A Robust Incremental Volt/VAR Control for Distribution Networks

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***Abstract*—This paper proposes an incremental Volt/VAR control for voltage regulation in distribution networks with high penetration of distributed energy resources. The Volt/VAR controller coefficients are obtained by solving a robust optimization problem, where reactive power is minimized. The proposed optimization problem is solved using a Successive Convex Approximation method. Communication requirements are minimal and restricted to the offline stage since the local controllers share the same gains. Numerical studies on a 42-node low voltage network demonstrate the improved performance of our local controllers against a traditional static Volt/VAR control strategy.**

***Index Terms*—Distributed energy resources, distribution networks, incremental Volt/VAR control, local control.**

I. Introduction

Traditionally, voltage regulation in Distribution Networks (DNs) is performed with load tap changers, switchgear, or other devices. However, the increasing variability introduced by Distributed Energy Resources (DERs) can shorten their lifespan, and they may become insufficient to resolve voltage issues. Advancements in power electronics converters offer new avenues for controlling DERs, thereby introducing novel opportunities for voltage regulation.

Different control architectures exist for the real-time operation of DERs. In the following we make a distinction between centralized, distributed and decentralized architectures. Centralized controllers require accurate network knowledge, which is challenging, particularly in networks with increased DER penetration. While recent feedback-based optimization methods, e.g. [1], alleviate the necessity for perfect knowledge of non-controllable power injections, they still require an advanced communication infrastructure, which is not always available in DNs [2]. Distributed controllers, e.g. [3], [4], can bridge the gap between decentralized and centralized methods. However, they still rely on communication infrastructures and are susceptible to communication delays and errors [2]. In contrast, decentralized strategies rely solely on local measurements to implement control actions [5], offering robustness, simplicity, and low cost. Local Volt/VAR controllers can be designed to achieve diverse objectives by solving dedicated optimization problems [6]. It is known that traditional static Volt/VAR controls may result in oscillatory behaviors [7], while incremental strategies incorporate voltage measurements and past reactive power setpoints [8] to overcome this problem.

Data-driven local Volt/VAR schemes can learn near-optimal controllers, thus closing the performance gap with centralized and distributed controllers [9]. Nonetheless, they may lack stability guarantees of the closed-loop system [10]. Some research works, e.g. [11], offer stability guarantees for specific control strategies. However, these methods often consider optimized local strategies for each DER, requiring an advanced offline communication infrastructure to determine DER locations and dispatch gains accordingly. Moreover, optimization-based methods frequently overlook the feasibility of the optimal reactive power flow (ORPF) problem, i.e., if there are enough reactive power reserves to satisfy voltage constraints under a controller architecture.

Statement of Contributions: We introduce a local incremental Volt/VAR controller scheme with optimized gains to minimize reactive power usage while ensuring that voltage constraints are met. Gains are predetermined based on forecasted power generation and loads, eliminating the need for historical data, and are then broadcasted to the local controllers, keeping the offline communication infrastructure as simple as possible. Leveraging the algorithm proposed in [12], we develop a tractable formulation of our optimal gain design using Successive Convex Approximation (SCA) methods. We also discuss the feasibility of the ORPF problem.

The text is structured as follows. In Section II, we define the DN model and our problem. Section III introduces the design of our controller. Section IV presents our optimal control design problem. Section V provides details regarding the implementation of the feedback controllers. Numerical simulations in Section VI compare the proposed approach to benchmark methods. We make some final comments in Section VII and conclude the paper with Section VIII.

