is clear that the element matrices are continuous functions of the uncertain properties as long as these stay within physical feasible bounds. Since all basic parameters are defined as completely physical plausible closed intervals, the exact range of each of the element matrix entries is a closed interval. Also the conservative procedures described above both for 1D and more complex elements result in closed interval matrices when all basic parameters are closed intervals. Therefore, we have:

$$\breve{K}^{e}\left(\{\mathbf{x}\}\right) = \left[\!\left[[\mathbf{K^e}\left(\{\mathbf{x}\}\right)] \right]\!\right]_{\{\mathbf{x}\}} \tag{48}$$

$$\breve{M}^{e}\left(\{\mathbf{x}\}\right) = \left[\!\left[[\mathbf{M^e}\left(\{\mathbf{x}\}\right)] \right]\!\right]_{\{\mathbf{x}\}} \tag{49}$$

5.3 IFEM system matrices assembly

The total interval system matrices calculation consists of the assembly of the interval element matrices. The interval counterpart of the deterministic assembly phase is very simple, since it consists of the addition of interval scalars for the entries of the interval element matrices. Representing this operation by $\breve{\cup}$, we obtain:

$$\left\langle \left[K\left(\left[K^{e_1}\right], \left[K^{e_2}\right], \ldots \left[K^{e_n}\right]\right)\right] \right\rangle_{\left[\mathbf{K^{e_i}}\right]_{i=1\ldots n}} = \breve{\bigcup_{i=1\ldots n}} \left[\mathbf{K^{e_i}}\right] \tag{50}$$

$$\left\langle \left[M\left(\left[M^{e_1}\right], \left[M^{e_2}\right], \ldots \left[M^{e_n}\right]\right)\right] \right\rangle_{\left[\mathbf{M^{e_i}}\right]_{i=1\ldots n}} = \breve{\bigcup_{i=1\ldots n}} \left[\mathbf{M^{e_i}}\right] \tag{51}$$

This phase combines independently the interval element matrices. It unlinks as such the physical properties that different elements might have in common. This is easily illustrated taking the example of an uncertain Young's modulus defined globally for all elements. The result of independently adding together interval stiffness matrix entries from different elements as in equation (50) implicitly encloses the result obtained from using different values for the Young's modulus in each element. This does not comply with the realistic interpretation of the uncertainty, where the Young's modulus is uncertain, but equal for all elements. Therefore, this phase increases the degree of conservatism for all total system matrix entries which result from the addition of element matrix entries with common uncertain properties. The resulting conservative approximations of the system matrices yield:

$$\left[\!\left[[\mathbf{K}] \right]\!\right]_{\{\mathbf{x}\}} = \breve{\bigcup_{i=1\ldots n}} \left[\mathbf{K^{e_i}}\right] \tag{52}$$

$$\left[\!\left[[\mathbf{M}] \right]\!\right]_{\{\mathbf{x}\}} = \breve{\bigcup_{i=1\ldots n}} \left[\mathbf{M^{e_i}}\right] \tag{53}$$

The system damping matrix follows from applying the proportional damping assumption of equation (13) on equations (52) and (53):

$$\left[\!\left[[\mathbf{C}] \right]\!\right]_{\{\mathbf{x}\}} = \alpha_K \left[\!\left[[\mathbf{K}] \right]\!\right]_{\{\mathbf{x}\}} + \alpha_M \left[\!\left[[\mathbf{M}] \right]\!\right]_{\{\mathbf{x}\}} \tag{54}$$

The above definition of the damping matrix couples the uncertainty on the system damping to the uncertainty on the stiffness and mass properties. This limits the possibilities of the uncertainty implementation on the damping. Defining the proportional constants as interval scalars enables an independent proportional damping uncertainty:

$$\left[\!\left[[\mathbf{C}] \right]\!\right]_{\{\mathbf{x}\}} = \boldsymbol{\alpha}_{\boldsymbol{K}} \left[\!\left[[\mathbf{K}] \right]\!\right]_{\{\mathbf{x}\}} + \boldsymbol{\alpha}_{\boldsymbol{M}} \left[\!\left[[\mathbf{M}] \right]\!\right]_{\{\mathbf{x}\}} \tag{55}$$

Both equations (54) and (55) introduce conservatism in the matrix entries for which the constituting stiffness and mass entries have basic parameters in common.

Another important source of conservatism arises at this point. It results from the fact that an interval matrix covers all possible combinations of its entries within their prescribed bounds. This definition adds to the matrix range matrices which might have been impossible to achieve taking into account the internal entry dependencies. Thus, the interval matrix neutralises every existing dependency between the entries of the matrix. Consequently, the interval matrices generally loose important specific matrix properties as symmetry or positive-definiteness since not all matrices within the defined set result from physical models. This means that the analysis algorithms which rely on these properties are not necessarily extendable to the domain of interval analysis. This phenomenon generally does occur in interval system matrices, since it is very likely that different matrix entries are based on common physical properties. For instance, for a mass matrix it is obvious that the entries in an element matrix are coupled through the density and geometrical properties of the element. Combining different values for material density in the different entries of a single element's mass matrix is artificial, yet implicitly enabled by the interval system matrix.

5.4 Dynamic analysis using IFEM

The interval system matrix entries resulting from the assembly phase are available for performing the analysis. This requires the application of a set algorithm corresponding to the deterministic analysis on the interval system matrices. This yields an approximation of the range of the required result of the FE analysis:

$$\left\langle\!\left\langle \{y\} \right\rangle\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}\left(\left[\!\left[[\mathbf{K}] \right]\!\right]_{\{\mathbf{x}\}}, \left[\!\left[[\mathbf{M}] \right]\!\right]_{\{\mathbf{x}\}}, \left[\!\left[[\mathbf{C}] \right]\!\right]_{\{\mathbf{x}\}} \right) \tag{56}$$

The final result is here generally stated as a set object, since the effect of the analysis function on the interval matrices depends on the considered type of analysis. Depending on the analysis, additional conservatism might be introduced resulting from the fact that equation (56) considers the interval system matrices independently. This unlinks their stiffness and mass properties, while they could be based on common uncertain properties, as for instance an uncertain geometry description.

5.5 Discussion

Figure 6 illustrates the total procedure described in the previous sections. This procedure's major advantage is its simplicity. The element and system interval matrix assembly proves to be a series of simple interval operations on the basic parameters. This is easy to implement using the basic interval operations. From a numerical viewpoint, it can be seen from these definitions that the basic interval operations

model definition with uncertainties $\{\mathbf{x}\}$

$$\swarrow \qquad \cdots \qquad \searrow$$

$$\left\llbracket [\mathbf{K^{e_1}}] \right\rrbracket_{\{\mathbf{x}\}} = \breve{K}^{e_1}(\{\mathbf{x}\}) \quad \cdots \quad \left\llbracket [\mathbf{K^{e_n}}] \right\rrbracket_{\{\mathbf{x}\}} = \breve{K}^{e_n}(\{\mathbf{x}\})$$

$$\left\llbracket [\mathbf{M^{e_1}}] \right\rrbracket_{\{\mathbf{x}\}} = \breve{M}^{e_1}(\{\mathbf{x}\}) \qquad \left\llbracket [\mathbf{M^{e_n}}] \right\rrbracket_{\{\mathbf{x}\}} = \breve{M}^{e_n}(\{\mathbf{x}\})$$

$$\searrow \qquad \cdots \qquad \swarrow$$

$$\begin{aligned} \left\llbracket [\mathbf{K}] \right\rrbracket_{\{\mathbf{x}\}} &= \breve{\bigcup_{i=1\ldots n}} \left\llbracket [\mathbf{K^{e_i}}] \right\rrbracket_{\{\mathbf{x}\}} \\ \left\llbracket [\mathbf{M}] \right\rrbracket_{\{\mathbf{x}\}} &= \breve{\bigcup_{i=1\ldots n}} \left\llbracket [\mathbf{M^{e_i}}] \right\rrbracket_{\{\mathbf{x}\}} \end{aligned}$$

$$\left\llbracket [\mathbf{C}] \right\rrbracket_{\{\mathbf{x}\}} = \alpha_K \left\llbracket [\mathbf{K}] \right\rrbracket_{\{\mathbf{x}\}} + \alpha_M \left\llbracket [\mathbf{M}] \right\rrbracket_{\{\mathbf{x}\}}$$

$$\downarrow$$

$$\left\langle\!\left\langle \{y\} \right\rangle\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}\left(\left\llbracket [\mathbf{K}] \right\rrbracket_{\{\mathbf{x}\}}, \left\llbracket [\mathbf{M}] \right\rrbracket_{\{\mathbf{x}\}}, \left\llbracket [\mathbf{C}] \right\rrbracket_{\{\mathbf{x}\}} \right)$$

Figure 6: Conceptual overview of the algorithm for dynamic IFE analysis based on interval system matrices

consist of the corresponding deterministic operations on some combination of the lower and upper bounds of the operands. Therefore, an interval matrix assembly requires roughly between two and four times the computational effort of the corresponding deterministic matrix assembly. The final IFE solution of the analysis as expressed in equation (56) also proves to be a simple numerical formulation based on a function of interval matrices. A number of solution schemes for different numerical problems based on interval matrices are available from the world of interval arithmetic.

The major drawback of this method is its repeated vulnerability to conservatism. In the total algorithm, there are four major sources of conservatism:

- neglecting the correlation between the terms which constitute the integrand in the numerical integration for the element matrix computation
- neglecting the correlation between element matrix entries of different elements during the assembly
- neglecting the internal correlation between total system matrix entries
- neglecting the correlation between the total system matrices during the analysis

Each of these is caused by wrongly unlinking intermediate variables of the algorithm which are in reality coupled through common uncertain physical properties. The

analysis phase	source of conservatism
1D element matrices	geometrical uncertainties
2D & 3D element matrices	geometrical uncertainties
	number of Gauss points
system matrices assembly	common uncertainties between elements
analysis phase	algorithm applied on system matrices

Table 1: Controllable sources of conservatism during an IFE analysis

impact of each of these on the final analysis result depends strongly on the type of analysis and the nature of the uncertainties.

- For the first source, the amount of conservatism depends strongly on the type and complexity of the element. Using simple elements, limiting the number of Gauss points during the numerical integration of more complex elements, and avoiding geometry uncertainties is the recipe to neutralise all conservatism in this phase of the algorithm. However, in particular the latter is inherent to the problem description and, therefore, not to be decided for by the analyst.
- The second source is present if there are uncertainties common to a group of elements. While theoretically possible, it seems very unlikely that in a realistic analysis each element has its own independent uncertainty description, which makes this source of conservatism present in nearly all analyses.
- The third source of conservatism is unavoidable, since the entries of a system matrix are mutually always closely related.
- The fourth source of uncertainty depends totally on the considered analysis.

Table 1 gives an overview of all the sources of conservatism which are to some extent controllable by the analyst. It could serve as a guideline for controlling the amount of conservatism for interval uncertainty modelling when applying the interval system matrices approach.

In order for the result to be used for design validation or optimisation, a thorough verification of the conservatism is advisable. This requires an extensive Monte Carlo simulation as discussed in section 4.4 and, therefore, could cancel out one of the most important advantages of the interval analysis over the probabilistic approach: its time efficiency.

6 Hybrid IFE analysis

In limited cases, a possible remedy to some sources of conservatism is to perform as much as possible of the deterministic FE procedure analytically. This neutralises all sources of conservatism which occur before the point where the uncertainties are introduced. The procedure to compute the local element stiffness matrix of 1D elements based on the analytical description rather than a numerical integration strategy as described in section 5.2 is an elementary illustration of this principle. A possible extension of this strategy is to perform both the deterministic element

$$
\begin{array}{ccc}
\text{basic parameters} & \longrightarrow & \{\mathbf{x}\} = \{\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}\} \\
\text{optimisation} & \longrightarrow & \left\langle \{z_i\} \right\rangle_{\{\mathbf{x}\}} = \left[\min_{\{x\}\in\{\mathbf{x}\}} z_i\big(\{x\}\big), \max_{\{x\}\in\{\mathbf{x}\}} z_i\big(\{x\}\big) \right] \\
\text{interval analysis} & \longrightarrow & \left\langle\!\left\langle \{z_{i+1}\} \right\rangle\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}_{i+1} \left(\left\langle \{z_i\} \right\rangle_{\{\mathbf{x}\}} \right) \\
\ldots & \longrightarrow & \ldots \\
 & \longrightarrow & \left\langle\!\left\langle \{y\} \right\rangle\!\right\rangle_{\{\mathbf{x}\}} = \breve{f}_{m} \left(\left\langle\!\left\langle \{z_{m-1}\} \right\rangle\!\right\rangle_{\{\mathbf{x}\}} \right)
\end{array}
$$

Figure 7: Conceptual description of the hybrid interval analysis procedure

matrix calculation and the total system matrix assembly analytically. This enables a simplification of the matrix to global properties which can be brought outside the matrix. While it is theoretically exact, this approach is limited to small models with uncertain parameters that allow for a parametrical expression of the system matrix. This approach was introduced by Enling [3] and later used by Dessombz et al. [4] to decrease the conservatism resulting from a globally defined uncertain Young's modulus. Mullen et al. [5] used a method based on an analytical description to introduce uncertainty on the load vector in the final step of an algorithm.

In order to extend the applicability of IFEM, a more general applicable remedy to excessive conservatism is easily derived from this principle. Instead of a partial analytical analysis, it consists of a partial optimisation in the first part of the analysis. This means that an optimisation is applied to calculate the interval result at some intermediate step of the total algorithm. In the second part, the interval analysis is performed on these intermediate results. The optimisation yields:

$$
\left\langle \{z_i\} \right\rangle_{\{\mathbf{x}\}} = \left[\min_{\{x\}\in\{\mathbf{x}\}} z_i\big(\{x\}\big), \max_{\{x\}\in\{\mathbf{x}\}} z_i\big(\{x\}\big) \right] \tag{57}
$$

Figure 7 clarifies this procedure. This method has two major advantages:

- because of the global optimisation, all conservatism prior to the optimised intermediate result is neutralised
- the performance of the optimisation step is controllable by adequately choosing the level on which to perform it

A simple numerical example illustrates the effect of the hybrid approach compared to a pure interval arithmetic approach. Consider an analytical function:

$$
f\big(\{x\}\big) = \big((x_1 + x_2)(x_1 - x_2)\big)^2 \tag{58}
$$

and the basic parameters for this function defined as:

$$
\{\mathbf{x}\} = \left\{ \begin{array}{c} [-1, 2] \\ [-2, 3] \end{array} \right\} \tag{59}
$$

Figure 8 gives a deterministic algorithm in three steps for this function. It also describes the intermediate results of the corresponding interval arithmetic procedure, and the hybrid procedure with an optimisation performed on the second substep of the deterministic algorithm. This optimisation is mathematically expressed as:

$$\langle z_2 \rangle_{\{\mathbf{x}\}} = \left[\min_{\{x\} \in \{\mathbf{x}\}} \left(x_1^2 - x_2^2 \right), \max_{\{x\} \in \{\mathbf{x}\}} \left(x_1^2 - x_2^2 \right) \right] \tag{60}$$

deterministic algorithm	interval arithmetic algorithm	hybrid algorithm
$z_{11} = x_1 + x_2$ $z_{12} = x_1 - x_2$	$\langle z_{11} \rangle_{\{\mathbf{x}\}} = [-3, 5]$ $\langle z_{12} \rangle_{\{\mathbf{x}\}} = [-4, 4]$	
$z_2 = z_{11} \times z_{12}$	$\langle\langle z_2 \rangle\rangle_{\{\mathbf{x}\}} = [-20, 20]$	$\langle z_2 \rangle_{\{\mathbf{x}\}} = [-9, 4]$
$y = z_2 \times z_2$	$\langle\langle y \rangle\rangle_{\{\mathbf{x}\}} = [-400, 400]$	$\langle\langle y \rangle\rangle_{\{\mathbf{x}\}} = [-36, 81]$

Figure 8: Comparison of the interval arithmetic and hybrid approach for the approximation of the result of an interval problem

The exact result of the interval problem calculated using the global optimisation approach yields:

$$\left\langle f\left(\{x\}\right) \right\rangle_{\{\mathbf{x}\}} = [0, 81] \tag{61}$$

Therefore, the hybrid approach clearly results in a substantial improvement of the conservative result approximation compared to the pure interval arithmetic approach. Annex 2.4 of this project illustrates how this hybrid procedure can be of use in the context of an IFE frequency response function analysis.

7 Conclusion

In order to study the conservatism of the interval system matrices approach, the set algorithm translation method is introduced. Its basic principle is the translation of every substep of a deterministic algorithm to interval analysis. It consequently embodies a procedure which is able to construct the equivalent interval counterpart of nearly any deterministic numerical algorithm. The main disadvantage of this approach, however, is its high vulnerability to conservatism due to the inclusion property. A simple example illustrates that it is extremely difficult to predict the amount of conservatism implicitly introduced by the translation procedure. The Monte Carlo simulation procedure is proposed as a verification tool which could give some qualitative information on the result of the translated algorithm.

Applying the set algorithm translation concept on the deterministic FE analysis yields the corresponding IFE procedure. The conservatism introduced by this IFE procedure keeps the result on the safe side as far as reliability analysis is concerned. For realistic models, however, there are numerous sources of conservatism already during the system matrix assembly phase in the IFE procedure. These strongly devaluate the result of the analysis. Therefore, though the guidelines of table 1 might serve well for some applications with limited complexity, it is concluded that the approach based on interval system matrices needs special attention when it is used for implementation in a software environment intended for general uncertainty analysis. Especially for large models, the unlinking of elements and internal system matrix entries lifts the conservatism in the interval matrices to a very high level. Depending on the effect of the intended analysis, this could increase the conservatism of the final result to an unacceptable level.

The hybrid IFE procedure divides the analysis in an optimisation step followed by an interval analysis applied on the result of the optimisation. The intended improvement is that an appropriate choice of the optimisation level could decrease the conservatism to an acceptable level, even for large realistic models.

The application of the interval arithmetic approach on problems of a very low dimension already illustrates that the conservatism grows beyond reasonable limits. This conservatism is mainly due to the matrix assembly phase. Therefore, it is concluded that a global optimisation or a hybrid form that cancels out the assembly conservatism are the only applicable implementation strategies for the IFE analysis.

References

[1] Zienkiewicz, O. C. and Taylor, R. L., *The Finite Element Method*, McGraw-Hill Book Company, London, 1989.

[2] Hitchings, D., *A Finite Element Dynamics Primer*, chap. A Finite Element Dynamics Primer, NAFEMS, Glasgow, 1992, pp. 170–196.

[3] Enling, L., "A New Solution to Structural Fuzzy Finite Element Equilibrium Equations (SFFEEE)," *Applied Mathematics and Mechanics*, Vol. 18, No. 4, 1997, pp. 385–391.

[4] Dessombz, O., Thouverez, F., Laîné, J.-P., and Jézéquel, L., "Analysis of Mechanical Systems Using Interval Computations Applied to Finite Element Methods," *Journal of Sound and Vibration*, Vol. 239, No. 5, 2001, pp. 949–968.

[5] Mullen, R. and Muhanna, R., "Bounds of Structural Response for All Possible Loading Combinations," *Journal of Structural Engineering*, Vol. 125, No. 1, 1999, pp. 98–106.

Task 3.4 Application of the fuzzy finite element method to problems with multiple independent uncertain parameters

The employment of fuzzy numbers in engineering applications requires a twofold base. On the one hand, one has to know their meaning, their advantages and their restrictions, in comparison with other methods aiming at the modelling of uncertainty or variability of any kind. On the other hand, also a calculational aspect is involved: after deciding that fuzzy numbers are the appropriate tool in a certain application and setting up the fuzzy input, one has to be capable of performing the fuzzy analysis itself.

The interpretation of fuzzy numbers and a comparison with other methods such as probability theory and partial safety factor analysis has been described extensively in other tasks reports [3.1, 3.3, 3.3a]. Under these tasks, the application examples were of such simplicity that exact solutions could be obtained, allowing to focus on the interpretation of fuzzy numbers. However, real-life applications often are of a much higher complexity, so it is certainly worthwhile to spend some considerations on the use of fuzzy numbers in non-trivial examples.

A high complexity can be defined in different ways; one could use the number of DOFs as a criterion, or the number of uncertain variables. However, also the behaviour of the relation between input parameters and response variables has a large influence on the choice of the calculational approach. Since finite element analysis is a well-established method, even for large systems with many DOFs, the examples will mainly focus on the number of uncertain variables and on non-trivial (non-monotonic) input-output relationships.

This report consists of two parts: the first part gives an overview of the possible (theoretical) approaches to fuzzy calculations, the second part will illustrate some of these methods with practical applications.

A Problem description and solution strategies

1 Theoretical background on fuzzy number calculations

Although other possibilities exist [1], Zadeh's extension principle is the most used base to perform mathematical operations on fuzzy numbers. It states that the membership function of the outcome $\widetilde{y}$ of a function f of fuzzy arguments $\widetilde{x_i}$ is given by

$$\widetilde{y} = f(\widetilde{x_1}, \widetilde{x_2}, \ldots)$$

iff

$$\mu_Y(y) = \sup_{y=f(x_1,x_2,\ldots)} \inf \{\mu_{X_1}(x_1), \mu_{X_2}(x_2), \ldots\} \tag{1}$$

The rightmost part, $\inf \{\mu_{X_1}(x_1), \mu_{X_2}(x_2), \ldots\}$, can be interpreted as the joint membership function of the independent fuzzy variables $\widetilde{x}_i$. Although very general, this expression is not very practical; one has to discretize with respect to y, determine the contour lines at these values, and then take the maximum value of the joint membership function along each contour. Certainly for non-monotonic functions, determining the contour lines is a difficult and expensive task.

Therefore, the problem is reformulated, by discretizing along different α-levels, instead of different values of y. It can be shown that the expression

$$\widetilde{y} = f(\widetilde{x_1}, \widetilde{x_2}, \ldots)$$

iff

$$[\widetilde{y}]_\alpha = f([\widetilde{x_1}]_\alpha, [\widetilde{x_2}]_\alpha, \ldots) \qquad \forall \alpha \in [0,\ 1] \tag{2}$$

is equivalent to the previous one. Note that the definition of a function of interval variables is given by

$$\begin{aligned}[y] &= f([x_1], [x_2], \ldots) \\ &= \{f(x_1, x_2, \ldots) \mid x_1 \in [x_1],\ x_2 \in [x_2], \ldots\}\end{aligned} \tag{3}$$

2 Solution of interval expressions

Since a fuzzy number expression can be reformulated as a set of interval expressions, the aim is now to perform these interval calculations in an efficient yet robust way. In general, two fundamental approaches exist to evaluate an interval expression. The first one makes use of interval arithmetics, the second one considers it as a double constrained optimization problem.

2.1 Interval arithmetics

The idea behind this approach is to generalize all basic mathematical operators $(+, -, \times, \div, \ldots)$ to interval operators:

$$\begin{aligned}[x_1] + [x_2] &= \Big[\ [x_1]^- + [x_2]^-\ ,\ [x_1]^+ + [x_2]^+\ \Big] \\ [x_1] - [x_2] &= \Big[\ [x_1]^- - [x_2]^+\ ,\ [x_1]^+ - [x_2]^-\ \Big] \\ &\ldots\end{aligned}$$

This approach is very attractive from a computational point of view: the evaluation of an interval expression with interval arithmetics typically will take only a few times longer than an expression with real numbers. However, there are two important drawbacks of this approach. The first one is the implementation effort: since all operations have to be generalized to interval operations, this method requires complete new finite element software. This explains why this method has been investigated for some small-scale academic structures [2], but not yet for real-life problems.

A second, more fundamental disadvantage of interval arithmetics is the dependency problem, which occurs if a variable appears multiple times in one expression. In such situation, each of them will be treated as an independent variable, giving raise to conservative results. Consider for example an interval variable $[x] = [1, 2]$, and the function $y = \frac{x+1}{x}$. Then the application of interval arithmetics leads to

$$[y] = \frac{[1,2]+1}{[1,2]} = \frac{[2,3]}{[1,2]} = [1,3]$$

whereas preceding simplification yields

$$[y] = 1 + \frac{1}{[x]} = 1 + [\frac{1}{2}, 1] = [\frac{3}{2}, 2]$$

This example clearly shows how the different occurrences of $[x]$ in the numerator and denominator are treated as independent variables which can take different values, leading to a conservative estimation of the actual bounds on the outcome.

Certainly for complex, large-scale structures, involving numerous operations to analyse, this may lead to an excessive conservatism, or even render the results meaningless. Efforts have been made by several authors [3] to reduce this artificial conservatism, but this is not yet completely successfull and it goes at the expense of the computational efficiency.

2.2 Double constrained optimization problem

Another way to evaluate interval expressions is to view them as a double optimization problem:

$$[y] = \{f(x_1, x_2, \ldots) \mid x_1 \in [x_1]\,,\ x_2 \in [x_2]\,, \ldots\} \tag{4}$$

iff

$$\begin{cases} [y]^- = \min f(x_1, x_2, \ldots) \\ [y]^+ = \max f(x_1, x_2, \ldots) \end{cases} \qquad \text{subject to } x_1 \in [x_1]\,, x_2 \in [x_2]\,, \ldots \tag{5}$$

When applying this method to a fuzzy finite element analysis, two subproblems should be tackled: first an interface should be developed, which reduces the FE-software to a (black-box) function, taking input values and returning output values. This allows to perform the fuzzy calculations on a higher level, as a shell around the FE-software. The development of and choice between appropriate optimization methods is the second subproblem.

The advantages of this approach are clear: since no modifications have to be performed on the actual FE-calculations, existing FE-software can be used. Secondly, the results will be exact and no artificial conservatism will be induced. However, this accurateness has a computational price, and in general an optimization-based fuzzy calculation will be far more expensive than one based on interval arithmetics.

2.3 Hybrid methods

As a way in between these two fundamental approaches, hybrid methods make a combination of both. Up till a certain point in the calculation, for example after the system matrices assembly, or after a modal analysis, an optimization method is applied. This will result in accurate (i.e. non-conservative) intermediate results, at a reasonable computational effort. Hereafter, the remainder of the analysis (which involves few operations) is done with interval arithmetics.

In a certain way, hybrid methods indeed combine the advantages of both approaches: they are less expensive than plain optimization, they can make use of existing FE-software, and they will be not as conservative as a complete interval-arithmetical analysis. However, also the disadvantages are combined: both optimization methods and interval operators have to be implemented, and even when only the very last step is performed with interval arithmetics, this may induce a large conservatism, which is hard to quantify. On top of that, they are not generally applicable, and it takes some judgement on which intermediate results are appropriate as a starting point for the interval analysis.

2.4 Choice of calculational approach

In the remainder of this task report, the focus will lie on the optimization-based fuzzy calculations. Three reasons exist for this choice.

First, this approach yields correct results (if the optimization is successful). Even in view of the subjective character of fuzzy numbers, accurate results are preferable, certainly because the degree of overestimation with interval arithmetics is impossible to quantify.

Secondly, it is the most general method. Once the interface and the optimization methods have been developed, any kind of analysis can be performed: static, modal, harmonic or transient, linear or nonlinear, problems where the results follow directly from the FE solution or problems where finite elements are used as a lower-level procedure (such as updating problems),...

Finally, this method is also generic. The division between optimization methods and finite element software makes it possible to extend to other optimization methods or other FE-packages. In fact, this division even allows for a complete abstraction of the finite elements, and any kind of calculational procedure which can be reduced to a black box can be subjected to a fuzzy analysis with the same optimization methods.

3 Interface with FE-programs

The interface which will described in this section was developed for the finite element software Ansys [4]. Like most FE packages, Ansys has the capability of running in batch mode, whereby an input file containing the subsequent commands should be specified. In order to improve the efficiency, it is desirable to gather different evaluations of a model into one analysis.

Therefore, use is made of a shell-inputfile, which reads the input variable combinations one-by-one, makes a call to the actual model-inputfile, and stores the results for each combination.

Example

Suppose one wants to use Ansys to perform the simple calculation $y = x_1 - x_2^2$ for $x_1 = x_2 = 0.0,\ 0.2, \ldots,\ 1.0$. First, the model-input file is created as

Model.inp

```
Result = _FV(1)-_FV(2)**2
```

Here, `_FV(i)` denotes fuzzy variable i (i.e. x_i), and `Result` is the fixed way to denote the outcome. Next, a file containing the different combinations of the input variables is created as

InpVals.dat

```
0.0   0.0
0.2   0.2
0.4   0.4
0.6   0.6
0.8   0.8
1.0   1.0
```

When calling the shell-inputfile, the operations are performed as follows:

Shellfile.inp

```
<Read x from Inpvals.dat>
*DO,i,1,6
   _FV(1) = x(i,1)
   _FV(2) = x(i,2)
   \INPUT, Model.inp
   y(i) = Result
*ENDDO
<Write y to Results.dat>
```

The results file will contain the results for the different combinations:

Results.dat

```
0.00
0.16
0.24
0.24
0.16
0.00
```

An additional advantage of this architecture, besides the evaluation of different input parameter combinations in one run, is that existing inputfiles for deterministic models can be modified to a blackbox model easily, simply by replacing numerical values by `_FV(i)`.

4 Optimization methods

This section describes and compares some commonly used optimization methods for fuzzy calculations. Each method will be illustrated with two test functions f_1 and f_2, being a monotonic and a non-monotonic function of the input respectively.

Monotonic test function

The first test function describes the elongation Δ of a two-bar truss structure under a static load P (figure 1). The uncertain variables are the cross sections of the two bars, $\widetilde{A_1}$ and $\widetilde{A_2}$.

With $P = 1$ and $E = 1$, the relation between the input variables and the response parameter is

$$\Delta = f_1(A_1, A_2) = \frac{1}{A_1} + \frac{1}{A_2} \tag{6}$$

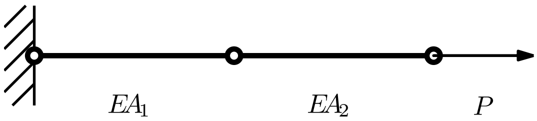
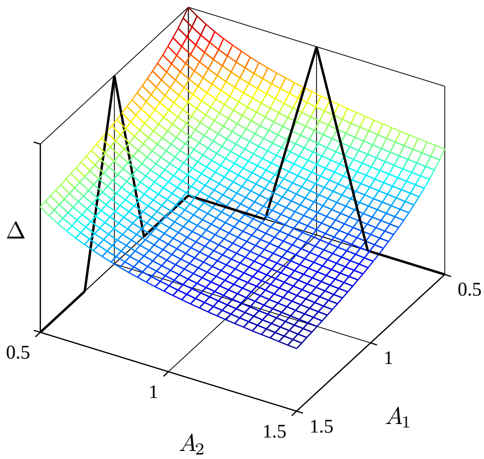
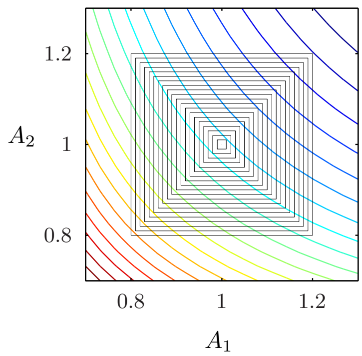


Figure 1: Two-bar truss structure

Both uncertain input variables are assigned a triangular membership function with unit centre value and a base width of 0.4. Figure 2 shows this monotonic function along with the membership functions.



(a) Surface plot of monotonic test function and marginal membership functions



(b) Contour lines of monotonic test function and joint membership function

Figure 2: Monotonic test function and input membership functions

Non-monotonic test function

The second test function describes the modulus of the displacement $\|U\|$ of a single degree of freedom mass-spring-damper system under a harmonic load $P = \sin \Omega t$, as shown in figure 3. The uncertain variables are the spring stiffness $\widetilde{k}$ and the damping ratio $\widetilde{\xi}$.

With $m = 1$ and $\Omega = 1$, the relation between the response amplitude and the input variables is given by

$$
\begin{aligned}
\|U\| &= f_2(k, \xi) \\
&= \left\| \frac{1}{-m\Omega^2 + ic\Omega + k} \right\| \\
&= \left\| \frac{1}{-1 + 2i\xi\sqrt{k} + k} \right\| \\
&= \frac{1}{\sqrt{k^2 + 2\left(2\xi^2 - 1\right)k + 1}}
\end{aligned}
\tag{7}
$$

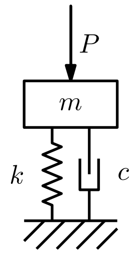
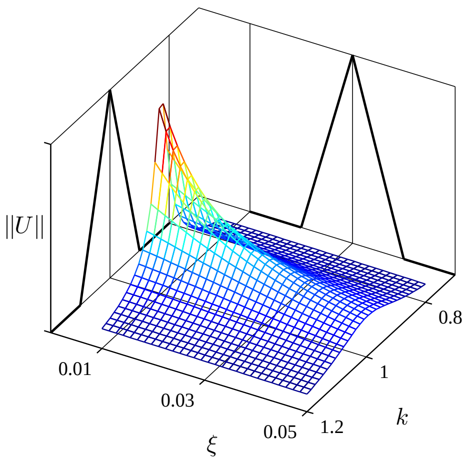
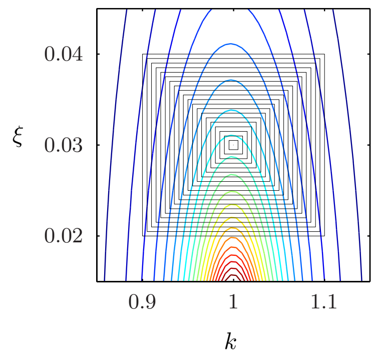


Figure 3: Mass-spring-damper system

The membership functions of $\widetilde{k}$ and $\widetilde{\xi}$ are symmetric triangular functions, with base intervals of $[0.9, 1.1]$ and $[2\%, 4\%]$, respectively. Figure 4 shows a surface plot and a contour plot of this test function and the membership functions.



(a) Surface plot of non-monotonic test function and marginal membership functions



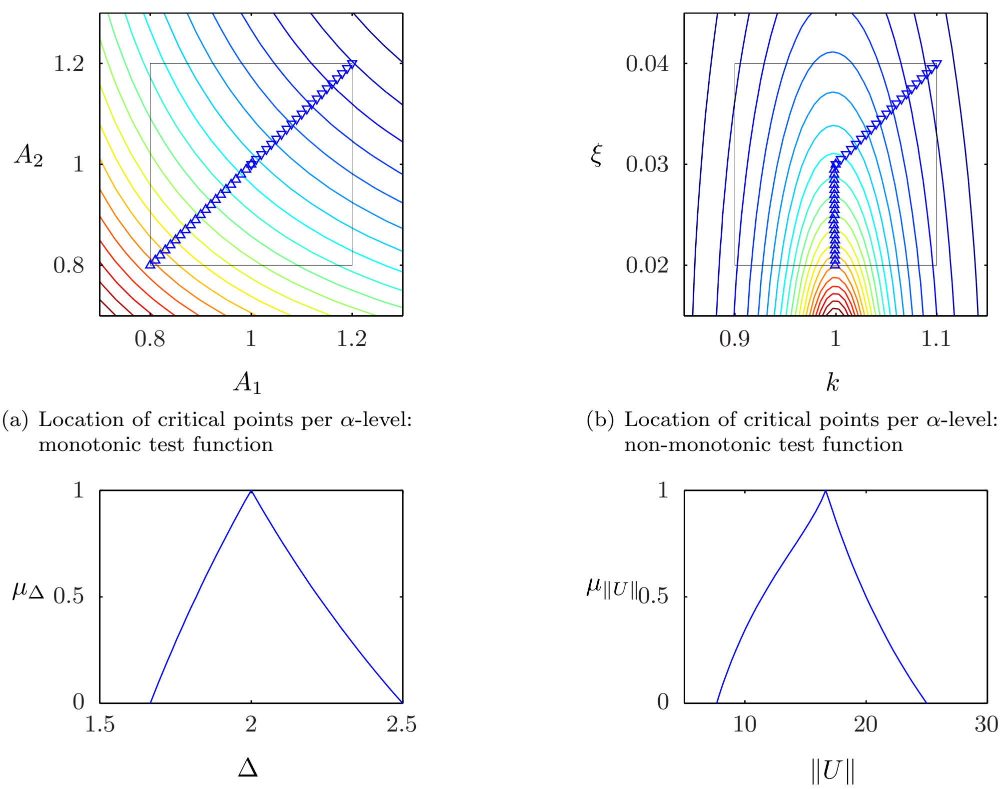
(b) Contour lines of non-monotonic test function and joint membership function

Figure 4: Non-monotonic test function and input membership functions

4.1 General purpose optimization methods

The optimization problem is given by eq. (5) and has to be solved for every α-level (eq. (2)). In fact, any algorithm capable of solving a constrained optimization problem is a candidate. In literature, pattern search methods [5], SLP [6] and Genetic Algorithms [7] are found amongst others. In this report, a sequential quadratic programming (SQP) algorithm is adopted.

