SLOW & RANDOM PHASE MODELS FOR VORTEX-INDUCED VIBRATIONS

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Abstract. Vortex-induced vibrations (VIV) present significant challenges in understanding and modeling the cross-wind behavior of structures subjected to fluid flow. This presentation has provided a review and comparison of some existing models, classified under the families A, B, and C as defined in [1]. After recalling the rationale behind the well-known spectral method, which falls within the B-family, some limitations of the wake-oscillator model from family C were highlighted. In particular, although several additional models have been developed since the seminal works of Hartlen and Currie [2], and Tamura [3], models from this family are seldom used today in the wind engineering community. One reason that could explain this reluctance to use such models is that, in their current form, they still usually operate in a deterministic setting.

The talk addressed stochasticity from two main sources within fluid-structure interaction. The first source is the oncoming wind flow, which characterizes turbulence in the atmospheric boundary layer and affects the vibration response of structures, relevant to tower and chimney design. The second source arises from turbulence in the near wake, creating randomness as vortices are shed, observable in a fixed cylinder in uniform flow with a slowly changing lift force envelope.

This short summary of the presentation gives a glimpse of the main messages and highlights some comments that were made during the discussions after the presentation.

1. Multiple Timescale Analysis of a Wake-Oscillator Model

A simple example of wake-oscillator model is the governed by the following equations

$$\mathcal{Y}'' + 2\xi\mathcal{Y}' + \mathcal{Y} = 2\varepsilon\mathcal{M}_0\Omega^2\mathcal{Q},$$
$$\mathcal{Q}'' + \varepsilon\Omega\left(\mathcal{Q}^2 - 1\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = 2\varepsilon\mathcal{A}_0\mathcal{Y}''.$$

It is the dimensionless version of the model proposed by Facchinetti et al. [4], where $\mathcal{Y} = y/D$ is the dimensionless cylinder motion, D is the transverse dimension of the cross-section, ξ is the total (structural + aerodynamic) damping, and $\mathcal{Q}$ represents the lift coordinate. The first equation represents the dynamics of a spring-mounted cylinder, with the lift force in the right hand side. It is written with a dimensionless time defined as $\tau = \omega_0 t$, with ω_0 being the natural frequency of the mass-spring system in still air. The second equation is a nonlinear van der Pol equation governing the dynamics of the lift force. The dimensionless vortex shedding frequency $\Omega = \frac{2\pi \mathrm{St} U}{\omega_0 D}$ is directly related to the ratio of the wind velocity to the critical wind velocity. Parameter ε is a small parameter of the model, responsible for the slow buildup of vibrations when observing the transient from initial conditions. The two dimensionless parameters $\mathcal{M}_0 = \frac{\mu}{8\pi^3\varepsilon^2}\frac{C_L^0}{\mathrm{St}^2}$ and $\mathcal{A}_0$ can be seen as gain parameters quantifying the importance of the coupling between the fluid and the structure equations.

By recognizing the smallness of ε it is possible to develop asymptotic solutions to this nonlinear problem using multiple scales techniques. At leading order, this results in slowly modulated responses,

$$\mathcal{Y} \sim R_y(T)\cos\left[\tau + \varphi(T)\right] \quad ; \quad \mathcal{Q} \sim R_q(T)\cos\left[\tau + \varphi(T) + \psi(T)\right] \tag{1.1}$$

where the slow amplitudes and the slow phase are given by [5]

$$\begin{aligned} R_q' &= \mathcal{A}_0 R_y \sin\psi - \frac{1}{8}R_q^3 + \frac{1}{2}R_q \\ R_y' &= \mathcal{M}_0 R_q \sin\psi - \xi_0 R_y \\ \psi' &= \left(\mathcal{A}_0\frac{R_y}{R_q} + \mathcal{M}_0\frac{R_q}{R_y}\right)\cos\psi + \xi_0\delta. \end{aligned} \tag{1.2}$$

It is easier to solve these three equations as they just depend on the slow timescale. Also, by solving these equations, the effort in the numerical approach is put on the actual essence of the problem (the slowly varying envelope and the phase) instead of engaging numerical technique with small time step to capture the fast dynamics. Solving these equations for various values of δ, defined as $\Omega = 1 + \varepsilon\delta$, provides the VIV response curve.