II. Problem formulation

A. Power system model

We consider a balanced three-phase DN with $N+1$ nodes and G DERs. The node 0 is taken to be the substation node or the point of common coupling, while $\mathcal{N} := \{1, ..., N\}$ is the set of remaining nodes. We consider a phasor representation of the single-phase equivalent DN and model the system using

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power flow equations that can be written in a compact form as:

$$\mathbf{s} = \mathsf{diag}(\mathbf{u})(\bar{\mathbf{y}}^* V_0 + \mathbf{Y}^* \mathbf{u}^*), \tag{1}$$

where $\mathbf{u} := \{u_k\}_{k\in\mathcal{N}}$, $\mathbf{i} := \{i_k\}_{k\in\mathcal{N}}$ collect the voltages, the injected currents at every node, respectively, and $\mathbf{s} = \mathbf{p}_{\text{av}} - \mathbf{p}_{\text{l}} + j(\mathcal{A}\mathbb{q} - \mathbf{q}_{\text{l}}) \in \mathbb{C}^N$ is the net power injection at nodes $n \in \mathcal{N}$. We define $\mathbf{p}_{\text{av}}, \mathbf{p}_{\text{l}}, \mathbf{q}_{\text{l}} \in \mathbb{R}^N$ the vectors collecting the non-controllable active power injection, the active and reactive power consumption at nodes $n \in \mathcal{N}$, respectively. We define $\mathbb{q} \in \mathbb{R}^G$ the vector collecting the controllable reactive power of the G DERs. Powers take positive (negative) values if they are injected into (absorbed from) the grid. We also define $\mathcal{A} \in \mathbb{R}^{N\times G}$ as the matrix that maps a DER index to the node where it is located. The voltage at node 0 is set to $u_0 = V_0$.

Let us define $\mathbf{z} := (\mathbf{p}_{\text{av}}, \mathbf{p}_{\text{l}}, \mathbf{q}_{\text{l}})$ as the concatenation of the non-controllable powers, the algebraic map $H := \mathbb{R}^{G+3N} \to \mathbb{R}^N$ and $\mathbf{v} := \{v_k\}_{k\in\mathcal{N}}$ the vector collecting the voltage magnitudes at every node. For convenience, we denote $\mathbf{v} = H(\mathbb{q}, \mathbf{z})$ where H relates the net power injections to the *practical* solution – high voltage, low line currents solution – of the power flow equations (1). Although there is no analytical formulation of H, its existence and uniqueness has been discussed in [13].

B. Problem setup

We want to minimize the total reactive power usage while ensuring that voltages remain inside a given feasible set. We formulate an ORPF problem as follows:

$$\begin{aligned} (\text{P0}(t)) \quad & \min_{\mathbb{q}\in\mathbb{R}^G} \quad f(\mathbb{q}) \\ & \quad \text{s.t.} \quad \mathbb{q} \in \mathcal{Q} \\ & \qquad\quad H(\mathbb{q}, \mathbf{z}(t)) \in \mathcal{V}, \end{aligned} \tag{2}$$

where $\mathcal{Q} \in \mathbb{R}^G$ is the set of feasible values for the reactive powers, $\mathcal{V} \in \mathbb{R}^N$ is the set of feasible values for voltage magnitudes, and $f := \mathbb{R}^G \to \mathbb{R}$ is a differentiable cost function. Notice that P0(t) is a time-varying optimization problem since the non-controllable power injections $\mathbf{z}(t)$, and thus the optimal reactive power injections, vary with time, usually within a second. Collecting data, solving the problem (P0(t)) and broadcasting the setpoints to the inverters every second is challenging because of the non-linear nature of the power flow equations (1) represented by H, and the communication burden associated with large DNs. Therefore, this paper aims at solving the following problem.

Problem 1: Design feedback controllers to approximate the solution of (P0(t)) with limited computational resources and in a decentralized fashion to restrict the communication burden to the offline stage.

Assumption 1: There is one, and only one, DER per node.

Assumption 2: The map H does not change with time.

Assumption 3: There exists a solution such that $\mathbf{v}(t) \in \mathcal{V}$ for $\mathbf{q} \in \mathcal{Q}$ at any time t.