The application of this method to the test functions is shown in figure 5. From the contour plots, it can be verified that this method yields correct results for both test functions.



(a) Location of critical points per α-level: monotonic test function

(b) Location of critical points per α-level: non-monotonic test function

(c) Membership function of output

(d) Membership function of output

Figure 5: Application of the SQP algorithm to the test functions. In (a) and (b), $\triangle$ denote the points where the maximal value is obtained within a certain α-cut, $\triangledown$ denote the points where the minimal value is obtained.

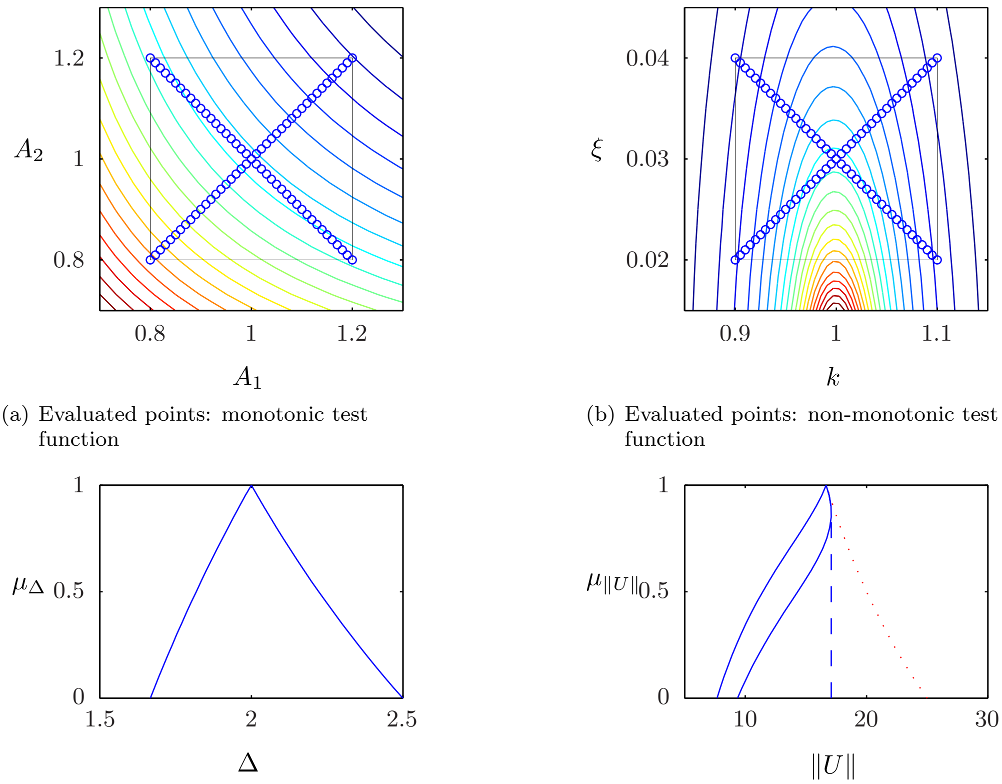
4.2 Transformation Method

This method, introduced by Hanss [8], and applied by many other researchers, is not really an optimization scheme. However, since it aims at finding the extreme values at each α-level by means of a series of deterministic function evaluations, it is treated in this section. The underlying idea is that combinations of extreme values of the input variables (the vertices of an n-dimensional hypercube), will lead to the extreme values of the output response parameters. In case of monotonic behaviour of the system, this leads to the exact solution, and for systems with few uncertain parameters, the computational effort is limited;

$$N = (m-1)2^n + 1$$

with N the number of evaluations needed, m the number of α-cuts and n the number of uncertain parameters. At $\alpha = 1$, only one evaluation is needed in case of normal fuzzy numbers.

Figure 6 shows the application of this method to both test functions. The results for the first test function are the same as those obtained by SQP, but for the non-monotonic test function, the Transformation Method yields incorrect results. Since fuzzy numbers are convex by definition (i.e. every α-cut must be a subset of any lower level α-cut), the output membership function of this function can be corrected (dashed line in figure 6(d)). However, this is no guarantee to obtain the real solution (as obtained from the optimization method).



(a) Evaluated points: monotonic test function

(b) Evaluated points: non-monotonic test function

(c) Membership function of output: monotonic test function

(d) Membership function of output for non-monotonic test function with Transformation Method (−−), after correction of Transformation Method (—) and with optimization method (· · ·)

Figure 6: Application of the Transformation Method to the test functions

Major drawbacks of this method are that the results may be unsafe (for non-monotonic systems), and that the number of evaluations needed increases rapidly (exponentially) as the number of uncertain variables increases. Factors in favour of this method are that its behaviour is predictable, that it is very easy to implement, and that multiple output components can be treated without additional costs.

4.3 Short Transformation Method

Like the Transformation Method, the Short Transformation Method [9] is not really an optimization scheme, but rather an attempt to generate the set of points leading to extremal values. The underlying idea is similar to that of the Transformation Method, but it tries to circumvent the fast increase of computational effort as the number of uncertain variables increases.

The method consists of two parts: first, a sensitivity analysis is performed with respect to all uncertain variables. This analysis allows to predict which combinations will be critical. Note that only the signs of the sensitivities are important; the actual magnitude does not matter. The second phase consists of the actual fuzzy calculation: the function is evaluated along the critical diagonal at several α-levels.

Hanss [10] proposes to obtain the sensitivities by performing a full factorial analysis at one α-level (e.g. $\alpha = 0$), and then selecting the critical combinations of the input variables. It is shown that this approach leads to quite comparable results as a sensitivity analysis based on finite differences at the center point. When comparing the calculational effort, the full factorial sensitivity calculation needs 2^n evaluations, whereas the finite difference approach requires $n + 1$ evaluations. Two of the evaluations of the full factorial method can be recycled (they immediately give the bounds of the result at that α-level), and one can be recycled for the finite difference method (the center point has to be evaluated anyway). This means that the finite difference method is cheaper if $n \leq 2^n - 2$, or $n \geq 2$. Therefore, throughout the remainder of this report, the finite difference based sensitivity analysis will be used.

Figure 7 illustrates the application of the Short Transformation Method to both test functions. The results are exactly the same as those obtained from the full Transformation Method, and a similar correction should be applied in order to obtain convex fuzzy numbers.

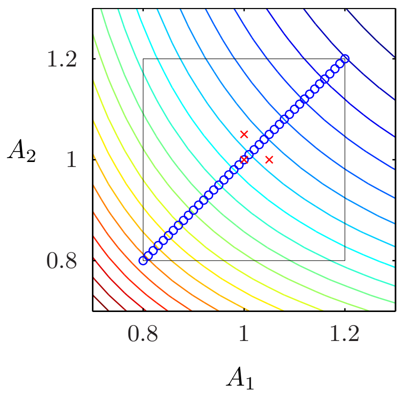
The most important aspect of this method is that it is independent of the number of uncertain variables (if the sensitivity analysis is not considered). Therefore, this method is an obvious candidate for problems with many uncertain variables. However, the results may be erroneous for non-monotonic functions, and therefore the application area is restricted to monotonic functions, or to problems with a small uncertainty range.

4.4 Gradual α-level Decreasing algorithm

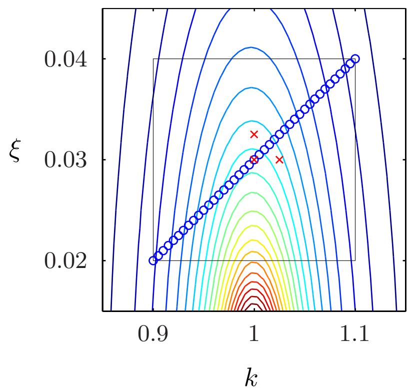
The Gradual α-level Decreasing (GαD) algorithm is an optimization method which was developed specifically for fuzzy number calculations. Its main difference with the previous methods is that it does not consider the different α-cut interval calculations as separate problems, but that it makes use of the fact that the target function remains the same while only the bounds vary.

The algorithm starts with a number of search paths at the center of the fuzzy numbers, which proceed in different directions. At each level, the partial derivatives are calculated, allowing to detect possible new search directions. Adaptive α-level stepsizing makes it possible to concentrate on interesting search paths and reducing the number of function evaluations.

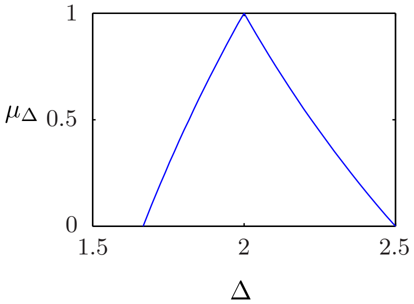
Depending on the choice of the adaptive stepsize parameters, the algorithm evolves from a very robust (but very expensive) method to a method which is comparable to the Short Transformation Method (i.e. independent of the number of fuzzy variables). Figure 8 shows the application of the GαD algorithm to the two test functions, with the adaptive stepsize parameters set to moderate values. From the evaluated points on the contour plots it is clear that the results are the same as with the general purpose optimization method, both for the monotonic and non-monotonic test functions.



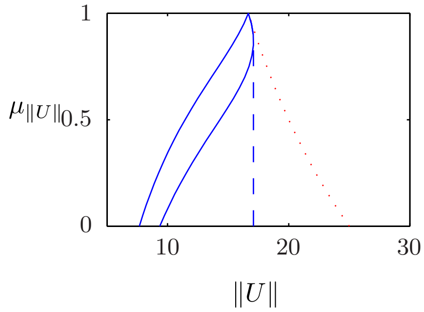
(a) Evaluated points: monotonic function



(b) Evaluated points: non-monotonic function

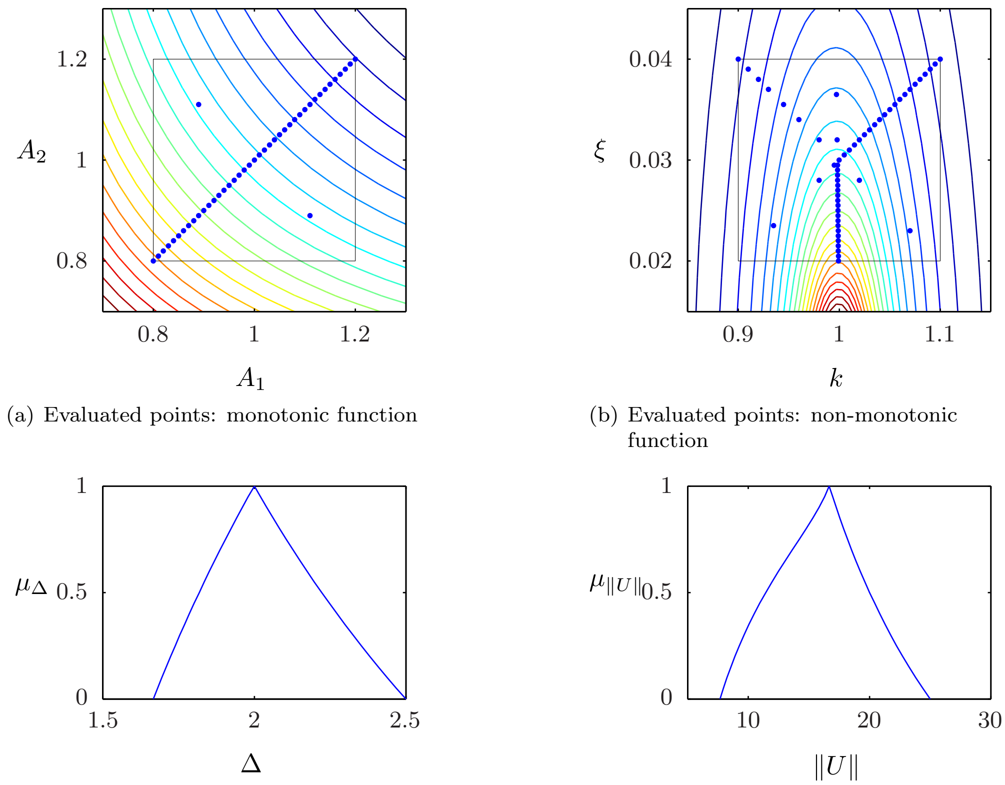


(c) Membership function of output: monotonic function



(d) Membership function of output for non-monotonic test function with Short Transformation Method (—), after correction of Short Transformation Method (– –) and with optimization method (· · ·)

Figure 7: Application of the Short Transformation Method to the test functions. In (a) and (b), × denote the points needed for the finite difference sensitivity analysis and ∘ denote the evaluated points along the critical diagonal.



(a) Evaluated points: monotonic function

(b) Evaluated points: non-monotonic function

(c) Membership function of output: monotonic function

(d) Membership function of output: non-monotonic test function

Figure 8: Application of the GαD algorithm to the test functions

4.5 Conclusions

As always in engineering problems, a trade-off has to be made between robustness and accuracy on the one hand, and efficiency and computational cost on the other hand. In order to make an optimal judgement between the different methods described above, some basic knowledge about the problem's behaviour is necessary.

In general, following rules can be maintained:

- in case of many uncertain input variables, the Transformation Method should be avoided. Also the non-adaptive version of GαD will require a lot of function evaluations. The Short Transformation Method, a general purpose optimization method or the adaptive version of GαD (with large maximal α-level stepsize and large aggression factor) are better candidates.
- also the number of output design variables is important. General purpose optimization methods deal with these different output components separately, and the computational effort increases linearly with the number of design variables. The Transformation Method, the Short Transformation Method and the GαD algorithm more efficient for such problems
- in case of expected non-monotonical behaviour, the Transformation Method and certainly the Short Transformation Method should be avoided in order not to obtain unsafe results. This aspect also has an influence on the choice of parameters in general purpose optimization algorithms, or on the choice of the adaptive stepsizing parameters in GαD
- general purpose optimization algorithms and GαD also perform more efficient if first-order derivative information is readily available, such that it hasn't to be determined by means of finite differences.

B Applications

1 Fuzzy Frequency Response Function of a composite floor

In this example, a 3385 DOF model with six uncertain parameter is subjected to a fuzzy analysis. The Fuzzy Frequency Response Function is calculated with (a modified version of) the Transformation Method. As has been shown above, a FRF of a system with uncertain stiffness and damping parameters is a non-monotonic function. Therefore, the Short Transformation Method may lead to unsafe results. This also holds for the Transformation Method, but it will be shown how this problem can be circumvented.

Although the Transformation Method is not the most efficient method for systems with a large number of uncertain variables, it is possible to reduce the calculational efforts substantially by means of the method of Component Mode Synthesis in combination with a fuzzy superelement.

1.1 Case description

The structure under investigation consists of an orthotropic concrete slab supported by cellular beams (figure 9). The eigenfrequencies and modeshapes of the first floor have been measured.

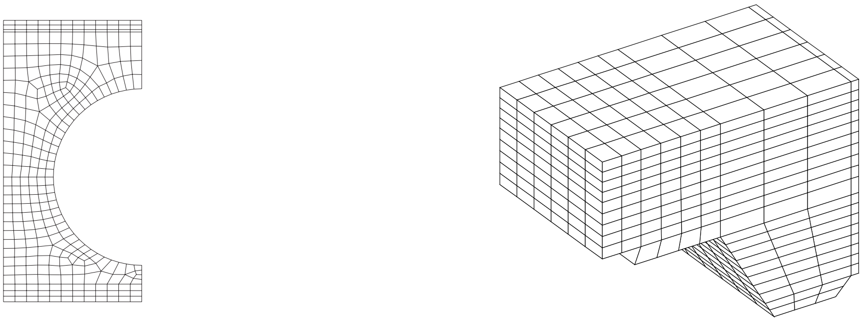
Figure 9: Composite floor

The cellular beams are modelled as beams with constant cross section and equivalent stiffness parameters, derived from a demand of equal strain energy under equal deformations applied to an elementary module of the beam (figure 10). The geometrically orthotropic slab (based on a steeldeck profile) is transformed into a slab with constant thickness and material orthotropy. These material characteristics again are derived from the principle of equal strain energy, but cannot be determined unambiguously because there are more elementary deformations on a slab than orthotropic material properties. It is therefore believed that the material properties of the equivalent slab carry more uncertainty than the properties of the beams.

An important factor in the dynamic behaviour of this floor are the boundary conditions. Along the long sides of the floor, the cellular beams are connected to columns with bolted connections. The stiffness of these connections depends on the type of connection, but also on the number of floors at that location, which varies from two up to six (figure 9). Therefore, the connection stiffnesses are also allowed to vary from one connection to another (K_{c1}, K_{c2} and K_{c3}), as indicated in figure 11. One of the short sides is simply supported on four columns, the opposite side is connected to a stiff concrete core in the building. This connection is modelled with rotational springs as well, but it is difficult to predict a value for its stiffness.

An updating procedure has been applied to estimate the connection stiffnesses and to check the orthotropic material properties. The updated values of the connection stiffnesses are respectively $K_{c1}^U = 33.0\,\mathrm{kNm/rad}$, $K_{c2}^U = 6.5\,\mathrm{kNm/rad}$, $K_{c3}^U = 38.0\,\mathrm{kNm/rad}$ and $K_{wall}^U = 1.72\,\mathrm{kN/rad}$, and the material properties of the slab are corrected with a factor 0.99.

Figure 12 shows the first four mode shapes of the floor.



(a) Elementary module of a cellular beam (b) Elementary module of the orthotropic slab

Figure 10: FE models used to determine equivalent properties of cellular beams and orthotropic slab

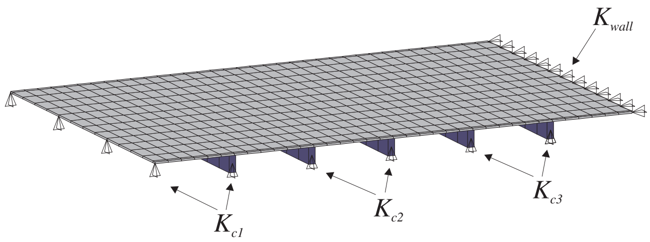
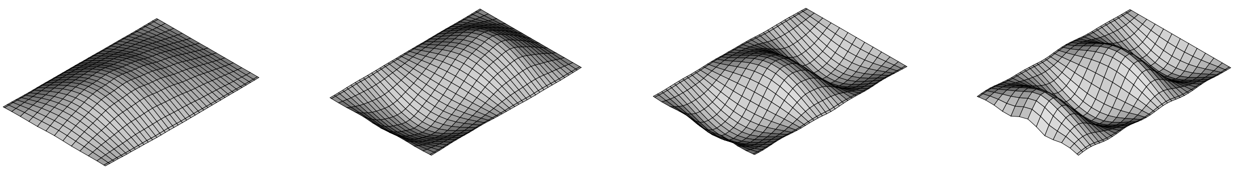


Figure 11: Model of the composite floor



(a) Mode 1 at 5.67 Hz (b) Mode 2 at 6.60 Hz (c) Mode 3 at 7.83 Hz (d) Mode 4 at 9.68 Hz

Figure 12: Eigenmodes of the floor

1.2 Uncertainty modelling

It is most interesting to investigate the influence of the uncertainty on the updated parameters on the modal data of the structure, as this may explain part of the remaining discrepancy between measurements and numerical data.

Six individual uncertain parameters are used: one for the orthotropic material properties ($\widetilde{E_x} = E_x^U \widetilde{X_1}$, $\widetilde{E_y} = E_y^U \widetilde{X_1}$ and $\widetilde{G_{xy}} = G_{xy}^U \widetilde{X_1}$), three for the column connection stiffnesses ($\widetilde{K_{c1}} = K_{c1}^U \widetilde{X_2}$, $\widetilde{K_{c2}} = K_{c2}^U \widetilde{X_3}$ and $\widetilde{K_{c3}} = K_{c3}^U \widetilde{X_4}$), one for the connection with the concrete core ($\widetilde{K_{wall}} = K_{wall}^U \widetilde{X_5}$), and finally also the damping coefficient is modelled as a fuzzy variable ($\widetilde{\xi} = 0.8\% \cdot \widetilde{X_6}$).

It is observed that most of the uncertainty acts very locally, but for this structure, no deterministic substructure exists. A considerable profit is however obtained when the floor is modelled with a fuzzy superelement (when using the Transformation Method): at a certain α-level, two superelements are generated, one for $\left[\widetilde{X_1}\right]_\alpha^-$, and one for $\left[\widetilde{X_1}\right]_\alpha^+$. While all other uncertain parameters change, x_1 always takes one of these two values. When taking 20 modes of the substructure into account, the total number of DOFs is reduced from 3385 to 119.

Secondly, when calculating an FRF with the modal superposition method, the damping ratio only comes in at a very late phase in the calculation. This means that when two analyses take equal values for $x_1, \ldots, x_5$, and only differ in their values for x_6 (as is the case when applying the Transformation Method), the main part of the calculation should only be performed once.

Figure 13 shows the flowchart of the calculation of the FRF at all vertices of one α-level. If the fuzzy numbers are discretized into $m = 11$ α-levels, the original set of $m2^n = 704$ modal analyses of 3385 DOFs is reduced to 22 CMS analyses of 3385 DOFs and 352 modal analyses of 119 DOFs. The number and complexity of the FRF calculations has not changed (i.e. 704 analyses of 20 modes).

1.3 Fuzzy frequency response function

The uncertain parameters are modelled as triangular fuzzy numbers, with base widths of 20% for the material properties, 40% for the connection stiffnesses, and 75% for the damping ratio (meaning that $\left[\widetilde{\xi}\right]_0 = [0.5\%, 1.1\%]$). A fuzzy FRF (figure 14) is determined between an excitation at the center of the plate and the response at a point near the excitation point with the Transformation Method. The FRF is determined with modal superposition of 20 modes.

Most striking observation on this graph is the local decrease (and increase) of the upper (and lower) FRF envelope at resonance frequencies at low α-levels. The reason for this behaviour can be found in figure 6(b), where also stiffness parameters and damping ratio are modelled as fuzzy numbers. When considering an excitation frequency near an eigenfrequency of the ‘mean’ model, the maximal response will be obtained for mean values of the stiffness parameters, combined with the lowest value of the damping ratio. However, this combination is not made by the Transformation Method: at high α-levels, the moderate values of stiffness parameters are combined with moderate values of the damping ratio and at low α-levels, extreme values of both stiffness parameters and damping coefficient are combined.

Keeping in mind that most of the calculational efforts are spent to the variation of the stiffness parameters, and that an evaluation of the structure at a new damping ratio only requires very limited efforts, the evaluation points as shown in figure 15 (simplified to 2D) can be chosen in order to improve the fuzzy FRF. Since the FRF varies monotonically with respect to the damping coefficient, not all combinations are required.

With these evaluation points, the number of modal analyses remains the same (i.e. 22 CMS analyses of 3385 DOFs and 352 modal analyses of 119 DOFs), but the number of harmonic analyses increases from 704 to 46464 (but even for this number of analyses, the time spent to the harmonic analyses (with 1000 excitation frequencies) is only about 1/3 of the time spent to the modal analyses). The corrected FRF is shown in figure 16.

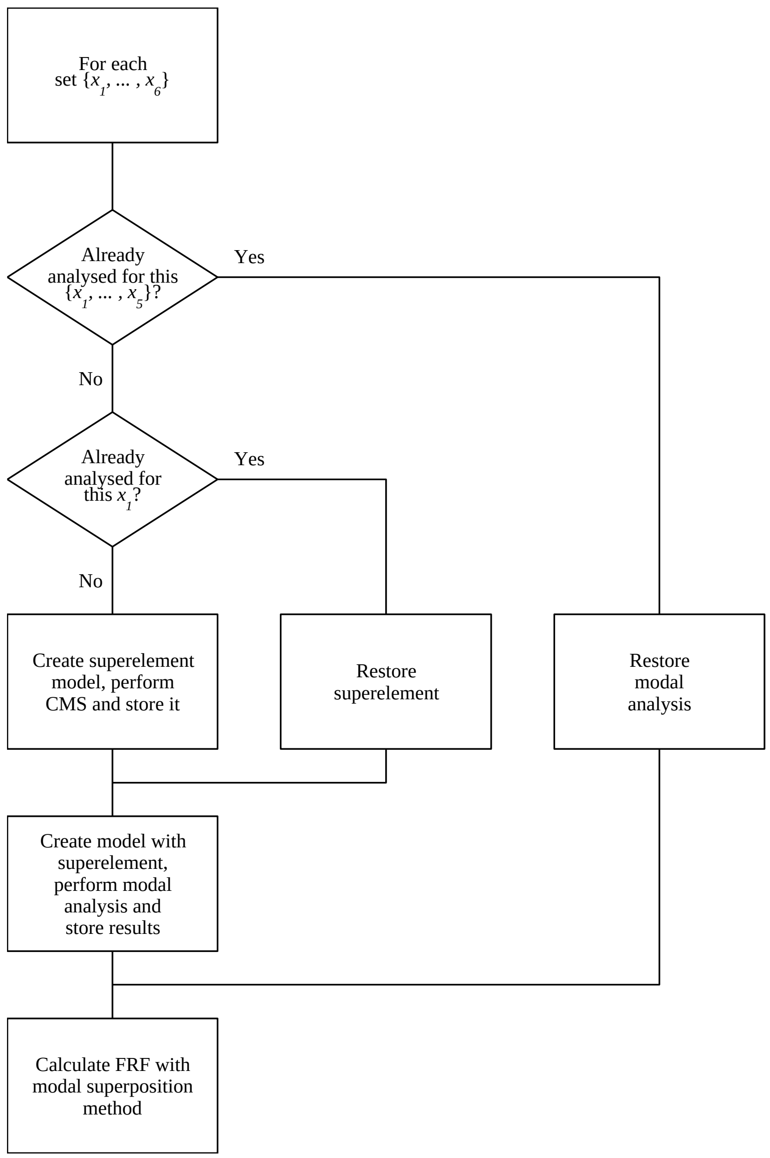


Figure 13: Flowchart for FRF calculation of composite floor

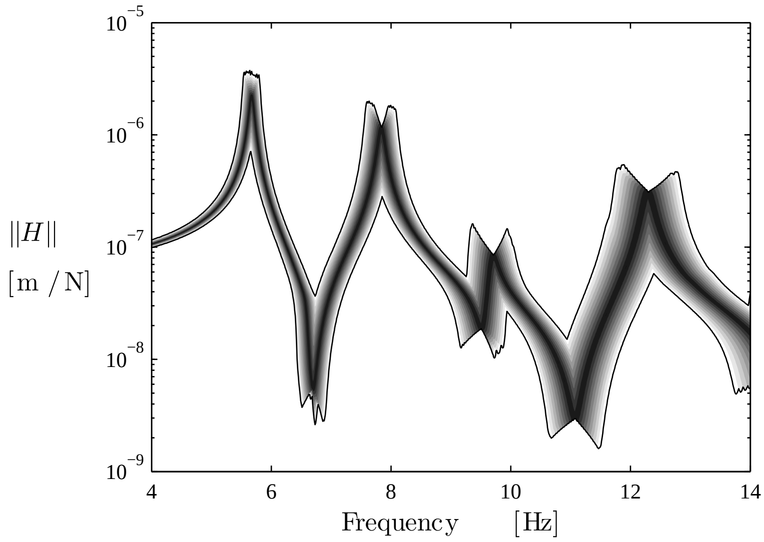


Figure 14: FRF of the composite floor

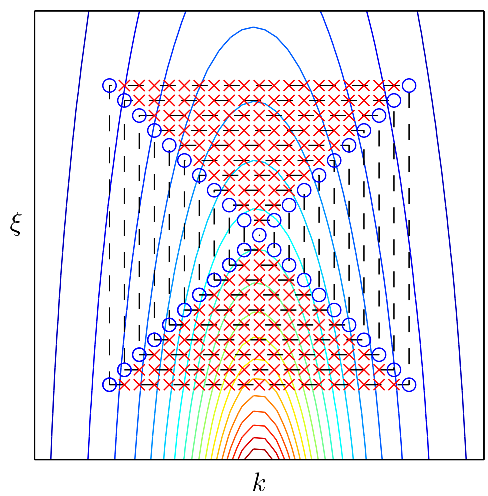


Figure 15: Extended version of the Transformation Method in case of uncertain damping: ○ denote the regular TM evaluation points, × denote the extra points

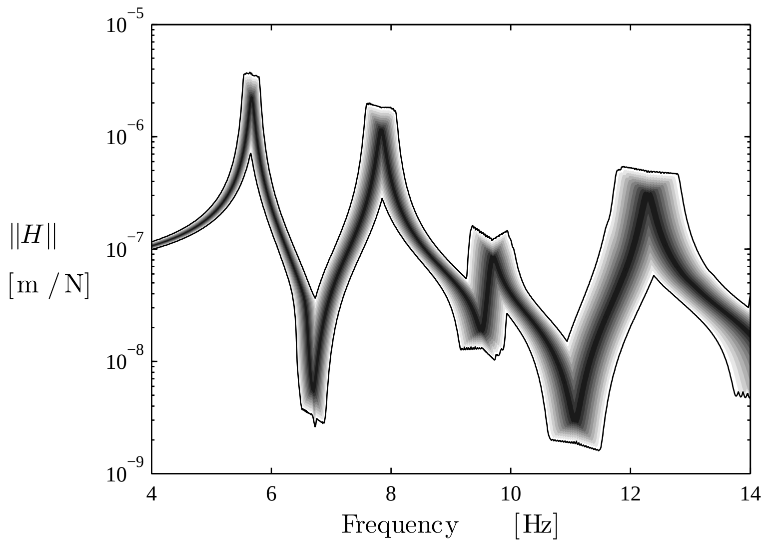


Figure 16: FRF of the composite floor, after correcting for fuzzy damping ratio

2 FE Model updating of a damaged beam

In this section, damage detection of a reinforced concrete beam was performed by a finite element (FE) model updating method in which the uncertainty, involved in the experimental modal data, was modelled using a fuzzy approach. First, vibration experiments are conducted in laboratory before and after application of a static load on the beam. The FE model results of the RC beam is adjusted to the test results, and the stiffness distribution along the beam is updated. FE model updating of the undamaged RC beam was performed by deterministic FE analysis, and the FE model updating of the damaged RC beam was performed by fuzzy FE analysis. In the present study, modal characteristics as natural frequencies and mode shapes are used as input parameters in the model updating.

Before the detail discussion, one should note an important difference between this application and the others: fuzzy analysis is applied on a problem in which FE method is used as a lower level procedure (FE method is directly used in the other applications).

2.1 Case description

The dimensions of the RC beam is shown in figure 17. The total mass of the beam is 750 kg. Vertical stirrups of 8 mm diameter are placed every 200 mm for shear reinforcement.

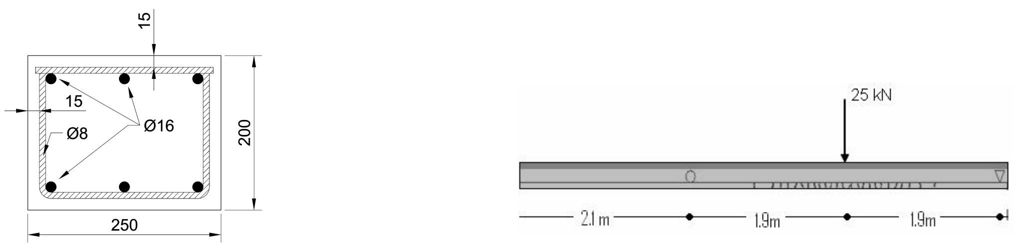


Figure 17: The dimensions of the RC beam and the static load experiment setup

A modal test is carried out on the beam supported by two flexible springs as shown in the figure 18. The load for excitation is an impact applied at one of the free ends of the beam. Dynamic response signals are taken from 62 sensors (accelerometers) placed at both longitudinal edges of the beam (31 sensors at each edge with a longitudinal interval of 20 cm). The dynamic modal parameters are extracted from measured dynamic response signals applying the stochastic subspace analysis [11]. After an initial (reference) dynamic test, a static point load of 25 kN is applied at 2 m away of the right end. The experimental setup of the static load application is shown in figure 17. The same modal test is applied again in order to obtain the experimental modal data of the damaged beam. The first four identified modes will be used in the computation section. The corresponding dynamic modal parameters of the reference and the damaged state of the beam are given in table 1 (eigenfrequencies) and figure 19 (modeshapes).

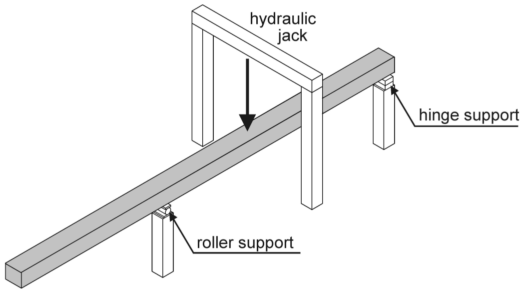


Figure 18: The vibration experiment setup

Mode	Undamaged	Damaged
1	22.02	19.35
2	63.44	56.90
3	123.27	111.64
4	201.92	185.22

Table 1: Experimental eigenfrequencies for the undamaged and damaged RC beam. All the values are in Hz

The reinforced concrete beam is modelled in ANSYS [4] with 30 (2 dimensional) beam elements. In principle, any parameter in the system matrices can be selected as the updated parameter [12]. The correction of the stiffness properties was performed in this work due to the fact that this property decreases significantly with applied damage. 8 damage elements are used instead of updating 30 beam elements [13]. Damage is represented as the sum of 7 linear damage functions multiplied by 7 updating parameters (design variables of the updating problem). The FE model of the beam, its damage elements and damage functions are shown in figure 20.

2.2 FE model updating

First, updating of the reference beam is performed with deterministic FE method. The initial FE model is characterized with $E_0 = 37.5$ GPa and $I_0 = 1.93 \times 10^{-4}\,\mathrm{m}^4$ being Young's modulus and moment of inertia respectively. Note that the initial FE model is the reference state for the updating of the undamaged (reference) beam. The objective function is set up with 4 frequency residuals, and 124 mode shape residuals. The initial and updated FE model results are shown in figure 21. As mentioned in the previous section, dynamic modal parameters are obtained from 62 measurement points (for both of the experiments). The obtained data is averaged to 31 values. Therefore, there are $31 \times 4 = 124$ (31 values for each one of the 4 modes) modal parameters, and 4 corresponding eigenfrequencies as inputs (128 inputs in total).

Updating of the damaged beam is similar with some differences. First of all, the reference state is the updated FE model of the undamaged beam. The other difference is that fuzzy uncertainty is applied on the inputs in this case (fuzzy uncertainty can also be applied to the updating of the initial FE model). The dynamic modal parameters are converted to fuzzy numbers to represent uncertainty. The number of α-levels is adjusted to 10 and $\pm$ 1% uncertainty is applied at zero α-level (support of fuzzy numbers). Triangular membership functions are used for simplicity [14, 15] where $\mu(x) = 1$ level has the deterministic value obtained from dynamic tests.