2. Turbulence in the Oncoming Wind

Turbulence of the oncoming wind can be modeled by replacing U with $U+u(t)$ in the original equations. By doing so, it turns out that the slow equations are exactly the same, except for the last equation, which now reads

$$\psi' = \left(\mathcal{A}_0 \frac{R_y}{R_q} + \mathcal{M}_0 \frac{R_q}{R_y}\right) \cos\psi + \xi_0 \delta + I_u \mathcal{U} \tag{2.1}$$

where I_u is the turbulence intensity and $\mathcal{U}(\tau)$ is a zero-mean, unit-variance stochastic process describing the low-frequency turbulence. It is characterized by its Power Spectral Density (PSD), $S_{\mathcal{U}}(\omega)$. Interestingly, this model dissociates two features of the turbulence: the turbulence intensity I_u and the characteristic low frequency of turbulence, represented here by a parameter α. Together with the first two equations of (1.2), (2.1) can be solved efficiently over very long simulation times since it involves only the slow dynamics. As such, they are able to provide neat statistical estimates of the envelope $R_y(T)$. Additionally, it is possible to further simplify the model by assuming that the dynamics of the system do not evolve too far from the limit cycle. This further simplifies the model, as it then consists of only one equation, the *slow phase equation,*

$$\psi' = \frac{\mathcal{A}_0 \mathcal{M}_0}{\xi_0} \sin\psi \cos\psi + \xi_0 \cot\psi + \xi_0 \delta + I_u \mathcal{U}. \tag{2.2}$$

During the discussions after the presentation, the question of having simple expressions for purposes such as codification was highlighted. It was noted that this equation possesses a semi-analytical solution [5]. This solution is accurate in the center of the lock-in region but tends to become less accurate further from the center of the lock-in region. Essentially, having a semi-analytical solution for this equation aligns with Approach 2 of the ENV1991-1-4 and could also serve as a basis for codification purposes.

The solutions of these equations express statistics of the response envelope, particularly the steady-state Probability Density Function (PDF) of the response envelope. Whether vortex detachment is locked or not, and whether large amplitudes are reached or not, are expressed in a probabilistic manner. This also aligns with the message of the presentation given by S. O. Hansen during another talk later at this VIV Symposium [6].

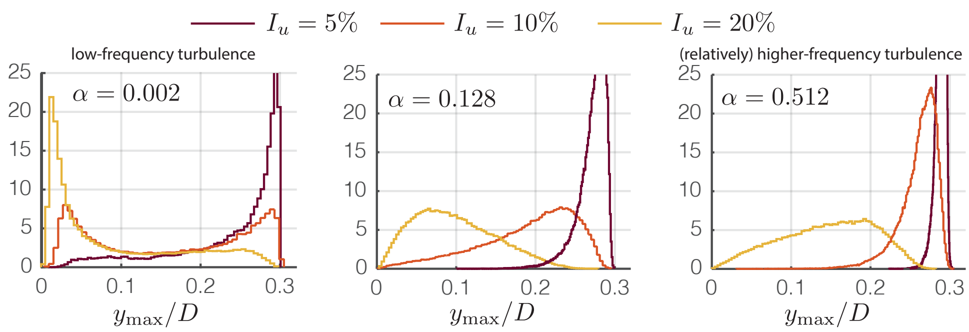

Figure 2.1. PDF of the response amplitude simulated for various turbulence intensities I_u and characteristic turbulence frequency α. Perfect lock-in conditions ($\Omega = 1$, $U = U_{\mathrm{cr}}$).