III. Design of feedback controllers

A. Incremental Volt/VAR control

Let us discretize the temporal domain as $t = k\tau$, where $k \in \mathbb{N}_{\geq 0}$ and $\tau \in \mathbb{R}_{>0}$ is a given time interval, small enough to resolve variations in the time-dependent disturbances, i.e., less than a second. We introduce the following feedback controller:

$$\mathbf{q}_{k+1} = \mathbf{q}_k + \eta(\mathbb{1} - \boldsymbol{\nu}_k) - (1-\eta)\alpha\mathbf{q}_k, \tag{3a}$$

$$\alpha \in \mathbb{R}_{\geq 0}, \quad \eta \in [0, 1], \tag{3b}$$

where $\boldsymbol{\nu}_k = X\mathbf{q}_k + \boldsymbol{\rho}_k$ with $X \in \mathbb{R}^{N\times N}$ is a linear approximation of the power flow equations and $\boldsymbol{\rho}_k = H(\mathbf{0}, \mathbf{z}_k)$ denotes the voltage profile obtained by setting the controllable reactive powers to 0. The linearized power flow equations can be derived from the branch flow model [14] such that it guarantees X being positive definite [15]. Substituting this approximation in (3a), the controller update law can be written as:

$$\begin{aligned} \mathbf{q}_{k+1} &= [(1 - (1-\eta)\alpha)\mathbb{I} - \eta X]\, \mathbf{q}_k + \eta(\mathbb{1} - \boldsymbol{\rho}) \\ &= A(\eta, \alpha)\mathbf{q}_k + B(\eta, \boldsymbol{\rho}), \end{aligned} \tag{4}$$

where $\mathbb{1}$ denotes a $N \times 1$ column vector and $\mathbb{I}$ is the $N \times N$ identity matrix.

B. Existence and uniqueness of the equilibrium

Denoting $\boldsymbol{\rho} = \boldsymbol{\rho}_k$ for a given k, the equilibrium for (4) is defined as:

$$\begin{aligned} \mathbf{q}^* &= [\eta X + (1-\eta)\alpha\mathbb{I}]^{-1}\, \eta(\mathbb{1} - \boldsymbol{\rho}) \\ \boldsymbol{\nu}^* &= X\,[\eta X + (1-\eta)\alpha\mathbb{I}]^{-1}\, \eta(\mathbb{1} - \boldsymbol{\rho}) + \boldsymbol{\rho}. \end{aligned} \tag{5}$$

Since X is positive definite and η, α satisfy (3b), the matrix $\eta X + (1-\eta)\alpha\mathbb{I}$ in equation (5) is always invertible and the equilibrium is unique. It is clear that increasing the gain α decreases the usage of reactive power, while increasing the gain η steers the voltage magnitudes to the nominal voltages.

C. Stability analysis

The controller defined in (4) is asymptotically stable if and only if $\rho(A) < 1$, where $\rho(\cdot)$ denotes the spectral radius. It follows that we can write $\rho(A) < 1$ as:

$$\mathbf{0} < (1-\eta)\alpha\mathbb{1} + \eta\boldsymbol{\lambda}_X < 2\mathbb{1}, \tag{6}$$

where $\boldsymbol{\lambda}_X \in \mathbb{R}^N$ is the vector containing the eigenvalues of the matrix X. Moreover the matrix X is positive definite by construction, and since η, α satisfy (3b), we always have $(1-\eta)\alpha\mathbb{1} + \eta\boldsymbol{\lambda}_X \geq \mathbf{0}$. The equality holds only if $\eta = \alpha = 0$, which guarantees a stable controller since $\mathbf{q}_{k+1} = \mathbf{q}_k$.

IV. Design of the controller gains

The performance of the controller defined in (4) depends on the choice of η and α. In the following section, we introduce our optimal gain design.

A. Time-varying formulation

Given a matrix X, a time-varying vector $\boldsymbol{\rho}_k = H(\mathbf{0}, \mathbf{z}_k)$, and feasible sets $\mathcal{Q}$ and $\mathcal{V}$, we formulate a discretized version of the problem (P0(t)) considering the linearized power flow equations and the controller (3):