The choice of optimization method for fuzzy calculations is limited due to the number of modal parameters (inputs). The SQP method minimizes and maximizes the function for each α-level. This corresponds to $7 \times 9 = 63$ times function optimization (7 design variables, 9 α-levels of intervals). This method was tried and it was observed that the developed program has a very high computation cost. The Transformation Method does not work for such a problem due to the high number of inputs. The number of function evaluations

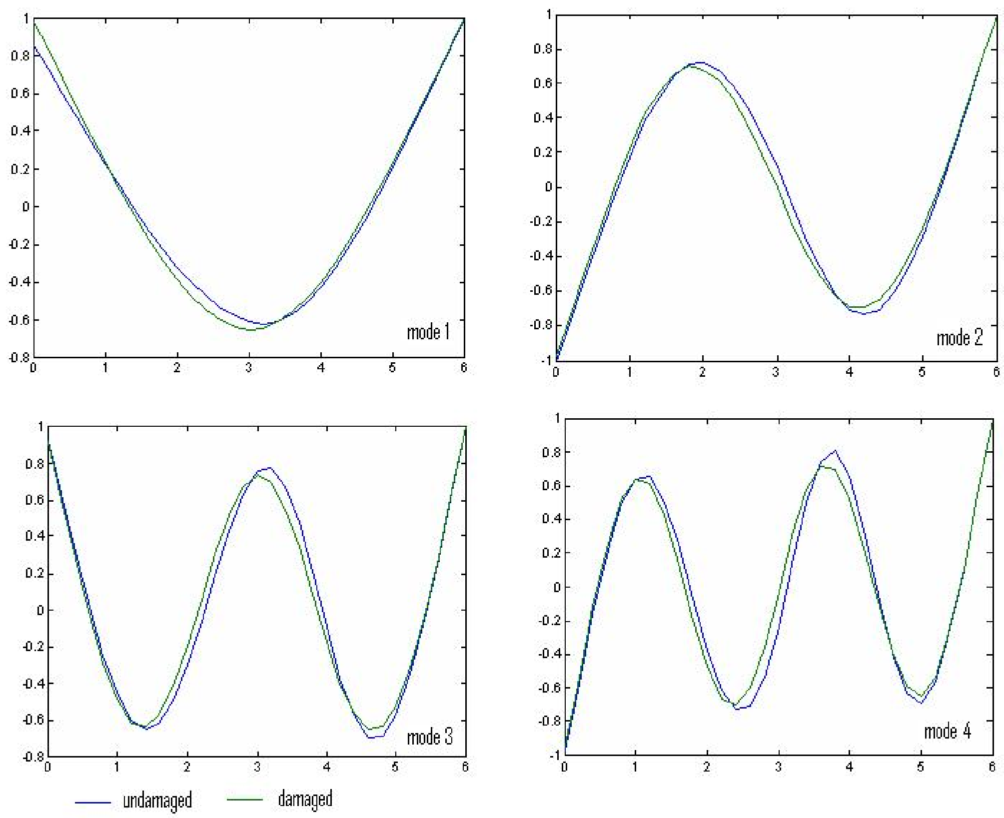


Figure 19: Experimental modeshapes of the undamaged and damaged beam

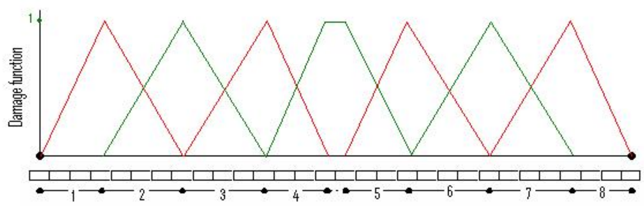


Figure 20: The FEM, damage elements and damage functions of the RC beam

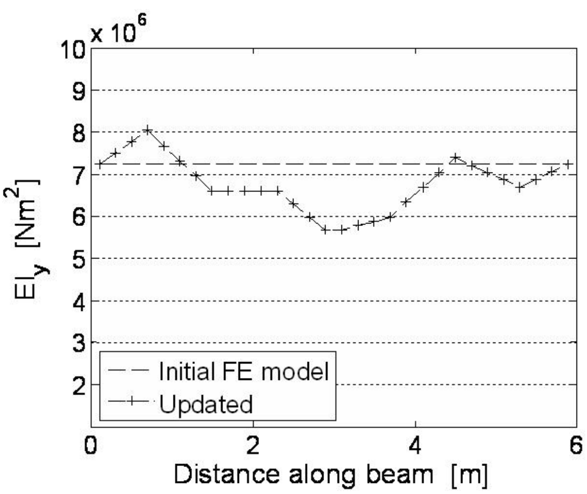


Figure 21: The stiffness distribution along the undamaged beam

needed for this method is $N_{TM} = (10-1) \times 2^{128} + 1 = 3.06 \times 10^{39}$, which means that the computation time is extraordinary for this problem. The last choice, Short Transformation Method, performs well for this updating problem. Due to the 7 design variables 14 critical combinations of upper and lower bound values are predicted by performing a sensitivity analysis (the value of the critical combinations can be any even number between 2 and 2 times the design variables depending on the problem), which corresponds to 127 function evaluations. It is obvious that Short Transformation Method is ideally suited for such a large problem. However, one should note that this method gives good results if the function is monotonic in its inputs. In a first approach the Short Transformation Method is applied on the current updating problem, with a low range of uncertainty ($\pm$ 1%) in order to obtain safe results. The results are shown in figure 22 .

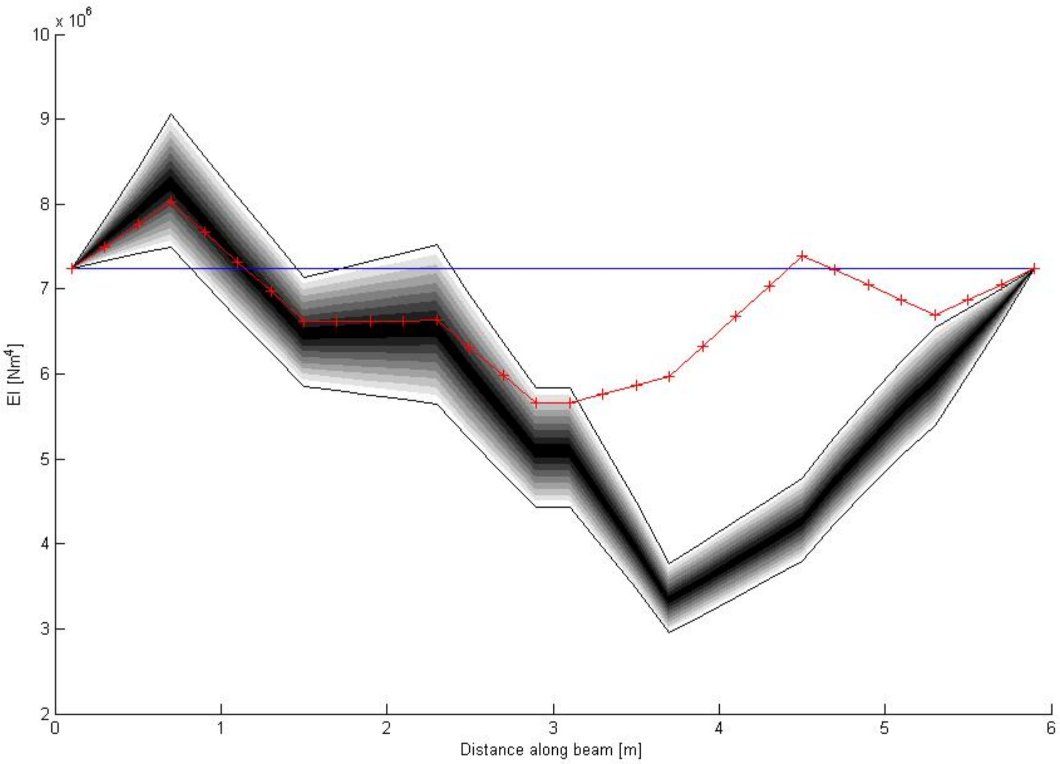


Figure 22: The stiffness distribution along the undamaged and damaged beam. $-$, indicates the initial FEM and $+$ indicates the updated FEM of the undamaged beam.

2.3 Comments on the results

FE model updating was performed including fuzzy uncertainty on the dynamic parameters obtained from vibration tests. The uncertainty was introduced by triangular membership functions for simplicity. At the end of the computation the fuzzy distribution of stiffness along the beam was obtained as output. When the output at each individual finite element is examined, it was observed that all of them are convex membership functions. Therefore, it can be concluded that the objective function is a more or less monotonic function of the eigenfrequencies and modeshapes. Consequently, the Short Transformation Method is a safe method for this problem. When the fuzzy stiffness distribution along the RC beam is studied, the corresponding graph shows wide intervals at zero α-level. Considering only $\pm$ 1% uncertainty at the input parameters at that α-level, it can be said that the objective function is very sensitive to the dynamic input parameters.

3 Static analysis of a frame

In this example, the maximal deflection of a frame under a static load (figure 23) is investigated. Different kinds of uncertainty are modelled by means of fuzzy numbers, including stiffness parameters (the rotational stiffnesses of the connections, K_1 and K_2), the load amplitude F, and the load location x.

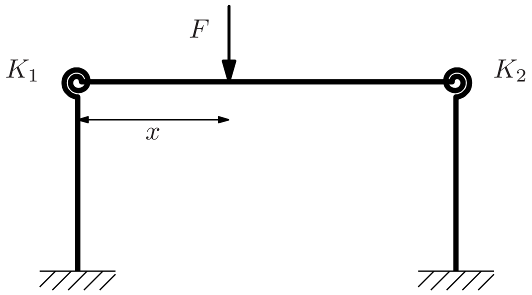


Figure 23: Static analysis of a frame

The frame has a height of $H = 5\,\text{m}$, a width of $20\,\text{m}$, the columns consist of IPE400 profiles and the girder is an IPE360 profile. Both spring stiffnesses are modelled as triangular fuzzy numbers with a centre value of $14.6{\times}10^6\,\text{Nm/rad}$, and a base width of 20 %. The load amplitude F has a centre value of 20 kN and a base width of 20 %, and the load location varies between 4 m and 12 m, with a centre value of 8 m. The membership functions of the uncertain model parameters are shown in figure 24.

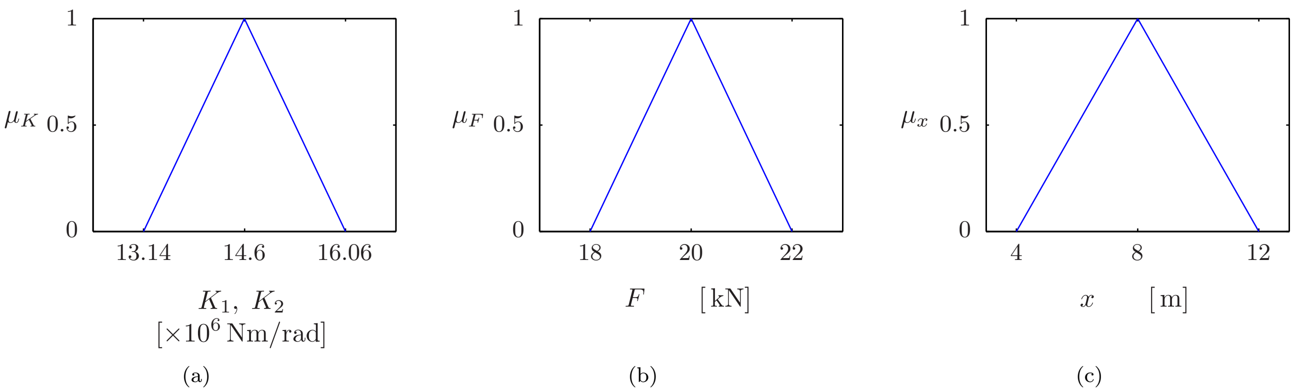


Figure 24: Uncertain input variables membership functions for frame example

Whereas the stiffness parameters and the load amplitude are expected to result in monotonical behaviour of the response variable (the maximal vertical deflection), the load position certainly does not, as it is clear that the maximal deflection will occur when the load takes place at midspan. Therefore, the Tranformation Method and the Short Transformation Method are not appropriate candidates to perform the fuzzy calculations, and a more sophisticated optimization scheme should be adopted.

Although non-monotonical, the input-output relation is believed to be a well-behaving function (continuous, differentiable), with only one local optimum at each α-level (namely for $x = 10\,\text{m}$). Therefore, the GαD algorithm, with the adaptive α-level stepsizing parameters set such that only two search paths remain, seems to be a well-suited choice for this problem.

In order to get an idea of the behaviour of this structure under these uncertainties, figure 25 gives a contour plot of the maximal vertical deflection as a function of the load location and the spring stiffnesses (which are assigned identical values here).

Finally, the fuzzy analysis itself is performed with the GαD algorithm, yielding results as shown in figure 26. It is observed that the extremal points indeed are formed by minimum and maximum values for the stiffness

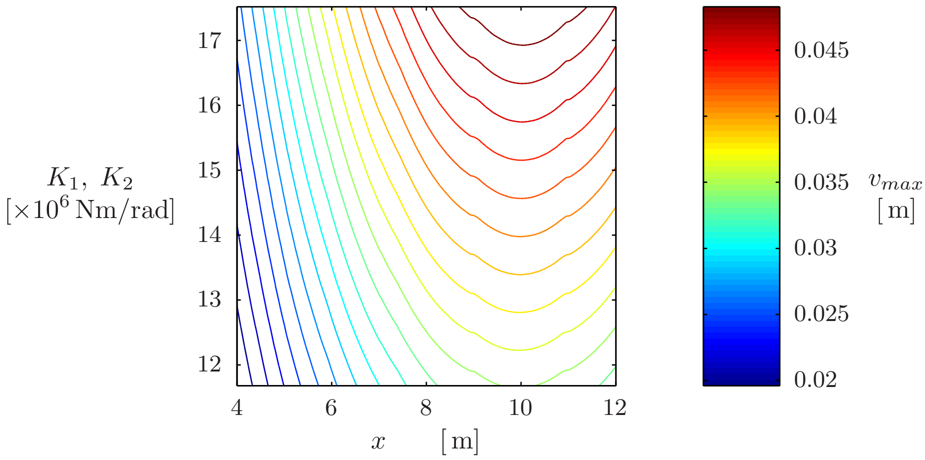


Figure 25: Maximal vertical deflection as a function of load location x and spring stiffnesses K_1 and K_2

parameters and the load amplitude, while for the load location, $x = 10$ m leads to the maximal deflection (if this value is inside the α-cut at that level).

References

[1] S. Lapointe, B. Bobée, Revision of possibility distributions: A Bayesian inference pattern, Fuzzy Sets and Systems 116 (2000) 119–140.

[2] R. Mullen, R. Muhanna, Bounds of structural response for all possible loading combinations, Journal of Seismic Exploration 125 (1) (1999) 98–106.

[3] A. Muhanna, R. Mullen, Uncertainty in mechanics problems – interval-based approach, Journal of Engineering Mechanics 127 (6) (2001) 557–566.

[4] ANSYS Inc., ANSYS v8.1 (2004).

[5] S. Rao, J. Sawyer, A fuzzy finite element approach for the analysis of imprecisely defined systems, AIAA J. 33 (12) (1995) 2364–2370.

[6] H. U. Köylüoğlu, A. Çakmak, S. Nielsen, Interval algebra do deal with pattern loading and structural uncertainties, Journal of Engineering Mechanics 121 (11) (1995) 1149–1157.

[7] F. Biondini, F. Bontempi, P. Malerba, Fuzzy reliability analysis of concrete structures, Computers and Structures 82 (13–14) (2004) 1033–1052.

[8] M. Hanss, The transformation method for the simulation and analysis of systems with uncertain parameters, Fuzzy Sets and Systems 130 (3) (2002) 277–289.

[9] S. Donders, D. Vandepitte, J. Van de Peer, W. Desmet, The Short Transformation Method to predict the FRF of dynamic structures subject to uncertainty, in: Proceedings of ISMA2004, Leuven, 2004, pp. 3034–3054.

[10] M. Hanss, A. Klimke, On the reliability of the influence measure in the transformation method of fuzzy arithmetic, Fuzzy Sets and Systems 143 (3) (2004) 371–390.

[11] B. Peeters, G. De Roeck, Reference-based stochastic subspace identification for output-only modal analysis, Mechanical Systems and Signal Processing 13 (6) (1999) 855–878.

[12] A. Teughels, Inverse modelling of civil engineering structures based on operational modal data, Ph.D. thesis, Katholieke Universiteit Leuven, Leuven, Belgium (2003).

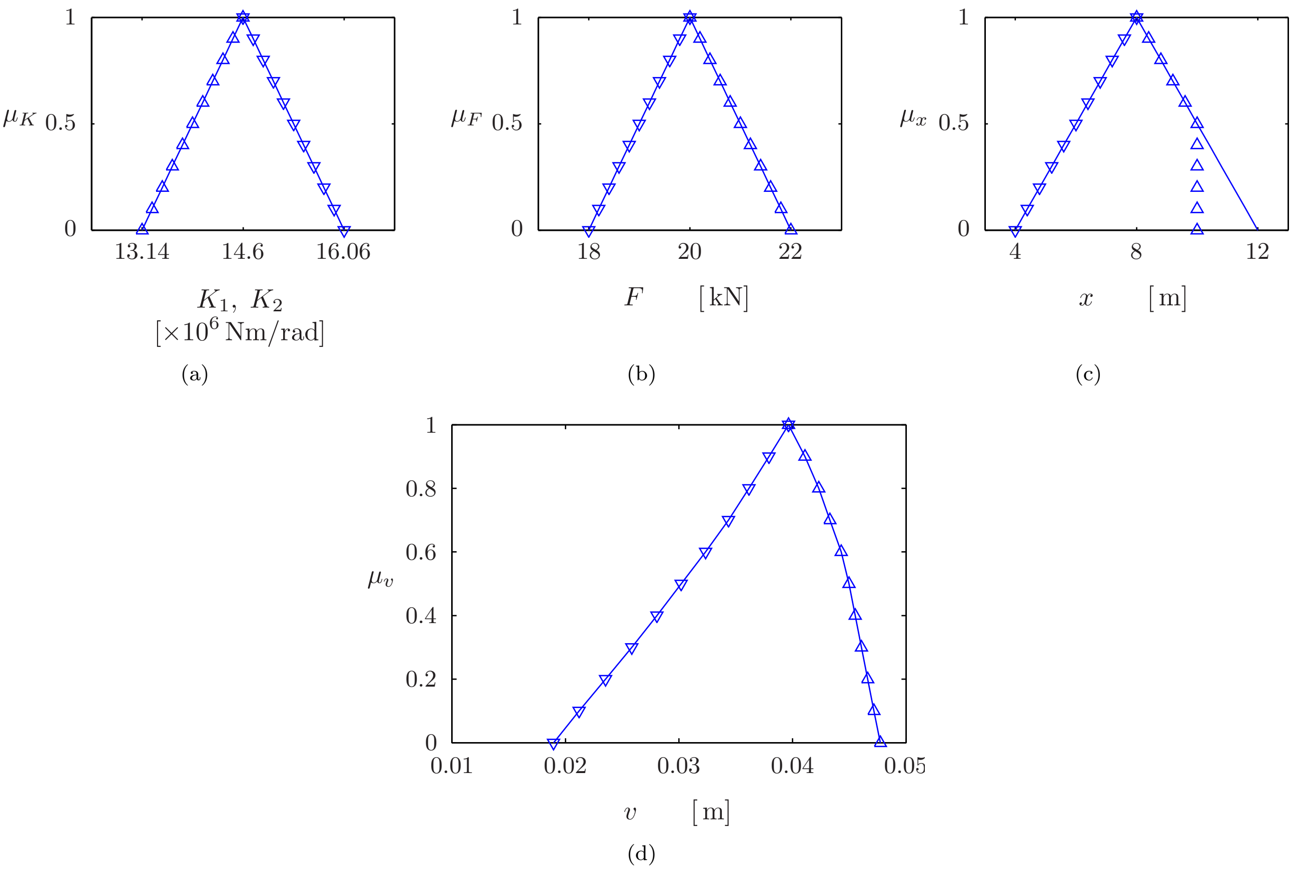


Figure 26: Results of fuzzy analysis of frame example. In (a), (b) and (c), $\bigtriangledown$ denote the combinations wherefore the minimum response is obtained at that α-level, $\bigtriangleup$ denote combinations wherefore the maximum response is obtained.

[13] A. Teughels, J. Maeck, G. De Roeck, Damage assessment by FE model updating using damage functions, Computers and Structures 80 (25) (2002) 1869–1879.

[14] D. Moens, D. Vandepitte, A survey of non-probabilistic uncertainty treatment in finite element analysis, Computer Methods in Applied Mechanics and Engineering 194 (12–16) (2005) 1527–1555.

[15] M. Hanss, K. Willner, A fuzzy arithmetical approach to the solution of finite element problems with uncertain parameters, Mechanics Research Communications 27 (3) (2000) 257–272.

TAP 31

Report part 3.5
Industrial testcase

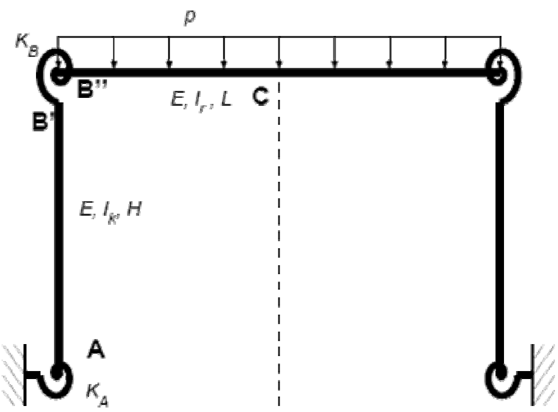
Stephan Masselis
WTCM-CRIF
Celestijnenlaan 300C, B-3001 Leuven, Belgium

1. Introduction

This part of the project will describe the application of uncertainty modeling on industrial test cases. As described in work package 1.5, different alternatives have been presented as possible industrial cases for static structural analysis.

2. civil engineering

A first proposal for industrial test case consists in evaluating the settlement of the footings of a symmetric 2-dimensional frame. In a first approximation, the structure was considered as well defined with only uncertainty in the (rotational) stiffness at the connection between the column and the beam and also an uncertain finite rotational stiffness at the connection with the soil. This real – but simple – industrial case is discussed in work package 3.3 *(Annex33a_scientific_report_2004_313_BWM.pdf)*.



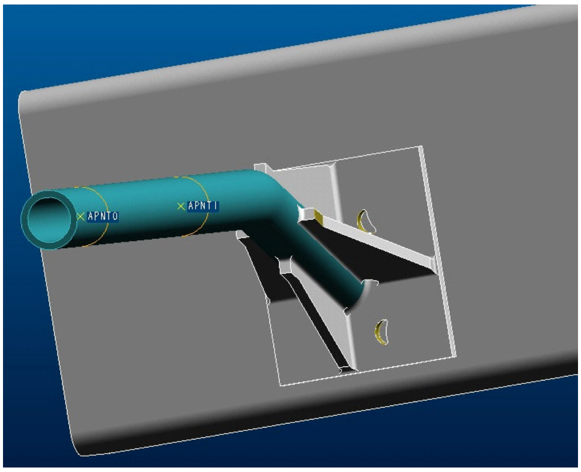
In a later discussion of this industrial test case, the soil is described more realistically by two parameters (cohesion and angle of friction) and non linear material behaviour for the soil (type elasto-plastic). Uncertainties concerning the soil parameters were deduced from extensive soil investigation data obtained at two test sites in Limelette (loam and sand soils) and in Sint-Katelijne-Waver (clay soil). As this extension of the basic industrial testcase also considers only 2 uncertain parameters, the report is also bundled in work package 3.3 *(Annex33a_scientific_report_2004_313_BWM.pdf)*.

In both studies, emphasis has been put on the comparison of different methodologies to model the uncertain behavior (deterministic with safety factors, probabilistic, possibilistic).

For an example in civil engineering with multiple uncertainties, an example of industrial background is discussed in workpackage 3.4.

3. mechanical engineering

A first industrial test case consists of a mud guard support (example made available by DAF Trucks). The main uncertainties are in fact design variables: wall thickness of stiffeners and base plate. Their values are discrete (e.g. 6 or 8 mm thick). Secondary uncertainties depend on tolerances and have possible values within narrower, but continuous ranges (e.g. wall thickness of tube between 4.5 and 5 mm, and the position of the connection bolts to the frame, which vary within a tolerance of 2 mm around the nominal position).

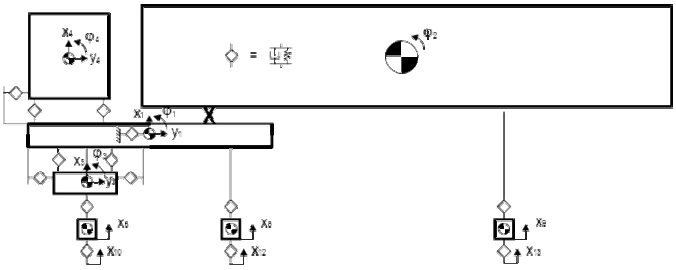


As an academic case, further investigation is done on the connection method of the base plate to the frame and the geometrical detail at the end of the stiffeners (triangle or trapezium), i.e. topological parameters.
As the validation of the results depend on the first natural frequency by way of a penalty function (the lower the eigenfrequency, the more vibrations with higher amplitude will take place), a modal analysis has also to be carried out. See details in chapter 3.1.

A second, more complex case from CNH (Case New Holland), consisting of a complex welded structure forming the frame of a harvesting machine, has not been executed. Instead, another case has been setup for evaluation by a design of experiments setup (see further 3.2).

A third case is rather a dynamic problem (courtesy of DAF-Trucks). It consists of a truck cabin placed by means of spring/dampers on the chassis, which again rests through spring/dampers on the wheel axles. The goal is to optimize spring/dampers to the most comfortable behavior for the driver. This is a model with limited degrees of freedom (point masses and springs in a 2-dimensional model), but many uncertainties:

- design variables or uncertainties for the stiffness & damping of all springs (including tyre stiffness).
- masses of cabin, chassis and trailer
- center of gravity for the masses of trailer and the cabin with respect to the chassis.

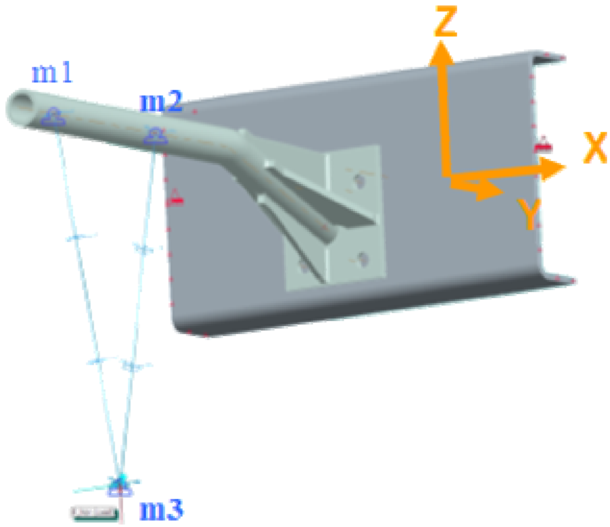


This case was presented at the meeting of the Users' Committee of 21june2005 by David Moens, and is further discussed in the work-package 4 (dynamic structural analysis).

3.1 Study of the mudguard support (DAF-Trucks)

The picture on next page shows the support as modeled in the CAD-system, connected to a [-shaped beam (modeled with limited length). That peace of chassis girder is considered rigid connected at its ends. De mudguard itself (with some additional components) is modeled as 3 pointmasses (m1=m2=3.35 kg; m3=6.1kg) connected to the tube.

Horizontal and vertical unit forces Fx=Fz=1kN act on the tube at points m1 and m2. The real forces are unknown. This study is used to compare new designs with an existing and tested design. The tests are performed on a test bed, and calculation results for a unit force are (logarithmic) scaled to a lifespan of 6000 km on the test bed.



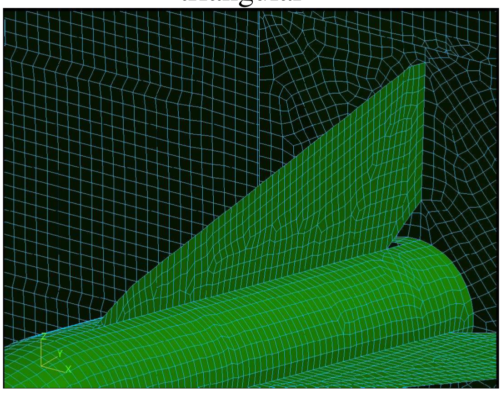
The model is calculated in FEA using shell elements. This allows for an easy implementation of a variable shell thickness for the tube, the stiffeners and the base plate. The model consists of 20746 plate elements (CQUAD4), 21049 nodes (6 dof per node).

<u>The uncertain model parameters:</u>

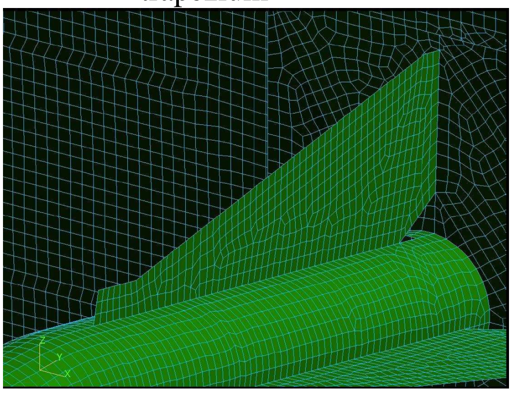
→ Young's modulus:	nominal=210 GPa	range [205 ; 211] GPa
→ thickness of the tube:	nominal= 5mm	range [4.5 ; 5] mm
→ stiffener thickness:	nominal=8mm	2 possible values: 8 or 10 mm

→ 2 possible shapes for the stiffener geometry:

triangular

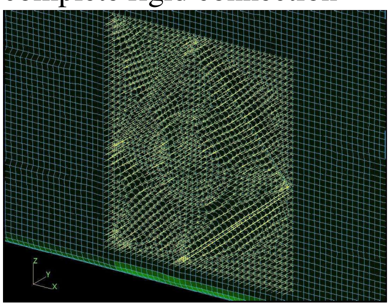


trapezium

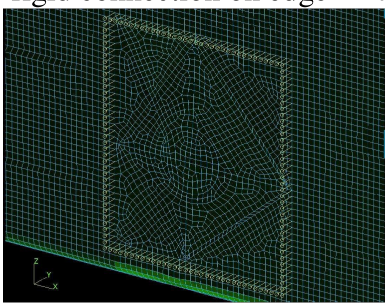


→ connection of the baseplate to the chassis by MPC (Multi Point Constraint or Coupling):
3 possibilities:

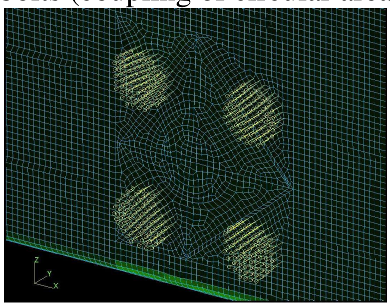
complete rigid connection



rigid connection on edge



4 bolts (coupling of circular areas)



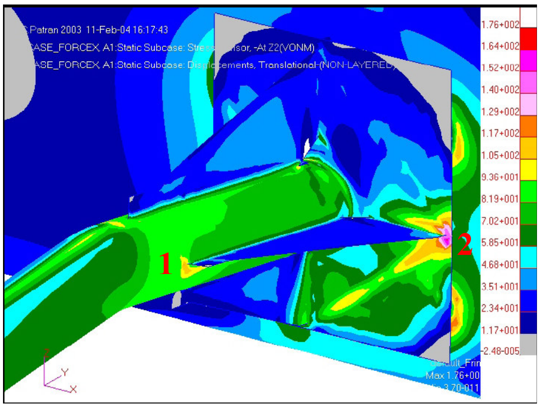
The 4 bolts are located within a tolerance of ±2mm in vertical and horizontal direction about the nominal value.

Most of the uncertain parameters have discrete values or mean topological differences. For the tube thickness, the nominal value corresponds to the maximum. For that reason, no membership functions have been set up. The calculations are performed using the DOE-approach to the transformation method. Because there are no α-cuts defined, this means that the study is limited to calculations at the vertex points (thus a full factorial DOE-approach). Only for the material property (Young's modulus), it could be interesting to define a membership function with α-cuts. Only the nominal value is used for a (deterministic) "center point" analysis, and for a first set of analyses where the response ranges are quantified for each uncertain parameter keeping al the other parameters at there nominal value. For the tolerances on the bolt-positions, the nominal position is the central location.

On the other hand, the different connections between baseplate and chassis that we consider, means that this parameter must be evaluated at different (>2) distinct values, and for the case of a bolted connection, we consider for each bolt separately an uncertainty in horizontal and vertical position, thus combined 8 uncertainties, this means that $2+2^8 = 258$ combinations must be considered for the connection (the first "2" refers to the other 2 totpologies).

The responses of interest depending upon the uncertain model inputs:

→ the first two eigenfrequencies in a modal analysis
→ maximum displacement for both static analys (with force along X resp. along Z) at the end of the tube in longitudinal (X) and vertical (Z) direction.
→ maximum stress responses for both static analyses:
  1) maximum Von Mises' stress in the connection stiffener-cylinder
  2) maximum Von Mises' stress in the connection stiffener-baseplate



The results of the modal analysis:

The first 2 modeshapes correspond to vibrations of the tube (with pointmasses !), the first in a quasi horizontal plane, the second normal to the first.
As explained higher, a first set of analyses is performed to quantify the response-ranges for each input parameter separately (keeping all the other parameters at their nominal values). During a second set of analyses, all the combinations of bolt-positions are considered, and finally, all possible combinations of the uncertain parameters at their extreme values.

		Trapezium		Triangle	
Variable	**Inputvalue**	$f_1(Hz)$	$f_2(Hz)$	$f_1(Hz)$	$f_2(Hz)$
Young's modulus	205000 N/mm^2	19.8781	21.6874	19.8594	21.6462
	211000 N/mm^2	20.1669	22.0025	20.1480	21.9607
Tube thickness	4.5 mm	19.8462	21.7554	19.8270	21.7118
	5.0 mm	20.1191	21.9503	20.1002	21.9086
Stiffener thickness	8 mm	20.1191	21.9503	20.1002	21.9086
	10 mm	20.3875	22.1473	20.3663	22.1059
Global MPC-set	1 (full contact)	21.9634	25.7598	21.9398	25.6897
	2 (bolt connection)	20.1191	21.9503	20.1002	21.9086
	8 (edge contact)	21.3010	25.1005	21.2794	25.0358
MPC1 (position bolt 1)	13 (-2Z)	20.1281	21.9775	20.1091	21.9358
	14 (+2Z)	20.1384	21.8463	20.1194	21.8053
	15 (-2X)	20.1539	21.866	20.1349	21.8249
	16 (+2X)	20.1212	21.9623	20.1023	21.9206
	17 (nom.)	20.1191	21.9503	20.1002	21.9086
Position bolts	Minimum	20.0283	21.6787	Analysis	
	Maximum	20.2181	22.2248	not completed	
Combination	Minimum	19.6085	21.4948	19.5895	21.4518
	Maximum	22.3416	26.1345	22.3147	26.0635

Overview of the results for the modal analysis (indicated numbers = nominal case).

As expected, results show monotonic eigenfrequency variation with respect to Young’s modulus, tube and stiffener thickness. And of course, the connection method of the baseplate to the chassis influences considerably the stiffness (thus the eigenfrequencies).

<u>The results of the static analysis:</u> the tables beneath show a short overview of the calculation results for the static analyses.

Horizontal load F_X:

Variable	Input	Ux(mm)	Uz(mm)	El.	Stress	El.	Stress
Young's	205 GPa	4.808	-0.311	51137	132.092	52091	170.929
modulus	211 GPa	4.671	-0.303	51137	132.092	52091	170.929
Tube	4.5 mm	4.818	-0.318	51137	142.905	52091	170.854
thickness	5.0 mm	4.693	-0.304	51137	132.092	52091	170.929
Stiffener	8 mm	4.693	-0.304	51137	132.092	52091	170.929
thickness	10 mm	4.575	-0.294	51137	136.314	52091	167.944
Global	1	3.768	-0.291	51137	131.997	52022	117.716
MPC-set	2	4.693	-0.304	51137	132.092	52091	170.929
	8	4.082	-0.288	51137	132.048	52090	151.184
MPC1	13	4.683	-0.311	51137	132.098	52091	170.892
	14	4.686	-0.291	51137	132.087	52091	169.648
	15	4.672	-0.297	51137	132.109	52091	169.524
	16	4.691	-0.306	51137	132.076	52091	170.835
	17	4.693	-0.304	51137	132.092	52091	170.929
Combination	Minimum	3.647	-0.272	51137	131.997	52022	117.716
	Maximum	4.936	-0.326	51119	148.432	52091	170.929

Vertical load F_Z:

Variable	Input	Ux(mm)	Uz(mm)	El.	Stress	El.	Stress
Young's	205 GPa	-0.068	3.888	51002	118.439	52178	132.508
modulus	211 GPa	-0.066	3.777	51002	118.439	52178	132.508
Tube	4.5 mm	-0.066	3.883	51002	132.059	52178	132.773
thickness	5.0 mm	-0.066	3.795	51002	118.439	52178	132.508
Stiffener	8 mm	-0.066	3.795	51002	118.439	52178	132.508
thickness	10 mm	-0.062	3.729	51002	126.447	52148	130.744
Global	1	-0.062	2.651	51002	118.229	52178	105.757
MPC-set	2	-0.066	3.795	51002	118.439	52178	132.507
	8	-0.052	2.806	51002	118.257	52178	173.164
MPC1	13	-0.073	3.789	51002	118.442	52178	132.533
	14	-0.053	3.828	51002	118.411	52178	132.900
	15	-0.059	3.824	51002	118.409	52178	133.020
	16	-0.068	3.792	51002	118.443	52178	132.427
	17	-0.066	3.795	51002	118.439	52178	132.508
Combination	Minimum	-0.047	2.574	51002	118.229	52178	105.752
	Maximum	-0.068	3.977	51002	140.826	52178	173.287

Comments on these results:

(expected monotonic variation of displacements with respect to Young’s modulus, tube and stiffener thickness

(edge and full rigid connection between baseplate and frame substantially increase stiffness, resulting in significantly lower displacements. The huge number of analyses for the consideration of tolerance of the bolt positions, is in contrast with the limited influence of that uncertainty.