Figure 2.1 shows examples of results from this model. These selected results highlight that the turbulence intensity and the frequency content of turbulence affect the synchronization capabilities in two independent ways. This feature of the model was pointed out by R. Höffer during the discussion of the presentation. Additionally, this aligns well with the field observations reported in this seminar by I. Kurniawati et al. [7] and P. Hémon [8] in two different talks on full-scale data. More precisely, the figure shows that increasing turbulence intensity results in smaller average vibration amplitudes. Furthermore, when the ratio α α of the characteristic frequency of turbulence to the natural frequency of the cylinder increases, the sensitivity to turbulence intensity weakens. In other words, in this model, VIV is more sensitive to small-scale turbulence than to large-scale turbulence

3. Turbulence in the Signature

The second type of randomness in the problem is identified as near-wake turbulence. To model this in a two-equation model, additive noise is introduced in the fluid equation. Notably, after the presentation, an informal discussion with A. Metrikine suggested that another (potentially more physical) way of modeling this phenomenon would be to introduce randomness in the coefficients of the fluid equation.

This option would be viable and similar to randomizing the kernels introduced in the model based on frequency-dependent coupling [9].

So, instead of considering the van der Pol type equation proposed in [4], a generalized version of the fluid equation was investigated [10, 11]. Adjustment on experimental data (in the subcritical regime) provided very good agreement on the stationary cylinder when the model

$$\mathcal{Q}'' - \Omega\left(a\mathcal{Q}^2 + c\Omega^2\mathcal{Q}'^2 + d\right)\mathcal{Q}' + \Omega^2\mathcal{Q} = \eta \tag{3.1}$$

was used with $a = -0.09$, $c = 0.009$ and $d = 0.063$, and where η is a zero-mean stationary random process with a PSD given by $\sigma_\eta^2 \frac{2L_\eta}{\pi U} / \left(1 + \left(1.339 L_\eta \frac{\omega}{U}\right)\right)^{5/6}$, with $\sigma_\eta = 1$ and $L_\eta = 1.1$ in the subcritical regime, see [10] for more details.

Figure 3.1 shows that this model predicts the fluctuating lift force on a fixed cylinder with reasonable accuracy. Notably, the non-Gaussian PDF of the lift coefficient (on the left) is remarkably well reproduced, more so than the Gaussian description offered by the Vickery-Clark model [12] shown in green. The PDF of the lift envelope R_q closely mimics the experimental data. Finally, the comparison of the PSD of the lift force on the right, resulting from the solution of (3.1), exhibits a wider distribution than the Vickery-Clark model, especially in the high-frequency range.

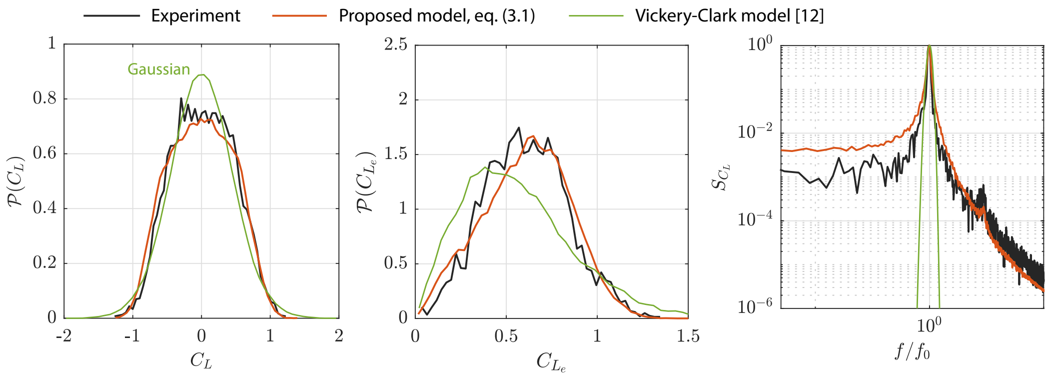

FIGURE 3.1. Comparison of the proposed model, the Vickery-Clark model and experimental data.

4. Conclusions

Two ways to randomize the wake-oscillator have been presented. They aim to model the turbulence of the oncoming wind and the turbulence of the near wake, respectively. Future work could aim to combine the two sources of randomness, extend these random models to the space-continuous case, and apply them to structures with additional damping, such as tuned mass dampers.

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