$$
\begin{aligned}
(\mathrm{P1}_k) \quad & \min_{\alpha,\eta} \quad \|\mathbf{q}_k^*(\alpha,\eta)\|^2 && \text{(7a)}\\
& \text{s.t.} \quad \mathbf{q}_k^*(\alpha,\eta) \in \mathcal{Q} && \text{(7b)}\\
& \qquad X\mathbf{q}_k^*(\alpha,\eta) + \boldsymbol{\rho}_k \in \mathcal{V} && \text{(7c)}\\
& \qquad (1-\eta)\alpha\mathbb{1} + \eta\boldsymbol{\lambda}_X < 2\mathbb{1} && \text{(7d)}\\
& \qquad \eta \in [0,1],\ \alpha \in \mathbb{R}_{\geq 0}, && \text{(7e)}
\end{aligned}
$$

where $\mathbf{q}_k^*(\alpha,\eta) = [\eta X + (1-\eta)\alpha\mathbb{I}]^{-1}\eta(\mathbb{1}-\boldsymbol{\rho}_k)$ is a non-linear function of η and α.

Remark 1: The function $\mathbf{q}_k^*(\alpha,\eta)$ represents the equilibrium of the controller (3) for given gains and voltages $\boldsymbol{\rho}_k$. In practice, $\boldsymbol{\rho}$ changes continuously, so the controller always pursues a new equilibrium. However, it is reasonable to assume that, given a sufficiently small τ, the controller is always close to its equilibrium.

Collecting measurements at every node, solving (P1$_k$), and then dispatching the controller gains in real-time is unfeasible because of the communication and the computational burden. Moreover, we would like to find optimal controller gains η and α over a longer time period, to avoid broadcasting new values at every time $k\tau$, or to avoid storing a large number of gains η, α in each controller. In the next subsection we address this issue and present a robust formulation of our problem.

B. Robust formulation

We consider the worst-case scenario such that, for given controller gains, the voltage and reactive power constraints are always satisfied for any $\boldsymbol{\rho}_k$ with $k \in \mathcal{T}_{h1-h2}$, where $\mathcal{T}_{h1-h2}$ is the set of time indices between hours $h1$ and $h2$. In particular, we consider solving the problem:

$$
\max_{\boldsymbol{\rho}_k, k\in\mathcal{T}_{h1-h2}} \left(\min_{\mathbb{q}\in\mathcal{C}(\boldsymbol{\rho}_k)} \|\mathbb{q}\|^2 \right), \tag{8}
$$

with $\mathcal{C}(\boldsymbol{\rho}_k) := \{\mathbb{q} : \mathbb{q} \in \mathcal{Q}, X\mathbb{q} + \boldsymbol{\rho}_k \in \mathcal{V}\}$. Let $\boldsymbol{\rho}_{h1-h2}^*$ denote an optimal solution to (8). The proposed heuristic (8) gives the $\boldsymbol{\rho}_{h1-h2}^*$ that leads to the largest objective value $\|\mathbb{q}\|^2$. Indeed, if $\mathbb{q} \in \mathcal{C}(\boldsymbol{\rho}_{h1-h2}^*)$, then $\mathbb{q} \in \mathcal{C}(\boldsymbol{\rho}_k)$ $\forall k \in \mathcal{T}_{h1-h2}$.

We next present a reduced form of (P1$_k$). The dependency on k is removed by replacing $\boldsymbol{\rho}_k$ with $\boldsymbol{\rho}_{h1-h2}^*$. From now on, the dependency on $\boldsymbol{\rho}_{h1-h2}^*$ will be omitted for notational simplicity. We introduce the optimization variable $\mathbf{x} = [\frac{\alpha}{\eta}, -\alpha]^\top \in \mathbb{R}^2$, write $\mathbf{q}(\mathbf{x}) = \left[X + \mathbf{1}^\top\mathbf{x}\mathbb{I}\right]^{-1}(\mathbb{1} - \boldsymbol{\rho}_{h1-h2}^*)$, specify $\mathcal{Q}$ and $\mathcal{V}$ by box constraints, and reformulate the problem (P1$_k$) as:

$$
\begin{aligned}
(\mathrm{P2}_{h1-h2}) \quad & \min_{\mathbf{x}\in\mathbb{R}^2} \quad h_0(\mathbf{x})\\
& \text{s.t.} \quad h_i(\mathbf{x}) \leq 0 \quad \forall i \in \{1,...,8\},
\end{aligned} \tag{9}
$$