(maximal Von Mises stress in the cylinder-stiffener connection is strongly influenced by the tube and stiffener thickness
(model of baseplate connection mainly influences maximal Von Mises stress in the baseplate-stiffener connection

Conclusions:

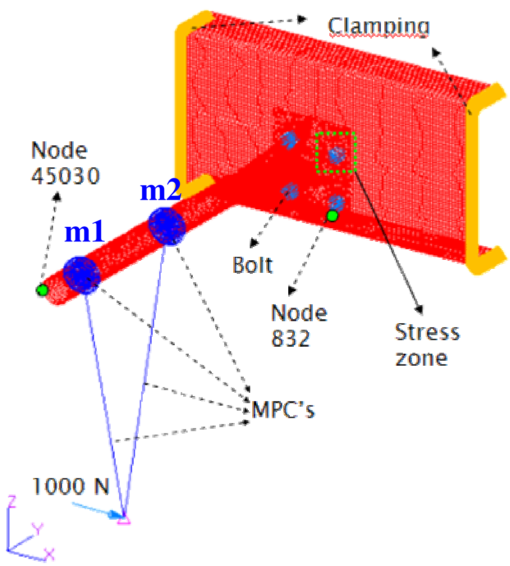
- this case study shows that material, geometrical and modeling uncertainties can be combined in a single uncertainty study
- the interval procedure identifies the model uncertainties that mainly influence the eigenfrequency, displacement and maximal Von Mises' stress
- material and element property uncertainties are clearly the easiest to handle in the automated procedure
- in order to analyze topological modeling uncertainties with more than two possibilities (as in this case for the baseplate-stiffener connection), an 'adjustable full factorial' is required

<u>Remark (see workpackage 2.2):</u> An experimental design where factors are set at more than two levels, is sometimes called an 'adjustable' factorial. An adjustable full factorial DOE for a model with k factors at 2 levels, m factors at 3 levels (n factors at 4 levels) will require $2^k * 3^m$ ($* 4^n$) experiments. In the case of the mudguard support, the adjustable full factorial uncertainty study required $2^4 * (2+2^8)^1 = 4128$ experiments for each laodcase (1 modal & 2 static cases).

2^4 → 4 parameters evaluated at 2 levels
$(2+2^8)$ → number of variants for the connection between baseplate and chassis

<u>Further investigations on the mudguard support:</u>

In order to compare the FFE (Fuzzy Finite Element) analysis with other tools, th model has been slightly simplified: only the bolted connection between baseplate and chassis is considered and tolerance on the position of the bolts is omitted (this had only a small influence), but the bolts are modeled as springs with variable axial stiffness and constant shear stiffness. Also only the trapezoidal shaped stiffener geometry is considered. Hence all uncertain parameters can vary in a continuous range, mostly symmetric.



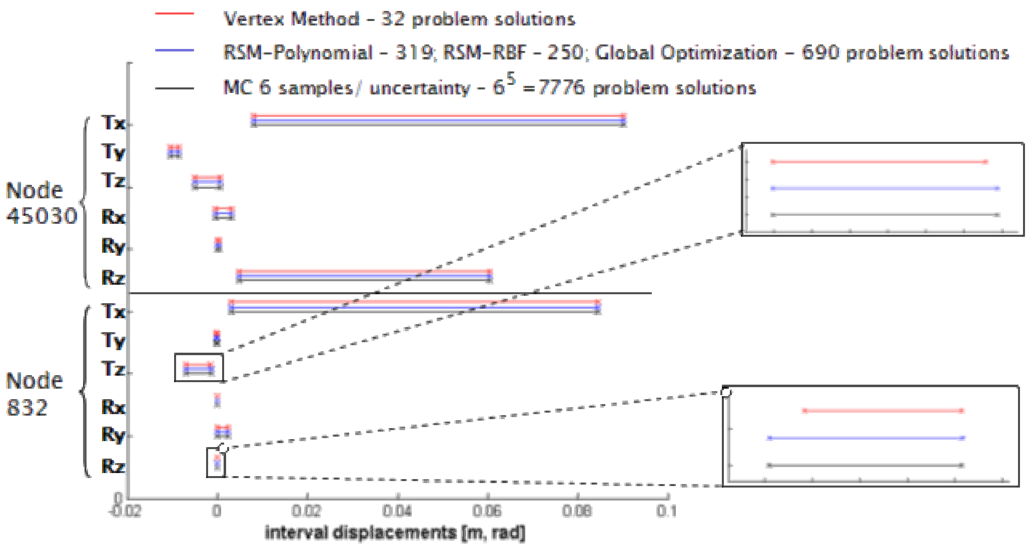
Uncertain parameters:

- unc1 – Young's modulus E ± 5% [210 GPa]
- unc2 – thickness stiffeners ±25% [8 mm]
- unc3 – thickness tube [0 ;-10]% [5 mm]
- unc4 – thickness baseplate [0; +25]% [8 mm]
- unc5 – axial stiffness kn of 1 bolt [1-10000] MN/m, nominal 100MN/m

Constant transverse stiffness horizontal and vertical: kt=500 MN/m. The model consists of ~100.000 DOF's

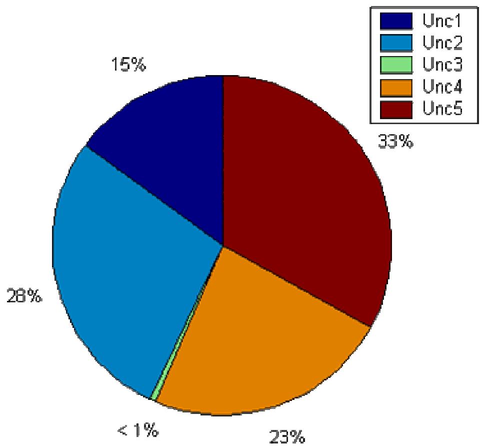
The responses are the displacements (translations & rotations) in node 45030 (end point of the tube) & 832 (lower right bolt, rather an academic control) for the horizontal force Fx.

The study is executed in several approaches (some using LMS-Optimus application):
→ FFE using transformation method with 1 α-level (=vertex method)
→ RSM (Response Surface Modeling) using polynomial
→ RSM using Radial Basis Functions (RBF)
→ Global optimization method
→ Monte Carlo simulations (MC)



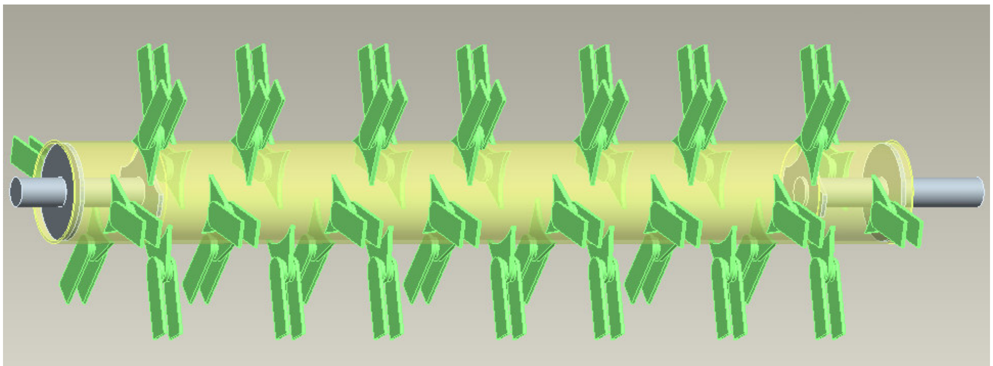
Despite the relative few solution runs (32) for the vertex method, the total range of outputs is covered, except for the academic output at node 832: there is a small part *at the lower amplitudes* of output covered by the other methods, that is missed by the vertex method for Tz (translation along Z) and Rz (rotation about Z-axis). This means that the response is not completely monotone for the variable input parameters. Although a detailed investigation has not been performed, one can expect that for some combinations of the parameters, the combination of bending & twisting can lead to situations where the total deformation for some DOF is to a higher level compensated than for the vertex points..

Finally, a numeric quantification of the influence of each parameter is performed (analog to the methodology of DOE, discussed more in detail in next industrial case (§3.2)).
The plot on the right shows the relative influence for each input variable on the displacement Tx of node 45030.
It seems surprising that parameter3 (thickness off the tube) has only a small influence (1%). This can be explained by the fact that the nodes at points m1 & m2 are connected by MPC's. Due to this modeling idealization, only the small part of the tube between point m2 and the stiffeners is really excitated. Also the range covered by uncertainty 3 is relative small compared with the other (unc5 covers a very large range !).



3.2 Study of the straw chopper (Case New Holland [*])

A straw chopper is a machine that cuts the straw of harvested wheat or other cereals in small pieces, in order to leave it on the ground as natural fertilizer. The main part of the machine is a drum (semi-transparent yellow on figure; ±ϕ194mm and more than 2m long) with triangular shaped plates welded on it (paddles). On each of these paddles two knifes are connected over



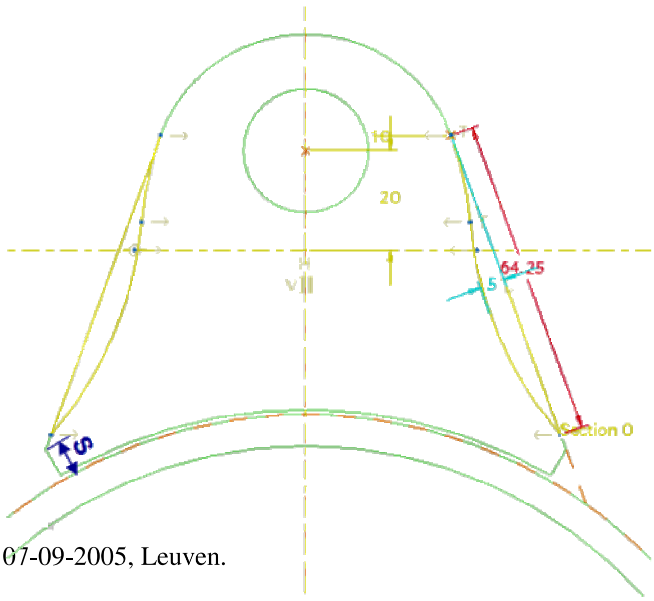
an axis with bushings, so they can rotate during a collision with a big part. The bushings are clamped by a bolt against the paddle, so that radial force is transferred by friction (according to the clamping force, this is true if the friction coefficient $\mu > 0.3$).
The radial distance between the axes of 2 opponent knife-pairs is 300 mm. This dimension is very strict and follows from the paddle geometry with very tight tolerances and the setup for the automatic welding robot.
This case considers the nominal centrifugal load. The rotational speed is 3500 rpm, leading to a radial force of 24kN per paddle.

<u>Uncertain input parameters</u>

The drum consists of a standard tube, with normalized tolerances for outer diameter (±1%) and wall thickness (approx ±5%):

unc A	***R_out*** (from ϕ193.7 ±1%)	nominal 96.85 mm	range [95.88 ; 97.82]mm
unc B	Thickness **T**	nominal 6.3 mm	range [5.95 ; 6.75] mm

The inner face of the paddle is dimensioned such that it fits the outer tube diameter for the maximum R_out. For smaller values of R_out a gap exists up to about 2mm that has to be filled with welding material.



Peak stresses are expected near the shoulder (**S**) in the weld . For that reason 2 design parameters are introduced that model an excavation in order to introduce less rigid behavior and hence less peak stresses near the shoulder S:

unc C = excavation depth h (blue dimension)
nominal 6mm range [1 ; 8] mm
unc D = excavation length L2 (red dimension)
nominal 67.4mm range [64.25 ; 68.9] mm

The maximum of D is given by the necessity to leave a minimum amount of material in order to form the weld.

[*] the case has been presented in a meeting of the partners on 07-09-2005, Leuven.

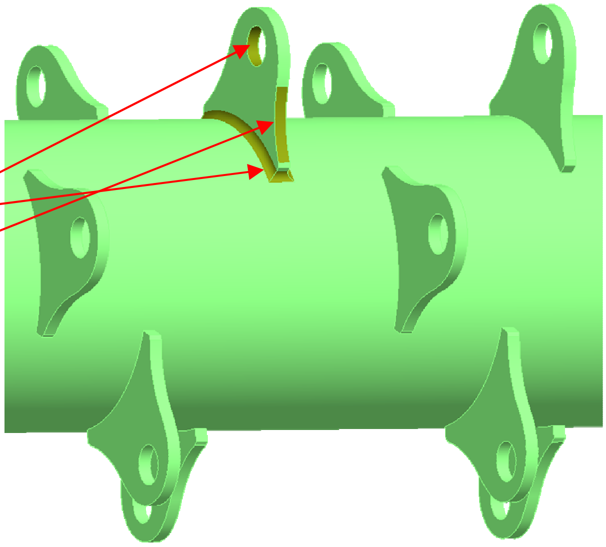
As an additional (academic) parameter, the way how forces are distributed from the bushings to the paddle, is taken into account by de force distribution parameter F_distr and is expressed as the rate (%) the force is transferred as bearing load (sinusoidal distributed on the outer half of the hole). The rest of the force is transferred by friction due to the clamping force on the bushings (equally distributed all over the hole). The nominal value for F_distr is unknown, but ideal and in normal circumstances the bearing rate = 0 (100% Clamping). For the study it is assumed that the bearing component (B) varies between 33% and 67% (the clamping component inversely varies from 67% to 33%)

unc E F_distr nominal (B): 33% range (B) [33 ; 67] %

Uncertain measured responses

The uncertain outputs that were considered are the maximum Von Mises' stresses (VM) in the dark colored areas on the plot. Only the fillets for one paddle representing the weld is modeled and meshed in detail.

Response 1: VM at the fillet (weld)
Response 2: VM in the hole
Response 3: VM in the excavation



Remarks:
- after executing the analyses, response 3 was no longer considered: stress levels are low. The expected raise of stresses in that area becomes relevant for higher values of **h.**
- the tightening force for clamping the bushings to the paddle is not modeled
- it was not the purpose to investigate the way how welds are exactly modeled. This would be a research project on itself.

Results of the uncertainty study

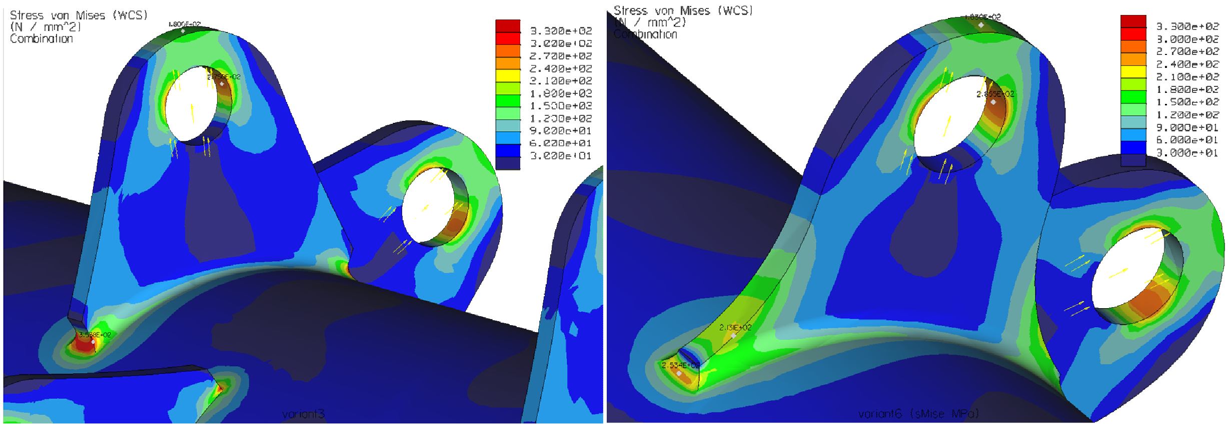
The purpose of this study is to verify the effect of different parameters on the level of the peak stresses. Based on these results, geometry could be optimized by setting the optimal values for the design variables.

As the ranges for the uncertainties are relative small (except for the force distribution), the number of calculations has further been reduced. The vertex method (only 1 α-level) would necessitate 2^5 calculations (+eventually a core level analysis).

For this study one has limited the number of analysis to 8 (fractional factorial study). The parameter combinations are set up based on the DOE-methodology using a so called L8-factorial. The table on next page shows the combinations and the results (for responses 1 and 2). The last 2 variants (NOMINAL_1 & NOMINAL_2) will be used afterwards for inspection & prediction.

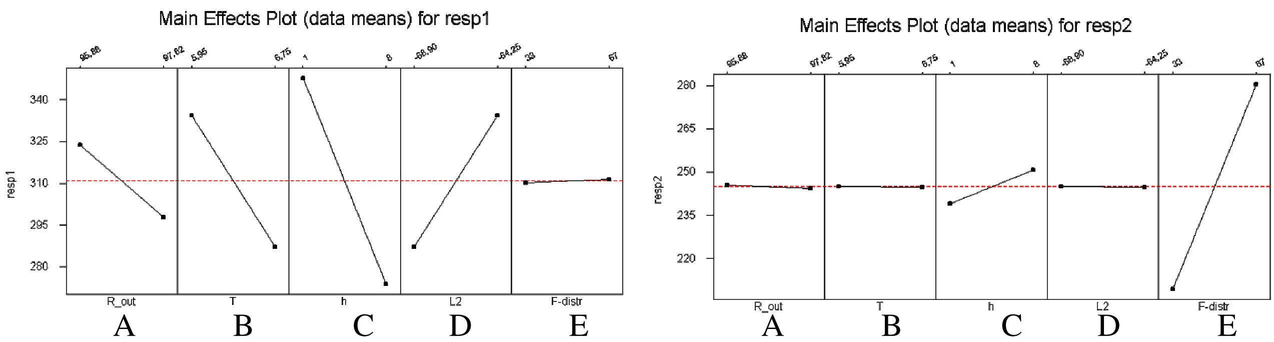
	A tube outer radius R_out	B tube thickness T	C excavation depth h	D excavation length L2	E Force distribution (Bearing & Clamped) F_distr	interactions A x C but also B x D	A x D but also B x C	respons1 sMise fillet (MPa)	respons2 sMise gat (MPa)
VARIANT1	- 95.88	- 5.95	- 1	- -68.9	+ B33% & C67%	+	+	358	204
VARIANT2	+ 97.82	- 5.95	- 1	+ -64.25	- B67% & C33%	-	+	390	274
VARIANT3	- 95.88	+ 6.75	- 1	+ -64.25	- B67% & C33%	+	-	354	275
VARIANT4	+ 97.82	+ 6.75	- 1	- -68.9	+ B33% & C67%	-	-	289	203
VARIANT5	- 95.88	- 5.95	+ 8	+ -64.25	+ B33% & C67%	-	-	336	216
VARIANT6	+ 97.82	- 5.95	+ 8	- -68.9	- B67% & C33%	+	-	254	286
VARIANT7	- 95.88	+ 6.75	+ 8	- -68.9	- B67% & C33%	-	+	247	287
VARIANT8	+ 97.82	+ 6.75	+ 8	+ -64.25	+ B33% & C67%	+	+	258	214
$\Sigma(resp1^{+})/4$	297.75	287	273.75	334.5	310.25	306	313.25	mean value	
$\Sigma(resp1^{-})/4$	323.75	334.5	347.75	287	311.25	315.5	308.25	310.75	244.875
effect on response 1	-26	-47.5	-74	47.5	-1	-9.5	5		
(analog) effect on resp 2	-1.25	-0.25	11.75	-0.25	71.25	-0.25	-0.25		
NOMINAL_2	96.85	6.3	6	-67.4	B0% & C100%			297	152
NOMINAL_1	96.85	6.3	6	-67.4	B50% & C50%			296	248

Combinations of parameter values & results for the “straw chopper study”



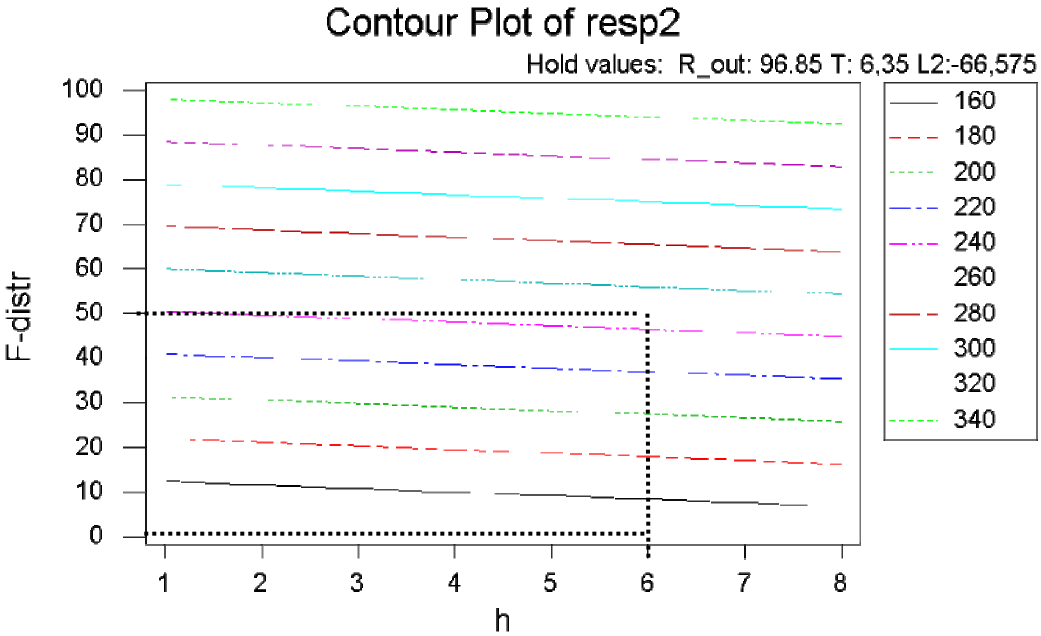
Detail of the stress plot for VARIANT3 (left) and for VARINT6 (right)

If we study the effects of each parameter on the response, one can conclude that the Force distribution has no effect on the maximum stress in the weld, but is the most influencing parameter on the hole-stress (as expected). Only the excavation depth (h) has also a limited effect on response 2. A quick visual overview of the responses is given in the figures below.



Response 1 is the most interesting concerning sensitivity analysis & geometrical optimization

(goal = minimal VM-stress in the fillet). From the main effects, it is clear that the excavation should be maximized (parameters C & D maximized).
The limited number of analyses allows to construct response surfaces. As only 2 parameters(C & E) influence response 2, it is easy to interpret the response surface that plots the response2 in function of these 2 parameters.



As the isolines (for constant stress response 2) are almost horizontal, the parameter on the horizontal axis has only small influence on the response.
As only the parameters C (excavation depth h) & D (Force distribution F_distr) influence response 2, a full prediction can be made based on this single response surface (extrapolated for the force distribution from 0% (zero bearing, thus full clamping) to 100%.
Example: h = Bearing force 50% → 240 < resp2< 250 MPa (see variant NOMINAL_1)
h = Bearing force 0% → resp2 ~150 MPa (see variant NOMINAL_2)

It is also possible to calculate (predict) the stress response1 for any combination of parameter values. For the parameter values of the combination NOMINAL_1, the prediction calculates:
response 1: 289.4 MPa (FEM-control calculation → 296 MPa)
response 2: 247.4 MPa (FEM-control calculation → 248 MPa)

Remarks:

- the methodology followed above is only valid for monotonic and (quasi) linear behavior of the responses in function of the uncertain parameters. This should always be checked by extra calculations (at least 1, e.g. "core level" analysis at the nominal values of the parameters).
- as known from the DOE-methodology, the main responses in the results table (page 10) also include possible interactions (which means that the effect of one parameter is strengthened or weakened by the variation of another parameter.

main effect A also includes interaction BxE
main effect B also includes interaction AxE
main effect C also includes interaction DxE
main effect D also includes interaction CxE

However, we do not expect any interaction between these parameters and the force distribution

main effect E also includes interaction AxB and CxD

TAP 31

Report part 3.6
Comparison of fuzzy finite element approach to the DOE approach.

Stephan Masselis
WTCM-CRIF
Celestijnenlaan 300C, B-3001 Leuven, Belgium

1. Introduction

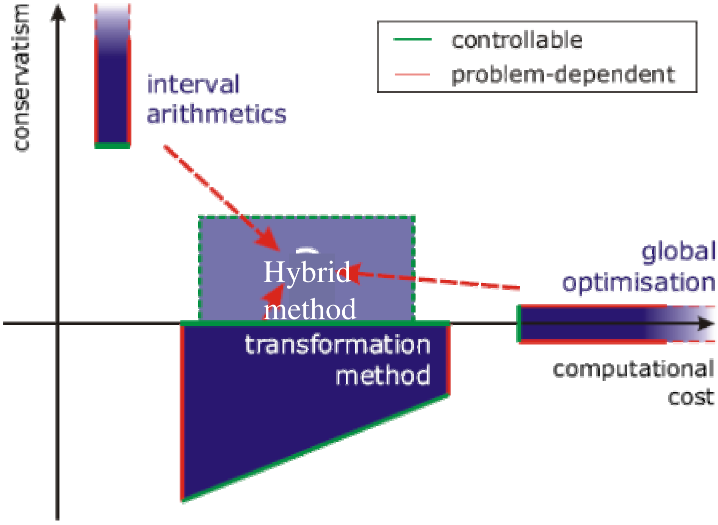
The approach using fuzzy arithmetic that is developed in task 2.3 should produce results that are similar to the results of the Design Of Experiments approach. However, the performance of the procedures is probably different. This task compares the results of both procedures and the performance of both methods.
Remark that also comparisons are made of the fuzzy method with different probabilistic methods. For these cases (see general description in workpackage 3.1 and examples in 3.3) a more ‘realistic’ membership function has been setup (possibility curve of the strength & necessity curve of the load). This allows to estimate the ‘possibilistic’ reliability of a civil structure, which is compared with the probability that a structure will resist (or fail) in a probabilistic analysis. This issue, as well as the comparison with deterministic calculations using safety factors, is described in more detail in workpackage 3.7.

2. Comparison of the fuzzy finite element approach (FFE) to the DOE approach.

The FFE methodology is implemented following different strategies ([*]):

- using interval arithmetic (→ IFFE = Interval Fuzzy Finite Element method).
- global optimization (i.e. optimization method in order to find the minimum & maximum responses (= fuzzy response interval) as function of the fuzzy input parameters and the considered ranges (respective ranges for each α-cut). The input parameter values corresponding with the maximum output is in general the *‘worst case’*.
- transformation method (reduced to vertex method in only 1 α-cut is considered)
- hybrid approach: combines global optimization & interval arithmetic in order to reduce computational cost and moderate the conservativeness of the interval approach.

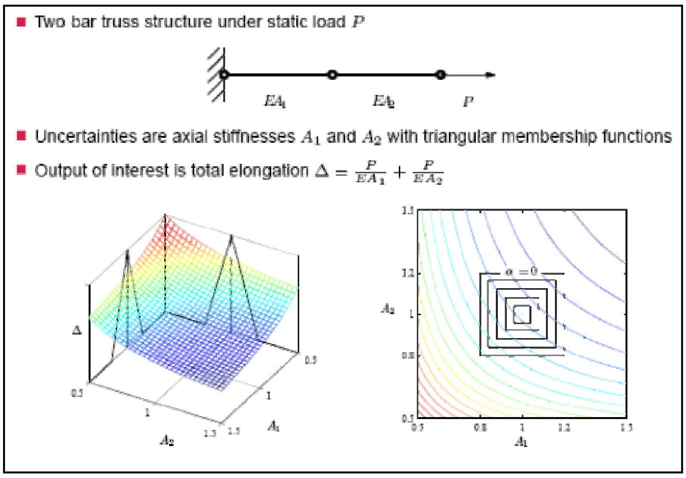
[*] see presentation “Implementation and Application of Finite element Method” of David Moens at the meeting of the Users’ Committee on 21/06/2005
see also workpackage 2.3 (& 2.4): Static (& dynamic) fuzzy FE development



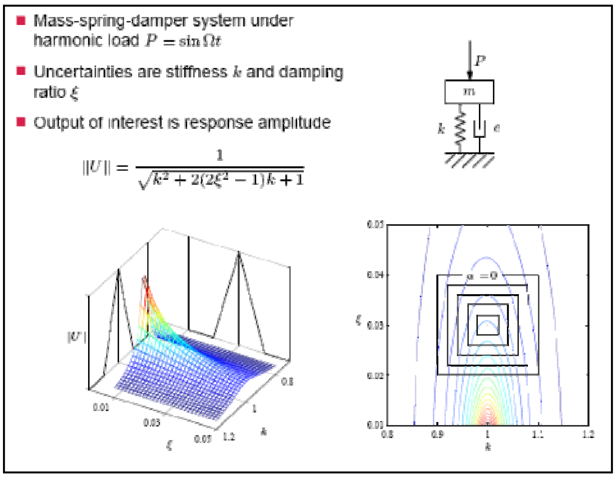
The hybrid approach as well as the interval arithmetic are implemented for dynamic problems (modal analysis & FRF-analysis), and is discussed more in detail in workpackage 4 (Dynamic structural analysis).

The transformation method is far most used for static (mechanical) structural analyses. The main disadvantage of this approach is the fact that it is only applicable in case of monotonic behavior of the responses with respect to the uncertain input parameters. If not, one risks to miss a range of the fuzzy output (and hence the worst case).

An analytical illustration of monotonic versus non-monotonic behavior is given by 2 simple systems with 2 fuzzy input parameters (**):



two-bar truss: monotonic behavior



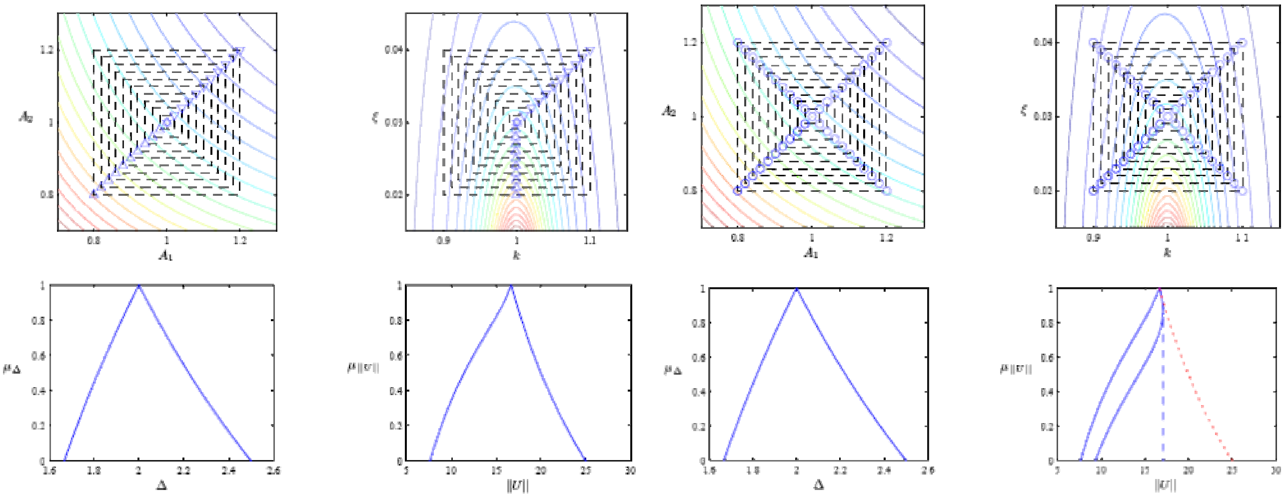
resonant spring-damper system: non-monotonic behavior

The plots on next page show the points where minima (▼) and maxima (▲) are found for each α-cut using the optimization approach (left) and the vertices (o) for calculations at each α-cut using the transformation method (right). The analytical results are plotted as iso-lines

** see presentation "Performing fuzzy analysis using optimization methods" by David Moens at the meeting of the Users' Committee on 21/06/2005 of the response surface on the background. It is clear the optimization method finds the extreme responses (best & worst case) for both cases, but the transformation method fails to detect the worst case for the spring-damper response (mid-bottom area of the response surface).

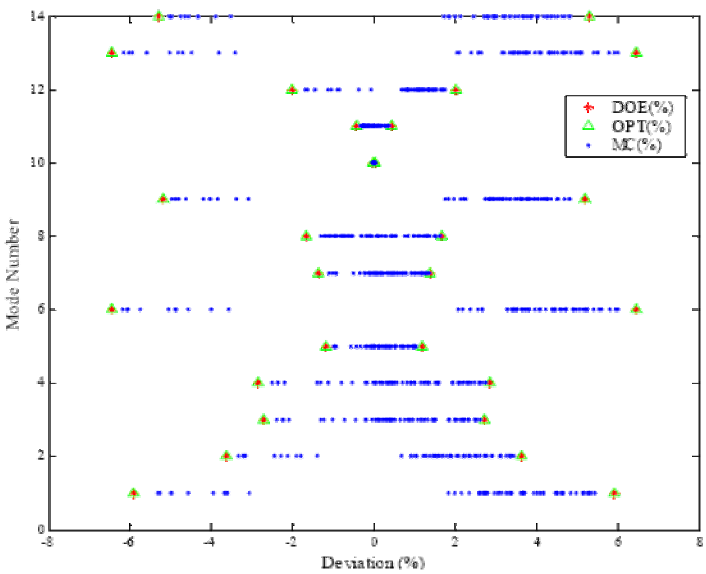
optimization method | transformation method

two-bar truss | spring damper | two-bar truss | spring-damper



As illustration, the membership function of the fuzzy output is also plotted (by interpolation of the minimum and the maximum responses at each α-cut).

<u>Remark:</u> the non-monotonic example is a dynamic calculation and indeed, resonance phenomena yield to non-linearity an non-monotonic behavior. Another dynamic example is demonstrated by the Garteur –model (airplane). In that case the eigenfrequencies wee calculated, not the harmonic response. This property is close related to stiffness & mass (not resonance responses) and hence rather monotonic behavior can be expected. Indeed, the results showed perfect correlation between the optimization method and the transformation method.



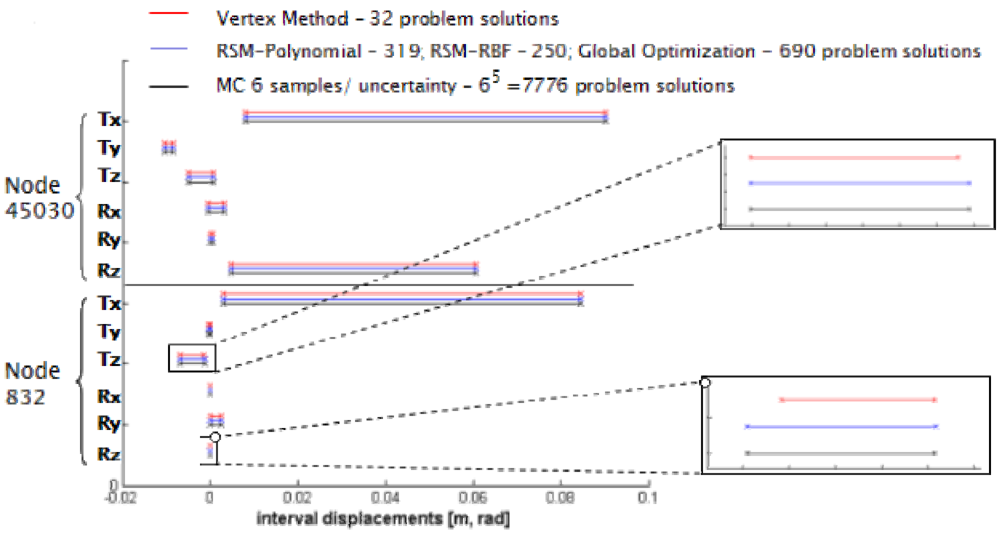
This plot shows the range of the fyzzy outputs (the 14 first eigenfrequencies) as deviation about their mean value for different approaches:

DOE = transformation method (vertex method)
→ 2^4 = 16 runs

OPT = optimization method
→ ±16 runs per mode

MC= Monte Carlo simulations
→ 60 runs (limited number, hence lack of finding the real extrema)

Another static structural example was the mudguard support (courtesy of DAF-Trucks). The plot below compares the results and computational effort (# of runs) for the different approaches for the simplified model with fuzzy spring stiffness for the connecting bolts. (5 parameters at 2 levels → 2^5=32 runs for the vertex method).

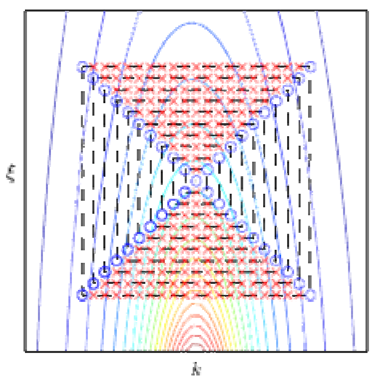


Only for 2 responses, the exact extrema were not found by the vertex method. These extrema are minima (Tz are negative values and the skipped extremum is at the richt side) and do not concern ‘worst case’.

