3D finite-element modeling for the magnetization of bulk high-Tc drilled superconductors

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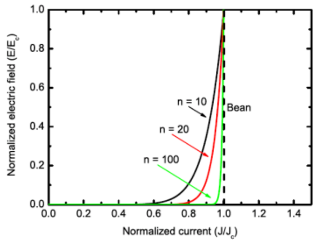
Introduction

HTS are modeled by a non-linear resistivity



$$\boxed{E(J) = E_c \left(\frac{J}{J_c}\right)^n}$$

For HTS : $n \approx 20$

Bean model : $n \to \infty$ **analytical** calculations are **available** on specific geometries **for comparison** with numerical results

Finite-element method (FEM) is widely used for solving HTS-based systems

Advantages

- **Many geometries** can be treated
- **No extensive writing of numerical codes** is required
- **Treatment of non-linear problems** available in most commercial packages

Drawbacks

- **Long calculation time** on fine meshing or in 3D geometry
- **Convergence problems** when *n* is **large**

Proposed improvements

Single time step *method implemented in an* ***open-source solver, GetDP***

- **Better control** of the algorithm parameters
- Used for simulating the **penetration of an external magnetic field** that varies linearly with time

1. FEM *A*-ϕ formulation

- The Maxwell equations are solved for two independent variables
  - **the vector potential A**
  - **the scalar potential** ϕ

$$\mathbf{B} = \mathbf{B}_{react} + \mathbf{B}_a = \text{curl}\,\mathbf{A} + \text{curl}\,\mathbf{A}_a$$

$$\mathbf{E} = -\,d\mathbf{A}/dt - d\mathbf{A}_a/dt - \text{grad}\,\phi$$

Magnetic flux density induced by the HTS

Applied magnetic flux density (uniform)

- ***A*** is approximated by a series of **edge functions** $\boldsymbol{A_i}$

$$\mathbf{A} = \sum_i a_i \mathbf{A}_i$$

(ensures the ***continuity of the tangential*** *component of A)*

- ϕ is approximated by a series of **node functions** ϕ_i

$$\phi = \sum_j b_j \phi_j$$

(ensures the ***continuity*** *of* ϕ*)*

- The Maxwell equations reduce to

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \sigma(\mathbf{A}, \phi) \left(-\dot{\mathbf{A}} - \dot{\mathbf{A}}_a - \nabla\phi\right)$$

Ampere's law (rot H = J)

$$\nabla \cdot \left\{\sigma(\mathbf{A}, \phi) \left(-\dot{\mathbf{A}} - \dot{\mathbf{A}}_a - \nabla\phi\right)\right\} = 0$$

Continuity equations (div J = 0)

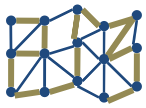
where $\sigma(\mathbf{A}, \phi)$ is derived from the non-linear E-J relationhisp

2. Gauge conditions

$$\mathbf{A}.\mathbf{w} = 0$$

(not a Coulomb gauge)

Set of meshing edges that **connects** all the nodes **without closed contours**

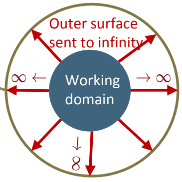


3. Boundary Conditions

- Use of **Jacobian transformation** for sending the outer surface of a spherical shell to infinity

Dirichlet conditions

$$\mathbf{A} = 0$$
$$\phi = 0$$

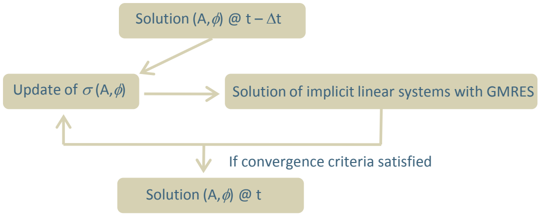


- Source field $\mathbf{A}_a$ corresponds to a **uniform magnetic flux density** . The source field $\mathbf{B}_a$ is a temporal **ramp** with a **constant sweep rate** *(mT/s)*

4. Numerical scheme

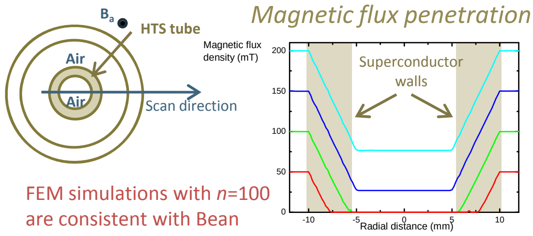
Implicit time-resolution and non-linear Picard iteration

- The equations are solved with a **Galerkin residual minimization method**
- We use the **Backward Euler method** at each time step
- Non-linear terms are treated with a **Picard iteration** loop



5. Comparison with Bean model for a HTS tube with an infinite extension

Magnetic flux penetration



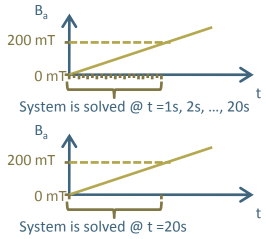
FEM simulations with *n*=100 are consistent with Bean model

Choice of the time step

For solving the problem with B_a = 200 mT with a sweep rate of 10 mT/s:

1. 20 time steps of 1s
2. 1 time step of 20s

Single time step method



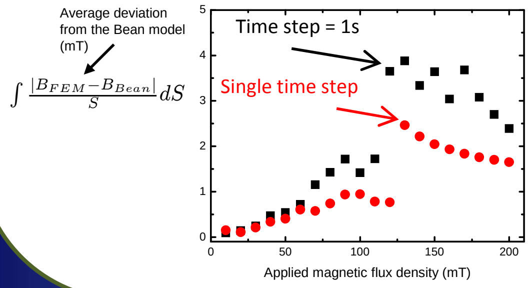
Comparison of single/multiple time steps methods

Magnetic field penetration of an external field increasing from 0 mT to 200 mT

1. **in a single simulation** with 20 time steps of 1s

2. **in 20 single time step simulations** with a time step of respectively 1s, 2s, …, 20s

Analysis of the deviation from the Bean model



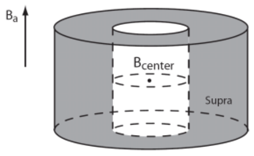
$$\int \frac{|B_{FEM}-B_{Bean}|}{S}dS$$

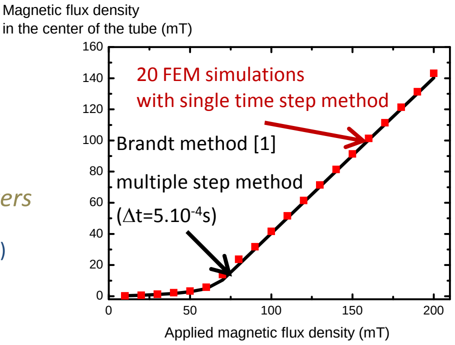
Calculation time

1 simulation with 20 time steps	2 days 3/4
20 single time step simulations	3 hours

Single time step method produces more accurate results in a shorter calculation time

5. Comparison with Brandt's method [1,2] for a HTS tube of finite extension



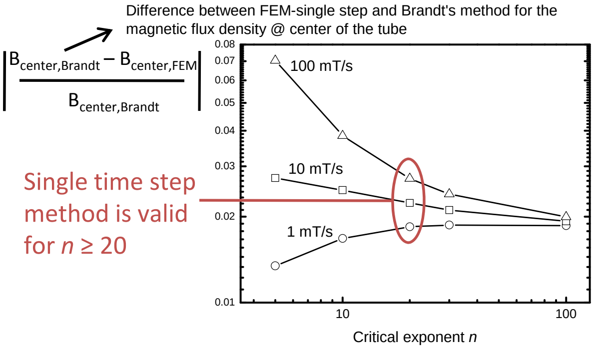


Simulation parameters

n = 100
B$_a$ = 0 – 200 mT (10 mT/s)
Single time step method
3D geometry

Accuracy of the single time step method with smaller *n* values

- Applied magnetic flux density : 200 mT with different sweep rates (1mTs, 10 mT/s and 100 mT/s)



$$\left|\frac{B_{center,Brandt}-B_{center,FEM}}{B_{center,Brandt}}\right|$$

Single time step method is valid for *n* ≥ 20

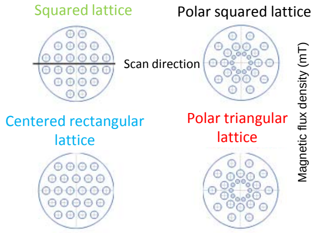
[1] Brandt E H *1998 Phys. Rev. B* **58** 6506
[2] Denis S *et al. 2007 Supercond. Sci. Technol.* **20** 192

6. Magnetization of drilled HTS cylinders

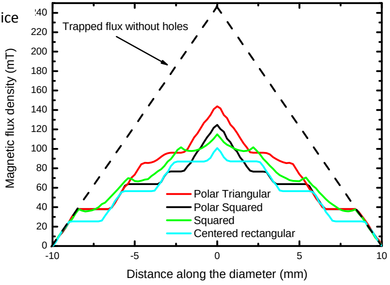
Simulation parameters

n = 25
B$_a$ = 0 → 200 mT in one time step and B$_a$ = 200 → 0 mT in another time step (two time steps method)
2D (infinite height) or 3D (finite height) geometry

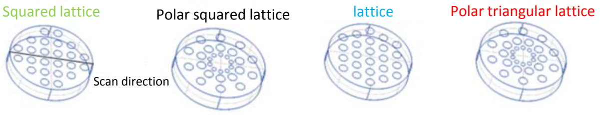
Cylinders of infinite height

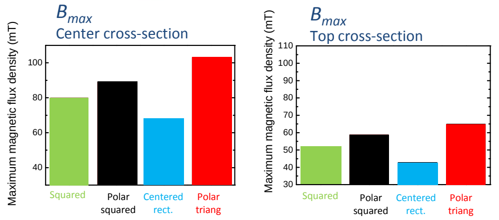


Trapped flux profile



Cylinders of finite height





Squared → Polar Triang

Maximum trapped flux increases by ~25 % (center) ~30 % (top)

Assuming the same J_c in both configurations

Acknowledgments

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