where

$$
\begin{aligned}
& h_0(\mathbf{x}) = \|\mathbf{q}(\mathbf{x})\|^2, \quad h_1(\mathbf{x}) = \mathbf{q}(\mathbf{x}) - \mathbf{q}_{\max},\\
& h_2(\mathbf{x}) = -\mathbf{q}(\mathbf{x}) + \mathbf{q}_{\min},\\
& h_3(\mathbf{x}) = X\mathbf{q}(\mathbf{x}) + \boldsymbol{\rho}_{h1-h2}^* - V_{\max}\mathbb{1},\\
& h_4(\mathbf{x}) = -X\mathbf{q}(\mathbf{x}) - \boldsymbol{\rho}_{h1-h2}^* + V_{\min}\mathbb{1},\\
& h_5(\mathbf{x}) = \left(\mathbf{1}^\top\mathbf{x} - 2\right)\mathbb{1} + \boldsymbol{\lambda}_X,\\
& h_6(\mathbf{x}) = -\mathbf{1}^\top\mathbf{x}, \quad h_7(\mathbf{x}) = -x_1, \quad h_8(\mathbf{x}) = x_2,
\end{aligned} \tag{10}
$$

with x_1, x_2 scalar components of $\mathbf{x}$ and $\mathbf{1} = [1,1]^\top \in \mathbb{R}^2$. Function $h_5(\mathbf{x})$ gives a tighter bound on (7d) with the equality reached for $\eta = 1$, such that the controller gains derived from (P2$_{h1-h2}$) ensure asymptotic stability of the controller defined in (3) as long as $\eta < 1$.

C. Tractable formulation

It is not straightforward to implement the inverse matrix $\left[X + \mathbf{1}^\top\mathbf{x}\mathbb{I}\right]^{-1}$ contained in $\mathbf{q}(\mathbf{x})$ in a computationally efficient way. Therefore, we use the algorithm proposed in [12] which follows the ideas of SCA methods. The method solves a sequence of strongly convex inner approximations of the initial problem. The problem (P2$_{h1-h2}$) can be approximated as a series of subproblems:

$$
\begin{aligned}
(\mathrm{P3}_{h1-h2}(\mathbf{x}_p)) \quad & \min_{\mathbf{x}\in\mathbb{R}^2} \quad \widetilde{h}_0(\mathbf{x};\mathbf{x}_p)\\
& \text{s.t.} \quad \widetilde{h}_{i,n}(\mathbf{x};\mathbf{x}_p) \leq 0\\
& \qquad \forall i \in \{1,...,4\}, n \in \mathcal{N},\\
& \qquad h_i(\mathbf{x}) \leq 0 \quad \forall i \in \{5,...,8\},
\end{aligned} \tag{11}
$$

where $\widetilde{h}_0(\mathbf{x};\mathbf{x}_p)$, $\widetilde{h}_{i,n}(\mathbf{x};\mathbf{x}_p)$ approximates $h_0(\mathbf{x})$, $h_{i,n}(\mathbf{x})$ around $\mathbf{x} = \mathbf{x}_p$. The problem (P3$_{h1-h2}(\mathbf{x}_p)$) is solved for successive values of $\mathbf{x}_p$ until convergence. The surrogate functions in (11) are defined as:

$$
\begin{aligned}
\widetilde{h}_0(\mathbf{x};\mathbf{x}_p) &= \|\mathbf{q}(\mathbf{x}_p) + (\mathbf{x}-\mathbf{x}_p)^\top\nabla\mathbf{q}(\mathbf{x}_p)\|^2 && \text{(12a)}\\
&\quad + \frac{d}{2}\|\mathbf{x}-\mathbf{x}_p\|^2,\\
\widetilde{h}_{i,n}(\mathbf{x};\mathbf{x}_p) &= h_{i,n}(\mathbf{x}_p) + (\mathbf{x}-\mathbf{x}_p)^\top\nabla h_{i,n}(\mathbf{x}_p) && \text{(12b)}\\
&\quad + (\mathbf{x}-\mathbf{x}_p)^\top M_{i,n}(\mathbf{x}-\mathbf{x}_p)\ \forall i \in \{1,...,4\}, n \in \mathcal{N},
\end{aligned}
$$