Remark: the computational effort for the vertex method is extremely low in relation to the other methods.

3. Further refining the results of the Transformation Method

In order to assess the behavior in non-linear or non-monotonic cases, one can increase the number calculations for each α-cut of any fuzzy parameter using multiple values within the range (thus not only the minimum and maximum value) as described at the end of workpackage 2.2 (“general transformation method”).



The illustration beside shows the evaluations points for the spring-damper system if only one parameter is evaluated at intermediate values:

- o → evaluation point for vertex method
- x → extra evaluation points

The procedure of calculating at one or more intermediate values can be limited to only one α-cut (level 0). This approach yields to the “adjustable” full factorial DOE. The number of calculations is then $\sim m_1^{n1} * m_2^{n2} * \ldots$ where m_1 parameters are evaluated at n_1 values etc.

4. Reducing computational effort.

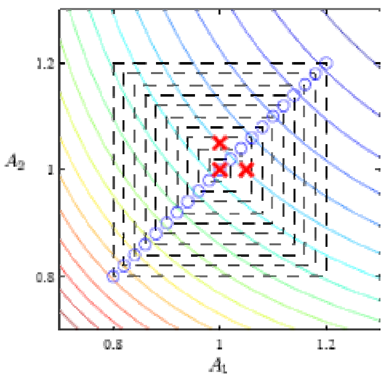
Although the computational effort for the transformation method is relative low, in many cases one can even reduce it further.

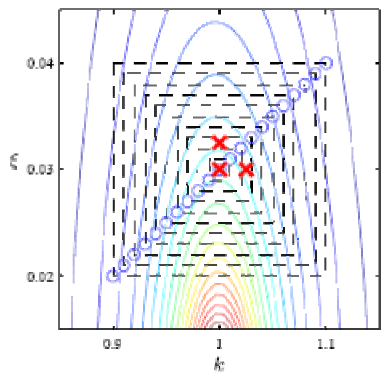
One approach is reducing the number of algebraic operations by reducing n_{dof} (number of degrees of freedom). A general used method in Finite Element systems is *substructuring*. Pieces of geometry that are not dependent on the variation of fuzzy parameters, can be reduced to one 'super'-element. Suppose that piece of geometry divided in a number of finite elements (and a corresponding number of nodes and hence number of degrees of freedom n_{sub}), then that piece of geometry can be reduced to 1 element with a limited set of DOF (these DOF can be a number of selected points where responses are to be measured, or the points/nodes where that substructure interfaces with the rest of the geometry).
Another approach that can be followed, is the reduction of 'active' DOF for dynamic (modal) analysis. That method is also called '*Component Mode Synthesis*' or '*Reduced Modal Analysis*'. The set of active DOF are called the master DOF. After a calculation of the eigenfrequencies and modeshapes with the reduced set of DOF, the results can be expanded to the complete model in order to visualize the modeshapes on the complete model.

Both approaches ([***]) for reducing the number of active DOF (substructuring & reduced modal analysis) are standard implementations in most commercial FE-systems and can in fact be used for any FFE-approach, as well as for other deterministic & probabilistic calculations.

Another approach for further reducing the computational effort is applying a fractional factorial DOE. When monotonic and quasi linear structural behavior can be expected, the computational effort of the transformation method (vertex method) can further be reduced:

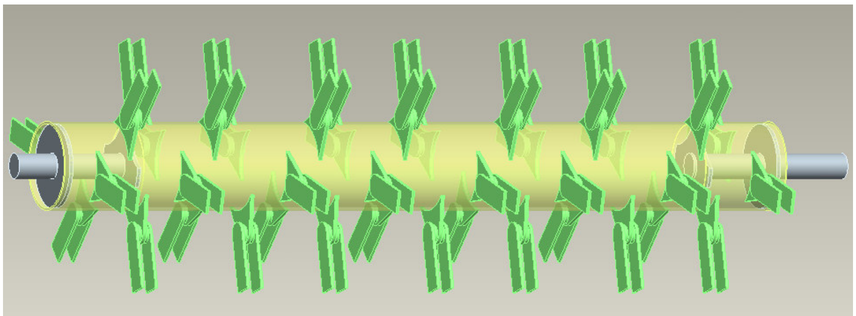
- *Short Transformation Method* (illustrated by way of the previous examples: two-rod truss & spring-damper): after a sensitivity analysis (calculations **x** on the plot) to determine the critical diagonal, the response is calculated for the vertices along that diagonal. It is obvious for this example that the worst case is detected for the two-truss beam, but not for the resonant spring-damper.





[***] examples showed in presentation "The application of substructures in fuzzy analyses" by Daan Degrauwe ea at the meeting of the Users' Committee on 21/06/056

- fractional factorial DOE: this approach is well known in the DOE-methodology. It allows to detect the worst case and to quantify the effects of the fuzzy input parameters on the outputs. The method is only applicable in case of monotonic and (quasi) linear behavior. However, it is also possible to apply adjustable fractional factorial DOE-experiments (one or more parameters are then evaluated at one or more intermediate values between the minimum and maximum).
  A fractional factorial DOE-experiment has been executed for the industrial case of the 'straw chopper' (courtesy of Case New Holland, see workpackage 3.5). For a problem with 5 uncertainties (each at 2 levels), only 8 solutions have been calculated (5^2 = 32 for a full factorial or vertex method).



5. Conclusion.

The FFE-analysis using the optimization-approach yields best results in terms of covering the complete output range and detecting the worst case. The computational cost is high relative to the transformation method, but lower than the probabilistic methods.
The transformation method yields to adequate results and is very powerful in terms of computational cost, but is limited to monotonic problems. In most cases, the transformation method can be limited to the vertex method, which is analogue to a full factorial DOE. However, this method can be extended to 'adjustable' full factorial when more accurate results are needed for non-linear or even non-monotonic behavior. On the other side, the method can further be optimized in terms of computational applying a 'fractional' factorial DOE or following the "short Transformation Method". A combination of increasing the output quality with limited extra computational cost, can be attained with application of the GαD-algorithm (discussed more in detail in task 3.4).
As the behavior of static structural models are is in most cases monothonic, and the ranges are in mahy cases relative narrow, the transformation method is very well suited for this kind of analysis. Dynamic problems, and certainly harmonic responses in the vicinity of resonance, are often strongly non-monotonic. For that reason, the optimization method (and hybrid method, see workpackage 4) is rather suited for dynamic problems.

TAP 31

Report part 3.7

Comparison to traditional design rules

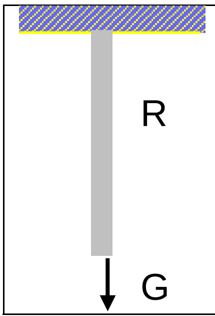
1. Introduction

This part of the report gives an overview of the safety margin obtained with probabilistic methods, the partial safety approach of the Eurocodes, and fuzzy methods applied to civil applications.

2. Overview of the applications

2.1 Tensile bar

The first application consists of a tensile bar as illustrated in the figure below and presented at the users meeting of 11 Oct. 2004.



G_k = 1 MN
f_{yk} = 235 N/mm²
A = 6319 mm²
γ_G = 1.35
γ_M = 1.1

One can verify that the design meets the Eurocode rules ($G_d < R_d$ with $G_d = G_k * \gamma_G$ and $R_d = f_{yk} * A / \gamma_M$)

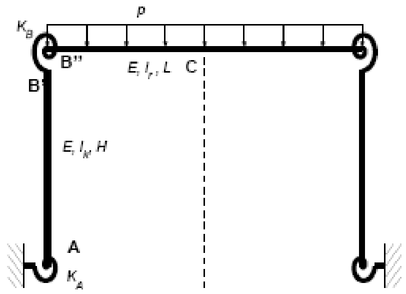
Supposing that $\mu_G = G_k$ and $\sigma_G = 0.1\ \mu_G$ and
$\mu_R = R_k + 2\ \sigma$ and $\sigma_R = 0.08\ \mu_R$,
one can determine the reliability index β = 4.4 and the probability of failure P_f = 4.5 E-6.

A design according to Eurocodes and with the partial factors given in their annexes is considered generally to lead to a structure with a β value greater than 3.8 for a 50 year reference period, which corresponds to a probability of failure of 5 E^{-5} / 50 year.

This design is indeed safe enough (β> 3.8). See also part 5.2 of this report.

2.2 Two dimensional portal frame

The second application consists of a two dimensional portal frame as shown in the figure below and already presented in part 3.3.



Beam : IPE 360, L = 20 m
Column : IPE 400, H = 5 m
P = 2.24 kN/m
K_A = 8.3 E^6 Nm / rad
K_B = 14.8 E^6 Nm / rad

The main goal was to compare the fuzzy approach with probabilistic methods for the angles ϕ_A and ϕ'_B and the midspan deflection v.

The problem is solved for two values of the standard deviation of the parameters : 10 % and 50 % of the mean value. Membership functions are defined from the CDFs. For more information see part 3.3 of this report.

The results show that the fuzzy method provides a safe approximation at the tails of the CDFs. The deviation between the analytical solution and the fuzzy one is much bigger for ϕ_A than for ϕ'_B and v because ϕ'_B and v are nearly independent of K_A and mainly depend on one fuzzy input variable K_B.

Method	$\phi_A^{0.01}$ (mrad)	${\phi'_B}^{0.99}$ (mrad)	$v^{0.01}$ (mm)
Analytical	-0.7896	1.8693	-58.87
Fuzzy	-0.8078	1.8854	-58.93
Difference	2.28%	0.86%	0.10%

Percentile values for σ = 10 %

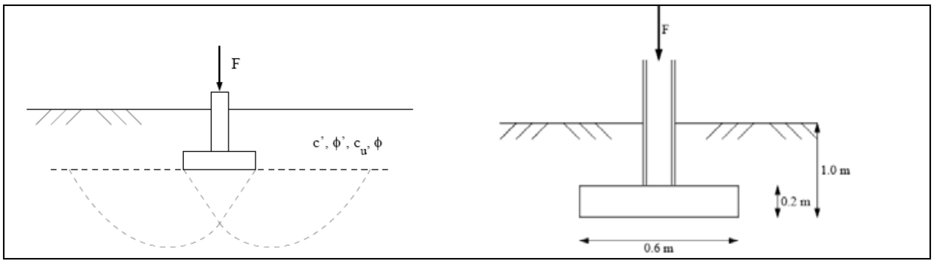
Method	$\phi_A^{0.01}$ (mrad)	$\phi_B^{0.99}$ (mrad)	$v^{0.01}$ (mrad)
Analytic	-0.9404	2.04	-87.64
Fuzzy	-1.0021	2.09	-87.75
Difference	6.65%	2.45%	0.12%

Percentile values for σ = 50 %

The fuzzy method leads to conservative results. Important parameters are the correlation between the different input parameters, the number of stochastic parameters and the sensitivity of the response to these parameters.

2.3 Shallow foundation

The third application consists of a shallow square foundation in Boom Clay, as shown in the figure below and presented in part 3.3a of this report. The load applies vertically and centrally on the footing. Load and soil are modelled as uncertain variables. Three different methods are applied to evaluate the reliability of the foundation : the Eurocode method with partial safety factors, a probabilistic analysis and a possibilistic approach based on fuzzy numbers. Soil characteristics are determined based on soil investigation results. For more information see part 3.3a of this report.



A summary of the results is given in the table below.

Eurocode method (partial safety factors)	Set 1 : F_d = 43.23 kN < R_d = 56.96 kN Set 2 : F_d = 32.02 kN < R_d = 40.69 kN → design OK
Probabilistic approach	β = 3.96 P_f = 3.7 E^{-5} Design point : F_d = R_d = 30.60 kN → OK
Possibilistic approach (fuzzy)	β = 3.22 Design point : c_u = 16.36 kPa F = 36.35 kN → NOK

As was illustrated already with the other examples, the reliability index obtained from the possibilistic method is more conservative and in this case leads to an inacceptable design.

3. Conclusions

The fuzzy method leads to more or less conservative results. Important parameters are the correlation between the different input parameters, the number of stochastic parameters and the sensitivity of the response to these parameters.

On the other hand, the fuzzy method has also his advantages. First of all, no direct assumption has to be made on the PDF of the input parameters, and only the upper and lower boundaries of all CDFs are required, which makes it very suitable for design problems where no sufficient statistical data are available.

TAP 31

Report part 3.7(B)
Comparison of the analysis to the traditional design rules with safety factors in mechanical engineering

Stephan Masselis
WTCM-CRIF
Celestijnenlaan 300C, B-3001 Leuven, Belgium

1. Introduction

This report aims at comparing the results of a FFE-analysis with traditional analysis based on design rules with safety factors.

2. Design rules

In civil applications, where safety is primordial, decades of experience and statistical data is available concerning loads (windload, snowload, seismic data, …) and resistance of materials & structures , leading to standards and normalization of calculation methods based on safety factors. The main goal of a civil design, is the requirement that a building or bridge lasts for 30, 50 or 100 years with a failure rate of less then e.g. 1E-6. (see also report 1.2, and examples in reports 3.2 & 3.3).
In mechanical applications however, design rules are based on functional performance and life-time prediction. The further essentially is related with allowed deformations, vibrational behavior and limits on plastic deformation or rupture; the latter is essentially related to stresses (stress concentrations) with respect to fatigue limits of the applied material and material treatment.

3. Validation methods for mechanical & automotive designs

Design in mechanic and automotive applications is mostly based on rules of thumb and practical experience (test bed validation). This was already mentioned in the beginning of the project (questionnaire, as basis for report 1.2 & 1.3).
In most cases, a new design is compared with an existing deterministic (and tested) design. That existing design is treated as a reference, often without knowing to which degree that part is 'over-dimensioned' (unless fatigue failure data is available). The new design and the reference are both analysed under a "unit" force when no accurate data is available. Validation is based on stress levels or deformation. The uncertainties on both designs are considered to be similar.

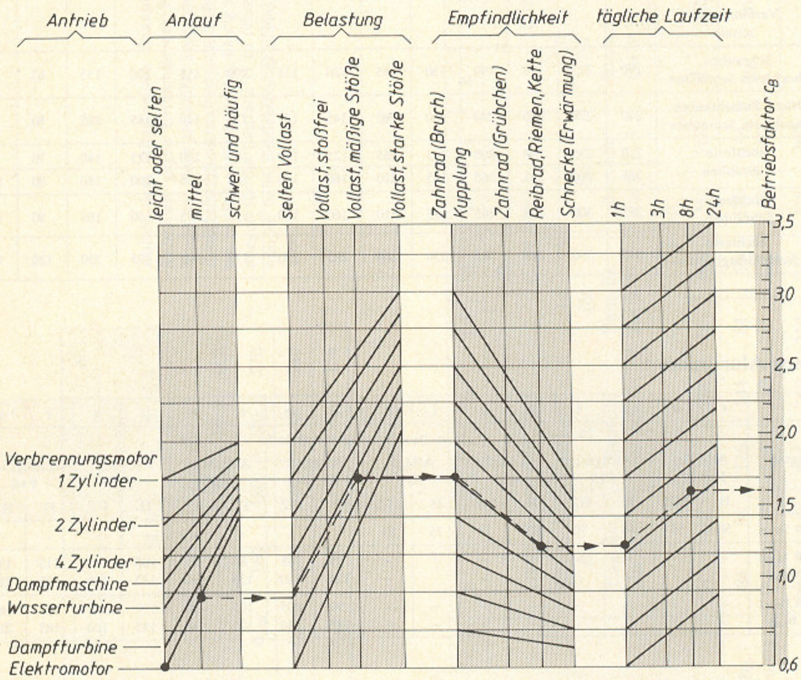
For vehicles and trucks, lots of track-data are available, but this is not the case e.g. for agricultural machines, and reciprocating machines in general. General rules of thumb are available, e.g. the use of a load factor to quantify the design load based on the nominal torque or forces. As one can see in the table below, the load factor ("Betriebsfactor") is very fuzzy for machines with e.g. hammering mechanisms, as is often the case for agricultural machines.

TB 3-6 Betriebsfaktor c_B

a) allgemein

Art der Maschine bzw. des Bauteiles (Beispiele)	Kennzeichnende Arbeitsweise	Art der Stöße	Betriebsfaktor c_B
Elektrische Maschinen, Turbinen, Gebläse, Schleifmaschinen	gleichförmig umlaufende Bewegungen	leicht	1,0 ... 1,1
Brennkraftmaschinen, Kolbenverdichter, Hobelmaschinen, Stoßmaschinen	hin- und hergehende Bewegungen	mittel	1,2 – 1,5
Pressen, Steinbrecher, Walzenständer	hin- und hergehende, stoßhafte Bewegungen	stark	1,6 ... 2,0
Hämmer, Steinbrecher, Walzenständer	schlagartige Bewegungen	sehr stark	2 ... 3,5

b) für Zahnrad-, Reibrad-, Riemen- und Kettentriebe (nach *Richter-Ohlendorf*)



Ablesebeispiel: Antrieb durch Elektromotor; mittlere Anlaufverhältnisse; Vollast, mäßige Stöße; 8 h tägliche Laufzeit. Hierfür wird bei einem Kettentrieb der Betriebsfaktor $c_B \approx 1,6$.

fig. 1: load factor ("Betriebsfaktor") (ref. "Maschinenelemente" by Roloff/Matek)

In some cases, design and validation is based on existing standards (imposed by authorities or insurance companies). This is the case for "standard parts" (like gear teeth or steel cables), in applications where safety and reliability are primordial (lifting devices, windmills (*)), and in application area where standards are available based on decades of

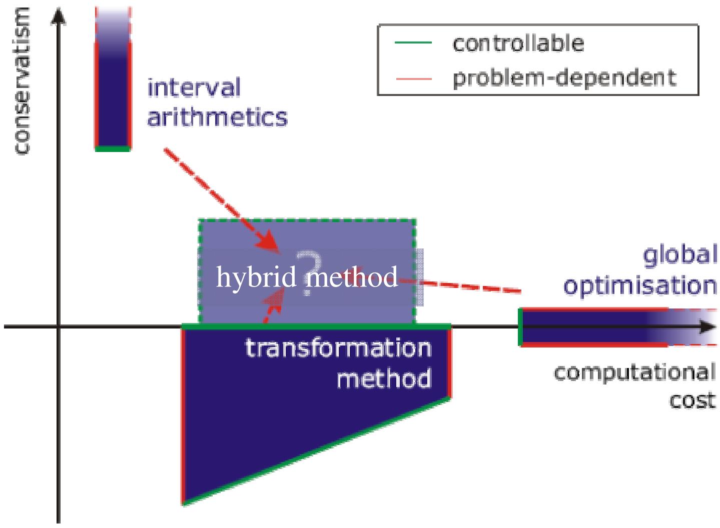
* see presentation concerning "Design of a Windmill" of Joris Peeters & Dirk Vandepitte at partner meeting of 11/10/2004

experience concerning the loads and material resistance (**). However, most of these parts are not analysed by means of FEM.
Other applications where standards are to be met are crashworthiness of vehicles and roll-over of buses. Such analyses can be done by FEM, but are extremely non-linear. Besides, these standards do not rely on safety factors, but describe experiments that have to be simulated and executed, with validation based on maximum accelerations and maximum deformation of the occupants' zone. Simulations are executed to analyse robustness of a design, and to prevent multiple iterations of the final physical test.

4. relevancy of uncertainty modeling for mechanical design.

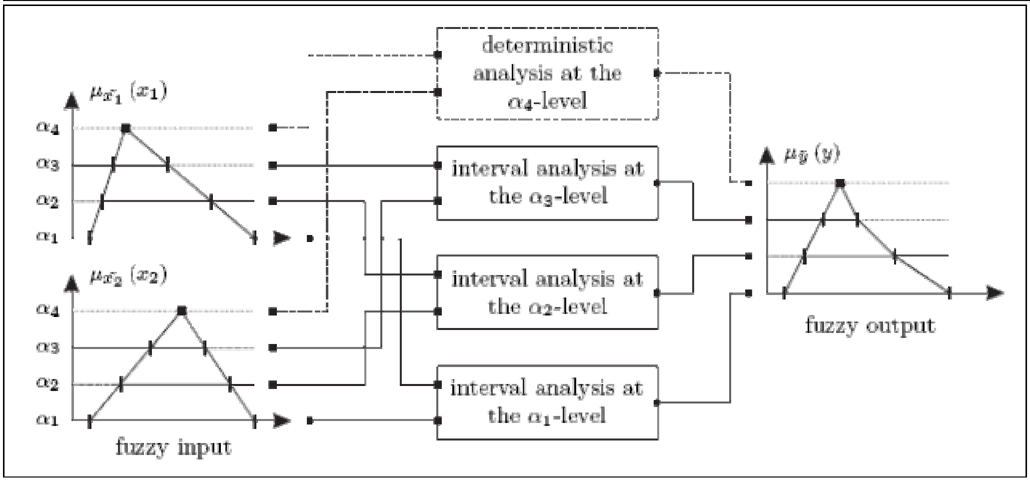
For all the reasons above, it is difficult to compare the fuzzy method as a validation tool with existing practices. However, *the methodologies investigated and developed, are very suitable as design tool for sensitivity analysis, optimisation and reliability analysis (robustness of a design*).
The different methods investigated have shown that (***)

- interval arithmetic approach is too conservative
- global optimisation is very time & computer resources consuming
- transformation method has an inherent risk of being under-conservative, especially if limited to the vertex method (full factorial with 1 α-level)
- hybrid method : more complex to implement on commercial FE-systems



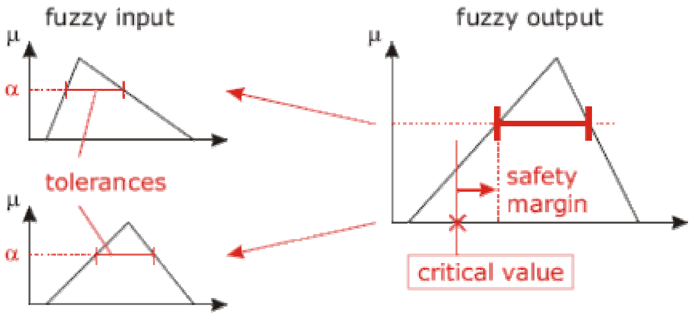
Especially the vertex method is easy to implement with existing FE-systems. In many cases, where variations are relative small (e.g. tolerances) or the behavior is quasi linear (at least monotonic), the method is very powerful tool for analyzing parameter sensitivity, optimization and reliability analysis.
An illustration of reliability analysis is the detection of allowable tolerances. Based on a study with at least one α-cut and a core-level analysis, one can construct the response as function of one or more variable parameters (see figure on next page).

** e.g. design of towercranes: DIN15018-15019
*** see presentation of David Moens at the meeting of the Users' Committee on 21/06/2005



presentation of the transformation method with multiple α-cuts.

The response can be validated based on experience concerning allowable stresses or deformations (critical value). Giving feed-back to the design parameters (or tolerance variations), it is possible to adapt the permitted tolerances.



TAP 31
Report part 4.5
Comparison of the vertex method and optimisation methods for envelope FRF calculation

Maarten De Munck
K.U.Leuven, Dept. Mechanical Engineering, PMA
Celestijnenlaan 300B, B-3001 Heverlee, Belgium
maarten.demunck@mech.kuleuven.be

1 Introduction

This annex of the project report compares the vertex method and optimisation methods for envelope FRF calculation. A first section gives a short overview of both methods, their advantages and disadvantages. A second section compares both methods on an envelope FRF calculation (described in annex 2.4 of this report) on the Garteur benchmark model (described in annex 1.4 of this report).

2 Description of the methods

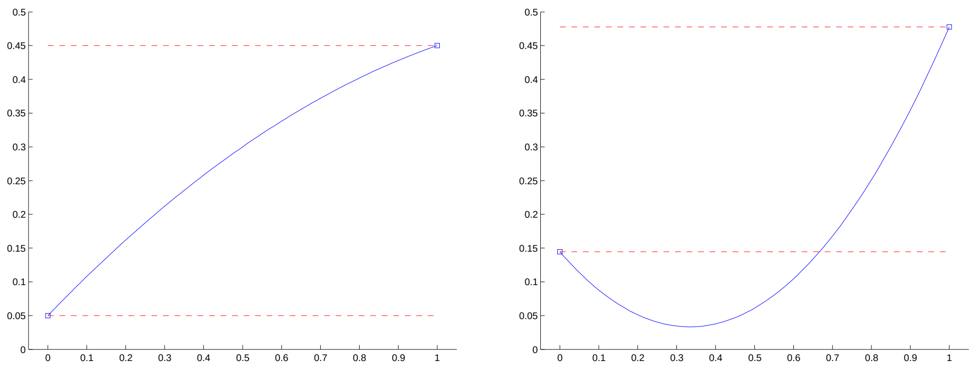
This section describes the vertex method and optimisation methods and their main advantages and disadvantages.

2.1 Vertex Method

The *vertex method*, also known as *transformation method* or *two level full factorial design of experiments*, calculates the response of every possible combination of minimum and maximum values for each uncertain parameter. A vertex analysis with n uncertain parameters requires 2^n deterministic analyses.

The vertex method has some clear advantages. It is by far the easiest interval finite element method to implement. Since the number of deterministic analyses and the values for all uncertain parameters for each analysis are known in advance, it is also easy to parallelise the computation on HPC (High Performance Computing) clusters.

The vertex method also has two important disadvantages. The number of deterministic analysis grows exponentially with respect to the number of uncertain parameters. A vertex analysis with 10 uncertain parameters requires 1024 deterministic analyses, which is feasible for most finite element models. An analysis with 20 uncertain parameter requires 1048576 deterministic analyses, which is unfeasible for all but the most simple finite element models. When the output is monotonous with respect to the uncertain parameters, the vertex method finds the extremes, but when the output is non-monotonous with respect to the uncertain parameters, the vertex method underestimates the extremes. Figures 1(a) and 1(b) illustrate this on a monotonous and a non-monotonous function with one uncertain parameter. The squares and the dashed lines show the extrema found by the vertex method. For the monotonous function, both the minimum and the maximum are found, but for the non-monotonous function, only the maximum is found.



(a) Monotonous function

(b) Non-monotonous function

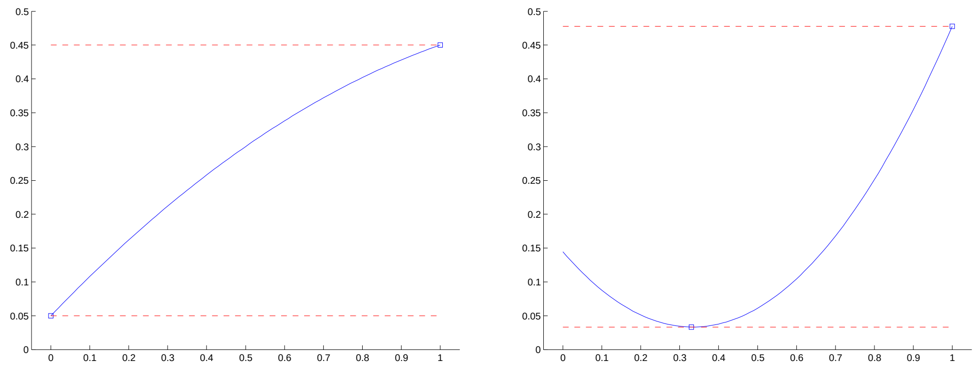
Figure 1: Vertex method and monotonicity

2.2 Optimisation Methods

Optimisation methods use a different approach. There are a lot of different optimisation algorithms, but most of them have in common that the number of deterministic analysis nor the values for the uncertain parameters in each analysis are known in advance. They require the user to specify a combination of uncertain parameters to start with (the *starting point*) and based on the function value and often also gradient of the results, they try to find a new combination of uncertain parameters, closer to the extreme. This is repeated until the algorithm estimates that the current combination is close enough to the real extrema.

The most important advantage of optimisation methods is of course that they are able to approximate the real extreme quite accurate, both for monotonous and non-monotonous functions. Figure 2(a) and 2(b) shows the extrema found by an optimisation method. For most optimisation algorithm, the choice of the starting point is important: for the function shown in figure 2(b), most optimisation algorithms will converge to the left side of the interval instead of to the right side when the user specifies a starting point left of the minimum.

Optimisation algorithms also have some disadvantages. The values for the uncertain parameters in each analysis are not known in advance, which makes it hard to implement a parallel version for HPC clusters, although there are some optimisation algorithms designed for parallel processing. Even the computational cost isn't known in advance, but in general optimisation methods are computationally more expensive than the vertex method, especially for a low number of uncertain parameters.



(a) Monotonous function

(b) Non-monotonous function

Figure 2: Optimisation methods and monotonicity

3 Example

The Garteur benchmark model is described in annex 1.4 of this report. The first analysis uses three uncertain parameters, as defined in that annex, i.e. the Young's modulus of the wing ($E = [67.5 \text{ GPa}, 68.5 \text{ GPa}]$), the stiffness of the fuselage-wing-connection ($k = [10^5 \text{ N/m}, 10^{11} \text{ N/m}]$ and the thickness of the visco-elastic layer on the wings ($t = [0.1 \text{ mm}, 1.6 \text{ mm}]$). The frequency response function is calculated using the algorithm described in annex 2.4 of this report.

The top graph in figure 3 shows the envelope frequency response functions for the vertex method and for an optimisation method, the vertex FRF in red and the optimised FRF in blue. The bottom graph shows the ratio between the upper bound of the vertex FRF to the upper bound of the optimised FRF in magenta and of the envelope with of the vertex FRF to the envelope width of the optimised FRF in cyan. A value of 100% indicates that the bounds of both methods are equal, a value ¡ 100% indicates the vertex method underestimates the responses. A value ¿ 100% would indicate that the vertex method overestimates the response, but from the description of both methods it's clear this would indicate a failing optimisation. On average, the vertex method underestimates the response by 1% or 2%, but for some frequencies, the vertex method underestimates the response with as much as 8%.

The second analysis uses only one uncertain parameter, the Poisson coefficient of the wing material. For metals, this coefficient is around 0.3, but for composite materials, it can be as low as 0.1 or as high as 0.4. This analysis uses values in the interval $[0.1, 0.4]$. Although the maximum underestimation of the upper bound by the vertex method is lower than in the first analysis, the average underestimation is significantly higher.

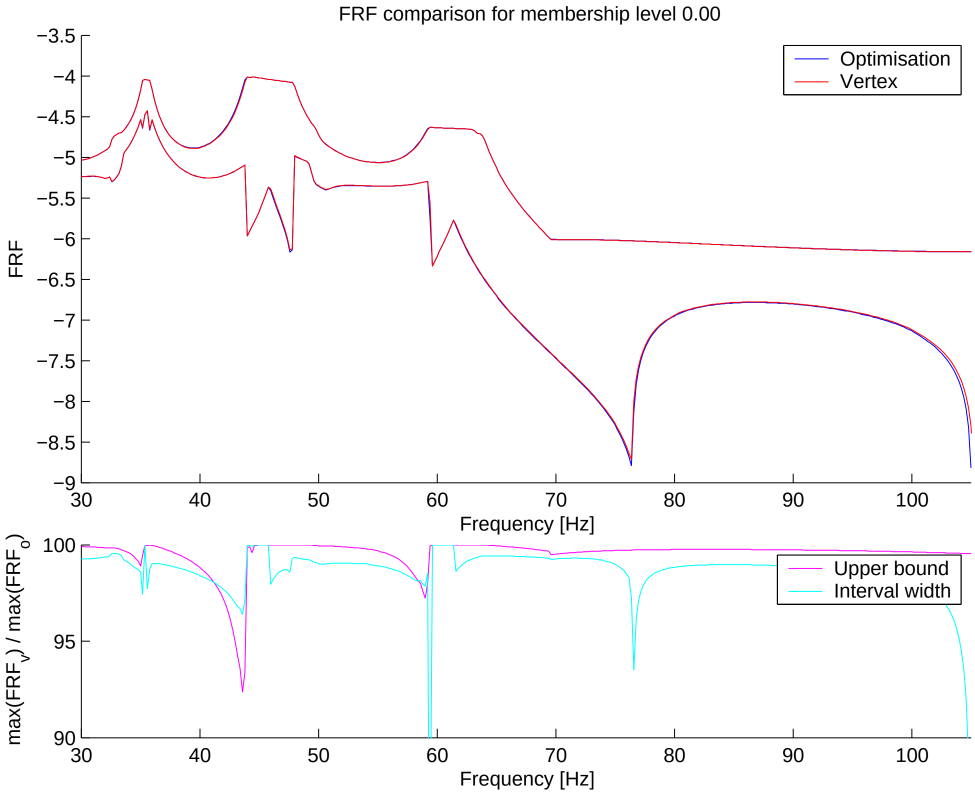


Figure 3: Garteur testcase

4 Conclusion

This annex compares the vertex method and optimisation methods for envelope FRF calculation. The vertex method underestimates responses that are not monotonous with respect to the uncertain parameters. Optimisation methods get more accurate results, but in general their computational cost is higher than that of the vertex method, especially for a lower number of parameters.

Two analyses on the Garteur benchmark model show that this theoretical underestimation can be important in real life models.

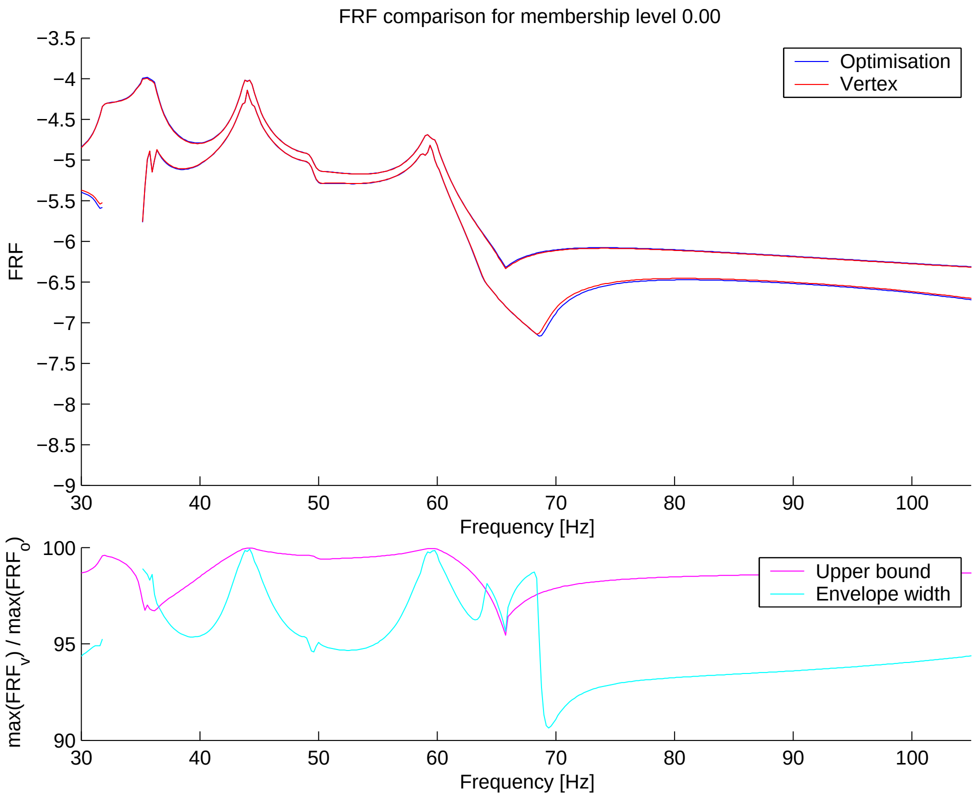


Figure 4: Garteur testcase

Task 5.1 Summary of the results of static test cases

1 Interpretation of fuzzy numbers

The first static test case involved the investigation of a 2D portal frame, with uncertain connection stiffnesses [Task 1.4]. A comparison has been made there between a classical probabilistic approach, executed with different probabilistic methods, and a possibilistic approach by means of fuzzy numbers. This comparison is based on the axioms of possibility theory, which introduce possibility and necessity as limiting cases of probability.

Regarding the calculational efforts, the possibilistic method performed quite well, mainly because the calculation of events with very low probabilities is quite expensive with Monte Carlo based methods, while this is not the case for events with very low possibilities.

It was however observed that the possibilistic method results in conservative estimations of the results. Further reasoning with respect to the cause of this conservatism leads to the observation that a possibilistic analysis comes down to an *implicit assumption of worst case correlation.* By this, it is meant that when interpreting fuzzy numbers, through possibility theory, in terms of probabilistic concepts, one neglects any existing or non-existing correlation between different uncertain variables, and performs an analysis as if they were correlated in the worst possible way.

In the second static test case [Task 3.3a], a foundation problem was solved with three methods: a probabilistic analysis, an analysis based on partial safety factors (as described by EuroCode 7), and a fuzzy-possibilistic method. Since the partial safety factors were originally calibrated in order to yield similar results as a probabilistic analysis, both of them indeed showed that this test case fulfills prescribed requirements. The possibilistic analysis however, resulted in an inacceptable value for the reliability index, indicating that this method indeed is conservative in comparison with probability theory.

Of course, it is not fair to compare the results of a possibilistic analysis with criteria which were originally intended for probabilistic analyses. However, in view of the lack of possibilistic criteria, this renders a possibilistic analysis rather useless as a method to check if a design fulfills prescribed norms. Instead of seeing it as a replacement for traditional design procedures, fuzzy numbers and possibility theory should be used as a complement to them. They doubtlessly are a usefull tool for design optimization, sensitivity analyses, worst case load combinations or linguistic uncertainty modelling, but one should avoid a direct comparison with probabilistic results or criteria based on possibility theory.

2 Calculational aspects

As illustrated by the two basic test cases, static analyses often behave very well. Response variables such as displacements, stresses or member forces usually are continuous, differentiable, monotonic functions of uncertain variables like stiffness parameters or load amplitudes.

This good behaviour suggests the application of the fastest method to perform the fuzzy calculations, namely the Short Transformation Method [Task 3.4]. With this method, the number of function evaluations is independent of the number of uncertain input variables, making it very interesting for large-scale problems. It also is capable of handling multiple response variables at the same time.

In some cases, non-monotonic behaviour is observed, for example when a load position is modelled as an uncertain parameter. In such situation, the GαD algorithm with a high maximal α-level stepsize, and a high aggression factor provides an efficient but robust alternative to the Short Transformation Method [Task 3.4, example 3].

TAP 31
Report part 5.2
Comparison to conventional rules

1. Introduction

All safety aspects of civil applications are based on the Construction Products Directive (CPD 89/106/CEE) in which are stated the basic requirements of all construction products. Two basic requirements are important in the framework of this project : (1) stability and mechanical resistance, and (2) safety at serviceability stage.

This part of the report gives an overview of the relation between the failure probability and traditional design rules for civil applications and the role fuzzy methods can play in this context.

2. Probability of failure

A construction is usually designed in accordance with a certain level of safety which is generally accepted. This means that the probability of failure of the construction during its design working life is limited to a certain value. This is often expressed by the reliability index β. The relation between the reliability index β and the probability of failure P_f is given in the table below.

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1,28	2,32	3,09	3,72	4,27	4,75	5,20

3. Design rules : global safety format

In civil applications, where safety is primordial, decades of experience and statistical data are available concerning loads (wind load, snow load, seismic data,…) and resistance of materials and structures, leading to standards of design methods based on global safety factors. E.g. by limiting the allowable stress to a fraction of the yield stress or the ultimate stress (at rupture) or by limiting the calculated loads to a fraction of the calculated resistance, thus only taking into acount the mean value and not the variation of the parameters. These are deterministic methods ; no quantfied relation between the probability failure and the global safety factors exists. The value of the global safety factor is determined based on experience.

4. Design rules : partial safety format of the Eurocodes

How do the Eurocodes apply the basic requirements of the CPD ?

For the purpose of reliability differentiation consequences classes may be established, as illustrated in the table below :

Consequences Class	Description	Examples of buildings and civil engineering works
CC3	**High** consequence for loss of human life, *or* economic, social or environmental consequences **very great**	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequences **considerable**	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	**Low** consequence for loss of human life, *and* economic, social or environmental consequences **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

EN 1990, Annex B - Table B1 : Definition of consequences classes

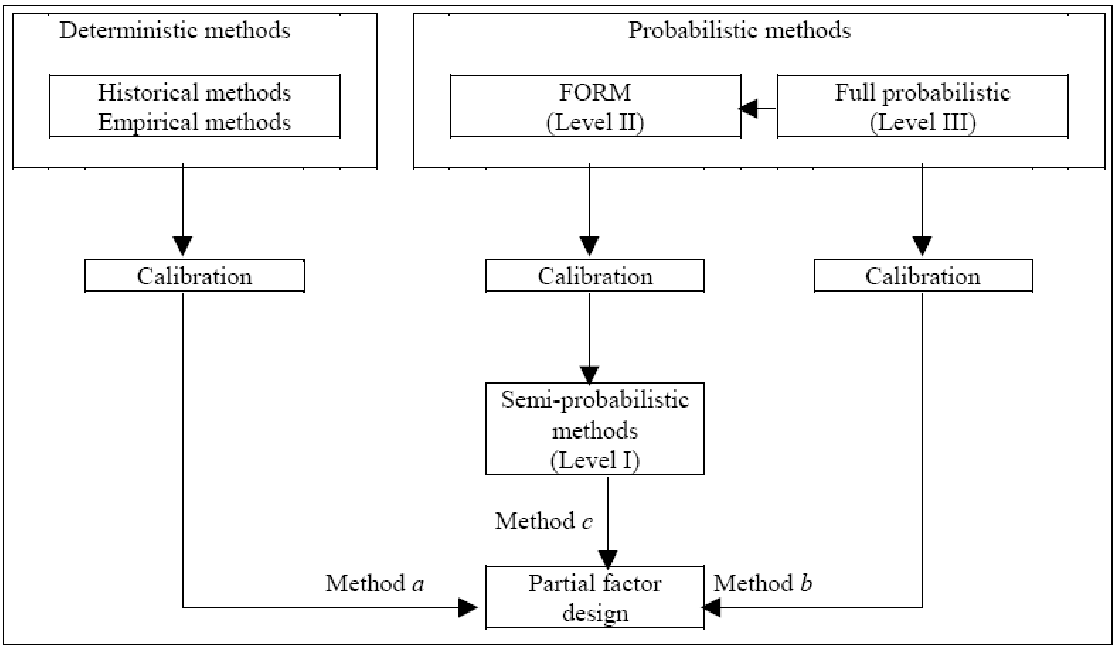
Reliability classes may be associated with the consequences classes (RC1 to CC1, RC2 to CC2, RC3 to CC3). The table below gives recommended minimum values for the reliability index β associated with reliability classes.

Reliability Class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

EN 1990, Annex B - Table B2 : Recommended minimum values for reliability index β (ultimate limit state)

A design according to Eurocodes and with the partial factors given in their annexes is considered generally to lead to a structure with a β value greater than 3.8 for a 50 year reference period, which corresponds to a probability of failure of 5 E^{-5} / 50 year.

The figure below illustrates the methods available for the calibration of the partial factor design :



EN 1990, Annex C – Figure C1 : Overview of reliability methods

Full probabilistic methods (Level III) give in principle correct answers to the reliability problem as stated. Level III methods are seldom used in the calibration of design codes because of the frequent lack of statistical data.

The level II methods make use of certain well defined approximations and lead to results which for most structural applications can be considered sufficiently accurate.

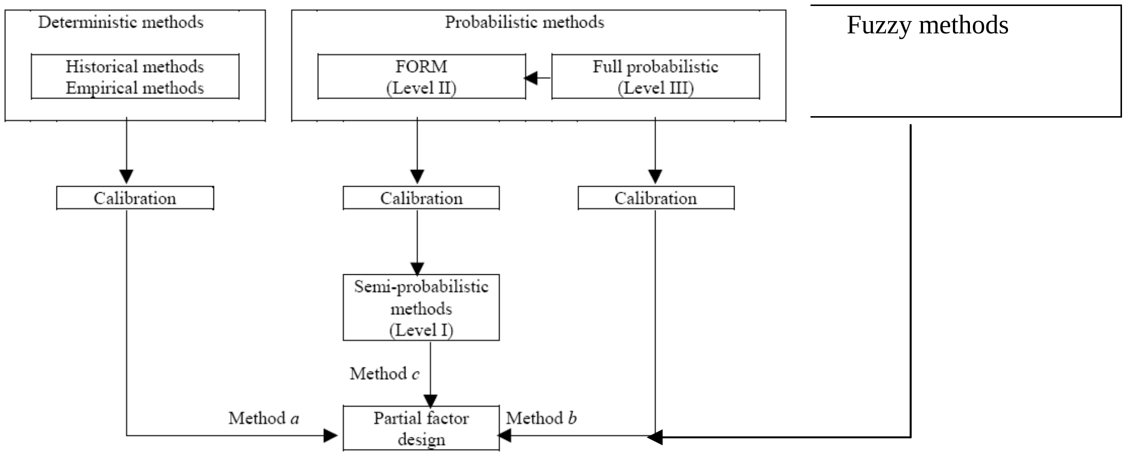
The Eurocodes have been primarily based on method *a*. Method *c* or equivalent methods have been used for further development of the Eurocodes. E.g. design values may be determined based on the formulas given in the table below. In these formulas α is the FORM (First Order Reliability Method) sensivity factor.

Distribution	**Design values**
Normal	$\mu - \alpha\beta\sigma$
Lognormal	$\mu \exp(-\alpha\beta V)$ for $V = \sigma/\mu < 0{,}2$
Gumbel	$u - \frac{1}{a}\ln\{-\ln\Phi(-\alpha\beta)\}$ where $u = \mu - \frac{0{,}577}{a}$; $a = \frac{\pi}{\sigma\sqrt{6}}$

EN 1990, Annex C – Table C3 : Design values for various distribution functions

5. Design rules : the role of fuzzy methods

As an alternative to the use of partial safety factors, a design according to Eurocodes may also be directly based on probabilistic methods (EN 1990 §3.5(5)). Since fuzzy methods lead to more conservative results than full probabilistic methods, they may play a role in a Eurocode design. They also may be used to calibrate the partial factors.



The fuzzy method leads to more or less conservative results. Important parameters are the correlation between the different input parameters, the number of stochastic parameters and the sensitivity of the response to these parameters.

On the other hand, the fuzzy method has also his advantages. First of all, no direct assumption has to be made on the PDF of the input parameters, and only the upper and lower boundaries of all CDFs are required, which makes it very suitable for design problems where no sufficient statistical data are available.

Interval Finite Element procedures can perform the core calculations in an automated fuzzy implementation.

Considering the optimisation approach, the effect of uncertain parameters on the analysis result is often unknown, which renders the numerical performance of the optimisation strategy unpredictable.

Considering the interval arithmetic approach, the conservatism grows quickly beyond reasonable limits.

A hybrid IFE procedure, which divides the analysis in an optimisation step and an interval analysis step, could decrease the conservatism to an acceptable level, even for large models.

This illustrates that fuzzy analysis is complementary to classical probabilistic approach, rather than competitive. If reliable probabilistic information is available, a probabilistic analysis remains the best choice. If probabilistic information is limited, fuzzy analysis is a valuable alternative.

TAP 31

Report part 5.3

Definition of design criteria for dynamically loaded structures with uncertain parameters

CSL

PARC SCIENTIFIQUE, B-4031 ANGLEUR, Belgium

The objective of this project is to develop an applicable and consistent methodology to predict the effect of parameter uncertainties on the dynamic response of a structure, to be able to propose verification criteria, that are

- safe, even in worst case conditions
- based on objective knowledge of structural characteristics
- economical, by minimizing artificial conservatism in the analysis

For static load cases, the objective is to develop a theoretical basis for the determination of safety factors. For dynamic load cases, the objective is to develop a theoretical basis that builds on dynamic response levels that must not be exceeded.

Although procedures for static design validation are very different for different domains of applications, the concept is always the same. For dynamic load cases however, there are no such rules. The most common requirements are expressed in terms of resonance frequencies which should not coincide with operating frequencies of the structure, but even these rules depend on the application. The design rules for components that are used on launchers are conceptually summarized to the following requirements that should be verified in a prescribed order:

1) The first resonance frequencies of the global modes (with effective masses greater than 10 % of the static mass) should at least be equal to a prescribed minimum value level
2) Criterion 1 being satisfied, the structure has to survive with some safety factors, to the Flight Loads which are accelerations applied to the instantaneous center of gravity.
3) The structure has also to survive with some safety factors, to the Sine Vibration, Random Vibration, Acoustic and Shock Qualification Tests, whose specifications correspond to imposed acceleration curves at the interface between the Spacecraft and the launcher (for a spacecraft) or between the instrument and the Spacecraft (for the instrument). This situation could some times be more severe than the precedent one even if flight loads are normally considered as dimensioning for the primary structure.
4) if the structure which results from the application of rule 1) would be too heavy, the designer is allowed to neglect rule 1), but he must take the dynamics of that

component into account and prove that the entire system does not exhibit excessive vibration levels. Coupled load analyses could be needed to determine the correct flight loads and the qualification tests levels.

5) if vibration levels are too high, the designer must modify and optimize the topology of his design or introduce additional damping.

Whereas verification of this set of rules is fairly straightforward for structures that are already well detailed, it is much more difficult to do so for structures that are characterized by a number of uncertainties.

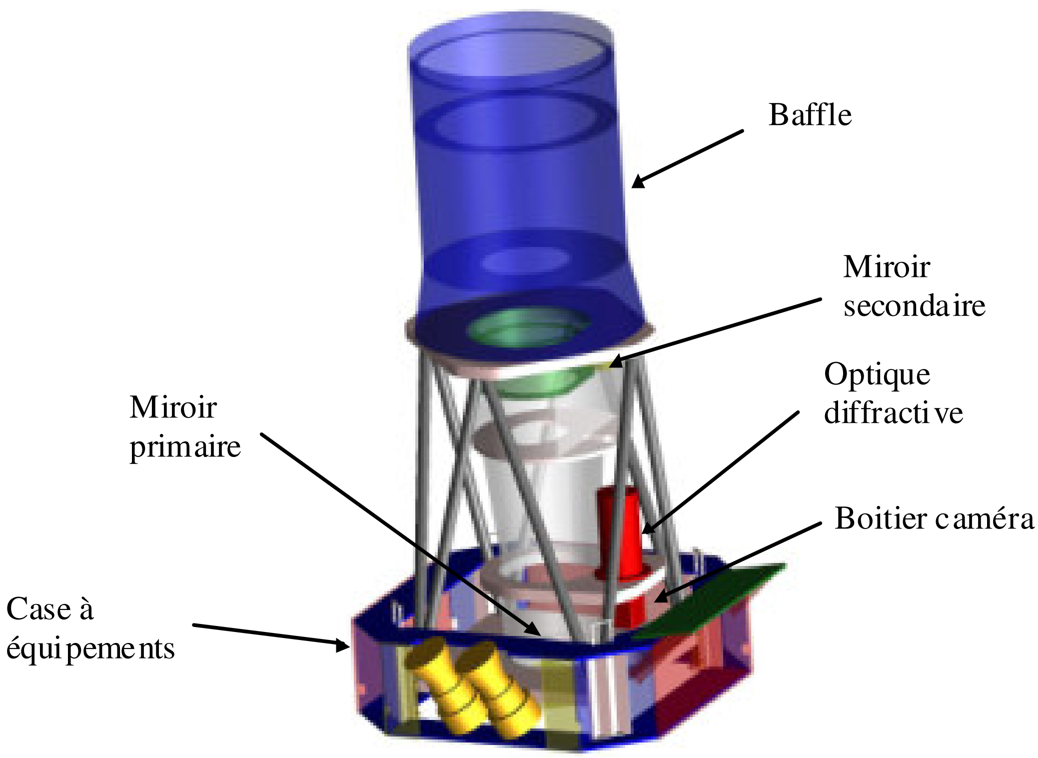
Spacecraft structural response to low frequency mechanical environment of the launch and ascent phase is simulated by spacecraft-launcher coupled analysis. The loads of the spacecraft issued from coupled dynamic analysis are taken as a basis to verify the dimensioning of the spacecraft. The quality of these loads depends on the quality of the mathematical models used for such simulations. Therefore, *assuming the launcher model and the loads are well known,* it is mandatory to take steps ensuring that the spacecraft model is adequately representative of the actual hardware.

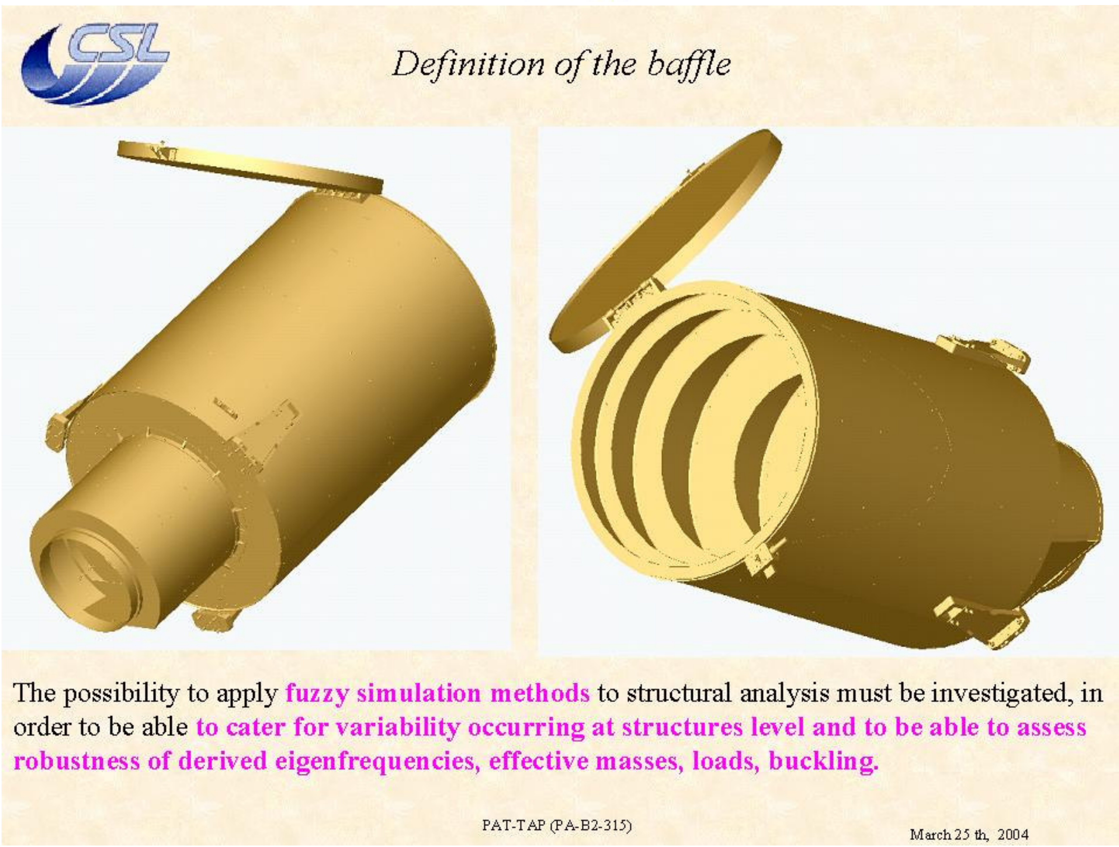
One way of minimizing artificial conservatism in the usually adopted procedure is to reduce the safety factors introduced in all the specification definition procedure and to study the response distribution compared to the acceptable values. The responses of interest can be stresses, deformations, displacements, … and the most important result of a fuzzy dynamic finite element analysis is to give upper bounds on the frequency response functions for these dimensioning parameters. These upper bounds take into account all kinds of uncertainties that can possibly occur, and their combinations. It covers the entire frequency range that is specified. The worst case situation that is within the design space is certainly included. If uncertainty on damping values can also be included, the prediction is certainly conservative.

Similar situations appear for a scientific payload instrument mounted on the Spacecraft or simply for a subsystem mounted on an instrument.

We decided in this project to analyze in this context, the effects of uncertainties on the COROT baffle flight loads and the resistance of this baffle with respect to qualification tests and launch. The criteria are manifold: no plastic deformation, integrity, no buckling of the vanes and limited displacements at some points of the baffle to avoid contact with the telescope. For this baffle, it is difficult to reach the requested 150 Hz and so a coupled analysis with the telescope, with the satellite, with the launcher (??) is required to evaluate the real Flight loads and the risks of contact with the telescope. For this type of analysis, a good accuracy of the eigenfrequencies, damping parameters, eigenmodes is required.

L'instrument COROT





COROT Baffle on jig and shaker

The generalized accelerations $\{a_i>$ at the D.O.F. of the interface Baffle / COROT Telescope are related to the generalized forces $\{F_i>$ at the D.O.F., by the following equation which is coming from the mathematical model of the structure on which the baffle is mounted (Launcher + Spacecraft + Telescope coupled mathematical model) :

(1) $\{a_i> = \{a_{i,\,Free}> + \{Z_{Launcher+Spacecraft+Telescope}\}\ \{F_i>$

The generalized forces $\{F_i>$ can also be deduced from the model of the baffle

(2) $\{F_i> = \{-\{L\text{»«}diag(\ldots\omega^2 H_k(\omega)/k_k\ldots)\text{»«}\phi] - \{\phi]\ ][F_0>$

$+ \{\ -\omega^2\{L\text{»«}diag(\ldots\omega^2 H_k(\omega)/k_k\ldots)\text{»«}L\} - \omega^2\{\phi M\phi\} + \{\phi K\phi\}\ \ \}\ \{q_i>$

or

(3) $\{F_i> = -\omega^2\{\ M_{dynamic\ Baffle}\}\ \{q_i>$

with the dynamic amplification factor :

(4) $$H_k(\omega) = \frac{1}{1 - \dfrac{\omega^2}{\omega_k^2} + 2\,i\,\varepsilon_k\,\dfrac{\omega}{\omega_k}}.$$

(5) $\{a_i> := -\ \omega^2\ \{q_i>$

The displacements are given by :

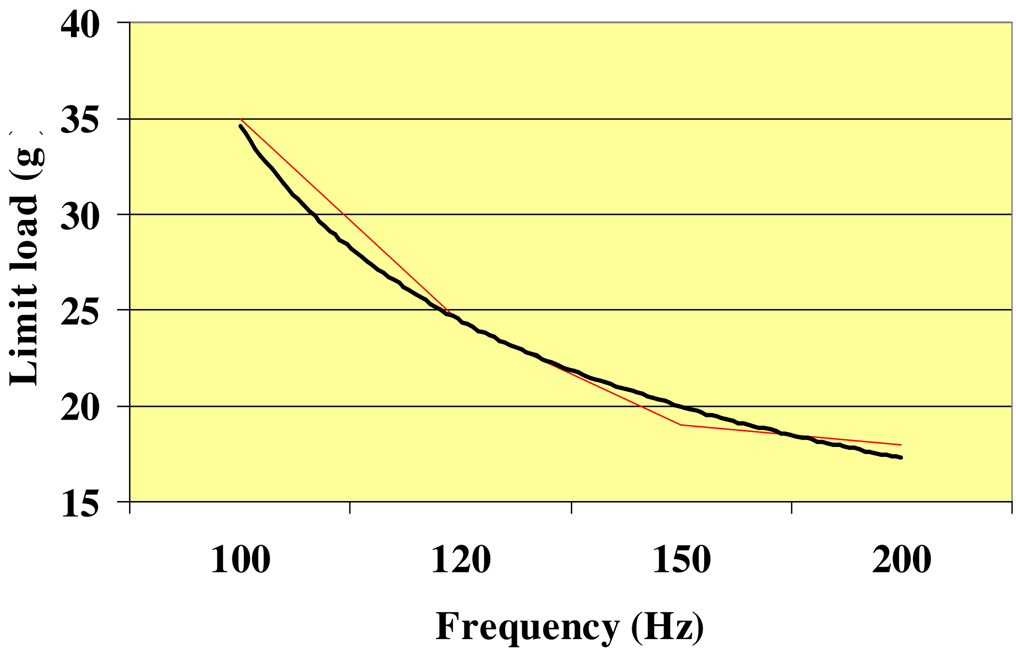
(6) $$\left[q_0\right> = \sum_k \frac{H_k(\omega)}{\omega_k^2} . \alpha_{al_k} a_0 \left[\phi_{eff,k}\right> + \left[\phi\right\} \left\{\frac{a_0}{\omega^2}\right>$$

The baffle has to have a first "significant [1] " eigenfrequency higher than **150Hz** to avoid coupling with the telescope.

Our main concern in this stage of the project is that the baffle manufacturer proposes us a first eigenfrequency of exactly 120Hz and tests show a frequency below 80 Hz. **There is no margin on the eigenfrequency (120 Hz could be acceptable).**

That means that all the uncertainties that could appear in the determination of the eigenfrequency are not taken into account. The consequence is that the loads that will appear in the baffle could be greater than the calculated loads at 120 Hz.

A first evaluation of the flight loads is shown on the graphic below.



To obtain a good estimation of $\{a_i>$ and $\{F_i>$ and consequently of the stresses, deformations and loads, the complete process need an excellent evaluation of the modal parameters. This is done by a correlation and update process, where test results (dynamic, static) are compared to predicted results and the mathematical model updated till representativity is judged satisfactory within a certain tolerance. When the distance between the model and the experimental results is sufficiently "close", the model is said

[1] effective mass greater than 10 % of total static mass

to be valid. (A similar situation appears at the interface between an instrument and the spacecraft).

1 Vibration Test / Mathematical model correlation

The correlation and mathematical model update process is based on deterministic approaches, i.e. is based on the attempt to match, as closely as possible, the results of a deterministic numerical analysis, with that of a single physical test, without taking into account the natural dispersion or scatter inherent to all physical structures. This approach can lead to unreliable results or be misleading and induce wrong conclusions on the quality of the model. This results in limited confidence in the model, which can only be partially compensated by the use of *uncertainty or safety factors*. It would be therefore highly desirable, if not necessary, to consider the scatter as an integral part of the model and to establish correlation and validation techniques that take this scatter into account. In this way, deterministic "point-to-point" comparison is replaced by a much more robust "cloud-to-cloud" comparison where each cloud contains a full stochastic description of the model including scatter among its observed variables (Figure 1). In order to do this, we have to perform the following tasks: (test-analysis) correlation, error localization, model updating and validation.

A key point to be developed is the validation of its models by experiments themselves subject to scatter.

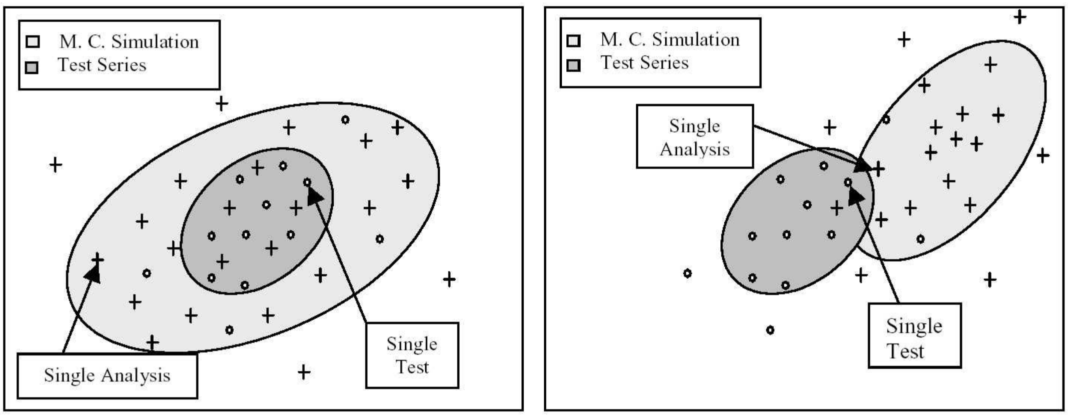


Figure 1 : Experimental and computed Meta-Models. Initial and after Updating clouds

Stochastic approach can on one side enhance the traditional design approach, but on another side procures (via the meta-model) a very important additional insight in the structural behaviour.

The lack of matching can come from:

- inaccuracies or uncertainties that may be present in analytical models and which are mainly due to:
  + The approximation of boundary conditions

+ The classical lumping of distributed parameter systems

+ The estimation of the different physical properties and design parameters of different structural materials

+ Lack or inappropriate damping representation

+ Inadequate modelling of joints and couplings

+ Classical assumption of linearities

- Shortcomings in test data which usually include one or more of the following problems:

  + The number of measurement degrees of freedom is limited and may be different from those of the analytical model.

  + Some types of degrees of freedom, such as rotational or internal ones, cannot be readily measured with present technology.

  + The number of identified mode shapes is limited. Thus the frequency range of test data is also limited.

  + Measured data or parameters contain levels of errors.

  + Some modes of the structure under test may not be excited in a certain test or, if excited, may not be identified.

  + Experimental simulation of boundary conditions.

  + Experimental simulation of actual environment (such as near vacuum and zero gravity for space structures).

  + Test non-linearities versus the linear assumption of the analytical model.

A further important reason of mismatch are mistakes resulting from human error (analysts, experimentalists, etc.). In addition model debugging is made difficult due to the large size, in terms of numbers of properties and degrees of freedom, of the actual models.

2 Uncertainties

It is clear that uncertainties play a key role in the model validation process. Uncertainties can be broadly classified as being either reducible or irreducible. *Irreducible uncertainties* are inherent in process being simulated and cannot be eliminated. *Reducible uncertainties* are a potential deficiency that is due to incomplete information, e.g., lack of knowledge, poor understanding of physical process, etc. We can summarize the main sources of uncertainty as follows:

- Experimental data (Text fixtures, Environmental conditions, Measurements)
- As-built systems and structures (Design tolerances, Material properties, Construction methods)
- Models (Modelers judgment, Modeling tools, Modeling resources)

3 Scatter and variability

Multiple sources of scatter and variability (not to be confused with absence of knowledge) affect the design and life of a structure. First of all, material & components variability (Table 1) as a result of the material nature, its production mode, acceptance testing, its implementation at component level.

Material	Strength Characteristic	Coefficient of Variation (%)
Metallic	Rupture	8
	Yield	8 to 15
	Buckling	14
CFRP	Rupture	10 to 17
		acceptance testing
Screw, rivet, weld	Rupture	8
Bonding:	Rupture	12 to 16
Adhesive film,		8 to 13
Metal-Metal join		acceptance testing
Honeycomb	Tension	16
	Shear/compression	10
	Face wrinkling	8
Structural Inserts	Axial rupture	12
Equipment Inserts	Axial rupture	16
Mirror glass	Static strength	10 to 30
		surface finish
Invar Superior	Rupture	4 to 9
		hammered/tempered
Fibrous Thermal Protection	Rupture	2 to 24
		loading/temperature

Table 1: Scatter and Variability in Material strength

Second, the loads by their nature and their knowledge exhibits a considerable randomness and variability, as shown in Table 2.

Load Type	Coefficient of Variation (%)	Source
Launcher thrust	5	STS, Ariane
Other flight events QSL	30	STS, Delta, Ariane
Transient events	60	Ariane
Thermal	8 to 20	Thermal tests (correlated or not)
Deployment shock	10	regulated mechanism
Thruster	2	Thruster calibration
Acoustic	30	STS, Ariane
Vibration	20	Test damping factors
Re-entry		No data available
Mech. Pre-load	5 to 15	Test results (with/without gauge)

Table 2: Scatter & Variability in loads

Other factors bring additional scatter elements, such as tolerances on prepregs and sheets, lay-up angle errors for composites, manufacturing tolerances, scatter of local stiffnesses due to assembly (friction coefficients, tooling, torques, …). Depending on the application and its related requirements, minor aspects can acquire an extreme importance (e.g. homogeneity of material for a high precision mirror) and need special measure to be accounted for during definition, design and analysis.

4 Safety factors

This paragraph defines and explains the use of factor of safety (FOS). The Table 3 is a list of the cases where FOS are used.

1. Safety factors are used in projects to cover possible uncertainties in:

– Load predictions, e.g. design limit loads, etc.
– Structural analyses,
– Manufacturing process, and
– Statistical variations in material properties

2. Different safety factors may be selected within a program, depending on the levels of uncertainty. Higher safety factors may be allocated to cover ill-defined loads, incomplete material characterization and/or a decision not to undertake structural testing. The uncertainties are either load or strength related. These depend on the space vehicle type. Uncertainties include:

LOAD RELATED	– Loads caused by pressure, e.g.: - Fuel containment, - Propulsion systems, and - cabins – Aerodynamic included load, e.g.: - Vehicle manoeuvers, - Wind gust, - Inertial change effects – Rocket motor induced or related loads, e.g.: - Thrust - Motor cut-off - Stage release – Launch loads, including survivable emergencies – Transportation and ground handling – Rocket and aerodynamic noise.
STRENGTH RELATED	– Material choice and properties. Particularly concerning composite anisotropy. Important issues include: - Effects of temperature and moisture, - Space specific: outgassing, LEO: atomic oxygen damage and corrosion – Standardisation of manufacturing processes, e.g common procedures for different sites – Structural analyses and verification testing, e.g: - Confidence in theoretical and experimental stress/strain prediction - Simulation of environmental effects, - Demonstration of surplus strength and/or residual strength by test to failure – Reduction in structural strength during operational life – Aeroelastic effects involving the interaction of loads, stiffness and strength consideration.

Table 3: uncertain parameters: output of work package 1.2

3. Special factors, also called "additional factors", can be applicable for joints, bearings and welds. Such additional factors shall be applied in combination with other factors of safety.
4. Determination of safety factors:

- Factors of safety shall be determined considering the uncertainty of all relevant load, design, material, manufacturing and verification parameters.
- As opposed to the empirical approach applied in the definition of deterministic factors of safety, a probabilistic approach can be followed. Factors of safety are calculated based on a statistical description of loads, materials and geometry, combined with a failure probability requirement.

4.1 Loads and factors relationship

Definition of the different loads used:

- Loads that depend on the environment:
  - Flight load (or limit load or design load): maximum launch load that may be applied during flight

- Qualification load: loads applied during qualification tests
- Loads that depend on the structure itself:
    - Ultimate load: maximum strength value that does not lead to yield deformation
    - Yield strength: strength value that lead to yield deformation

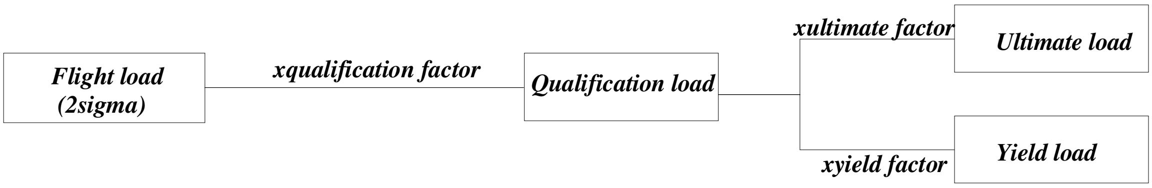


Figure 2: *loads and factors relationship, definition of qualification, ultimate and yield factors*

The following values are used for the mechanical design of structure, to check the resistance of materials used in the structure:

1. Ultimate load: Design load multiplied by the ultimate safety factor
2. Yield load: Design load multiplied by the yield safety factor

It is clear that the **Factors of Safety** nowadays called **factors of uncertainty**

- which are identical for static and dynamic calculations !
- and only depend on the type of material (Metals and non-metals[2]) and the type of failure (static, buckling, …)

don't take into account all the inevitable uncertainties in the presented complex process of calculation.

Historically, 1.5 safety factor was derived from a ratio of aluminum ultimate to yield stress, but this is just a coincidence which tended to support the selection of 1.5. This factor did not evolve as the result of some concentrated effort to derive a useful factor. Rather, it evolved together with other design requirements as part of an overall desire to rationalize structural design criteria. Its use is accepted by most engineers without question.

When problems have arisen or structural failures have occurred, changes were made to design specifications, load prediction techniques, manufacturing techniques, etc., but the factor of safety value has never been changed.

The 1.5 factor is rational because it is based on what were considered to be representative ratios of design to operating maneuver load factors experienced during the 1920s and 1930s. Yet at the same time it is arbitrary because we do not know the exact design, manufacturing, and operating intricacies and variations it protects against or how to

[2] : The amount of scatter observed in composite material testing tends to be high relative to metals. Variability in composite material property data results from a number of sources, including variability in laying up the material, batch-to-batch variability of raw materials, and material testing methods. In addition, composite properties show higher (compared to metals) degradation due to environmental effects, which creates the need for testing at different temperatures and moisture absorption levels.

quantify them. Neither can the degree of inflight safety provided by the 1.5 factor be quantified; but its successful history cannot be lightly dismissed.

Interest in replacing the factor of safety approach with probabilistic interpretations of structural safety initiated in the late 1950s. The continued application of the factor of safety approach is challenged by some engineers, but there is reluctance to undergo a major change in design philosophy, especially one which could encourage legal entanglements. The factor of safety still covers many unknown contingencies, and for this reason, some engineers believe there will always be a need for some such factor.