where $M_{i,n} \in \mathbb{R}^{2\times 2}$ is derived to ensure that $\widetilde{h}_{i,n}(\mathbf{x};\mathbf{x}_p)$ is a global majorizer of $h_{i,n}(\mathbf{x}_p)$. The functions $\widetilde{h}_0$ and $\widetilde{h}_{i,n}$ defined in (12a–12b) satisfy the assumptions of [12].

Algorithm 1 *Optimal Gain Design*

Initialization: $\gamma_p \in (0,1]$, $\mathbf{x}_0 \in \mathcal{X}$. Set $p = 0$.
[1.] If $\|\mathbf{x}_p - \mathbf{x}_{p-1}\| < e$ with $e > 0$, then STOP.
[2.] Compute the solution $\mathbf{x}^*(\mathbf{x}_p)$ of (P3$_{h1-h2}(\mathbf{x}_p)$).
[3.] Set $\mathbf{x}_{p+1} = \mathbf{x}_p + \gamma_p(\mathbf{x}^*(\mathbf{x}_p) - \mathbf{x}_p)$
[4.] $p \leftarrow p + 1$ and go to step 1.

Let us define the convex set $\mathcal{K} = \{\mathbf{x} : h_i(\mathbf{x}) \leq 0\ \forall i \in \{5,...,8\}\}$, and the feasible set of problem (P2$_{h1-h2}$), $\mathcal{X} = \{\mathbf{x} : h_i(\mathbf{x}) \leq 0\ \forall i \in \{1,...,8\}\}$, such that $\mathcal{X} \subset \mathcal{K}$. The algorithm 1 is guaranteed to converge towards a stationary solution of problem (P3$_{h1-h2}(\mathbf{x}_p)$) under the assumptions specified in [12].

V. Implementation of feedback controllers

We assume that each controller is capable to measure the voltage magnitude at the node where it is located. For any given DER g connected to node $n \in \mathcal{N}$, the following incremental Volt/VAR control is implemented:

$$q_{g,k+1} = q_{g,k} + \eta(1 - v_{n,k}) - (1-\eta)\alpha q_{g,k}, \tag{13a}$$
$$p_{g,k+1} = \min\left(p_{g,k}, \sqrt{s_g^2 - q_{g,k+1}^2}\right), \tag{13b}$$

with s_g the rated power of DER g. We implemented a feedback through the voltage measurement $v_{n,k}$. Equation (13b) indicates that we prioritize reactive power to further mitigate overvoltage issues.

We assume that the forecasts of $\mathbf{z}_k$ are available, and therefore $\boldsymbol{\rho}_k = H(\mathbf{0}, \mathbf{z}_k)$ can be retrieved by solving the power flow equations. For each time interval $h1 - h2$, the system operator can run algorithm 1 offline until convergence, then broadcast the same gains to all DERs connected to the network.

We can relax assumption 1 by considering a reduced version of matrix X, only taking into account entries where a DER is located. We can then only guarantee voltage satisfaction for a subset of nodes $\mathcal{N}_{\text{red}} \subset \mathcal{N}$. However, our methodology can be combined with other traditional regulation methods, embedded in the map H and impacting $\boldsymbol{\rho}$. A combination of slow acting controllers, with our fast acting controllers can guarantee voltage satisfaction at every node in $\mathcal{N}$. For assumption 2, planned topological changes (maintenance, reconfiguration) can be integrated by properly selecting the matrix X and the time intervals $h1 - h2$ for gains update. Our controller can also deal with unplanned changes since the grid conditions are taken into account as a feedback. For assumption 3, one first needs to make sure that the set $\mathcal{C}(\boldsymbol{\rho}_k)$ in equation (8) is not empty for some values of k. In that case, one can either consider a larger set $\mathcal{V}$, or pick $\boldsymbol{\rho}^*_{h1-h2}$ following equation (8) for all $k \in \{k : k \in \mathcal{T}_{h1-h2}, \mathcal{C}(\boldsymbol{\rho}_k) \neq \emptyset\}$. This guarantees feasibility of the ORPF problem for a given $\boldsymbol{\rho}^*_{h1-h2}$. However, given our controller architecture, the feasible set of (P2$_{h1-h2}$) may still be empty. In such cases, one should pick a different value of $\boldsymbol{\rho}^*_{h1-h2}$ to ensure that the set $\mathcal{X}$ (see Section IV-C) is not empty.