Recently, the factor of safety was renamed to “factor of uncertainty.” A draft (June 1995) of the Joint Service Structures Specification Handbook states “The selection of the factor of uncertainty, formerly called the factor of safety, should be made by assessing the factors that have been used on similar air vehicles performing similar missions. The value for manned aircraft has been 1.5.... The selected value of the factor of uncertainty should be increased to account for above normal uncertainty in the design, analysis, and fabrication methods when the inspection methods have reduced accuracy or are limited by new materials and fabrication methods and where the usage of the air vehicle is significantly different.... The use of reduced factors of uncertainty needs to be carefully defined and justified.” If variability in design, manufacturing, and operating environments can be reduced, then a reduction of the 1.5 factor of uncertainty could be justified. If, however, the introduction of new material systems (for example) actually increases the variability, then the 1.5 factor may be unconservative and have to be increased. In either case, probabilistic analysis can be used to quantify these effects, hence serving a useful purpose. It would not necessarily replace the factor of safety as a design criterion, but would help to establish the optimum factor of uncertainty level.

5 Traditional approach to the Scatter / Variability Problem

The above mentioned variability were recognized in the past, and tackle with the general concept of Factor of Safety and similar factors. Regarding buckling for example, the knock-down factor was introduced to cater for the difference between the theoretical critical load as computed for a perfect structure and the actual values resulting from experiments (Figure 3).

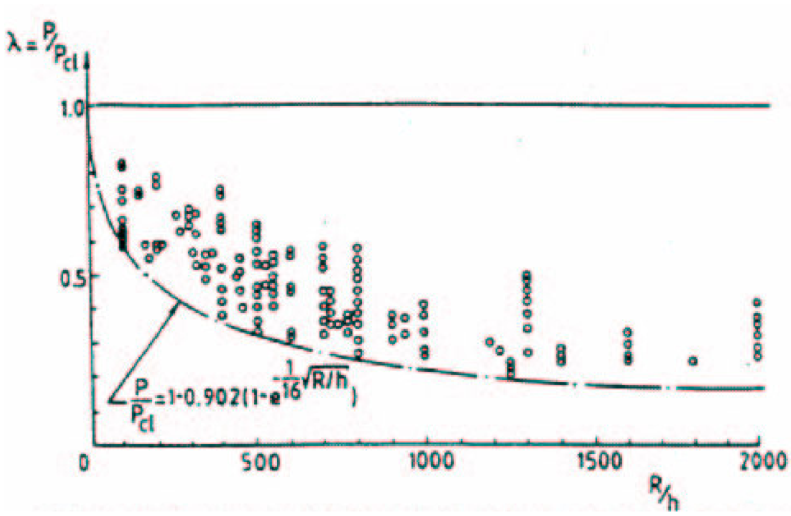


Figure 3: Test data for isotropic cylinders under axial compression

For stressing purposes, the well known factor of 1.5 was defined at the time aluminium alloys started to be used, based on the ultimate to yield ratio of the alloy, with the objective to keep the structure operating in the linear range. Such factors (with various values) have been used since now; associated to the requirement for the design to exhibit a positive margin of safety (MOS), the latter being defines as:

$$MOS = \frac{allowable\ stress}{computed\ stress * FOS} - 1$$

However, considering the actual scatter of loads and components as shown in Table 1 and Table 2, and assuming the simple Stress-Strength approach (Figure 4) it can be easily shown that, using a fixed FOS, very different structural reliability as achieved depending on considered loads and materials (Figure 5). This is a problem for some new materials.

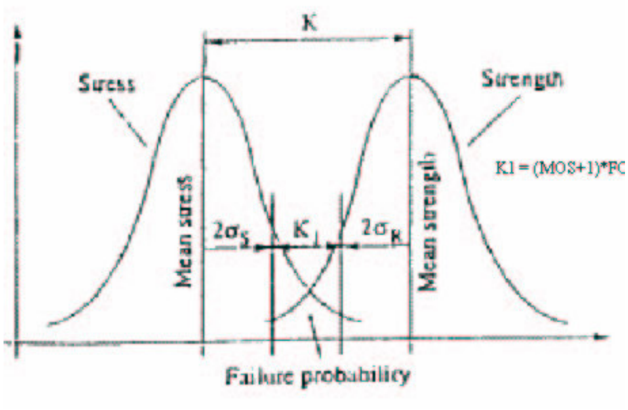


Figure 4: The Stress-Strength Approach

	Metallic Material (σ/μ = 8 %)	Metallic Material: Yield Strength when R/Yield = 1.2 (σ/μ = 15 %)	Buckling Strength of Conical or Cylindrical Metallic Shells (σ/μ = 14 %)	Carbon Fiber Composites (σ/μ = 10 %)	Junction by Screw, Rivet Welding (σ/μ = 8 %)	Bonding Structural Insert (Axial Loading), (σ/μ = 12 %)	Honeycomb: Tension (σ/μ = 16 %)	Honeycomb: Shear, Compression (σ/μ = 10 %)	Honeycomb: Face Wrinkling (σ/μ = 8 %)	Equipment Insert (in Honeycomb Axial Loading), (σ/μ = 16 %)
Launch Vehicle Thrust (σ/μ = 5 %)	1.29	2.24	1.98	1.42	1.29	1.64	2.60	1.42	1.29	2.60
Launch Vehicle Other Static Loads (σ/μ = 30 %)	1.45	1.98	1.82	1.51	1.45	1.62	2.21	1.51	1.45	2.21
Transient Loads (σ/μ = 50 %)	1.59	2.00	1.87	1.62	1.59	1.71	2.19	1.62	1.59	2.19
Thermal Loads (correlated) (σ/μ = 7.5 %)	1.29	2.17	1.93	1.41	1.29	1.61	2.51	1.41	1.29	2.51
Deployment Shock (σ/μ = 10 %)	1.29	2.11	1.89	1.41	1.29	1.58	2.45	1.41	1.29	2.45
Thruster Loads (σ/μ = 2 %)	1.31	2.34	2.07	1.48	1.31	1.71	2.73	1.48	1.31	2.73
Acoustic Loads (σ/μ = 40 %)	1.53	1.98	1.84	1.57	1.53	1.67	2.19	1.57	1.53	2.19
Vibration Loads - Thermal Loads (uncorrelated) (σ/μ = 20 %)	1.37	2.01	1.82	1.45	1.37	1.59	2.28	1.45	1.37	2.28

Failure probability = $10^{-6} \rightarrow K_1 = \frac{\text{Minimum guaranteed strength } (2\sigma \text{ value})}{\text{Limit Load } (2\sigma \text{ value})}$; $(1.29 < K_1 < 2.73)$

Figure 5: FOS for a target probability of failure 1E-6 at 2 sigmas

As a result, various actions were undertaken in a recent past to better take into account, along different routes, the variability of the various structural and related parameters.

6 Reliability Based Refinement of the Present Approach

An action was undertaken by ESA to identify a set of adequate FOS on the basis of accepted structural probability of failure (identified itself during the study). For this purpose, the number of critical parts in a typical spacecraft was identified in relation to the related failure cases (materials, components) and loading cases and their respective scatter. Also considered was the vehicle/spacecraft model philosophy (e.g. QM, PFM) in relation to the possible re-use of structures during different tests as well as possibly for the flight model (PFM), and the objectives of the various tests and their accepted risks. The output was a set of consistent FOS as shown in Table 4. One shall note that these FOS exhibit strong similarities with the currently used factors. However the difference lies in the knowledge now gained with respect to their applicability conditions, and the consequent possibility to modify these factors when considering different / new materials with other scatters for which heuristic information is not available yet. These approach and results are the basis for an ECSS Standard on FOS in preparation.

	FOS wrt qualification loads FOS wrt limit loads		
	yield / functional	ultimate	buckling
QM with no yield exceedance requirement	Standard material scatter Qualification factor = 1.4		
Conventional material metallic	1.25 1.75	1.40 1.9	1.30 1.85
Conventional material non metallic	1.05 1.45	1.15 1.60	1.30 1.85
Non-conventional mat.	1.70 2.35	1.85 2.60	1.30 1.85
Insert / bonding	1.70 2.35	1.85 2.60	
QM with no yield exceedance requirement	Reduced material scatter (U=5%,Y=8%) Qualification factor = 1.25		
Conventional material metallic	1.05 1.30	1.15 1.45	1.40 1.75
Conventional material non metallic	1.00 1.25	1.10 1.40	1.40 1.75
Non-conventional mat.	1.80 2.25	2.00 2.50	1.40 1.75
Insert / bonding	1.80 2.25	2.00 2.50	
Note: For a classical automatic spacecraft, 2.3 sigma loads and A-values for materials			

Table 4: Typical set of reliability based Factors of Safety

In parallel, the possibility to use reliability methods applied to fracture Analysis and Fatigue was investigated using simulation approach (Monte Carlo).

The objective was to feed into the analysis the scatter derived from test measured fracture / fatigue parameters, and to replace the go/no-go life statement for a given load spectrum by a probability of failure statement (Figure 6). The attempt was successful, and resulted in probabilistic prototypes of the ESACrack and ESAFatigue applications, and confirmed the possibility to consider partial factors of safety (wrt the parameters of fracture / fatigue equations) and load spectrum dependent FOS.

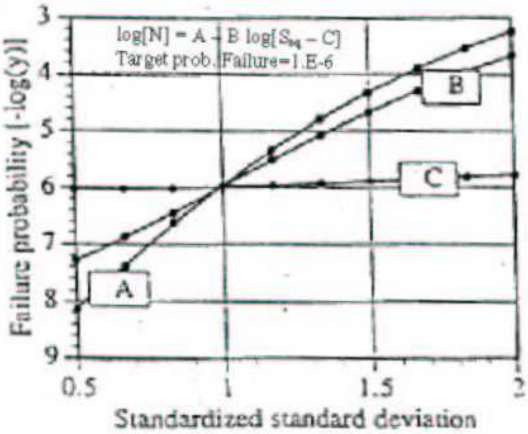


Figure 6: Variation of the probability to fail in fatigue versus the scatter of the parameters A, B, C

7 Evaluation of Scattered Structural Responses

In a second step, the possibility to apply simulation methods to structural analysis was investigated, in order to be able to cater for variability occurring at structures level and to be able to assess robustness of derived frequencies, effective masses, loads, buckling knockdown factors (manufacturing geometric imperfections) for example, and to enable evaluation of full structural reliability of critical parts (single point failure items).

Specific modules were generated (and are available) for general purpose software such as MSC-Nastran for covering the identified need for probabilistic based analysis, including launcher coupled analysis and vibroacoustic analysis. Where relevant, specific matrix perturbation, modal space projection and load transformation methods were used to reduce the volume of computations.

Investigations were led on the technological scatter of spacecraft elements (e.g. cylinder and panels face sheets, cleat stiffness, members inertia) in industrial environment.

8 New trends and perspectives for structural analysis in Space

The Stochastic Structural Analysis by the virtue of its merit is gaining momentum in the space community. The approach has already contributed efficiently to take into account real life facts and scatter, to rationalize the Factors of Safety, to highlight the importance of reliability as compared to Margin of Safety, to improve definition of load envelopes and design requirements. With the advent of new materials, and increased requirements for lightweight and better-controlled behaviors, the experience behind these factors is put in default. **We also have to evaluate possibilities to drastically reduce computational effort by using advanced fast response evaluation methods. This will be tested in this PAT project.**

The approach has without doubt key role and a very high potential with respect to reliable prediction of the behaviour of complex systems whether structural or *multi-physics*.

However, one shall pay good attention that fundamental aspects of Physics need

always to be properly understood, and cannot be replaced by stochastics.

Stochastic / reliability based design is now needed: Although structural computed / target reliability might be 0.999, as a result of implementation aspects by humans, manufacturing, assembly and verification, experienced structural reliability is only 0.95. That difference needs to enter in the design decision, theoretical probability of failure / reliability being only relative values.

The natural question is therefore: is all the future of structural design, analysis, and verification on the stochastic route?

An element of answer for the Space field might be obtained by considering structural design of elements of a space project. Such projects encompassing multiple factors, their respective requirements and commitments must be (contractually) well defined at individual level (Technical Specifications, ICD's (interface control document)…). It is noted that no heritage exists in this area. How should one define a Technical Specification / an ICD in a stochastic / multiple contractor approach? How should one perform or define / perform a structural analysis (what are the deliverables, how to couple?) in a stochastic / multiple contractor approach? How will the sub-system verification be done? What are the qualification / acceptance cases? What is the responsibility of the subcontractors at system level? How will it be verified? Similarly, the actual benefit of such approach has not been quantified yet. **This PAT project will certainly improve our knowledge of the subject.**

It is clear that, in addition to its more complex interface aspects, owning to its novelty and more demanding interpretation skills, the stochastic method is not going to rush into all levels of the design process (e.g. down to equipment / component level). With exception of special cases, it might actually not be necessary there since physics / behavioural options are limited. **At these lower levels, reliability based and calibrated refined traditional methods (e.g. FOS, partial-FOS) will still have their niches.**

On the contrary, where physics / behavioural options are widely open, and hence where sensitivity / jumps / discontinuities are possible, there efforts should be made to promote the application of the method. This addresses also the system level.

With respects to open physics / behavioural options, coupled (multidisciplinary) problems appear to be natural privileged candidates for such approach (e.g. aero-structural interaction, aero-thermal interaction), in general all multi-physics problems.

With regards to computational aspects, the perspective of analyzing complete systems in their full details (to which level?) requires teraflop level machines.

Although these are (nearly) available on some very specific places, they will hardly be available at the level of all design offices. Hence, there is still scope for improvement of computational methods in support to popular size computers. The recent successful development of Higher Order Derivatives Methods opens interesting perspectives in a near future.

With regard to industrial implementation, a set of tools is available (commercial or not), certainly adequate to experiment the method. Application of stochastic approach is particularly important in automotive industry (crash), but momentum is increasing in aerospace industry, see applications to Artemis , Ariane 5 EPS, composite fiber lay-up, separation shock analyses.

9 Safety Factors or Uncertainty factors are no more representative

Current aerospace design analysis methods generally do not directly account for the random nature of most input parameters. The result of treating parameters such as material properties, geometry, environment, and loads as singly determined (deterministic) values is a design of unknown reliability, or conversely, unknown risk.

New developments (e.g., reusable launch vehicle, high-speed civil transport) are departing dramatically from traditional environments. Application of historical uncertainty factors may not be sufficient to provide adequate safety. Conversely, the trend to design to all possible unfavorable events occurring simultaneously could produce an unacceptable weight.

The aerospace industry has seen a steady rise in the percentage of composite airframe structures. These materials have more intrinsic variables than metals due to their heterogeneity and are subjected to more manufacturing process sources of variation. To account for uncertainties, relatively large knockdown factors are employed, which reduce the material allowable. This results in a substantial weight increase without a quantifiable increase in structural reliability.

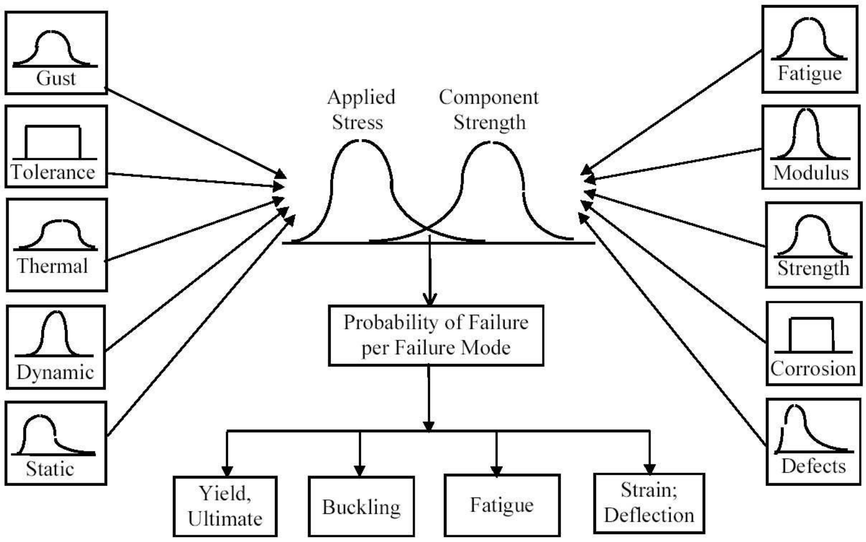
The foundation of probabilistic design involves basing design criteria on reliability targets instead of deterministic criteria. Design parameters such as applied loads, material strength, and operational parameters are researched and/or measured, then statistically defined. A probabilistic analysis model is developed for the entire system and solutions performed to yield failure probabilities.

The solution includes a number of locations and failure modes. Each location requires corresponding applied stress and material strength distributions. The applied stress is usually obtained from finite element modeling, coupled with conventional structural mechanics approaches. Mathematically, the applied stress and material strength distributions are generally assumed to be independent. The general concept is to integrate the joint probability of applied stress and material strength over the region where stress exceeds strength. The result of this integration is the probability of structural failure.

10 Optimization is only really meaningful under a probabilistic approach

Sensitivity analysis and/or optimization can be performed once the probabilistic model has been established. The concept is that once design driver contributions are identified, the design can be optimized for the given constraints, while maintaining the overall failure probability at an acceptable level. **Sensitivity analysis reveals the major contributors to risk; this allows the analyst to vary the design parameters to produce acceptable reliability at minimum weight, for example.**

The basic probabilistic approach can be summarized as the statistical definition of all input variables required for structural analysis methods, statistical definition of the resulting stress and strength of the structure associated with predicted failure modes, and evaluation of the resulting probability of structural failure. The Figure below illustrates this process. The left-hand side shows the input data to determine the applied stress distribution, with each having a statistical distribution, while the right-hand side depicts the various capabilities of the structure. The middle shows the output of the process, that being an applied stress and resistive component strength distribution, per failure mode, with an associated probability of failure.



PROBABILISTIC ANALYSIS CONCEPT

One missing element is probably the definition of an acceptable probability of failure.

Optimally, probabilistic analysis codes should be interfaced to these structural analysis programs and procedures so that the structural analysis output can be directly fed to the probabilistic program and vice-versa. This is one of the objectives of the present study

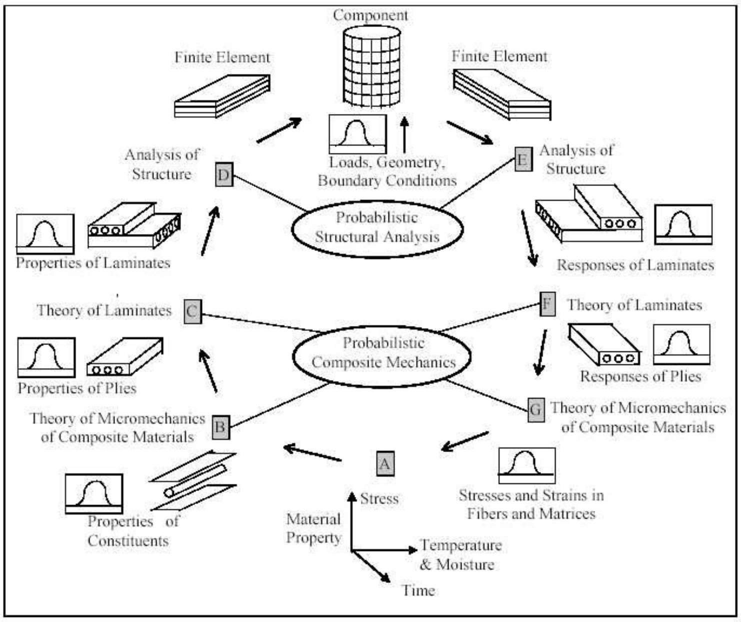


FIGURE 4-2. NASA LEWIS COMPOSITES PROBABILISTIC ANALYSIS

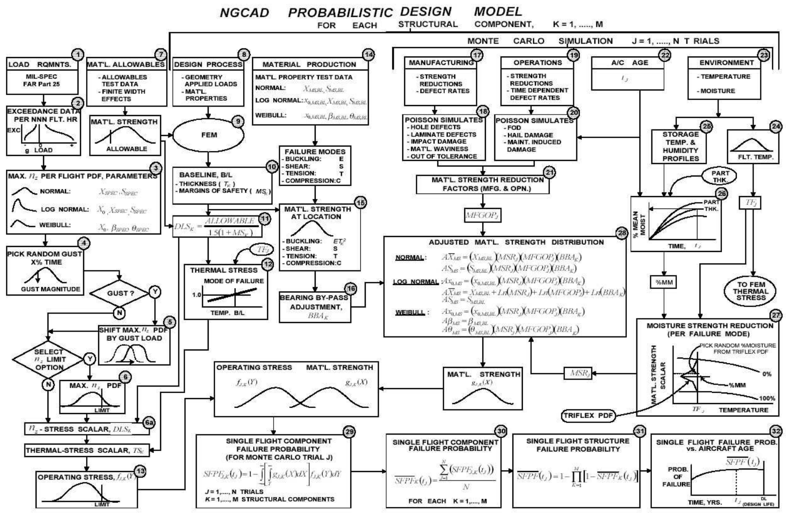


FIGURE 5-2. NGCAD PROBABILISTIC DESIGN MODEL FLOWCHART

11 BENEFITS OF PROBABILISTIC ANALYSIS.

Uncertainties in the definition of loads, geometry, assembly procedures, manufacturing processes, engineering models, material properties, and maintenance or operational environments as well as uncertainties in testing lead to uncertainty in structural design and ultimately safety.

Here is a list of the benefits of a probabilistic approachJ:

a. *Enables quantification of the design risk or reliability.* The classical deterministic analysis approach accounts for design uncertainties via an uncertainty factor multiplying the maximum expected stress. Probabilistic analysis, on the other hand, models most or all design parameters as being variable and combined with established structural analysis, yields a quantitative measure of reliability. This is obviously advantageous if reliability is specified as a basic contractual requirement. NASA design requirements for future space vehicles and structures are expected to be specified in reliability terms.

Employing probabilistic methods will aid reliability engineers in improving their analyses.

b. *Identifies regions of high risk in a design.* The total structural risk is typically a function of a series of reliability values at specific locations within the structure. Should a particular region be shown to drive the overall risk, measures can be taken to reduce that risk via design change, and/or manufacturing inspection procedures can be implemented to minimize the occurrence of defects in critical zones.

c. *Allows determination of design variable importance to reliability.* The reports were unanimous in identifying this benefit of probabilistic analysis. A powerful attribute of probabilistic analysis is the information gained in understanding the interactions, effects, and sensitivities of design variables. This information can be used to optimize testing for various purposes and can highlight the need to tighten (or relax) design or manufacturing tolerances. For instance, if it was shown that minor variation in stringer thickness had a major effect on resultant stress, tightening tolerances may be advantageous. Most probabilistic analysis software provides an output of design parameter sensitivity.

d. *Provides a means to compare competing designs*. In addition to comparing overall reliability values of competing designs, the probabilistic analysis can point out specific features or locations in which the reliability significantly differs among designs. This can increase the understanding of the structure's behavior and lead to design improvements.
e. *Provides a metric for design optimization*. Aerospace structures are operated in harsh and uncertain environments and yet must meet minimum weight, high performance, and stringent reliability requirements. Safety must be maintained at a high level. Reduced weight tends to reduce reliability and therefore must be implemented judiciously. Probabilistic analysis provides the measure of structural reliability, which can then be optimized by changing certain design variables. That is, design parameters are varied to minimize weight, but the overall reliability must meet a specified level.

f. *Can reduce unnecessary conservatism.* This is particularly true with composite aircraft and spacecraft structure design, which is governed by compounded conservatism illustrated by the following criteria:

Worst case temperature and moisture

Worst case damage, undetected

Reduced design allowables

This approach translates to a design philosophy that assumes the structure will simultaneously experience the worst case temperature, moisture, and damage conditions and will be composed of low-strength material. These worst case assumptions often lead to an excessively conservative design. The probabilistic analysis approach accounts for the expected occurrence of such events and combines them statistically.

Incorporating probabilistic methods eventually leads to a better design approach in that the engineer develops a more comprehensive understanding of the problem encompassing many disciplines. The probabilistic evaluation gives the designer an idea of the inherent risk, but just as important, provides a means of evaluating design parameter sensitivities. In general, probabilistic methods require more detailed analysis, which can ultimately lead to **an improved, more efficient design.**

12 Overview of non-deterministic modelling techniques

The standard FE code is a deterministic procedure. Standard codes however have options to calculate the effect of deviations from nominal conditions for quite a number of years already. Next to options that are incorporated in commercial FE software, several codes exist to work with results calculated from FE codes. The approaches that are listed are generic approaches that can be used for all kinds of FE analysis.

- The first additional component that was developed to analyse the effect of non-nominal conditions was sensitivity analysis. This type of analysis predicts the effect of the variation of one parameter to the output quantity. This type of analysis is fully deterministic. The variation of the output quantity is directly coupled to the variation of one or more input quantities.
- Sensitivity calculations are used in optimisation. In the optimisation procedure an objective function is defined, together with a number of minimum and maximum values on individual input parameters and/or output quantities. The procedure searches for the combination of input parameters that lies within the allowable range, and that optimises the objective functions. Several FE codes have this built-in.
- External optimisation procedures are built to work with any other analysis package, and they combine multiple types of analysis. Research on Multidisciplinary Design Optimisation (MDO) has developed very powerful analysis procedures that are used in the design of (extremely) complicated systems. Different sets of specifications on structural, thermal, aerodynamic, ... behaviour are combined into a generic approach.
- Response Surface Modelling (RSM) is a technique which calculates a certain characteristic of the response of a structure for various discrete parameter combinations. A surface with a prescribed (usually polynomial) form is then fitted through all the calculated data points. The numerical description of this surface is considered to represent the response of the system. It is then no longer needed to repeat extensive calculations on the actual system. RSM is often used in conjunction with Design Of Experiments (DOE). DOE is an approach that originates from experimental procedures. It is now used in conjunction with numerical simulation systems as well. DOE is a methodology to efficiently plan a series of experiments (individual analysis runs) for systems that require the definition of several independent (input) parameters. The strategy is to obtain the maximum of information from the minimum number of experiments. The effect of individual parameters is calculated, next to the effect of parameter combinations. The output of the analysis ranks the effects in order of importance. Statistical analysis of data is involved, including analysis of variance (ANOVA).

- Monte Carlo (MC) analysis is widely used as a statistical approach to evaluate the behaviour of systems that are characterised by a number of parameters of which some are defined by a statistical distribution. A sample of random numbers is generated within the distribution of each uncertain parameter, and these samples are combined into a random set of parameter settings for the entire model. One analysis is run for each combination of parameter settings, and after all individual analyses are completed, all response data are gathered to represent the stochastic response of the uncertain model. After some statistical interpretation of the entire response data set, the maximum value is considered to be the worst case, and design criteria are applied. A requirement for using MC simulation is that statistical data on input parameters are available. A serious disadvantage of the approach is the fact that running many individual cases with separate parameter combinations requires a very large computational effort. Much research has been done and is still being done on this type of approach. Many recent research efforts focus precisely on procedures to reduce the computational effort. Another important requirement that has to be met in order to effectively apply MC analysis is that an actual statistical distribution of model parameters must be available. Elishakoff has shown that it is unsafe to use MC analysis when parameter distributions are not clearly defined.
- Fuzzy arithmetic and interval analysis employ the concept of imprecisely defined numbers. Fuzzy arithmetic is a recently developed mathematical theory that is able to represent and operate on imprecise or ill-known quantities. These quantities are represented by means of fuzzy closed intervals (i.e. graded extensions of classical crisp intervals) and fuzzy numbers (i.e. extension of crisp real numbers). All classical arithmetic operations such as addition, multiplication, subtraction and division have been extended theoretically as well as practically to fuzzy numbers.
- Next to the generic approaches, many techniques have been developed that are specific to certain fields of application, such as updating methods and perturbation techniques.

13 The fuzzy finite element method

The fuzzy FE method was introduced in 1995 by Rao in a very simple case of linear static analysis on one-dimensional rod elements. Unknown degrees of freedom are calculated as a solution of the system of equations. As inputs are imprecise, outputs are imprecise too. The general principles are more or less established, but only in theory. Up to now, numerical implementations of fuzzy arithmetic have always been based on a decomposition of the fuzzy numbers into intervals at different levels of memberships, and a subsequent application of the theory of interval arithmetic.

There are several methods to perform numerical operations on fuzzy numbers. Straightforward application may however lead to overestimation of the result. For the purpose of safe design, overestimation is not inappropriate, but excessive overestimation should be avoided by all means. Artificial conservatism masks real physical results, and reduces the quality of the predictions.

14 The fuzzy concept for design validation with uncertainties

Zadeh introduced the theory of fuzzy sets as a basis for reasoning with possibility. From this point of view, the membership function is considered as a possibility distribution function, providing information on the values that the described quantity can adopt. More generally, the possibility is defined as a subjective measure that expresses the degree to which a person considers that an event can occur. As such, it provides in a system of defining intermediate possibilities between strictly impossible and strictly possible events. The choice of the possibility distribution of a quantity of which no statistical data is available is subjective and can only be based on expert opinion. On the other hand, a number of methods exist to derive a possibility distribution corresponding to a known probability density function. However, apart from the probability distribution, these conversion techniques always rely on some sort of subjective judgement. This is why a possibilistic analysis can only be interpreted in reference to the input possibilities.

Fuzzy analysis in this case is a sort of large-scale sensitivity analysis of the combined effect of design variables and uncertainties on design requirements. It enables the analyst to calculate design variable ranges for which the design meets the requirements with a certain degree of possibility. This means that the interpretation of the result of the analysis is only meaningful by referring to the considered input possibility distributions. A different possibility distribution for the design variables will yield a different possibility distribution of the analysis result, and consequently also different allowable ranges for the design variables. The design based on this analysis however is equally safe. As such, the possibility distribution for a designer is merely a useful tool to control the allowable range for the uncertainties than an absolute quality measure.

15 Description of some probabilistic methods

15.1 Monte-Carlo Simulation

Monte-Carlo method is time consuming but leads to reference predictions. Its precision largely depends on the number of draws and on the algorithm generating the random numbers. Generally, a very large number of draws is needed to insure & good convergence of the standard errors of the uncertain parameters.

15.2 Taylor's Development and Perturbations

This method is based on the Taylor development, often up to the second order of the dynamical response around mean parameters. The mathematical expectation of this development gives the statistical moments of the response. The efficiency of the Taylor development method strongly depends on the ratio fluctuation/mean value of the uncertain parameters and on the sensitivity of the system to the uncertain parameters.

When the analytical evaluation of the partial derivatives of the response is not feasible, the efficiency of the method will also depend on numerical evaluation of the derivates.

15.3 Taguchi Method

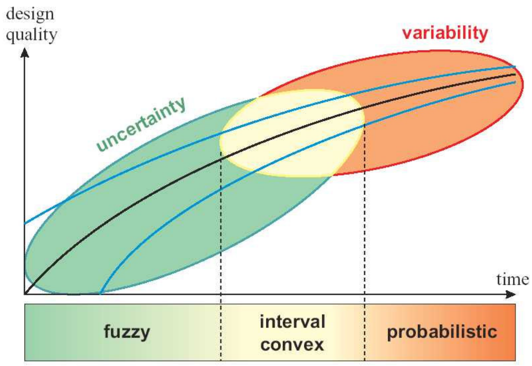
Taguchi method is based on numerical integration methods of the Gauss quadrature type. It allows a simple estimation of the statistical moments of a function of several independent random variables the probability densities of which are known. The principle consists in the discretization of each probability density in a finite number of points suitably chosen and in the computation of the function for whole the possible configurations. The statistical moments of the function are obtained by a weighted sum of the results of each configuration. The ponderation depends on the type of probability density of each random variable. The precision of the method rapidly increases with the number of points. The main advantages of this method are simplicity and small computation times compared to Monte-Carlo simulation.

Reference: D'Errico J.R., Zaino, jr. N.A.. Statistical tolerancing using a modification of Taguchi's method. *Technometrics*, Vol 30, no 4, pp 397-405, 1988.

15.4 Fuzzy Logic Method

The Fuzzy Logic method followed in this project should be more general and more efficient than the above listed methods.

In any case, we believe that at the early stage of a new project with uncertain parameters for which we don't have an idea of the distribution law, a Fuzzy Logic FEM must be applied; later on, when a better knowledge of the distribution of the unknown parameters is obtained, a more classical probabilistic (Monte Carlo) method can be applied.



16 Activities at CSL and GDTECH

In the context of a PAT contract n° PA-12-315, of CSL with SSTC : Static and dynamic analysis procedures for structures with uncertain parameters, GDTech has been identified as a subcontractor of CSL for specific FEM calculations preparation and evaluation on different aerospace uncertain models on which a Fuzzy Logic FEM will also be applied.

With the help of GDTech, for the FEM modelisation, definition and calculations, and with some inputs from the Users, CSL will define, test cases in dynamics, for space activities.

The activities identified for GDTech at the beginning of the project, are :

1. Define test cases FEM
2. Test the validity of the FEM of the test cases.
3. If possible, we will try to compare the results obtained with the fuzzy logic developments by the partners, with "our classical" way to do this type of calculations. There are different ways to do the classical calculations: sensitivity analysis (suitable

if linear and not to many parameters), Monte-Carlo analysis, ... The way we will do it, will be discussed with GDTech on a case by case basis.

4. If possible, we will introduce in someway, the developments of the fuzzy method in our SAMCEF methods.

These activities are subject to modifications due to the possible interactions from the partners and especially from the users.

Due to all these uncertainties (the last two activities are not contractual but are interesting for both parties CSL and GDTech) and since the time and budget allocated to these activities are limited, CSL and GDTECH will define the priority activities on a regular basis. An extension of this project could easily be justified.

TAP 31
Report part 5.4
Formulation of guidelines on taking parameter uncertainties into account during Finite Element analysis

Joris Peeters, David Moens, Hilde De Gersem
K.U.Leuven, Dept. Mechanical Engineering, PMA
Celestijnenlaan 300B, B-3001 Heverlee, Belgium

1 Introduction

Both the probabilistic and the non-probabilistic finite element method aim at the investigation of the effect of non-determinism on selected finite element output quantities. Each of the non-deterministic methods has its strengths and weaknesses, and requires a specific input. It is up to the engineering analyst to select an appropriate method for the analysis of the problem at hand. In some cases, the selection of a specific approach might be straightforward, but in other cases the pros and cons of different techniques have to be weighted against each other. Therefore, this report tends to formulate some guidelines on the application possibilities of the different non-deterministic approaches.

Section 2 of this report gives a general overview of the applicability of both the probabilistic and non-probabilistic method, based on the type of the non-determinism present in the problem, and based on the purpose of the presumed non-deterministic analysis. Section 3 gives some guidelines regarding the uncertainty assessment of some frequently encountered non-determinism in FE models of engineering problems.

2 Selection of a non-deterministic approach

2.1 Selection based on the type of non-determinism

For the selection of an appropriate non-deterministic FE approach, the type of non-determinism and the amount of available non-deterministic input data has to be considered. In this part, distinction is made between certain variabilities, uncertain variabilities and invariable uncertainties. This terminology and classification of non-determinism is clearly described in the second section of the report of project task 1.1.

2.1.1 Probabilistic variability and uncertainty representation

The probabilistic concept is most appropriate to represent *certain variabilities*, as for this type of non-determinism information on both the range and the likelihood can unambiguously be incorporated in the required probability density function. However, care has to be taken when multiple certain variabilities are present, as the information on the joint probability density function is then required. In case of *uncertain variabilities*, mostly the information on the likelihood is missing. The probabilistic concept can be used, but it is mandatory to apply a number of different probability density functions in order to examine the effect of the chosen PDFs on the result. In case of *invariable uncertainties*, some PDF support and distribution is chosen by the analyst, but it does not represent available objective information. Hence care should be taken when interpreting the resulting probabilistic outcome.

Nowadays, the probabilistic concept is by far the most often used approach to describe non-determinism in finite element models, and is included in most available non-deterministic FEM software codes. However, criticism has arisen concerning the use of the probabilistic model for the numerical representation of both variabilities and uncertainties. When subjective input information on the non-deterministic properties has to be included, non-probabilistic techniques could give an additional insight. In this case, the limited (objective) added value of a probabilistic analysis might not justify the required computational effort, which can be very high.

2.1.2 Interval variability and uncertainty representation

The use of an interval for the numerical description of *certain variabilities* is not mandatory, as only the information on the range is used, while the information on the likelihood is lost. Hence, not all available information is used, which is an important disadvantage. The interval concept is mainly appropriate for *uncertain variabilities* of which the upper and lower bound is known but the information on hte likelihood is missing. In this case an interval is a very good representation of the available information, without the inclusion of subjectiveness. In case only limited or no information is available on a given PDF support, the non-determinism is best modelled probabilistically. For *invariable uncertainties* a subjective interval is chosen by the analyst, which represents the range of values he/she considers possible at the time the analysis is performed. Hence the representation is subjective, but requires less information than a probabilistic representation.

It can be concluded that the interval concept is most valuable for the description of uncertain variabilities with known support but unknown distribution, and for the description of invariable uncertainties.

2.1.3 Fuzzy variability and uncertainty representation

The fuzzy concept uses a membership function for the description of each of the non-deterministic properties, and hence uses subjective input as these membership functions are mostly completely based on the subjective knowledge or believes of the analyst. Therefore, the results of a fuzzy analysis may only be interpreted in reference to the assumed fuzzy input.

In case of a *certain variability*, the known probability density function has to be converted into a compatible membership function. A number of methods are available for this purpose, but they all insert subjectivity. Hence the objective data is replaced by subjective data, which is highly irrational and should not be done. For *uncertain variabilities*, the fuzzy concept can be used for a hybrid uncertainty model, as is described in the report of task 1.1. In this approach, a fuzzy number is used to simultaneously examine the effect of a set of different probability density functions in a single analysis. Hence this method seems very powerful, but is has only rarely been applied yet. The fuzzy concept is most useful for the description of *uncertainties*. In this case, the fuzzy membership function represents the subjective expectation of the analyst, eg. the possible values of a design parameter that is still left open for optimisation. The more objective information becomes available on a non-deterministic model property, the less the fuzzy concept becomes appropriate to describe it.

2.2 Selection based on analysis pupose and the design phase

From the discussion above, it is clear that non-probabilistic approaches can be very valuable to model non-deterministic properties in a finite element model in absence of crucial probabilistic information. Still, the decision regarding which non-deterministic concept to use should not be based exclusively on the available information at the analysis input. The clear definition of the objective of the analysis is at least equally important in the determination of the most appropriate non-deterministic analysis tool. Therefore, this section now focusses on a number of practical non-deterministic analysis types that concern a design engineer. In order to evaluate the possibilities of the non-probabilistic approaches in specific applications, references will be made to the corresponding probabilistic treatment of the non-determinism.

2.2.1 Numerical non-determinism in a design process

The main objective of the application of numerical tools in a design process is to assess the product quality at a specific design stage by simulation of its realistic physical behaviour. Still, an exact quantification of the design quality based on the numerical predictions is not always straightforward. This is mainly due to the non-determinism implicitly contained in the numerical analysis results. Analysing the design quality over time, very often an evolution as illustrated in figure 1 is observed [1].

The design quality is expected to increase over time. Still, there always is a scatter on the predicted design quality, represented by the grey area in the figure. This scatter tends to decrease over the process, since additional information acquired over time will decrease the amount of uncertainty. On the other hand, the scatter will generally not disappear because of the presence of irreducible variability. Figure 1 indicates the evolution of the applicability of design analysis tools over time. The upper bound on the useful range of numerical methods is induced by the limit on the realism in numerical simulation of physics. There are currently two fundamentally different approaches that aim at moving this upper bound forward into the design process. On the one hand, there is a tendency towards integrated numerical analysis,

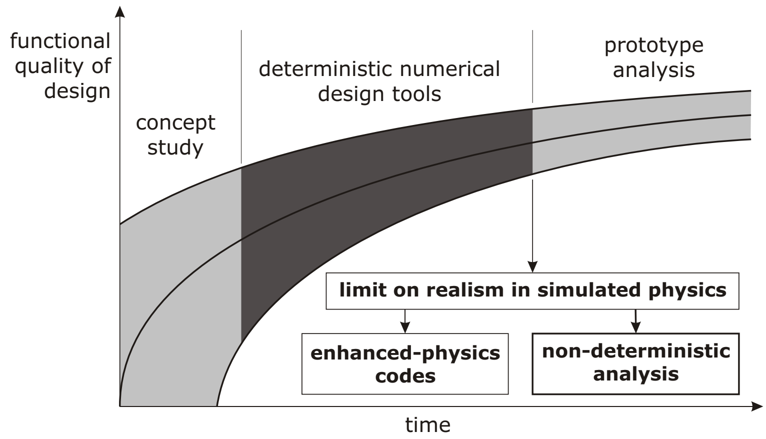


Figure 1: Limits on the application of numerical analysis in design

aiming mainly at multi-physics simulations that incorporate all physics relevant for the design into a single simulation. On the other hand, it is more and more acknowledged that the introduction of non-determinism in the numerical analysis is equally, if not more important in influencing this upper bound. Therefore, it is imperative to have insight in the numerical analysis tools that can be of use to incorporate non-determinism in the analysis during design. This will not only extend the useful range of numerical methods, but should simultaneously lead to a better understanding of the sources of the scatter on the predicted behaviour. This information on its turn can be extremely important to improve the design quality obtained after the numerical design cycle. Figure 2 summarises these envisaged effects of numerical analysis of non-determinism in a design process.

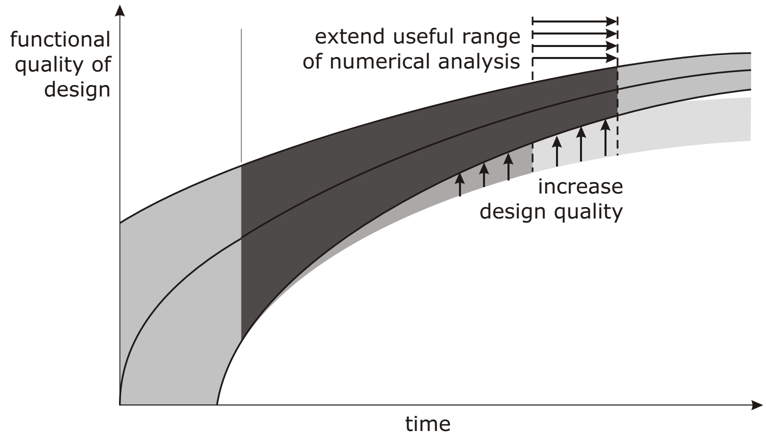


Figure 2: Extending the applicability of numerical methods and improving design quality through the application of non-deterministic analysis techniques in the numerical design cycle

Generally, there is an evolution of the type of non-determinism encountered during a typical design process, or as formulated by Ross *et al.* [2]: *As more information about a problem becomes available, the mathematical description of non-determinism can transform from one theory to the next in the characterization of the uncertainty* *as the uncertainty diminishes or, alternatively, as the information granularity increases and becomes specific.* In an early stage, objective information on model properties is often difficult to obtain, since a large number of model properties have yet to be defined. Some design decisions are even intentionally postponed in order to be able to study their effect on the design quality. Furthermore, early design improvements are commonly the result of expert knowledge rather than detailed numerical procedures. This means that the amount of objective information on average is low, and therefore subjectiveness is substantially present in the analysis. This leads to the conclusion that in early design stages, most non-determinism belongs to the uncertainty class. Through the course of a design process, the amount of information generally increases. In some cases, the non-deterministic properties can be more objectively described, e.g., when certain design aspects are fixed, or component manufacturers are chosen. The classification of the remaining non-determinism gradually moves towards to what has been defined as model variability, i.e., design independent variations in the product or its environment.

The evolution of non-determinism in a typical design process as described above is illustrated in figure 3. This figure also indicates the evolution of the numerical concepts that are most appropriate for the dominant class of the occurring non-determinism. In the early stages, the non-determinism in the numerically predicted design quality is mainly driven by model uncertainties, which leads to the conclusion that non-probabilistic concepts are most appropriate in these early stages. Later in the design process, variability becomes more important, leading to a more prominent application of the typical variability modelling tool, i.e., the probabilistic concept.

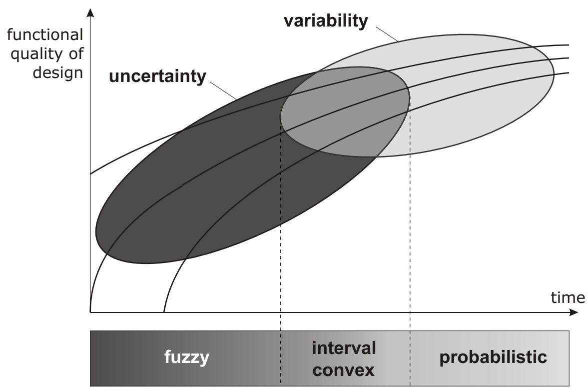


Figure 3: Typical occurrence of non-determinism in the product quality predictions during a design process

The evolution of a property from one class of non-determinism to another can be clarified using a simple example. Take for instance the design of a new car body. The start point of the structural design is generally a conceptual design inspired esthetically rather than mechanically. In this initial design, there is a lot of non-determinism on the dimensions of structural components, such as for instance plate thickness. Since there is no information whatsoever on the exact plates that will be used, numerical analysis in this phase can only incorporate subjective knowledge based on other designs. Alternatively, a designer could be interested in the impact of a certain plate thickness on the behaviour of the design. In that case, a preferred range could be defined for the thickness in order to identify the most appropriate value. In either case, there is no clear objective information on the actual property in the final product. Hence, if non-determinism on this property is to be taken into account, this can only be achieved through modelling of subjective knowledge. Later, at a certain point in the design process, a specific reference value will be chosen for the thickness of the plates in the car body structure. Tolerances are chosen, which define the allowable region for these properties in the actual product. At this point, the range of the thickness in the actual product is known, but there's no information on the likelihood inside the range. The property has clearly evolved to an interval. Finally, when the design is finalised up to the detailed description of the manufacturing process, information on the variation of the plate thickness within the bounds of the tolerances could become available. The value for the thickness then becomes a variability.

2.2.2 Probabilistic reliability analysis

The reliability of a product is defined as the likelihood that it will successfully fulfil its intended task over a predefined period in time under specific environmental conditions. Numerical reliability analysis is very popular in a structural design context because it can provide a designer with crucial information on the likelihood of failure of the analysed design. As such, it can be usefully applied in an economical product analysis taking into account the cost associated with failure.

Reliability analysis of non-deterministic structures using the probabilistic concept has been studied extensively in literature. Very powerful software codes exist supplying the analyst with a vast arsenal of probabilistic reliability analysis procedures. See Casciati *et al.* [3] for a comprehensive overview of probabilistic reliability methods. Most commonly, the probabilistic reliability analysis results in a probability of failure, defined as *the likelihood that the structure will successfully fulfil its intended task over a predefined period in time under specified environmental conditions*. This probabilistic reliability analysis is broadly applied and already incorporated in generally accepted design specifications in civil engineering. However, its application in mechanical engineering is far less standardised. This is mainly due to the plentitude of different mechanical products, which all require a different amount of reliability under very different environmental and loading conditions. Hence, there are very few standards for reliability in mechanical design. Each product designer applies rules which are based on experience rather than on general engineering standards.

Mathematically, the probabilistic reliability analysis requires the definition of a performance criterion based on the relevant load and resistance parameters. This performance function generally is referred to as the *limit state* function and is described as:

$$Z = g(X_1, X_2, \dots, X_n) \tag{1}$$

The *failure surface* is then defined as $Z = 0$. It represents the boundary between what are considered to be unsafe and safe design regions in the parameter space. The limit state can be an explicit or implicit function of the parameters. This characteristic has an important influence on the analysis procedure. Using the definition of Eq. (1) the probability of failure P_f equals:

$$P_f = \int \dots \int_{g(X_1, X_2, \dots, X_n) < 0} f_X(x_1, x_2, \dots, x_n) \; dx_1 dx_2 \dots dx_n \tag{2}$$

with $f_X(x_1, x_2, \dots, x_n)$ the joint probability density function for the considered parameters. This equation forms the basis of probabilistic reliability analysis. However, it is in most cases impossible to solve because the necessary information to describe the joint probability density function is missing. But even if it was available, evaluating the multiple integral is extremely difficult. In this context, approximation methods have been developed. Each of these methods has its own requirements concerning the performance function. Only the most common are listed here:

- **First Order Reliability Methods (FORM)**: After transformation to a standard normal parameter space, each limit state function is replaced with a first-order polynomial approximation at a specific point in the parameter space. This point is usually the point on the failure surface nearest to the origin, and is generally referred to as the *design point* or *most probable point*. The probability of failure follows directly from the distance from the origin to the design point.

- **Second Order Reliability Methods (SORM)**: This method is completely similar to FORM, with the exception that a second-order polynomial is used for the limit state function approximation. (see [4] for a general introduction to FORM and SORM)

- **Mean Value Based Methods (MVBM)**: This method constructs a first-order Taylor series expansion of the limit state function around the mean values of the random variables.

- **Simulation Methods (SM)**: The approximation of the probability of failure results directly from a series of analysis runs using samples of each variable.

The FORM, SORM and MVBM require information on the derivatives of the limit state function to the parameters. Therefore, they are most appropriate when an analytical closed-form expression of the limit state function is available. This is generally not the case for reliability assessment based on finite element analysis, where the relation between the model parameters and the limit state function is implicit. This has led to the development of specific algorithms for sensitivity analysis which directly aim at the calculation of these derivatives, either analytically or based on numerical approximations. This is already provided in a number of commercial finite element codes nowadays. When there is no explicit relation between design parameters and the limit state, response surface methods are commonly applied to approximate the limit state function in the design space. With these, a limited number of analysis runs is performed at several points in the design space based on a design of experiments strategy. The approximation of the true limit state function then generally results from a second-order polynomial fitted through the resulting points. These developments induced implementations of FORM, SORM and MVBM around a finite element code. The computational burden for these implementations, however, remains large. Furthermore, the exactness of these methods decreases rapidly when the range of the parameter variabilities increases because the approximations are based on local information.

Currently, the simulation methods are by far the most popular numerical tool to predict the probability of failure of a given design. This is mainly due to the fact that they are easy to use, straightforward, and require little background in probability theory. Their main disadvantage is that they are computational expensive. However, in combination with a response surface approximation of the limit state, their efficiency can be increased. The probability of failure can be derived numerically based on a Monte Carlo simulation by rewriting Eq. (2) to:

$$P_f = \int_{-\infty}^{+\infty} I(g(x)) f_X(x) dx = E\{I(g(x))\} \tag{3}$$

with:

$$I(g(x)) = \begin{cases} 1 & \text{if} \quad g(x) \leq 0 \\ 0 & \text{if} \quad g(x) > 0 \end{cases} \tag{4}$$

Therefore, it can be estimated from N Monte Carlo samples using:

$$P_f \approx \frac{1}{N} \sum_{i=1}^{N} I(g(X_i)) \tag{5}$$

with X_i the numerical value of the samples. From this approximation it is also clear how a good preceding response surface procedure could greatly improve the efficiency of Monte Carlo simulation. Recently, Schuëller *et al.* [5] gave a clear overview of the recent advances in Monte Carlo based simulation procedures for application in reliability analysis of high dimensional problems.

According to its definition, reliability belongs clearly to the probabilistic framework in the frequentist context. On the one hand, this complicates probabilistic analysis of designs intended for limited production, since the fact that the product is only produced in limited quantity strongly complicates a decent aposterior verification of the non-deterministic numerical predictions. Furthermore, for most designs intended for limited production nowadays, an unverifiably high reliability is requested (e.g. spacecraft). The current tendency towards designing for 6-σ clearly illustrates this evolution. However, such specifications require an extremely high accuracy of the predicted probabilistic behaviour, especially in the tails of the obtained probability density functions. This is extremely difficult to achieve. Furthermore, even if a mass production is envisaged, such high reliability requirements can never be verified. Therefore, it is the authors opinion that it is rather irrational to attach any objective meaning to reliability values of 1-10^9 or more. The reliability specification in this case comes down to requiring an extremely reliable product, which is a clear step towards treating reliability in a subjective non-deterministic context.

As discussed in section 2.1, while applying the probabilistic concept for the representation of subjective information is possible, results from such an analysis should definitely not be interpreted as indication for an absolute frequency of occurrence. This means that the subjectiveness devaluates the use of the probabilistic results in a reliability context. It is important to note that the subjectiveness incorporated in the information on which the analysis is based is not always detected. For instance, neglecting unknown correlation between properties by assuming them as independent is a common simplification that is sometimes implicitly made, but that can have important consequences. This implicit assumption of independence between probabilistic quantities was one of the important errors that were the source of the Challenger space shuttle disaster [6]. In this case, the impact of different extreme weather conditions on the launch was analysed for each condition individually beforehand. The impact of a combination of more than one of these events, however, was never checked. Although each of the events had a very low probability of occurring, the probability of their combination proved to be not simply a multiplication of the probabilities of the single events. The correlation between the conditions was clearly misjudged, leading to a plausible but unaccounted probability for a weather situation with disastrous consequences.

The lack of credibility of numerical predictions of reliability is generally compensated by safety factors. However, one could argue that using these safety factors after applying sophisticated and computationally expensive numerical procedures is not a really economical situation. Much effort is spent on a numerical prediction, which, in the end, still has to be corrected based on practical experience. In this context, the non-probabilistic approaches could prove their value. The remainder of this section briefly discusses possible applications of the non-probabilistic concepts for numerical reliabilty analysis.

2.2.3 Non-probabilistic reliability analysis

The application of the interval concept in numerical reliability studies is often referred to as *anti-optimisation*. This name stems from the fact that from all numerical models within the interval input boundaries, the one with the least favourable analysis result is the most interesting from reliability point of view. Finding this least favourable result is mathematically equivalent to performing a numerical optimisation aimed at the worst-case result with respect to the input intervals.

The concept of anti-optimisation has been introduced as the basis for a non-probabilistic reliability framework [7]. This requires an evolution from a reliability concept as *probability of failure* towards *range of acceptable behaviour*. This means that the design must assure that the performance remains within an acceptable domain, without specifying a likelihood of failure. Reliability then becomes a crisp criterion distinguishing between either acceptable or unacceptable designs. The most important benefit of the anti-optimisation concept is that it broadens the objectivity of reliability studies to uncertain variabilities with known range, because the interval model perfectly represents these uncertainties without the need for subjective input. For instance, this enables a fast assessment of dimension tolerances on a design, without knowing the actual distribution of the dimension within the bounds of the prescribed tolerance. For some cases, it can be shown that the anti-optimisation procedure results in the same choice of design parameters as a probabilistic analysis if the required reliability tends to one [8]. The anti-optimisation in this case proves to be far less expensive in computation time.

The numerical implementation of the anti-optimisation approach is subject to an important requirement. Since the result of the analysis is the source of a crisp decision between acceptable and unacceptable designs, approximate results should always be kept on the safe side of the exact result. This means that if approximate solution procedures are used in the numerical implementation, they should guarantee conservatism in their result. On the other hand, this conservatism should not be excessively high in order for the result to be of any practical value.

Also the fuzzy concept has been introduced as a numerical reliability assessment tool [9]. In the interpretation of the membership function as a degree of possibility, the fuzzy outcome of an analysis could be used to define a possibility of failure. This possibility is clearly influenced by the subjectiveness that is implicitly incorporated in the fuzzy input of the analysis. This means that for the same problem, different analysts can and generally will end up with different possibilities of failure. This could be compensated by defining a personal threshold value for the allowed possibility of failure in the final decision on acceptable or unacceptable designs. However, due to the necessary amount of personal interpretation of the analyst, possibility of failure only has a relative value. Therefore, this approach is extremely difficult to standardise in a general reliability framework.

Still, based on the α-sublevel technique, the fuzzy approach becomes very useful when the effect of interval bounds on the anti-optimisation result has to be analysed. In this context, the fuzzy analysis can serve as a tool to derive the α-level on which the required safety margins are reached on the crisp failure modes. The input intervals derived from the input membership functions intersected at this α-level then define the allowable range for the non-deterministic input properties. The fuzzy reliability analysis as proposed by Biondini *et al.* [10] is based on this principle. The same approach was applied by Catallo [11] for reliability assessment based on a fuzzy analysis of limit state load multipliers of a precast concrete structure.

A different application of the fuzzy concept in reliability analysis is based on the use of the membership function as limit cumulative density functions as explained in the report of project task 1.1. It was shown by Ferrari *et al.* [12] that, if the input membership functions represent boundaries on the cumulative density functions of the input parameters, the membership function resulting from fuzzy analysis on this input forms reliable boundaries on the actual cumulative density function of the result. Therefore, the fuzzy result of a fuzzy finite element analysis can be used to derive bounds on the probability of failure. A simple example illustrates this. Suppose that a fuzzy finite element analysis results in a membership function $\mu_{\tilde{\lambda}}(\lambda)$ representing a crucial eigenfrequency of a design as illustrated in figure 4. Suppose furthermore that a crisp criterion states that the design is acceptable if this eigenfrequency is kept below the value λ^*. The fuzzy result envelopes the exact cumulative density function of the eigenfrequency. This means that the bounds on the probability that the eigenfrequency of the design lies below λ^* can be derived from the fuzzy result. The probability interval is obtained from taking the value of the envelope curves at λ^* as indicated in the figure by $\underline{P_f^{'}}$ and $\overline{P_f^{'}}$. The most conservative statement resulting from the analysis is that the probability of failure equals $(1 - \underline{P_f^{'}})$ in the worst case.

It is clear that also the above non-probabilistic reliability methods are subject to the limitation that whenever there is subjective information involved in the problem definition, the results can not be interpreted as absolute measures of design quality. In an absolute reliability context, the amount of expert knowledge required in the distinction between a good or bad design is proportional to the amount of subjectiveness incorporated in the description of the non-determinism. Still, subjective analysis can be of great value when used in a relative framework, as for instance a

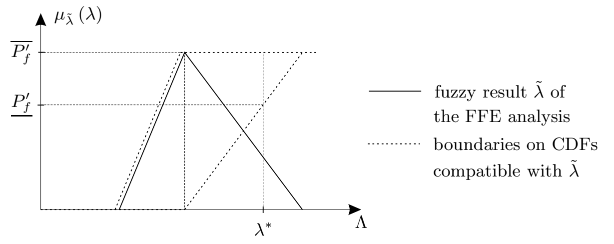


Figure 4: Example of the application of the fuzzy outcome of a fuzzy finite element analysis to predict bounds on the probability of failure

design optimisation procedure. This will be discussed in the next section.

2.2.4 Numerical design optimisation

The principal goal of design optimisation is to define the best possible product under certain restrictions. These restrictions can be anything from manufacturing cost to limitations placed on physical properties of the design. The ingredients of the goal function and their relative weights determine the final result of the optimisation. Reliability can be used as an indication for the design quality, and therefore can be an important part of the goal function. Classicaly, this is approached from a probabilistic viewpoint, and referred to as *reliability based design optimisation.* YOUN *et al.* [13] gives an overview of different approaches that aim at an increase in design quality or robustness through an optimisation based on numerical reliability predictions.

Still, when reliability is used as a design quality indicator in an iterative design optimisation process, the demands on the objectivity are much lower than when it is used for absolute design assessment. A relative reliability improvement during an optimisation process can already be very valuable, even though the absolute reliability is only roughly approximated. This means that also subjective analysis can be usefully applied in a design optimisation context. While applying subjective probability for this purpose is possible, it is not always the most advisable approach. In some cases, especially in design optimisation, a probabilistic reliability measure is not required. For instance, if the range on some parameters is all information that is available, placing subjective probability density functions on these ranges only complicates the numerical problem, while it doesn't necessarily add any valuable meaning to the analysis. In that case, it doesn't really make sense to transform the problem to the probabilistic concept. Or as formulated by ROSS *et al.* [14]: *Sometimes, striving for precision can be expensive, or adds little or no useful information, or both.* This indeed holds for the application of reliability calculations in an iterative optimisation procedure, where the numerical efficiency becomes very important. It is now discussed to what extend the non-probabilistic approaches can be considered as valuable alternatives for design analysis in an optimisation framework.

For the interval concept, the most useful application lies in modelling invariable uncertainties. Though they are assumed to be constant, they could play an important role during design optimisation. The analyst may ask the question whether the defined ranges for the invariable uncertainties result in an allowable range for the behaviour, without really being interested in the likelihood of occurrence within the defined interval bounds. Or, alternatively, the invariable uncertainty represents an open design decision, i.e., a model property that has yet to be quantified, and the value of which will be optimised. Pure probabilistic analysis in both cases seems like an unnatural thing to do, since it requires information that is not available (probabilistic input) to produce information that is not requested (probabilistic output). The interval procedure is limited to the definition of the intervals on the uncertainties the analyst would like to take into account. Subsequently, the design can be assessed from an interval analysis by reassuring that the worst-case output is still within the range of acceptable physical behaviour. This comes down to a worst-case oriented design optimisation.

A commonly formulated criticism on this approach is that the worst-case behaviour generally results from the combination of extremely rare events. Taking these combinations into account in a design assessment procedure could lead to severe over-dimensioning. This criticism only holds if you can objectively verify the actual probability of occurrence of the model properties which are considered to be extreme events. But even more important, if you want to give a realistic weight to the actual occurrence of such an extreme combination of events, it is imperative to incorporate the exact mutual interdependence between these extreme events in the procedure, as discussed for the Challenger case in section 2.2.2. In such cases, worst-case analysis could be a tool for identification of extreme events which lead to failure, without the need for a prediction of the actual probability of this extreme event. This identification should not necessarily lead to adapted designs and the generally associated over-dimensioning. In the Challenger case, accustomed launch protocols incorporating identification of possible disastrous extreme weather conditions would already have been of great value.

As discussed in the previous section, due to its implicit subjective nature, the value of fuzzy finite element analysis as an absolute reliability analysis tool is rather limited. In an optimisation procedure, however, the complete process is generally conducted or followed up by one and the same analyst. This means that the subjective possibility measure can be interpreted in a consistent manner throughout the optimisation procedure. Therefore, the possibility of failure can be used as a quality measure in an optimisation procedure. In this context, Choi *et al.* [15] recently introduced a possibility-based design optimisation procedure based on a fuzzy representation of the uncertain design aspects.

Apart from reliability optimisation, an important aspect of designing under uncertainty is to define a robust design, i.e., a design whose critical properties have a minor sensitivity to changes in the uncertain influences like for instance external loading. Also in this context, the fuzzy approach can be of value. By placing fuzzy membership functions as loading factors on the crucial loading components, the sensitivity of some design quality indicators to these external influences can be analysed. Using this approach, the robustness of the design can be assessed by measuring the width of the resulting membership function on the critical design quality indicators.

Another practical approach of the fuzzy analysis is in the study and choice of tolerances placed on design dimensions. From the α-cut strategy, it is clear that the fuzzy finite element analysis is actually a large-scale sensitivity analysis of the combined effect of the bounds defined on some interval design variables on critical design properties. By placing membership functions on the design properties subject to tolerances, the effect of their range on the design behaviour can be analysed. This can be helpful in defining tolerance intervals in the model. For instance, at a certain α-level, an allowable range could be identified in the fuzzy outcome of the analysis. The corresponding input intervals at this α-level can then be chosen as the set of tolerances on the analysed design properties. This procedure is clarified in figure 5, where the design specification is assumed to be an upper bound λ^* on an eigenfrequency. The analyst can control the analysis by defining the possibility distributions on the input according to personal preference or practical limitations. A different possibility distribution for the design variables will yield a different possibility distribution of the analysis result, and consequently also different tolerances for the design variables. The design based on these alternative allowable ranges, however, is equally safe. In this context, again, the possibility distribution is rather a useful tool to control the allowable range for the uncertainties than an absolute quality measure.

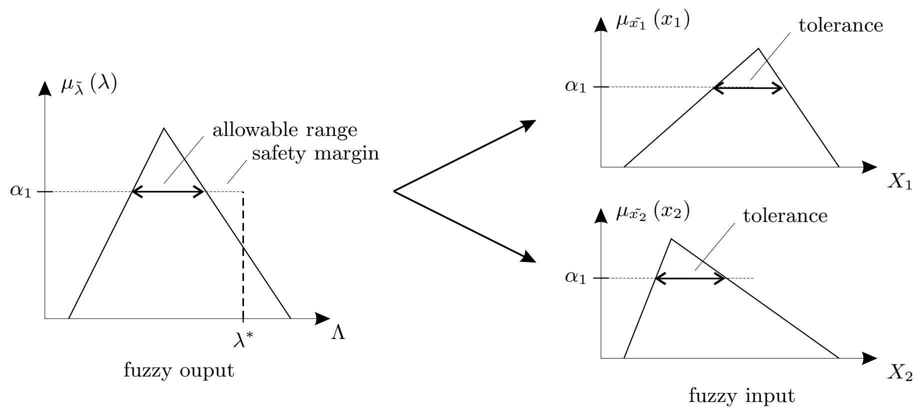


Figure 5: Illustration of the application of the fuzzy concept for design tolerance analysis

2.3 How are probabilistic, interval and possibilistic analyses related?

Elishakoff [16] compares the concepts of probabilistic analysis, fuzzy sets and anti-optimisation applied on finite element analysis. He concludes that each of the methods has its own advantages and could be preferred above the others under specific circumstances. De Lima *et al.* [17] compared the result of a probabilistic finite element analysis and an equivalent fuzzy finite element analysis on a simple example. He concludes that the fuzzy method leads to less expensive qualitative results which are adequate for practical engineering purposes.

To compare the applicability of the different methods, it is of interest to study the performance of each of the different concepts when applied to the same design problem. After all, they are all aimed at providing the analyst with enough information on the influence of the non-deterministic input on the numerical analysis to draw conclusions regarding the performance of the design. Maglaras *et al.* [18]

compares experimentally the designs resulting from optimising reliability in both a probabilistic and possibilistic framework. He concludes that the design acquired through probabilistic analysis is better when there is enough information to describe the probabilistic data realistically. Another comparison of probabilistic and possibilistic design under uncertainty by NIKOLAIDIS *et al.* [19] demonstrates that a fuzzy set method yields safer designs than probabilistic design methods when very limited information is available. Both these conclusions confirm the main drawback of probabilistic analysis, i.e., the fast devaluation of its result with increasing lack of information on the non-deterministic input.

It could be useful to do an analysis using a mixture of different uncertainty models, for instance when there is sufficient statistical data to describe some variabilities, but also uncertainties are present in the model. For this purpose, a hybrid finite element analysis has been developed by LANGLEY [20]. It consists of a single mathematical algorithm to analyse all three models of non-deterministic quantities simultaneously, based on a SORM or FORM approach for reliability. A different approach for combined uncertainty and variability analysis was proposed by RAO *et al.* [21]. It is based on a separate probabilistic and non-probabilistic analysis run, after which both results are unified to a hybrid-uncertainty mean value and variance.

Based on the above discussion, it is concluded that the mutual relationship between the probabilistic and the non-probabilistic approaches is rather weak. While both can be put to use in a numerical design procedure, their application field is strongly dependent on the available information and the intention of the numerical analysis. Considering a design process as given in figure 3, this leads to the conclusion that the non-probabilistic approaches should be regarded as complementary rather than competitive to the probabilistic approach.

3 Typical non-determinism in FE models of mechanical engineering structures

This section gives some practical guidelines for the assessment of non-determinism typically encountered in finite element models built for the static and/or dynamical analysis of mechanical structures. Non-determinism is frequently found affecting several parameters, such as material characteristics (Young's modulus, mass density), damping characteristics, geometry, loading and connection characteristics (boundary conditions, joints, etc). Depending on the available information and the purpose of the analysis, one non-deterministic approach can be more appropriate than others.

For *material characteristics* such as the Young's modulus, the Poisson coefficient and the mass density, both probabilistic and non-probabilistic methods can be applied. In case there is a sufficient amount of statistical data available on both the range and the likelihood of the parameters, the probabilistic approach is advisable. In case only the range is (objectively/subjectively) known to the analyst, the interval or fuzzy method is preferable. For *loading conditions*, the same considerations can be made.

For non-determinism affecting the *geometry*, a distinction has to be made between small geometric changes (eg. variabilities caused by manufacturing tolerances) and larger geometric changes (eg. due to open design decisions). In most cases of pro-

duction tolerances, statistical data is available on the range and likelihood of the geometric characteristics, and hence the probabilistic approach is a logical choice. In case of large uncertainties affecting the geometry, eg. in case of not yet decided design dimensions, the interval and fuzzy methods are far more advisable, as hardly no objective data is available in this early design stage.
The most difficult sources of non-determinism in finite element models arise from *damping*, *joint characteristics* (eg. spot welds) and *non-ideal boundary conditions*, as these properties are inherently difficult to model numerically, and hardly any information is available. Hence the uncertainty intervals are large and the ranges are often subjective. Therefore, the interval and certainly the fuzzy method are best suited to account for these non-deterministic model properties.

In case a non-probabilistic approach is chosen, special attention has to be paid to an appropriate implementation of the interval method. In this project, a *Design of Experiments* (DOE) approach and a *global optimisation* approach have been proposed. The DOE approach is the most computationally efficient method, but has as important drawback that is does not guarantee that the worst-case scenario is detected and conservative results are produced. Especially the effect of large uncertainty intervals and uncertain parameters affecting only a small part of the structure is very hard to predict, and mostly non-monotonic. In these cases it is necessary to apply a global optimisation approach in order to detect the extreme static and/or dynamic response values.

4 Conclusion

This report describes some guidelines regarding the selection of an appropriate non-deterministic method in order to investigate the influence of variabilities and uncertainties on the static and/or dynamic behaviour of engineering structures. Several important factors influencing this selection are detected: the sources and types of non-determinism, the available objective information (probabilistic/non-probabilistic), the purpose of the non-deterministic analysis, the design phase, the available computer resources, etc.

References

[1] T.A. Zang, M.J. Hemsch, M.W. Hilburger, S.P. Kenny, J.M. Luckring, P. Maghami, and S.L. Padula. Needs and opportunities for uncertainty-based multidisciplinary design methods for aerospace vehicles. Technical Report NASA/TM-2002-211462, Langley Research Center, Hampton, Virginia, 2002.

[2] T. Ross, K. Sellers, and J. Booker. Considerations for using fuzzy set theory and probability theory. In *Fuzzy Logic and Probability Applications: Bridging the Gap*, chapter 5, pages 87–104. SIAM Philadelphia (PA) and ASA Alexandria (VA), 2002.

[3] F. Casciati and J. Roberts. *Mathematical Models for Structural Reliability Analysis.* CRC Press, New York, 1996.

[4] A. Haldar and S. Mahadevan. First-order/second-order reliability methods (FORM/SORM). Probabilistic Structural Mechanics Handbook, 1993. pp. 27-52.

[5] G. Schuëller, H. Pradlwarter, and P. Koutsourelakis. A critical appraisal of reliability estimation procedures for high dimensions. *Probabilistic Engineering Mechanics*, 19(4):463–474, 2004.

[6] M. Salita. Shuttle disasters: a common cause? *Aerospace America*, 4:41–43, 2004.

[7] Y. Ben Haim. A non-probabilistic measure of reliability of linear systems based on expansion of convex models. *Structural Safety*, 17(2):91–109, 1995.

[8] M. Zingales and I. Elishakoff. Anti-optimization versus probability in an applied mechanics problem: vector uncertainty. *Transactions of the ASME, Journal of Applied Mechanics*, 67(3):472–483, 2000.

[9] J. Sawyer and S. Rao. Strength-based reliability and fracture assessment of fuzzy mechanical and structural systems. *AIAA Journal*, 37(1):84–92, 1999.

[10] F. Biondini, F. Bontempi, and P. Malerba. Fuzzy reliability analysis of concrete structures. *Computers & Structures*, 82(13-14):1033–1052, 2004.

[11] L. Catallo. Genetic anti-optimization for reliability structural assessment of precast concrete structures. *Computers & Structures*, 82(13-14):1053–1065, 2004.

[12] P. Ferrari and M. Savoia. Fuzzy number theory to obtain conservative results with respect to probability. *Computer Methods in Applied Mechanics and Engineering*, 160(3-4):205–222, 1998.

[13] B. Youn and K. Choi. Selecting probabilistic approaches for reliability-based design optimization. *AIAA Journal*, 42(1):124–131, 2004.

[14] T. Ross, K. Sellers, and J. Booker. Introduction. In *Fuzzy Logic and Probability Applications: Bridging the Gap*, chapter 1, pages 3–27. SIAM Philadelphia (PA) and ASA Alexandria (VA), 2002.

[15] K. Choi, L. Du, and B. Youn. Vertex-domain fuzzy analysis method for possibility-based design optimization. In *Proceedings of the 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2004.

[16] I. Elishakoff. Three versions of the finite element method based on concepts of either stochasticity, fuzziness, or anti-optimization. *ASME Applied Mechanics Reviews*, 51(3):209–218, 1998.

[17] B. de Lima and N. Ebecken. A comparison of models for uncertainty analysis by the finite element method. *Finite Elements in Analysis and Design*, 34(2):211–232, 2000.

[18] G. Maglaras, E. Nikolaidis, T. Haftka, and H. Cudney. Analytical-experimental comparison of probabilistic methods and fuzzy set based methods for designing under uncertainty. *Structural Optimization*, 13(2-3):69–80, 1997.

[19] E. Nikolaidis, H. Cudney, and Q. Chen. Bayesian and fuzzy set methods for design under uncertainty. In *Proceedings of the 13th ASCE Engineering Mechanics Division Specialty Conference*, 1999.

[20] R. Langley. Unified approach to probabilistic and possibilistic analysis of uncertain systems. *Journal of Engineering Mechanics*, 126(11):1163–1172, 2000.

[21] S. Rao, L. Chen, and E. Mulkay. Unified finite element method for engineering systems with hybrid uncertainties. *AIAA Journal*, 36(7):1291–1299, 1998.