VI. Numerical experiments

We consider a modified low-voltage (0.4 kV) 42-nodes network from [16], in which photovoltaic power (PV) plants have been placed at each node, with inverter-rated size picked randomly among $\{20, 25, 31\}$ kVA. Note that our strategy can be applied to any type of inverter-interfaced generation with controllable reactive power. The DERs dynamics are neglected, because of the time-scale separation between the power system phenomena and the different control loops [10]. Thus, a new reactive power setpoint is directly implemented by the DER. We used the power system analysis tool PandaPower [17] for our numerical simulations. The data are from the Open Power System Data[1], and have been modified to match the initial nominal values of loads and PV plants. The reactive power demand is set such that the power factor is 0.95 (lagging).

A. Simulation setup

In the following, we assume that the controllable DERs are equipped with an overvoltage protection, i.e., the plant is disconnected from the grid if the voltage goes above 1.06 pu, or stays above 1.05 pu for 10 minutes. The DER reconnects if the voltage stays at least for 1 minute below 1.05 pu.

The voltage service limits are set to 0.95 and 1.05 pu. The load and PV production profiles change every second. The time horizon $h1 - h2$ is set to 1 hour. The reactive power setpoints update τ is set to 100 ms. We compare our proposed controller based on algorithm 1 (OGD) with: (a) a static Volt/VAR control (VoltVar); and, (b) no control (ON/OFF). They are defined as:

a) Static Volt/VAR control: It is inspired by the standard *IEEE Std 1547-2018*, with maximum reactive power consumed/absorbed set to 44% of the nominal power of the DER and reached for voltages 1.05/0.95 pu, respectively. The deadband ranges between 0.99 and 1.01 pu.

b) No control: We set reactive powers to 0.

B. Results

In Fig. 1, we show the cumulative distribution functions (CDF) for maximum and minimum voltages for the entire duration of the simulation. We define the vectors $\mathbf{V}_{\max} = \{\max v_{i,k}\}_{i\in\mathcal{N},k\in\mathcal{T}}$ and $\mathbf{V}_{\min} = \{\min v_{i,k}\}_{i\in\mathcal{N},k\in\mathcal{T}}$, where $\mathcal{T}$ is the set of time indices for the entire simulation time. Our method slightly exceeds the voltage limits because of some transients, while the VoltVar and the ON/OFF strategies withstand more important voltage violations, though limited by the overvoltage protection.

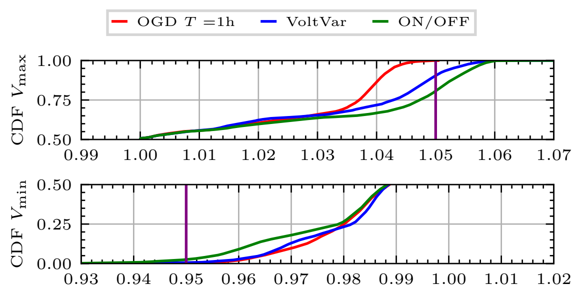

Fig. 1: CDF for maximum and minimum voltages $\mathbf{V}_{\max}$ and $\mathbf{V}_{\min}$ for the entire duration of the simulation.

In Fig. 2, we show the total energy lost and the reactive energy usage. VoltVar and our method are equivalent in terms of active power losses, in particular VoltVar suffers from larger curtailment while our method induces larger losses in the lines. Our method uses more reactive power, but keeps the voltages within fixed limits. If the cost of reactive power usage

[1] Data available at https://data.open-power-system-data.org/household_data/2020-04-15

exceeds the cost of violating voltage constraints, one could either change the voltage limits, or pick a different $\boldsymbol{\rho}^{*}_{h1-h2}$ to reduce the total reactive energy usage.

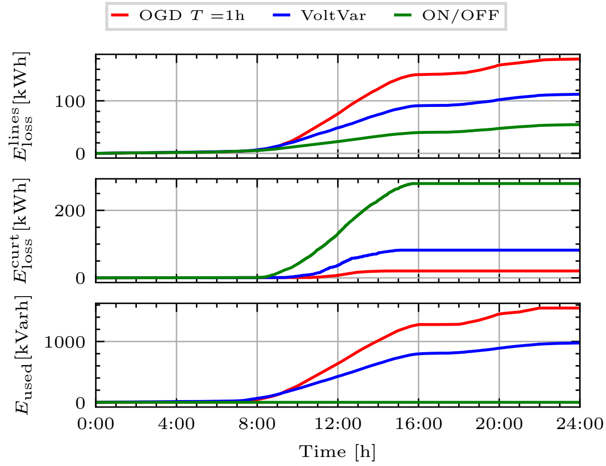

Fig. 2: Energy lost in the lines and because of curtailment, and reactive energy usage.

VII. Discussion

The validity of our heuristic (8) is illustrated using a simple example. Consider a two-node network; the voltage magnitude of node 1 is set to 1 pu, and node 1 is connected to node 2 through a line of inductance 1 pu. The linearized power flow equation gives $\nu = q + \rho$, where the quantities are defined for node 2. We consider three different values of ρ, for different $k \in \{1, 2, 3\}$; $\rho_1 = 1.04, \rho_2 = 1.06, \rho_3 = 0.93$ and infinite reactive power reserve. Solving the ORPF problem $\min_{\mathcal{C}(\rho_k)} q^2$ for $k \in \{1, 2, 3\}$, one gets: $q_1 = 0, q_2 = -0.01, q_3 = 0.02$. Following (8), we have $\rho^* = \rho_3 = 0.93$. The equilibrium function of our controller is:

$$q^* = \frac{1}{\eta + (1-\eta)\alpha}\eta(1-\rho). \tag{14}$$

Replacing ρ by ρ^*, fixing $\eta = 0.8$, and knowing that $q^* = q_3 = 0.02$ minimizes the reactive power usage and satisfies the voltage constraints, we have $\alpha = 10$. The expression (14), with the fixed controller gains, becomes $q^* = 0.2857(1-\rho)$. Then, we can plug ρ_2 in (14) with the fixed controller gains, and we have $q^* = -0.017$, and $\nu^* = 1.043 \leq 1.05$. The voltage constraint is also verified for ρ_1, and shows that the constraint is satisfied for all ρ_k with $k \in \{1, 2, 3\}$ by picking $\rho^* = \rho_3$ according to (8) to design the controller gains.

Now let us consider some reactive power constraints with $q_{\min} = -0.015$ and $q_{\max} = 0.03$. Nothing changes; $\rho^* = \rho_3$ and the equilibrium function for ρ_2 gives $q^* = -0.017$. Notice that $q^* \leq q_{\min}$: the reactive power constraint is not satisfied. However, in the real implementation of our controller, the DER will inject $q = -0.015$, which ensures $\nu = 1.045 \leq 1.05$. In the case where $q_{\min} > -0.01$, then the initial ORPF for ρ_2 is not feasible, since $q_2 \leq -0.01$ is required to meet the voltage constraint.

VIII. Conclusion

We proposed an incremental Volt/VAR control strategy for voltage regulation in DNs. We showed that our controller is stable, and we introduced an optimal gain design to minimize the reactive power usage. Our methodology only needs limited offline communications. Our local controllers perform better compared to static Volt/VAR curves with fixed parameters on a 42-nodes low-voltage network. Future works will investigate the combination of our fast-acting controller with slower traditional regulation devices.

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